# Tidal Stripping of Two-Component Systems

*The goal of this project is to understand better how two-component systems are tidally stripped in energy space.*

In minor mergers, smaller dark matter halos fall into larger host halos and lose mass through tidal stripping, and the remaining bound material may exist as substructure within the host halo. There are many interesting cosmological problems related to this small scale structure (missing satellites problem, core-cusp problem, too-big-to-fail).

Isolated simulations are often used to develop empirical models to describe the evolution of subhalos, in which an $N$-body realization of a subhalo is placed in a fixed background potential representing the host halo and allowed to evolve. These descriptions are used to develop galaxy formation models, dark matter annihilation constraints and lensing predictions.

I have been developing a model that describes how a subhalo evolves as it is tidally stripped, based on physical reasoning (Drakos et al., 2017, 2020b,a). I am interested in how well this works if there are stars inside the subhalo; i.e. I want to simulate dark matter particles with one density profile, as well as star particles with another density profile. Presumably, since the dark matter is more extended, it will be stripped earlier. I would also like to see how this depends on the central density of the dark matter halo; since there are numerical effects at the high density central regions of dark matter halos, these regions aren't well constrained by simulations.

# 1 Dark Matter Halos

Most of understanding of the large scale structure of the universe comes from cosmological dark matter only simulations. In these simulations, there are $N$ particles (each corresponding to a large amount of dark matter) that interact gravitationally in an expanding universe. As the universe expands, overdense regions collapse to form dark matter halos, which are connected by filaments and sheets. (for further reading see, e.g. Taylor, 2011; Diemand & Moore, 2011; Drakos, 2019).

One main finding from these simulations is that dark matter halos have a universal density profile; all halos have a scaled version of the same density profile, regardless of mass or redshift. The original parameterization of this density profile is the Navarro-Frenk-White (NFW) profile (Navarro et al., 1996, 1997), which has two free parameters, $r_s$ and $\rho_0$.

$$\rho(r) = \frac{\rho_0}{r(r+r_s)^2} \quad . \tag{1}$$

To get a better physical understanding of how individual dark matter halos evolve, isolated simulations are sometimes used; rather than simulate the creation and evolution of millions of halos within the entire cosmological context, one (or a few) halos are simulated in high resolution, to study a specific process in isolation.

To run these isolated simulations, you need initial conditions (ICs). For example, if you want to study a NFW profile with $N$ particles, need to assign positions to each particle so that they have the desired density profile. Also, you need to assign velocities to each particle, so that the density profile is preserved as the $N$ particles interact gravitationally.

I have code online, ICICLE `https://github.com/ndrakos/ICICLE`, to create ICs for isolated simulations. One complication is that for NFW profiles, you can't actually create a stable isolated $N$-body realization; this is because if you solve for the total mass of the system, it diverges as radius goes to infinity. One solution to this is to define a cut-off radius $r_{\rm cut}$, and only generate particles within this radius. However, if you do this, the profile is no longer stable; therefore there is an option in the code TRUNCATE, which if set to True, will alter the output so that there is a stable profile that resembles an NFW profile.

## Exercises

1. Plot an NFW density profile.
2. Download ICICLE, and generate an NFW profile with $10^5$ particles (you can choose the mass, scale radius and cut-off radius: set TRUNCATE to False).
3. With these particle positions, calculate the density profile and compare to the theoretical NFW profile (you will have to figure out how to calculate $\rho_0$ given $M(< r_{\rm cut})$).
4. Now try creating ICs for the truncated NFW profile, and compare the density profile.

## 2 $N$ Body Simulations

Generally, $N$-body simulations are used to simulation the evolution of collisionless systems (a system in which individual collisions between particles are negligible). Typically, each particle is meant to represent a random sampling of some volume in a smooth potential. One of the best references for understanding these types of systems is Binney & Tremaine (1987), though it is more at a graduate level.

One numerical problem in $N$ body simulations, is that since the gravitational potential is proportional to $1/r$, when two point particles are very close the force between them diverges to infinity. The solution that is used is softening length, $\epsilon$, which modifies the potential (e.g. change $r$ to $\sqrt{r^2+\epsilon^2}$, or something similar). Effectively this gives each particle a radius on the order of the softening length, preventing particles from getting too close to one another. When running $N$ body simulations, you need to select an appropriate softening length. I usually use a softening length of $\epsilon = 0.5 r_h N^{-1/3}$ (van Kampen, 2000), where $r_h$ is the radius which encloses half the mass of the system.

Another concept you should be aware of is numerical relaxation. This happens because of interactions between two massive particles can greatly effect their velocities. Eventually, the velocities will change significantly, and particles will be ejected from the system ("evaporation"). Therefore, the longer you run a simulation, the more objects will become disrupted through these relaxation effects. The only way to reduce this issue is by increasing the resolution of the simulation (i.e. use more particles).

The relaxation time, $t_{\rm rel}$, is defined as the time in which an average particle's velocity has changed by order of itself. Given a system of mass $M$, and size $R$, with $N$ particles,

$$t_{\rm rel} \approx \frac{0.1N}{\ln N}\sqrt{\frac{R^3}{GM}} \tag{2}$$

### Exercises

I will provide code to convert ICs to a GADGET file, and to read in the GADGET outputs

1. Download and install GADGET-2 (there are great instructions here: `https://astrobites.org/2011/04/02/installing-and-running-gadget-2/`)
2. Convert ICs file to GADGET format
3. Calculate the softening length, and make parameter file for GADGET. Be careful that your ICs and GADGET have the same units.
4. Run the simulation (you decide how long to run it; think about the relaxation time)
5. Read in the snapshot files
6. Check that the density profile is stable (doesn't evolve in time).
7. Can you calculate the radius at which you expect relaxation effects to be a problem?

# 3 Orbits

Eventually, we are going to place our $N$-body realization of our satellite halo within a fixed potential that represents the background, host halo. First, think a bit about what trajectory the satellite will follow in the host halo

Consider a point mass orbiting in a potential, $\phi(r)$ (with a corrsponding mass profile $M(r)$). In general, the trajectory of a point mass can be calculated by solving the following system of ODEs:

$$\begin{aligned} \frac{\mathrm{d}\mathbf{r}}{dt} &= \mathbf{v} \\ \frac{\mathrm{d}\mathbf{v}}{dt} &= -\frac{GM(<r)}{\mathbf{r}^3} \end{aligned} \quad . \tag{3}$$

However, If we restrict our attention to a circular orbit, things become a lot simpler: the velocity of a circular orbit is $v = \sqrt{GM/r}$.

## Exercises

1. Consider a point mass in an NFW potential. Given that you want it to orbit in a circular orbit at radius $r_c$, what initial position and velocity could you give the point mass? What is the period of this orbit? What would happen if you increased or decreased this initial velocity?
2. Edit the ICs from previous step so that they have these initial positions and velocities.
3. ~~If you are interested, write an ODE solver that tracks a point mass potential in an NFW potential, given an initial velocity and position (this isn't required for the next steps).~~

# 4 Fixed Background Potential

One approach to study the evolution of a satellite that is tidally stripped is to evolve it in a static background potential. Hayashi et al. (2003) is the first paper to look at the evolution of dark matter halos in these type of simulations.

## Exercises

I will provide a modified version of GADGET-2 (originally from Andrew Benson), that allows you to place an NFW background potential into the simulation.

1. Install this new version of GADGET-2, and alter the parameter file to have the background potential you want.
2. Run the simulation for a few orbits.
3. Plot the particle output for different snapshots. How do they look?
4. Compare the satellite orbit to your predictions.
5. If you are interested, try making an animation (I use FFmpeg).

# 5 Analyze Simulations

Next, we want to look at how the density profile of the satellite evolved, as well as look at the bound mass evolution. We need to (1) find the frame of the satellite and (2) decide which particles are still bound to the satellite. See, e.g., Drakos et al. (2017) for how to determine the self-bound particles. I can provide more notes on that (and potentially give you code) if you need help.

## Exercises

1. Calculate the frame of the satellite in each snapshot (compare this to the expected orbit).
2. Once in the frame of the satellite, calculate and plot the density profile (using all the particles).
3. Now, find out which particles are bound, and plot the density profile using only these particles. How does it compare?.
4. Plot the bound mass as a function of time.

# 6 Two-Component Model

Next, we want to repeat this all, but with a two-component model.

I have made code to add to Icicle to create two-component systems. Right now I just have it working for two Hernquist profiles (Hernquist, 1990), but I might have more options by the time you get to this step.

## Exercises

1. Generate ICs, and check they are stable (check the total density profile, as well as the density profile of the individual components).
2. Redo the fixed-potential simulation with the new ICs.
3. Calculate the frame and self-bound mass.
4. Plot density profiles and mass loss curves (total profile, as well as individual components).

# 7 Stripping in Energy Space

Traditionally, tidal stripping has been thought of as an outside-in process in radius; i.e. particles at large radii get removed first. However, as first indicated in Choi et al. (2009). it might be better described as an outside-in process in energy space; particles that are the least bound get removed first.

I have had a lot of success predicting the evolution of subhalos using this concept (Drakos et al., 2017, 2020b,a). I want to think about what this means for multiple component systems.

## Exercises

1. Plot energy stripping for one-component simulation (see Fig. 5 in Drakos et al. (2020b))
2. Now look at the two-component system, and make the same plot for each component, as well as the total profile.
3. Do both components get stripped to the same energy?

# 8 Predictions

Can we predict the bound mass and density profiles of the individual components using the model in (Drakos et al., 2020b)? If we get this working, we should run a suite of simulations with different ICs and orbits.

# References


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