Structural Breaks and the Bank of Canada Equation

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1 Introduction

The Bank of Canada exchange rate equation is a USD/CAD forecasting model that was developed in the early 1990s by Robert Amano and Simon van Norden at the Bank of Canada (Amano & van Norden, 1992). It described the behavior of the exchange rate "remarkably well" over the 20 years preceding their study (Laidler & Aba, 2001). Laidler and Aba, using a modified version of the original equation, found that the model also continued to perform well with the addition of data from the 1990s.

They did, however, find some evidence of parameter instability in their model. In particular, the relationship between energy prices and the exchange rate seems to have changed between the 1980s and 1990s. This is confirmed in work done by Issa, Lafrance, and Murray, who identied a structural change in the determinants of the exchange rate in the early 1990s (Issa & Murray, 2008). There also appears to have been a structural change in 2003 (Aguilar, n.d.). This paper looks to confirm these findings using Laidler and Aba's model.

My approach is to first look at how the model performs using recent data. The presence of structural breaks will be implied if the model fails to explain the movements of the exchange rate. I then perform statistical structural break tests on the model. The results confirm structural changes in 1979, 1992, and 2003. I then see how the model performs when the structural changes are taken into account.

The structure of the paper is as follows. I begin by providing some background on the Bank of Canada equation, and on the structural breaks. I then outline the general model used by Laidler and Aba, as well my data set. The next sections outline my findings in regards to model performance and the results of the structural break tests. Finally, I discuss my results and the associated implications.

2 Background

One of the defining characteristics of Canada's economy is its status as a large exporter of commodities – particularly energy commodities. The vast majority of these exports are purchased by the United States. As a consequence, changes in commodity prices impact the U.S demand for Canadian dollars.

This relationship between commodity prices and the USD/CAD exchange rate is the basic underlying principle of the Bank of Canada equation. By capturing this relationship, it initially worked well in describing movements in the exchange rate. The first period used to estimate the model was 1972 to 1993. Later, Laidler and Aba found that the model continued to describe exchange rate movements up to the 2000s.

They did, however, find evidence that the relationship between commodity prices and the exchange rate was not stable over time. In econometrics this is referred to as parameter, or model, instability. It can have two different implications: (1) that an important variable has been omitted from the model, and (2) that changes occurred in the relationship between the explanatory variables and the dependent variable.

In the first case, a different model is likely called for – something is fundamentally missing. In the second, the model's parameters are more effective if allowed to change after some point in the data set.

This point is called a structural break, and it signifies a structural change. A structural change is a long-term shift in an economy's fundamental structure. Two such changes have been identified that impacted the behaviour of the USD/CAD exchange rate. The first occurred in 1992, and the second in 2003.

1992 Structural Break

The structural break in 1992 was a change in the relationship between energy prices and the exchange rate. More specifically, Issa, Lafrance, and Murray found that the sign of the relationship changed from negative to positive (Issa & Murray, 2008). This is also supported by the work of Laidler and Aba. The cause is thought to have been changes in Canada's energy policies, and changes in energy related cross-border trade and investment.

This is an example of a model performing better when coefficient values are allowed to change. That is, two different regressions using the same model are better than one. The structural break in 2003, however, seems to be unexplainable within model.

2003 Structural Break

From 2003 to 2005 there was a sudden appreciation of the Canadian dollar. It increased in value by roughly 25 cents U.S. Maier and DePratto have found that this sudden appreciation represents a structural break (Maier & DePratto, 2008). In contrast to 1992, this break seems to be more than just a relationship sign change, which I address later in the paper.

Before performing my own structural break analyses, I outline the formulation of the Bank of Canada equation, and the modified version used in this paper.

3 The Bank of Canada Equation

The Bank of Canada equation is an error correction model that forecasts the US/CAD exchange rate one quarter into the future. This paper uses the modified version of the equation proposed by Laidler and Aba (Laidler & Aba, 2001). Error correction models describe economic relationships that tend to some long-run equilibrium, but fluctuate around that equilibrium due to the movement of short-run components (Engle & Granger, 1987).

In the case of the BoC equation, the long-run equilibrium is determined by the price levels of non-energy commodities and energy commodities. Short-run fluctuations are due to interest rate differentials between the United States and Canada. In formal terms, the model is expressed:

$$\Delta\mathrm{RFX} = \alpha(\mathrm{RFX}_{t-1} - \beta_0 - \underbrace{\beta_c\mathrm{COM}_{t-1} - \beta_e\mathrm{ENE}_{t-1})}_{\text{Long-run determinants}} + \underbrace{\gamma\mathrm{RDIFF}_{t-1}}_{\text{Short-run determinant}}$$

where ΔRFX is the change in average exchange rate from one period to the next, RFX_{t-1} is the average exchange rate, COM_{t-1} is the level of the non-energy commodity price index, ENE_{t-1} is the level of the energy commodity price index, and RDIFF_{t-1} is the difference between Canadian and US 90-day prime commercial paper rates. Each of the explanatory variables are of course lagged one period.

In order to estimate the model parameters, the equation is expanded and we define $-\alpha\beta_0 = \Omega_0$, $-\alpha\beta_c = \Omega_c$, and $-\alpha\beta_e = \Omega_e$, yielding:

$$\Delta\mathrm{RFX} = \Omega_0 + \alpha\mathrm{RFX}_{t-1} + \Omega_c\mathrm{COM}_{t-1} + \Omega_e\mathrm{ENE}_{t-1} + \gamma\mathrm{RDIFF}_{t-1}$$

which can then be regressed using the OLS method. In the actual estimated equation, each explanatory variable, other than RDIFF, is expressed as a logarithm. The long-run relationship is based on the "well-determined positive effect" that the non-energy commodity price index has on the exchange rate, and vice versa for the energy price index (Amano & van Norden, 1992). [1]

The basic idea behind these interactions is that a rise in non-energy commodity prices causes an increase in demand for the CAD, which results in the CAD appreciating against the USD, while the inverse holds true for energy prices. The idea behind the short-run determinant – the interest rate differential – is that money should flow in the direction of the higher real interest rate, and cause appreciation and depreciation accordingly.

The next section covers the data used to form the time series for each of the aforementioned variables. I outline the data sources and the steps necessary to express the observations in real terms.

4 The Data

As previously mentioned, all of the time series are quarterly. The observations cover the period 1972:Q1 to 2013:Q4. I will cover each of the terms in turn, beginning with the US/CAD exchange rate.

US/CAD Exchange Rate (RFX)

The exchange rate data is a series released by Statistics Canada. Each quarter is the average of daily noon rates. To express the exchange rates in real terms, they are multiplied by the ratio of Canadian to US GDP deflators.

Non-Energy and Energy Commodity Price Indexes ($\mathbf{COM}_{t-1}$, $\mathbf{ENE}_{t-1}$)

Both price index series are produced by the Bank of Canada, are chain Fisher price indexes, and have a base year of 1972. (*Bank of Canada Commodity Price Indexes*, n.d.) In order to express each series in real terms, they are divided by the US GDP deflator.

Interest Rate Differential (RDIFF)

The interest rate differential is the difference between the Canadian 90-day prime commercial paper rate, and the max of the US prime non-financial and financial commercial paper rates. The Canadian series is from Statistics Canada, and the U.S series is from the U.S Federal Reserve.

[1] Whether or not these relationships are still "well-determined" is one of the questions this paper seeks to answer.

In the next section, I outline the various tests I perform to assess the model's performance and identify structural breaks.

5 Structural Break Analysis

My analysis has four parts. First, I look at the model's performance using recent data. In Laidler and Aba's model estimates, they only had data available up to the third quarter of 2000. I follow their approach, but use data from up to the forth quarter of 2013. The idea is to look for initial signs of structural breaks. Because the model works for earlier data, it failing with more recent data could indicate a structural change. In the second part, I perform actual structural break tests on the model using a statistical method called a Chow test. Next, I perform a series of Bai-Perron global breakpoint tests. Finally, I revisit the Bank of Canada equation, using flexible parameters based on the structural break tests.

Model performance with recent data

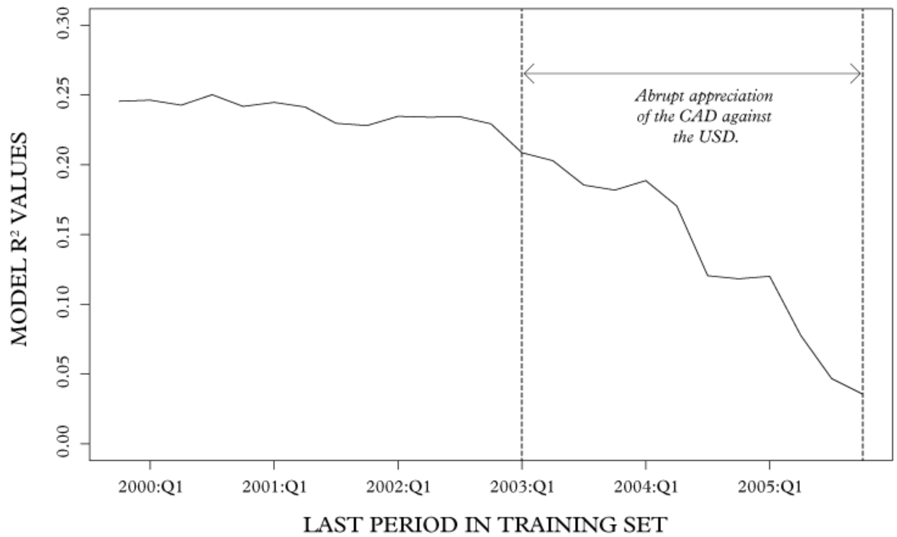
To test the model's performance with recent data, I use two separate regressions. In the first, the coefficients for COM_{t-1} and ENE_{t-1} are fixed, while in the second, separate coefficients are estimated for each decade. This allows for a preliminary look at whether or not the exchange rate determinants vary in nature over time. Laidler and Aba found that allowing for parameter flexibility resulted in a better model fit, indicating to them the possibility of structural changes.

The results of the first regression, outlined in Table 1, show clearly that the model breaks down with the more recent data. None of the terms maintain any statistical significance. The equation no longer effectively describes the behaviour of the exchange rate. This is also evident in the extremely low R^2 value of 0.008. Figure 1 illustrates how the model captures less and less of the exchange rate movement over time. It gives the resulting R^2 values of regressions run using data up to the date given on the y-axis.

It is clear to see that the model cannot account for the appreciation of the Canadian dollar between 2003 and 2005. The R^2 values decrease steeply, indicating a change in the factors determining the exchange rate movements.

The second regression, as previously mentioned, has different commodity price index coefficients for each decade. So, for example, $\mathrm{COM70s}_{t-1}$ has a value of zero for any quarter outside of the 1970s. This allows for the possibility that the model can still describe the movements if the parameters are allowed to change. The results, outlined in Table 2, are an improvement over the fixed parameter model, but still indicate a breakdown of the model. Only two of the

FIGURE 1.
Model R^2 values against the last period used in the regression estimate.



coefficients show any statistical significance. The signs also do not fit to the assumptions the model is based upon.

TABLE 1.
Fixed Parameter Regression Results

	Dependent variable:
	ΔRFX
COM_{t-1}	-0.003
	(0.011)
ENE_{t-1}	-0.003
	(0.005)
RFX_{t-1}	-0.007
	(0.021)
RDIFF_{t-1}	0.0001
	(0.001)
Constant	0.039
	(0.081)
Observations	167
R^2	0.008
Adjusted R^2	-0.016
Residual Std. Error	0.025 (df = 162)
F Statistic	0.331 (df = 4; 162)
Note:	*p<0.1; **p<0.05; ***p<0.01

TABLE 2.
Flexible Parameter Regression Results

	Dependent variable:
	ΔRFX
COM70s_{t-1}	0.025
	(0.025)
COM80s_{t-1}	0.020
	(0.028)
COM90s_{t-1}	0.020
	(0.032)
COM00s_{t-1}	0.017
	(0.029)
ENE70s_{t-1}	-0.028^{**}
	(0.013)
ENE80s_{t-1}	-0.021^{*}
	(0.011)
ENE90s_{t-1}	-0.023
	(0.029)
ENE00s_{t-1}	-0.016
	(0.018)
RFX_{t-1}	-0.021
	(0.033)
$RDIFF_{t-1}$	0.002
	(0.002)
Constant	0.018
	(0.150)
Observations	167
R^2	0.105
Adjusted R^2	0.048
Residual Std. Error	0.024 (df = 156)
F Statistic	1.829^{*} (df = 10; 156)
Note:	$^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

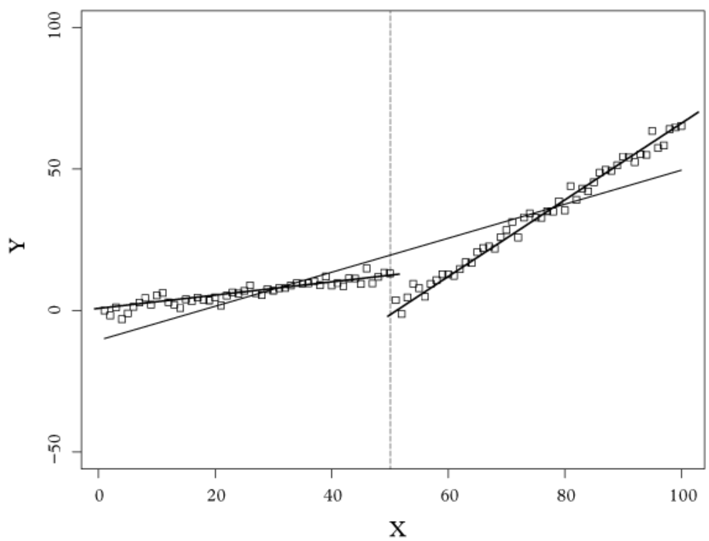


FIGURE 2.
A simple example of a structural change in a single variable linear model.

Structural break tests: Chow

To confirm the existence of structural breaks, I use a Chow test. Chow tests work by splitting a data set into two subsets, running a regression on each, and testing against the null hypothesis that the coefficients in each regression are equal. The null hypothesis can then be rejected at some given confidence level. Where the data set is split is based on the timing of a hypothesized structural break.

Figure 2 illustrates a structural change in a single variable linear model using simulated data. The structural break is clearly visible at $X = 50$. It is also apparent that the data is better described using two different regressions rather than one. In an actual model, of course, structural changes are not so easily identifiable, which is why a statistical test is required.

In Laidler and Aba's work with the Bank of Canada equation, they state that there may be structural breaks between decades (parameter instability), but that a "complete econometric analysis that addresses this problem is well beyond the scope [of their paper]" (Laidler & Aba, 2001). To address this gap in their analysis, I've used the Chow test to determine whether or not there are structural breaks between each decade.

The results of the Chow tests, which are provided in Table 3, confirm structural breaks at 1979:Q4 and 1991:Q4. This confirms the findings of Issa, Lafrance, and Murray (Issa & Murray, 2008). It also confirms the existence of parameter instabilities in Laider and Aba's model.

Structural Break	F-statistic	p-value
1979:Q4	5.5638	0.0001439
1991:Q4	3.2914	0.008538

TABLE 3.
Results of Chow tests (1972:Q2 - 1999:Q4 Model)

The model I estimated using data up to 2013 also exhibits structural breaks. The results of the relevant Chow tests are outlined in Table 4. All breaks are significant at the 95% confidence level.

Structural Break	F-statistic	p-value
1980:Q1	2.2761	0.04961
1991:Q4	2.3931	0.04006
2002:Q4	13.7179	0.00000

TABLE 4.
Results of Chow tests (1972:Q2 - 2013:Q4 Model)

The next section uses a series of Bai-Perron tests to further confirm these results.

Structural break tests: Bai-Perron

A Bai-Perron test is different from a Chow test in that it does not require the input of a hypothesized break point. It allows for the estimation of simultaneous breakpoints, and can determine the optimal breakpoints in a time series (Bai & Perron, 2003). This provides a more robust test for the structural breaks this paper looks to confirm.

The test requires two input parameters: (1) a minimum segment size (it wont "break" the series into subsets smaller than this), and (2) a maximum number of breaks. In order to ensure the robustness of the results, I perform three different tests using different values for these parameters.

In the first test, I set the number of maximum breaks equal to 4, and set the minimum segment size at two years (8 periods). In the second, I once again use minimum segments of two years, but allow for only 3 breaks. In the third, I allow for 3 breaks but use a five year minimum segment size. The results of the tests are outlined in Table 5.

The results of all three tests confirm the presence of structural breaks. The first test appears to capture the sudden appreciation of the CAD between 2003

TABLE 5.
Bai-Perron test results

Test #	Number of Breaks	Minimum Segment Size	Results / Structural Break Points
1	4	2 years	1979:Q2 1991:Q4 2002:Q4 2005:Q3
2	3	2 years	1979:Q2 1991:Q2 2002:Q4
3	3	5 years	1979:Q2 1991:Q2 2003:Q1

– 2005. It also confirms the break at the end of the 1970s and the break in the early 1990s. The next two tests also confirm these results. The only change between setting the minimum segment size at 2 and 5 years is that the 2002 structural break occurs one period later at 2003:Q1.

In the next section, I run another regression using the BoC model, this time allowing the parameters to change based on the structural breaks indicated in the Bai-Perron tests.

Model performance with flexible break point parameters

The purpose of this section is to determine whether or not the model can account for the structural changes if the parameters are allowed to change. In the previous regression, the parameters simply changed by decade. This time I change the parameters based on the findings of the structural break tests. This requires defining four periods:

Period	Time Span
A	1972:Q2 – 1979:Q2
B	1979:Q3 – 1991:Q4
C	1992:Q1 – 2003:Q1
D	2003:Q2 – 2006:Q4

Each period has a different variable, and thus coefficient, for ENE and COM. The regression results are outlined in Table 6. The subscripts of ENE and COM indicate the period they represent. It is interesting to note that the R^2 value is much higher than before. The results also suggest that the model cannot account for the structural change in 2003. Neither ENE_D nor COM_D are significant at the 95% level, though they do reverse their signs. The signs of the coefficients for each of the other periods are what we would expect given the assumptions the model are based upon.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-39.3765	4.4135	-8.92	0.0000
breakfactor(log_gnp_breakpoints)segment2	-101.8940	6.9853	-14.59	0.0000
breakfactor(log_gnp_breakpoints)segment3	38.5478	6.7631	5.70	0.0000
breakfactor(log_gnp_breakpoints)segment4	-116.8294	7.4016	-15.78	0.0000
breakfactor(log_gnp_breakpoints)segment5	-163.8289	6.6081	-24.79	0.0000
breakfactor(log_gnp_breakpoints)segment6	-44.9355	6.0747	-7.40	0.0000
dat$Year	0.0243	0.0023	10.35	0.0000
breakfactor(log_gnp_breakpoints)segment2:dat$Year	0.0537	0.0037	14.57	0.0000
breakfactor(log_gnp_breakpoints)segment3:dat$Year	-0.0194	0.0035	-5.49	0.0000
breakfactor(log_gnp_breakpoints)segment4:dat$Year	0.0609	0.0038	15.83	0.0000
breakfactor(log_gnp_breakpoints)segment5:dat$Year	0.0847	0.0034	24.75	0.0000
breakfactor(log_gnp_breakpoints)segment6:dat$Year	0.0248	0.0031	7.90	0.0000

6 Conclusion

The results of all four of my analyses support the existence of structural breaks in the determinants of the USD/CAD exchange rate. This confirms the findings of other researchers. I've also determined that the structural change in 1991:Q2 can be explained in-model. That is, the Bank of Canada equation can account for the exchange rate if parameter flexibility is allowed. The structural break at 2002:Q4, however, cannot be explained by the model even when allowing parameter flexibility.

This strongly suggests that, after this point, the determinants of the exchange rate change, and a different model is required. That is, a variable, or variables, are missing from the equation.

These findings are not groundbreaking, but rather they provide more support for the findings of others. The novelty of my work comes from using a different model, and different statistical tests, than those researchers.

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