Assignment 2

1. Programming Problems - Matrix Transpose, Matrix Multiplication (80 pts)

1.1 (10 pts): Matrix Transpose

- (5 pts) Implement two parallel transpose algorithms using both PyCUDA and PyOpenCL. The input is a 2D matrix with any size, the output is the transpose of the input.
- (1 pts) Implement a serial transpose algorithm. The input is a 2D matrix with any size, the output is the transpose of the input.
- (2 pts) Choose any two integers M and N from 1 to 10. Then, randomly generate matrices with size M x N, 2M x 2N, 3M x 3N,.... Calculate the transpose of them using 3 transpose algorithms (2 parallel, 1 serial) respectively. Record the running time of each call for each of the algorithm.
- (2 pts) Plot running time vs. matrix size in one figure, **compare and analyze** the results in your report. You need to plot one for only kernel execution and another for including memory copy time (both host to device & device to host) in each of PyCUDA.py and PyOpenCL.py

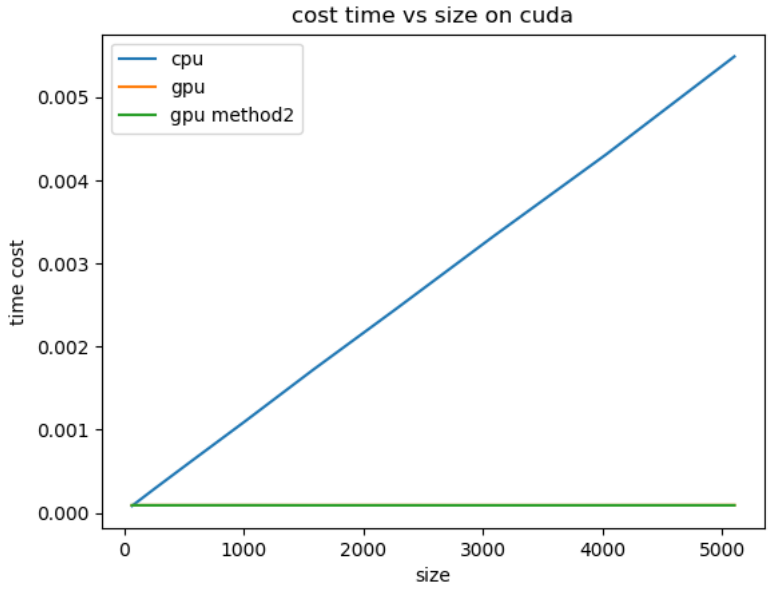
Report:

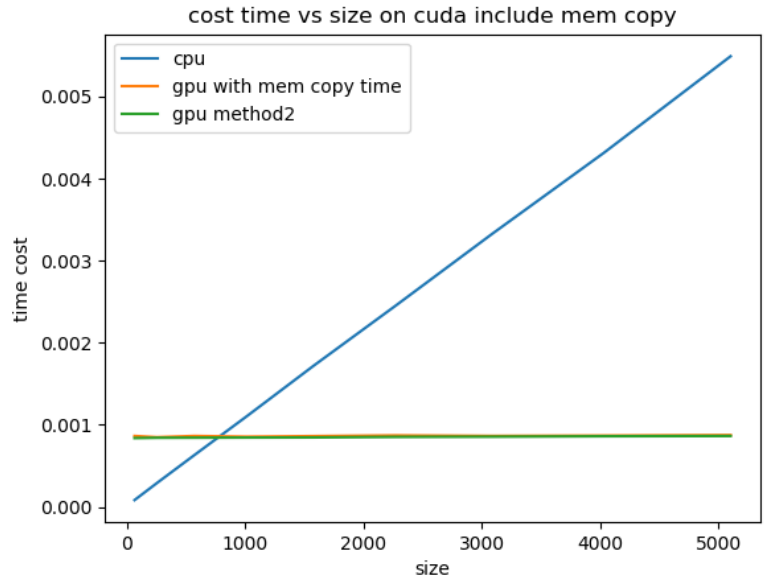
In the first task to transpose an any-size matrix, I implemented one serial and two GPU algorithm in the `class Transpose`. Two GPU methods are called `transpose` and `transpose2`. The serial (CPU) one is called `transpose_serial`. I firstly implemented these two GPU algorithms in CUDA, then translated to OpenCL.

To help myself do record running time, I set an additional argument called flag for every method. This flag is set default as 0, this will let the method return the result matrix only. If set it to some number other than 0, these methods will also return the running time. Then I can record this. In this assignment, I record the averaged running time and record them in a CSV file.

The plot is using same method to generate just as the last assignment.

For CUDA:



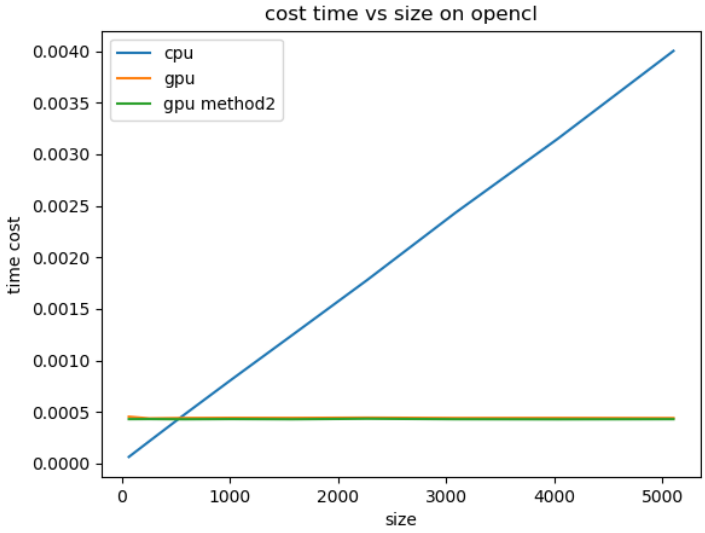


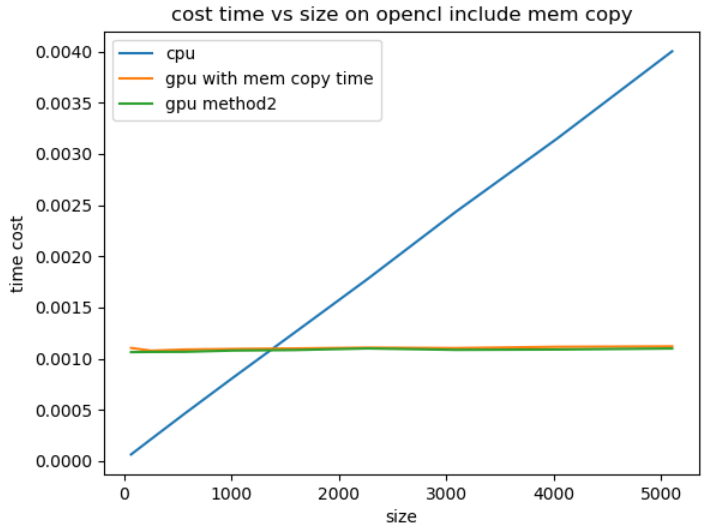
And the recorded data looks like this:

i	time serial	time	time2	time include mem	time2 include mem
1	8.28E-05	9.03E-05	9.03E-05	0.00086403	0.00086403
2	0.00028998	9.12E-05	9.12E-05	0.00084901	0.00084901
3	0.00062895	9.08E-05	9.08E-05	0.00086403	0.00086403

4	0.00109971	9.11E-05	9.11E-05	0.00085521	0.00085521
5	0.00171709	9.11E-05	9.11E-05	0.00086224	0.00086224
6	0.00244856	9.13E-05	9.13E-05	0.00087102	0.00087102
7	0.00332654	9.08E-05	9.08E-05	0.00086445	0.00086445
8	0.00431436	9.08E-05	9.08E-05	0.0008685	0.0008685
9	0.00549042	9.18E-05	9.18E-05	0.000875	0.000875

For OpenCL:





The recorded data:

i	time serial	time	time2	time include mem	time2 include mem
1	6.37E-05	0.00045371	0.00045371	0.00110404	0.00110404
2	0.00021537	0.00043702	0.00043702	0.00107757	0.00107757
3	0.00046635	0.00044092	0.00044092	0.00108902	0.00108902
4	0.00081031	0.00044266	0.00044266	0.00109442	0.00109442
5	0.0012455	0.00044203	0.00044203	0.00109935	0.00109935
6	0.00177932	0.00044417	0.00044417	0.0011073	0.0011073
7	0.00243354	0.00044099	0.00044099	0.00110292	0.00110292
8	0.00314999	0.00044171	0.00044171	0.00111532	0.00111532
9	0.00400241	0.00044036	0.00044036	0.00111969	0.00111969

We can see the CPU method's cost time goes proportional with matrix size as expected. Two GPU methods shows no effect on time cost as matrix becoming larger. And they have very similar performance. Moreover, we can see time cost on memory operation cost a lot of time that is much more than actual kernel execution time.

1.2 (70 pts): Matrix Multiplications

To simplify the algorithm, the input is only **one** M x N matrix. The output is the multiplication of the input matrix and its transpose. The result should be a M x M matrix.

- (20 pts) Implement a naïve version of the matrix multiplication algorithm, in each of PyCUDA & PyOpenCL.
- (15 pts) Optimize the algorithms using shared memory.
- (15 pts) Optimize the algorithms further using any method.
- (10 pts) Choose any two integers M and N from 1 to 10. Then, randomly generate matrices with size M x N, 2M x 2N, 3M x 3N,.... Calculate the multiplication of them and their transpose using your 6 multiplication algorithms (2 naïve, 2 optimized, 2 further optimized) respectively. Record the running time of each call for each algorithm.
- (8 pts, for CUDA only) Using the Nvidia Visual Profiler (nvvp) to compare your 3 algorithms written in CUDA.
- (2 pts) Plot all running time-matrix size profile in one figure, compare and analyze the results in your report. Insert the profiling results of NVVP into your report.You need to plot one for only kernel execution and another for including memory copy

time(both host to device & device to host) in each of PyCUDA.py and PyOpenCL.py

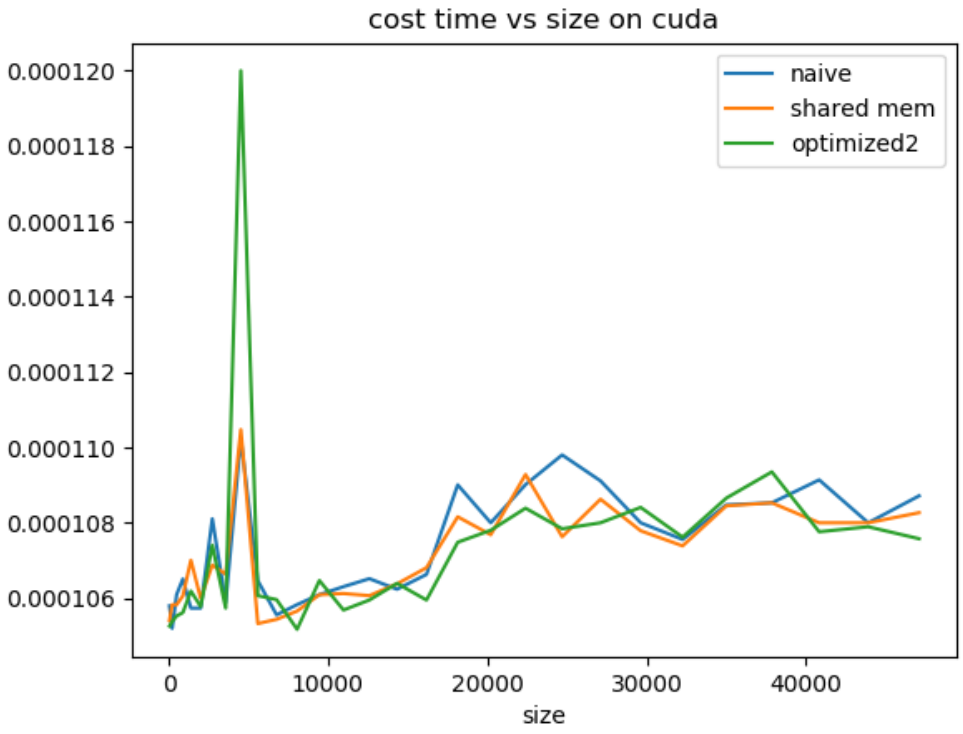
Report:

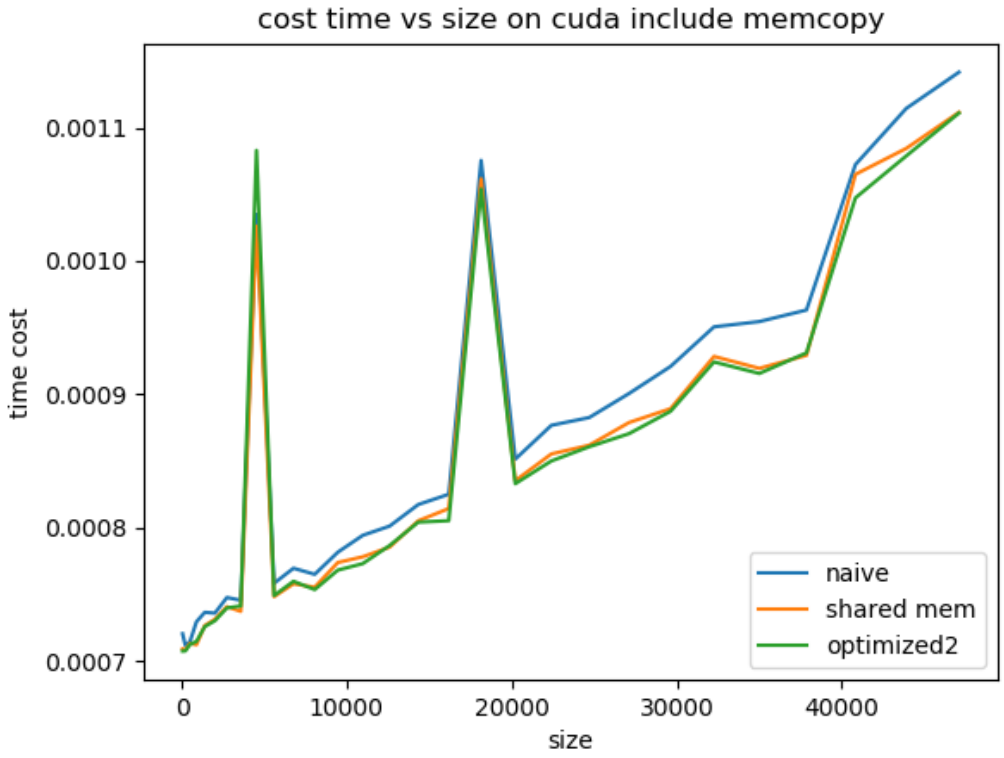
Matrix multiplication is a little complicated than transpose. There are three methods in `class MatrixMultiply`: `matrix_mul_naive`, `matrix_mul_optimized1`, and `matrix_mul_optimized2`.

Naïve version is quite straight forward, just use block to handle any-size matrix and take care of matrix boundary problem. Then the optimized with shared memory one use shared block memory to accelerate the algorithm just as told in the lecture. Then I use global memory coalescing accessing to accelerate tile block accessing and avoided shared memory bank conflict. This is further optimized algorithm.

I used same procedure to record time cost and produce figures.

For CUDA:





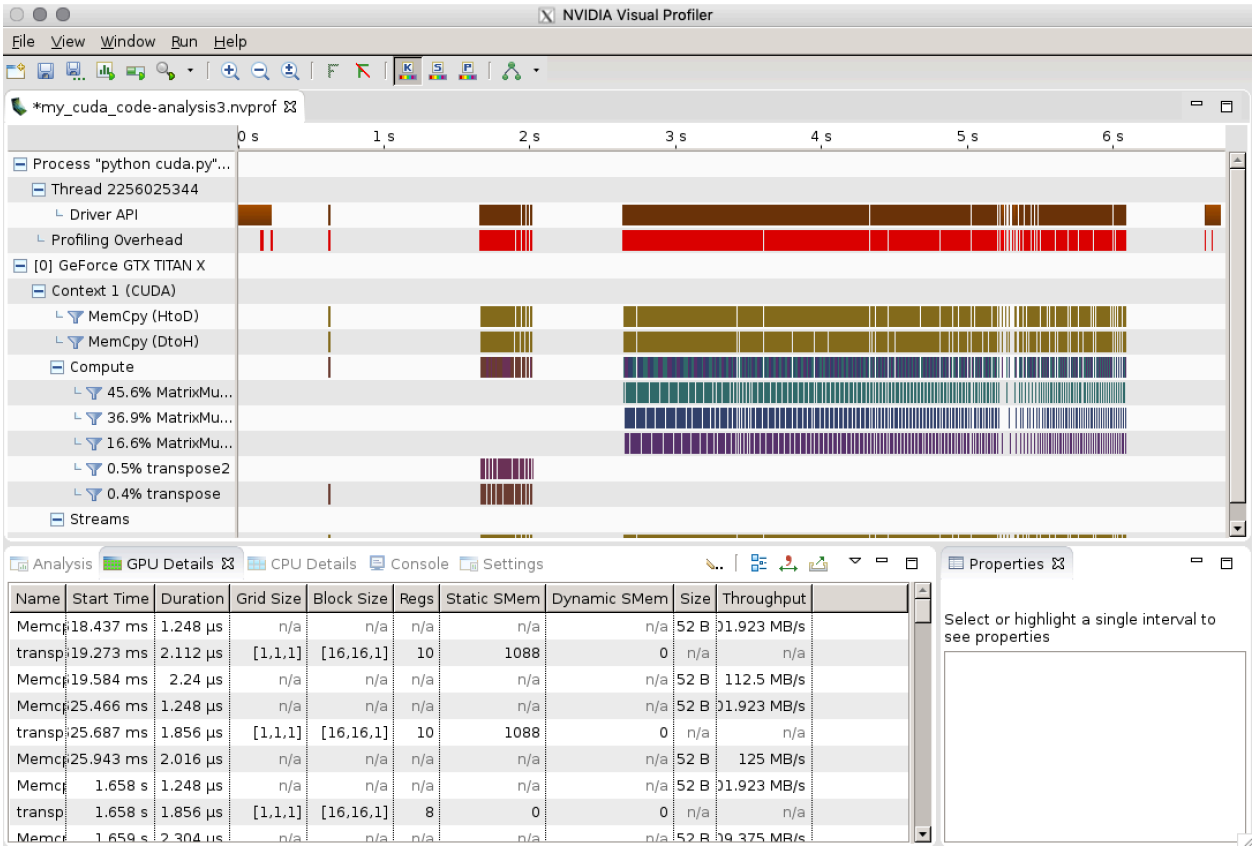
i	time	time optm1	time optm2	time include mem	time optm1 include mem	time optm2 include mem
1	0.0001058	0.00010541	0.00010526	0.00072046	0.000709	0.00070744
2	0.0001052	0.00010583	0.00010538	0.00071179	0.0007084	0.00070749
3	0.0001061	0.00010583	0.00010553	0.00071446	0.00071332	0.00071239
4	0.00010651	0.00010606	0.00010562	0.0007294	0.00071219	0.0007147
5	0.00010574	0.00010701	0.00010619	0.00073647	0.00072654	0.00072575
6	0.00010574	0.00010599	0.00010578	0.00073591	0.00073131	0.00073017
7	0.00010811	0.00010688	0.00010741	0.00074762	0.0007407	0.00073979
8	0.00010589	0.00010664	0.00010574	0.00074553	0.00073719	0.00074128
9	0.00011024	0.00011048	0.00012	0.00103489	0.00102609	0.00108282
10	0.00010647	0.00010533	0.00010607	0.00075825	0.00074824	0.00074914
11	0.00010556	0.00010544	0.00010596	0.00076954	0.00075785	0.00075974
12	0.00010583	0.00010566	0.00010518	0.000765	0.00075534	0.00075352
13	0.0001061	0.0001061	0.00010647	0.00078157	0.00077385	0.00076813
14	0.00010631	0.00010613	0.00010569	0.00079423	0.00077813	0.00077302
15	0.00010652	0.00010607	0.00010595	0.00080112	0.00078535	0.00078658
16	0.00010624	0.00010638	0.00010639	0.0008173	0.00080511	0.00080408
17	0.00010663	0.00010681	0.00010595	0.00082502	0.0008143	0.00080519
18	0.00010901	0.00010816	0.00010749	0.00107535	0.00106149	0.00105378

19	0.000108	0.00010769	0.0001078	0.00085137	0.00083521	0.00083293
20	0.00010902	0.00010929	0.00010839	0.0008767	0.00085523	0.00084996
21	0.00010981	0.00010763	0.00010784	0.00088252	0.00086178	0.00086063
22	0.00010912	0.00010863	0.000108	0.00090054	0.00087878	0.00087044
23	0.000108	0.00010779	0.00010841	0.00092086	0.00088918	0.00088722
24	0.00010756	0.00010739	0.00010762	0.00095055	0.00092835	0.00092421
25	0.00010848	0.00010845	0.00010866	0.00095448	0.00091941	0.00091556
26	0.00010853	0.00010853	0.00010935	0.00096314	0.00092916	0.00093102
27	0.00010914	0.000108	0.00010777	0.00107222	0.00106487	0.00104724
28	0.000108	0.000108	0.0001079	0.00111431	0.00108433	0.00107869
29	0.00010872	0.00010827	0.00010758	0.00114147	0.00111139	0.00111079

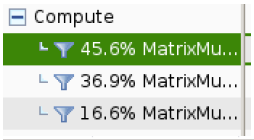
We can see the time cost is as our expectation. The performance of further optimized one is better than shared memory one and naïve version.

An interesting fact is that kernel execution time doesn't grow as much as matrix size grows. While the including-memory-copy time grows almost proportionally to the size.

The NVIDIA visual profile looks like this:



It shows I called each method for about 290 times. Just as showed in graph, naïve version cost most time. Shared memory does progress on optimization. Further optimized one has best performance.



Name	Start Time	Duration	Grid Size	Block Size	Regs	Static SMem	Dynamic SMem
MatrixMulNaive	2.638 s	2.72 µs	[1,1,1]	[16,16,1]	25	0	0
MatrixMulNaive	2.644 s	2.688 µs	[1,1,1]	[16,16,1]	25	0	0
MatrixMulNaive	2.653 s	2.688 µs	[1,1,1]	[16,16,1]	25	0	0
MatrixMulNaive	2.662 s	2.656 µs	[1,1,1]	[16,16,1]	25	0	0
MatrixMulNaive	2.67 s	2.656 µs	[1,1,1]	[16,16,1]	25	0	0
MatrixMulNaive	2.679 s	2.656 µs	[1,1,1]	[16,16,1]	25	0	0
MatrixMulNaive	2.687 s	2.72 µs	[1,1,1]	[16,16,1]	25	0	0
MatrixMulNaive	2.696 s	2.688 µs	[1,1,1]	[16,16,1]	25	0	0

MatrixMulNaive

Duration	
Session	6.737 s (6,736,742,
Kernel	9.754 ms (9,754,12
Invocations	291
Importance	45.6%

Name	Start Time	Duration	Grid Size	Block Size	Regs	Static SMem	Dynamic SMem
MatrixMulNaive	5.955 s	110.08 µs	[13,13,1]	[16,16,1]	25	0	0
MatrixMulNaive	5.97 s	109.888 µs	[13,13,1]	[16,16,1]	25	0	0
MatrixMulNaive	5.985 s	108.897 µs	[13,13,1]	[16,16,1]	25	0	0
MatrixMulNaive	6 s	109.472 µs	[13,13,1]	[16,16,1]	25	0	0
MatrixMulNaive	6.015 s	109.217 µs	[13,13,1]	[16,16,1]	25	0	0
MatrixMulNaive	6.03 s	109.312 µs	[13,13,1]	[16,16,1]	25	0	0
MatrixMulNaive	6.045 s	109.729 µs	[13,13,1]	[16,16,1]	25	0	0
MatrixMulNaive	6.061 s	109.536 µs	[13,13,1]	[16,16,1]	25	0	0
MatrixMulNaive	6.076 s	110.464 µs	[13,13,1]	[16,16,1]	25	0	0

Name	Start Time	Duration	Grid Size	Block Size	Regs	Static SMem	Dynamic SMem
MatrixMulShare	5.959 s	84.417 µs	[13,13,1]	[16,16,1]	30	2048	0
MatrixMulShare	5.974 s	83.776 µs	[13,13,1]	[16,16,1]	30	2048	0
MatrixMulShare	5.989 s	83.936 µs	[13,13,1]	[16,16,1]	30	2048	0
MatrixMulShare	6.004 s	83.904 µs	[13,13,1]	[16,16,1]	30	2048	0
MatrixMulShare	6.019 s	83.808 µs	[13,13,1]	[16,16,1]	30	2048	0
MatrixMulShare	6.034 s	84.288 µs	[13,13,1]	[16,16,1]	30	2048	0
MatrixMulShare	6.05 s	83.808 µs	[13,13,1]	[16,16,1]	30	2048	0
MatrixMulShare	6.065 s	84.192 µs	[13,13,1]	[16,16,1]	30	2048	0
MatrixMulShare	6.08 s	83.936 µs	[13,13,1]	[16,16,1]	30	2048	0

MatrixMulSharedMem

Duration	
Session	6.737 s (6,736,742,
Kernel	7.902 ms (7,901,77
Invocations	290
Importance	36.9%

Name	Start Time	Duration	Grid Size	Block Size	Regs	Static SMem	Dynamic SMem
MatrixMulOptm	5.963 s	31.872 µs	[13,13,1]	[16,16,1]	27	2048	0
MatrixMulOptm	5.978 s	31.776 µs	[13,13,1]	[16,16,1]	27	2048	0
MatrixMulOptm	5.993 s	31.712 µs	[13,13,1]	[16,16,1]	27	2048	0
MatrixMulOptm	6.008 s	32.48 µs	[13,13,1]	[16,16,1]	27	2048	0
MatrixMulOptm	6.023 s	31.904 µs	[13,13,1]	[16,16,1]	27	2048	0
MatrixMulOptm	6.037 s	31.968 µs	[13,13,1]	[16,16,1]	27	2048	0
MatrixMulOptm	6.054 s	32.032 µs	[13,13,1]	[16,16,1]	27	2048	0
MatrixMulOptm	6.069 s	31.52 µs	[13,13,1]	[16,16,1]	27	2048	0
MatrixMulOptm	6.084 s	31.616 µs	[13,13,1]	[16,16,1]	27	2048	0

MatrixMulOptm

Duration	
Session	6.737 s (6,736,742,
Kernel	3.562 ms (3,561,76
Invocations	290
Importance	16.6%

For kernel time, naïve one may cost around 109 µs, the shared memory one costs around 83.7 µs and the further optimized one costs around 32 µs.

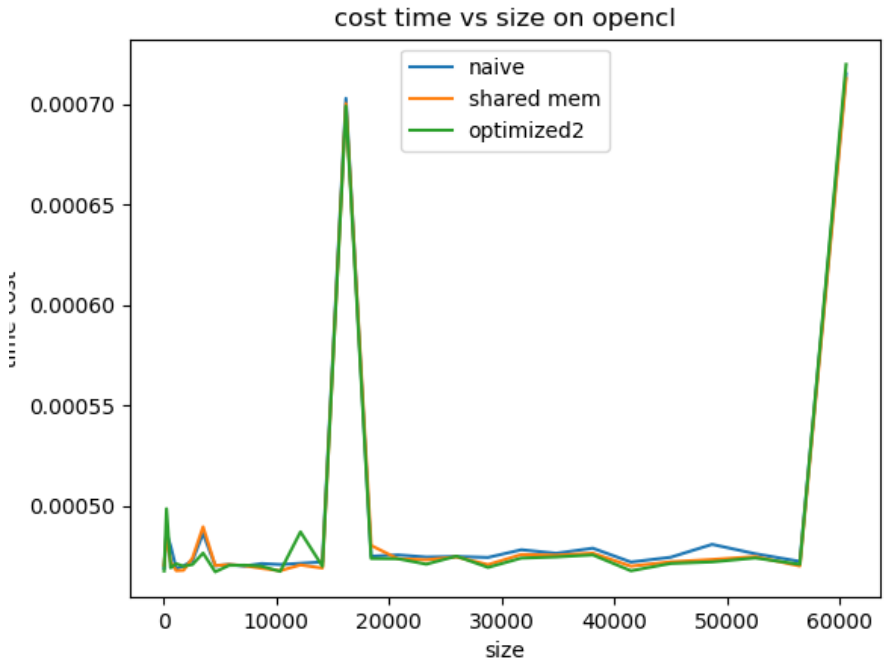
Memcpy HtoD [sync]	3.889 s	5.984 µs	n/a	n/a	n/a	n/a	n/a	37.856 kB	6.326
Memcpy HtoD [sync]	3.889 s	5.984 µs	n/a	n/a	n/a	n/a	n/a	37.856 kB	6.326
MatrixMulNaive	3.889 s	16.992 µs	[6,6,1]	[16,16,1]	25	0	0	n/a	
Memcpy DtoH [sync]	3.889 s	4.8 µs	n/a	n/a	n/a	n/a	n/a	33.124 kB	6.901
Memcpy HtoD [sync]	3.892 s	5.952 µs	n/a	n/a	n/a	n/a	n/a	37.856 kB	6.36
Memcpy HtoD [sync]	3.892 s	5.952 µs	n/a	n/a	n/a	n/a	n/a	37.856 kB	6.36
MatrixMulSharedMem	3.892 s	14.592 µs	[6,6,1]	[16,16,1]	30	2048	0	n/a	
Memcpy DtoH [sync]	3.892 s	4.768 µs	n/a	n/a	n/a	n/a	n/a	33.124 kB	6.947
Memcpy HtoD [sync]	3.895 s	5.952 µs	n/a	n/a	n/a	n/a	n/a	37.856 kB	6.36
Memcpy HtoD [sync]	3.895 s	5.952 µs	n/a	n/a	n/a	n/a	n/a	37.856 kB	6.36
MatrixMulOptm	3.895 s	8.48 µs	[6,6,1]	[16,16,1]	27	2048	0	n/a	
Memcpy DtoH [sync]	3.895 s	4.768 µs	n/a	n/a	n/a	n/a	n/a	33.124 kB	6.947

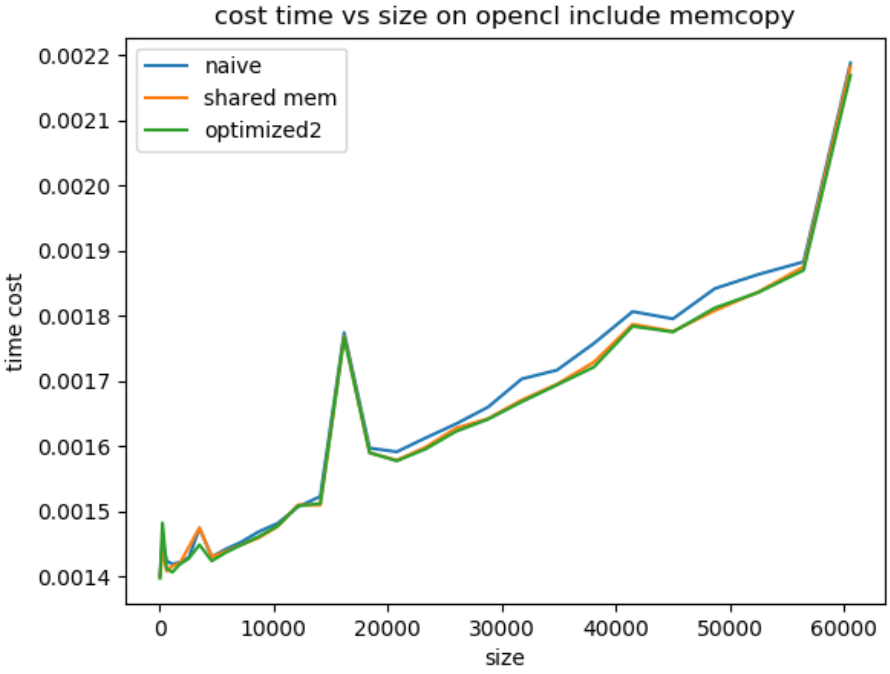
For example, when grid size is [6,6,1], memory copy time can cost 12 µs for host to device and 5 µs from device to host. Memory copy time is much more than execution time.

Memcpy HtoD [sync]	6.076 s	19.616 µs	n/a	n/a	n/a	n/a	n/a	188.384 kB	9.604
Memcpy HtoD [sync]	6.076 s	18.688 µs	n/a	n/a	n/a	n/a	n/a	188.384 kB	10.08
MatrixMulNaive	6.076 s	110.464 µs	[13,13,1]	[16,16,1]	25	0	0	n/a	
Memcpy DtoH [sync]	6.076 s	14.848 µs	n/a	n/a	n/a	n/a	n/a	164.836 kB	11.102
Memcpy HtoD [sync]	6.079 s	19.424 µs	n/a	n/a	n/a	n/a	n/a	188.384 kB	9.699
Memcpy HtoD [sync]	6.08 s	19.009 µs	n/a	n/a	n/a	n/a	n/a	188.384 kB	9.91
MatrixMulSharedMem	6.08 s	83.936 µs	[13,13,1]	[16,16,1]	30	2048	0	n/a	
Memcpy DtoH [sync]	6.08 s	14.88 µs	n/a	n/a	n/a	n/a	n/a	164.836 kB	11.078
Memcpy HtoD [sync]	6.083 s	19.104 µs	n/a	n/a	n/a	n/a	n/a	188.384 kB	9.861
Memcpy HtoD [sync]	6.083 s	18.944 µs	n/a	n/a	n/a	n/a	n/a	188.384 kB	9.944
MatrixMulOptm	6.084 s	31.616 µs	[13,13,1]	[16,16,1]	27	2048	0	n/a	
Memcpy DtoH [sync]	6.084 s	14.848 µs	n/a	n/a	n/a	n/a	n/a	164.836 kB	11.102

When grid size grows to [13,13,1] (matrix size is around 200*230), memory copy time may be not so significant according to the execution time of naïve version.

For OpenCL:





Performances for three algorithms on OpenCL do not show much difference. To use shared memory (__local in OpenCL), I used `LocalMemory` class in PyOpenCL. One confusing issue I got is naïve algorithm has much less time cost than optimized one. To solve this, I tried to also input local memory objects in to kernel function of naïve version (of course not used in kernel), then the result comes out as expectation. So, I keep this in my code. My guess is that the

`LocalMemory` operation in PyOpenCL costs much time, so I need to balance this by adding same procedure to naïve version. This is not like CUDA, shared memory is directly declared in the kernel, OpenCL requires you to treat it as a input of kernel function.

i	time include mem	time optm1 include mem	time optm2 include mem	time	time optm1	time optm2
1	0.00140035	0.00140035	0.00139686	0.00046915	0.00047067	0.00046775
2	0.00145602	0.00144139	0.00148201	0.00048664	0.00048572	0.00049874
3	0.00142342	0.00140792	0.00141147	0.00048029	0.00047231	0.00046936
4	0.00141889	0.00141573	0.00140619	0.00046888	0.00046793	0.00047138
5	0.00142121	0.00141975	0.00141826	0.00047028	0.00046812	0.00047007
6	0.00142878	0.00144476	0.00142726	0.00047296	0.00047424	0.000471
7	0.00147364	0.00147444	0.00144833	0.00048637	0.00048974	0.00047669
8	0.00142986	0.0014298	0.00142348	0.0004704	0.00047046	0.00046733
9	0.0014416	0.00143844	0.00143641	0.000471	0.00047097	0.00047055
10	0.00145271	0.00144836	0.00144824	0.00047021	0.00047043	0.00047064
11	0.00146833	0.00145897	0.00146085	0.00047147	0.00046909	0.00047013
12	0.00148124	0.00147626	0.00147715	0.000471	0.00046793	0.00046757
13	0.00150672	0.00150973	0.00150821	0.00047159	0.00047076	0.00048724
14	0.0015226	0.00150929	0.00151157	0.00047225	0.00046921	0.00047013
15	0.00177419	0.00176865	0.00176704	0.00070289	0.00070027	0.00069898
16	0.0015969	0.00158954	0.00158975	0.00047508	0.00048047	0.00047398
17	0.00159112	0.00157815	0.0015772	0.00047576	0.00047383	0.00047386
18	0.0016124	0.00159812	0.00159532	0.00047469	0.00047332	0.00047115
19	0.00163392	0.00162733	0.00162238	0.00047496	0.00047478	0.00047514
20	0.00165975	0.00164214	0.00164109	0.00047451	0.00047094	0.00046955
21	0.00170302	0.00167063	0.00166786	0.00047833	0.00047597	0.00047418
22	0.00171661	0.00169513	0.00169386	0.00047656	0.00047565	0.00047475
23	0.00175777	0.00172889	0.00172145	0.0004791	0.00047663	0.00047576
24	0.00180656	0.00178713	0.001784	0.00047222	0.00047016	0.00046786
25	0.00179538	0.00177613	0.00177521	0.0004746	0.00047234	0.0004715
26	0.00184187	0.00180805	0.00181219	0.00048101	0.0004735	0.00047227
27	0.0018636	0.00183672	0.00183597	0.00047636	0.00047493	0.00047424
28	0.00188294	0.00187543	0.00186998	0.00047255	0.00047025	0.00047109
29	0.00218821	0.0021826	0.00216949	0.00071514	0.00071284	0.00071979

2. Theory Problems (20 pts, 4 points each)

2.1 If we need to use each thread to calculate one output element of a vector addition, what would be the expression for mapping the thread/block indices to data index: (A) i=threadIdx.x + threadIdx.y; (B) i=blockIdx.x + threadIdx.x; (C) i=blockIdx.x*blockDim.x + threadIdx.x; (D) i=blockIdx.x * threadIdx.x.

Answer:

(C) should be the correct one.

2.2 We want to use each thread to calculate two (adjacent) output elements of a vector addition. Assume that variable i should be the index for the first element to be processed by a thread. What would be the expression for mapping the thread/block indices to data index of the first element?(A) i=blockIdx.x*blockDim.x + threadIdx.x +2; (B) i=blockIdx.x*threadIdx.x*2; (C) i=(blockIdx.x*blockDim.x + threadIdx.x)*2; (D) i=blockIdx.x*blockDim.x*2 + threadIdx.x.

Answer:

(C) should be the correct one.

2.3 Assume that a kernel is launched with 1000 thread blocks each of which has 512 threads. If a variable is declared as a shared memory variable, how many versions of the variable will be created through the lifetime of the execution of the kernel? a. 1; ;b. 1,000; c. 512; d. 512,000.

Answer:

(b) should be the correct one.

2.4 For our tiled matrix-matrix multiplication kernel, if we use a 32X32 tile, what is the reduction of memory bandwidth usage for input matrices A and B? a. 1/8 of the original usage; b. 1/16 of the original usage; c. 1/32 of the original usage; d. 1/64 of the original usage.

Answer:

(c) should be the correct one.

2.5 For the tiled single-precision matrix multiplication kernel as shown in one of the lectures, assume that the tile size is 32X32 and the system has a DRAM burst size of 128 bytes. How many DRAM bursts will be delivered to the processor as a result of loading one A-matrix tile by a thread block?a. 16; b. 32; c. 64; d. 128.

Answer:

There are `sizeof(float)*32*32=2048` bytes in a tile. Burst size is 128 bytes. There are `2048/128=16` bursts in total.

(a) should be the correct one.