Signal Noise removal

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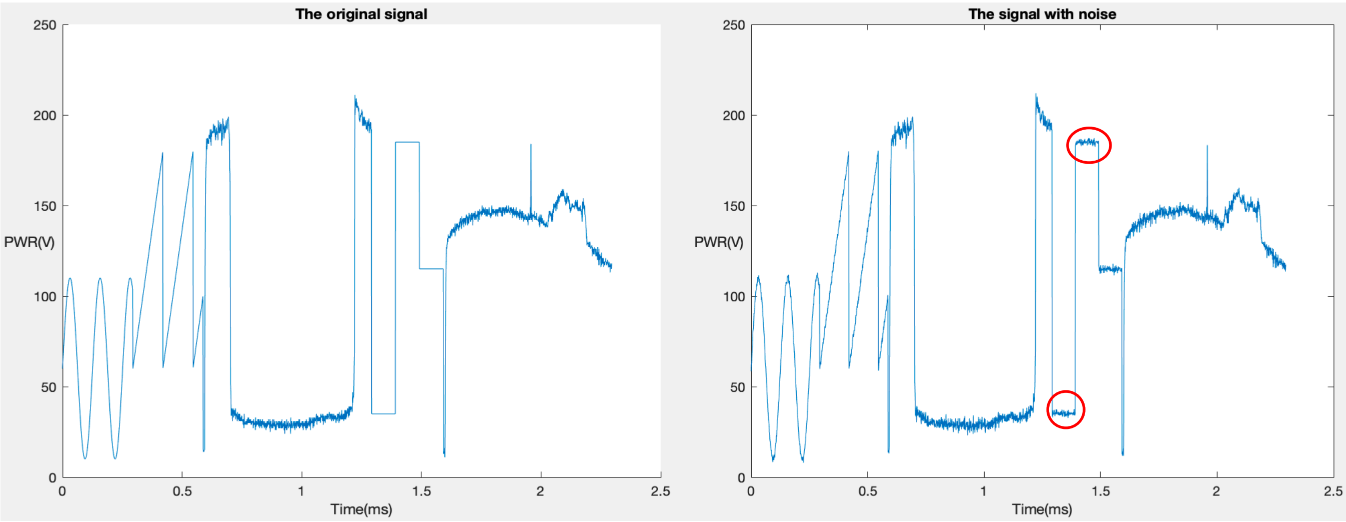
I. Background

Nowadays, the rapid growth in graphics performance. As numerous devices are operating with a screen, peoples are expecting a stable screen display, those factors make image denoising becomes a significant task in analysis and image processing. Also, it plays an important role in different application fields such as preprocessing for computer vision and medical imaging [1].



(a) **(b)**

Figure 1. (a) the given signal (b) the signal with noise

1.1 Aim of the paper

This paper presents the analysis of the given signal number 31 and noise **(Figure 1)**, which was mostly done by MATLAB programing language, and applied Peak Signal to noise ratio (PSNR) to analyze the denoising performance. Based on the characteristic of the frequency domain and signal in time domain, the denoising process is able to be initialized. There are two different denoising task has been discussed in this paper. For the first task **(Task 2)**, it applied Fourier domain noise removal technic to recover the signal, and analyzed the results with the changing the threshold's parameter to understand the performance of frequency domain. For the second task **(Task 3)**, remove the noise by DCT, and observe the performance in different conditions. Finally, those technics will be compared to see which possess better performance.

II. Methodology

2.1 Analysis of the given signal (Task 1)

To remove the noise from the noise signal, it is significant to analyze the noise signal in time domain and frequency domain. While time-domain analyze a signal changes over time, and frequency domain presents the distribution of signal's energy. Also the majority of frequency component can be obtained. In order to do this, Fast Fourier transform (FFT) is the quickest way to transform from time domain to frequency domain. As the result of using FFT algorithm, the length of the analyzed signal must be power of two as shown in the second step below. In addition, the frequency-domain represents the information of signal carries related to the signal's phase and magnitude at each frequency, resulting in a complex number outcome of FFT [2], which needs to use MATLAB function **ABS** to plot the image.

According to Nyquist's theorem half the sampling frequency must contain the majority of frequency component in the signal, the frequencies exceed this are transformed to the lower frequencies. It results in the mirror image occurs, while the second half of the spectrum is the reflection of the first half.

1. Set up the length of the "noise single" and divide it by two
2. Power of two the divided signal
3. Utilize FFT function in MATLAB
4. Set up the sample range of x-axis
5. Display the FFT result

For segment determination is based on using a Moving difference filter, which is a simplest high pass filter with convolution to analyze the successive component in the signal, also observe the edge of the signal to determine the rang of each segment.

2.2 Fourier domain noise removal (Task 2)

After understanding the performance of the noise signal in frequency domain and time domain, the noise is able to be removed by filtering out the noise frequency from frequency domain, and remain the original signal frequency. It enables Fourier inverse transform the signal back.

1. Set up the threshold in the frequency domain, remove the noise frequency
2. Inverse the outcome by ifft
3. Display the ifft result

Above steps shows the method of denoising the signal. Firstly, the threshold is set up based on the analysis of frequency domain from **Task 1**. Secondly, utilize Fourier inverse with times the length of signal to subset the attenuation during Fourier transform. Finally, the new signal can be obtained.

2.3 Discrete Cosine Transform (DCT)

To explore more transform domain processing for signal denoising, Discrete Cosine Transform (DCT) has considered in this paper. According to the working principle, the way of DCT denoising is similar to the Fourier denoising, while both of them remove the noise frequency component after transformation. In practical, it is working in a segmented manner as the for loop in below step (**Figure 2**), while the N determine the range of data points, and the segment is transformed with N point transform as Y=HX below [1].

1. Set up the value of N
2. Apply DCT to the identity NxN matrix save as H
3. Set up X as input signal, and sample range as SR
4. Set up the length of the noise signal
5. If the reminder of the length and N >0
   for loop until the divided number of length and noise
   Y=HX
   Y(SR:end)=0 means the parameter only taken until number SR, and remove rest of them
   Inverse Y by using idct, and put it to a new variable YDCT
   End
   Redo the same process as above but only for the rest of the column as the reminder is not 0
   Else
   Do the process in for loop, as there is no reminder
   End
6. Display YDCT

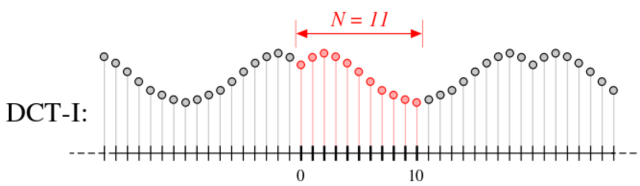
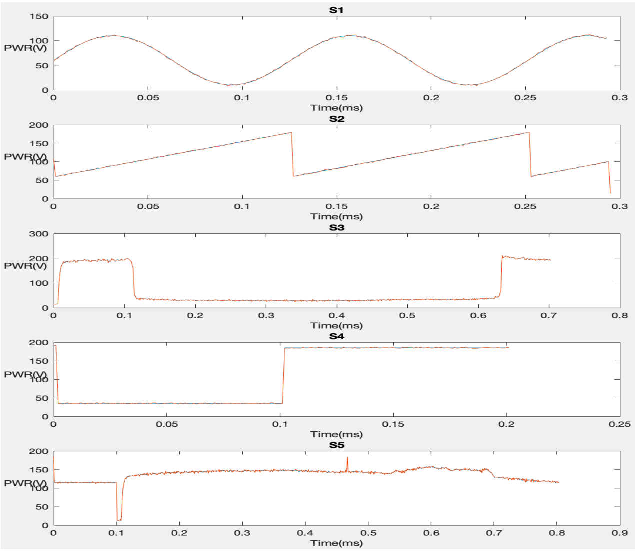


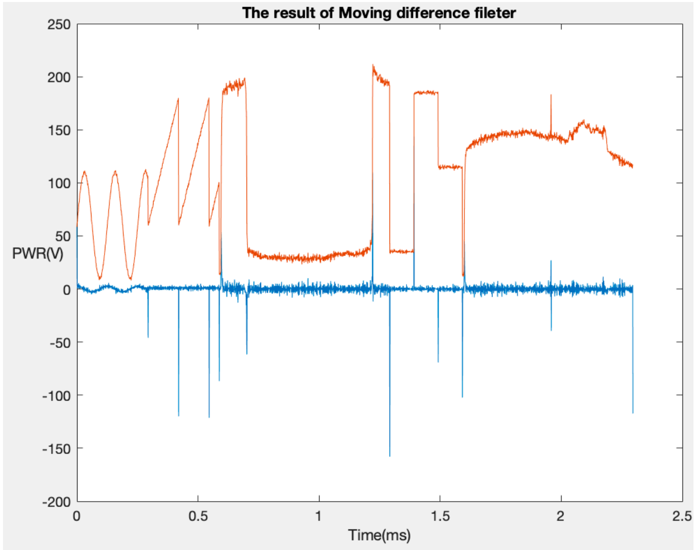
Figure 2. working principle of DCT

III. Analysis and Results

3.1 Task 1

From time domain perspective (**Figure1(b)**), the noise fluctuated the signal widely, especially at the red circle marked range. Due to the signal consists of different type of signal with different frequencies, the signal can be roughly divided into 5 different segments as shown in **Figure 3 (a),** also based on the result of Moving difference filter, whereas **Figure 3 (b)** shows five large magnitude. Therefore, the type of signal can be classified as Sine wave (s1), Triangular wave (s2), Square wave (s4) and irregular wave (s3, s5). After this, the analysis of the signal is able to proceed, and obtain a better value of Peak Signal to noise ratio (PSNR) at the end.



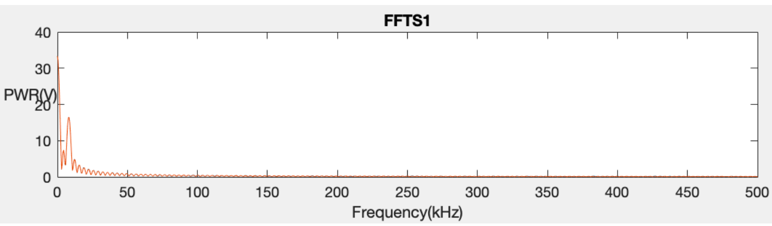


(a) Five different segment (S1~S2) **(b) the result of Moving difference filter**

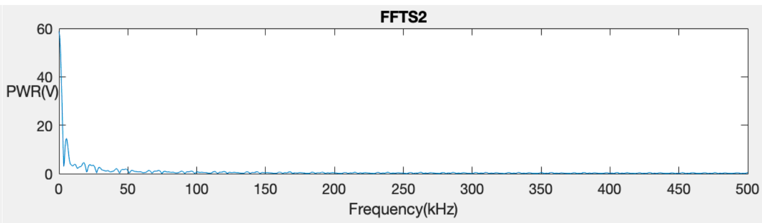
Figure 3. the way of dividing the signal with noise into segment

For the first segment, in time domain the time period of the sine wave was 0.126 second, which is roughly equivalent to 7.9 Hz. **Figure 6** shows the frequency domain of the signal peak is located at 8.301 Hz. Hence, the value obtained from the frequency domain is close to the calculated value, which means the code was accurate and applicable to any signal. In addition, (N/2) +1 points cover the frequency range from 0 to the half of sampling frequency, where N represents the window size. Therefore, At 1000Hz speech sample with 2048 point FFT resulting in 0 to 500 Hz is covered by 1025 spectral points [2].

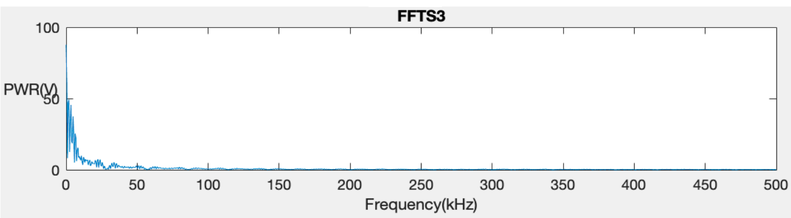
The highest peak of the second segment (s2) is located at 14.58 kHz, then 5.3, 3.4, 12.7 and 1.95 (kHz) for s3, s4, s5 respectively. Due to s3 and s5 are irregular signal, the distance between the signal peak is close to each other. Therefore, it is harder to observe the majority of frequency compoment. In addition, **Figure 4** presents the signal peak in frequency domain is mostly located in 0 to 100 range. Hence, design a low pass filter might be useful for filtering out the noise signal. In addition, the PSNR value between the signal with noise and the original signal is 48.09 dB.



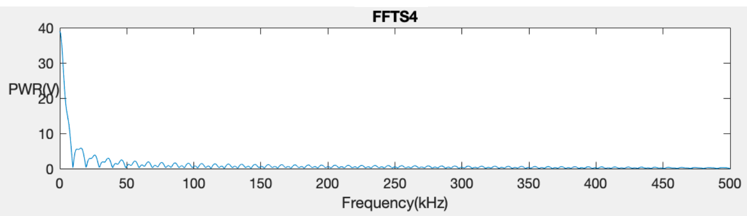
(a)



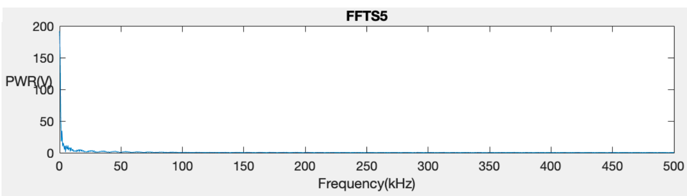
(b)



(c)



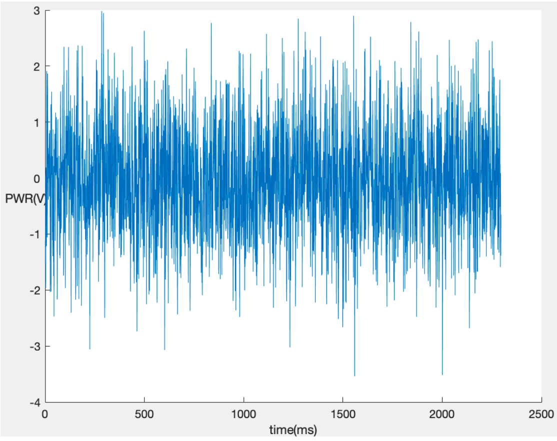
(d)



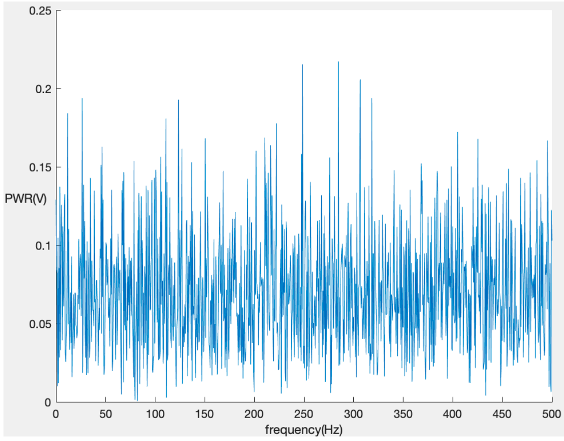
(e)

Figure 4. the frequency domain of five different segments (s1~s6)

3.2 Task 2



(a) the time domain of noise signal



(b) the frequency domain of noise signal

Figure 5. the noise signal

Figure 5 (b) shows the signal with noise in frequency domain. As the noise signal is thought-out all frequency range, we can assume the noise has influenced in the all frequencies. Based on the obtained information from **Task 1,** the threshold for filtering the noise can be set up. As the noise signal consists of 5 different types of format, the threshold should be designed individually for each segment. **Figure 4 (a)** shows that after the highest peak, the others Y values attenuate to 1.12, 1.024 and 0.71 respectively. Theoretically, the original frequency is more likely at the highest peak, so by filtering out other lower Y value, the signal is able to be recovered.

Signal	Performance	without threshold	mean(noiseFFT)	mean(signalFFT)	first wave trough	second wave trough	third wave trough
S1	Threshold (v)	0	0.0013	0.45	1.12	1.02	0.71
	PSNR (dB)	48.08	20.85	20.91	20.97	20.96	20.96
S2	Threshold (v)	0	0.0025	0.53	2.18	1.9	0.96
	PSNR (dB)	47.97	17.93	17.22	16.22	16.3	16.91
S3	Threshold (v)	0	0.0085	0.25	5.067	4.3	2.74
	PSNR (dB)	48.18	17.26	17.18	14.68	14.68	14.68
S4	Threshold (v)	0	0.0017	0.6	0.47	1.49	0.25
	PSNR (dB)	47.74	12.068	11.8	11.88	11.35	12.03
S5	Threshold (v)	0	0.0085	0.39	3.54	1.92	0.63
	PSNR (dB)	48.11	21.57	21.55	21.12	21.43	21.7

Table1. the result of different threshold setting

Table1 shows the result of different threshold, presents in Peak Signal to noise ratio (PSNR). The best performance of S1 is while the threshold is set up at 1.12. For other segments is 0.0025 in S2, 0.0085 in S3, 0.0017 in S4 and 0.0085 in S5. Overall, taking average from the frequency domain of noise possesses a better PSNR result than others. However, the denoising performance is even worse than before denoising, the reason of this is probably because of the noise magnitude is too small, which between -3 to 3 as shown in **Figure 5(a)**. Therefore, the impact of the noise does not make a big different compare with the original signal. In addition, **Table 1** shows that taking the average of noise in frequency domain has a better results of PSNR, which has been marked with red color, and reconstructed as **Figure 6 (b).**

3.3 Task 3

The result of Discrete Cosine Transform (DCT) has been shown in the below table. Intend to obtain a comprehensive analysis, this part has been divided into two different testing way. Theoretically, the value of N plays a significant role in DCT, which determine the sample rate for transformation (like the resolution of television). Thus, the PSNR should be increasing along with the larger N value. However, **Table 2 (a)** shows a different pattern, while the PSNR is decreasing with the increasing N from the second to the fifth signal (**S2~S5**), exclude the first segment signal (**S1**). As the working principle of DCT is operating in a segment manner with N point transform, also from the analysis of **Task 1** demonstrates S1 is mostly consisted in low frequency, but for S2~S4 the frequency distribution is throughout the whole frequency range equally. Therefore, it is more likely the original signal frequency was removed by DCT during the denoising process for S2~S4. **Table 2 (b)** presents the relation between different threshold with constant N, the results shows the PSNR is increasing along with greater threshold value, but maximumly, it only able to achieve the same number as before denoising. As the result of the value of threshold is increasing, it collects more sample from the signal with noise, instead of filtering out the noise part. Therefore, if the N value is constant, the DCT barely is a compressing technique, which compresses the data and recover it at the end. To recover the original signal, the set up value with best PSNR performance has been marked with red color, and reconstructed as **Figure 8(a)**.

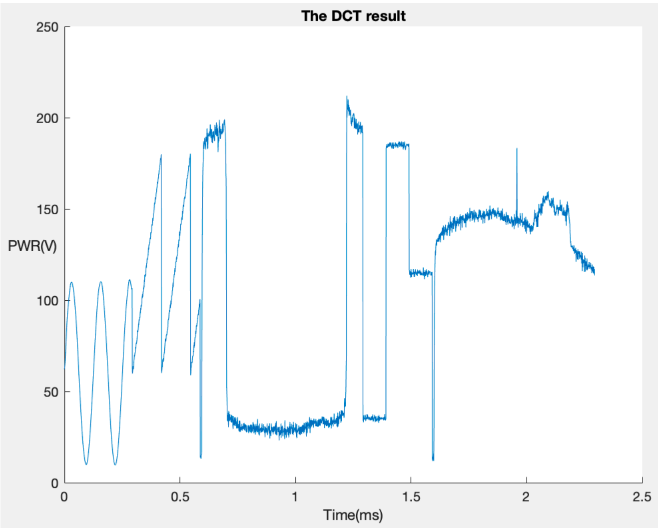
Signal	Performance	#1	#2	#3	#4	#5
S1	Threshold (v)	40	40	40	40	40
	N	10	60	150	220	296
	PSNR (dB)	48.08	49.51	53.02	53.35	54.65
S2	Threshold (v)	40	40	40	40	40
	N	10	60	150	220	296
	PSNR (dB)	47.97	37.57	31.06	31.62	28.34
S3	Threshold (v)	40	40	40	40	40
	N	10	290	430	570	703
	PSNR (dB)	48.18	34.05	31.36	29.88	26.52
S4	Threshold (v)	40	40	40	40	40
	N	10	80	120	155	202
	PSNR (dB)	47.74	32.32	30.11	28.19	26.63
S5	Threshold (v)	40	40	40	40	40
	N	200	350	520	650	804
	PSNR (dB)	48.11	33.47	31.03	30.2	29.46

(a) constant threshold with different N value

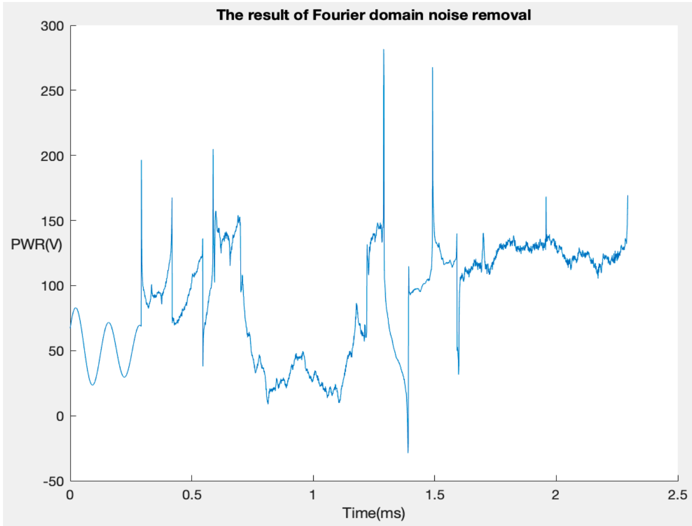
Signal	Performance	#1	#2	#3	#4	#5
S1	Threshold (v)	30	60	90	120	150
	N	150	150	150	150	150
	PSNR (dB)	54.22	51.66	49.85	48.82	48.09
S2	Threshold (v)	30	60	90	120	150
	N	150	150	150	150	150
	PSNR (dB)	29.61	33.22	36.31	40.24	47.87
S3	Threshold (v)	70	140	210	280	350
	N	350	350	350	350	350
	PSNR (dB)	34.94	39.11	41.47	44.85	48.14
S4	Threshold (v)	20	40	60	80	100
	N	100	100	100	100	100
	PSNR (dB)	25.96	31.92	32.72	37.27	47.79
S5	Threshold (v)	80	160	240	320	400
	N	400	400	400	400	400
	PSNR (dB)	36.63	38.98	41.24	44.33	48.1

(b) constant N with different threshold

Table 2. the result of different testing method in DCT



(a) the recoverd signal from Task 3



(b) the recovered signal form Task 2

Figure 6. the recoverd signal

IV. DICUSSION

Figure 6 presents the recovered signal from the test above (DCT and Fourier domain noise removal), the recovered signal by DCT possesses a better PSNR in **Table 3,** which had even exceeded the PSNR before denoising. In contrast, the threshold of Fourier domain noise removal was set up manually, it results in the threshold was not able to filter out the noise frequency component accurately. Meanwhile, it might accidentally erase the frequency components of original single, which make it even worse. Therefore, the result of PSNR only achieved 17.77 dB at the end, while it is even hard to recognize the signal of **Figure 6 (b).**

Denoising methods	PSNR (dB)
Fourier domain noise removal(Task 2)	17.77
DCT (Task 3)	48.51
Before denoising	48.09

Table 3. the PSNR result from Task 2 and Task 3

V. CONCLUSION

To conclude, this paper has presented the analysis of different denoising method, and compared the performance at the end. Even though the Fourier domain noise removal is not as practical as DCT. However, it is an easier way of getting knowledge about the working principle of frequency domain, while it has been widely used in analyzing a signal. Moreover, those knowledge enables us to design a customized filter, also understand what kind of filter will be suitable for a certain type of signal.

VI. REFERENCES

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[2] E. Dubois, "Frequency-domain methods for demosaicking of bayer-sampled color images," *IEEE Signal Process. Lett.*, 2005.