Computer Vision, Homework 2: Structure from Motion (SfM)

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3.5 Report

Problems:

3.1 Camera Pose from Essential Matrix (20%)

In [ ]:
```
'''
ESTIMATE_INITIAL_RT from the Essential Matrix, we can compute 4 initial
guesses of the relative RT between the two cameras
Arguments:
    E - the Essential Matrix between the two cameras
Returns:
    RT: A 4x3x4 tensor in which the 3x4 matrix RT[i,:,:] is one of the
        four possible transformations
'''
```

In [ ]:
```
def estimate_initial_RT(E):
    U, sigma, V_transpose = np.linalg.svd(E)          # 將 essential matrix 做 SVD 分解為 U, sigma, V_transpose

    #Z = np.array([[0, 1, 0], [-1, 0, 0], [0, 0, 0]])
    W = np.array([[0, -1, 0], [1, 0, 0], [0, 0, 1]])  # 定義 W 矩陣

    Q1 = U.dot(W.dot(V_transpose))                    # Q1 = U W VT
    Q2 = U.dot(W.T.dot(V_transpose))                  # Q2 = U WT VT

    T1 = U[:, 2]                                      # T = u3
    T2 = -U[:, 2]                                     # T = -u3

    R1 = (np.linalg.det(Q1) * Q1).T                   # R1 = det(Q1) · Q1
    R2 = (np.linalg.det(Q2) * Q2).T                   # R2 = det(Q2) · Q2

    RT = np.array([                                   # vstack rotation (R) and translation (T)
        np.vstack([R1, T1]).T,
        np.vstack([R1, T2]).T,
        np.vstack([R2, T1]).T,
        np.vstack([R2, T2]).T
    ])

    return RT                                         # RT
```

estimate_initial_RT(E)將essential matrix做SVD，可以得到U、Σ和V^T，再根據所給的W矩陣和$Q = UWV^T$ or $Q = UW^TV^T$得到兩個Q，接著以$R = det(Q) \cdot Q$就能得到兩個rotation (R)，再和translation (T)，也就是 $T = u_3$和$T = -u_3$合併得到所求的四個RT。

這是用來評估相機的位置，即旋轉的角度和位移。

3.2 Linear 3D Points Estimation (20%)

In [ ]:
```
'''
LINEAR_ESTIMATE_3D_POINT given a corresponding points in different images,
compute the 3D point is the best linear estimate
Arguments:
    image_points - the measured points in each of the M images (Mx2 matrix)
    camera_matrices - the camera projective matrices (Mx3x4 tensor)
Returns:
    point_3d - the 3D point
'''
```

In [ ]:
```python
def linear_estimate_3d_point(image_points, camera_matrices):
    pi = image_points.copy()                              # pi = image_points
    Mi = camera_matrices.copy()                           # Mi = camera_matrices

    A1 = (pi[:, 1] * Mi[:, 2, :].T).T - Mi[:, 1, :]       # v_n M_n^3 - M_n^2
    A2 = Mi[:, 0, :] - (pi[:, 0] * Mi[:, 2, :].T).T       # M_n^1 - u_n M_n^3
    A = np.vstack([A1, A2])

    U, sigma, V_transpose = np.linalg.svd(A)              # solve P by SVD
    point_3d = V_transpose[3, :].copy()
    point_3d /= point_3d[-1]
    return point_3d[:-1]
```

透過p_i和M_i以外積推導$p_i \times M_i P = 0$，得到$\begin{bmatrix} v_1 M_1{}^3 - M_1{}^2 \\ M_1{}^1 - u_1 M_1{}^3 \\ . \\ . \\ . \\ v_n M_n{}^3 - M_n{}^2 \\ M_n{}^1 - u_n M_n{}^3 \end{bmatrix} \cdot P = 0$。

再對其左邊的矩陣做SVC解P得到U、Σ和V^T，將V^T的最後一個row除以V^T的最後一個row的最後一個數即為3D點的位置。

3.3 Non-Linear 3D Points Estimation (20%)

In [ ]:
```python
'''
JACOBIAN given a 3D point and its corresponding points in the image
planes, compute the reprojection error vector and associated Jacobian
Arguments:
    point_3d - the 3D point corresponding to points in the image
    camera_matrices - the camera projective matrices (Mx3x4 tensor)
Returns:
    jacobian - the 2Mx3 Jacobian matrix
'''
```

In [ ]:
```python
def jacobian(point_3d, camera_matrices):
    P = np.hstack([point_3d.copy(), 1])                   # P = [point_3d\\ 1]
    Mi = camera_matrices.copy()                           # Mi = camera_matrices

    numerator = (np.matmul(Mi[:, 2, :], P)) ** 2          # jacobian 所有偏微分的共同分母

    Jx1 = Mi[:, 0, 0] * np.matmul(Mi[:, 2, [1, 2, 3]], P[[1, 2, 3]]) - Mi[:, 2, 0] * \
        np.matmul(Mi[:, 0, [1, 2, 3]], P[[1, 2, 3]])
    # jacobian 所有偏微分中每一個 partial X 元素的第一項
    Jx2 = Mi[:, 0, 1] * np.matmul(Mi[:, 2, [0, 2, 3]], P[[0, 2, 3]]) - Mi[:, 2, 1] * \
        np.matmul(Mi[:, 0, [0, 2, 3]], P[[0, 2, 3]])
    # jacobian 所有偏微分中每一個 partial Y 元素的第一項
    Jx3 = Mi[:, 0, 2] * np.matmul(Mi[:, 2, [0, 1, 3]], P[[0, 1, 3]]) - Mi[:, 2, 2] * \
        np.matmul(Mi[:, 0, [0, 1, 3]], P[[0, 1, 3]])
    # jacobian 所有偏微分中每一個 partial Z 元素的第一項

    Jy1 = Mi[:, 1, 0] * np.matmul(Mi[:, 2, [1, 2, 3]], P[[1, 2, 3]]) - Mi[:, 2, 0] * \
        np.matmul(Mi[:, 1, [1, 2, 3]], P[[1, 2, 3]])
    # jacobian 所有偏微分中每一個 partial X 元素的第二項
    Jy2 = Mi[:, 1, 1] * np.matmul(Mi[:, 2, [0, 2, 3]], P[[0, 2, 3]]) - Mi[:, 2, 1] * \
        np.matmul(Mi[:, 1, [0, 2, 3]], P[[0, 2, 3]])
    # jacobian 所有偏微分中每一個 partial Y 元素的第二項
    Jy3 = Mi[:, 1, 2] * np.matmul(Mi[:, 2, [0, 1, 3]], P[[0, 1, 3]]) - Mi[:, 2, 2] * \
        np.matmul(Mi[:, 1, [0, 1, 3]], P[[0, 1, 3]])
    # jacobian 所有偏微分中每一個 partial Z 元素的第二項

    Jx = np.vstack([[Jx1], [Jx2], [Jx3]])                 # 組合 jacobian 所有偏微分的第一項
    Jy = np.vstack([[Jy1], [Jy2], [Jy3]])                 # 組合 jacobian 所有偏微分的第二項
    Jx = np.divide(Jx, numerator).T                       # 同除共同分母
    Jy = np.divide(Jy, numerator).T                       # 同除共同分母

    jacobian = np.zeros((2 * Jx.shape[0], Jy.shape[1])) # 組合 jacobian 矩陣
    jacobian[0::2, :] = Jx
    jacobian[1::2, :] = Jy

    return jacobian
```

為獲得jacobian矩陣，需先推導jacobian中所有偏微分元素的組合順序，以$\frac{\partial e_i}{\partial X_i}$為例，

$$\frac{\partial e_i}{\partial X_i} = \frac{\partial e_i}{\partial p_i'} \times \frac{\partial p_i'}{\partial X_i} = \frac{\partial (p_i' - p_i)}{\partial p_i'} \times \frac{\partial p_i'}{\partial X_i} = 1 \times \frac{\partial p_i'}{\partial X_i}$$

也就是$\frac{\partial e_i}{\partial X_i}$其實等於$\frac{\partial p_i'}{\partial X_i}$。

$$J = \begin{bmatrix} \frac{\partial p'_1}{\partial X_1} & \frac{\partial p'_1}{\partial Y_1} & \frac{\partial p'_1}{\partial Z_1} \\ . & & \\ . & & \\ . & & \\ \frac{\partial p'_{2n}}{\partial X_n} & \frac{\partial p'_{2n}}{\partial Y_n} & \frac{\partial p'_{2n}}{\partial Z_n} \end{bmatrix}$$

接著計算$\frac{\partial p'_i}{\partial X_i}$， $M_i = \begin{bmatrix} m_{i00} & m_{i01} & m_{i02} & m_{i03} \\ m_{i10} & m_{i11} & m_{i12} & m_{i13} \\ m_{i20} & m_{i21} & m_{i22} & m_{i23} \end{bmatrix}$， $P = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$

以公式得知

$$y_i = M_i P = \begin{bmatrix} m_{i00} & m_{i01} & m_{i02} & m_{i03} \\ m_{i10} & m_{i11} & m_{i12} & m_{i13} \\ m_{i20} & m_{i21} & m_{i22} & m_{i23} \end{bmatrix} \times \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} m_{i00}X + m_{i01}Y + m_{i02}Z + m_{i03} \\ m_{i10}X + m_{i11}Y + m_{i12}Z + m_{i13} \\ m_{i20}X + m_{i21}Y + m_{i22}Z + m_{i23} \end{bmatrix}$$

再以公式得知

$$p'_i = \frac{1}{y_{i3}} \begin{bmatrix} y_{i1} \\ y_{i2} \end{bmatrix} = \begin{bmatrix} \frac{m_{i00}X+m_{i01}Y+m_{i02}Z+m_{i03}}{m_{i20}X+m_{i21}Y+m_{i22}Z+m_{i23}} \\ \frac{m_{i10}X+m_{i11}Y+m_{i12}Z+m_{i13}}{m_{i20}X+m_{i21}Y+m_{i22}Z+m_{i23}} \end{bmatrix}$$

所以

$$\frac{\partial p'_i}{\partial X} = \begin{bmatrix} \frac{m_{i00}(m_{i20}X+m_{i21}Y+m_{i22}Z+m_{i23}) - m_{i20}(m_{i00}X+m_{i01}Y+m_{i02}Z+m_{i03})}{(m_{i20}X+m_{i21}Y+m_{i22}Z+m_{i23})^2} \\ \frac{m_{i10}(m_{i20}X+m_{i21}Y+m_{i22}Z+m_{i23}) - m_{i20}(m_{i10}X+m_{i11}Y+m_{i12}Z+m_{i13})}{(m_{i20}X+m_{i21}Y+m_{i22}Z+m_{i23})^2} \end{bmatrix}$$
$$\frac{\partial p'_i}{\partial Y} = \begin{bmatrix} \frac{m_{i01}(m_{i20}X+m_{i21}Y+m_{i22}Z+m_{i23}) - m_{i21}(m_{i00}X+m_{i01}Y+m_{i02}Z+m_{i03})}{(m_{i20}X+m_{i21}Y+m_{i22}Z+m_{i23})^2} \\ \frac{m_{i11}(m_{i20}X+m_{i21}Y+m_{i22}Z+m_{i23}) - m_{i21}(m_{i10}X+m_{i11}Y+m_{i12}Z+m_{i13})}{(m_{i20}X+m_{i21}Y+m_{i22}Z+m_{i23})^2} \end{bmatrix}$$
$$\frac{\partial p'_i}{\partial Z} = \begin{bmatrix} \frac{m_{i02}(m_{i20}X+m_{i21}Y+m_{i22}Z+m_{i23}) - m_{i22}(m_{i00}X+m_{i01}Y+m_{i02}Z+m_{i03})}{(m_{i20}X+m_{i21}Y+m_{i22}Z+m_{i23})^2} \\ \frac{m_{i12}(m_{i20}X+m_{i21}Y+m_{i22}Z+m_{i23}) - m_{i22}(m_{i10}X+m_{i11}Y+m_{i12}Z+m_{i13})}{(m_{i20}X+m_{i21}Y+m_{i22}Z+m_{i23})^2} \end{bmatrix}$$

$(m_{i20}X + m_{i21}Y + m_{i22}Z + m_{i23})^2$即為共同分母，將此些算式組合，即為jacobian (J)。

In [ ]:
```
'''
REPROJECTION_ERROR given a 3D point and its corresponding points in the image
planes, compute the reprojection error vector and associated Jacobian
Arguments:
    point_3d - the 3D point corresponding to points in the image
    image_points - the measured points in each of the M images (Mx2 matrix)
    camera_matrices - the camera projective matrices (Mx3x4 tensor)
Returns:
    error - the 2M reprojection error vector
'''
```

In [ ]:
```
def reprojection_error(point_3d, image_points, camera_matrices):
    pi = image_points.copy()                          # pi = image_points
    Mi = camera_matrices.copy()                       # Mi = camera_matrices
    P = np.hstack([point_3d.copy(), 1])               # P = [point_3d\\ 1]

    y = np.matmul(Mi, P)                              # y = M_i P
    y = y.T
    pi_prime = y / y[-1, :]                           # pi_prime = 1/y_3 [y_1\\ y_2]

    ei = (pi_prime[:-1, :].T - pi).reshape(2 * pi.shape[0], )       # ei = pi_prime - pi
    return ei
```

根據公式$y = M_i P$計算出每個y_i，再以$p'_i = \frac{1}{y_{i3}} \begin{bmatrix} y_{i1} \\ y_{i2} \end{bmatrix}$計算出$p'_i$，最後$e_i = p'_i - p_i$。

In [ ]:
```
'''
NONLINEAR_ESTIMATE_3D_POINT given a corresponding points in different images,
compute the 3D point that iteratively updates the points
Arguments:
    image_points - the measured points in each of the M images (Mx2 matrix)
    camera_matrices - the camera projective matrices (Mx3x4 tensor)
Returns:
    point_3d - the 3D point
'''
```

In [ ]:
```
def nonlinear_estimate_3d_point(image_points, camera_matrices):
    pi = image_points.copy()                              # pi = image_points
    Mi = camera_matrices.copy()                           # Mi = camera_matrices

    iterations = 10                                       # run the optimization for 10 iterations

    estimated_3d_point = linear_estimate_3d_point(pi, Mi)
    # linear_estimate_3d_point()

    for i in range(iterations):
        J = jacobian(estimated_3d_point, Mi)
        # jacobian()
        reprojection_error_ = reprojection_error(estimated_3d_point, pi, Mi)
        # reprojection error()
        estimated_3d_point = estimated_3d_point - \
            np.matmul(np.matmul(np.linalg.inv(J.T.dot(J)), J.T), reprojection_error_)
        # P = P - (J^T J)^-1 J^T e

    return estimated_3d_point
```

先以linear_estimate_3d_point(pi, Mi)作為P的initial guess，再以公式$P = P - (J^T J)^{-1} J^T e$ 迭代10次。

3.4 Decide the Correct RT (20%)

In [ ]:
```
'''
ESTIMATE_RT_FROM_E from the Essential Matrix, we can compute  the relative RT
between the two cameras
Arguments:
    E - the Essential Matrix between the two cameras
    image_points - N measured points in each of the M images (NxMx2 matrix)
    K - the intrinsic camera matrix
Returns:
    RT: The 3x4 matrix which gives the rotation and translation between the
        two cameras
'''
```

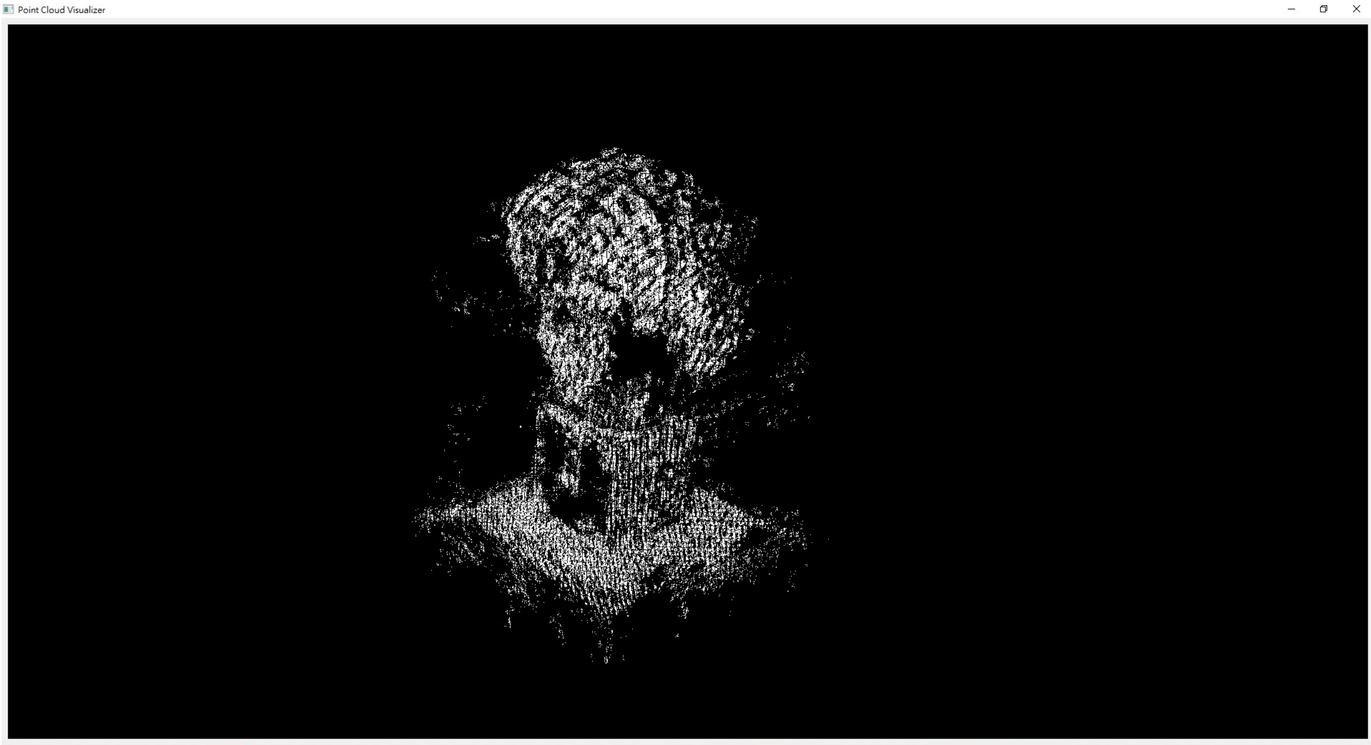
In [ ]:
```
def estimate_RT_from_E(E, image_points, K):
    estimate_initial_RT_ = estimate_initial_RT(E)        # 呼叫四種可能的 RT

    correct_RT_temp = [0, 0, 0, 0]                        # 建立投票矩陣

    M1 = K.dot(np.append(np.eye(3), np.zeros((3, 1)), axis=1))      # 第一個相機的相機矩陣

    for i in range(image_points.shape[0]):
        for j in range(estimate_initial_RT_.shape[0]):
            M2 = K.dot(estimate_initial_RT_[j])                     # 透過四種 RT 轉換的第二個相機的相機矩陣
            M = np.array((M1, M2))                                  # 合併兩種相機矩陣

            X = linear_estimate_3d_point(image_points[i], M)        # 估計第一個3D點
            X2 = estimate_initial_RT_[j].dot(np.append(X, 1).T)     # 透過四種 RT 轉換的第二個3D點

            if X2[2] > 0 and X[2] > 0:                              # 檢查兩個3D點的Z軸是否都為正
                correct_RT_temp[j] += 1                             # 在四種 RT 的投票矩陣投票

    RT = estimate_initial_RT_[np.argmax(correct_RT_temp)]           # 得到真正的RT

    return RT
```

先呼叫estimate_initial_RT(E)得到四種可能的RT，並建立一個針對各RT的投票矩陣，透過四種RT的轉換，得到真正的RT。

3.5 Result

In [ ]:

```
--------------------------------------------------------------------------------
Part A: Check your matrices against the example R,T
--------------------------------------------------------------------------------
Example RT:
 [[ 0.9736 -0.0988 -0.2056  0.9994]
  [ 0.1019  0.9948  0.0045 -0.0089]
  [ 0.2041 -0.0254  0.9786  0.0331]]

Estimated RT:
 [[[ 0.98305251 -0.11787055 -0.14040758  0.99941228]
  [-0.11925737 -0.99286228 -0.00147453 -0.00886961]
  [-0.13923158  0.01819418 -0.99009269  0.03311219]]

 [[ 0.98305251 -0.11787055 -0.14040758 -0.99941228]
  [-0.11925737 -0.99286228 -0.00147453  0.00886961]
  [-0.13923158  0.01819418 -0.99009269 -0.03311219]]

 [[ 0.97364135 -0.09878708 -0.20558119  0.99941228]
  [ 0.10189204  0.99478508  0.00454512 -0.00886961]
  [ 0.2040601  -0.02537241  0.97862951  0.03311219]]

 [[ 0.97364135 -0.09878708 -0.20558119 -0.99941228]
  [ 0.10189204  0.99478508  0.00454512  0.00886961]
  [ 0.2040601  -0.02537241  0.97862951 -0.03311219]]]
--------------------------------------------------------------------------------
Part B: Check that the difference from expected point
is near zero
--------------------------------------------------------------------------------
Difference:  0.0029243053036762667
--------------------------------------------------------------------------------
Part C: Check that the difference from expected error/Jacobian
is near zero
--------------------------------------------------------------------------------
Error Difference:  8.301300414891369e-07
Jacobian Difference:  1.8171196103367038e-08
--------------------------------------------------------------------------------
Part D: Check that the reprojection error from nonlinear method
is lower than linear method
--------------------------------------------------------------------------------
Linear method error: 98.73542356894183
Nonlinear method error: 95.59481784846034
--------------------------------------------------------------------------------
Part E: Check your matrix against the example R,T
--------------------------------------------------------------------------------
Example RT:
 [[ 0.9736 -0.0988 -0.2056  0.9994]
 [ 0.1019  0.9948  0.0045 -0.0089]
 [ 0.2041 -0.0254  0.9786  0.0331]]

Estimated RT:
 [[ 0.97364135 -0.09878708 -0.20558119  0.99941228]
 [ 0.10189204  0.99478508  0.00454512 -0.00886961]
 [ 0.2040601  -0.02537241  0.97862951  0.03311219]]
--------------------------------------------------------------------------------
Part F: Run the entire SFM pipeline
--------------------------------------------------------------------------------
Save results to results.npy!
```



心得:

相較於上次的作業，這次的作業可以說是難上許多，但其主題也很有趣，就像自己在建造一個3D掃描儀一樣，且經過這次的作業，也讓我對於相機的成像、translation和Rotation之類的原理更加了解。