

# Numerical Optimization: Final presentation

A Comparative Study of CO2 Emission Forecasting in the Gulf Countries  
Using Autoregressive Integrated Moving Average,  
Artificial Neural Network,  
and Holt-Winters Exponential Smoothing Models

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### Research Article

## A Comparative Study of CO<sub>2</sub> Emission Forecasting in the Gulf Countries Using Autoregressive Integrated Moving Average, Artificial Neural Network, and Holt-Winters Exponential Smoothing Models

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Forecasting is the process of making predictions based on past and present data, with the most common method being trend analysis. Forecasting models are becoming increasingly critical in uncovering the intricate linkages between large amounts of imprecise data and uncontrollable variables. The main purpose of this article is to compare CO<sub>2</sub> emission forecasts in Gulf countries. In this study, the autoregressive integrated moving average (ARIMA), artificial neural network (ANN), and Holt-Winters exponential smoothing (HWES) forecasting models are used to anticipate CO<sub>2</sub> emissions in the Gulf countries on an annual basis. This study attempts to predict time series data on CO<sub>2</sub> emissions in the Gulf countries using statistical tools. The current analysis relied on secondary data gathered from the United States Energy Information Administration (EIA). The study's findings show that the ARIMA (1,1,1), Holt-Winters exponential smoothing, ARIMA (0,1,2), and ARIMA (2,1,2) models do not outperform the artificial neural network model in estimating CO<sub>2</sub> emissions in the Gulf countries. This study gives information on the current state of CO<sub>2</sub> emission forecasts. This study will aid the researcher's understanding of CO<sub>2</sub> emissions forecasts. In addition, government agencies can use the findings of this study to develop strategic plans.

### 1. Introduction

This research aims to forecast CO<sub>2</sub> emissions in Gulf countries. Gulf countries have long dominated the oil and gas industry. They produce around 35 and 25 percent of the world's natural gas and crude oil, respectively, and are its major crude oil producers. CO<sub>2</sub> is the most significant greenhouse gas emitted by human activity. Carbon dioxide exists in the atmosphere naturally as part of the Earth's carbon cycle (the natural circulation of carbon among the atmosphere, oceans, soil, plants, and animals) [1].

According to Saudi government plans, multiple measures have been taken towards predicting the country's future. ARIMA (1,0,0), ARIMA (0,1,1), ARIMA (1,1,2), and ANN suitable models were used for predicting the total

revenue and expenditure of Saudi Arabia [2]. The expected growth of CO<sub>2</sub> emissions of China has suddenly increased throughout the selected period of the study [3]. The regression analyses had been employed for 25 countries, and the statistical analyses indicated that eleven countries had a significant trend [4]. The gray prediction method was used to forecast the future CO<sub>2</sub> emissions for the period of 2010–2021 in Taiwan, and the study showed that CO<sub>2</sub> emissions would increase over the next three years [5]. The CO<sub>2</sub> data of 1999–2009 had been used to predict the future trend by using gray method (GM) [6].

Recently, many studies have combined ARIMA, Holt-Winters exponential smoothing, and ANN methods for CO<sub>2</sub> emission predictions. Prediction of CO<sub>2</sub> emissions based on the time series data has been analyzed, compared, and

- 因應環保議題，此篇論文的研究目的是預測海灣國家的 $CO_2$ 排放，使用的預測方法是ARIMA(1,1,1)、ARIMA(1,1,2)、ARIMA(2,1,2)、ANN和Holt-Winters method[1]。
- 預測方法與預測目標的相關論文引用：
  - 沙烏地阿拉伯使用ARIMA(1,0,0)、ARIMA(0,1,1)、ARIMA(1,1,2)和ANN來預測國家的總收入和支出[2]。
  - 中國的 $CO_2$ 排放在研究進行期間突然增加[3]。
  - 對25個國家進行迴歸分析(Regression Analysis)，其中11個國家的 $CO_2$ 排放將有明顯升高的趨勢[4]。
  - 使用灰色預測方法(gray prediction method)對於臺灣在2010年到2012年的 $CO_2$ 排放量進行預測，結果表明未來三年的排放量將增加[5]。

自迴歸(Autoregressive, AR)模型，此模型假設同一變數 $X$ 的當期值等於一個或數個落後期的線性組合，再加上常數項和隨機誤差。

## AR model

$$X_{AR_t} = c + \sum_{i=1}^p \varphi_i X_{AR_{t-i}} + \varepsilon_t$$

其中

$X_{AR_t}$  自迴歸模型之預測值

$c$  常數項

$\varphi_i$  自迴歸模型之權重

$X_{AR_{t-i}}$  落後 $i$ 期之歷史資料

$\varepsilon_t$  隨機誤差、白雜訊

在定義上，自迴歸模型是從線性迴歸(Linear regression)發展而來，因是以同一變數進行預測(以 $X$ 預測 $X$ )，所以稱自迴歸。

移動平均(Moving Average, MA)模型，類似於AR模型，MA模型假設序列可以由同期與數個落後期的隨機項乘上不同的權重來組合。

## MA model

$$X_{MA_t} = \mu + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \dots + \theta_q \varepsilon_{t-q}$$

其中

$X_{MA_t}$  移動平均模型之預測值

$\mu$  平均值

$\theta_i$  移動平均模型之權重

$\varepsilon_t$  隨機誤差、白雜訊

$\varepsilon_{t-i}$  落後*i*期之模型誤差

與AR模型不同，MA模型總是平穩的。

# 研究方法-ARMA[8, 10]

自迴歸移動平均(Autoregressive moving average, ARMA)模型，由p個自迴歸項和q個移動平均項組合而成。

## ARMA model

$$(1 - \sum_{i=1}^p \varphi_i L^i) X_t = (1 - \sum_{i=1}^q \theta_i L^i) \varepsilon_t$$

其中

$X_t$  自迴歸移動平均模型之預測值

$\varphi_i$  自迴歸模型之權重

$\theta_i$  移動平均模型之權重

$\varepsilon_t$  隨機誤差、白雜訊

$L$  滯後算子(一階滯後算子代表落後一期，以此類推)

不過此模型僅可預測平均值和方差不發生明顯變化之平穩數據，但多數預測情形下資料未必是平穩的。

差分整合移動平均自迴歸

(Autoregressive Integrated Moving Average model, ARIMA)模型，  
為ARMA模型再乘上d階的差分項而成。

## 差分

一階差分：

$$\Delta x_t = x_t - x_{t-1} = x_t - Lx_t = (1 - L)x_t$$

二階差分：

$$\Delta^2 x_t = \Delta x_t - \Delta x_{t-1} = (1 - L)x_t - (1 - L)x_{t-1} = (1 - L)^2 x_t$$

d階差分：

$$\Delta^d x_t = (1 - L)^d x_t = w_t$$

差分可以使非穩定的序列轉為穩定序列，

「差分」一詞雖未出現在ARIMA的英文名稱中，  
卻是使時間序列得以平穩關鍵的步驟。

ARIMA模型由p個自迴歸項、d個差分項、q個移動平均項組合而成。

## ARIMA model

ARIMA(p,d,q)模型可以表示為

$$(1 - \sum_{i=1}^p \varphi_i L^i)(1 - L)^d X_t = (1 - \sum_{i=1}^q \theta_i L^i) \varepsilon_t$$

其中

$X_t$  差分整合移動平均自迴歸模型之預測值

$\varphi_i$  自迴歸模型之權重

$\theta_i$  移動平均模型之權重

$\varepsilon_t$  隨機誤差、白雜訊

$L$  滯後算子(一階滯後算子代表落後一期，以此類推)



# 研究方法-ARIMA[1]

此篇論文並無ARIMA(p,d,q)模型的推導過程，  
而是直接給予整理後的數學式如下：

## ARIMA model

ARIMA(1,1,1):

$$\hat{Y} = \varnothing_0 + Y_{t-1} + \varnothing_1(Y_{t-1} - Y_{t-2}) - \omega_1\mathcal{E}_{t-1}$$

ARIMA(1,1,2):

$$\hat{Y} = \varnothing_0 + Y_{t-1} + \varnothing_1(Y_{t-1} - Y_{t-2}) - \omega_1\mathcal{E}_{t-1} - \omega_2\mathcal{E}_{t-2}$$

ARIMA(2,1,2):

$$\hat{Y} = \varnothing_0 + Y_{t-1} + \varnothing_1(Y_{t-1} - Y_{t-2}) + \varnothing_2(Y_{t-2} - Y_{t-3}) - \omega_1\mathcal{E}_{t-1} - \omega_2\mathcal{E}_{t-2}$$

推導過程以ARIMA(1,1,2)為例：

$$(p, d, q) = (1, 1, 2)$$

$$(1 - \sum_{i=1}^p \varphi_i L^i)(1 - L)^d X_t = (1 - \sum_{i=1}^q \theta_i L^i) \varepsilon_t$$

$$\Rightarrow (1 - \varphi_1 L)(1 - L)X_t = (1 - \theta_1 L - \theta_2 L^2)\varepsilon_t$$

$$\Rightarrow X_t - LX_t - \varphi_1 LX_t + \varphi_1 L^2 X_t = \varepsilon_t - \theta_1 L\varepsilon_t - \theta_2 L^2 \varepsilon_t$$

$$\Rightarrow X_t = X_{t-1} + \varphi_1 X_{t-1} - \varphi_1 X_{t-2} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2}$$

$$= \varepsilon_t + X_{t-1} + \varphi_1 (X_{t-1} - X_{t-2}) - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2}$$

$$\Rightarrow \hat{Y} = \varnothing_0 + Y_{t-1} + \varnothing_1 (Y_{t-1} - Y_{t-2}) - \omega_1 \varepsilon_{t-1} - \omega_2 \varepsilon_{t-2}$$

# 研究方法-SARIMA[10]

季節性差分整合移動平均自迴歸

(Seasonal autoregressive integrated moving average, SARIMA)模型，  
不同於普通的ARIMA模型，

SARIMA模型額外加入了和季節性相關的週期參數，  
更適合用於負載預測。

## SARIMA model

$$\begin{aligned} (1 - \sum_{i=1}^p \varphi_i L^i)(1 - \sum_{i=1}^p \Phi_i L^{S*i})(1 - L)^d(1 - L)^{S*d} X_t \\ = (1 - \sum_{i=1}^q \theta_i L^i)(1 - \sum_{i=1}^q \Theta_i L^{S*i}) \mathcal{E}_t \end{aligned}$$

其中

$S$  週期性

# 研究方法-ARIMAX[10]

增加外生輸入差分整合移動平均自迴歸

(Autoregressive integrated moving average with exogenous input, ARIMAX)模型，

不同於普通的ARIMA模型，

ARIMAX模型額外加入了外生變數作為輸入，

因為ARIMA模型中只使用過去的歷史資料做預測，

但某些情形下，不可預測的外在變因也會影響結果。

## ARIMAX model

$$(1 - \sum_{i=1}^p \varphi_i L^i)(1 - L)^d X_t = (1 - \sum_{i=1}^q \theta_i L^i) \varepsilon_t + \beta \mu_t$$

其中

$\beta$  外生變數之權重

$\mu_t$  外生變數

# 研究方法-ARIMA python實作[10]

因ARIMA之參數設定較為困難，最佳模型尋找不易，  
所以實作上使用Auto ARIMA函式庫來尋找合適的參數組合：  
使用ADF test來求d項，p項和q項則是透過AIC來評斷優劣。

## Augmented Dickey Fuller(ADF) Test

ADF Test用來檢測時序是否穩定，  
若非穩定則可計算差分項d的階數致使序列平穩。

## 赤池訊息量準則(Akaike Information Criterion, AIC)

AIC是用來評估統計模型的複雜度和衡量模型擬合資料之優良性的一種標準，可以表示為：

$$AIC = 2(p + q + k + 1) - 2\log(L)$$

其中 $L$ 為概似函數，  
且當AR模型中的常數 $c \neq 0$ 時， $k = 1$ ； $c = 0$ 時， $k = 0$ 。  
透過最小化AIC，可以得到最佳的模型參數。

# 研究方法-ANN[10]

人工神經網路(Artificial Neural Network, ANN)利用不同的輸入資料去預測未來的資訊。其架構為許多「神經元」相互連接而成，彼此間以權重(weight)連接，並加上誤差項與經過激勵函數(activation function)成為新的輸出傳至下一個神經元，而整體的學習過程即是持續更新這些參數直至達到預期的預測值。

## ANN

ANN的更新式可由下列方程式表達：

$$A_i = g\left(\sum_{i=0}^n W_{ij} * a_i\right)$$

其中

$A_i$  神經網路的輸出

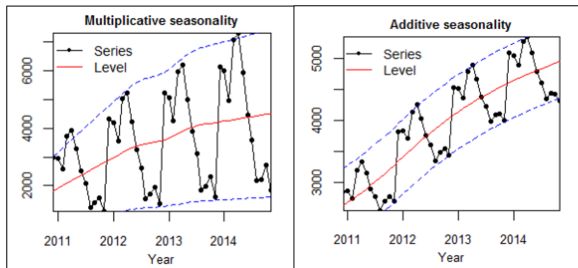
$g(*)$  激勵函數

$W_{ij}$  第*i*個神經元至第*j*個神經元的權重

$a_i$  神經元的輸入

# 研究方法-Holt-Winters method[10]

Holt-Winters 方法是從三次指數平滑延伸而來，分別對Level(水平)、Trend(趨勢)、Seasonality(季節性)做指數平滑的預測方程，應用於有季節性的時序預測。其又分為Multiplicative 和Additive 兩種預測公式，分別對應不同的seasonality 情況。



而此篇論文採用的是Multiplicative的預測公式。

# 研究方法-Holt-Winters(Multiplicative)[1, 10]

## Holt-Winters(Multiplicative)

$$Y_{t+1} = (L_t + hT_t)S_{t-M-h}$$

$$L_t = \alpha\left(\frac{Y_t}{S_{t-M}}\right) + (1 - \alpha)(L_{t-1} + T_{t-1})$$

$$T_t = \beta(L_t - L_{t-1}) + (1 - \beta)T_{t-1}$$

$$S_t = \gamma\left(\frac{Y_t}{L_t}\right) + (1 - \gamma)S_{t-M}$$

其中

$Y_t$	實際值
$Y_{t+1}$	預測值
$L_t$	Level(水平)
$T_t$	Trend(趨勢)
$S_t$	Sensonal(季節性)
$M$	週期
$h$	預測的範圍數(ex:1,2,3)
$\alpha, \beta, \gamma$	平滑參數[0,1]



## Holt-Winters(Additive)

$$Y_{t+1} = L_t + hT_t + S_{t-M-h}$$

$$L_t = \alpha(Y_t - S_{t-M}) + (1 - \alpha)(L_{t-1} + T_{t-1})$$

$$T_t = \beta(L_t - L_{t-1}) + (1 - \beta)T_{t-1}$$

$$S_t = \gamma(Y_t - L_t) + (1 - \gamma)S_{t-M}$$

其中

$Y_t$  實際值

$Y_{t+1}$  預測值

$L_t$  Level(水平)

$T_t$  Trend(趨勢)

$S_t$  Seasonal(季節性)

$M$  週期

$h$  預測的範圍數(ex:1,2,3)

$\alpha, \beta, \gamma$  平滑參數[0,1]

# 論文研究結果-誤差指標[1]

## 均方誤差(mean-square error, MSE)

$$MSE = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

其中

$y_i$  實際值

$\hat{y}_i$  預測值

## 均方根誤差(root-mean-square error, RMSE)

$$RMSE = \sqrt{MSE} = \sqrt{\frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{n}}$$

其中

$y_i$  實際值

$\hat{y}_i$  預測值

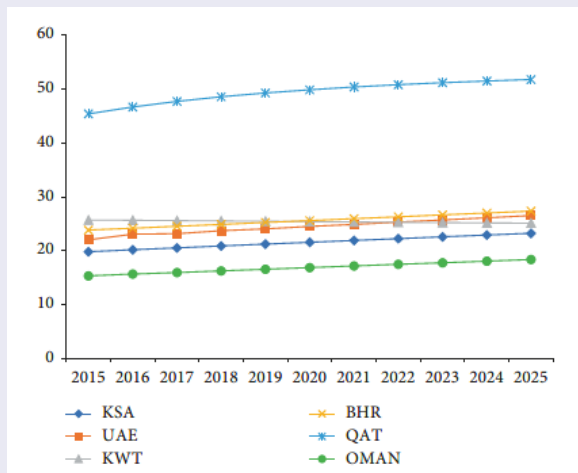
# 論文研究結果-預測結果[1]

## Predicted values of ARIMA (1,1,1)

	KSA	UAE	KWT	BHR	QAT	OMAN
2015	19.81	22.03	25.69	23.84	45.41	15.34
2016	20.18	23.07	25.65	24.19	46.68	15.68
2017	20.54	23.18	25.59	24.55	47.72	15.97
2018	20.89	23.73	25.53	24.90	48.57	16.27
2019	21.24	24.07	25.47	25.25	49.29	16.57
2020	21.58	24.51	25.41	25.60	49.88	16.87
2021	21.92	24.90	25.35	25.95	50.39	17.17
2022	22.25	25.32	25.29	26.30	50.82	17.48
2023	22.59	25.72	25.24	26.66	51.18	17.78
2024	22.92	26.13	25.18	27.01	51.50	18.08
2025	23.24	26.54	25.12	27.36	51.79	18.38
MSE	2.73	191.13	39.47	7.51	199.11	1.58
RMSE	1.65	13.82	6.28	2.74	14.11	1.26

# 論文研究結果-預測結果[1]

## Predicted values of ARIMA (1,1,1)



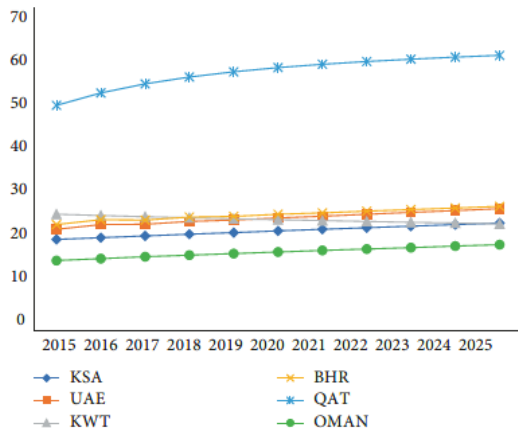
# 論文研究結果-預測結果[1]

## Predicted values of ARIMA (1,1,2)

	KSA	UAE	KWT	BHR	QAT	OMAN
2015	19.83	22.03	25.30	23.07	48.96	15.25
2016	20.21	23.06	25.02	24.13	51.64	15.67
2017	20.59	23.14	24.77	24.03	53.66	16.06
2018	20.96	23.72	24.53	24.67	55.12	16.42
2019	21.32	24.03	24.32	24.84	56.26	16.77
2020	21.68	24.49	24.11	25.30	57.15	17.10
2021	22.03	24.87	23.92	25.58	57.87	17.43
2022	22.37	25.29	23.74	25.98	58.47	17.75
2023	22.71	25.69	23.56	26.30	58.98	18.06
2024	23.05	26.10	23.38	26.67	59.43	18.37
2025	23.39	26.51	23.21	27.01	59.83	18.68
MSE	2.70	191.09	36.73	6.53	174.72	1.44
RMSE	1.64	13.82	6.06	2.56	13.22	1.20

# 論文研究結果-預測結果[1]

## Predicted values of ARIMA (1,1,2)

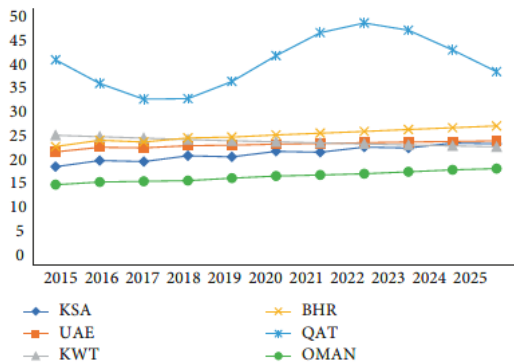


# 論文研究結果-預測結果[1]

## Predicted values of ARIMA (2,1,2)

	KSA	UAE	KWT	BHR	QAT	OMAN
2015	19.25	22.14	25.37	23.18	40.06	15.75
2016	20.44	23.02	25.08	24.42	35.45	16.28
2017	20.25	22.93	24.80	24.07	32.40	16.40
2018	21.37	23.33	24.55	24.85	32.52	16.56
2019	21.18	23.43	24.31	25.01	35.85	17.02
2020	22.24	23.65	24.09	25.42	40.89	17.44
2021	22.05	23.78	23.88	25.79	45.35	17.64
2022	23.07	23.94	23.68	26.11	47.23	17.87
2023	22.89	24.06	23.49	26.50	45.80	18.28
2024	23.87	24.18	23.30	26.83	41.96	18.64
2025	23.70	24.28	23.12	27.20	37.74	18.88
MSE	2.48	181.59	36.79	6.34	197.99	1.38
RMSE	1.58	13.48	6.07	2.52	14.07	1.18

## Predicted values of ARIMA (2,1,2)





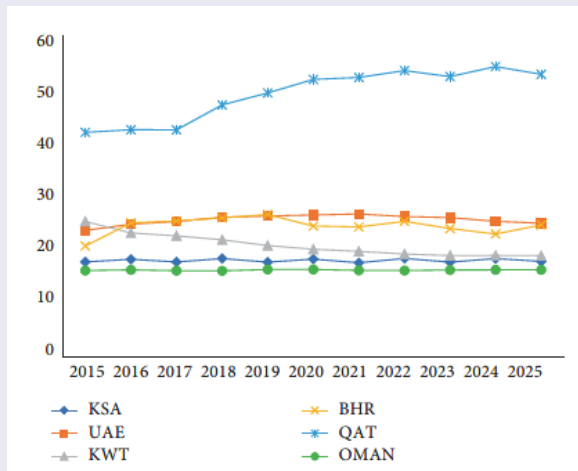
# 論文研究結果-預測結果[1]

Predicted values of ANN

	KSA	UAE	KWT	BHR	QAT	OMAN
2015	17.78	23.68	25.35	20.77	42.04	16.17
2016	18.29	24.85	23.21	25.09	42.52	16.32
2017	17.77	25.35	22.64	25.44	42.51	16.14
2018	18.42	26.16	21.94	26.01	47.17	16.12
2019	17.76	26.37	20.86	26.66	49.42	16.36
2020	18.30	26.57	20.15	24.50	51.96	16.37
2021	17.63	26.76	19.76	24.32	52.30	16.20
2022	18.43	26.29	19.28	25.38	53.58	16.18
2023	17.74	26.07	18.96	24.04	52.47	16.26
2024	18.41	25.40	19.00	23.00	54.34	16.30
2025	17.88	25.02	18.95	24.68	52.90	16.29
MSE	2.06	13.94	14.85	2.93	24.18	0.76
RMSE	1.44	3.73	3.85	1.71	4.92	0.87

# 論文研究結果-預測結果[1]

## Predicted values of ANN

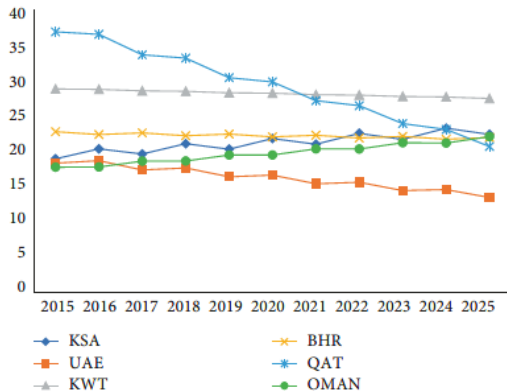


# 論文研究結果-預測結果[1]

Predicted values of Holt-Winters(Multiplicative)

	KSA	UAE	KWT	BHR	QAT	OMAN
2015	18.96	18.37	28.86	22.81	36.96	17.78
2016	20.39	18.70	28.81	22.40	36.60	17.82
2017	19.65	17.39	28.59	22.65	33.72	18.65
2018	21.12	17.68	28.54	22.24	33.23	18.67
2019	20.35	16.42	28.33	22.48	30.47	19.51
2020	21.85	16.66	28.27	22.08	29.87	19.51
2021	21.04	15.44	28.06	22.32	27.22	20.38
2022	22.58	15.64	28.00	21.91	26.50	20.36
2023	21.73	14.47	27.79	22.15	23.97	21.25
2024	23.31	14.62	27.73	21.75	23.14	21.21
2025	22.42	13.49	27.52	21.99	20.72	22.11
MSE	5.30	457.08	129.27	38.33	496.42	3.80
RMSE	2.30	21.38	11.37	6.19	22.28	1.95

## Predicted values of Holt-Winters(Multiplicative)

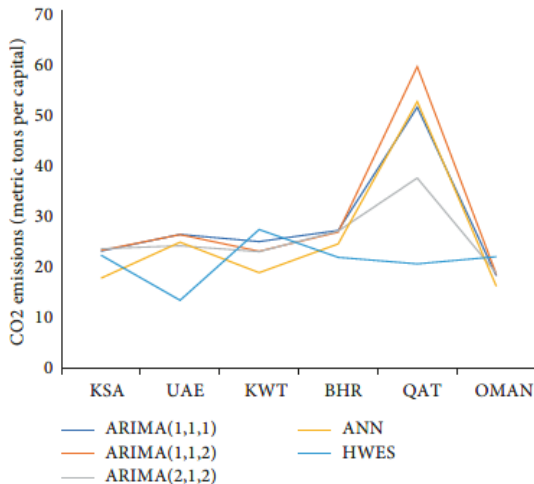


## Predicted values of $CO_2$ emissions for the year 2025

Countries	ARIMA(1,1,1)	ARIMA(1,1,2)	ARIMA(2,1,2)	ANN	HWES
KSA	23.24	23.39	23.70	17.88	22.42
UAE	26.54	26.51	24.28	25.02	13.49
KWT	25.12	23.21	23.12	18.95	27.52
BHR	27.36	27.01	27.20	24.68	21.99
QAT	51.79	59.83	37.74	52.90	20.72
OMAN	18.38	18.68	18.88	16.29	22.11

# 論文研究結果-預測結果[1]

## Predicted values of $CO_2$ emissions for the year 2025



# 論文研究結果-預測結果[1]

根據論文研究顯示，

綜合各方法的預測結果，除了KWT，其他國家的CO<sub>2</sub>排放都將增加，且各方法對於各國家依照準確度的排序為：

ARIMA(1,1,1)： OMAN→KSA→BHR→KWT→UAE→QAT；

ARIMA(1,1,2)： OMAN→KSA→BHR→KWT→QAT→UAE；

ARIMA(2,1,2)： OMAN→KSA→BHR→KWT→UAE→QAT；

ANN： OMAN→KSA→BHR→UAE→KWT→QAT；

HWES： OMAN→KSA→BHR→KWT→UAE→QAT，

其中又以ANN的預測誤差為最小。

## 國家代碼

KSA	沙烏地阿拉伯
UAE	阿拉伯聯合大公國
KWT	科威特
BHR	巴林
QAT	卡達
OMAN	阿曼

# 工研院計畫結果-誤差指標[10]

於計畫中，為顯示預測誤差結果，使用MAPE進行預測效能評估，其評估每一小時預測值與真實值之差距，另一評估使用MAPE(sum)為計算該日預測誤差對於全天之需量影響。

平均絕對百分比誤差(mean absolute percentage error, MAPE)

$$MAPE = \frac{100\%}{n} \sum_{t=1}^n \left| \frac{A_t - F_t}{A_t} \right|$$

$$MAPE(sum) = 100\% \left| \frac{\sum_{t=1}^n |A_t - F_t|}{\sum_{t=1}^n |A_t|} \right|$$

其中

$A_t$  實際值

$F_t$  預測值



# 工研院計畫結果-預測結果(工作日)(節錄)[10]

根據計畫結果顯示，  
在Area1，ANN的預測結果較準確，  
但在Area2和Area3，反而是Holt-Winters method較為準確。

## Predicted MAPEs of three methods for three areas

	SARIMAX	ANN	HWES
Area1-MAPE	17.19	14.08	15.30
Area1-MAPE(sum)	14.34	13.61	14.86
Area2-MAPE	43.23	12.90	7.31
Area2-MAPE(sum)	10.35	13.17	8.45
Area3-MAPE	67.14	44.51	22.34
Area3-MAPE(sum)	27.10	38.53	12.67

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