

## Problem Set 3: Differential Calculus

### CS&SS Math Camp 2021

1. Plot the function  $f(x) = 3x + 2$ .

(a) By eye, what is the derivative of  $f(x)$  at  $x = 4$ , i.e. what is the slope of the tangent line to the curve at  $x = 4$ ?

(b) Compute the derivative using the appropriate formula.

Compute the derivative:

2.  $f(x) = x^5$

3.  $f(x) = 10x - 30$

4.  $f(x) = 2x^4 + x^2$

5.  $f(x) = \tan(x)$

6.  $f(x) = e^{\sin(x)}$

7.  $f(x) = xe^x + \log(\sin(x))$

We can also have a function of a different variable besides  $x$ . This is just changing the variable name and you will see this a lot in your statistics methods classes.

8. Compute the derivative of  $g(\theta) = \theta^2 - \theta^4$

9. Find the global minimum of  $f(z) = z^2 - 6z + 8$

10. In the following function, treat  $x$  as a constant, and differentiate with respect to  $\mu$ :  
 $h(\mu) = x\mu^2$

Bonus In the following function of  $\mu$ , treat  $X_1, X_2, \dots, X_n$  (your data) as constants. Maximize the function over  $\mu$  (the population mean). In other words, find the value of  $\mu$ , expressed in terms of  $X_1, X_2, \dots, X_n$ , at which the function reaches its global maximum. HINT: review your rules of logarithms and exponents to simplify the expression before taking the derivative.

$$L(\mu) = \log\left(\frac{1}{\sqrt{2\pi}} e^{-\sum_{i=1}^n (X_i - \mu)^2}\right)$$