Problem Set 2: Matrix Algebra
CS&SS Math Camp 2021

$$A = \begin{pmatrix} 2 & 7 \\ 1 & 8 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 3 & 7 \\ 4 & 2 & 6 \end{pmatrix} \quad C = \begin{pmatrix} 4 & 5 & 2 \\ 3 & 7 & 1 \\ 2 & 1 & 6 \end{pmatrix} \quad D = \begin{pmatrix} 3 & 4 \\ 5 & 1 \end{pmatrix} \quad E = \begin{pmatrix} 1 & 4 \\ 9 & 3 \\ 7 & 2 \end{pmatrix}$$

Compute the following (if you can):

- B^t

- $A + D$

- $C + D$

- D^{-1}

- $C \cdot B$

- $B \cdot E$

- $E \cdot B$

- $A \cdot E$

- $A \cdot E^t$

Find the determinants. Do the inverses exist?

$$A = \begin{pmatrix} 4 & 1 \\ 3 & 6 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 3 \\ 5 & 10 \end{pmatrix} \qquad C = \begin{pmatrix} 3 & 7 \\ 9 & 21 \end{pmatrix}$$

Solve the following systems of equations using matrices.

- $4x + 9y = 31,\ 2x + 3y = 11$

- $5y - 2x = 10,\ 4x - 3y = 6$

You ask 5 people 4 questions about their high school academics and note their answers (i.e. number of math classes in high school, number of extra curricular activities). You also write down their college GPA. You want to find the relationship between the high school academics questions (independent variables, x) and their college GPA (dependent variable, y). i.e. you would like to find the β matrix in $y = X\beta$.

Person 1: $x_1 = 8$, $x_2 = 12$, $x_3 = 2$, $x_4 = 16$; $y = 3.6$
Person 2: $x_1 = 9$, $x_2 = 7$, $x_3 = 3$, $x_4 = 18$; $y = 3.3$
Person 3: $x_1 = 5$, $x_2 = 13$, $x_3 = 1$, $x_4 = 15$; $y = 3.9$
Person 4: $x_1 = 4$, $x_2 = 9$, $x_3 = 2$, $x_4 = 20$; $y = 3.7$
Person 5: $x_1 = 7$, $x_2 = 11$, $x_3 = 3$, $x_4 = 21$; $y = 3.8$

Write down y, X.