

# Estimation Calibration

```
In[1]:= clear["Global`*"];
```

Sensor Constants

```
In[19]:= gamma = 30.0; theta = 20.0; xmax = 315; ymax = 207;
```

Constant Matrices

```
In[20]:= T = {{Tan[gamma Degree], 0, 0}, {0, Tan[theta Degree], 0}, {0, 0, 1}};
```

```
In[21]:= DELTA = {{2 / xmax, 0}, {0, 2 / ymax}, {0, 0}};
```

```
In[23]:= ivec = {{1}, {1}, {-1}};
```

Sensor Position Vector

```
In[2]:= s[s1_, s2_, s3_] = {s1, s2, s3};
```

Sensor Frame z-axis Definition

```
In[3]:= z[s1_, s2_, s3_] = -s[s1, s2, s3] / Sqrt[s1^2 + s2^2 + s3^2];
```

Sensor Frame x-axis Definition

```
In[4]:= xtmp[s1_, s2_, s3_] = Cross[z[s1, s2, s3], {0, 0, 1}];
```

```
In[5]:= x[s1_, s2_, s3_] = xtmp[s1, s2, s3] /  
  Sqrt[xtmp[s1, s2, s3][[1]]^2 + xtmp[s1, s2, s3][[2]]^2 + xtmp[s1, s2, s3][[3]]^2];
```

Sensor Frame y-axis Definition

```
In[6]:= y[s1_, s2_, s3_] = Cross[z[s1, s2, s3], x[s1, s2, s3]];
```

Sensor Frame Definition Matrix

```
In[7]:= S[s1_, s2_, s3_] = {x[s1, s2, s3], y[s1, s2, s3], z[s1, s2, s3]};
```

Marker Position Vector

```
In[8]:= xhat[x1_, x2_, x3_] = Transpose[{{x1, x2, x3}}];
```

Defined A Vector

In[163]:= **A[s1\_, s2\_, s3\_, x1\_, x2\_, x3\_] = S[s1, s2, s3].xhat[x1, x2, x3]**

$$\text{Out[163]} = \left\{ \left\{ -\frac{s2 x1}{\sqrt{s1^2 + s2^2 + s3^2} \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} + \frac{s1 s3 x2}{(s1^2 + s2^2 + s3^2) \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} - \frac{s1 x3}{\sqrt{s1^2 + s2^2 + s3^2}} \right\}, \left\{ \frac{s1 x1}{\sqrt{s1^2 + s2^2 + s3^2} \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} + \frac{s2 s3 x2}{(s1^2 + s2^2 + s3^2) \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} - \frac{s2 x3}{\sqrt{s1^2 + s2^2 + s3^2}} \right\}, \left\{ -\frac{s1^2}{(s1^2 + s2^2 + s3^2) \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} - \frac{s2^2}{(s1^2 + s2^2 + s3^2) \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} \right\} x2 - \frac{s3 x3}{\sqrt{s1^2 + s2^2 + s3^2}} \right\}$$

Defined B Vector

In[164]:= **B[s1\_, s2\_, s3\_] = S[s1, s2, s3].s[s1, s2, s3]**

$$\text{Out[164]} = \left\{ -\frac{s1 s3}{\sqrt{s1^2 + s2^2 + s3^2}} + \frac{s1 s2 s3}{(s1^2 + s2^2 + s3^2) \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} - \frac{s1 s2}{\sqrt{s1^2 + s2^2 + s3^2} \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}}, -\frac{s2 s3}{\sqrt{s1^2 + s2^2 + s3^2}} + \frac{s2^2 s3}{(s1^2 + s2^2 + s3^2) \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} + \frac{s1^2}{\sqrt{s1^2 + s2^2 + s3^2} \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}}, -\frac{s3^2}{\sqrt{s1^2 + s2^2 + s3^2}} + s2 \left( -\frac{s1^2}{(s1^2 + s2^2 + s3^2) \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} - \frac{s2^2}{(s1^2 + s2^2 + s3^2) \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} \right) \right\}$$

Defined Scalar a

In[165]:= **a[s1\_, s2\_, s3\_, x1\_, x2\_, x3\_] = z[s1, s2, s3].(xhat[x1, x2, x3] - s[s1, s2, s3])**

$$\text{Out[165]} = \left\{ -\frac{s1 (-s1 + x1)}{\sqrt{s1^2 + s2^2 + s3^2}} - \frac{s2 (-s2 + x2)}{\sqrt{s1^2 + s2^2 + s3^2}} - \frac{s3 (-s3 + x3)}{\sqrt{s1^2 + s2^2 + s3^2}} \right\}$$

1

a

In[166]:= **ooa**[s1\_, s2\_, s3\_, x1\_, x2\_, x3\_] = 1 / a[s1, s2, s3, x1, x2, x3][[1]]

Out[166]= 
$$\frac{1}{-\frac{s1(-s1+x1)}{\sqrt{s1^2+s2^2+s3^2}} - \frac{s2(-s2+x2)}{\sqrt{s1^2+s2^2+s3^2}} - \frac{s3(-s3+x3)}{\sqrt{s1^2+s2^2+s3^2}}}$$

Partial Derivative  $\frac{\partial(1/a)}{\partial s}$

In[167]:= **dooads**[s1\_, s2\_, s3\_, x1\_, x2\_, x3\_] =  
 {{D[**ooa**[s1, s2, s3, x1, x2, x3][[1]], s1], D[**ooa**[s1, s2, s3, x1, x2, x3][[1]], s2],  
 D[**ooa**[s1, s2, s3, x1, x2, x3][[1]], s3]}}

Out[167]= 
$$\left\{ \left\{ \frac{s1}{\sqrt{s1^2+s2^2+s3^2}} + \frac{s1^2(-s1+x1)}{(s1^2+s2^2+s3^2)^{3/2}} - \frac{-s1+x1}{\sqrt{s1^2+s2^2+s3^2}} + \right. \right.$$

$$\frac{s1 s2(-s2+x2)}{(s1^2+s2^2+s3^2)^{3/2}} + \frac{s1 s3(-s3+x3)}{(s1^2+s2^2+s3^2)^{3/2}}, \frac{s2}{\sqrt{s1^2+s2^2+s3^2}} + \frac{s1 s2(-s1+x1)}{(s1^2+s2^2+s3^2)^{3/2}} +$$

$$\frac{s2^2(-s2+x2)}{(s1^2+s2^2+s3^2)^{3/2}} - \frac{-s2+x2}{\sqrt{s1^2+s2^2+s3^2}} + \frac{s2 s3(-s3+x3)}{(s1^2+s2^2+s3^2)^{3/2}}, \frac{s3}{\sqrt{s1^2+s2^2+s3^2}} +$$

$$\left. \left. \frac{s1 s3(-s1+x1)}{(s1^2+s2^2+s3^2)^{3/2}} + \frac{s2 s3(-s2+x2)}{(s1^2+s2^2+s3^2)^{3/2}} + \frac{s3^2(-s3+x3)}{(s1^2+s2^2+s3^2)^{3/2}} - \frac{-s3+x3}{\sqrt{s1^2+s2^2+s3^2}} \right\} \right\}$$

Complete Nonlinear Measurement Function h(s, x)

In[168]:= **yhat[s1\_, s2\_, s3\_, x1\_, x2\_, x3\_] =**

**Inverse[Transpose[DELTA].DELTA].Transpose[DELTA].(ooa[s1, s2, s3, x1, x2, x3] ×**  
**Inverse[T].(A[s1, s2, s3, x1, x2, x3] - B[s1, s2, s3]) + ivec)**

$$\text{Out[168]} = \left\{ \left\{ \frac{315}{2} \left( 1 + \left( 0. + 1.73205 \left( \frac{s1 s3}{\sqrt{s1^2 + s2^2 + s3^2}} - \frac{s1 s2 s3}{(s1^2 + s2^2 + s3^2) \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} + \right. \right. \right. \right.$$

$$\frac{s1 s2}{\sqrt{s1^2 + s2^2 + s3^2} \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} - \frac{s2 x1}{\sqrt{s1^2 + s2^2 + s3^2} \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} +$$

$$\left. \left. \frac{s1 s3 x2}{(s1^2 + s2^2 + s3^2) \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} - \frac{s1 x3}{\sqrt{s1^2 + s2^2 + s3^2}} \right) \right) /$$

$$\left( - \frac{s1 (-s1 + x1)}{\sqrt{s1^2 + s2^2 + s3^2}} - \frac{s2 (-s2 + x2)}{\sqrt{s1^2 + s2^2 + s3^2}} - \frac{s3 (-s3 + x3)}{\sqrt{s1^2 + s2^2 + s3^2}} \right) \left. \right\},$$

$$\left\{ \frac{207}{2} \left( 1 + \left( 0. + 2.74748 \left( \frac{s2 s3}{\sqrt{s1^2 + s2^2 + s3^2}} - \frac{s2^2 s3}{(s1^2 + s2^2 + s3^2) \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} - \right. \right. \right.$$

$$\frac{s1^2}{\sqrt{s1^2 + s2^2 + s3^2} \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} + \frac{s1 x1}{\sqrt{s1^2 + s2^2 + s3^2} \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} +$$

$$\left. \left. \frac{s2 s3 x2}{(s1^2 + s2^2 + s3^2) \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} - \frac{s2 x3}{\sqrt{s1^2 + s2^2 + s3^2}} \right) \right) /$$

$$\left( - \frac{s1 (-s1 + x1)}{\sqrt{s1^2 + s2^2 + s3^2}} - \frac{s2 (-s2 + x2)}{\sqrt{s1^2 + s2^2 + s3^2}} - \frac{s3 (-s3 + x3)}{\sqrt{s1^2 + s2^2 + s3^2}} \right) \left. \right\} \left. \right\}$$

Nonlinear Measurement Function - Excluding Constants

```
In[169]:= yhatpartial[s1_, s2_, s3_, x1_, x2_, x3_] =
  (ooa[s1, s2, s3, x1, x2, x3] (A[s1, s2, s3, x1, x2, x3] - B[s1, s2, s3]))
```

$$\text{Out[169]} = \left\{ \left\{ \left( \frac{s1 s3}{\sqrt{s1^2 + s2^2 + s3^2}} - \frac{s1 s2 s3}{(s1^2 + s2^2 + s3^2) \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} + \right. \right. \right.$$

$$\frac{s1 s2}{\sqrt{s1^2 + s2^2 + s3^2} \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} - \frac{s2 x1}{\sqrt{s1^2 + s2^2 + s3^2} \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} +$$

$$\left. \frac{s1 s3 x2}{(s1^2 + s2^2 + s3^2) \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} - \frac{s1 x3}{\sqrt{s1^2 + s2^2 + s3^2}} \right) /$$

$$\left( -\frac{s1 (-s1 + x1)}{\sqrt{s1^2 + s2^2 + s3^2}} - \frac{s2 (-s2 + x2)}{\sqrt{s1^2 + s2^2 + s3^2}} - \frac{s3 (-s3 + x3)}{\sqrt{s1^2 + s2^2 + s3^2}} \right),$$

$$\left\{ \left( \frac{s2 s3}{\sqrt{s1^2 + s2^2 + s3^2}} - \frac{s2^2 s3}{(s1^2 + s2^2 + s3^2) \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} - \right. \right.$$

$$\frac{s1^2}{\sqrt{s1^2 + s2^2 + s3^2} \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} + \frac{s1 x1}{\sqrt{s1^2 + s2^2 + s3^2} \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} +$$

$$\left. \frac{s2 s3 x2}{(s1^2 + s2^2 + s3^2) \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} - \frac{s2 x3}{\sqrt{s1^2 + s2^2 + s3^2}} \right) /$$

$$\left( -\frac{s1 (-s1 + x1)}{\sqrt{s1^2 + s2^2 + s3^2}} - \frac{s2 (-s2 + x2)}{\sqrt{s1^2 + s2^2 + s3^2}} - \frac{s3 (-s3 + x3)}{\sqrt{s1^2 + s2^2 + s3^2}} \right), \left\{ \frac{s3^2}{\sqrt{s1^2 + s2^2 + s3^2}} - \right.$$

$$s2 \left( -\frac{s1^2}{(s1^2 + s2^2 + s3^2) \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} - \frac{s2^2}{(s1^2 + s2^2 + s3^2) \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} \right) +$$

$$\left( -\frac{s1^2}{(s1^2 + s2^2 + s3^2) \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} - \frac{s2^2}{(s1^2 + s2^2 + s3^2) \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} \right) x2 -$$

$$\frac{s3 x3}{\sqrt{s1^2 + s2^2 + s3^2}} \left. \right) / \left( -\frac{s1 (-s1 + x1)}{\sqrt{s1^2 + s2^2 + s3^2}} - \frac{s2 (-s2 + x2)}{\sqrt{s1^2 + s2^2 + s3^2}} - \frac{s3 (-s3 + x3)}{\sqrt{s1^2 + s2^2 + s3^2}} \right) \Big\} \Big\}$$

# Jacobian $\frac{\partial h}{\partial s}$

Excluding Constants

```
In[170]:= Htildepart[s1_, s2_, s3_, x1_, x2_, x3_] =
  {{D[yhatpartial[s1, s2, s3, x1, x2, x3][[1]][[1]], s1],
    D[yhatpartial[s1, s2, s3, x1, x2, x3][[1]][[1]], s2],
    D[yhatpartial[s1, s2, s3, x1, x2, x3][[1]][[1]], s3]},
  {D[yhatpartial[s1, s2, s3, x1, x2, x3][[2]][[1]], s1],
    D[yhatpartial[s1, s2, s3, x1, x2, x3][[2]][[1]], s2],
    D[yhatpartial[s1, s2, s3, x1, x2, x3][[2]][[1]], s3]},
  {D[yhatpartial[s1, s2, s3, x1, x2, x3][[3]][[1]], s1],
    D[yhatpartial[s1, s2, s3, x1, x2, x3][[3]][[1]], s2],
    D[yhatpartial[s1, s2, s3, x1, x2, x3][[3]][[1]], s3]}}
```

$$\text{Out[170]} = \left\{ \left\{ -\frac{1}{\left( -\frac{s1(-s1+x1)}{\sqrt{s1^2+s2^2+s3^2}} - \frac{s2(-s2+x2)}{\sqrt{s1^2+s2^2+s3^2}} - \frac{s3(-s3+x3)}{\sqrt{s1^2+s2^2+s3^2}} \right)^2} \right. \right.$$

$$\left. \left( \frac{s1 s3}{\sqrt{s1^2+s2^2+s3^2}} - \frac{s1 s2 s3}{(s1^2+s2^2+s3^2) \sqrt{\frac{s1^2}{s1^2+s2^2+s3^2} + \frac{s2^2}{s1^2+s2^2+s3^2}}} + \right. \right.$$

$$\left. \frac{s1 s2}{\sqrt{s1^2+s2^2+s3^2} \sqrt{\frac{s1^2}{s1^2+s2^2+s3^2} + \frac{s2^2}{s1^2+s2^2+s3^2}}} - \frac{s2 x1}{\sqrt{s1^2+s2^2+s3^2} \sqrt{\frac{s1^2}{s1^2+s2^2+s3^2} + \frac{s2^2}{s1^2+s2^2+s3^2}}} + \right.$$

$$\left. \frac{s1 s3 x2}{(s1^2+s2^2+s3^2) \sqrt{\frac{s1^2}{s1^2+s2^2+s3^2} + \frac{s2^2}{s1^2+s2^2+s3^2}}} - \frac{s1 x3}{\sqrt{s1^2+s2^2+s3^2}} \right) \left( \frac{s1}{\sqrt{s1^2+s2^2+s3^2}} + \right.$$

$$\left. \frac{s1^2(-s1+x1)}{(s1^2+s2^2+s3^2)^{3/2}} - \frac{-s1+x1}{\sqrt{s1^2+s2^2+s3^2}} + \frac{s1 s2(-s2+x2)}{(s1^2+s2^2+s3^2)^{3/2}} + \frac{s1 s3(-s3+x3)}{(s1^2+s2^2+s3^2)^{3/2}} \right) +$$

$$-\frac{1}{\sqrt{s1^2+s2^2+s3^2}} - \frac{s1(-s1+x1)}{\sqrt{s1^2+s2^2+s3^2}} - \frac{s2(-s2+x2)}{\sqrt{s1^2+s2^2+s3^2}} - \frac{s3(-s3+x3)}{\sqrt{s1^2+s2^2+s3^2}} \left( -\frac{s1^2 s3}{(s1^2+s2^2+s3^2)^{3/2}} + \frac{s3}{\sqrt{s1^2+s2^2+s3^2}} + \right.$$

$$\frac{s1 s2 s3 \left( -\frac{2 s1^3}{(s1^2+s2^2+s3^2)^2} - \frac{2 s1 s2^2}{(s1^2+s2^2+s3^2)^2} + \frac{2 s1}{s1^2+s2^2+s3^2} \right)}{2 (s1^2+s2^2+s3^2) \left( \frac{s1^2}{s1^2+s2^2+s3^2} + \frac{s2^2}{s1^2+s2^2+s3^2} \right)^{3/2}} -$$

$$\frac{s1 s2 \left( -\frac{2 s1^3}{(s1^2+s2^2+s3^2)^2} - \frac{2 s1 s2^2}{(s1^2+s2^2+s3^2)^2} + \frac{2 s1}{s1^2+s2^2+s3^2} \right)}{2 \sqrt{s1^2+s2^2+s3^2} \left( \frac{s1^2}{s1^2+s2^2+s3^2} + \frac{s2^2}{s1^2+s2^2+s3^2} \right)^{3/2}} +$$

$$\begin{aligned}
& \frac{2 s_1^2 s_2 s_3}{(s_1^2 + s_2^2 + s_3^2)^2 \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} - \frac{s_1^2 s_2}{(s_1^2 + s_2^2 + s_3^2)^{3/2} \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} - \\
& \frac{s_2 s_3}{(s_1^2 + s_2^2 + s_3^2) \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} + \frac{s_2}{\sqrt{s_1^2 + s_2^2 + s_3^2} \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} + \\
& \frac{s_2 \left( -\frac{2 s_1^3}{(s_1^2 + s_2^2 + s_3^2)^2} - \frac{2 s_1 s_2^2}{(s_1^2 + s_2^2 + s_3^2)^2} + \frac{2 s_1}{s_1^2 + s_2^2 + s_3^2} \right) x_1}{2 \sqrt{s_1^2 + s_2^2 + s_3^2} \left( \frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2} \right)^{3/2}} + \frac{s_1 s_2 x_1}{(s_1^2 + s_2^2 + s_3^2)^{3/2} \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} - \\
& \frac{s_1 s_3 \left( -\frac{2 s_1^3}{(s_1^2 + s_2^2 + s_3^2)^2} - \frac{2 s_1 s_2^2}{(s_1^2 + s_2^2 + s_3^2)^2} + \frac{2 s_1}{s_1^2 + s_2^2 + s_3^2} \right) x_2}{2 (s_1^2 + s_2^2 + s_3^2) \left( \frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2} \right)^{3/2}} - \frac{2 s_1^2 s_3 x_2}{(s_1^2 + s_2^2 + s_3^2)^2 \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} + \\
& \left. \frac{s_3 x_2}{(s_1^2 + s_2^2 + s_3^2) \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} + \frac{s_1^2 x_3}{(s_1^2 + s_2^2 + s_3^2)^{3/2}} - \frac{x_3}{\sqrt{s_1^2 + s_2^2 + s_3^2}} \right), \\
& - \frac{1}{\left( -\frac{s_1 (-s_1 + x_1)}{\sqrt{s_1^2 + s_2^2 + s_3^2}} - \frac{s_2 (-s_2 + x_2)}{\sqrt{s_1^2 + s_2^2 + s_3^2}} - \frac{s_3 (-s_3 + x_3)}{\sqrt{s_1^2 + s_2^2 + s_3^2}} \right)^2} \\
& \left( \frac{s_1 s_3}{\sqrt{s_1^2 + s_2^2 + s_3^2}} - \frac{s_1 s_2 s_3}{(s_1^2 + s_2^2 + s_3^2) \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} + \right. \\
& \frac{s_1 s_2}{\sqrt{s_1^2 + s_2^2 + s_3^2} \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} - \frac{s_2 x_1}{\sqrt{s_1^2 + s_2^2 + s_3^2} \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} + \\
& \left. \frac{s_1 s_3 x_2}{(s_1^2 + s_2^2 + s_3^2) \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} - \frac{s_1 x_3}{\sqrt{s_1^2 + s_2^2 + s_3^2}} \right) \\
& \left( \frac{s_2}{\sqrt{s_1^2 + s_2^2 + s_3^2}} + \frac{s_1 s_2 (-s_1 + x_1)}{(s_1^2 + s_2^2 + s_3^2)^{3/2}} + \frac{s_2^2 (-s_2 + x_2)}{(s_1^2 + s_2^2 + s_3^2)^{3/2}} - \frac{-s_2 + x_2}{\sqrt{s_1^2 + s_2^2 + s_3^2}} + \right. \\
& \left. \frac{s_2 s_3 (-s_3 + x_3)}{(s_1^2 + s_2^2 + s_3^2)^{3/2}} \right) + \frac{1}{-\frac{s_1 (-s_1 + x_1)}{\sqrt{s_1^2 + s_2^2 + s_3^2}} - \frac{s_2 (-s_2 + x_2)}{\sqrt{s_1^2 + s_2^2 + s_3^2}} - \frac{s_3 (-s_3 + x_3)}{\sqrt{s_1^2 + s_2^2 + s_3^2}}} \\
& \left( -\frac{s_1 s_2 s_3}{(s_1^2 + s_2^2 + s_3^2)^{3/2}} + \frac{s_1 s_2 s_3 \left( -\frac{2 s_1^2 s_2}{(s_1^2 + s_2^2 + s_3^2)^2} - \frac{2 s_2^3}{(s_1^2 + s_2^2 + s_3^2)^2} + \frac{2 s_2}{s_1^2 + s_2^2 + s_3^2} \right)}{2 (s_1^2 + s_2^2 + s_3^2) \left( \frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2} \right)^{3/2}} - \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{s1 s2 \left( -\frac{2 s1^2 s2}{(s1^2+s2^2+s3^2)^2} - \frac{2 s2^3}{(s1^2+s2^2+s3^2)^2} + \frac{2 s2}{s1^2+s2^2+s3^2} \right)}{2 \sqrt{s1^2+s2^2+s3^2} \left( \frac{s1^2}{s1^2+s2^2+s3^2} + \frac{s2^2}{s1^2+s2^2+s3^2} \right)^{3/2}} + \frac{2 s1 s2^2 s3}{(s1^2+s2^2+s3^2)^2 \sqrt{\frac{s1^2}{s1^2+s2^2+s3^2} + \frac{s2^2}{s1^2+s2^2+s3^2}}} - \\
& \frac{s1 s2^2}{(s1^2+s2^2+s3^2)^{3/2} \sqrt{\frac{s1^2}{s1^2+s2^2+s3^2} + \frac{s2^2}{s1^2+s2^2+s3^2}}} - \frac{s1 s3}{(s1^2+s2^2+s3^2) \sqrt{\frac{s1^2}{s1^2+s2^2+s3^2} + \frac{s2^2}{s1^2+s2^2+s3^2}}} + \\
& \frac{s1}{\sqrt{s1^2+s2^2+s3^2} \sqrt{\frac{s1^2}{s1^2+s2^2+s3^2} + \frac{s2^2}{s1^2+s2^2+s3^2}}} + \frac{s2 \left( -\frac{2 s1^2 s2}{(s1^2+s2^2+s3^2)^2} - \frac{2 s2^3}{(s1^2+s2^2+s3^2)^2} + \frac{2 s2}{s1^2+s2^2+s3^2} \right) x1}{2 \sqrt{s1^2+s2^2+s3^2} \left( \frac{s1^2}{s1^2+s2^2+s3^2} + \frac{s2^2}{s1^2+s2^2+s3^2} \right)^{3/2}} + \\
& \frac{s2^2 x1}{(s1^2+s2^2+s3^2)^{3/2} \sqrt{\frac{s1^2}{s1^2+s2^2+s3^2} + \frac{s2^2}{s1^2+s2^2+s3^2}}} - \frac{x1}{\sqrt{s1^2+s2^2+s3^2} \sqrt{\frac{s1^2}{s1^2+s2^2+s3^2} + \frac{s2^2}{s1^2+s2^2+s3^2}}} - \\
& \frac{s1 s3 \left( -\frac{2 s1^2 s2}{(s1^2+s2^2+s3^2)^2} - \frac{2 s2^3}{(s1^2+s2^2+s3^2)^2} + \frac{2 s2}{s1^2+s2^2+s3^2} \right) x2}{2 (s1^2+s2^2+s3^2) \left( \frac{s1^2}{s1^2+s2^2+s3^2} + \frac{s2^2}{s1^2+s2^2+s3^2} \right)^{3/2}} - \\
& \left( \frac{2 s1 s2 s3 x2}{(s1^2+s2^2+s3^2)^2 \sqrt{\frac{s1^2}{s1^2+s2^2+s3^2} + \frac{s2^2}{s1^2+s2^2+s3^2}}} + \frac{s1 s2 x3}{(s1^2+s2^2+s3^2)^{3/2}} \right), \\
& - \frac{1}{\left( -\frac{s1 (-s1+x1)}{\sqrt{s1^2+s2^2+s3^2}} - \frac{s2 (-s2+x2)}{\sqrt{s1^2+s2^2+s3^2}} - \frac{s3 (-s3+x3)}{\sqrt{s1^2+s2^2+s3^2}} \right)^2} \\
& \left( \frac{s1 s3}{\sqrt{s1^2+s2^2+s3^2}} - \frac{s1 s2 s3}{(s1^2+s2^2+s3^2) \sqrt{\frac{s1^2}{s1^2+s2^2+s3^2} + \frac{s2^2}{s1^2+s2^2+s3^2}}} + \right. \\
& \frac{s1 s2}{\sqrt{s1^2+s2^2+s3^2} \sqrt{\frac{s1^2}{s1^2+s2^2+s3^2} + \frac{s2^2}{s1^2+s2^2+s3^2}}} - \frac{s2 x1}{\sqrt{s1^2+s2^2+s3^2} \sqrt{\frac{s1^2}{s1^2+s2^2+s3^2} + \frac{s2^2}{s1^2+s2^2+s3^2}}} + \\
& \left. \frac{s1 s3 x2}{(s1^2+s2^2+s3^2) \sqrt{\frac{s1^2}{s1^2+s2^2+s3^2} + \frac{s2^2}{s1^2+s2^2+s3^2}}} - \frac{s1 x3}{\sqrt{s1^2+s2^2+s3^2}} \right) \\
& \left( \frac{s3}{\sqrt{s1^2+s2^2+s3^2}} + \frac{s1 s3 (-s1+x1)}{(s1^2+s2^2+s3^2)^{3/2}} + \frac{s2 s3 (-s2+x2)}{(s1^2+s2^2+s3^2)^{3/2}} + \frac{s3^2 (-s3+x3)}{(s1^2+s2^2+s3^2)^{3/2}} - \right. \\
& \left. \frac{-s3+x3}{\sqrt{s1^2+s2^2+s3^2}} \right) + \frac{1}{-\frac{s1 (-s1+x1)}{\sqrt{s1^2+s2^2+s3^2}} - \frac{s2 (-s2+x2)}{\sqrt{s1^2+s2^2+s3^2}} - \frac{s3 (-s3+x3)}{\sqrt{s1^2+s2^2+s3^2}}}
\end{aligned}$$



$$\begin{aligned}
& \left( -\frac{s1 s3^2}{(s1^2 + s2^2 + s3^2)^{3/2}} + \frac{s1}{\sqrt{s1^2 + s2^2 + s3^2}} + \frac{s1 s2 s3 \left( -\frac{2 s1^2 s3}{(s1^2 + s2^2 + s3^2)^2} - \frac{2 s2^2 s3}{(s1^2 + s2^2 + s3^2)^2} \right)}{2 (s1^2 + s2^2 + s3^2) \left( \frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2} \right)^{3/2}} - \right. \\
& \frac{s1 s2 \left( -\frac{2 s1^2 s3}{(s1^2 + s2^2 + s3^2)^2} - \frac{2 s2^2 s3}{(s1^2 + s2^2 + s3^2)^2} \right)}{2 \sqrt{s1^2 + s2^2 + s3^2} \left( \frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2} \right)^{3/2}} + \frac{2 s1 s2 s3^2}{(s1^2 + s2^2 + s3^2)^2 \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} - \\
& \frac{s1 s2 s3}{(s1^2 + s2^2 + s3^2)^{3/2} \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} - \frac{s1 s2}{(s1^2 + s2^2 + s3^2) \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} + \\
& \frac{s2 \left( -\frac{2 s1^2 s3}{(s1^2 + s2^2 + s3^2)^2} - \frac{2 s2^2 s3}{(s1^2 + s2^2 + s3^2)^2} \right) x1}{2 \sqrt{s1^2 + s2^2 + s3^2} \left( \frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2} \right)^{3/2}} + \frac{s2 s3 x1}{(s1^2 + s2^2 + s3^2)^{3/2} \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} - \\
& \frac{s1 s3 \left( -\frac{2 s1^2 s3}{(s1^2 + s2^2 + s3^2)^2} - \frac{2 s2^2 s3}{(s1^2 + s2^2 + s3^2)^2} \right) x2}{2 (s1^2 + s2^2 + s3^2) \left( \frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2} \right)^{3/2}} - \frac{2 s1 s3^2 x2}{(s1^2 + s2^2 + s3^2)^2 \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} + \\
& \left. \frac{s1 x2}{(s1^2 + s2^2 + s3^2) \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} + \frac{s1 s3 x3}{(s1^2 + s2^2 + s3^2)^{3/2}} \right\}, \\
& \left\{ -\frac{1}{\left( -\frac{s1 (-s1+x1)}{\sqrt{s1^2+s2^2+s3^2}} - \frac{s2 (-s2+x2)}{\sqrt{s1^2+s2^2+s3^2}} - \frac{s3 (-s3+x3)}{\sqrt{s1^2+s2^2+s3^2}} \right)^2} \right. \\
& \left( \frac{s2 s3}{\sqrt{s1^2 + s2^2 + s3^2}} - \frac{s2^2 s3}{(s1^2 + s2^2 + s3^2) \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} - \right. \\
& \frac{s1^2}{\sqrt{s1^2 + s2^2 + s3^2} \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} + \frac{s1 x1}{\sqrt{s1^2 + s2^2 + s3^2} \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} + \\
& \left. \frac{s2 s3 x2}{(s1^2 + s2^2 + s3^2) \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} - \frac{s2 x3}{\sqrt{s1^2 + s2^2 + s3^2}} \right) \\
& \left( \frac{s1}{\sqrt{s1^2 + s2^2 + s3^2}} + \frac{s1^2 (-s1+x1)}{(s1^2 + s2^2 + s3^2)^{3/2}} - \frac{-s1+x1}{\sqrt{s1^2 + s2^2 + s3^2}} + \frac{s1 s2 (-s2+x2)}{(s1^2 + s2^2 + s3^2)^{3/2}} + \right. \\
& \left. \frac{s1 s3 (-s3+x3)}{(s1^2 + s2^2 + s3^2)^{3/2}} \right) + \frac{1}{-\frac{s1 (-s1+x1)}{\sqrt{s1^2+s2^2+s3^2}} - \frac{s2 (-s2+x2)}{\sqrt{s1^2+s2^2+s3^2}} - \frac{s3 (-s3+x3)}{\sqrt{s1^2+s2^2+s3^2}}}
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{s1 s2 s3}{(s1^2 + s2^2 + s3^2)^{3/2}} + \frac{s2^2 s3 \left( -\frac{2 s1^3}{(s1^2 + s2^2 + s3^2)^2} - \frac{2 s1 s2^2}{(s1^2 + s2^2 + s3^2)^2} + \frac{2 s1}{s1^2 + s2^2 + s3^2} \right)}{2 (s1^2 + s2^2 + s3^2) \left( \frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2} \right)^{3/2}} + \right. \\
& \frac{s1^2 \left( -\frac{2 s1^3}{(s1^2 + s2^2 + s3^2)^2} - \frac{2 s1 s2^2}{(s1^2 + s2^2 + s3^2)^2} + \frac{2 s1}{s1^2 + s2^2 + s3^2} \right)}{2 \sqrt{s1^2 + s2^2 + s3^2} \left( \frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2} \right)^{3/2}} + \frac{2 s1 s2^2 s3}{(s1^2 + s2^2 + s3^2)^2 \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} + \\
& \frac{s1^3}{(s1^2 + s2^2 + s3^2)^{3/2} \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} - \frac{2 s1}{\sqrt{s1^2 + s2^2 + s3^2} \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} - \\
& \frac{s1 \left( -\frac{2 s1^3}{(s1^2 + s2^2 + s3^2)^2} - \frac{2 s1 s2^2}{(s1^2 + s2^2 + s3^2)^2} + \frac{2 s1}{s1^2 + s2^2 + s3^2} \right) x1}{2 \sqrt{s1^2 + s2^2 + s3^2} \left( \frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2} \right)^{3/2}} - \frac{s1^2 x1}{(s1^2 + s2^2 + s3^2)^{3/2} \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} + \\
& \frac{x1}{\sqrt{s1^2 + s2^2 + s3^2} \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} - \frac{s2 s3 \left( -\frac{2 s1^3}{(s1^2 + s2^2 + s3^2)^2} - \frac{2 s1 s2^2}{(s1^2 + s2^2 + s3^2)^2} + \frac{2 s1}{s1^2 + s2^2 + s3^2} \right) x2}{2 (s1^2 + s2^2 + s3^2) \left( \frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2} \right)^{3/2}} - \\
& \left. \frac{2 s1 s2 s3 x2}{(s1^2 + s2^2 + s3^2)^2 \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} + \frac{s1 s2 x3}{(s1^2 + s2^2 + s3^2)^{3/2}} \right), \\
& - \frac{1}{\left( -\frac{s1 (-s1+x1)}{\sqrt{s1^2+s2^2+s3^2}} - \frac{s2 (-s2+x2)}{\sqrt{s1^2+s2^2+s3^2}} - \frac{s3 (-s3+x3)}{\sqrt{s1^2+s2^2+s3^2}} \right)^2} \\
& \left( \frac{s2 s3}{\sqrt{s1^2 + s2^2 + s3^2}} - \frac{s2^2 s3}{(s1^2 + s2^2 + s3^2) \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} - \right. \\
& \frac{s1^2}{\sqrt{s1^2 + s2^2 + s3^2} \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} + \frac{s1 x1}{\sqrt{s1^2 + s2^2 + s3^2} \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} + \\
& \left. \frac{s2 s3 x2}{(s1^2 + s2^2 + s3^2) \sqrt{\frac{s1^2}{s1^2 + s2^2 + s3^2} + \frac{s2^2}{s1^2 + s2^2 + s3^2}}} - \frac{s2 x3}{\sqrt{s1^2 + s2^2 + s3^2}} \right) \\
& \left( \frac{s2}{\sqrt{s1^2 + s2^2 + s3^2}} + \frac{s1 s2 (-s1 + x1)}{(s1^2 + s2^2 + s3^2)^{3/2}} + \frac{s2^2 (-s2 + x2)}{(s1^2 + s2^2 + s3^2)^{3/2}} - \frac{-s2 + x2}{\sqrt{s1^2 + s2^2 + s3^2}} + \right. \\
& \left. \frac{s2 s3 (-s3 + x3)}{(s1^2 + s2^2 + s3^2)^{3/2}} \right) + \frac{1}{-\frac{s1 (-s1+x1)}{\sqrt{s1^2+s2^2+s3^2}} - \frac{s2 (-s2+x2)}{\sqrt{s1^2+s2^2+s3^2}} - \frac{s3 (-s3+x3)}{\sqrt{s1^2+s2^2+s3^2}}}
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{s_2^2 s_3}{(s_1^2 + s_2^2 + s_3^2)^{3/2}} + \frac{s_3}{\sqrt{s_1^2 + s_2^2 + s_3^2}} + \frac{s_2^2 s_3 \left( -\frac{2 s_1^2 s_2}{(s_1^2 + s_2^2 + s_3^2)^2} - \frac{2 s_2^3}{(s_1^2 + s_2^2 + s_3^2)^2} + \frac{2 s_2}{s_1^2 + s_2^2 + s_3^2} \right)}{2 (s_1^2 + s_2^2 + s_3^2) \left( \frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2} \right)^{3/2}} + \right. \\
& \frac{s_1^2 \left( -\frac{2 s_1^2 s_2}{(s_1^2 + s_2^2 + s_3^2)^2} - \frac{2 s_2^3}{(s_1^2 + s_2^2 + s_3^2)^2} + \frac{2 s_2}{s_1^2 + s_2^2 + s_3^2} \right)}{2 \sqrt{s_1^2 + s_2^2 + s_3^2} \left( \frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2} \right)^{3/2}} + \frac{2 s_2^3 s_3}{(s_1^2 + s_2^2 + s_3^2)^2 \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} + \\
& \frac{s_1^2 s_2}{(s_1^2 + s_2^2 + s_3^2)^{3/2} \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} - \frac{2 s_2 s_3}{(s_1^2 + s_2^2 + s_3^2) \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} - \\
& \frac{s_1 \left( -\frac{2 s_1^2 s_2}{(s_1^2 + s_2^2 + s_3^2)^2} - \frac{2 s_2^3}{(s_1^2 + s_2^2 + s_3^2)^2} + \frac{2 s_2}{s_1^2 + s_2^2 + s_3^2} \right) x_1}{2 \sqrt{s_1^2 + s_2^2 + s_3^2} \left( \frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2} \right)^{3/2}} - \frac{s_1 s_2 x_1}{(s_1^2 + s_2^2 + s_3^2)^{3/2} \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} - \\
& \frac{s_2 s_3 \left( -\frac{2 s_1^2 s_2}{(s_1^2 + s_2^2 + s_3^2)^2} - \frac{2 s_2^3}{(s_1^2 + s_2^2 + s_3^2)^2} + \frac{2 s_2}{s_1^2 + s_2^2 + s_3^2} \right) x_2}{2 (s_1^2 + s_2^2 + s_3^2) \left( \frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2} \right)^{3/2}} - \frac{2 s_2^2 s_3 x_2}{(s_1^2 + s_2^2 + s_3^2)^2 \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} + \\
& \left. \frac{s_3 x_2}{(s_1^2 + s_2^2 + s_3^2) \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} + \frac{s_2^2 x_3}{(s_1^2 + s_2^2 + s_3^2)^{3/2}} - \frac{x_3}{\sqrt{s_1^2 + s_2^2 + s_3^2}} \right), \\
& - \frac{1}{\left( -\frac{s_1 (-s_1 + x_1)}{\sqrt{s_1^2 + s_2^2 + s_3^2}} - \frac{s_2 (-s_2 + x_2)}{\sqrt{s_1^2 + s_2^2 + s_3^2}} - \frac{s_3 (-s_3 + x_3)}{\sqrt{s_1^2 + s_2^2 + s_3^2}} \right)^2} \\
& \left( \frac{s_2 s_3}{\sqrt{s_1^2 + s_2^2 + s_3^2}} - \frac{s_2^2 s_3}{(s_1^2 + s_2^2 + s_3^2) \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} - \right. \\
& \frac{s_1^2}{\sqrt{s_1^2 + s_2^2 + s_3^2} \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} + \frac{s_1 x_1}{\sqrt{s_1^2 + s_2^2 + s_3^2} \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} + \\
& \left. \frac{s_2 s_3 x_2}{(s_1^2 + s_2^2 + s_3^2) \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} - \frac{s_2 x_3}{\sqrt{s_1^2 + s_2^2 + s_3^2}} \right) \\
& \left( \frac{s_3}{\sqrt{s_1^2 + s_2^2 + s_3^2}} + \frac{s_1 s_3 (-s_1 + x_1)}{(s_1^2 + s_2^2 + s_3^2)^{3/2}} + \frac{s_2 s_3 (-s_2 + x_2)}{(s_1^2 + s_2^2 + s_3^2)^{3/2}} + \frac{s_3^2 (-s_3 + x_3)}{(s_1^2 + s_2^2 + s_3^2)^{3/2}} - \right. \\
& \left. \frac{-s_3 + x_3}{\sqrt{s_1^2 + s_2^2 + s_3^2}} \right) + \frac{1}{-\frac{s_1 (-s_1 + x_1)}{\sqrt{s_1^2 + s_2^2 + s_3^2}} - \frac{s_2 (-s_2 + x_2)}{\sqrt{s_1^2 + s_2^2 + s_3^2}} - \frac{s_3 (-s_3 + x_3)}{\sqrt{s_1^2 + s_2^2 + s_3^2}}}
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{s_2 s_3^2}{(s_1^2 + s_2^2 + s_3^2)^{3/2}} + \frac{s_2}{\sqrt{s_1^2 + s_2^2 + s_3^2}} + \frac{s_2^2 s_3 \left( -\frac{2 s_1^2 s_3}{(s_1^2 + s_2^2 + s_3^2)^2} - \frac{2 s_2^2 s_3}{(s_1^2 + s_2^2 + s_3^2)^2} \right)}{2 (s_1^2 + s_2^2 + s_3^2) \left( \frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2} \right)^{3/2}} + \right. \\
& \frac{s_1^2 \left( -\frac{2 s_1^2 s_3}{(s_1^2 + s_2^2 + s_3^2)^2} - \frac{2 s_2^2 s_3}{(s_1^2 + s_2^2 + s_3^2)^2} \right)}{2 \sqrt{s_1^2 + s_2^2 + s_3^2} \left( \frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2} \right)^{3/2}} + \frac{2 s_2^2 s_3^2}{(s_1^2 + s_2^2 + s_3^2)^2 \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} + \\
& \frac{s_1^2 s_3}{(s_1^2 + s_2^2 + s_3^2)^{3/2} \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} - \frac{s_2^2}{(s_1^2 + s_2^2 + s_3^2) \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} - \\
& \frac{s_1 \left( -\frac{2 s_1^2 s_3}{(s_1^2 + s_2^2 + s_3^2)^2} - \frac{2 s_2^2 s_3}{(s_1^2 + s_2^2 + s_3^2)^2} \right) x_1}{2 \sqrt{s_1^2 + s_2^2 + s_3^2} \left( \frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2} \right)^{3/2}} - \frac{s_1 s_3 x_1}{(s_1^2 + s_2^2 + s_3^2)^{3/2} \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} - \\
& \frac{s_2 s_3 \left( -\frac{2 s_1^2 s_3}{(s_1^2 + s_2^2 + s_3^2)^2} - \frac{2 s_2^2 s_3}{(s_1^2 + s_2^2 + s_3^2)^2} \right) x_2}{2 (s_1^2 + s_2^2 + s_3^2) \left( \frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2} \right)^{3/2}} - \frac{2 s_2 s_3^2 x_2}{(s_1^2 + s_2^2 + s_3^2)^2 \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} + \\
& \left. \frac{s_2 x_2}{(s_1^2 + s_2^2 + s_3^2) \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} + \frac{s_2 s_3 x_3}{(s_1^2 + s_2^2 + s_3^2)^{3/2}} \right\}, \\
& \left\{ -\frac{1}{\left( -\frac{s_1 (-s_1 + x_1)}{\sqrt{s_1^2 + s_2^2 + s_3^2}} - \frac{s_2 (-s_2 + x_2)}{\sqrt{s_1^2 + s_2^2 + s_3^2}} - \frac{s_3 (-s_3 + x_3)}{\sqrt{s_1^2 + s_2^2 + s_3^2}} \right)^2} \left( \frac{s_3^2}{\sqrt{s_1^2 + s_2^2 + s_3^2}} - \right. \right. \\
& s_2 \left( -\frac{s_1^2}{(s_1^2 + s_2^2 + s_3^2) \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} - \frac{s_2^2}{(s_1^2 + s_2^2 + s_3^2) \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} \right) + \\
& \left( -\frac{s_1^2}{(s_1^2 + s_2^2 + s_3^2) \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} - \frac{s_2^2}{(s_1^2 + s_2^2 + s_3^2) \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} \right) x_2 - \\
& \left. \frac{s_3 x_3}{\sqrt{s_1^2 + s_2^2 + s_3^2}} \right) \left( \frac{s_1}{\sqrt{s_1^2 + s_2^2 + s_3^2}} + \frac{s_1^2 (-s_1 + x_1)}{(s_1^2 + s_2^2 + s_3^2)^{3/2}} - \frac{-s_1 + x_1}{\sqrt{s_1^2 + s_2^2 + s_3^2}} + \right. \\
& \left. \frac{s_1 s_2 (-s_2 + x_2)}{(s_1^2 + s_2^2 + s_3^2)^{3/2}} + \frac{s_1 s_3 (-s_3 + x_3)}{(s_1^2 + s_2^2 + s_3^2)^{3/2}} \right) + \frac{1}{-\frac{s_1 (-s_1 + x_1)}{\sqrt{s_1^2 + s_2^2 + s_3^2}} - \frac{s_2 (-s_2 + x_2)}{\sqrt{s_1^2 + s_2^2 + s_3^2}} - \frac{s_3 (-s_3 + x_3)}{\sqrt{s_1^2 + s_2^2 + s_3^2}}}
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{s_1 s_3^2}{(s_1^2 + s_2^2 + s_3^2)^{3/2}} - s_2 \left( \frac{s_1^2 \left( -\frac{2 s_1^3}{(s_1^2 + s_2^2 + s_3^2)^2} - \frac{2 s_1 s_2^2}{(s_1^2 + s_2^2 + s_3^2)^2} + \frac{2 s_1}{s_1^2 + s_2^2 + s_3^2} \right)}{2 (s_1^2 + s_2^2 + s_3^2) \left( \frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2} \right)^{3/2}} + \right. \\
& \quad \frac{s_2^2 \left( -\frac{2 s_1^3}{(s_1^2 + s_2^2 + s_3^2)^2} - \frac{2 s_1 s_2^2}{(s_1^2 + s_2^2 + s_3^2)^2} + \frac{2 s_1}{s_1^2 + s_2^2 + s_3^2} \right)}{2 (s_1^2 + s_2^2 + s_3^2) \left( \frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2} \right)^{3/2}} + \frac{2 s_1^3}{(s_1^2 + s_2^2 + s_3^2)^2 \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} + \\
& \quad \left. \frac{2 s_1 s_2^2}{(s_1^2 + s_2^2 + s_3^2)^2 \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} - \frac{2 s_1}{(s_1^2 + s_2^2 + s_3^2) \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} \right) + \\
& \left( \frac{s_1^2 \left( -\frac{2 s_1^3}{(s_1^2 + s_2^2 + s_3^2)^2} - \frac{2 s_1 s_2^2}{(s_1^2 + s_2^2 + s_3^2)^2} + \frac{2 s_1}{s_1^2 + s_2^2 + s_3^2} \right)}{2 (s_1^2 + s_2^2 + s_3^2) \left( \frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2} \right)^{3/2}} + \frac{s_2^2 \left( -\frac{2 s_1^3}{(s_1^2 + s_2^2 + s_3^2)^2} - \frac{2 s_1 s_2^2}{(s_1^2 + s_2^2 + s_3^2)^2} + \frac{2 s_1}{s_1^2 + s_2^2 + s_3^2} \right)}{2 (s_1^2 + s_2^2 + s_3^2) \left( \frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2} \right)^{3/2}} + \right. \\
& \quad \frac{2 s_1^3}{(s_1^2 + s_2^2 + s_3^2)^2 \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} + \frac{2 s_1 s_2^2}{(s_1^2 + s_2^2 + s_3^2)^2 \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} - \\
& \quad \left. \frac{2 s_1}{(s_1^2 + s_2^2 + s_3^2) \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} \right) x_2 + \frac{s_1 s_3 x_3}{(s_1^2 + s_2^2 + s_3^2)^{3/2}}, \\
& - \frac{1}{\left( -\frac{s_1 (-s_1 + x_1)}{\sqrt{s_1^2 + s_2^2 + s_3^2}} - \frac{s_2 (-s_2 + x_2)}{\sqrt{s_1^2 + s_2^2 + s_3^2}} - \frac{s_3 (-s_3 + x_3)}{\sqrt{s_1^2 + s_2^2 + s_3^2}} \right)^2} \left( \frac{s_3^2}{\sqrt{s_1^2 + s_2^2 + s_3^2}} - \right. \\
& \quad s_2 \left( -\frac{s_1^2}{(s_1^2 + s_2^2 + s_3^2) \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} - \frac{s_2^2}{(s_1^2 + s_2^2 + s_3^2) \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} \right) + \\
& \quad \left( -\frac{s_1^2}{(s_1^2 + s_2^2 + s_3^2) \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} - \frac{s_2^2}{(s_1^2 + s_2^2 + s_3^2) \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} \right) x_2 - \\
& \quad \frac{s_3 x_3}{\sqrt{s_1^2 + s_2^2 + s_3^2}} \left( \frac{s_2}{\sqrt{s_1^2 + s_2^2 + s_3^2}} + \frac{s_1 s_2 (-s_1 + x_1)}{(s_1^2 + s_2^2 + s_3^2)^{3/2}} + \frac{s_2^2 (-s_2 + x_2)}{(s_1^2 + s_2^2 + s_3^2)^{3/2}} - \right. \\
& \quad \left. \frac{-s_2 + x_2}{\sqrt{s_1^2 + s_2^2 + s_3^2}} + \frac{s_2 s_3 (-s_3 + x_3)}{(s_1^2 + s_2^2 + s_3^2)^{3/2}} \right) + \frac{1}{-\frac{s_1 (-s_1 + x_1)}{\sqrt{s_1^2 + s_2^2 + s_3^2}} - \frac{s_2 (-s_2 + x_2)}{\sqrt{s_1^2 + s_2^2 + s_3^2}} - \frac{s_3 (-s_3 + x_3)}{\sqrt{s_1^2 + s_2^2 + s_3^2}}}
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{s_2 s_3^2}{(s_1^2 + s_2^2 + s_3^2)^{3/2}} + \frac{s_1^2}{(s_1^2 + s_2^2 + s_3^2) \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} + \right. \\
& \frac{s_2^2}{(s_1^2 + s_2^2 + s_3^2) \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} - s_2 \left( \frac{s_1^2 \left( -\frac{2 s_1^2 s_2}{(s_1^2 + s_2^2 + s_3^2)^2} - \frac{2 s_2^3}{(s_1^2 + s_2^2 + s_3^2)^2} + \frac{2 s_2}{s_1^2 + s_2^2 + s_3^2} \right)}{2 (s_1^2 + s_2^2 + s_3^2) \left( \frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2} \right)^{3/2}} + \right. \\
& \frac{s_2^2 \left( -\frac{2 s_1^2 s_2}{(s_1^2 + s_2^2 + s_3^2)^2} - \frac{2 s_2^3}{(s_1^2 + s_2^2 + s_3^2)^2} + \frac{2 s_2}{s_1^2 + s_2^2 + s_3^2} \right)}{2 (s_1^2 + s_2^2 + s_3^2) \left( \frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2} \right)^{3/2}} + \frac{2 s_1^2 s_2}{(s_1^2 + s_2^2 + s_3^2)^2 \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} + \\
& \left. \frac{2 s_2^3}{(s_1^2 + s_2^2 + s_3^2)^2 \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} - \frac{2 s_2}{(s_1^2 + s_2^2 + s_3^2) \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} \right) + \\
& \left( \frac{s_1^2 \left( -\frac{2 s_1^2 s_2}{(s_1^2 + s_2^2 + s_3^2)^2} - \frac{2 s_2^3}{(s_1^2 + s_2^2 + s_3^2)^2} + \frac{2 s_2}{s_1^2 + s_2^2 + s_3^2} \right)}{2 (s_1^2 + s_2^2 + s_3^2) \left( \frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2} \right)^{3/2}} + \frac{s_2^2 \left( -\frac{2 s_1^2 s_2}{(s_1^2 + s_2^2 + s_3^2)^2} - \frac{2 s_2^3}{(s_1^2 + s_2^2 + s_3^2)^2} + \frac{2 s_2}{s_1^2 + s_2^2 + s_3^2} \right)}{2 (s_1^2 + s_2^2 + s_3^2) \left( \frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2} \right)^{3/2}} + \right. \\
& \frac{2 s_1^2 s_2}{(s_1^2 + s_2^2 + s_3^2)^2 \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} + \frac{2 s_2^3}{(s_1^2 + s_2^2 + s_3^2)^2 \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} - \\
& \left. \frac{2 s_2}{(s_1^2 + s_2^2 + s_3^2) \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} \right) x_2 + \frac{s_2 s_3 x_3}{(s_1^2 + s_2^2 + s_3^2)^{3/2}}, \\
& - \frac{1}{\left( -\frac{s_1 (-s_1 + x_1)}{\sqrt{s_1^2 + s_2^2 + s_3^2}} - \frac{s_2 (-s_2 + x_2)}{\sqrt{s_1^2 + s_2^2 + s_3^2}} - \frac{s_3 (-s_3 + x_3)}{\sqrt{s_1^2 + s_2^2 + s_3^2}} \right)^2} \left( \frac{s_3^2}{\sqrt{s_1^2 + s_2^2 + s_3^2}} - \right. \\
& s_2 \left( -\frac{s_1^2}{(s_1^2 + s_2^2 + s_3^2) \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} - \frac{s_2^2}{(s_1^2 + s_2^2 + s_3^2) \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} \right) + \\
& \left( -\frac{s_1^2}{(s_1^2 + s_2^2 + s_3^2) \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} - \frac{s_2^2}{(s_1^2 + s_2^2 + s_3^2) \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} \right) x_2 - \\
& \left. \frac{s_3 x_3}{\sqrt{s_1^2 + s_2^2 + s_3^2}} \right) \left( \frac{s_3}{\sqrt{s_1^2 + s_2^2 + s_3^2}} + \frac{s_1 s_3 (-s_1 + x_1)}{(s_1^2 + s_2^2 + s_3^2)^{3/2}} + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{s_2 s_3 (-s_2 + x_2)}{(s_1^2 + s_2^2 + s_3^2)^{3/2}} + \frac{s_3^2 (-s_3 + x_3)}{(s_1^2 + s_2^2 + s_3^2)^{3/2}} - \frac{-s_3 + x_3}{\sqrt{s_1^2 + s_2^2 + s_3^2}} \Bigg) + \\
& - \frac{s_1 (-s_1 + x_1)}{\sqrt{s_1^2 + s_2^2 + s_3^2}} - \frac{s_2 (-s_2 + x_2)}{\sqrt{s_1^2 + s_2^2 + s_3^2}} - \frac{s_3 (-s_3 + x_3)}{\sqrt{s_1^2 + s_2^2 + s_3^2}} \Bigg) - \frac{s_3^3}{(s_1^2 + s_2^2 + s_3^2)^{3/2}} + \frac{2 s_3}{\sqrt{s_1^2 + s_2^2 + s_3^2}} - s_2 \\
& \left( \frac{s_1^2 \left( -\frac{2 s_1^2 s_3}{(s_1^2 + s_2^2 + s_3^2)^2} - \frac{2 s_2^2 s_3}{(s_1^2 + s_2^2 + s_3^2)^2} \right)}{2 (s_1^2 + s_2^2 + s_3^2) \left( \frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2} \right)^{3/2}} + \frac{s_2^2 \left( -\frac{2 s_1^2 s_3}{(s_1^2 + s_2^2 + s_3^2)^2} - \frac{2 s_2^2 s_3}{(s_1^2 + s_2^2 + s_3^2)^2} \right)}{2 (s_1^2 + s_2^2 + s_3^2) \left( \frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2} \right)^{3/2}} + \right. \\
& \left. - \frac{2 s_1^2 s_3}{(s_1^2 + s_2^2 + s_3^2)^2 \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} + \frac{2 s_2^2 s_3}{(s_1^2 + s_2^2 + s_3^2)^2 \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} \right) + \\
& \left( \frac{s_1^2 \left( -\frac{2 s_1^2 s_3}{(s_1^2 + s_2^2 + s_3^2)^2} - \frac{2 s_2^2 s_3}{(s_1^2 + s_2^2 + s_3^2)^2} \right)}{2 (s_1^2 + s_2^2 + s_3^2) \left( \frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2} \right)^{3/2}} + \frac{s_2^2 \left( -\frac{2 s_1^2 s_3}{(s_1^2 + s_2^2 + s_3^2)^2} - \frac{2 s_2^2 s_3}{(s_1^2 + s_2^2 + s_3^2)^2} \right)}{2 (s_1^2 + s_2^2 + s_3^2) \left( \frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2} \right)^{3/2}} + \right. \\
& \left. - \frac{2 s_1^2 s_3}{(s_1^2 + s_2^2 + s_3^2)^2 \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} + \frac{2 s_2^2 s_3}{(s_1^2 + s_2^2 + s_3^2)^2 \sqrt{\frac{s_1^2}{s_1^2 + s_2^2 + s_3^2} + \frac{s_2^2}{s_1^2 + s_2^2 + s_3^2}}} \right) \\
& x_2 + \frac{s_3^2 x_3}{(s_1^2 + s_2^2 + s_3^2)^{3/2}} - \frac{x_3}{\sqrt{s_1^2 + s_2^2 + s_3^2}} \Bigg) \Bigg\} \Bigg\}
\end{aligned}$$

Complete Jacobian

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In[172]:= Htilde[s1_, s2_, s3_, x1_, x2_, x3_] = Inverse[Transpose[DELTA].DELTA].
Transpose[DELTA].Inverse[T].Htildepart[s1, s2, s3, x1, x2, x3];
```