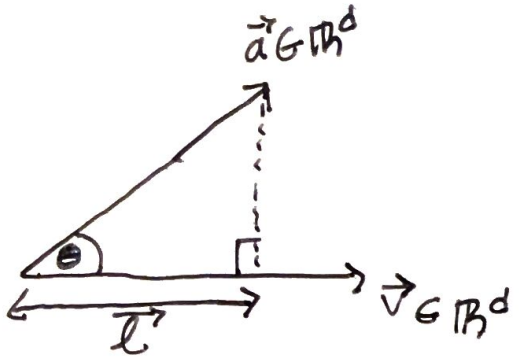
Math 342W Lecture 9



$\vec{\ell}$ = orthogonal projection of $\vec{a}$ onto $\vec{v}$

$\vec{\ell} = \text{proj}_{\vec{v}}(\vec{a})$

Law of cosines

$$\cos\theta = \frac{\vec{a}\cdot\vec{v}}{\|\vec{a}\|\|\vec{v}\|} = \frac{\|\vec{\ell}\|}{\|\vec{a}\|} \Rightarrow \|\vec{\ell}\| = \frac{\vec{a}\cdot\vec{v}}{\|\vec{v}\|}$$

$$= \|\vec{\ell}\| \,\text{direction}(\vec{v}) = \|\vec{\ell}\| \frac{\vec{v}}{\|\vec{v}\|} = \frac{\vec{a}\cdot\vec{v}}{\|\vec{v}\|^2}\vec{v}$$

$$= \frac{1}{\|\vec{v}\|^2}\vec{v}\vec{v}^T\vec{a}$$

$$= \underbrace{\frac{\vec{v}\vec{v}^T}{\|\vec{v}\|^2}}_{H}\vec{a} = H\vec{a} = \vec{\ell}$$

The projection matrix $\text{proj}_{\vec{v}}(\vec{v}) = H\vec{v} = \vec{v}$

$\text{proj}_{\vec{v}}(\vec{v}) = H\vec{v} = \vec{v}$

$\text{Proj}_{\vec{v}}(\vec{a}-\vec{\ell}) = H(\vec{a}-\vec{\ell})$
$= H\vec{a} - H\vec{\ell} = \vec{\ell} - \vec{\ell} = \vec{0}_d$

$\text{proj}_{\vec{v}}(\text{proj}_{\vec{v}}(\vec{a})) = HH\vec{a} = H\vec{a} = \vec{\ell}$

$\Downarrow$

$HH = H$ (Idempotent)

$V = [\vec{v}_1 \mid \vec{v}_2 \mid \cdots \mid \vec{v}_n] \in \mathbb{R}^{d\times n}$, $n < d$