Bid Price Optimization for Simultaneous Truckload Transportation Procurement Auctions

Özlem Ergun, Gültekin Kuyzu, Martin Savelsbergh
H. Milton Stewart School of Industrial Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0205

Abstract

We study simultaneous transportation procurement auctions from a truckload carrier's perspective. We formulate a stochastic bid price optimization problem with the objective of maximizing a carrier's expected profit. The formulation takes into account the synergies among the lanes and the competing carriers' bidding strategies. For solving this stochastic optimization problem, we develop an iterative coordinate search algorithm that finds good solutions efficiently. The effectiveness of the developed algorithm and the overall strategy is demonstrated through computational experiments.

1. Introduction

Transportation marketplaces are Internet-enabled environments in which shippers and carriers procure and sell transportation services. Transportation marketplaces are ideally suited to provide a spot market for short-term transportation contracts, often simply the movement of a single truckload shipment. There are various mechanisms that can be employed for matching shippers and carriers. We focus on procurement auctions because of their practical importance and because the body of literature on the subject is small. More specifically, we study the design and analysis of bidding strategies for a carrier participating in multiple simultaneous single-lane auctions. The challenge for the carrier is to determine bid prices, and thus potential revenues, while operating costs are uncertain as they depend on the lanes won, which is not known until the end of the auctions. In determining bid prices the carrier has to consider the potential revenues, the probable costs, and the possible competitors' bids, which directly or indirectly impact the carrier's profit.

Our focus is on incorporating the synergies between the lanes up for auction and the carrier's existing network and anticipating bidding behavior of competitors when determining bid prices. Competitors' bids greatly affect the outcome of the procurement auctions and therefore the set of lanes the carrier ends up winning. The set of lanes won, in turn, determines the carrier's revenue and operating cost. A bid price optimization problem with an objective of maximizing a carrier's expected profit is formulated and algorithms for its solution are designed and implemented. The value of bid price optimization for carriers is empirically evaluated on randomly generated instances simulating a real-life spot market environment.

The contributions of the research presented in this paper can be summarized as follows:

- We provide optimization-based bidding (or dynamic pricing) algorithms for carriers participating in multiple simultaneous truckload procurement auctions. We are the first to study simultaneous truckload procurement auctions from a carrier's perspective. Most of the literature has focused on the design or the selection of auction mechanisms with desirable properties or on the characterization of equilibrium strategies for carriers. The literature that does study transportation procurement auctions from a carrier's perspective has focused on combinatorial or sequential auction settings.

- We formulate a stochastic bid price optimization problem, with the objective of maximizing a carrier's expected profit, and develop an iterative coordinate search algorithm for its solution. The coordinate search algorithm produces optimal or near-optimal solutions for small to medium size practical instances with little computational effort.

- We conduct a simulation study to demonstrate the value of using bid price optimization. In an environment in which the competition determines bids on a lane-by-lane bases without considering synergies between the lanes up for auction, a carrier relying on bid price optimization outperforms the competition, in terms of profit, especially in situations where exploiting network synergies is critical.

The remainder of the paper is organized as follows. In Section 2, we present an overview of transportation procurement and discuss the relevant literature. In Section 3, we introduce the bid price optimization problem. In Section 4, we propose a coordinate search algorithm for the solution of the bid price optimization problem. In Section 5, we analyze the bid price optimization problem for small sets of lanes. Finally, in Section 6, we present the results of an extensive simulation study.

2. Transportation Procurement and Literature Review

The developments in information and communication technologies, especially wide-spread availability and use of the internet, facilitate quick, convenient, and cost-effective transactions between shippers and carriers. Many shippers form private transportation exchanges (or marketplaces) to procure transportation services where only a few carriers selected by the shipper compete for the lanes of the shipper. These private marketplaces enable shippers to work with a few trusted carriers, and thus are predominantly used for procurement of long term contracts. Alternatively, the shippers can procure transportation services using public exchanges, such as www.freemarkets.com, where many shippers and many carriers are involved. Mostly, these public exchanges act as a spot market enabling the shippers to satisfy immediate and short term transportation needs resulting from fluctuations in shipment volume in a cost effective way. Regan and Nandiraju ([16]) provide a review of transportation marketplaces and their characteristics. Shippers and carriers participating in transportation market places are typically matched through auctions.

Auctions have been used by people for thousands of years as market mechanisms. Today, auctions account for an enormous amount of economic activity. Governments use auctions to sell treasury bills, radio frequency spectrums, foreign exchange, mineral rights including oil fields, and other assets such as firms to be privatized. Government contracts are usually awarded using procurement auctions. Firms seeking to buy inputs or subcontracting work also use procurement auctions. Houses, cars, antiques, artwork, agricultural produce and livestock are commonly sold through auctions. Internet auction web-sites such as Ebay and Yahoo are used to sell almost anything.

Auctions are simple but useful and practical price discovery mechanisms to extract buyers' or sellers' valuations, especially when there is uncertainty about the value of an object or service. If there were no uncertainty, the seller/buyer would just transact with the buyer/seller that had the highest valuation. The term *auction* usually refers to the case that involves a seller and several buyers. Note that procurement auctions are *reverse* auctions because sellers (carriers) bid instead of buyers (shippers); prices are bid down instead of up. Most of the auctions studied in the literature are forward auctions. On the other hand, models and intuition derived for most auctions can be easily reversed and applied to reverse auctions and vice versa ([20]).

Procurement auctions existed in the logistics and transportation industry in the form of Request for Proposals (RFPs) or Request for Quotations (RFQs). With the advent of internet technologies and their widespread adoption, internet emerged as a platform for facilitating business transactions such as buying and selling of goods, integrating supply chains, and interacting with partners and employees. Following the success stories of the early adopters, firms are investing billions of dollars in electronic procurement (e-procurement) technologies (e-markets) to achieve greater efficiency. See Elmaghraby ([5]) for a review of the current state of procurement in the industry.

The strategic significance and practical implications of using e-procurement attracted significant research efforts from the operations research and management science community. It is important to note that most researchers take the perspective of the buyer (or auctioneer), focusing on the design of new auction mechanisms or analyzing existing ones ([2, 14]). Particular attention has been paid to combinatorial procurement auctions which allow the suppliers (bidders) to express volume discounts, incompatible products/services, and complementary products/services. Elmaghraby and Keskinocak ([6, 7]) provide an overview of the state-of-the-art in combinatorial procurement auctions. To the best of the authors' knowledge, relatively little research literature exists about bidding strategies in procurement auctions ([29]).

Because of inherent significance of synergies between shipping lanes for transportation carrier operations, the use of combinatorial auctions has been proposed for procurement of transportation services. Again, these auctions have mostly been analyzed from the shippers' perspective by researchers and practitioners alike. Moore et al. ([15]) describe the formulation of an optimization-based bidding approach for Reynolds Metals which was not fully implemented in practice due to limited computer capabilities available at the time. Ledyard et al. ([11]) describe the use of a "combined value" auction for purchasing transportation services for Sears Logistics Services in the early 1990s. Elmaghraby and Keskinocak ([6]) discuss how several companies, including Home Depot, Wal-Mart Stores, Compaq Computer Co., Staples Inc., the Limited Inc., and others, utilized optimization-based procurement tools to procure transportation services. Sheffi ([22]) provides a survey of the literature and practical results on the use of combinatorial auctions in procurement of truckload transportation services.

On the other hand, determining the optimal bid values for a carrier poses several challenging research questions. In essence, the problem that the carrier faces in this context is a pricing problem. Most of the literature on carrier operations treated pricing and cost minimization problems separately, and focused solely on minimizing the cost of serving the customers by solving fleet management and vehicle routing problems (VRPs). See Crainic and Laporte ([3]) for a review of fleet management problems and Toth and Vigo ([28]) for a survey of the VRP and its variants. Some of the past work considers the problem of how much the total profit would change when an additional load is introduced into a dynamic fleet management system ([17, 18, 19, 27]). Although these works certainly are valuable, today's market conditions call for a more direct and integrated approach, especially in the case of auctions. Recently, the study of transportation procurement auctions from the carrier perspective has received some attention. In combinatorial auctions, Song and Regan ([23, 24]) propose optimization-based bid construction strategies for carriers. An et al. [1] propose a generic synergy model for constructing combinatorial bids. In these studies, the lane bundles are formed based on anticipated cost of serving each of the bundles and then the bid on each bundle is obtained by adding a fixed markup to this anticipated cost. They ignore the interactions of the carrier with other shippers and competing carriers.

Also receiving attention are bidding strategies in sequential transportation procurement auctions where multiple auctions take place in sequence, i.e., one at a time, ruling out the possibility of having to bid on two or more auctions simultaneously. In this setting, the carrier must incorporate pricing decisions into dynamic fleet management decisions. Figliozzi ([9]) in his thesis studies the impact of different dynamic pricing and fleet management technologies on the revenues and operating costs of a carrier in a spot truckload procurement market. Mes et al. ([13]) consider a real-time dynamic pickup and delivery problem and propose a pricing strategy based on dynamic programming where the impact of a job insertion on future opportunities is taken into account. Douma et al. ([4]) consider a multi-company, less-than-truckload, dynamic VRP and use revenue management concepts to price the loads dynamically under capacity restrictions and taking into account possible future revenues.

3. A Bid Price Optimization Problem

We study simultaneous transportation procurement auctions from a truckload carrier's perspective. Our goal is neither to design simultaneous auction mechanisms with desirable properties, nor to investigate if the auction mechanisms in place have such properties, nor to characterize equilibrium strategies of carriers in such auctions. Our goal is to develop bidding algorithms for a carrier which account for synergies among lanes and which anticipate bidding behavior of competitors.

We take the perspective of a carrier whom we assume to have a network which consists of long term contracts. We assume that the carrier wants to complement his network with short term contracts from an auction-based spot procurement market in order to recover some of his repositioning costs. In this spot procurement market, the carrier must place bids in multiple independent single lane auctions, which are run in parallel. These auctions take place at the beginning of every period (e.g., a week, a month, etc.), and involve lanes that must be served in the period that follows. Each of the auctions is a single-round, sealed-bid, first-price auction. We assume that the carrier has sufficient capacity to handle all the lanes he gets from the auctions.

Without loss of generality, we assume that there is a single auctioneer running all the auctions. The auctioneer may be a single shipper or a third party facilitating the communication between the shippers and the carriers. The following stages take place in sequence:

1. The auctioneer announces which lanes are going to be auctioned.

2. Carriers place bids on the lanes being auctioned.

3. The auctioneer receives the bids and determines which carrier will be given the right to serve each of the lanes.

4. The carrier routes his vehicles to serve the lanes won

The carrier makes decisions in Stage 2 and 4. The decisions in Stage 2 affect the carrier's revenue while the decisions in Stage 4 affect the carrier's costs. The stochastic optimization model is designed to determine optimal bids in Stage 2 taking into account potential outcomes and thus costs in Stage 4. In order to account for the uncertainty regarding the outcomes of the auctions, the lowest bid of the competing carriers for each lane is modeled as an independent continuous random variable which is uniformly distributed on an interval. We make this simplifying assumption because it is easier for a carrier to determine the range of values of his competitors' bids than to characterize the actual distribution of such bids. In addition, as will be seen later, it is not hard to incorporate other probability distributions into our solution approach.

Let L_0 denote the set of lanes representing the carrier's existing network. Let $L = \{1, 2, \ldots, m\}$ denote the set of lanes being auctioned. For each lane i, the random variable representing the lowest bid of the competing carriers on lane i is denoted by X_i which is uniformly distributed on the interval (ℓ_i, u_i).

The expected profit of the carrier is a function of his bids on the lanes being auctioned. Note that it is not necessary for the carrier to bid outside the interval $[\ell_i, u_i]$ for lane i. Since bidding ℓ_i on lane i guarantees winning the lane, the carrier need not bid lower than ℓ_i. Similarly, since bidding u_i on lane i guarantees losing the lane, the carrier does not need to consider bidding higher than u_i. So, we restrict the carrier's bids to the values within the interval $[\ell_i, u_i]$. Therefore, the optimization problem has the following form:

$$\begin{aligned} \max \quad & \pi(b) = \sum_{S \subseteq L} \Big\{ P(S,b)\ Q(L-S,b)\ \big[R(S,b) - C(S)\big] \Big\} \\ \text{subject to} \quad & b_i \in [\ell_i, u_i] \quad \forall i \in L \end{aligned}$$

where

$$\begin{aligned} b &= \text{Vector of bids for the lanes (decision variables)} \\ P(S,b) &= \text{Probability of winning the set of lanes } S \text{ with bids } b \\ Q(L-S,b) &= \text{Probability of losing the set of lanes } L-S \text{ with bids } b \\ R(S,b) &= \text{Revenue obtained from the set of lanes } S \text{ with bids } b \\ C(S) &= \text{Incremental cost of serving the set of lanes } S \end{aligned}$$

Note in the above formulation that the product $P(S,b)Q(L-S,b)$ gives the probability of winning exactly the set of lanes S with a given vector of bids b for the carrier. Since the carrier wins a lane if his bid is lower than the competitor's bid and the competitor's bids are uniformly and independently distributed, we have

$$P(S,b) = \prod_{i \in S} \mathbb{P}\{X_i \geqslant b_i\} = \prod_{i \in S} \left(\frac{u_i - b_i}{u_i - \ell_i} \right). \tag{1}$$

Similarly, since the carrier loses a lane if his bid is higher than the competitor's bid, we have:

$$Q(L-S,b) = \prod_{i \in L-S} \mathbb{P}\{X_i \leqslant b_i\} = \prod_{i \in L-S} \left(\frac{b_i - \ell_i}{u_i - \ell_i} \right). \tag{2}$$

Since the auctions are assumed to be first price, the revenue that can be obtained from a given set of lanes S with a given vector of bids b is given by:

$$R(S,b) = \sum_{i \in S} b_i. \tag{3}$$

Note that the incremental cost of serving the set of lanes in S, denoted by $C(S)$, is independent of the vector of bids b. It however depends on the way the carrier routes his trucks over his existing network and the set of lanes S. In other words, the carrier must solve a routing problem to determine his costs. We assume that the carrier solves a Lane Covering Problem (LCP) to determine his costs. Here the LCP is defined to be the problem of covering a set of given lanes with a set of cycles such that the total driving cost is minimized. LCP can be solved in polynomial time and provides a good approximation of a truckload carrier's costs ([8]).

4. A Bid Price Optimization Algorithm

The objective function in the formulation given in the previous section involves the enumeration of all possible outcomes of the auctions, i.e., $\forall S \subseteq L$. The number of potential outcomes grows exponentially as the number of lanes increases, which makes the problem difficult to solve. Even evaluating the objective function may become computationally prohibitive for a large number of lanes let alone solving the bid price optimization problem. Therefore, we focus on designing effective and efficient heuristic algorithms.

The proposed algorithm is based on coordinate search and can be described as follows. Given a vector of bids (with some arbitrary feasible values), the algorithm cycles through the coordinates of the vector. Each component (or coordinate) of the vector is updated, keeping the other components fixed, in a way that results in the maximum improvement in the objective function. This process is repeated until none of the components of the bid vector can be changed by more than a pre-specified value. A more formal description is presented in Algorithm 1.

Note that the coordinate search algorithm involves solving the optimization problem

$$\max \left\{ \pi(b) : b_i \in [\ell_i, u_i], b_j = b_j^k \quad \forall j \in L - \{i\} \right\}$$

for a given feasible bid vector b^k and a coordinate i. If this componentwise optimization problem always has a unique solution, then the coordinate search algorithm increases the objective function value each time it moves from one solution to another. Thus, the algorithm never visits the same solution more than once, i.e., it does not cycle. Note also that the coordinate search algorithm is a hill-climbing algorithm, which only utilizes local information about the objective function. Therefore, it does not guarantee finding a global optimum solution when the objective function is not concave.

Therefore, in the next section, we analyze the bid price optimization problem to identify properties of its objective function and its optimal solutions. We first analyze the optimization problem to investigate concavity and other properties of the objective function. Our analysis shows that the objective function is not concave in general but it is concave under certain conditions. Since coordinate search is applicable only if we can maximize the objective function with respect to a

Algorithm 1 Coordinate search algorithm.

Given: Lane set $L = \{1, 2, .., n\}$, intervals $[\ell_i, u_i] \quad \forall i \in L$, an initial feasible solution vector b^0 (e.g. $b_i^0 = u_i \quad \forall i \in L$), and $\epsilon > 0$.

$k \leftarrow 0$
repeat
  $\gamma \leftarrow false$
  for all $i \in L$ **do**
    $\bar{b} \leftarrow \arg\max \left\{ \pi(b) : b_i \in [\ell_i, u_i], b_j = b_j^k \quad \forall j \in L - \{i\} \right\}$
    if $|\overline{b_i} - b_i^k| > \epsilon$ **then**
      $\gamma \leftarrow true$
      $k \leftarrow k + 1$
      $b_i^k \leftarrow \overline{b_i}$
      for all $j \in L - \{i\}$ **do**
        $b_j^k \leftarrow b_j^{k-1}$
      end for
    end if
  end for
until $\gamma = false$

single component of the bid vector, we also derive the expression for maximizing the objective function with respect to an arbitrary component of the bid vector. We then use this expression to derive an upper bound on the optimal solutions which may be used in fine tuning of the solution algorithms.

5. Analysis

To obtain a better understanding of the bid price optimization problem, we first analyze the situation in which only a small number of lanes are being auctioned, since this is expected to be easier than analyzing the problem for an arbitrarily large number of lanes. We then use the insights obtained from this analysis to identify properties of the problem in more general settings.

5.1 Single Lane

Suppose the carrier is bidding on a single lane, i.e. $L = \{1\}$. There are two possible outcomes. The carrier either wins the lane or loses it. If the carrier loses the auction, he incurs zero additional cost and earns zero additional revenue. If he wins the auction, his additional revenue is equal to his bid b_1.

In this case, the optimization problem to be solved is:

$$\begin{aligned} \max \quad & \left(\frac{u_1 - b_1}{u_1 - \ell_1}\right)(b_1 - C(\{1\})) \\ \text{subject to} \quad & \ell_1 \leq b_1 \leq u_1. \end{aligned}$$

Note that π is concave in this case. In fact, it is a second degree polynomial with a negative second derivative and roots u_1 and $C(\{1\})$.

$$\begin{aligned} \pi(b) &= \left(\frac{u_1 - b_1}{u_1 - \ell_1}\right)(b_1 - C(\{1\})) \\ &= -\frac{1}{u_1 - \ell_1}(b_1 - u_1)(b_1 - C(\{1\})). \end{aligned}$$

The optimal bid can easily be computed in this case as:

$$b_1^* = \begin{cases} \frac{u_1 + C(\{1\})}{2} & \frac{u_1 + C(\{1\})}{2} \in [\ell_1, u_1] \\ \ell_1 & \frac{u_1 + C(\{1\})}{2} < \ell_1 \\ u_1 & \frac{u_1 + C(\{1\})}{2} > u_1. \end{cases} \tag{4}$$

See figures 1(a), 1(b) and 1(c) for illustrations.

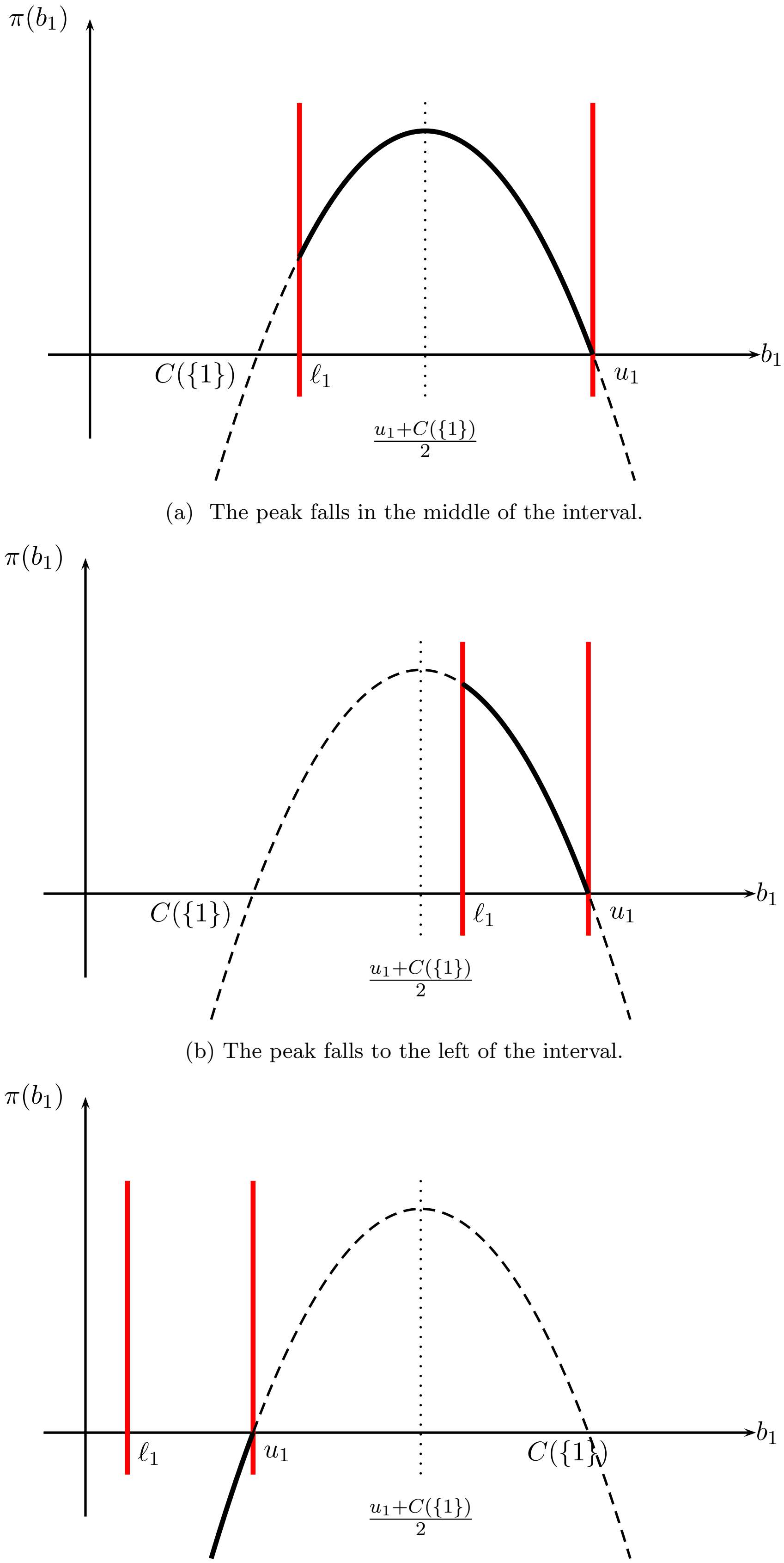
5.2 Two Lanes

Here, it is necessary to assume that the two lanes have positive synergy, that is

$$C(\{1,2\}) < C(\{1\}) + C(\{2\}),$$

since otherwise the auctions for the two lanes can be analyzed separately as two independent single lane auctions. Let $\Delta = C(\{1\}) + C(\{2\}) - C(\{1,2\})$. In this case, the objective function is a quadratic function which can be written in the form:

$$\pi(b) = \frac{1}{2} b^T H b + b^T f + g \tag{5}$$



(a) The peak falls in the middle of the interval.

(b) The peak falls to the left of the interval.

(c) The peak falls to the right of the interval.

Figure 1: Optimal solutions in the single lane case.

where

$$H = \frac{1}{(u_1 - \ell_1)(u_2 - \ell_2)} \begin{bmatrix} -2(u_2 - \ell_2) & \Delta \\ \Delta & -2(u_1 - \ell_1) \end{bmatrix}$$

which is, in fact, the Hessian of $\pi(b)$, and

$$f = \frac{1}{(u_1 - \ell_1)(u_2 - \ell_2)} \begin{bmatrix} (u_2 - \ell_2)C(\{1\}) + u_2(u_1 - \Delta) \\ (u_1 - \ell_1)C(\{2\}) + u_1(u_2 - \Delta) \end{bmatrix}$$

$$g = \frac{u_1 \ell_2 C(\{1\}) + \ell_1 u_2 C(\{2\}) - u_1 u_2 C(\{1,2\})}{(u_1 - \ell_1)(u_2 - \ell_2)}.$$

Theorem 1 *Let $L = \{1, 2\}$ and $\Delta = C(\{1\}) + C(\{2\}) - C(\{1, 2\})$. Then, $\pi(b)$ given by (5) is concave if and only if*

$$4(u_1 - \ell_1)(u_2 - \ell_2) - \Delta^2 \geqslant 0. \qquad (6)$$

Proof: It is well known that a real valued function is concave if and only if its Hessian is negative semi-definite. Thus, the function $\pi(b)$ is concave if and only if H is a negative semi-definite matrix. Note that H is a symmetric real-valued square matrix. We are going to make use of the following[1]:

Lemma 2 *Let $\mathbf{A}$ be a symmetric real $n \times n$ matrix, and let*

$$\chi(\lambda) = \det(\lambda \mathbf{I} - \mathbf{A}) = \lambda^n + a_{n-1}\lambda^{n-1} + a_{n-2}\lambda^{n-2} + \cdots + a_1\lambda + a_0.$$

Then

1. $\mathbf{A}$ *is positive definite* $\iff$ *all the a_i satisfy* $a_i(-1)^{n-i} > 0$.
2. $\mathbf{A}$ *is positive semidefinite* $\iff$ *all the a_i satisfy* $a_i(-1)^{n-i} \geq 0$.
3. $\mathbf{A}$ *is negative definite* $\iff$ *all the* $a_i > 0$.
4. $\mathbf{A}$ *is negative semidefinite* $\iff$ *all the* $a_i \geq 0$.
5. $\mathbf{A}$ *is indefinite* $\iff$ *none of the previous four conditions on the a_i holds.*

[1] Proof can be found in Sydsaeter and Hammond ([25]) and Loftin ([12]).

For H,

$$
\begin{aligned}
\chi(\lambda) &= det(\lambda\mathbf{I} - H) \\
&= det\left(\begin{bmatrix} \lambda + \frac{2}{u_1-\ell_1} & \frac{\Delta}{(u_1-\ell_1)(u_2-\ell_2)} \\ \frac{\Delta}{(u_1-\ell_1)(u_2-\ell_2)} & \lambda + \frac{2}{u_2-\ell_2} \end{bmatrix}\right) \\
&= \lambda^2 + 2\left(\frac{1}{u_1-\ell_1} + \frac{1}{u_2-\ell_2}\right)\lambda + \frac{4}{(u_1-\ell_1)(u_2-\ell_2)} - \left(\frac{\Delta}{(u_1-\ell_1)(u_2-\ell_2)}\right)^2 \\
&= \lambda^2 + \underbrace{2\left(\frac{1}{u_1-\ell_1} + \frac{1}{u_2-\ell_2}\right)}_{\geq 0}\lambda + \underbrace{\frac{4(u_1-\ell_1)(u_2-\ell_2) - \Delta^2}{(u_1-\ell_1)^2(u_2-\ell_2)^2}}_{?}
\end{aligned}
$$

It follows that H is negative semidefinite $\Leftrightarrow 4(u_1-\ell_1)(u_2-\ell_2) - \Delta^2 \geq 0$.

□

If (6) does not hold, then H is indefinite which means that $\pi(b)$ is saddle shaped. If $\pi(b)$ is concave, the optimal solution may be in the interior of the feasible region. If $\pi(b)$ is saddle shaped, then the optimal solution is certainly on the boundary of the feasible region. Also, when $\pi(b)$ is concave, the coordinate search algorithm is guaranteed to find the optimal solution. Otherwise, coordinate search may get stuck in a local optimum failing to find the global optimum.

5.3 Componentwise Optimization for an Arbitrary Number of Lanes

The coordinate search algorithm relies on our ability to maximize expected profit when only a given component of the bid vector is allowed to vary and all other components are kept at their current values. In other words, for each component b_i of b, we treat $\pi(b)$ as if it is only a function of b_i and maximize $\pi(b)$ with respect to b_i. We first write $\pi(b)$ as a function of a single bid to see if we can easily maximize it with respect to that bid.

Proposition 3 *If the objective function $\pi(b)$ is written as a function of a single bid b_i, we get:*

$$
\begin{aligned}
\pi_i(b) = \left(\frac{u_i - b_i}{u_i - \ell_i}\right)\left(b_i - \sum_{\substack{V \subseteq L \\ i \notin V}} P(V,b)\ Q(L - V - \{i\}, b)\left(C(V + \{i\}) - C(V)\right)\right) \\
+ \sum_{\substack{V \subseteq L \\ i \notin V}} P(V,b)\ Q(L - V - \{i\}, b)[R(V,b) - C(V)].
\end{aligned}
\tag{7}
$$

Proof: See Appendix. □

Let

$$
\sigma_i(b) = \sum_{\substack{V \subseteq L \\ i \notin V}} P(V,b)\ Q(L - V - \{i\}, b)\left(C(V + \{i\}) - C(V)\right). \tag{8}
$$

Then it is easy to see that $\pi_i(b)$ is equal to the sum of a constant and a second degree concave polynomial with roots u_i and $\sigma_i(b)$. Therefore, given an arbitrary feasible vector of bids b and an index i, the best improvement in the objective function is achieved, keeping all the other bids constant, when we set b_i to:

$$\overline{b_i} = \begin{cases} \frac{u_1+\sigma_i(b)}{2} & \frac{u_1+\sigma_i(b)}{2} \in [\ell_i, u_i] \\ \ell_i & \frac{u_1+\sigma_i(b)}{2} < \ell_i \\ u_i & \frac{u_1+\sigma_i(b)}{2} > u_i. \end{cases} \tag{9}$$

It is important to note that $\overline{b_i}$ is the unique maximizer of $\pi_i(b)$ on $[\ell_i, u_i]$. Because of the uniqueness of the componentwise maximizer, the coordinate search algorithm moves from one solution to another if and only if the objective function value increases as a result of that move. Consequently, the algorithm never cycles.

Note that $\sigma_i(b)$ is a measure of synergy between lane i and the rest of the lanes. It is equal to the expected increase in the cost of the carrier if the carrier wins lane i given his bids on the other lanes. It can also be used to find an upper bound on the value of the optimal bids. The worst scenario when lane i is added to a given set V of lanes is that there are no synergies with the lanes in V and thus lane i has to be served by itself, i.e. in a cycle that consists of a loaded move along the lane and an empty move in the opposite direction. Let c_i be the one-way cost of the lane. So, the worst case incremental cost of serving the lane if the carrier wins the auction is equal to $2c_i$. Mathematically, this means that:

$$C(V + \{i\}) - C(V) \leqslant 2c_i.$$

Then,

$$\sigma_i(b) \leqslant \sum_{\substack{V \subseteq L \\ i \notin V}} P(V, b)\ Q(L - V - \{i\}, b)\ 2c_i = 2c_i$$

and

$$\overline{b_i} = \frac{u_i + \sigma_i(b)}{2} \leqslant \frac{u_i + 2c_i}{2}.$$

Therefore, an upper bound for the optimal bid is identified. Note that the cost of serving a single lane when the carrier does not have an existing network is also equal to $2c_i$. This information can be used to speed up the algorithm. When we obtain the initial bid value for each lane by replacing $C(\{i\})$ with $2c_i$ in (4), then the coordinate search can be performed ignoring the coordinates for which the initial values are equal to their lower bounds.

6. Computational Experiments

We conducted two sets of experiments. First, we investigate the performance of the Coordinate Search Algorithm. Second, we analyze the value bid price optimization can bring to a carrier in a simulation study.

6.1 Performance of the Coordinate Search Algorithm

The objective function $\pi(b)$ of the bid price optimization problem is in general not concave and thus may have multiple local optima. Our coordinate search algorithm, therefore, may get stuck in a local optimum failing to find a global optimal solution.

In order to measure the effectiveness of our algorithm, we conducted a computational experiment in which we compared the quality of its solutions with the solutions obtained by various other global optimization solvers: *BARON* [26, 21], Matlab's *pattern search*, and a Matlab implementation of *multi-level coordinate search* ([10]) that combines pattern search with a branching scheme.

For the experiments with *BARON*, we used 50 Euclidean instances with points randomly generated inside a 300×300 region. The carrier does not have an existing network and ten lanes are being auctioned. The bid interval for each lane i is randomly set such that $[\ell_i, u_i] \subseteq [0, 4c_i]$. *BARON* was run with default parameter settings, e.g., it terminates after 1000 CPU seconds or when the optimality gap is smaller than 10%. Our coordinate search algorithm started with $b_i = u_i$ for all lanes i and was set to terminate when it was no longer possible to move more than a distance of 1×10^{-6} from the current solution.

For each instance, *BARON* terminated because it reached the time limit of 1000 CPU seconds and it returned the solution it had computed during its preprocessing phase as the best solution found. The coordinate search algorithm returned the same solution as *BARON* for *all* instances doing so in under five CPU seconds. (The time for the coordinate search algorithm includes instance generation, solving LCPs for all possible subsets of lanes, and writing the input files for *BARON*; the coordinate search portion itself takes a negligible amount of CPU time.) It is important to note that the final optimality gaps reported by *BARON* are huge despite computational efforts. This demonstrates the inherent difficulty of computing verifiably global optimal solutions to the bid price optimization problem.

For the experiments with Matlab's pattern search and the Matlab implementation of multi-coordinate search, we used 100 Euclidean instances with points randomly generated inside a $300 \times$

300 region. The carrier does not have an existing network and five lanes are being auctioned. We used 5 lanes instead of ten lanes to keep computation times of the Matlab routines reasonable. The bid interval for each lane i is randomly set such that $[\ell_i, u_i] \subseteq [0, 4c_i]$. All algorithms, i.e., pattern search, multi-level coordinate search, and coordinate search, returned the same solution. Again, the coordinate search algorithm finished in a few seconds, orders of magnitudes faster than the other algorithms.

6.2 A Simulation Study

In order to investigate the value for a carrier of using bid price optimization, we simulate a transportation procurement marketplace over 52 periods under various settings. The simulated marketplace has the following properties:

1. Origin-destination pairs come from 270 fixed points randomly generated in a 1.0×1.0 square at the beginning of the simulation. Each point has two coordinates and the distance between any two points is computed using the Euclidean metric.

2. The 1.0×1.0 square is divided into 9 (3 by 3) equal sized regions, referred to as South West (SW), South (S), South East (SE), West (W), Central (C), East (E), North West (NW), North (N), North East (NE).

3. There is a single auctioneer who runs 10 simultaneous independent single-lane auctions in each period. Each of the single-lane auctions is for a contract for a single period. There is no reserve price.

4. There are two carriers competing for the contracts. Both carriers have existing networks consisting of lanes with long term contracts (for the entire planning horizon). Both carriers compute the operating costs for the existing networks using LCP. One of the carriers solves the bid price optimization problem to determine his bids and will be referred to as SB (Smart Bidder). The other carrier computes the marginal cost, i.e., insertion cost into the existing network, of a lane and adds a 40% markup to the marginal cost to determine his bid for that lane. (The choice of a 40% markup may be slightly on the high side, but not unheard of in practice.) We will refer to this carrier as MM (Margin plus Markup bidder).

5. Both carriers place bids on every lane being auctioned in the marketplace.

Even though we assume that there is only a single competitor of SB in the market, MM may represent several competitors from SB's perspective. In this setting, the competitor MM is not completely naive. MM uses LCP to compute operating costs for the existing network. In practice, it is not unusual to encounter carriers with less sophisticated routing technologies.

In the simulation study, we vary three characteristics of the setting described above to explore their impact on the performance of the carriers. We vary the concentration of the existing network of lanes with long-term contracts for both carriers, i.e., the regions in which the carriers operate, we vary the size of the existing network of lanes with long-term contracts for both carriers, and we vary the assumption of SB concerning the bids of his competitors, i.e., the bounds of the bid intervals used.

With respect to the concentration of the existing networks of the carriers, we consider three situations:

E1. **Similar carrier networks:** The long-term lane contracts of the existing network for both carriers have origins and destinations that are drawn with equal probability from each of the regions.

    The origins and destinations of the short-term lane contracts being auctioned are also drawn with equal probability from each of the regions.

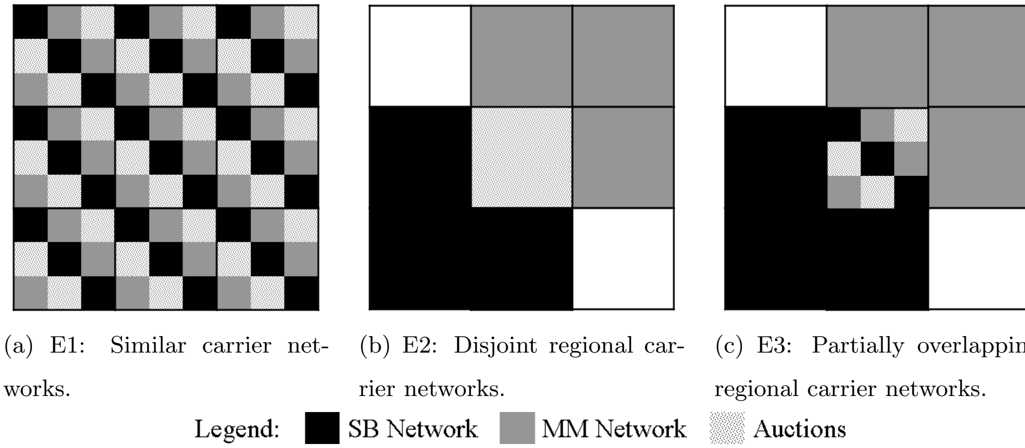
E2. **Disjoint regional carrier networks:** The origins and destinations of SB's long-term lane contracts are drawn with equal probability from regions SW, S, and W and the origins and destinations of MM's long-term lane contracts are drawn with equal probability from regions E, N, and NE. Note that the existing networks of the carriers do not overlap.

    The origins and destinations of the short-term lane contracts being auctioned are drawn with probability 0.68 from region C and with probability 0.04 from the remaining regions.

E3. **Partially overlapping regional carrier networks:** The origins and destinations of SB's long-term lane contracts are drawn with equal probability from regions SW, S, W, and C and the origins and destinations of MM's long-term lane contracts are drawn with equal probability from regions E, N, NE, and C. Note that the existing networks of the carriers overlap in region C.

    The origins and destinations of the short-term lane contracts being auctioned are drawn with probability 0.68 from region C and with probability 0.04 from the remaining regions.

Figure 2 illustrates the concentrations of carrier networks and auctioned lanes in the experiments.



(a) E1: Similar carrier networks.

(b) E2: Disjoint regional carrier networks.

(c) E3: Partially overlapping regional carrier networks.

Figure 2: Concentrations of carrier networks and auctioned lanes for the simulation study.

For each of these three situations, we have varied the size of the existing networks, 30, 45, or 90 lanes, and the bid intervals assumed by SB, $\frac{\ell_i}{c_i} \in \{0.5, 1.0\}$ and $\frac{u_i}{c_i} \in \{1.5, 2.0, 2.5\}$. The results of the experiments are summarized in Tables 1, 2, and 3. The statistics reported are average carrier profits, average carrier profit margins, and average number of auctions won by the carriers. The profit amounts reported refer to the (additional) profit due to participating in the auctions; it does not include any profits from the existing network of long-term contracts. Therefore, the profit in a period is computed as the revenue from the lanes won minus the difference between the operating costs of existing network plus the lanes won and the operating cost of the existing network. The profit margin in a period is computed as ratio of the profit in the period and the the difference between the the operating costs of existing network plus the lanes won and the operating cost of the existing network. Each of the Tables 1, 2, and 3 consists of two parts. In the first part, the averages are taken over different carrier network size combinations. In the second part, the averages are taken over different bid intervals. In what follows, we analyze and interpret these results.

6.2.1 Similar Carrier Networks (E1)

As mentioned above, the results of the simulation experiment for the setting in which the carriers have similar existing networks are summarized in Table 1.

Table 1(a) contains average values of carrier profits, carrier profit margins, and the number of auctions won by each carrier computed over different distribution intervals. We observe that

Table 1: E1: Similar carrier networks.

(a) Network size.

Network Size		**Profit**			**Margin**			**#Auctions won**		
SB	**MM**	**SB**	**MM**	**diff**	**SB**	**MM**	**diff**	**SB**	**MM**	**diff**
30	30	\$46.53	\$32.17	\$14.36	28.69%	26.94%	1.75%	332.5	187.5	145.0
30	45	\$48.81	\$51.00	-\$2.19	38.73%	39.68%	-0.95%	189.3	330.7	-141.3
30	90	\$50.66	\$53.15	-\$2.50	42.65%	41.30%	1.35%	259.7	260.3	-0.7
45	30	\$59.55	\$37.52	\$22.03	37.93%	34.96%	2.97%	308.5	211.5	97.0
45	45	\$47.30	\$39.39	\$7.91	39.05%	28.38%	10.66%	247.0	273.0	-26.0
45	90	\$41.60	\$50.10	-\$8.50	40.73%	35.41%	5.32%	232.7	287.3	-54.7
90	30	\$68.70	\$32.03	\$36.67	46.50%	30.36%	16.14%	306.3	213.7	92.7
90	45	\$65.37	\$30.61	\$34.76	47.41%	29.12%	18.29%	306.2	213.8	92.3
90	90	\$63.26	\$34.17	\$29.09	48.09%	35.74%	12.34%	250.0	270.0	-20.0

(b) Uniform distribution interval.

Bid Interval		**Profit**			**Margin**			**#Auctions won**		
$\frac{\ell_i}{c_i}$	$\frac{u_i}{c_i}$	**SB**	**MM**	**diff**	**SB**	**MM**	**diff**	**SB**	**MM**	**diff**
0.5	1.5	\$34.50	\$20.42	\$14.08	16.62%	32.51%	-15.89%	330.8	189.2	141.6
0.5	2	\$62.70	\$38.28	\$24.43	40.33%	33.05%	7.28%	275.6	244.4	31.1
0.5	2.5	\$66.18	\$62.14	\$4.03	66.27%	35.45%	30.83%	210.8	309.2	-98.4
1	1.5	\$34.66	\$19.94	\$14.72	16.64%	32.23%	-15.59%	316.4	203.6	112.9
1	2	\$63.17	\$37.76	\$25.41	40.49%	32.77%	7.72%	276.6	243.4	33.1
1	2.5	\$66.62	\$61.55	\$5.08	66.15%	35.25%	30.91%	211.3	308.7	-97.3

SB almost always outperforms MM in terms of profit margin. SB also achieves higher profits and wins more auctions when the size of his existing network is greater than or equal to the size of MM's existing network. This indicates that SB's use of bid price optimization, which allows him to effectively exploit potential synergies between his existing network and the lanes being auctioned, gives him an advantage, but that this advantage is not sufficient to overcome a difference in existing network size. We conclude that economies of scale are more important than economies of scope.

Table 1(b) demonstrates the effect of SB's assumptions on his competitor's behavior, i.e., the impact of the bid interval (ℓ_i, u_i), on SB's profits. Recall that for each lane $i \in L$, the lower bound ℓ_i and the upper bound u_i are chosen such that $\frac{\ell_i}{c_i} \in \{0.5, 1.0\}$ and $\frac{u_i}{c_i} \in \{1.5, 2.0, 2.5\}$, respectively. The first thing to note is that the difference in profits for $\frac{\ell_i}{c_i} = 0.5$ and $\frac{\ell_i}{c_i} = 1.0$ is negligible. On the other hand, we observe that the use of a smaller ratio $\frac{u_i}{c_i}$ enables SB to win more auctions, but leads to a decrease in profits and profit margins of *both* carriers. This is interesting and counter-intuitive. Since SB and MM are competing carriers in the same market, one would expect to see an increase in MM's profits if SB's decisions cause SB's profits to decrease, and vice versa. However, we see that the carriers' strategies impact the entire market. By using a smaller ratio $\frac{u_i}{c_i}$, SB is bidding more aggressively and increasing his chances of winning auctions while forgoing some of the profits. This situation is analogous to a price cut initiated by SB to increase his market share. In the end, SB increases his market share but ends up with less profit because of a disproportionate increase in costs. (Of course the shippers benefit from this increased competition because they spend less on transportation services.)

6.2.2 Disjoint Regional Carrier Networks (E2)

As mentioned above, the results of the simulation experiment for the setting in which the carriers have disjoint existing networks are summarized in Table 2.

In this situation, we expect the carriers to compete more fiercely for the lanes being auctioned. Since the lanes being auctioned complement the carriers' networks, we also expect the ability to exploit synergies to be critical. In other words, we expect SB to perform much better than MM.

The results presented in Table 2 confirm our expectations. There is a significant difference between the performance of SB and MM. SB outperforms MM in terms of all three statistics, i.e. profit, profit margin, and the number of auctions won, regardless of existing network size, even when SB has a mcuh smaller existing network than MM (30 long-term contract lanes vs. 90 long-term contract lanes). There is also an increase in SB's profit margins compared to the profit margins

Table 2: E2: Disjoint regional carrier networks.

(a) Effect of carrier network size

Network Size		**Profit**			**Margin**			**#Auctions won**		
SB	**MM**	**SB**	**MM**	**diff**	**SB**	**MM**	**diff**	**SB**	**MM**	**diff**
30	30	$47.20	$10.70	$36.49	23.01%	39.98%	-16.98%	467.3	52.7	414.7
30	45	$39.25	$16.39	$22.87	23.15%	34.27%	-11.12%	419.5	100.5	319.0
30	90	$40.39	$18.13	$22.26	23.58%	39.86%	-16.28%	416.2	103.8	312.3
45	30	$45.29	$12.38	$32.91	23.84%	36.86%	-13.02%	452.7	67.3	385.3
45	45	$47.39	$11.02	$36.37	24.08%	37.89%	-13.81%	459.2	60.8	398.3
45	90	$43.73	$15.75	$27.99	25.34%	37.34%	-12.00%	425.8	94.2	331.7
90	30	$47.52	$11.61	$35.91	24.68%	34.95%	-10.26%	447.8	72.2	375.7
90	45	$45.35	$13.16	$32.19	24.80%	35.17%	-10.37%	445.0	75.0	370.0
90	90	$49.48	$9.12	$40.36	23.88%	40.38%	-16.50%	472.8	47.2	425.7

(b) Effect of uniform distribution interval

Bid Interval		**Profit**			**Margin**			**#Auctions won**		
$\frac{\ell_i}{c_i}$	$\frac{u_i}{c_i}$	**SB**	**MM**	**diff**	**SB**	**MM**	**diff**	**SB**	**MM**	**diff**
0.5	1.5	$10.12	$4.23	$5.90	4.44%	36.68%	-32.23%	485.9	34.1	451.8
0.5	2	$51.89	$14.02	$37.87	26.24%	37.20%	-10.96%	438.4	81.6	356.9
0.5	2.5	$75.27	$22.86	$52.41	43.17%	38.69%	4.47%	404.7	115.3	289.3
1	1.5	$9.33	$4.21	$5.13	4.10%	36.64%	-32.54%	486.0	34.0	452.0
1	2	$49.14	$12.27	$36.87	24.35%	36.89%	-12.54%	445.6	74.4	371.1
1	2.5	$74.64	$21.26	$53.39	41.95%	38.37%	3.58%	410.3	109.7	300.7

observed when both carriers had similar existing networks (see Table 1).

Note that the different bid intervals used by SB show a similar effect as before. Again, the difference between $\frac{\ell_i}{c_i} = 0.5$ and $\frac{\ell_i}{c_i} = 1.0$ is negligible. Similarly, the use of a smaller ratio $\frac{u_i}{c_i}$ enables SB to win more auctions, but leads to a decrease in the profits and profit margins of both carriers. However, the chosen ratio $\frac{u_i}{c_i}$ has a greater impact on SB's profits. Due to the characteristics of existing carrier networks and the lanes being auctioned, the carriers must carefully choose the lanes they want and the lanes they do not want. If SB bids too aggressively (by using a small ratio $\frac{u_i}{c_i}$), he ends up with lanes he should not have won because they have little synergies with his existing network.

6.2.3 Partially Overlapping Regional Carrier Networks (E3)

As mentioned above, the results of the simulation experiment for the setting in which the carriers have overlapping existing networks are summarized in Table 3. Note that this setting is a crossover between the setting with disjoint existing networks and similar existing networks. Therefore, we expect the results to be somewhere between the results of the two previously discussed simulation experiments.

Once again, the results summarized in Table 3 confirm our expectations. SB outperforms MM regardless of network size, however the differences are not as large as for the setting with disjoint existing networks. The ability to exploit synergies is still significant but not as critical.

The bid interval used by SB has similar effects on the carriers' statistics (see Table 3(b)). We again observe that setting $\frac{\ell_i}{c_i} = 0.5$ or $\frac{\ell_i}{c_i} = 1.0$ makes little difference and that setting a smaller ratio $\frac{u_i}{c_i}$ enables SB to win more auctions, but leads to a decrease in profits and profit margins for both carriers.

Our simulation study indicates that the use of bid price optimization does give SB an advantage in terms of achieving economies of scope by exploiting synergies between lanes. The performance of SB is affected by SB's existing set of long-term lane contracts as well as MM's existing set of long-term lane contracts; there are clear economies of scale. The study also indicates that the pricing decisions of a carrier have system-wide effects. A carrier with an overly aggressive bidding strategy risks degrading the profitability of not only himself but also the other carriers in the market. The ratio $\frac{u_i}{c_i}$ determines the level of aggressiveness of SB; the lower the ratio, the more aggressive. Therefore, choosing the right bid interval is of critical importance for the performance of bid price optimization. Recall that the bid interval represents SB's assessment of his competitors' behavior.

Table 3: E3: Partially overlapping regional carrier networks

(a) Effect of network size.

Network Size		**Profit**			**Margin**			**#Auctions Won**		
SB	**MM**	**SB**	**MM**	**diff**	**SB**	**MM**	**diff**	**SB**	**MM**	**diff**
30	30	$30.46	$23.96	$6.51	25.07%	34.31%	-9.24%	299.8	220.2	79.7
30	45	$27.37	$25.10	$2.27	24.72%	35.34%	-10.63%	276.2	243.8	32.3
30	90	$27.43	$25.65	$1.79	24.56%	35.28%	-10.71%	277.2	242.8	34.3
45	30	$32.54	$21.21	$11.33	28.07%	31.89%	-3.82%	290.7	229.3	61.3
45	45	$32.01	$25.14	$6.87	29.45%	35.56%	-6.11%	287.2	232.8	54.3
45	90	$34.89	$21.35	$13.54	30.09%	30.76%	-0.67%	305.5	214.5	91.0
90	30	$42.33	$11.02	$31.32	30.28%	17.37%	12.91%	350.5	169.5	181.0
90	45	$34.56	$12.44	$22.11	23.44%	22.68%	0.76%	354.0	166.0	188.0
90	90	$36.87	$17.28	$19.59	25.19%	35.94%	-10.75%	365.0	155.0	210.0

(b) Effect of distribution interval

Bid Interval		**Profit**			**Margin**			**#Auctions won**		
$\frac{\ell_i}{c_i}$	$\frac{u_i}{c_i}$	**SB**	**MM**	**diff**	**SB**	**MM**	**diff**	**SB**	**MM**	**diff**
0.5	1.5	$10.79	$11.02	-$0.23	6.30%	29.40%	-23.10%	366.9	153.1	213.8
0.5	2	$39.44	$20.27	$19.17	28.38%	31.18%	-2.80%	308.8	211.2	97.6
0.5	2.5	$52.70	$31.80	$20.89	49.24%	33.25%	15.99%	249.6	270.4	-20.9
1	1.5	$10.69	$10.92	-$0.24	6.23%	29.25%	-23.02%	366.8	153.2	213.6
1	2	$38.63	$19.69	$18.94	27.56%	30.82%	-3.26%	310.8	209.2	101.6
1	2.5	$49.89	$32.26	$17.63	48.18%	33.01%	15.17%	241.7	278.3	-36.6

Thus, the effectiveness of bid price optimization depends on SB's ability to properly assess his competitors' behavior.

7. Final Remarks and Future Research

We have demonstrated that carriers participating in multiple simultaneous single lane auctions in transportation market places can benefit from the use of bid price optimization. Effective bid price optimization has to take into account both the synergies among lanes, existing lanes and lanes being auctioned, and the competitors' behavior. The latter, of course, is challenging. We have chosen to model the competitors' bidding strategies as a uniform random variable in some price interval. Although this seems reasonable, other choices should be investigated as well. Similarly, we have chosen to model the carrier's decision making process once the set of lanes to be served in a period is known as a lane covering problem. Here too, other options are possible and require further exploration. We believe that transportation market places will continue to increase in popularity and that therefore research along the lines indicated above will not only be scientifically interesting, but also practically valuable.

References

[1] Na An, Wedad Elmaghraby, and Pinar Keskinocak. Bidding strategies and their impact on revenues in combinatorial auctions. *Journal of Revenue and Pricing Management*, 3(4):337–357, 2005.

[2] G. Anandalingam, Robert W. Day, and S. Raghavan. The landscape of electronic market design. *Management Science*, 51(3):316–327, 2005.

[3] Teodor Gabriel Crainic and Gilbert Laporte, editors. *Fleet Management and Logistics*. Center for Transportation Research 25th anniversary series. Kluwer Academic Publishers, Norwell, MA, USA, 1998.

[4] Albert Douma, Peter Schuur, and Matthieu van der Heijden. Applying revenue management to agent-based transportation planning. *Beta Working Paper Series, WP-169*, 2006.

[5] Wedad Elmaghraby. Pricing and auctions in e-marketplaces. In David Simchi-Levi, S. David Wu, and Z. Max Shen, editors, *Handbook of Quantitative Supply Chain Analysis: Modeling*

in the E-Business Era, International Series in Operations Research and Management Science, Norwell, MA, 2004. Kluwer Academic Publishers.

[6] Wedad Elmaghraby and Pinar Keskinocak. Combinatorial auctions in procurement. In C. Billington, T. Harrison, H. Lee, and J. Neale., editors, *The Practice of Supply Chain Management*, Norwell,MA, 2003. Kluwer Academic Publishers.

[7] Wedad Elmaghraby and Pinar Keskinocak. Dynamic pricing: Research overview, current practices and future directions. *Management Science*, 49(10):1287–1309, 2003.

[8] Ozlem Ergun, Gultekin Kuyzu, and Martin Savelsbergh. Shipper collaboration. *Computers & Operations Research*, 34(6):1551–1560, July 2007.

[9] Miguel Figliozzi. *Performance and Analysis of Spot Truckload Procurement Markets Using Sequential Auctions*. PhD thesis, University of Maryland, 2004.

[10] W. Huyer and A. Neumaier. Global Optimization by Multilevel Coordinate Search. *Journal of Global Optimization*, 14:331–355, 1999.

[11] John O. Ledyard, Mark Olson, David Porter, Joseph A. Swanson, and David P. Torma. The first use of a combined value auction for transportation services. *Interfaces*, 32(5):4–12, September - October 2002.

[12] John Loftin. Definiteness of a symmetric matrix. http://www.math.columbia.edu/ loftin/ao/polyroot/polyroo
Accessed December 3, 2006.

[13] Martijn Mes, Matthieu van der Heijden, and Peter Schuur. Opportunity costs calculation in agent-based vehicle routing and scheduling. *Beta Working Paper Series, WP-168*, 2006.

[14] Debasis Mishra. *Auction Design for Multi-Item Procurement*. PhD thesis, University of Wisconsin, Madison, WI, USA, 2004.

[15] E. W. Moore, J. M. Warmke, and L. R. Gorban. The indispensable role of management science in centralizing freight operations at reynolds metals company. *Interfaces*, 21(1):107–129, 1991.

[16] S. Nandiraju and A. Regan. Freight transportation electronic marketplaces: A survey of market clearing mechanisms and exploration of important research issues. In *Proceedings 84th Annual Meeting of the Transportation Research Board*, Washington, D.C., January 2005.

[17] Warren B. Powell. Marginal cost pricing of truckload services: A comparison of two approaches. *Transportation Research, Part B: Methodological*, 19(5):433–445, 1985.

[18] Warren B. Powell. A review of sensitivity results for linear networks and a new approximation to reduce the effects of degeneracy. *Transportation Science*, 23(4):231–243, 1989.

[19] Warren B. Powell, Yossi Sheffi, K. Nickerson, K. Butterbaugh, and S. Atherton. Maximizing profits for north american van lines' truckload division: A new framework for pricing and operations. *Interfaces*, 18:21–41, 1988.

[20] M. Rothkopf. *Models of auctions and competitive bidding*, volume 6 of *Handbooks in Operations Research and Management Science*, chapter 19. Elsevier Science Publishing Co., New York, 1994.

[21] N. V. Sahinidis and M. Tawarmalani. *BARON 7.2.5: Global Optimization of Mixed-Integer Nonlinear Programs,* User's Manual, 2005. Available at http://www.gams.com/dd/docs/solvers/baron.pdf.

[22] Yossi Sheffi. Combinatorial auctions in the procurement of transportation services. *Interfaces*, 34(4):245–252, July-August 2004.

[23] J. Song and A. Regan. Combinatorial auctions for transportation service procurement: the carrier perspective. *Transportation Research Record*, 1833:40–46, 2002.

[24] J. Song and A. Regan. Approximation algorithms for the bid valuation and structuring problem in combinatorial auctions for the procurement of freight transportation contracts. *Transportation Research, Part B: Methodological*, 39(10):914–933, 2005.

[25] Knut Sydsaeter and Peter J. Hammond. *Mathematics for Economic Analysis*. Prentice Hall, 1995.

[26] M. Tawarmalani and N. V. Sahinidis. Global optimization of mixed-integer nonlinear programs: A theoretical and computational study. *Mathematical Programming*, 99(3):563–591, 2004.

[27] Huseyin Topaloglu and Warren B. Powell. Sensitivity analysis of a dynamic fleet management model using approximate dynamic programming. Technical report, Cornell University, School of Operations Research and Industrial Engineering, Ithaca, NY, USA, 2004.

[28] P. Toth and D. Vigo. *The Vehicle Routing Problem.* SIAM Monographs on Discrete Mathematics and Applications. Society for Industrial and Applied Mathematics, Philadelphia, PA, USA, 2002.

[29] Daniel D. Zeng, James C. Cox, and Moshe Dror. Coordination of purchasing and bidding activities across markets. In *HICSS '04: Proceedings of the 37th Annual Hawaii International Conference on System Sciences (HICSS'04) - Track 7*, page 70167.1, Washington, DC, USA, 2004. IEEE Computer Society.

Appendix

Proposition If the objective function $\pi(b)$ is written as a function of a single bid b_i, we get:

$$\pi_i(b) = \left(\frac{u_i - b_i}{u_i - \ell_i}\right)\left(b_i - \sum_{\substack{V \subseteq L \\ i \notin V}} P(V,b)\ Q(L - V - \{i\}, b)\,(C(V + \{i\}) - C(V))\right)$$
$$+ \sum_{\substack{V \subseteq L \\ i \notin V}} P(V,b)\ Q(L - V - \{i\}, b)[R(V,b) - C(V)].$$

Proof:

Recall that

$$\begin{aligned} P(S,b) &= \prod_{i \in S} \mathbb{P}\{X_i \geqslant b_i\} = \prod_{i \in S}\left(\frac{u_i - b_i}{u_i - \ell_i}\right) \\ Q(L - S, b) &= \prod_{i \in L - S} \mathbb{P}\{X_i \leqslant b_i\} = \prod_{i \in L - S}\left(\frac{b_i - \ell_i}{u_i - \ell_i}\right) \\ R(S,b) &= \sum_{i \in S} b_i \end{aligned}$$

Then,

$$\begin{aligned} \pi(b) &= \sum_{S \subseteq L} P(S,b)\ Q(L - S, b)\ [R(S,b) - C(S)] \\ &= \sum_{\substack{U \subseteq L \\ i \in U}} P(U,b)\ Q(L - U, b)\ [R(U,b) - C(U)] \\ &\quad + \sum_{\substack{V \subseteq L \\ i \notin V}} P(V,b)\ Q(L - V, b)\ [R(V,b) - C(V)] \\ &= P(\{i\},b)\ \sum_{\substack{V \subseteq L \\ i \notin V}} P(V,b)\ Q(L - V - \{i\}, b)\ [R(V + \{i\}, b) - C(V + \{i\})] \\ &\quad + Q(\{i\},b)\ \sum_{\substack{V \subseteq L \\ i \notin V}} P(V,b)\ Q(L - V - \{i\}, b)\ [R(V,b) - C(V)] \\ &= P(\{i\},b)\ \sum_{\substack{V \subseteq L \\ i \notin V}} P(V,b)\ Q(L - V - \{i\}, b)\ [R(V + \{i\}, b) - C(V + \{i\})] \\ &\quad + [1 - P(\{i\},b)]\ \sum_{\substack{V \subseteq L \\ i \notin V}} P(V,b)\ Q(L - V - \{i\}, b)\ [R(V,b) - C(V)] \\ &= P(\{i\},b)\ \sum_{\substack{V \subseteq L \\ i \notin V}} P(V,b)\ Q(L - V - \{i\}, b)\ [b_i + R(V,b) - C(V + \{i\}) + C(V) - C(V)] \end{aligned}$$

$$
\begin{aligned}
&+ [1 - P(\{i\}, b)] \sum_{\substack{V \subseteq L \\ i \notin V}} P(V, b)\ Q(L - V - \{i\}, b)\ [R(V, b) - C(V)] \\
&= P(\{i\}, b) \sum_{\substack{V \subseteq L \\ i \notin V}} P(V, b)\ Q(L - V - \{i\}, b)\ [b_i - C(V + \{i\}) + C(V)] \\
&\quad + P(\{i\}, b) \sum_{\substack{V \subseteq L \\ i \notin V}} P(V, b)\ Q(L - V - \{i\}, b)\ [R(V, b) - C(V)] \\
&\quad + [1 - P(\{i\}, b)] \sum_{\substack{V \subseteq L \\ i \notin V}} P(V, b)\ Q(L - V - \{i\}, b)\ [R(V, b) - C(V)] \\
&= P(\{i\}, b) \sum_{\substack{V \subseteq L \\ i \notin V}} P(V, b)\ Q(L - V - \{i\}, b)\ [b_i - C(V + \{i\}) + C(V)] \\
&\quad + \sum_{\substack{V \subseteq L \\ i \notin V}} P(V, b)\ Q(L - V - \{i\}, b)\ [R(V, b) - C(V)] \\
&= P(\{i\}, b) \left(b_i \underbrace{\sum_{\substack{V \subseteq L \\ i \notin V}} P(V, b)\ Q(L - V - \{i\}, b)}_{=1} \right. \\
&\qquad \left. - \sum_{\substack{V \subseteq L \\ i \notin V}} P(V, b)\ Q(L - V - \{i\}, b)\ [C(V + \{i\}) - C(V)] \right) \\
&\quad + \sum_{\substack{V \subseteq L \\ i \notin V}} P(V, b)\ Q(L - V - \{i\}, b)\ [R(V, b) - C(V)] \\
&= P(\{i\}, b) \left(b_i - \sum_{\substack{V \subseteq L \\ i \notin V}} P(V, b)\ Q(L - V - \{i\}, b) \left(C(V + \{i\}) - C(V) \right) \right) \\
&\quad + \sum_{\substack{V \subseteq L \\ i \notin V}} P(V, b)\ Q(L - V - \{i\}, b) [R(V, b) - C(V)]
\end{aligned}
$$

Since in our case $P(\{i\}, b) = \frac{u_i - b_i}{u_i - \ell_i}$ we get:

$$\pi_i(b) = \left(\frac{u_i - b_i}{u_i - \ell_i}\right) \left(b_i - \sum_{\substack{V \subseteq L \\ i \notin V}} P(V, b)\ Q(L - V - \{i\}, b)\, (C(V + \{i\}) - C(V)) \right)$$
$$+ \sum_{\substack{V \subseteq L \\ i \notin V}} P(V, b)\ Q(L - V - \{i\}, b)[R(V, b) - C(V)]$$

□