In[15]:= `$Assumptions = {d > 0}`

Out[15]=

{d > 0}

In[16]:= `p = r^3 - 3/2*(1+d)^(1/3)*r^2 + 1/2`

Out[16]=

$$\frac{1}{2} - \frac{3}{2} (1+d)^{1/3} r^2 + r^3$$

In[17]:= `r1 = 1/2*(1+d)^(1/3)*(1+2*Cos[2/3*ArcCot[Sqrt[d]]])`

Out[17]=

$$\frac{1}{2} (1+d)^{1/3} \left(1 + 2\,\mathrm{Cos}\left[\frac{2\,\mathrm{ArcCot}\left[\sqrt{d}\right]}{3}\right]\right)$$

In[18]:= `r2 = 1/2*(1+d)^(1/3)*(1+2*Cos[2*Pi/3+2/3*ArcCot[Sqrt[d]]])`

Out[18]=

$$\frac{1}{2} (1+d)^{1/3} \left(1 - 2\,\mathrm{Sin}\left[\frac{\pi}{6} + \frac{2\,\mathrm{ArcCot}\left[\sqrt{d}\right]}{3}\right]\right)$$

In[19]:= `r3 = 1/2*(1+d)^(1/3)*(1+2*Cos[4*Pi/3+2/3*ArcCot[Sqrt[d]]])`

Out[19]=

$$\frac{1}{2} (1+d)^{1/3} \left(1 - 2\,\mathrm{Sin}\left[\frac{\pi}{6} - \frac{2\,\mathrm{ArcCot}\left[\sqrt{d}\right]}{3}\right]\right)$$

In[20]:= `{FullSimplify[p /. r → r1], FullSimplify[p /. r → r2], FullSimplify[p /. r → r3]}`

Out[20]=

{0, 0, 0}

In[21]:= `c1 = (1/3) (1+(d+1)^(1/2)*Sin[1/3*ArcCot[d^(1/2)]])`

Out[21]=

$$\frac{1}{3} \left(1 + \sqrt{1+d}\ \mathrm{Sin}\left[\frac{\mathrm{ArcCot}\left[\sqrt{d}\right]}{3}\right]\right)$$

In[22]:= `c2 = (1/3) (1+(d+1)^(1/2)*Sin[-2*Pi/3+1/3*ArcCot[d^(1/2)]])`

Out[22]=

$$\frac{1}{3} \left(1 - \sqrt{1+d}\ \mathrm{Cos}\left[\frac{\pi}{6} - \frac{\mathrm{ArcCot}\left[\sqrt{d}\right]}{3}\right]\right)$$

In[23]:= `c3 = (1/3) (1+(d+1)^(1/2)*Sin[2*Pi/3+1/3*ArcCot[d^(1/2)]])`

Out[23]=

$$\frac{1}{3} \left(1 + \sqrt{1+d}\ \mathrm{Cos}\left[\frac{\pi}{6} + \frac{\mathrm{ArcCot}\left[\sqrt{d}\right]}{3}\right]\right)$$

In[24]:=
```
{FullSimplify[c1 + c2 + c3], FullSimplify[c1 * r1 + c2 * r2 + c3 * r3],
 FullSimplify[c1 * r1 ^ 2 + c2 * r2 ^ 2 + c3 * r3 ^ 2, Assumptions → d > 0]}
```

Out[24]=

$$\left\{1,\ (1+d)^{1/3},\ (1+d)^{2/3}\right\}$$