Chapter 10: Sinusoidal Steady-State Power Calculations

EEL 3112C – Circuits-II

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Topics to be Covered in this Chapter

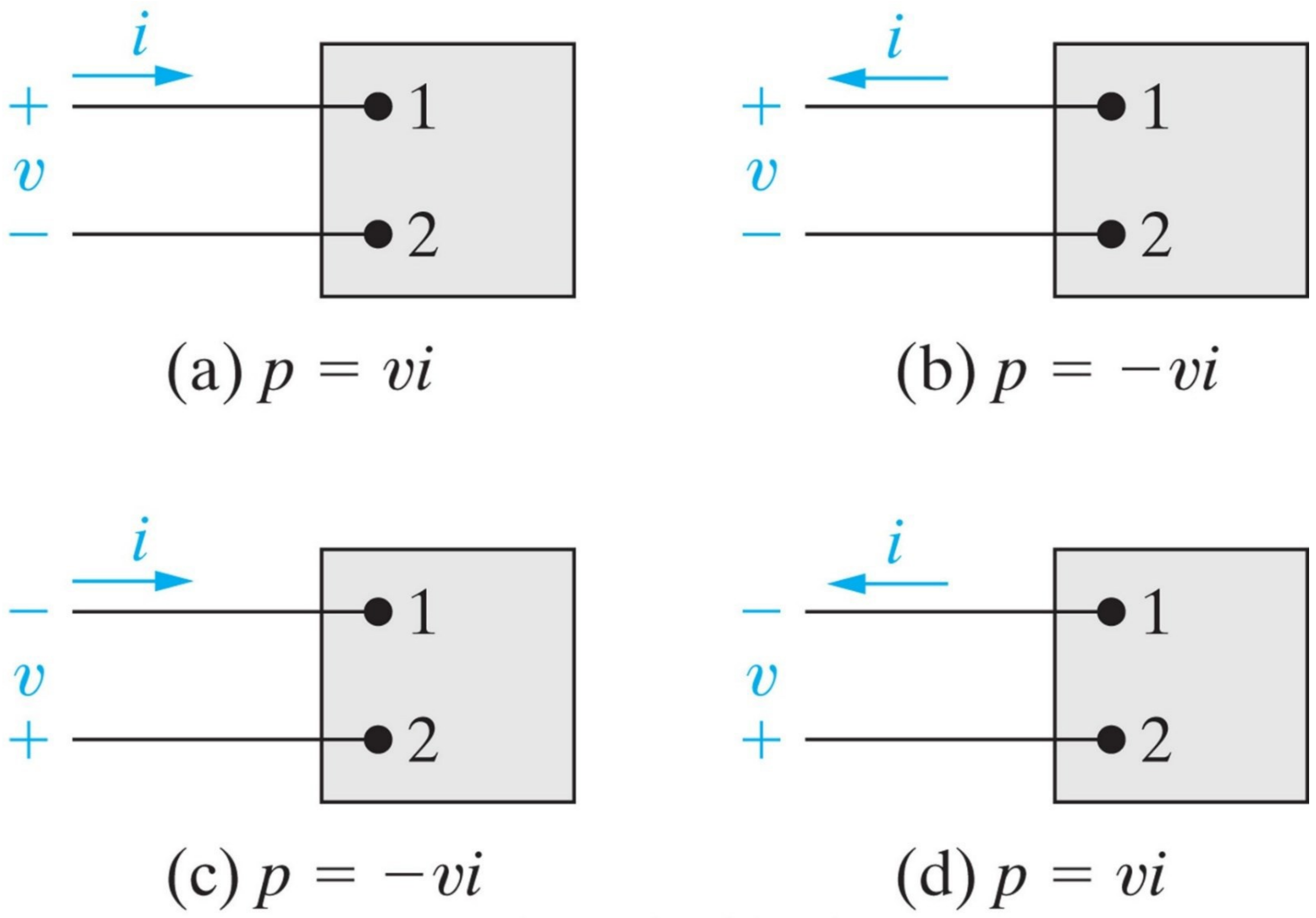
- In this chapter we will discuss:
  - Instantaneous power
  - Average & reactive power
  - The RMS values and power calculations
  - Power calculations & complex power
  - Maximum power transfer

- We will cover sections 10.1 – 10.6

Introduction

- In Chapter 9 we calculated the steady-state voltages and currents in electric circuits driven by sinusoidal sources
- In this chapter we will consider power in these circuits
- The techniques we will develop are useful for analyzing many of the electrical devices we encounter daily
    - Sinusoidal sources are the predominant means of providing electric power

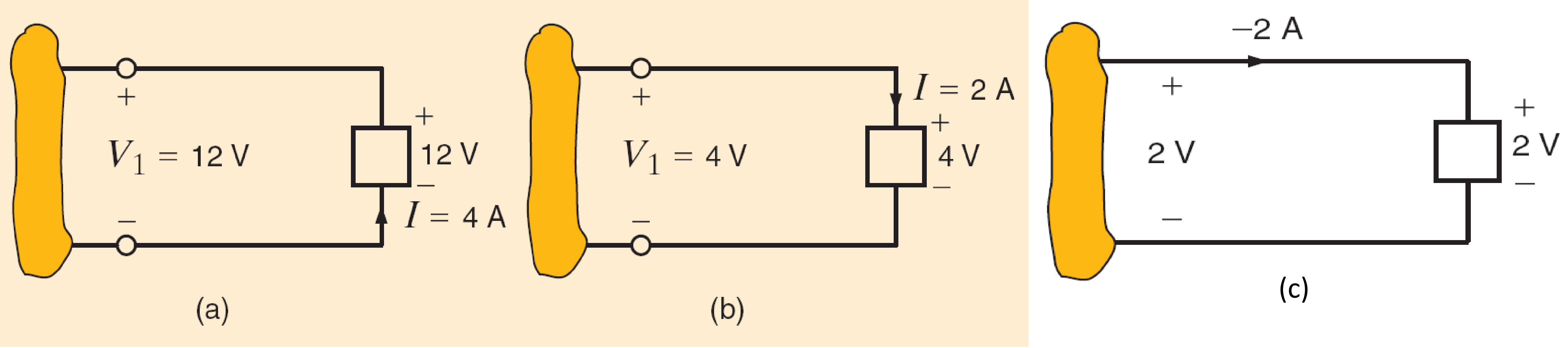
Review of Polarity References & Power Expression



-ve power means the element is delivering power to the circuit
+ve power means the source is absorbing power from the circuit

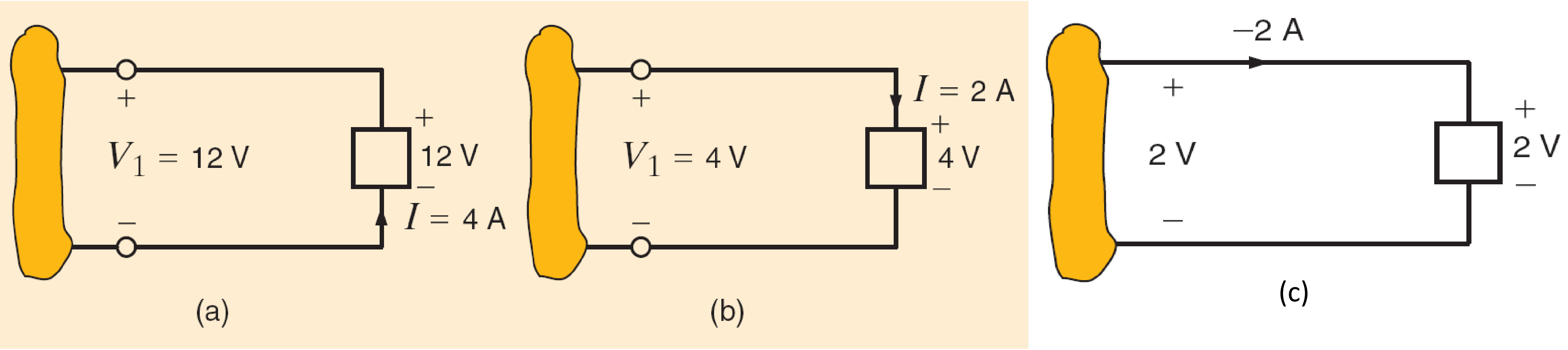
Example 1

- Find power P in Watts across the square circuit component?



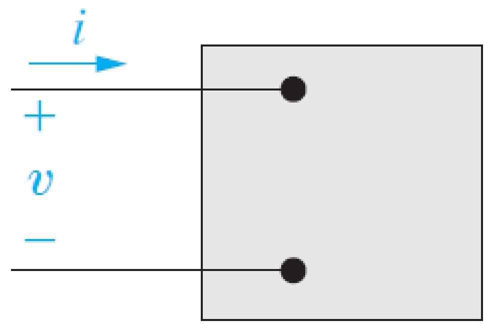
Example 1

Find power P in Watts across the square circuit component?



$$a)\ P = -iv = -4 \times 12 = -48\ W$$
$$b)\ P = iv = 2 \times 4 = 8\ W$$
$$c)\ P = iv = (-2) \times 2 = -4\ W$$

Instantaneous Power



Now we know that using the passive sign convention, we get the following expression for the power at any instant of time for the circuit shown here

$$p = vi.$$

We also know that the general form of sinusoidal current and voltage is given by:

$$v = V_m \cos(\omega t + \theta_v),$$

$$i = I_m \cos(\omega t + \theta_i),$$

Where θ_v is the voltage phase angle and θ_i is the current phase angle

For mathematical convenience, the current is chosen to be the reference (zero phase angle), so we can rewrite the above equations as:

$$v = V_m \cos(\omega t + \theta_v - \theta_i),$$

$$i = I_m \cos \omega t.$$

Shift both quantities by the phase angle θ_i

Then, we can write the expression for the instantaneous power as follows:

$$p = V_m I_m \cos(\omega t + \theta_v - \theta_i) \cos \omega t.$$

Using the trigonometric identity:

$$\cos\alpha \cos\beta = \frac{1}{2}\cos(\alpha - \beta) + \frac{1}{2}\cos(\alpha + \beta)$$

Where $\alpha = \omega t + \theta_v - \theta_i$ and $\beta = \omega t$, we get:

$$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(2\omega t + \theta_v - \theta_i).$$

Instantaneous Power – cont.

We can further simplify this equation by using the following trigonometric identity:

$$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(\overbrace{2\omega t}^{\alpha} + \overbrace{\theta_v - \theta_i}^{\beta}).$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\cos 2\omega t$$

$$- \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)\sin 2\omega t.$$

You can see that the frequency of the instantaneous power is twice the frequency of the voltage or current. Therefore, the instantaneous power goes through two complete cycles for every cycle of either the voltage or the current

We have 2 +ve peaks for v and i, while we have 4 +ve peaks for P. This is due to the fact that P has double the frequency

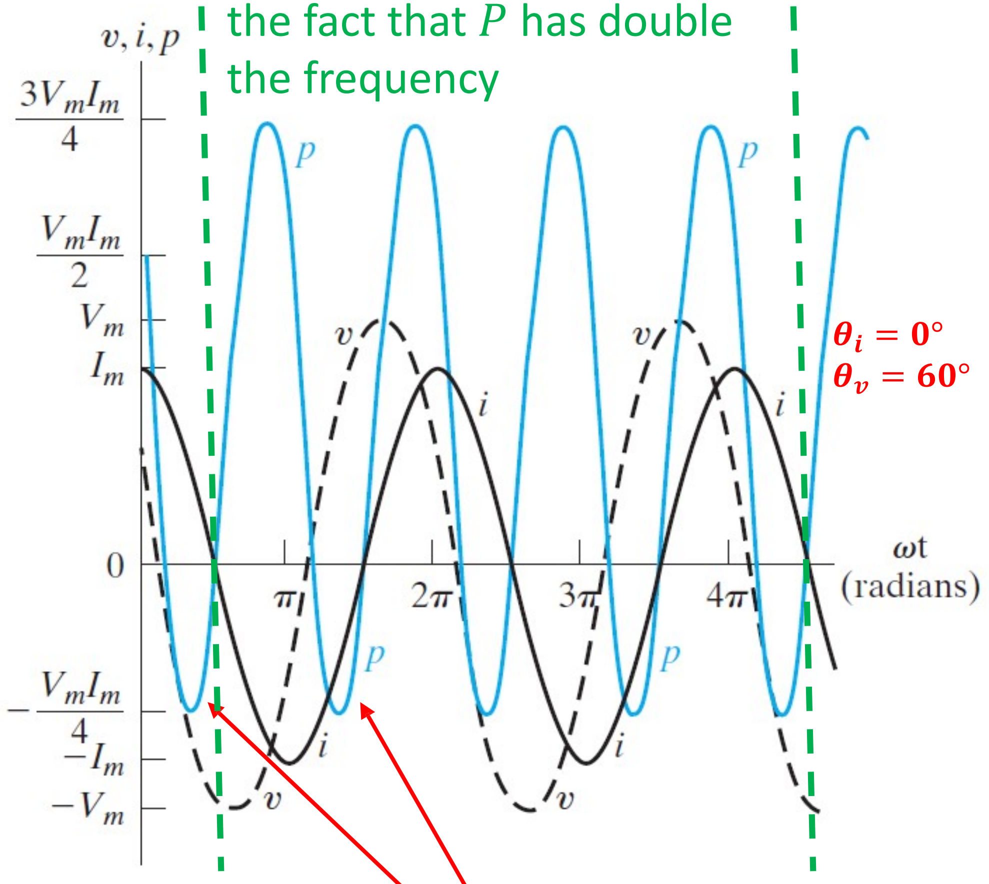


Figure 10.2 ▲ Instantaneous power, voltage, and current versus ωt for steady-state sinusoidal operation.

-ve power implies that energy stored in the inductors or capacitors is now being extracted

Average & Reactive Power

- If we look back at the equation for instantaneous power:

$$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\cos 2\omega t - \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)\sin 2\omega t.$$

- We can notice that it can be written as: $p = P + P\cos 2\omega t - Q\sin 2\omega t$

Average (real) power ▶

$$\boxed{P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i),}$$

$$\Rightarrow \quad P = \frac{1}{T}\int_{t_0}^{t_0+T} p\, dt$$

This is true because the integral of both $cos2\omega t$ and $sin2\omega t$ over one period is zero

Reactive power ▶

$$\boxed{Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i).}$$

- Note that average power (P) and reactive power (Q) carry the same dimension
  - To distinguish between average and reactive power, we use the units ***watt* (W) for average power** and **var (*volt-amp reactive,* or VAR) for reactive power**

Power for Purely Resistive Circuits

- If the circuit between the terminals is purely resistive, the voltage and current are in phase
  - This means that $\theta_v = \theta_i$
- Thus the instantaneous power (p) equation reduces to: $p = P + Pcos(2\omega t)$
  - Q will be equal to zero
  - This will be a real power, thus it is called instantaneous real power
- Note that the instantaneous real power can never be negative
  - Power can not be delivered by a resistor
  - All electric energy will be dissipated in the form of thermal energy

Just by looking at the graph, we can realize that the average power (P) for this circuit = 1

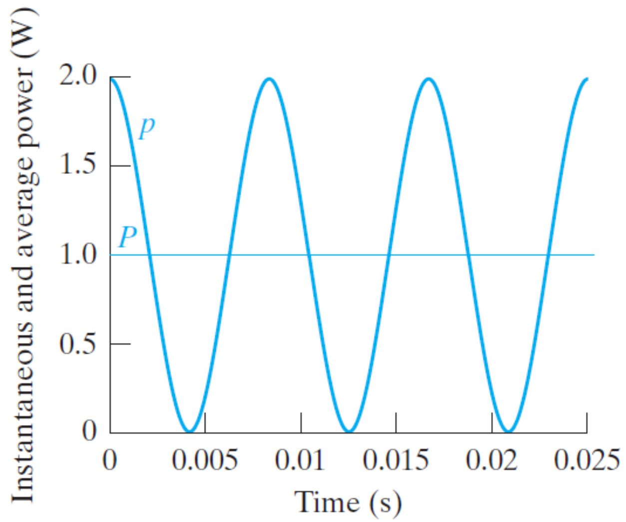


Figure 10.3 ▲ Instantaneous real power and average power for a purely resistive circuit.

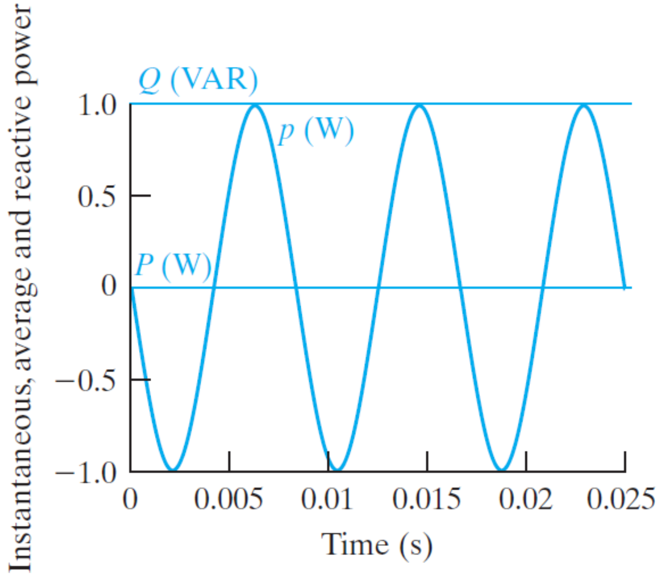
Power for Purely Inductive Circuits

- If the circuit between the terminals is purely inductive, the voltage and current are out of phase by precisely 90°
  - The current lags the voltage by 90° → $\theta_i = \theta_v - 90° \rightarrow \theta_v - \theta_i = 90°$
- Thus the expression for the instantaneous power (p) reduces to: $p = -Q \sin 2\omega t.$
  - Q will be a positive value in purely inductive load ($Q = \frac{V_m I_m}{2} \sin(90°) =$ +ve value)
- The instantaneous power (p) at the terminals in a purely inductive circuit is continually exchanged between the circuit and the source driving the circuit, at a frequency of 2ω
  - In other words, when (p) is positive, energy is being stored in the magnetic fields associated with the inductive elements, and when (p) is negative, energy is being extracted from the magnetic fields

Power for Purely Inductive Circuits – cont.

- A measure of the power associated with purely inductive circuits is the reactive power (Q)
  - The name *reactive power* comes from the characterization of an inductor as a reactive element; its impedance is purely reactive
- Note that average power (P) and reactive power (Q) carry the same dimension
  - To distinguish between average and reactive power, we use the units ***watt* (W) for average power** and **var (*volt-amp reactive*, or VAR) for reactive power**

In this plot, we are given that $Q = 1$, average power (P) equals 0 because this is a purely inductive circuit, and instantaneous power(p) equals $-Qsin2\omega t$, thus it oscillates between $1\ and\ -1$ at a frequency twice the frequency of the current or the voltage



Power for Purely Capacitive Circuits

- If the circuit between the terminals is purely capacitive, the voltage and current are out of phase by precisely 90°
  - The current leads the voltage by 90° → $\theta_i = \theta_v + 90°\ \rightarrow \theta_v - \theta_i = -90°$
- Thus the expression for the instantaneous power (p) reduces to: $p = -Q\sin 2\omega t.$
  - Q will be a negative value in purely capacitive load ($Q = \frac{V_m I_m}{2}\sin(-90°) =$ -ve value)
- Note that the decision to use the current as the reference leads to (Q) being positive for inductors (that is $\theta_v - \theta_i = 90°$), and negative for capacitors (that is $\theta_v - \theta_i = -90°$)

In this plot, we are given that $Q = -1$, average power (P) equals 0 because this is a purely capacitive circuit, and instantaneous power(p) equals $-Qsin2\omega t$, thus it oscillates between $1\ and\ -1$ at a frequency twice the frequency of the current or the voltage

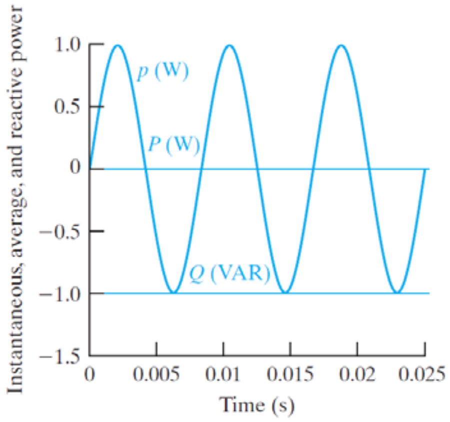


Figure 10.5 ▲ Instantaneous real power and average power for a purely capacitive circuit.

The Power Factor

- The angle $\theta_v - \theta_i$ plays a role in the computation of both average & reactive power and is referred to as the **power factor angle**
- The cosine of this angle is called the **power factor**, abbreviated **pf**, and the sine of this angle is called the **reactive factor**, abbreviated **rf**

$$\text{pf} = \cos(\theta_v - \theta_i) \;\; \& \;\; \text{rf} = \sin(\theta_v - \theta_i)$$

- Note: Knowing the value of the power factor does not tell you the value of the power factor angle, because $\cos(\theta_v - \theta_i) = \cos(\theta_i - \theta_v)$
  - To completely describe this angle, we use phrases **lagging power factor** and **leading power factor**
    - Lagging power factor implies that current lags voltage—hence an inductive load (+ve Q)
    - Leading power factor implies that current leads voltage—hence a capacitive load (-ve Q)

pf =1 means that the load is purely resistive $(\theta_v = \theta_i) \rightarrow \cos(0) = 1$

Example 10.1

a) Calculate the average power and the reactive power at the terminals of the network shown in Fig. 10.6 if

$$v = 100\cos(\omega t + 15^\circ)\text{ V},$$

$$i = 4\sin(\omega t - 15^\circ)\text{ A}.$$

b) State whether the network inside the box is absorbing or delivering average power.

c) State whether the network inside the box is absorbing or supplying magnetizing vars.

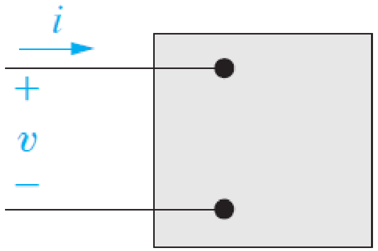


Figure 10.6 ▲ A pair of terminals used for calculating power.

Answer:

a) Because i is expressed in terms of the sine function, the first step in the calculation for P and Q is to rewrite i as a cosine function:

$$\sin(\theta) = \cos(\theta - 90^\circ)$$

$$i = 4\cos(\omega t - 105^\circ)\text{ A}.$$

We now calculate P and Q directly from Eqs. 10.10 and 10.11. Thus

$$P = \frac{1}{2}(100)(4)\cos[15 - (-105)] = -100\text{ W},$$

$$Q = \frac{1}{2}100(4)\sin[15 - (-105)] = 173.21\text{ VAR}.$$

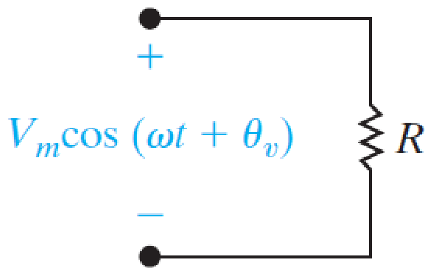
b)The negative value of $-100W$ means that the network inside the box is delivering average power to the terminals

c)Because Q is positive, the network inside the box is absorbing magnetizing vars at its terminals

Also, check example 10.2 in the book for general knowledge

The RMS Value & Power Calculations

- In introducing the **rms** value of a sinusoidal voltage (or current) in Section 9.1, we mentioned that it would play an important role in power calculations
- Assume that a **sinusoidal** voltage is applied to the terminals of a resistor, and that we want to determine the average power delivered to the resistor



$$P_{avg} = \frac{v_{avg}^2}{R} \Longrightarrow P = \frac{1}{T}\int_{t_0}^{t_0+T} \frac{V_m^2 \cos^2(\omega t + \phi_v)}{R}dt$$

$$= \frac{1}{R}\underbrace{\left[\frac{1}{T}\int_{t_0}^{t_0+T} V_m^2 \cos^2(\omega t + \phi_v)dt\right]}_{\text{From Ch9 we know this is } v_{rms}^2} \Longrightarrow \boxed{P = \frac{V_{\text{rms}}^2}{R}}$$

- If the resistor is carrying a sinusoidal current, say $I_m\cos(\omega t + \phi_i)$, the average power delivered to the resistor is $\boxed{P = I_{\text{rms}}^2 R}$

The RMS Value & Power Calculations – cont.

- The rms value is also referred to as the **effective value** of the sinusoidal voltage (or current). The rms value has an interesting property:
  - Given an equivalent resistive load, *R*, and an equivalent time period, *T*, the rms value of a sinusoidal source delivers the same energy to *R* as does a dc source of the same value
  - For example, a dc source of 100 V delivers the same energy in *T* seconds that a sinusoidal source of rms value 100 V delivers, assuming equivalent load resistances
  - This has led to the term ***effective value*** being used interchangeably with ***rms*** *value*

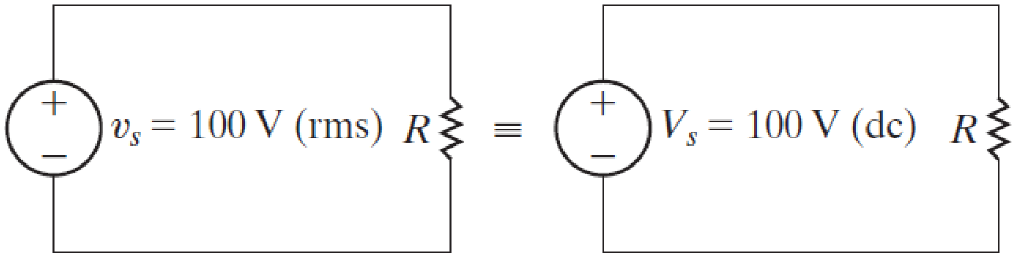


Figure 10.8 ▲ The effective value of v_s (100 V rms) delivers the same power to R as the dc voltage V_s (100 V dc).

$$
\begin{aligned}
P &= \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) \\
&= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(\theta_v - \theta_i) \\
&= V_{\text{eff}} I_{\text{eff}} \cos(\theta_v - \theta_i);
\end{aligned}
$$

$$
Q = V_{\text{eff}} I_{\text{eff}} \sin(\theta_v - \theta_i).
$$

We know that from our average and reactive power discussion we had earlier

Example 10.3

a) A sinusoidal voltage having a maximum amplitude of 625 V is applied to the terminals of a 50 Ω resistor. Find the average power delivered to the resistor.

b) Repeat (a) by first finding the current in the resistor.

Solution

a) The rms value of the sinusoidal voltage is $625/\sqrt{2}$, or approximately 441.94 V. From

Eq. 10.19, the average power delivered to the 50 Ω resistor is

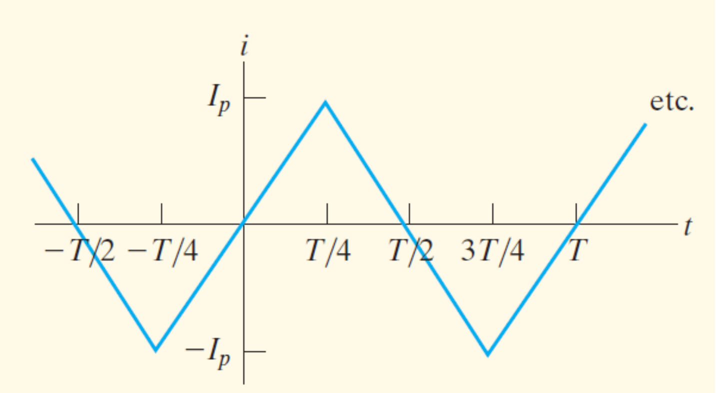
$$P = \frac{(441.94)^2}{50} = 3906.25 \text{ W}.$$

b) The maximum amplitude of the current in the resistor is 625/50, or 12.5 A. The rms value of the current is $12.5/\sqrt{2}$, or approximately 8.84 A. Hence the average power delivered to the resistor is

$$P = (8.84)^2 50 = 3906.25 \text{ W}.$$

Assessment Problem 10.3

The periodic triangular current in Example 9.4, repeated here, has a peak value of 180 mA. Find the average power that this current delivers to a 5 kΩ resistor.



Solution

$$\text{From Ex. 9.4} \quad I_{\text{eff}} = \frac{I_\rho}{\sqrt{3}} = \frac{0.18}{\sqrt{3}} \text{ A}$$

$$P = I_{\text{eff}}^2 R = \left(\frac{0.0324}{3}\right)(5000) = 54 \text{ W}$$

Complex Power

- Before proceeding to the various methods of calculating real and reactive power in circuits operating in the sinusoidal steady state, we need to introduce and define complex power
- **Complex power** is the complex sum of real power and reactive power

$$\boxed{S = P + jQ.}$$

$$P = \Re\{S\}$$

$$Q = \Im\{S\}$$

(the unit for complex power is **volt-amps** (VA))

TABLE 10.2 Three Power Quantities and Their Units

Quantity	Units
Complex power	volt-amps
Average power	watts
Reactive power	var

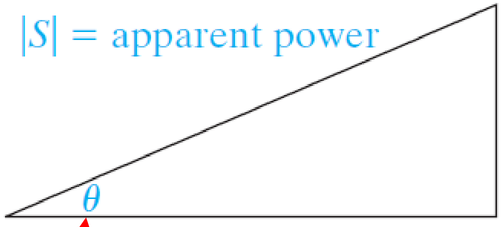


Figure 10.9 ▲ A power triangle.

cos(θ) is the power factor

$$\frac{Q}{P} = \frac{(V_m I_m/2)\sin(\theta_v - \theta_i)}{(V_m I_m/2)\cos(\theta_v - \theta_i)} = \tan(\theta_v - \theta_i).$$

$$\Rightarrow \tan\theta = \frac{Q}{P}.$$

The magnitude of complex power is referred to as **apparent power**

$$|S| = \sqrt{P^2 + Q^2}$$

Example 10.4

An electrical load operates at 240 V rms. The load absorbs an average power of 8 kW at a lagging power factor of 0.8.

a) Calculate the complex power of the load.

b) Calculate the impedance of the load.

Take a minute to think about it

Example 10.4

An electrical load operates at 240 V rms. The load absorbs an average power of 8 kW at a lagging power factor of 0.8.

a) Calculate the complex power of the load.

b) Calculate the impedance of the load.

Solution

a) The power factor is described as lagging, so we know that the load is inductive and that the algebraic sign of the reactive power is positive. From the power triangle shown in Fig. 10.10,

$$P = |S|\cos\theta,$$

$$Q = |S|\sin\theta.$$

Now, because $\cos\theta = 0.8$, $\sin\theta = 0.6$. Therefore

$$|S| = \frac{P}{\cos\theta} = \frac{8\text{ kW}}{0.8} = 10\text{ kVA},$$

$$Q = 10\sin\theta = 6\text{ kVAR},$$

and

$$S = 8 + j6\text{ kVA}.$$

b) From the computation of the complex power of the load, we see that $P = 8$ kW. Using Eq. 10.21,

$$\begin{aligned} P &= V_{\text{eff}} I_{\text{eff}} \cos(\theta_v - \theta_i) \\ &= (240) I_{\text{eff}}(0.8) \\ &= 8000\text{ W}. \end{aligned}$$

Solving for I_{eff},

$$I_{\text{eff}} = 41.67\text{ A}. \longrightarrow |Z| = \frac{|V_{\text{eff}}|}{|I_{\text{eff}}|} = \frac{240}{41.67} = 5.76$$

We already know the angle of the load impedance, because it is the power factor angle:

$$\theta = \cos^{-1}(0.8) = 36.87^\circ. \longrightarrow Z = 5.76\,\underline{/36.87^\circ}\ \Omega = 4.608 + j3.456\ \Omega.$$

- Alternative approach:
  - Find V_m which equals $V_{eff} \times \sqrt{2} \rightarrow \mathbf{V} = V_m \angle \theta_v$
  - Find I_m which equals $I_{eff} \times \sqrt{2} \rightarrow \mathbf{I} = I_m \angle \theta_i$
  - Then, $\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{(V_m \angle \theta_v)}{(I_m \angle \theta_i)} = \frac{V_m}{I_m} \angle(\theta_v - \theta_i)$
  - We can find $(\theta_v - \theta_i)$ from the power factor

Power Calculations

- We are now ready to develop additional equations that can be used to calculate real, reactive, & complex power

$$S = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + j\frac{V_m I_m}{2}\sin(\theta_v - \theta_i)$$

$$= \frac{V_m I_m}{2}[\cos(\theta_v - \theta_i) + j\sin(\theta_v - \theta_i)]$$

$$= \boxed{\frac{V_m I_m}{2}e^{\,j(\theta_v - \theta_i)} = \frac{1}{2}V_m I_m \underline{/(\theta_v - \theta_i)}.}$$

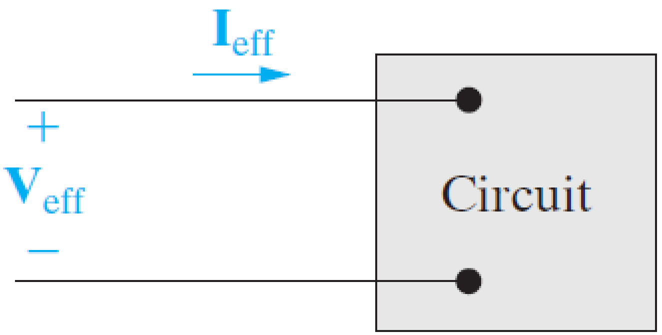


Figure 10.11 ▲ The phasor voltage and current associated with a pair of terminals.

- If we use the effective values of the sinusoidal voltage and current, then:

$$\boxed{S = V_{\text{eff}} I_{\text{eff}} \underline{/(\theta_v - \theta_i)}.}$$

Power Calculations – cont.

- The previous equation for S, we can be written as:

$$S = V_{\text{eff}} I_{\text{eff}} \underline{/ (\theta_v - \theta_i)}$$

$$= V_{\text{eff}} I_{\text{eff}} e^{\,j(\theta_v - \theta_i)}$$

$$= V_{\text{eff}} e^{\,j\theta_v} I_{\text{eff}} e^{-j\theta_i}$$

$$\boxed{= \mathbf{V}_{\text{eff}} \mathbf{I}^*_{\text{eff}}.}$$

Note that $\mathbf{I}^*_{\text{eff}} = I_{\text{eff}} e^{-j\theta_i}$ follows from Euler's identity and the trigonometric identities $\cos(-\theta) = \cos(\theta)$ and $\sin(-\theta) = -\sin(\theta)$:

$$I_{\text{eff}} e^{-j\theta_i} = I_{\text{eff}} \cos(-\theta_i) + jI_{\text{eff}} \sin(-\theta_i)$$

$$= I_{\text{eff}} \cos(\theta_i) - jI_{\text{eff}} \sin(\theta_i)$$

$$= \mathbf{I}^*_{\text{eff}}.$$

- Using the same derivation technique, we can also write S as:

$$S = \frac{1}{2} \mathbf{V}\mathbf{I}^*.$$

Note that we follow the polarity for power we discussed at the beginning of the chapter

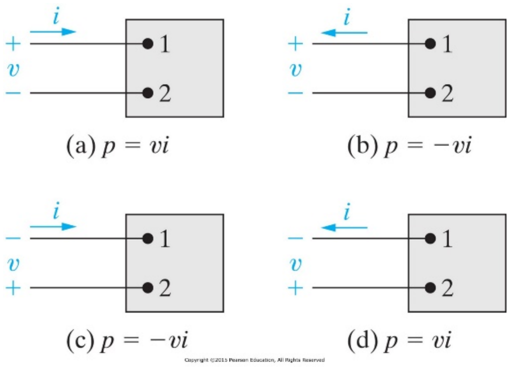


Illustration Example

- Assume the following waveforms for the voltage v and current i

$$v = 100\cos(\omega t + 15^\circ)\ \text{V},$$
$$i = 4\sin(\omega t - 15^\circ)\ \text{A}.$$

$$\mathbf{V} = 100\ \angle 15^\circ\ \text{V},$$
$$\mathbf{I} = 4\angle -105^\circ\ \text{A}.$$

To get this representation for current, write the current waveform using *cos*

$$i = 4\cos(\omega t - 15^\circ - 90^\circ) = 4\cos(\omega t - 105^\circ)$$

- Knowing that $S = \frac{1}{2}\mathbf{V}\mathbf{I}^*$, then:

$$S = \frac{1}{2}(100\ \angle 15^\circ)(4\ \angle +105^\circ) = 200\ \angle 120^\circ$$
$$= -100 + j173.21\ \text{VA}.$$

$$P = -100\ \text{W},$$
$$Q = 173.21\ \text{VAR}.$$

Alternate Forms for Complex Power

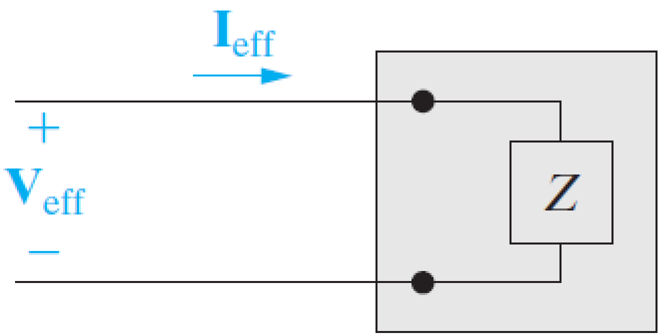


Figure 10.12 ▲ The general circuit of Fig. 10.11 replaced with an equivalent impedance.

$$\mathbf{V}_{\text{eff}} = Z\mathbf{I}_{\text{eff}}.$$

$$S = Z\underbrace{\mathbf{I}_{\text{eff}}\mathbf{I}^*_{\text{eff}}}$$

$$= |\mathbf{I}_{\text{eff}}|^2 Z$$

$$= |\mathbf{I}_{\text{eff}}|^2 (R + jX)$$

$$= |\mathbf{I}_{\text{eff}}|^2 R + j|\mathbf{I}_{\text{eff}}|^2 X = P + jQ$$

$$P = |\mathbf{I}_{\text{eff}}|^2 R = \frac{1}{2} I_m^2 R,$$

$$Q = |\mathbf{I}_{\text{eff}}|^2 X = \frac{1}{2} I_m^2 X.$$

Another useful variation of the power equation comes from replacing the current with the voltage divided by the impedance:

$$S = \mathbf{V}_{\text{eff}}\left(\frac{\mathbf{V}_{\text{eff}}}{Z}\right)^* = \frac{|\mathbf{V}_{\text{eff}}|^2}{Z^*} = P + jQ.$$

$$P = \frac{|\mathbf{V}_{\text{eff}}|^2}{R} \quad \& \quad Q = \frac{|\mathbf{V}_{\text{eff}}|^2}{X}$$

Example 10.5

In the circuit shown in Fig. 10.13, a load having an impedance of $39 + j26\ \Omega$ is fed from a voltage source through a line having an impedance of $1 + j4\ \Omega$. The effective, or rms, value of the source voltage is 250 V.

a) Calculate the load current $\mathbf{I}_L$ and voltage $\mathbf{V}_L$.

b) Calculate the average and reactive power delivered to the load.

c) Calculate the average and reactive power delivered to the line.

d) Calculate the average and reactive power supplied by the source.

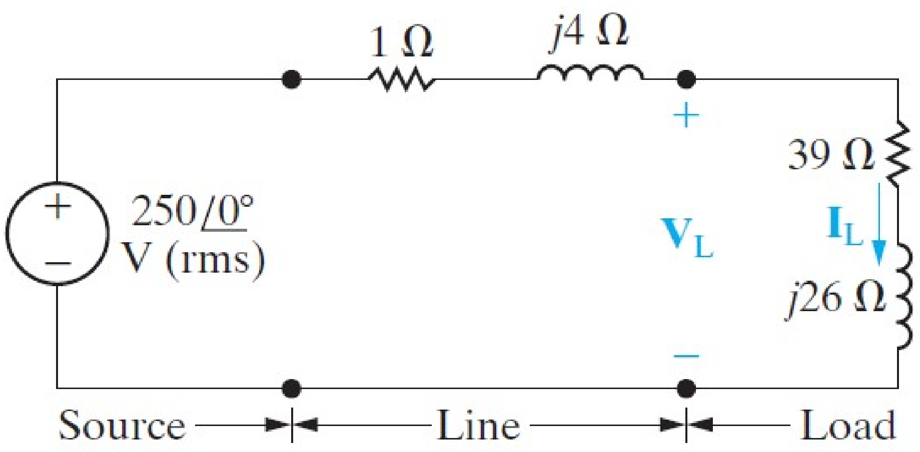


Figure 10.13 ▲ The circuit for Example 10.5.

Also, check examples 10.6 & 10.7

Example 10.5 – cont.

Solution

a) The line and load impedances are in series across the voltage source, so the load current equals the voltage divided by the total impedance, or

$$\mathbf{I}_L = \frac{250\,\underline{/0^\circ}}{40 + j30} = 4 - j3 = 5\,\underline{/-36.87^\circ}\text{ A (rms).}$$

Because the voltage is given in terms of its rms value, the current also is rms. The load voltage is the product of the load current and load impedance:

$$\mathbf{V}_L = (39 + j26)\mathbf{I}_L = 234 - j13$$

$$= 234.36\,\underline{/-3.18^\circ}\text{ V (rms).}$$

b) The average and reactive power delivered to the load can be computed using Eq. 10.29. Therefore

$$S = \mathbf{V}_L\mathbf{I}_L^* = (234 - j13)(4 + j3)$$

$$= 975 + j650\text{ VA.}$$

c) The average and reactive power delivered to the line are most easily calculated from Eqs. 10.33 and 10.34 because the line current is known. Thus

$$P = (5)^2(1) = 25\text{ W,}$$

$$Q = (5)^2(4) = 100\text{ VAR.}$$

Note that the reactive power associated with the line is positive because the line reactance is inductive.

Example 10.5 – cont.

- Solution – cont. :
  - d) One way to calculate the average and reactive power delivered by the source, is to add the complex power absorbed by the line to that absorbed by the load
    - Note that we need to pay attention to the sign after we add these powers. What we calculate here is the total power absorbed by the line and the load
    - To convert this to a total power supplied by the source, we need to change the sign to make it negative, since negative power means the power is supplied

$$S = 25 + j100 + 975 + j650$$

$$= 1000 + j750 \text{ VA}.$$

This is the total power absorbed by the line and the load. Thus, the total power supplied by the source is: $-1 \times (1000 + j750)\ VA$

Or

$$S_s = -250\mathbf{I}_L^*. \longrightarrow S_s = -250(4 + j3) = -(1000 + j750) \text{ VA}.$$

Example 10.6:

The two loads in the circuit shown in Fig. 10.14 can be described as follows: Load 1 absorbs an average power of 8 kW at a leading power factor of 0.8. Load 2 absorbs 20 kVA at a lagging power factor of 0.6.

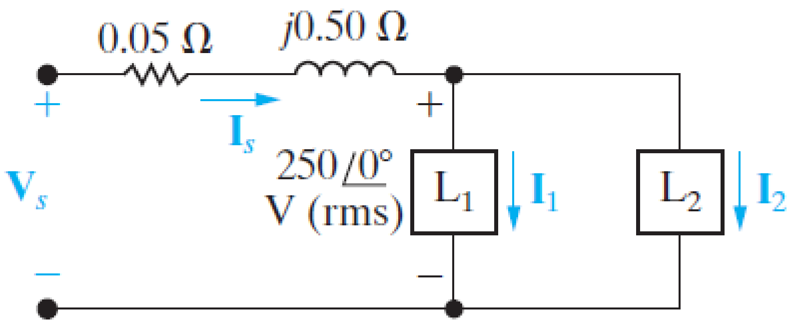


Figure 10.14 ▲ The circuit for Example 10.6.

a) Determine the power factor of the two loads in parallel.

b) Determine the apparent power required to supply the loads, the magnitude of the current, $\mathbf{I}_s$, and the average power loss in the transmission line.

c) Given that the frequency of the source is 60 Hz, compute the value of the capacitor that would correct the power factor to 1 if placed in parallel with the two loads.

Check file:**Power_Calc_Example1_Ch10.pdf,** for detailed answer

Maximum Power Transfer

- Maximum power transfer theorem denotes that a voltage source which is in series with a resistance delivers the maximum power to a load resistance if load resistance is equal to circuit resistance
- In the context of sinusoidal steady state analysis, we must determine the load impedance that results in the delivery of maximum average power to terminals a and b
- Thus the task reduces to finding the value of Z_L that results in maximum average power delivered to Z_L

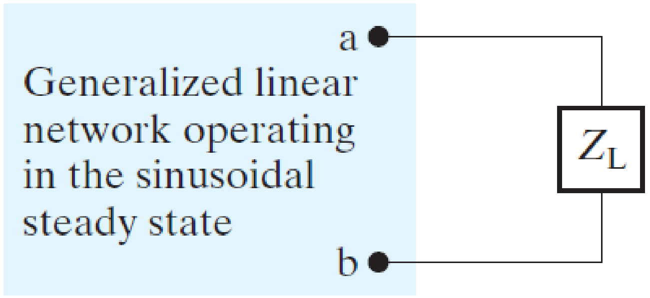


Figure 10.18 ▲ A circuit describing maximum power transfer.

Maximum Power Transfer – cont.

$$Z_{Th} = R_{Th} + jX_{Th},$$

$$Z_L = R_L + jX_L.$$

In these two equations, the reactance term carries its own algebraic sign. +ve for inductance and –ve for capacitance

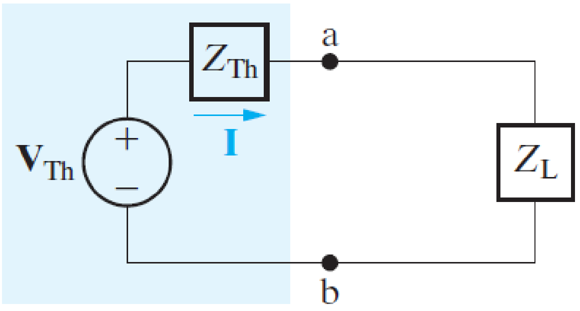


Figure 10.19 ▲ The circuit shown in Fig. 10.18, with the network replaced by its Thévenin equivalent.

- Because we are making an average-power calculation, we assume that the amplitude of the Thévenin voltage is expressed in terms of its rms value
- We also use the Thévenin voltage as the reference phasor

$$\mathbf{I} = \frac{\mathbf{V}_{Th}}{(R_{Th} + R_L) + j(X_{Th} + X_L)}. \quad \text{Eqn*}$$

- The average power delivered to the load is: $P = |\mathbf{I}|^2 R_L$ Eqn**

- Substituting eqn* into eqn** we get:

$$P = \frac{|\mathbf{V}_{Th}|^2 R_L}{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2}$$

When working with this equation, remember that $V_{TH}, R_{Th}, and\ X_{Th}$ are fixed quantities, whereas $R_L\ and\ X_L$ are independent variables depend on the load

Maximum Power Transfer – cont.

- Thus the tasks reduces now to find the values of R_L and X_L that will maximize the average power P
  - We must find the values of R_L and X_L where $\partial P/\partial R_L$ and $\partial P/\partial X_L$ are both zero

$$\boxed{\begin{aligned} \frac{\partial P}{\partial X_L} &= \frac{-|\mathbf{V}_{Th}|^2 2R_L(X_L + X_{Th})}{[(R_L + R_{Th})^2 + (X_L + X_{Th})^2]^2}, \\ \frac{\partial P}{\partial R_L} &= \frac{|\mathbf{V}_{Th}|^2[(R_L + R_{Th})^2 + (X_L + X_{Th})^2 - 2R_L(R_L + R_{Th})]}{[(R_L + R_{Th})^2 + (X_L + X_{Th})^2]^2} \end{aligned}}$$

$\partial P/\partial R_L$ is zero when:

$\partial P/\partial X_L$ is zero when:

$$\underbrace{R_L = \sqrt{R_{Th}^2 + (X_L + X_{Th})^2}. \qquad\qquad X_L = -X_{Th}.}$$

Combining these two equations, we see that maximum power transfer occurs when $\boldsymbol{Z_L = Z_{Th}^*}$

Maximum Power Transfer – cont.

- Thus, from previous analysis, the maximum average power that can be delivered to Z_L when it is set equal to the conjugate of Z_{Th}

$$\mathbf{I} = \underbrace{\frac{\mathbf{V}_{\mathrm{Th}}}{(R_{\mathrm{Th}} + R_{\mathrm{L}}) + j(X_{\mathrm{Th}} + X_{\mathrm{L}})}}.$$

Because $Z_L = Z_{Th}^*$, the imaginary term will be zero, and the real term will reduce to $2R_L$

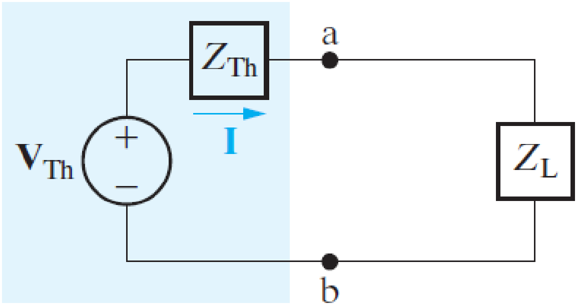


Figure 10.19 ▲ The circuit shown in Fig. 10.18, with the network replaced by its Thévenin equivalent.

- Thus, the RMS load current will equal to → $\mathbf{I} = \dfrac{\mathbf{V}_{Th}}{2R_L}$
- And the maximum **average power** delivered to the load is: $P_{\max} = \dfrac{|\mathbf{V}_{\mathrm{Th}}|^2 R_{\mathrm{L}}}{4R_{\mathrm{L}}^2} = \dfrac{1}{4}\dfrac{|\mathbf{V}_{\mathrm{Th}}|^2}{R_{\mathrm{L}}}$
- If the Thevenin voltage is expressed in terms of its maximum amplitude rather than its RMS, then the previous equation becomes:

$$P_{\max} = \frac{1}{8}\frac{\mathbf{V}_m^2}{R_{\mathrm{L}}}$$

Remember: $\mathbf{V}_{Th} = \dfrac{\mathbf{V}_m}{\sqrt{2}}$

Maximum Power Transfer When Z is Restricted

- In some situations, it is not possible to change the load impedance to match the Thevenin impedance of the circuit
- The optimum situation is to adjust X_L as near to $-X_{Th}$ as possible, then adjust R_L as close to $\sqrt{R_{Th}^2 + (X_L + X_{Th})^2}$ as possible
- Another type of restriction is when the magnitude of Z_L can be adjusted but the phase can not
  - In this case, the maximum power is transferred to the load when magnitude of Z_L is set equal to the magnitude of Z_{Th}

$$|Z_{\mathrm{L}}| = |Z_{\mathrm{Th}}|.$$

Example 10.8

a) For the circuit shown in Fig. 10.20, determine the impedance Z_L that results in maximum average power transferred to Z_L.

b) What is the maximum average power transferred to the load impedance determined in (a)?

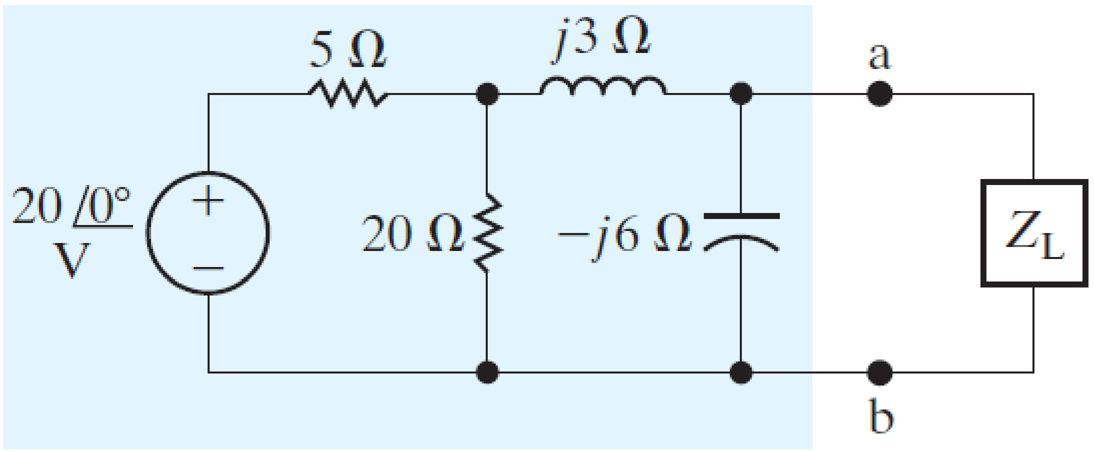


Figure 10.20 ▲ The circuit for Example 10.8.

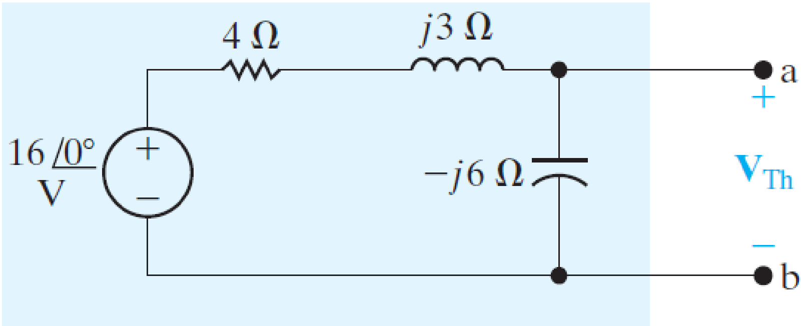
Example 10.8 – cont.

Solution

a) We begin by determining the Thévenin equivalent with respect to the load terminals a, b. After two source transformations involving the 20 V source, the 5 Ω resistor, and the 20 Ω resistor, we simplify the circuit shown in Fig. 10.20 to the one shown in Fig. 10.21. Then,

$$\mathbf{V}_{\text{Th}} = \frac{16\,\angle 0^\circ}{4 + j3 - j6}(-j6)$$

$$= 19.2\,\angle -53.13^\circ = 11.52 - j15.36 \text{ V}.$$



We find the Thévenin impedance by deactivating the independent source and calculating the impedance seen looking into the terminals a and b. Thus,

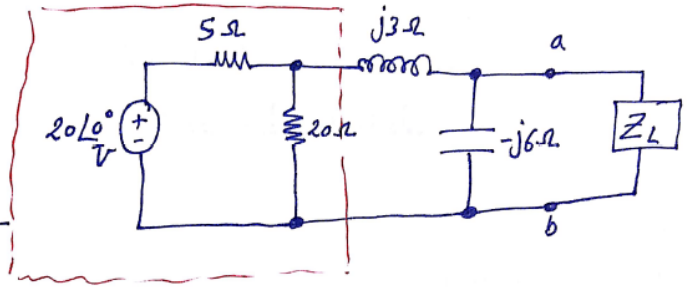
$$Z_{\text{Th}} = \frac{(-j6)(4 + j3)}{4 + j3 - j6} = 5.76 - j1.68\ \Omega.$$

For maximum average power transfer, the load impedance must be the conjugate of Z_{Th}, so

$$Z_L = 5.76 + j1.68\ \Omega.$$

Detailed answer in the following two slides

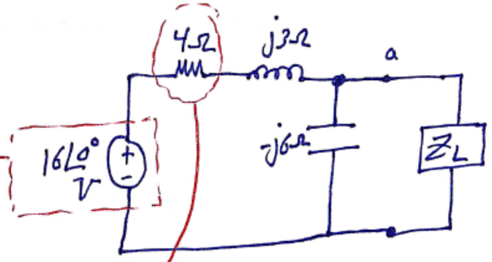
Example 10.8 – cont.



⊛ source Transformation for this block will lead to the following:

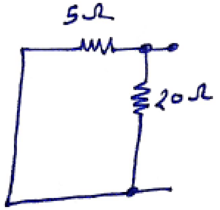
voltage divider

$$\frac{20\angle 0^\circ * 20}{5+20} = 16\angle 0^\circ \ V$$



⊛ To find This resistor, short circuit the voltage source & find Req for resistors inside The red bloc

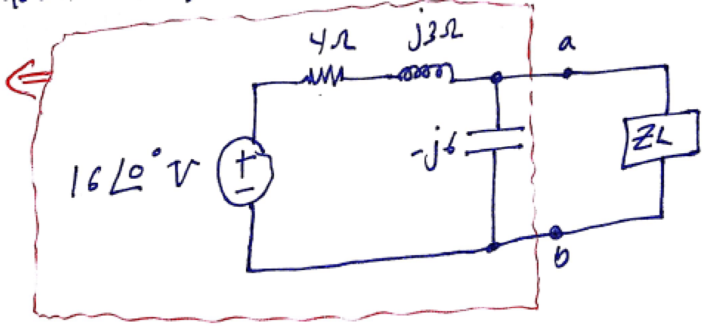
⇒



⇒ $5\Omega \,||\, 20\Omega \Rightarrow R_{eq} = 4\Omega$

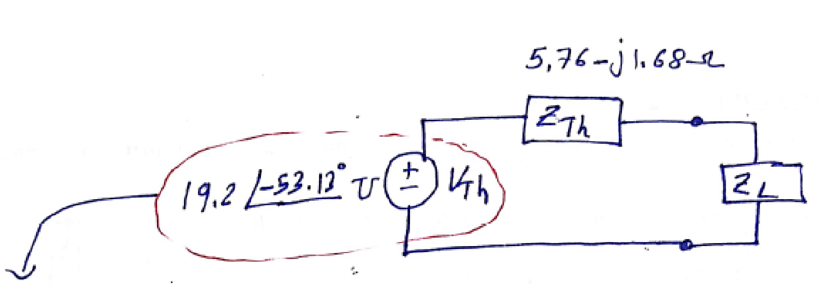
⊛ Then as shown before, the circuit simplifies to look like This:

⊛ now, we perform one more source transformation for This block



Example 10.8 – cont.

(2)



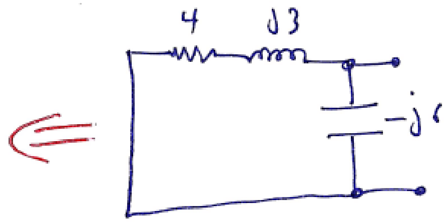
⊛ Voltage divider between the capacitor (−j6) & the resistor and the inductor (4 + j3 Ω) will lead to:

The −j is a −90° phase shift.

$$V_{Th} = \frac{16\angle 0° \,(-j6)}{4 + j3 - j6}$$

$$V_{Th} = \frac{96\angle -90°}{5\angle -36.87} = 19.2\angle -53.13° = 11.52 - j15.36 \text{ V}.$$

⊛ Now, to find Z_{Th}, we short circuit the voltage source:



$$(4 + j3) \parallel -j6$$

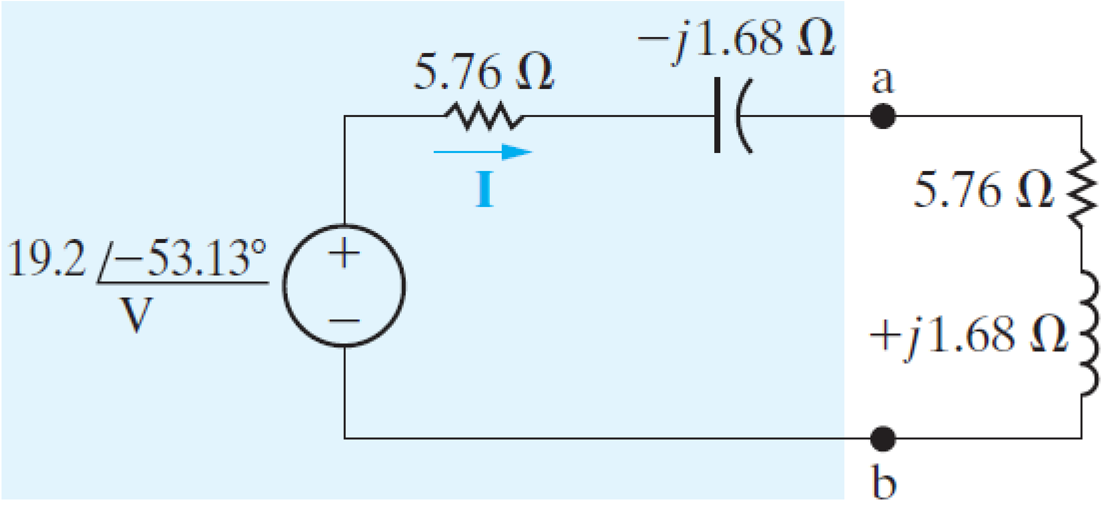
$$\Rightarrow Z_{Th} = \frac{(-j6)(4 + j3)}{4 + j3 - j6}$$

$$= 5.76 - j1.68\ \Omega.$$

Example 10.8 – cont.

b) We calculate the maximum average power delivered to Z_L from the circuit shown in Fig. 10.22, in which we replaced the original network with its Thévenin equivalent. From Fig. 10.22, the rms magnitude of the load current **I** is

$$I_{eff} = \frac{19.2/\sqrt{2}}{2(5.76)} = 1.1785 \text{ A}.$$



The average power delivered to the load is

$$P = I_{eff}^2(5.76) = 8 \text{ W}.$$

Example 10.9

a) For the circuit shown in Fig. 10.23, what value of Z_L results in maximum average power transfer to Z_L? What is the maximum power in milliwatts?

b) Assume that the load resistance can be varied between 0 and 4000 Ω and that the capacitive reactance of the load can be varied between 0 and −2000 Ω. What settings of R_L and X_L transfer the most average power to the load? What is the maximum average power that can be transferred under these restrictions?

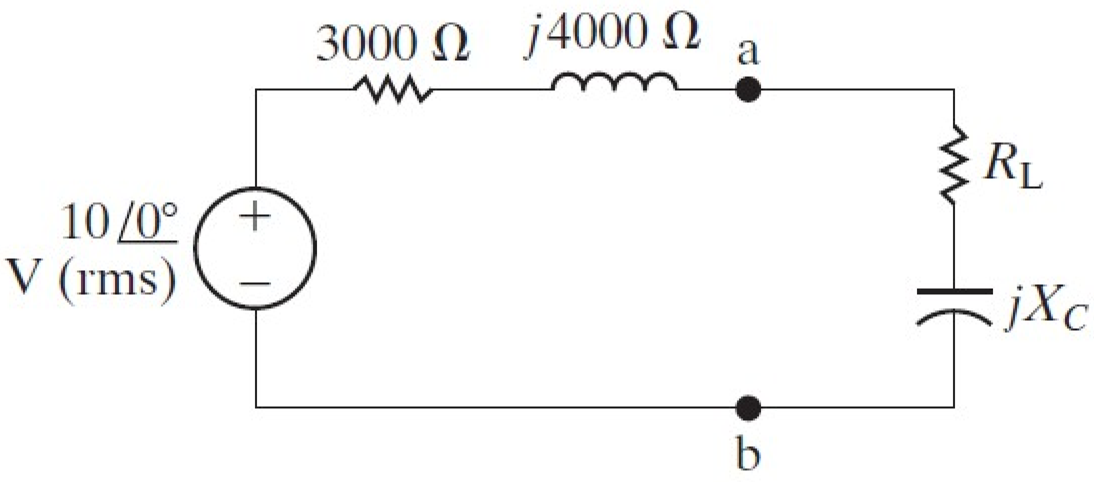


Figure 10.23 ▲ The circuit for Examples 10.9 and 10.10.

Example 10.9 – cont.

Solution

a) If there are no restrictions on R_L and X_L, the load impedance is set equal to the conjugate of the output or the Thévenin impedance. Therefore we set

$$R_L = 3000\ \Omega \quad \text{and} \quad X_L = -4000\ \Omega,$$

or

$$Z_L = 3000 - j4000\ \Omega.$$

Because the source voltage is given in terms of its rms value, the average power delivered to Z_L is

$$P = \frac{1}{4}\frac{10^2}{3000} = \frac{25}{3}\ \text{mW} = 8.33\ \text{mW}.$$

- Thus, the RMS load current will equal to → $\mathbf{I} = \frac{\mathbf{V}_{Th}}{2R_L}$
- And the maximum **average power** delivered to the load is: $P_{\max} = \frac{|\mathbf{V}_{\mathrm{Th}}|^2 R_L}{4R_L^2} = \frac{1}{4}\frac{|\mathbf{V}_{\mathrm{Th}}|^2}{R_L}$

b) Because R_L and X_L are restricted, we first set X_L as close to $-4000\ \Omega$ as possible; thus $X_L = -2000\ \Omega$. Next, we set R_L as close to $\sqrt{R_{\mathrm{Th}}^2 + (X_L + X_{\mathrm{Th}})^2}$ as possible. Thus

$$R_L = \sqrt{3000^2 + (-2000 + 4000)^2} = 3605.55\ \Omega.$$

Now, because R_L can be varied from 0 to 4000 Ω, we can set R_L to 3605.55 Ω. Therefore, the load impedance is adjusted to a value of

$$Z_L = 3605.55 - j2000\ \Omega.$$

With Z_L set at this value, the value of the load current is

$$\mathbf{I}_{\mathrm{eff}} = \frac{10\angle 0^\circ}{6605.55 + j2000} = 1.4489\angle -16.85^\circ\ \text{mA}.$$

The average power delivered to the load is

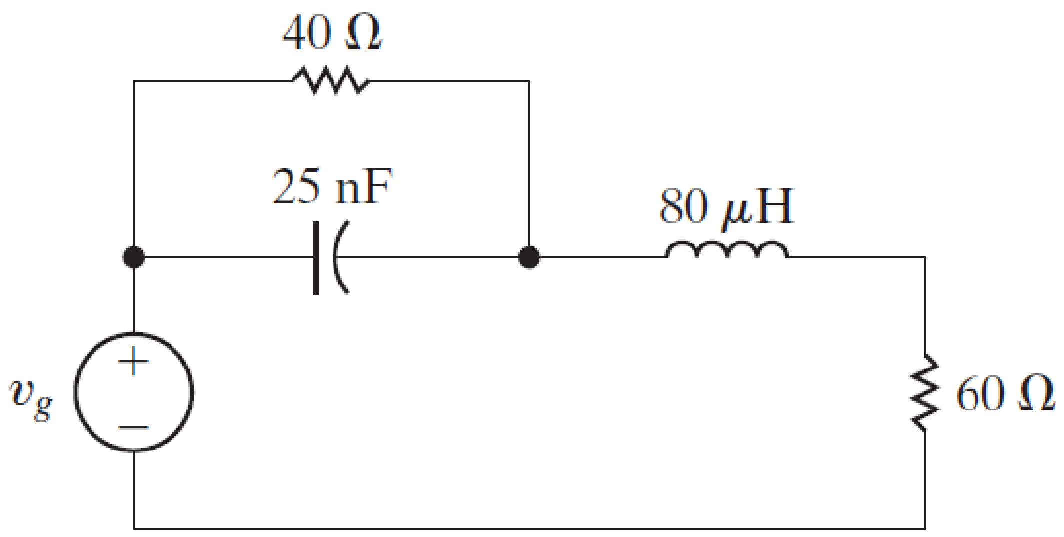
$$P = |\mathbf{I}|^2 R_L$$

$$P = (1.4489 \times 10^{-3})^2(3605.55) = 7.57\ \text{mW}.$$

Additional Example:

- For the circuit shown below find the following knowing that $v_g = 40\cos(10^6 t)$ V.
  - The average power (P) supplied by the voltage source
  - The reactive power (Q) supplied by the voltage source
  - The apparent power $(|S|)$ supplied by the voltage source

- **Micro: 10^{-6}**
- **Nano: 10^{-9}**
- **Pico: 10^{-12}**



Check file:**Power_Calc_Example2_Ch10.pdf,** for detailed answer

Summary of Topics Covered in this Chapter

- In this chapter we discussed:
  - Instantaneous power
  - Average & reactive power
  - The RMS values of power calculations
  - Power calculations & complex power
  - Maximum power transfer
- We covered sections 10.1 – 10.6
- Next chapter (Ch11) we will talk about balanced three-phase circuits
  - Balanced Three-phase Circuits