

Find best a such that $|f| \cong af^2$

EEE4520 HW4

Peter A. Dranishnikov

for f values between $0 < f < 1$. Namely, we want to approximate the abs. val. of f , by an f^2 over the freq. range (0 to 1). By using the integral version of the least squares technique, write the penalty function. Then differentiate with respect to a , set the derivative to zero and find a .

$$\int_0^1 (|f| - af^2)^2 df \quad (\text{penalty function})$$

$$\frac{d}{da} \int_0^1 (|f| - af^2)^2 df$$

$$0 = \int_0^1 \frac{d}{da} (|f| - af^2)^2 df$$

$$0 = \int_0^1 2f - 6af^2 + 4a^2f^3 df$$

$$0 = \left[f^2 - 2af^3 + a^2f^4 \right]_0^1$$

$$0 = (1 - 2a + a^2) - (0)$$

$$-1 = -2a + a^2$$

$$-1 = a(-2 + a)$$

$$a = 1$$

$$\frac{d}{da} (|f| - af^2)^2$$

$$= 2(|f| - af^2)(1 - 2af)$$

(for positive valued f)

$$= 2(|f| - 2af|f| - af^2 + 2a^2f^3)$$

$$= 2(f - 2af^2 - af^2 + 2a^2f^3)$$

$$= 2f - 6af^2 + 4a^2f^3$$