Circuits II

Home Work # 1 (Ch8) Solution

AP 8.3 From the given values of R, L, and C, $s_1 = -10\,\text{krad/s}$ and $s_2 = -40\ \text{krad/s}$.

[a] $v(0^-) = v(0^+) = 0, \qquad \text{therefore} \quad i_{\text{R}}(0^+) = 0$

[b] $i_{\text{C}}(0^+) = -(i_L(0^+) + i_R(0^+)) = -(-4+0) = 4\,\text{A}$

[c] $C\dfrac{dv_c(0^+)}{dt} = i_c(0^+) = 4, \qquad \text{therefore} \quad \dfrac{dv_c(0^+)}{dt} = \dfrac{4}{C} = 4 \times 10^8\,\text{V/s}$

[d] $v = [A_1 e^{-10{,}000t} + A_2 e^{-40{,}000t}]\,\text{V}, \qquad t \geq 0^+$

$$v(0^+) = A_1 + A_2, \qquad \frac{dv(0^+)}{dt} = -10{,}000A_1 - 40{,}000A_2$$

Therefore $\quad A_1 + A_2 = 0, \qquad -A_1 - 4A_2 = 40{,}000; \qquad A_1 = 40{,}000/3\,\text{V}$

[e] $A_2 = -40{,}000/3\,\text{V}$

[f] $v = [40{,}000/3][e^{-10{,}000t} - e^{-40{,}000t}]\,\text{V}, \qquad t \geq 0$

AP 8.4 **[a]** $\dfrac{1}{2RC} = 8000, \qquad \text{therefore} \quad R = 62.5\,\Omega$

[b] $i_{\text{R}}(0^+) = \dfrac{10\,\text{V}}{62.5\,\Omega} = 160\,\text{mA}$

$$i_{\text{C}}(0^+) = -(i_L(0^+) + i_R(0^+)) = -80 - 160 = -240\,\text{mA} = C\frac{dv(0^+)}{dt}$$

Therefore $\dfrac{dv(0^+)}{dt} = \dfrac{-240\,\text{m}}{C} = -240\,\text{kV/s}$

[c] $B_1 = v(0^+) = 10\,\text{V}, \qquad \dfrac{dv_c(0^+)}{dt} = \omega_d B_2 - \alpha B_1$

Therefore $6000B_2 - 8000B_1 = -240{,}000, \qquad B_2 = (-80/3)\,\text{V}$

[d] $i_{\text{L}} = -(i_{\text{R}} + i_{\text{C}}); \qquad i_{\text{R}} = v/R; \qquad i_{\text{C}} = C\dfrac{dv}{dt}$

$$v = e^{-8000t}[10\cos 6000t - \frac{80}{3}\sin 6000t]\,\text{V}$$

Therefore $i_{\text{R}} = e^{-8000t}[160\cos 6000t - \dfrac{1280}{3}\sin 6000t]\,\text{mA}$

$$i_{\text{C}} = e^{-8000t}[-240\cos 6000t + \frac{460}{3}\sin 6000t]\,\text{mA}$$

$$i_{\text{L}} = 10e^{-8000t}[8\cos 6000t + \frac{82}{3}\sin 6000t]\,\text{mA}, \qquad t \geq 0$$

AP 8.5 [a] $\left(\frac{1}{2RC}\right)^2 = \frac{1}{LC} = \frac{10^6}{4}$, therefore $\frac{1}{2RC} = 500$, $R = 100\,\Omega$

[b] $0.5CV_0^2 = 12.5 \times 10^{-3}$, therefore $V_0 = 50\,\text{V}$

[c] $0.5LI_0^2 = 12.5 \times 10^{-3}$, $I_0 = 250\,\text{mA}$

[d] $D_2 = v(0^+) = 50$, $\frac{dv(0^+)}{dt} = D_1 - \alpha D_2$

$i_{\text{R}}(0^+) = \frac{50}{100} = 500\,\text{mA}$

Therefore $i_{\text{C}}(0^+) = -(500 + 250) = -750\,\text{mA}$

Therefore $\frac{dv(0^+)}{dt} = -750 \times \frac{10^{-3}}{C} = -75{,}000\,\text{V/s}$

Therefore $D_1 - \alpha D_2 = -75{,}000$; $\alpha = \frac{1}{2RC} = 500$, $D_1 = -50{,}000$ V/s

[e] $v = [50e^{-500t} - 50{,}000te^{-500t}]$ V

$i_{\text{R}} = \frac{v}{R} = [0.5e^{-500t} - 500te^{-500t}]\,\text{A}$, $t \geq 0^+$

AP 8.6 [a] $i_{\text{R}}(0^+) = \frac{V_0}{R} = \frac{40}{500} = 0.08\,\text{A}$

[b] $i_{\text{C}}(0^+) = I - i_{\text{R}}(0^+) - i_{\text{L}}(0^+) = -1 - 0.08 - 0.5 = -1.58\,\text{A}$

[c] $\frac{di_{\text{L}}(0^+)}{dt} = \frac{V_o}{L} = \frac{40}{0.64} = 62.5\,\text{A/s}$

[d] $\alpha = \frac{1}{2RC} = 1000$; $\frac{1}{LC} = 1{,}562{,}500$; $s_{1,2} = -1000 \pm j750$ rad/s

[e] $i_{\text{L}} = i_f + B_1'e^{-\alpha t}\cos\omega_d t + B_2'e^{-\alpha t}\sin\omega_d t$, $i_f = I = -1\,\text{A}$

$i_{\text{L}}(0^+) = 0.5 = i_f + B_1'$, therefore $B_1' = 1.5\,\text{A}$

$\frac{di_{\text{L}}(0^+)}{dt} = 62.5 = -\alpha B_1' + \omega_d B_2'$, therefore $B_2' = (25/12)\,\text{A}$

Therefore $i_{\text{L}}(t) = -1 + e^{-1000t}[1.5\cos 750t + (25/12)\sin 750t]\,\text{A}$, $t \geq 0$

[f] $v(t) = \frac{\text{L}di_{\text{L}}}{dt} = 40e^{-1000t}[\cos 750t - (154/3)\sin 750t]V$ $t \geq 0$

AP 8.7 **[a]** $i(0^+) = 0$, since there is no source connected to L for $t < 0$.

[b] $v_c(0^+) = v_C(0^-) = \left(\dfrac{15\,\text{k}}{15\,\text{k} + 9\,\text{k}}\right)(80) = 50\,\text{V}$

[c] $50 + 80i(0^+) + L\dfrac{di(0^+)}{dt} = 100, \qquad \dfrac{di(0^+)}{dt} = 10{,}000\,\text{A/s}$

[d] $\alpha = 8000; \qquad \dfrac{1}{LC} = 100 \times 10^6; \qquad s_{1,2} = -8000 \pm j6000\ \text{rad/s}$

[e] $i = i_f + e^{-\alpha t}[B_1' \cos\omega_d t + B_2' \sin\omega_d t]; \qquad i_f = 0, \quad i(0^+) = 0$

Therefore $\quad B_1' = 0; \qquad \dfrac{di(0^+)}{dt} = 10{,}000 = -\alpha B_1' + \omega_d B_2'$

Therefore $\quad B_2' = 1.67\,\text{A}; \qquad i = 1.67e^{-8000t} \sin 6000t\,\text{A}, \qquad t \geq 0$