Circuits II

Home Work # 4 (Ch11) Solution

AP 11.6 [a] $$\mathbf{I}_{\mathrm{AB}} = \left[\left(\frac{1}{\sqrt{3}}\right) \underline{/30^\circ}\right] [69.28\underline{/-10^\circ}] = 40\underline{/20^\circ}\,\mathrm{A}$$

Therefore $$Z_\phi = \frac{4160\underline{/0^\circ}}{40\underline{/20^\circ}} = 104\underline{/-20^\circ}\,\Omega$$

[b] $$\mathbf{I}_{\mathrm{AB}} = \left[\left(\frac{1}{\sqrt{3}}\right) \underline{/-30^\circ}\right] [69.28\underline{/-10^\circ}] = 40\underline{/-40^\circ}\,\mathrm{A}$$

Therefore $$Z_\phi = 104\underline{/40^\circ}\,\Omega$$

AP 11.8 [a] $$|S| = \sqrt{3}(208)(73.8) = 26{,}587.67\,\mathrm{VA}$$

$$Q = \sqrt{(26{,}587.67)^2 - (22{,}659)^2} = 13{,}909.50\ \mathrm{VAR}$$

[b] $$\mathrm{pf} = \frac{22{,}659}{26{,}587.67} = 0.8522 \quad \text{lagging}$$

P 11.1 [a] First, convert the cosine waveforms to phasors:

$$\mathbf{V}_a = 137\underline{/63^\circ}; \qquad \mathbf{V}_b = 137\underline{/-57^\circ}; \qquad \mathbf{V}_c = 137\underline{/183^\circ}$$

Subtract the phase angle of the a-phase from all phase angles:

$$\underline{/\mathbf{V}'_a} = 63^\circ - 63^\circ = 0^\circ$$

$$\underline{/\mathbf{V}'_b} = -57^\circ - 63^\circ = -120^\circ$$

$$\underline{/\mathbf{V}'_c} = 183^\circ - 63^\circ = 120^\circ$$

Compare the result to Eqs. 11.1 and 11.2:

Therefore abc

[b] First, convert the cosine waveforms to phasors, making sure that all waveforms are represented as cosines:

$$\mathbf{V}_a = 820\underline{/-36^\circ}; \qquad \mathbf{V}_b = 820\underline{/84^\circ}; \qquad \mathbf{V}_c = 820\underline{/-156^\circ}$$

Subtract the phase angle of the a-phase from all phase angles:

$$\underline{/\mathbf{V}'_a} = -36^\circ + 36^\circ = 0^\circ$$

$$\underline{/\mathbf{V}'_b} = 84^\circ + 36^\circ = 120^\circ$$

$$\underline{/\mathbf{V}'_c} = -156^\circ + 36^\circ = -120^\circ$$

Compare the result to Eqs. 11.1 and 11.2:

Therefore acb

P 11.8 $Z_{ga} + Z_{la} + Z_{La} = 60 + j80\,\Omega$

$$Z_{gb} + Z_{lb} + Z_{Lb} = 90 + j120\,\Omega$$

$$Z_{gc} + Z_{lc} + Z_{Lc} = 30 + j40\,\Omega$$

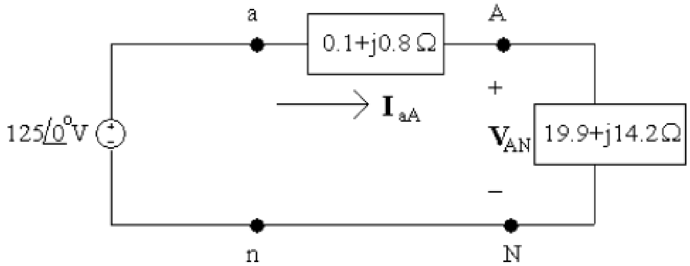
$$\frac{\mathbf{V}_N - 320}{60 + j80} + \frac{\mathbf{V}_N - 320\underline{/-120^\circ}}{90 + j120} + \frac{\mathbf{V}_N - 320\underline{/120^\circ}}{30 + j40} + \frac{\mathbf{V}_N}{20} = 0$$

Solving for $\mathbf{V}_N$ yields

$$\mathbf{V}_N = 49.47\underline{/75.14^\circ}\text{ V (rms)}$$

$$\mathbf{I}_o = \frac{\mathbf{V}_N}{20} = 2.47\underline{/75.14^\circ}\text{ A (rms)}$$

P 11.12 Make a sketch of the a-phase:



[a] Find the a-phase line current from the a-phase circuit:

$$\mathbf{I}_{\mathrm{aA}} = \frac{125\underline{/0^\circ}}{0.1 + j0.8 + 19.9 + j14.2} = \frac{125\underline{/0^\circ}}{20 + j15}$$

$$= 4 - j3 = 5\underline{/-36.87^\circ} \text{ A (rms)}$$

Find the other line currents using the acb phase sequence:

$$\mathbf{I}_{\mathrm{bB}} = 5\underline{/-36.87^\circ + 120^\circ} = 5\underline{/83.13^\circ} \text{ A (rms)}$$

$$\mathbf{I}_{\mathrm{cC}} = 5\underline{/-36.87^\circ - 120^\circ} = 5\underline{/-156.87^\circ} \text{ A (rms)}$$

[b] The phase voltage at the source is $\mathbf{V}_{\mathrm{an}} = 125\underline{/0^\circ}$ V. Use Fig. 11.9(b) to find the line voltage, $\mathbf{V}_{\mathrm{an}}$, from the phase voltage:

$$\mathbf{V}_{\mathrm{ab}} = \mathbf{V}_{\mathrm{an}}(\sqrt{3}\underline{/-30^\circ}) = 216.51\underline{/-30^\circ} \text{ V (rms)}$$

Find the other line voltages using the acb phase sequence:

$$\mathbf{V}_{\mathrm{bc}} = 216.51\underline{/-30^\circ + 120^\circ} = 216.51\underline{/90^\circ} \text{ V (rms)}$$

$$\mathbf{V}_{\mathrm{ca}} = 216.51\underline{/-30^\circ - 120^\circ} = 216.51\underline{/-150^\circ} \text{ V (rms)}$$

[c] The phase voltage at the load in the a-phase is $\mathbf{V}_{\mathrm{AN}}$. Calculate its value using $\mathbf{I}_{\mathrm{aA}}$ and the load impedance:

$$\mathbf{V}_{\mathrm{AN}} = \mathbf{I}_{\mathrm{aA}} Z_{\mathrm{L}} = (4 - j3)(19.9 + j14.2) = 122.2 - j2.9 = 122.23\underline{/-1.36^\circ} \text{ V (rms)}$$

Find the phase voltage at the load for the b- and c-phases using the acb sequence:

$$\mathbf{V}_{\mathrm{BN}} = 122.23\underline{/-1.36^\circ + 120^\circ} = 122.23\underline{/118.64^\circ} \text{ V (rms)}$$

$$\mathbf{V}_{\mathrm{CN}} = 122.23\underline{/-1.36^\circ - 120^\circ} = 122.23\underline{/-121.36^\circ} \text{ V (rms)}$$

[d] The line voltage at the load in the a-phase is $\mathbf{V}_{\mathrm{AB}}$. Find this line voltage from the phase voltage at the load in the a-phase, $\mathbf{V}_{\mathrm{AN}}$, using Fig, 11.9(b):

$$\mathbf{V}_{\mathrm{AB}} = \mathbf{V}_{\mathrm{AN}}(\sqrt{3}\underline{/-30^\circ}) = 211.72\underline{/-31.36^\circ}\ \mathrm{V\ (rms)}$$

Find the line voltage at the load for the b- and c-phases using the acb sequence:

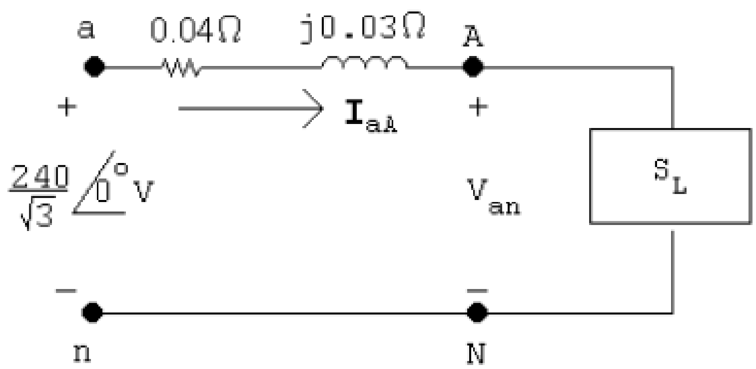
$$\mathbf{V}_{\mathrm{BC}} = 211.72\underline{/-31.36^\circ + 120^\circ} = 211.72\underline{/88.64^\circ}\ \mathrm{V\ (rms)}$$

$$\mathbf{V}_{\mathrm{CA}} = 211.72\underline{/-31.36^\circ - 120^\circ} = 211.72\underline{/-151.36^\circ}\ \mathrm{V\ (rms)}$$

P 11.37 [a] $S_{g/\phi} = \frac{1}{3}(41.6)(0.707 + j0.707) \times 10^3 = 9803.73 + j9803.73\,\mathrm{VA}$

$$\mathbf{I}_{\mathrm{aA}}^* = \frac{9803.73 + j9803.73}{240/\sqrt{3}} = 70.76 + j70.76\,\mathrm{A\ (rms)}$$

$$\mathbf{I}_{\mathrm{aA}} = 70.76 - j70.76\,\mathrm{A\ (rms)}$$



$$\mathbf{V}_{\mathrm{AN}} = \frac{240}{\sqrt{3}} - (0.04 + j0.03)(70.76 - j70.76)$$
$$= 133.61 + j0.71 = 133.61\underline{/0.30^\circ}\ \mathrm{V\ (rms)}$$

$$|\mathbf{V}_{\mathrm{AB}}| = \sqrt{3}(133.61) = 231.42\,\mathrm{V\ (rms)}$$

[b] $S_{L/\phi} = (133.61 + j0.71)(70.76 + j70.76) = 9404 + j9504.5\,\text{VA}$

$S_L = 3S_{L/\phi} = 28{,}212 + j28{,}513\,\text{VA}$

Check:

$S_g = 41{,}600(0.7071 + j0.7071) = 29{,}415 + j29{,}415\,\text{VA}$

$P_\ell = 3|\mathbf{I}_{\text{aA}}|^2(0.04) = 1202\,\text{W}$

$P_g = P_L + P_\ell = 28{,}212 + 1202 = 29{,}414\,\text{W} \quad \text{(checks)}$

$Q_\ell = 3|\mathbf{I}_{\text{aA}}|^2(0.03) = 901\,\text{VAR}$

$Q_g = Q_L + Q_\ell = 28{,}513 + 901 = 29{,}414\,\text{VAR} \quad \text{(checks)}$