Chapter 6: Inductance, Capacitance, & Mutual Inductance

EEL 3112c – Circuits-II

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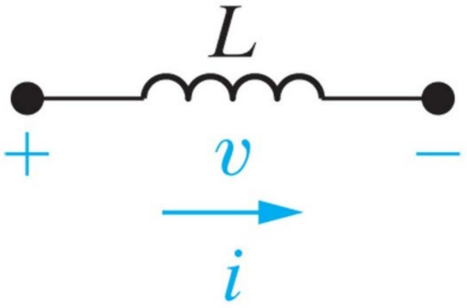
The Inductor

- Inductance is the circuit parameter used to describe an inductor
- Inductance is symbolized by the letter ***L***, is measured in henrys (**H**), and is represented graphically as a coil

(a)

- Assigning the reference direction of the current in the direction of the voltage drop across the terminals of the inductor, as shown in below, yields



(b)

The inductor $v - i$ equation ▶

$$v = L\frac{di}{dt},$$

Where v is measured in volts, L in henrys, i in amperes, and t in seconds

The Inductor – cont.

- Note from the inductor $v - i$ equation, the voltage across the terminals of an inductor is proportional to the rate of change of the current in the inductor
- We can make two important observations here
  - First, if the current is constant, the voltage across the ideal inductor is zero
    - Thus the inductor behaves as a short circuit in the presence of a constant, or dc, current
  - Second, current cannot change instantaneously in an inductor; that is, the current cannot change by a finite amount in zero time
    - This change would require an infinite voltage, and infinite voltages are not possible

The instantaneous change is represented as a vertical line at the point of change, and the slope (i.e. derivative of a vertical line is infinity. Thus, instantaneous change in current will require infinite voltage which is not possible)

Current in an Inductor in Terms of the Voltage Across the Inductor

$$v = L\frac{di}{dt} \Rightarrow v\,dt = L\,di \xrightarrow{\text{Integrate}} L\int_{i(t_0)}^{i(t)} dx = \int_{t_0}^{t} v\,d\tau$$

$$\Downarrow$$

$$\boxed{i(t) = \frac{1}{L}\int_0^t v\,d\tau + i(0)} \xleftarrow{t_0 = 0} i(t) = \frac{1}{L}\int_{t_0}^{t} v\,d\tau + i(t_0)$$

Power and Energy in the Inductor

$$p = vi \Rightarrow \boxed{p = Li\frac{di}{dt}}$$

◀ **Power in an inductor**

$$\Rightarrow p = v\left[\frac{1}{L}\int_{t_0}^{t} v\, d\tau + i(t_0)\right].$$

$$p = \frac{dw}{dt} = Li\frac{di}{dt} \Rightarrow dw = Li\, di$$

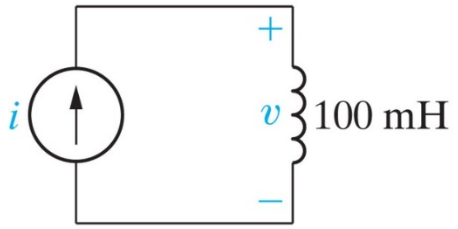
ω: *is energy in Joules*

$$\int_0^w dx = L\int_0^i y\, dy \Rightarrow \boxed{w = \frac{1}{2}Li^2}$$

◀ **Energy in an inductor**

Example 6.3

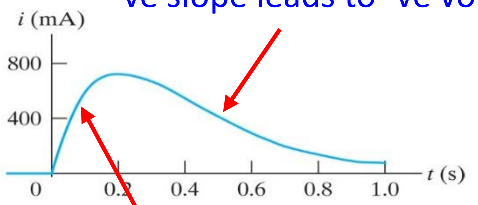
- The independent current source in the circuit shown below generates zero current for $t < 0$ and a pulse $10te^{-5t}$ A, for $t > 0$ (Example 6.1)

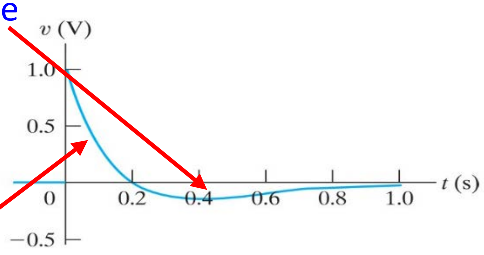


$$i = 0, \qquad t < 0$$

$$i = 10te^{-5t}\text{A}, \qquad t > 0$$

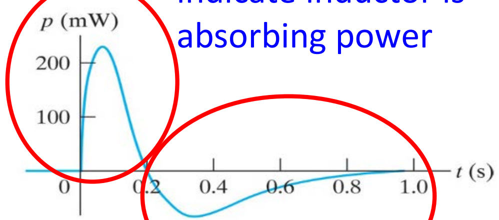
-ve slope leads to -ve voltage



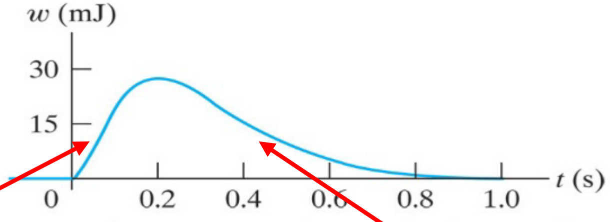


+ve slope leads to +ve voltage

+ve power to indicate inductor is absorbing power



-ve power to indicate inductor is generating power

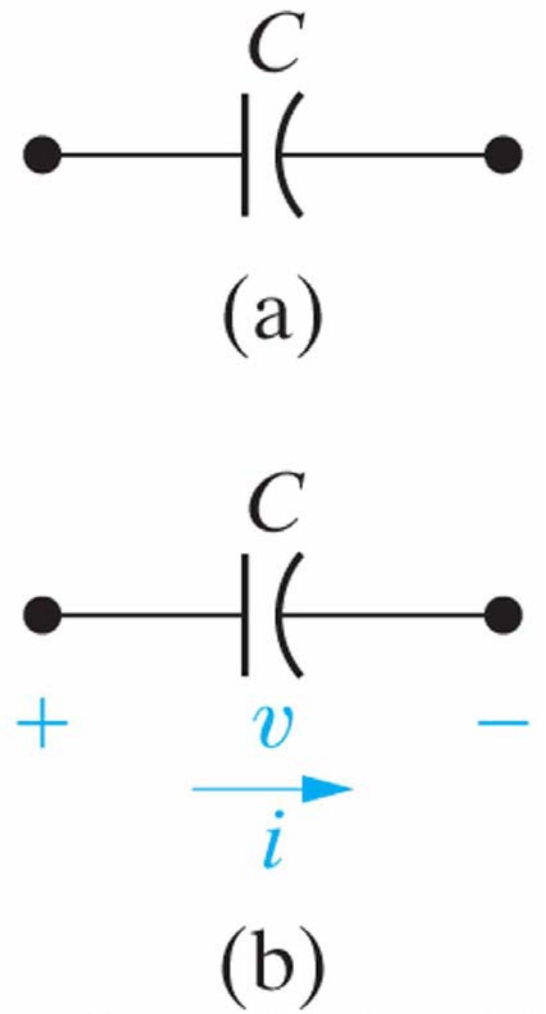


An increasing energy curve indicates that energy is being stored. Note that this corresponds to the interval when $p > 0$ (+ve power indicates the circuit element is absorbing power, thus the inductor is absorbing power by storing it)

A decreasing energy curve indicates that energy is being extracted. Note that this corresponds to the interval when $p < 0$ (-ve power indicates the circuit element is generating power, thus the inductor is supplying back the power it stored previously

The Capacitor

- The circuit parameter of capacitance
  - Represented by the letter ***C***
  - Measured in farads (**F**)
  - Symbolized graphically by two short parallel conductive plates
- Because the farad is an extremely large quantity of capacitance, practical capacitor values usually lie in the picofarad (pF) to microfarad (μF) range



The Capacitor – cont.

- The graphic symbol for a capacitor is a reminder that capacitance occurs whenever electrical conductors are separated by a dielectric, or insulating, material
  - Dielectrics are very poor conductors
- This condition implies that electric charge is not transported through the capacitor
  - Although applying a voltage to the terminals of the capacitor cannot move a charge through the dielectric, it can displace a charge within the dielectric
  - As the voltage varies with time, the displacement of charge also varies with time, causing what is known as the **displacement current**

The Capacitor – cont.

- The current is proportional to the rate at which the voltage across the capacitor varies with time

Capacitor $i - v$ equation ▶ $$i = C\frac{dv}{dt},$$

where i is measured in amperes, C in farads, v in volts, and t in seconds

- Two important observations follow from the capacitor $i - v$ equation
  - First, if the voltage across the terminals is constant, the capacitor current is zero
    - **Thus a capacitor behaves as an open circuit in the presence of a constant voltage**
  - Second, voltage cannot change instantaneously across the terminals of a capacitor; that is, the voltage cannot change by a finite amount in zero time
    - Such a change would produce infinite current, a physical impossibility

The Capacitor – cont.

- The capacitor $i - v$ equation gives the capacitor current as a function of the capacitor voltage
- Expressing the voltage as a function of the current is also useful

$$\boxed{i = C\frac{dv}{dt},} \Rightarrow \quad i\,dt = C\,dv \quad \xrightarrow{\text{Integrate}} \quad \int_{v(t_0)}^{v(t)} dx = \frac{1}{C}\int_{t_0}^{t} i\,d\tau.$$

$$\Downarrow$$

$$v(t) = \frac{1}{C}\int_{0}^{t} i\,d\tau + v(0). \quad \xleftarrow{\text{Let } t_0 = 0} \quad \boxed{v(t) = \frac{1}{C}\int_{t_0}^{t} i\,d\tau + v(t_0).}$$

◀ **Capacitor $v - i$ equation**

Capacitor Energy & Power

$$p = vi = Cv\frac{dv}{dt},$$

(6.16) ◀ **Capacitor power equation**

$$w = \frac{1}{2}Cv^2.$$

(6.18) ◀ **Capacitor energy equation**

Example 6.4:

The voltage pulse described by the following equations is impressed across the terminals of a 0.5 μF capacitor:

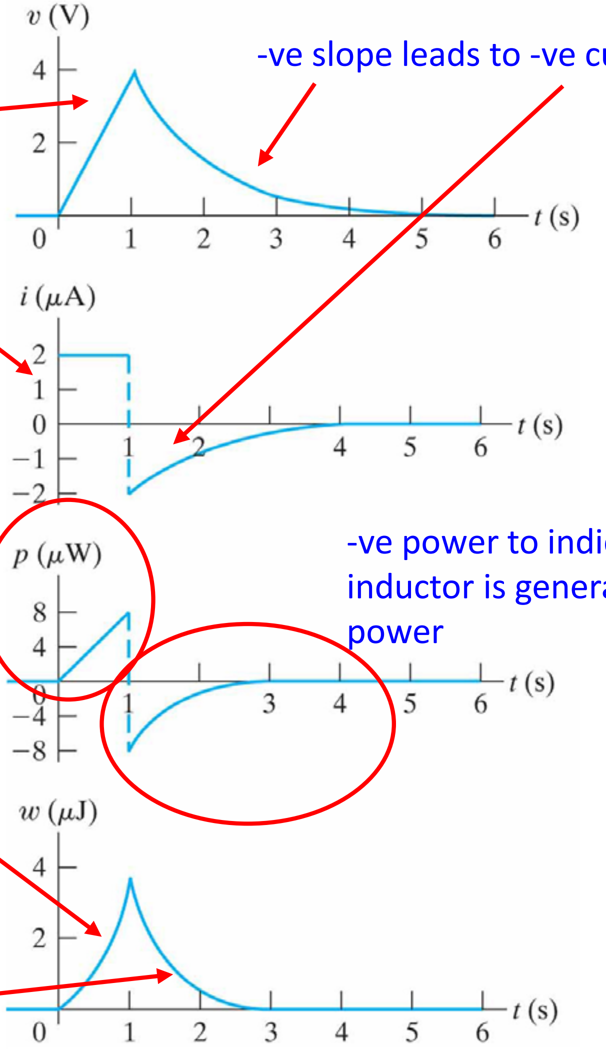
$$v(t) = \begin{cases} 0, & t \leq 0 \text{ s}; \\ 4t \text{ V}, & 0 \text{ s} \leq t \leq 1 \text{ s}; \\ 4e^{-(t-1)} \text{ V}, & t \geq 1 \text{ s}. \end{cases}$$

+ve slope leads to +ve current

-ve slope leads to -ve current

+ve power to indicate inductor is absorbing power

-ve power to indicate inductor is generating power



An increasing energy curve indicates that energy is being stored. Note that this corresponds to the interval when $p > 0$ (+ve power indicates the circuit element is absorbing power, thus the capacitor is absorbing power by storing it)

A decreasing energy curve indicates that energy is being extracted. Note that this corresponds to the interval when $p < 0$ (-ve power indicates the circuit element is generating power, thus the capacitor is supplying back the power it stored previously

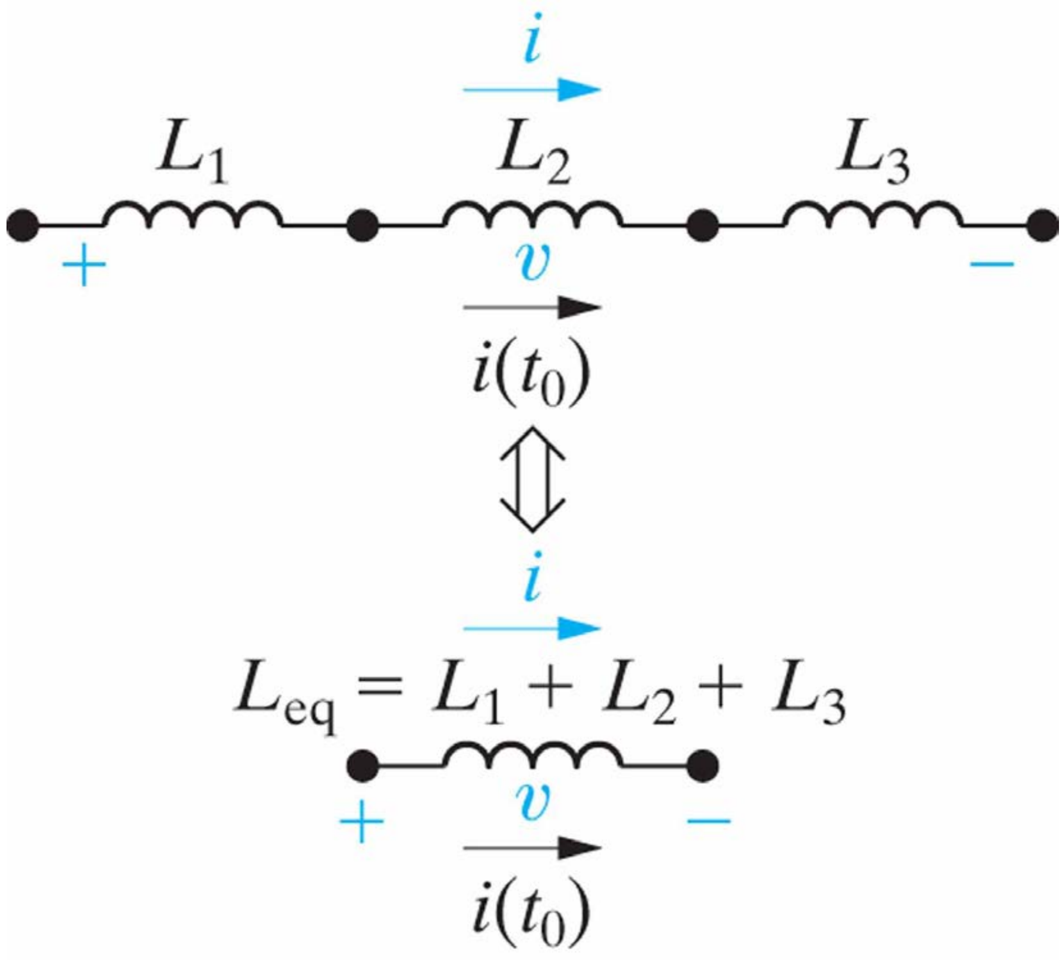
Series-Parallel Combinations of L & C

- Just as series-parallel combinations of resistors can be reduced to a single equivalent resistor, series-parallel combinations of inductors or capacitors can be reduced to a single inductor or capacitor

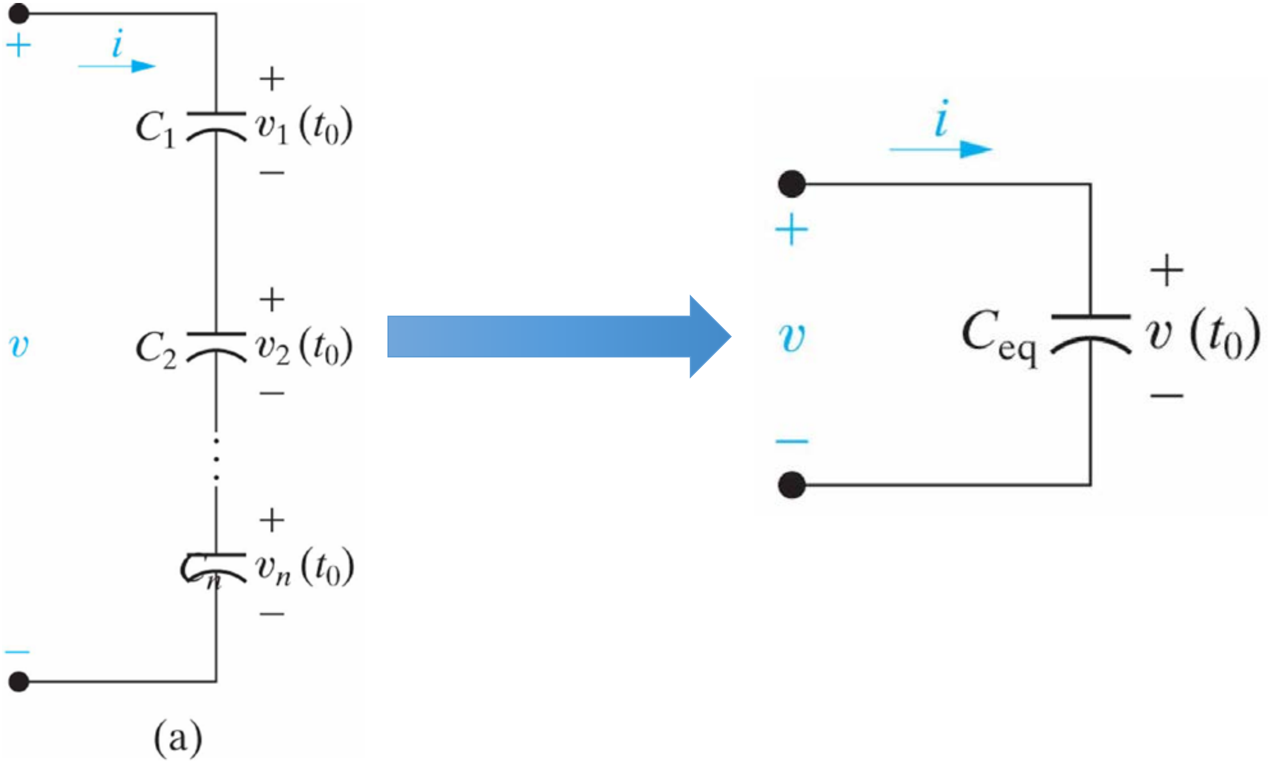
- **Elements connected in series are forced to carry the same current**

- **Elements connected in parallel are forced to have the same voltage**

Inductors & Capacitors in Series

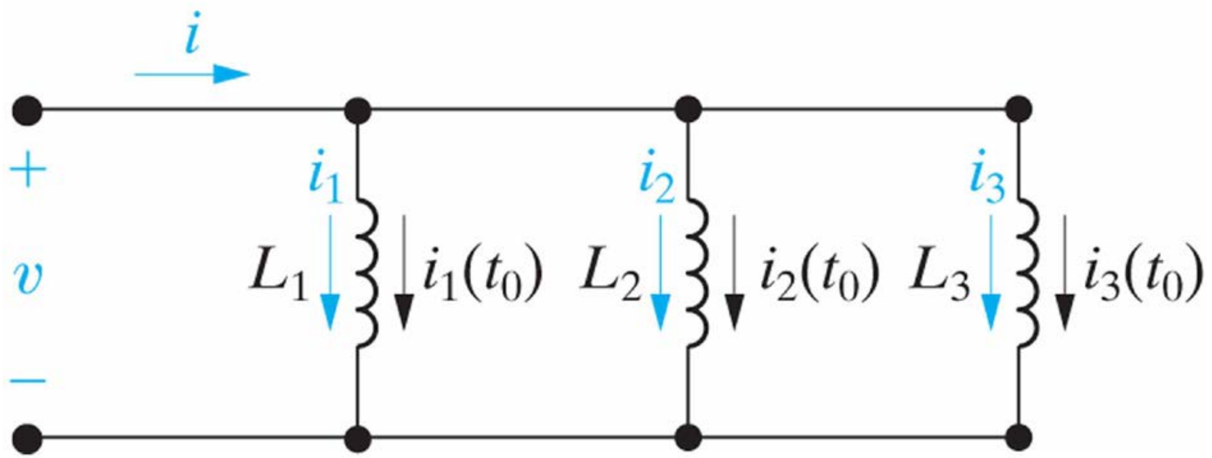


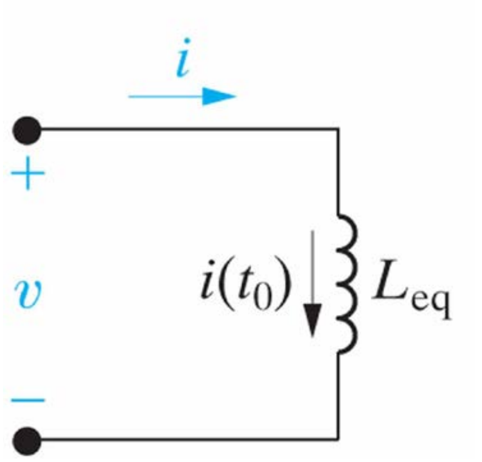
$$L_{eq} = L_1 + L_2 + L_3 + \cdots + L_n.$$



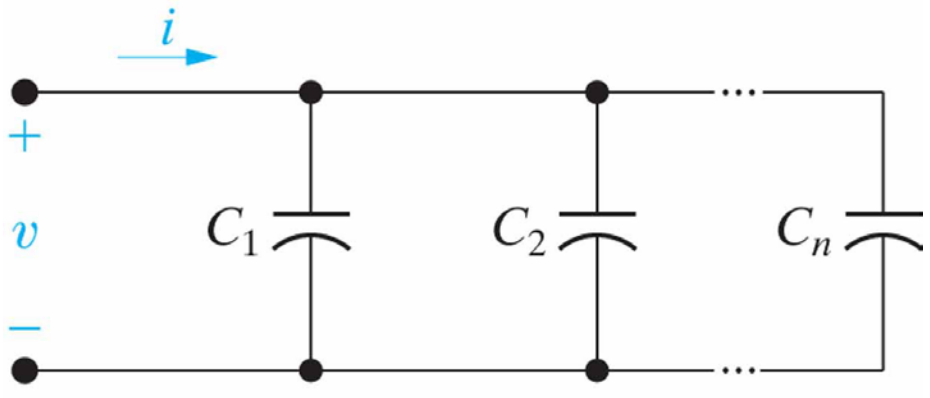
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$$

Inductors & Capacitors in Parallel

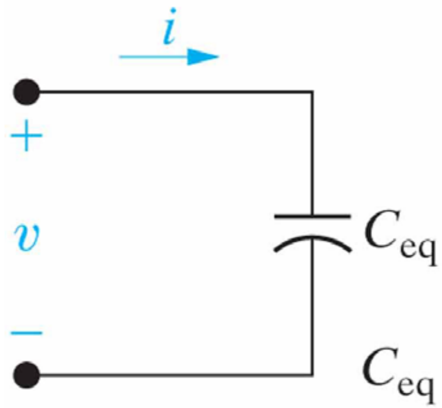




$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$



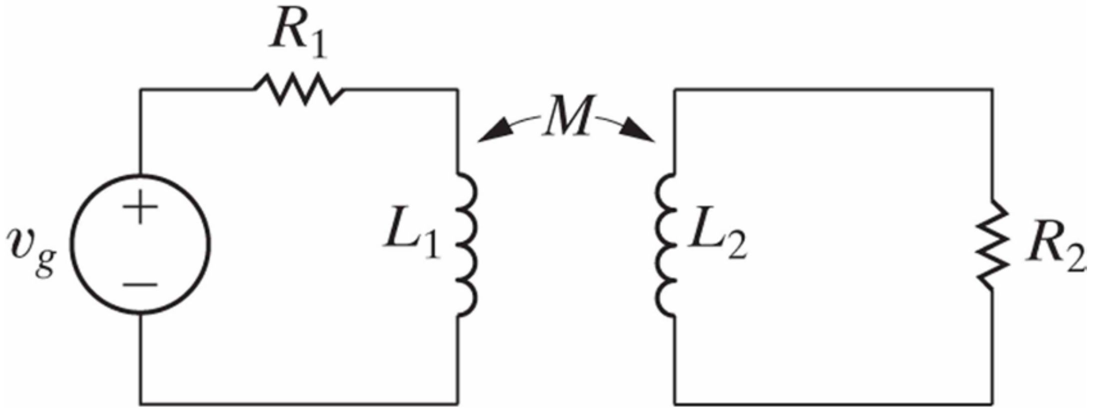
Combining capacitors in parallel ▶



$$C_{eq} = C_1 + C_2 + \cdots + C_n.$$

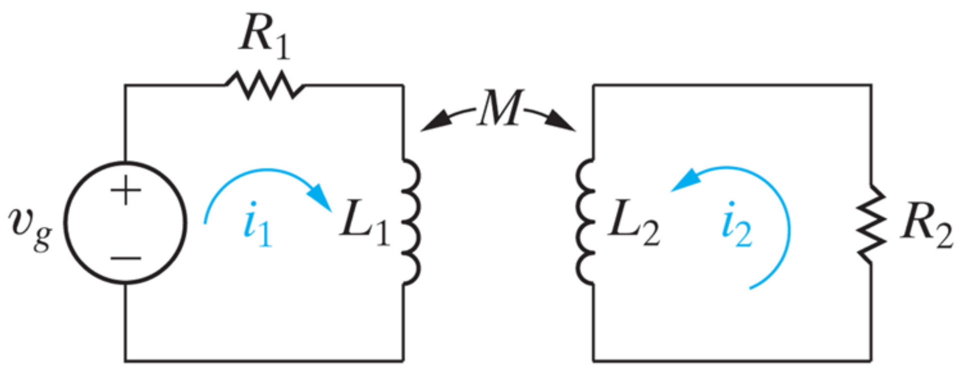
Mutual Inductance

- We now consider the situation in which two circuits are linked by a magnetic field (magnetically coupled)
  - Any current flowing in a an inductor will generate a magnetic field
- In this case, the voltage induced in the second circuit can be related to the time-varying current in the first circuit by a parameter known as mutual inductance
  - The self-inductances of the two coils are labeled L_1 and L_2 and the mutual inductance is labeled M
  - The double headed arrow adjacent to M indicates the pair of coils with this value of mutual inductance



Mutual Inductance – cont.

- The easiest way to analyze circuits containing mutual inductance is to use mesh currents
  - First select the reference direction for each coil
  - Then sum the voltages around each closed path
- Because of the mutual inductance *M*, there will be two voltages across each coil
  - Self-induced voltage
  - Mutually-induced voltage



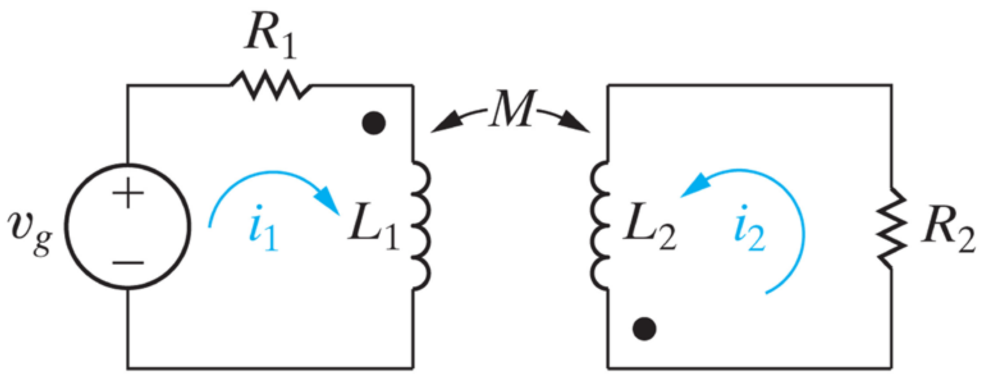
Left-hand circuit

Self-induced voltage: $L_1 \dfrac{di_1}{dt}$

Mutually-induced voltage: $M \dfrac{di_2}{dt}$

Note that the mutually induced voltage in one coil depends on the mutual inductance and the current of the other coil

Mutual Inductance – cont.



- Dot convention is used to determine polarity
  - Dot is placed on one terminal of each winding
  - These dots carry the sign information and allow us to draw the coils schematically rather than showing how they wrap around a core structure
- The rule for using the dot convention to determine the polarity of mutually induced voltage can be summarized as follows:
  - When the reference direction for a current enters the dotted terminal of a coil, the reference polarity of the voltage that it induces in the other coil is positive at its dotted terminal

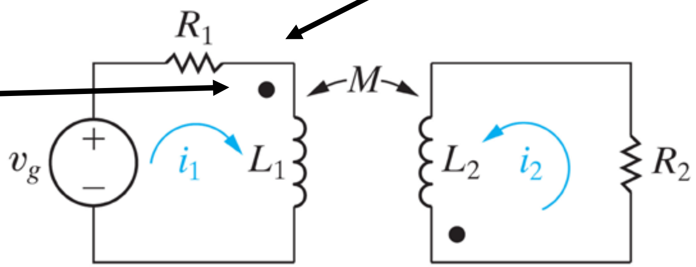
  OR:

  - When the reference direction for a current leaves the dotted terminal of a coil, the reference polarity of the voltage that it induces in the other coil is negative at its dotted terminal

Mutual Inductance – cont.

$$M\frac{di_2}{dt}$$

Result: reference polarity for voltage induced by current, i_2, is negative at the dotted terminal of coil 1.

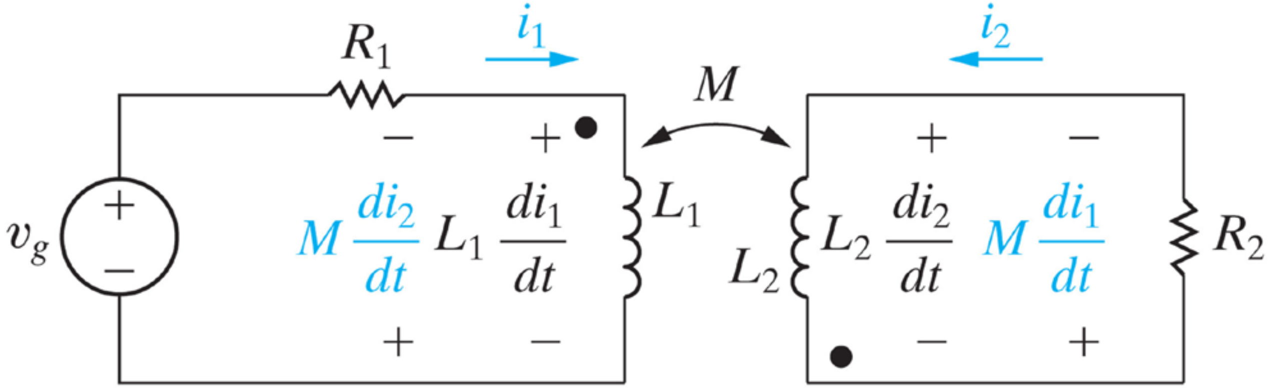


i_1 enters the dotted terminal of coil 1

$$M\frac{di_1}{dt}$$

Result: reference polarity for voltage induced by current, i_1, is positive at the dotted terminal of coil 2.

i_2 leaves the dotted terminal of coil 2



$$-v_g + i_1R_1 + L_1\frac{di_1}{dt} - M\frac{di_2}{dt} = 0$$

$$i_2R_2 + L_2\frac{di_2}{dt} - M\frac{di_1}{dt} = 0$$

Voltage polarity was negative at dotted terminal; results in a voltage rise wrt to i_1.

Voltage polarity was positive at dotted terminal; results in a voltage rise wrt to i_2.

Next Class

- Today we Reviewed Chapter 6
  - Inductance and capacitance
- Next class we will review chapter 7
  - Response of RL and RC circuits