HW 5 EEL 3112C §01 Peter A. Dravishnikov

AP 12.6, 12.8, 12.20

req. 12.14, 12.29

AP 12.6 find f(t) given

$$F(s) = \frac{(4s^2+7s+1)}{s(s+1)^2} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{(s+1)^2}$$

$$4s^2+7s+1 = A(s+1)^2 + B\cdot s(s+1) + C\cdot s$$

$$= A(s^2+2s+1) + B(s^2+s) + Cs$$

$$= (A+B)s^2 + (2A+B+C)s + A$$

$$\text{rref}\left(\left[\begin{array}{ccc|c} 1 & 1 & 0 & 4 \\ 2 & 1 & 1 & 7 \\ 1 & 0 & 0 & 2 \end{array}\right]\right)$$

$A=1 \quad B=3 \quad C=2$

$$F(s) = \frac{1}{s} + \frac{3}{s+1} + \frac{2}{(s+1)^2}$$

$$f(t) = u(t) + 3e^{-t}u(t) + 2te^{-t}u(t)$$

#12.8 Find f(t) given $F(s) = \frac{(5s^2+29s+32)}{(s+2)(s+4)}$

$$F(s) = \frac{(5s^2+29s+32)}{(s+2)(s+4)} = \frac{5s^2+29s+32}{s^2+6s+8} = 5 + \frac{s+8}{(s+2)(s+4)}$$

$$s^2+6s+8 \overset{5}{\overline{\big)\, 5s^2+29s+32}}$$

$$s+8$$

$$\frac{s+8}{(s+2)(s+4)} = \frac{A}{s+2} + \frac{B}{s+4}$$

$$-(s+8) = A(s+4) + B(s+2)$$

$$(A+B)s + (4A+2B)$$

$$\text{rref}\left(\begin{bmatrix} 1 & 1 & -1 \\ 4 & 2 & -8 \end{bmatrix}\right)$$

$$A = -3 \qquad B = +2$$

$$F(s) = 5 + \frac{-3}{s+2} + \frac{+2}{s+4}$$

$$f(t) = 5\delta(t) + 3e^{-2t}u(t) + 2e^{-4t}u(t)$$

AP 12.10 Use initial- & final-value theorems to find resp. values of $f(t)$

Note/
(12.7) $F(s) = \frac{40}{(s^2+4s+5)^2} \rightarrow \lim_{s\to\infty} \frac{40s}{(s^2+4s+5)^2} = 0$ initial

$\lim_{s\to 0^+} \frac{40s}{(s^2+4s+5)^2} = 0$ final

(12.6) $F(s) = \frac{(4s^2+7s+1)}{s(s+1)^2}$

(12.4) $F(s) = \frac{7s^2+63s+134}{(s+3)(s+4)(s+5)}$ (tutor)

$$\lim_{s\to\infty} \frac{s(4s^2+7s+1)}{s(s+1)^2} = \frac{(4s^3+7s^2+s)}{s(s^2+2s+1)}$$

$$= \lim_{s\to\infty} \frac{4s^3+7s^2+s}{s^3+2s^2+s} = 4/1 = 4 \text{ initial}$$

(please forgive my stupidity)

$$\lim_{s\to 0^+} \frac{4s^3+7s^2+s}{s^3+2s^2+s} = \frac{4s^2+7s+1}{s^2+2s+1} = 1$$

$$\lim_{s\to\infty} \frac{s(7s^2+63s+134)}{(s+3)(s+4)(s+5)} = \frac{s(7s^2+63s+134)}{(s^2+7s+12)(s+5)} = \frac{7s^3+63s^2+134s}{s^3+12s^2+47s+60}$$

s^3+7s^2+12s
$+5s^2+35s+60$

$= 7$ initial

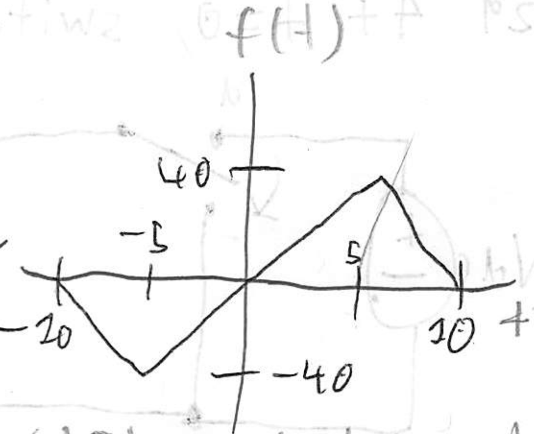
$$\lim_{s\to 0^+} \frac{7s^3+63s^2+134s}{s^3+12s^2+47s+60} = 0$$

but (tutor) answer of Hmm?

12.14 a) Find Laplace of chart here

b) Find Laplace of first derivative of chart

c) Find Laplace of second derivative of chart



Using bilateral transform

a)

$$f(t) = \begin{cases} -8t & -10 \leq t < 5 \\ 8t & -5 \leq t < 5 \\ -8t & 5 \leq t < 10 \end{cases}$$

$$= -8t\,(u(t+10) - u(t+5))$$

$$+ 8t\,(u(t+5) - u(t-5)$$

$$+ 8t\,(u(t-5) - u(t-10))$$

$$= 8t(-u(t+10) + u(t+5) + u(t+5) - u(t-5) - u(t-5) + u(t-10)$$

$$\mathcal{L}(f(t)) = \frac{8(-e^{10t} + 2e^{5t} - 2e^{-5t} + e^{-10t})}{s^2}$$

b)

$$f'(t) = -8(u(t+10) - u(t+5))$$

$$+ 8(u(t+5) - u(t-5)$$

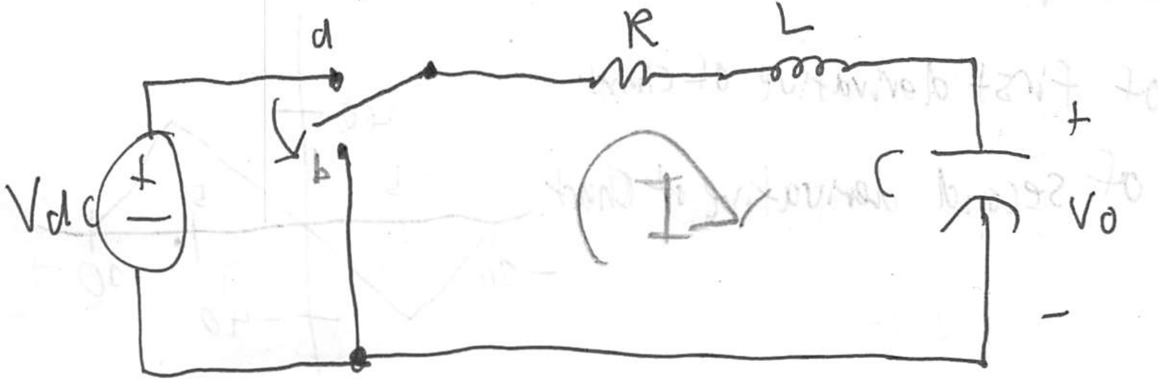
$$- 8(u(t-5) - u(t-10))$$

$$\mathcal{L}(f'(t)) = \frac{8(-e^{10t} + 2e^{5t} - 2e^{-5t} + e^{-10t})}{s}$$

c)

$$\mathcal{L}(f''(t)) = 8(-e^{10t} + 2e^{5t} - 2e^{-5t} + e^{-10t})$$

12.29 At $t=0$, switch to b



a) derive the diffyq that governs Vo for $t \geq 0^+$

b) ~~swo~~ show that $V_o(S) = \dfrac{Vdc[s+(R/L)]}{[s^2+(R/L)s+(1/LC)]}$

a) $V_0^{0^+} = Vdc$

$i(t) = C\dfrac{dv}{dt}$

$Ri + L\dfrac{di}{dt} + \dfrac{1}{C}\int_0^t i\,dt = 0$

$\dfrac{di}{dt} = C\dfrac{d^2v}{dt^2}$

$RC\dfrac{dv}{dt} + LC\dfrac{d^2v}{dt^2} + V(t) = 0 = R\dfrac{dv}{dt} + L\dfrac{d^2v}{dt^2} + \dfrac{v_o}{c} = 0$

b)

$\mathcal{L}\left\{R\dfrac{dv}{dt} + L\dfrac{d^2v}{dt^2} + \dfrac{V(t)}{C}\right\}$

$R(sV_o(s) - Vdc) + L(s^2V_o(s) - sVdc - 0) + \dfrac{V_o(S)}{C} = 0$

$0 = RV_o(s)s - RVdc + LV_o(s)s^2 - LsVdc + \dfrac{V_o(s)}{C}$

$V_o(s)\left(Rs + Ls^2 + \dfrac{1}{C}\right) = RVdc + LsVdc$

$V_o(S) = \dfrac{Vdc(R + Ls)}{(s^2 + \frac{R}{L}s + \frac{1}{LC})}$