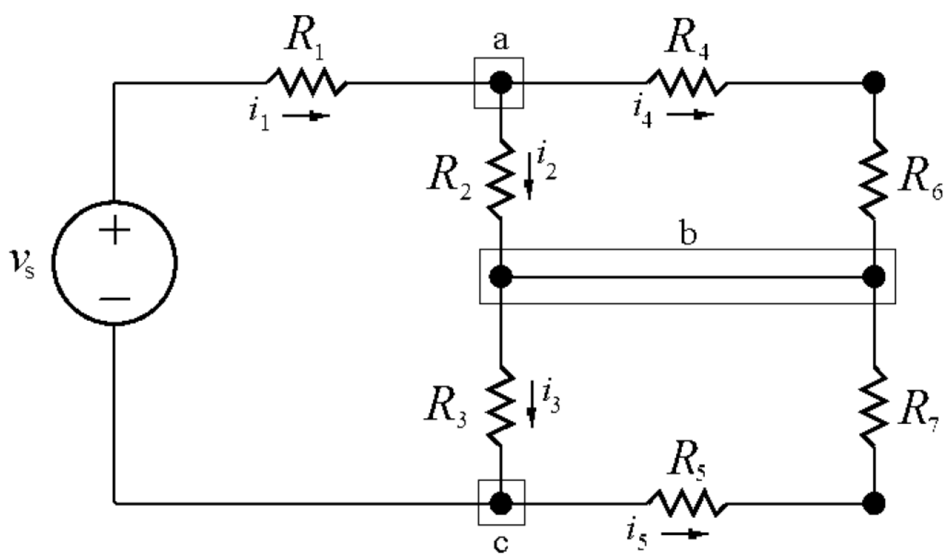
Circuits 2

Home Work # 0 Solution

P 4.3 [a] There are eight circuit components, seven resistors and the voltage source. Therefore there are **eight** unknown currents. However, the voltage source and the R_1 resistor are in series, so have the same current. The R_4 and R_6 resistors are also in series, so have the same current. The R_5 and R_7 resistors are in series, so have the same current. Therefore, we only need 5 equations to find the 5 distinct currents in this circuit.

[b]



There are three essential nodes in this circuit, identified by the boxes. At two of these nodes you can write KCL equations that will be independent of one another. A KCL equation at the third node would be dependent on the first two. Therefore there are **two** independent KCL equations.

[c] Sum the currents at any two of the three essential nodes a, b, and c. Using nodes a and c we get

$$-i_1 + i_2 + i_4 = 0$$

$$i_1 - i_3 + i_5 = 0$$

[d] There are three meshes in this circuit: one on the left with the components v_s, R_1, R_2 and R_3; one on the top right with components R_2, R_4, and R_6; and one on the bottom right with components R_3, R_5, and R_7. We can write KVL equations for all three meshes, giving a total of **three** independent KVL equations.

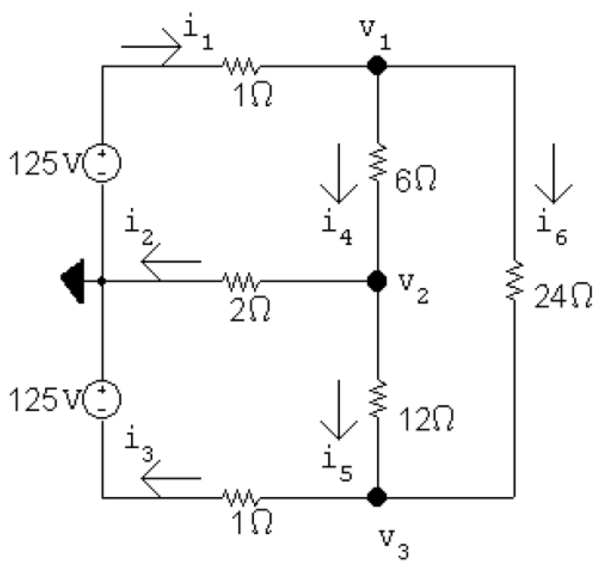
P 4.10 **[a]** $\dfrac{v_o - v_1}{R} + \dfrac{v_o - v_2}{R} + \dfrac{v_o - v_3}{R} + \cdots + \dfrac{v_o - v_n}{R} = 0$

$$\therefore \quad nv_o = v_1 + v_2 + v_3 + \cdots + v_n$$

$$\therefore \quad v_o = \frac{1}{n}[v_1 + v_2 + v_3 + \cdots + v_n] = \frac{1}{n}\sum_{k=1}^{n} v_k$$

[b] $v_o = \dfrac{1}{3}(100 + 80 - 60) = 40\,\text{V}$

P 4.15 **[a]**



$$\begin{aligned}
\frac{v_1 - 125}{1} + \frac{v_1 - v_2}{6} + \frac{v_1 - v_3}{24} &= 0 \\
\frac{v_2 - v_1}{6} + \frac{v_2}{2} + \frac{v_2 - v_3}{12} &= 0 \\
\frac{v_3 + 125}{1} + \frac{v_3 - v_2}{12} + \frac{v_3 - v_1}{24} &= 0
\end{aligned}$$

In standard form:

$$\begin{aligned}
v_1\left(\frac{1}{1}+\frac{1}{6}+\frac{1}{24}\right)+v_2\left(-\frac{1}{6}\right)+v_3\left(-\frac{1}{24}\right) &= 125\\
v_1\left(-\frac{1}{6}\right)+v_2\left(\frac{1}{6}+\frac{1}{2}+\frac{1}{12}\right)+v_3\left(-\frac{1}{12}\right) &= 0\\
v_1\left(-\frac{1}{24}\right)+v_2\left(-\frac{1}{12}\right)+v_3\left(\frac{1}{1}+\frac{1}{12}+\frac{1}{24}\right) &= -125
\end{aligned}$$

Solving, $v_1 = 101.24\,\text{V}$; $\quad v_2 = 10.66\,\text{V}$; $\quad v_3 = -106.57\,\text{V}$

Thus, $i_1 = \dfrac{125 - v_1}{1} = 23.76\ \text{A} \qquad i_4 = \dfrac{v_1 - v_2}{6} = 15.10\,\text{A}$

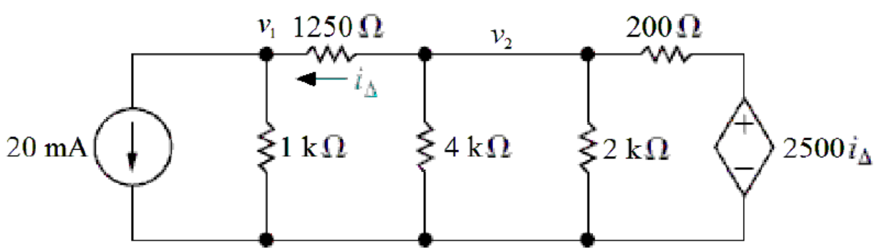
$i_2 = \dfrac{v_2}{2} = 5.33\ \text{A} \qquad i_5 = \dfrac{v_2 - v_3}{12} = 9.77\,\text{A}$

$i_3 = \dfrac{v_3 + 125}{1} = 18.43\ \text{A} \qquad i_6 = \dfrac{v_1 - v_3}{24} = 8.66\,\text{A}$

[b] $\sum P_{\text{dev}} = 125i_1 + 125i_3 = 5273.09\,\text{W}$

$\sum P_{\text{dis}} = i_1^2(1) + i_2^2(2) + i_3^2(1) + i_4^2(6) + i_5^2(12) + i_6^2(24) = 5273.09\,\text{W}$

P 4.19



[a] $0.02 + \dfrac{v_1}{1000} + \dfrac{v_1 - v_2}{1250} = 0$

$\dfrac{v_2 - v_1}{1250} + \dfrac{v_2}{4000} + \dfrac{v_2}{2000} + \dfrac{v_2 - 2500i_\Delta}{200} = 0$

$i_\Delta = \dfrac{v_2 - v_1}{1250}$

Solving,

$v_1 = 60\,\text{V}; \qquad v_2 = 160\,\text{V}; \qquad i_\Delta = 80\ \text{mA}$

$P_{20\text{mA}} = (0.02)v_1 = (0.02)(60) = 1.2\,\text{W}$ (absorbed)

$i_{\text{ds}} = \dfrac{v_2 - 2500i_\Delta}{200} = \dfrac{160 - (2500)(0.08)}{200} = -0.2\,\text{A}$

$P_{\text{ds}} = (2500i_\Delta)i_{\text{ds}} = 2500(0.08)(-0.2) = -40\,\text{W}$ (40 W developed)

$P_{\text{developed}} = 40\,\text{W}$

[b] $P_{1\text{k}} = \dfrac{v_1^2}{1000} = \dfrac{60^2}{1000} = 3.6\,\text{W}$

$P_{1250} = 1250 i_\Delta^2 = 1250(0.08)^2 = 8\,\text{W}$

$P_{4\text{k}} = \dfrac{v_2^2}{4000} = \dfrac{160^2}{4000} = 6.4\,\text{W}$

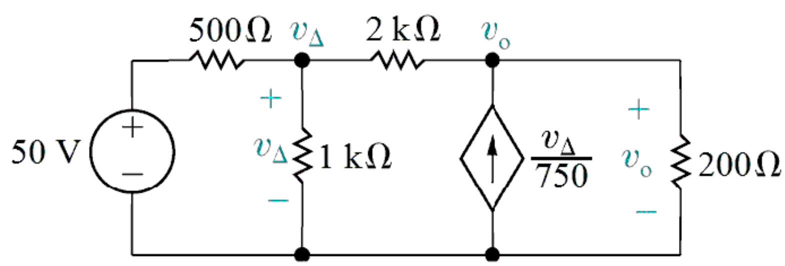
$P_{2\text{k}} = \dfrac{v_2^2}{2000} = \dfrac{160^2}{2000} = 12.8\,\text{W}$

$P_{200} = 200 i_{\text{ds}}^2 = 200(-0.2)^2 = 8\,\text{W}$

$$P_{\text{absorbed}} = P_{20\text{mA}} + P_{1\text{k}} + P_{1250} + P_{4\text{k}} + P_{2\text{k}} + P_{200}$$

$$= 1.2 + 3.6 + 8 + 6.4 + 12.8 + 8 = 40\,\text{W (check)}$$

P 4.20



[a] $\dfrac{v_\Delta - 50}{500} + \dfrac{v_\Delta}{1000} + \dfrac{v_\Delta - v_o}{2000} = 0$

$\dfrac{v_o - v_\Delta}{2000} - \dfrac{v_\Delta}{750} + \dfrac{v_o}{200} = 0$

Solving,

$v_\Delta = 30\,\text{V}; \qquad v_o = 10\,\text{V}$

[b] $i_{50\text{V}} = \dfrac{v_\Delta - 50}{500} = \dfrac{30 - 50}{500} = -0.04\,\text{A}$

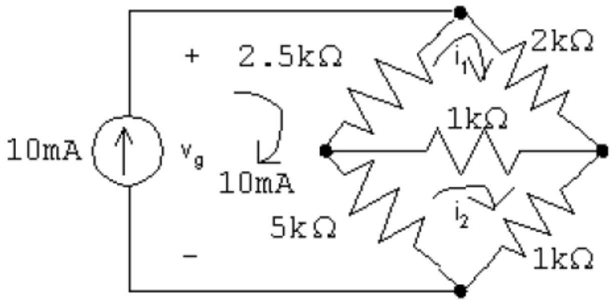
$P_{50\text{V}} = 50 i_{50\text{V}} = 50(-0.04) = -2\,\text{W} \qquad \text{(2 W supplied)}$

$P_{\text{ds}} = -v_o\left(\dfrac{v_\Delta}{750}\right) = -(10)(30/750) = -0.4\,\text{W} \qquad \text{(0.4 W supplied)}$

$P_{\text{total}} = 2 + 0.4 = 2.4\,\text{W supplied}$

P 4.54 [a] There are three unknown node voltages and only two unknown mesh currents. Use the mesh current method to minimize the number of simultaneous equations.

[b]



The mesh current equations:

$$2500(i_1 - 0.01) + 2000i_1 + 1000(i_1 - i_2) = 0$$

$$5000(i_2 - 0.01) + 1000(i_2 - i_1) + 1000i_2 = 0$$

Place the equations in standard form:

$$i_1(2500 + 2000 + 1000) + i_2(-1000) = 25$$

$$i_1(-1000) + i_2(5000 + 1000 + 1000) = 50$$

Solving, $i_1 = 6$ mA; $\quad i_2 = 8$ mA

Find the power in the $1\,\text{k}\Omega$ resistor:

$i_{1\text{k}} = i_1 - i_2 = -2\,\text{mA}$

$p_{1\text{k}} = (-0.002)^2(1000) = 4$ mW

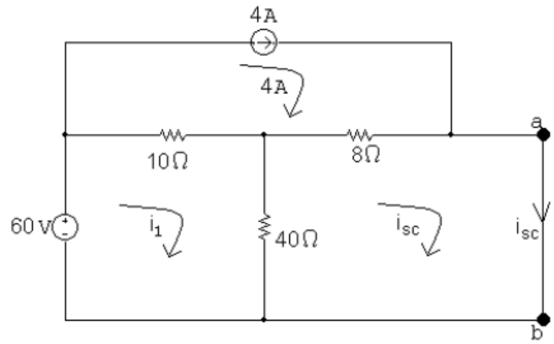
[c] No, the voltage across the 10 A current source is readily available from the mesh currents, and solving two simultaneous mesh-current equations is less work than solving three node voltage equations.

[d] $v_g = 2000i_1 + 1000i_2 = 12 + 8 = 20\,\text{V}$

$p_{10\text{mA}} = -(20)(0.01) = -200\,\text{mW}$

Thus the 10 mA source develops 200 mW.

P 4.66

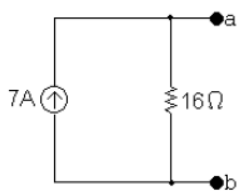


$$50i_1 - 40i_{\rm sc} = 60 + 40$$

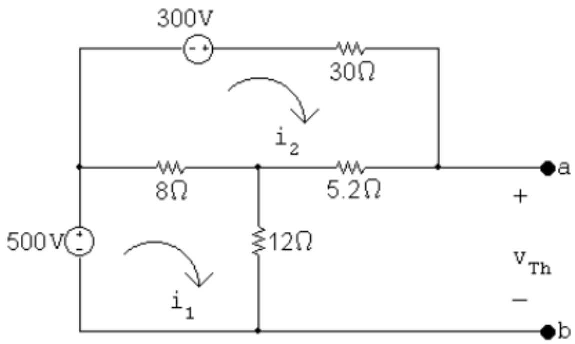
$$-40i_1 + 48i_{\rm scs} = 32$$

Solving, $\quad i_{\rm sc} = 7\,\text{A}$

$$R_{\rm Th} = 8 + \frac{(10)(40)}{50} = 16\,\Omega$$



P 4.67 After making a source transformation the circuit becomes



$$500 = 20i_1 - 8i_2$$

$$300 = -8i_1 + 43.2i_2$$

$$\therefore \quad i_1 = 30\,\text{A} \text{ and } i_2 = 12.5\,\text{A}$$

$$v_{\text{Th}} = 12i_1 + 5.2i_2 = 425\,\text{V}$$

$$R_{\text{Th}} = (8\|12 + 5.2)\|30 = 7.5\,\Omega$$