Circuits II

Home Work # 5 (Ch12) Solution

AP 12.6

$$F(s) = \frac{4s^2 + 7s + 1}{s(s+1)^2} = \frac{K_0}{s} + \frac{K_1}{(s+1)^2} + \frac{K_2}{s+1}$$

$$K_0 = \frac{1}{(1)^2} = 1; \qquad K_1 = \frac{4 - 7 + 1}{-1} = 2$$

$$K_2 = \frac{d}{ds}\left[\frac{4s^2 + 7s + 1}{s}\right]_{s=-1} = \left.\frac{s(8s+7) - (4s^2 + 7s + 1)}{s^2}\right|_{s=-1}$$

$$= \frac{1+2}{1} = 3$$

Therefore $\quad f(t) = [1 + 2te^{-t} + 3e^{-t}]\,u(t)$

AP 12.8

$$F(s) = \frac{5s^2 + 29s + 32}{(s+2)(s+4)} = \frac{5s^2 + 29s + 32}{s^2 + 6s + 8} = 5 - \frac{s+8}{(s+2)(s+4)}$$

$$\frac{s+8}{(s+2)(s+4)} = \frac{K_1}{s+2} + \frac{K_2}{s+4}$$

$$K_1 = \frac{-2+8}{2} = 3; \qquad K_2 = \frac{-4+8}{-2} = -2$$

Therefore,

$$F(s) = 5 - \frac{3}{s+2} + \frac{2}{s+4}$$

$$f(t) = 5\delta(t) + [-3e^{-2t} + 2e^{-4t}]u(t)$$

AP 12.10

$$\lim_{s\to\infty} sF(s) = \lim_{s\to\infty}\left[\frac{7s^3[1+(9/s)+(134/(7s^2))]}{s^3[1+(3/s)][1+(4/s)][1+(5/s)]}\right] = 7$$

$$\therefore \quad f(0^+) = 7$$

$$\lim_{s\to 0} sF(s) = \lim_{s\to 0}\left[\frac{7s^3+63s^2+134s}{(s+3)(s+4)(s+5)}\right] = 0$$

$$\therefore \quad f(\infty) = 0$$

$$\lim_{s\to\infty} sF(s) = \lim_{s\to\infty}\left[\frac{s^3[4+(7/s)+(1/s)^2]}{s^3[1+(1/s)]^2}\right] = 4$$

$$\therefore \quad f(0^+) = 4$$

$$\lim_{s\to 0} sF(s) = \lim_{s\to 0}\left[\frac{4s^2+7s+1}{(s+1)^2}\right] = 1$$

$$\therefore \quad f(\infty) = 1$$

$$\lim_{s\to\infty} sF(s) = \lim_{s\to\infty}\left[\frac{40s}{s^4[1+(4/s)+(5/s^2)]^2}\right] = 0$$

$$\therefore \quad f(0^+) = 0$$

$$\lim_{s\to 0} sF(s) = \lim_{s\to 0}\left[\frac{40s}{(s^2+4s+5)^2}\right] = 0$$

$$\therefore \quad f(\infty) = 0$$

P 12.14 [a] $f(t) = (-8t-80)[u(t+10)-u(t+5)]$

$$+8t[u(t+5)-u(t-5)]$$

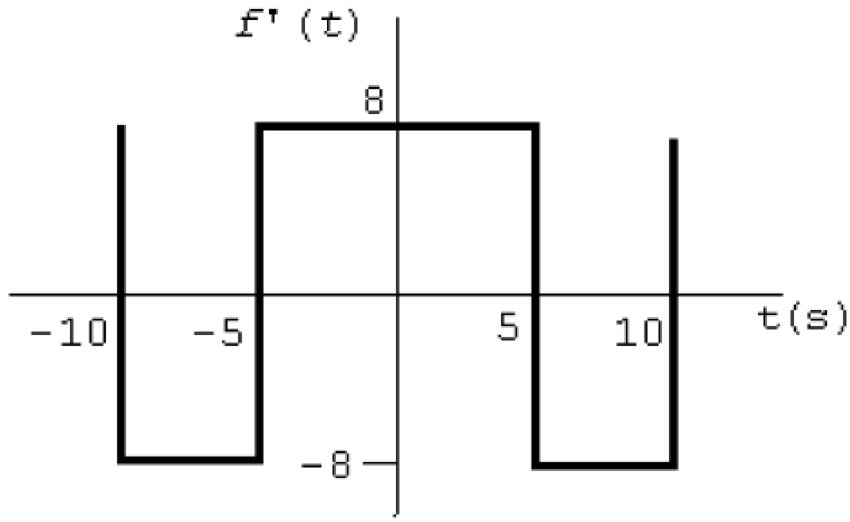
$$+(-8t+80)[u(t-5)-u(t-10)]$$

$$= -8(t+10)u(t+10)+16(t+5)u(t+5)$$

$$-16(t-5)u(t-5)+8(t-10)u(t-10)$$

$$\therefore \quad F(s) = \frac{8[-e^{10s}+2e^{5s}-2e^{-5s}+e^{-10s}]}{s^2}$$

[b]



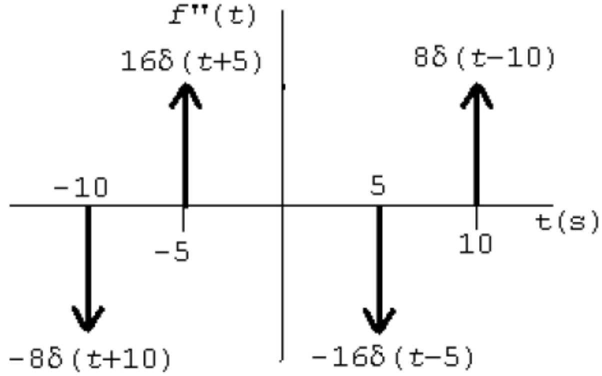
$$f'(t) = -8[u(t+10)-u(t+5)]+8[u(t+5)-u(t-5)]$$

$$+(-8)[u(t-5)-u(t-10)]$$

$$= -8u(t+10)+16u(t+5)-16u(t-5)+8u(t-10)$$

$$\mathcal{L}\{f'(t)\} = \frac{8[-e^{10s}+2e^{5s}-2e^{-5s}+e^{-10s}]}{s}$$

[c]



$$f''(t) = -8\delta(t+10) + 16\delta(t+5) - 16\delta(t-5) + 8\delta(t-10)$$

$$\mathcal{L}\{f''(t)\} = 8[-e^{10s} + 2e^{5s} - 2e^{-5s} + e^{-10s}]$$

P 12.29 [a] For $t \geq 0^+$:

$$Ri_o + L\frac{di_o}{dt} + v_o = 0$$

$$i_o = C\frac{dv_o}{dt} \qquad \frac{di_o}{dt} = C\frac{d^2v_o}{dt^2}$$

$$\therefore \quad RC\frac{dv_o}{dt} + LC\frac{d^2v_o}{dt^2} + v_o = 0$$

or

$$\frac{d^2v_o}{dt^2} + \frac{R}{L}\frac{dv_o}{dt} + \frac{1}{LC}v_o = 0$$

[b] $s^2V_o(s) - sV_{\rm dc} - 0 + \frac{R}{L}[sV_o(s) - V_{\rm dc}] + \frac{1}{LC}V_o(s) = 0$

$$V_o(s)\left[s^2 + \frac{R}{L}s + \frac{1}{LC}\right] = V_{\rm dc}(s + R/L)$$

$$V_o(s) = \frac{V_{\rm dc}[s + (R/L)]}{[s^2 + (R/L)s + (1/LC)]}$$