Circuits II

Ch12 Additional Problems Solution

AP 12.1 [a] $\cosh \beta t = \dfrac{e^{\beta t} + e^{-\beta t}}{2}$

Therefore,

$$\mathcal{L}\{\cosh \beta t\} = \frac{1}{2} \int_{0^-}^{\infty} [e^{(s-\beta)t} + e^{-(s-\beta)t}] dt$$

$$= \frac{1}{2} \left[\frac{e^{-(s-\beta)t}}{-(s-\beta)} \Big|_{0^-}^{\infty} + \frac{e^{-(s+\beta)t}}{-(s+\beta)} \Big|_{0^-}^{\infty} \right]$$

$$= \frac{1}{2} \left(\frac{1}{s-\beta} + \frac{1}{s+\beta} \right) = \frac{s}{s^2 - \beta^2}$$

[b] $\sinh \beta t = \dfrac{e^{\beta t} - e^{-\beta t}}{2}$

Therefore,

$$\mathcal{L}\{\sinh \beta t\} = \frac{1}{2} \int_{0^-}^{\infty} \left[e^{-(s-\beta)t} - e^{-(s+\beta)t} \right] dt$$

$$= \frac{1}{2} \left[\frac{e^{-(s-\beta)t}}{-(s-\beta)} \right]_{0^-}^{\infty} - \frac{1}{2} \left[\frac{e^{-(s+\beta)t}}{-(s+\beta)} \right]_{0^-}^{\infty}$$

$$= \frac{1}{2} \left(\frac{1}{s-\beta} - \frac{1}{s+\beta} \right) = \frac{\beta}{(s^2 - \beta^2)}$$

AP 12.2 [a] Let $f(t) = te^{-at}$:

$$F(s) = \mathcal{L}\{te^{-at}\} = \frac{1}{(s+a)^2}$$

$$\text{Now,} \quad \mathcal{L}\{tf(t)\} = -\frac{dF(s)}{ds}$$

$$\text{So,} \quad \mathcal{L}\{t \cdot te^{-at}\} = -\frac{d}{ds}\left[\frac{1}{(s+a)^2}\right] = \frac{2}{(s+a)^3}$$

[b] Let $f(t) = e^{-at}\sinh \beta t$, then

$$\mathcal{L}\{f(t)\} = F(s) = \frac{\beta}{(s+a)^2 - \beta^2}$$

$$\mathcal{L}\left\{\frac{df(t)}{dt}\right\} = sF(s) - f(0^-) = \frac{s(\beta)}{(s+a)^2 - \beta^2} - 0 = \frac{\beta s}{(s+a)^2 - \beta^2}$$

[c] Let $f(t) = \cos \omega t$. Then

$$F(s) = \frac{s}{(s^2 + \omega^2)} \quad \text{and} \quad \frac{dF(s)}{ds} = \frac{-(s^2 - \omega^2)}{(s^2 + \omega^2)^2}$$

$$\text{Therefore} \quad \mathcal{L}\{t\cos \omega t\} = -\frac{dF(s)}{ds} = \frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$$

AP 12.3

$$F(s) = \frac{6s^2 + 26s + 26}{(s+1)(s+2)(s+3)} = \frac{K_1}{s+1} + \frac{K_2}{s+2} + \frac{K_3}{s+3}$$

$$K_1 = \frac{6 - 26 + 26}{(1)(2)} = 3; \qquad K_2 = \frac{24 - 52 + 26}{(-1)(1)} = 2$$

$$K_3 = \frac{54 - 78 + 26}{(-2)(-1)} = 1$$

$$\text{Therefore} \quad f(t) = \left[3e^{-t} + 2e^{-2t} + e^{-3t}\right]u(t)$$

AP 12.5

$$F(s) = \frac{10(s^2 + 119)}{(s+5)(s^2 + 10s + 169)}$$

$$s_{1,2} = -5 \pm \sqrt{25 - 169} = -5 \pm j12$$

$$F(s) = \frac{K_1}{s+5} + \frac{K_2}{s+5-j12} + \frac{K_2^*}{s+5+j12}$$

$$K_1 = \frac{10(25 + 119)}{25 - 50 + 169} = 10$$

$$K_2 = \frac{10[(-5+j12)^2 + 119]}{(j12)(j24)} = j4.17 = 4.17\underline{/90°}$$

Therefore

$$f(t) = [10e^{-5t} + 8.33e^{-5t}\cos(12t + 90°)]\,u(t)$$

$$= [10e^{-5t} - 8.33e^{-5t}\sin 12t]\,u(t)$$