HW3 EEE 4510 Peter A. Dranishnikov

Design a DSP system running the difference equation $y[n] = a\,x[n] + x[n-1] + a\,x[n-2]$ with $T_s = 0.01\,s$ and "a" is the parameter you are asked to choose. The objective is to block 5 Hz sinusoidal signals completely. Find whether this is technically possible, and if yes, find the value of "a".

Blocking ~~5~~ 5Hz completely: $H(e^{j\cdot 2\pi\cdot 5\cdot 0.01}) = 0$

$H(z) = a + z^{-1} + a z^{-2}$ by difference equation, is BIBO stable since denominator is a unit (1) value

$H(e^{j2\pi\cdot 5\cdot 0.01}) = a + e^{-j2\pi 0.05} + a e^{-j4\pi 0.05} = 0$

~~$H(f=0): a + 1 + a = 0 \quad a = -\frac{1}{2}$~~ ~~complex part of "a"~~ - rest of solution on next page

~~Complex: $0 + j\sin(-2\pi 0.05) + j\sin(-4\pi 0.05) = 0$~~

~~$\mathrm{Im}\{a\} = \frac{-j\sin(-2\pi 0.05)}{-j\sin(-4\pi 0.05)}$~~ ← forgot how to complex number

~~Conjugate: $a^* + e^{j2\pi 0.05} + a^* e^{j4\pi 0.05}$~~

$\mathrm{Re}\{x\} = \frac{1}{2}(x + x^*)$ x^* is conjugate of x complex#

$\mathrm{Im}\{x\} = \frac{1}{2j}(x - x^*)$

real: $a + \frac{e^{-j2\pi 0.05} + e^{j2\pi 0.05}}{2} + a\frac{e^{-j4\pi 0.05} + e^{j4\pi 0.05}}{2}$

$= a + \frac{1 + e^{j2\pi 0.05 + j2\pi 0.05}}{e^{j2\pi 0.05}} +$

$$a + e^{-j2\pi 0.05} + a e^{-j4\pi 0.05} = 0$$

$$a + a e^{-j4\pi 0.05} = -e^{-j2\pi 0.05}$$

$$a(1 + e^{-j4\pi 0.05}) = -e^{-j2\pi 0.05}$$

$$a = \frac{-e^{-j2\pi 0.05}}{1 + e^{-j4\pi 0.05}} = \frac{-(\cos(-2\pi 0.05) - j\sin(-2\pi 0.05))}{1 + \cos(-4\pi 0.05) + j\sin(-4\pi 0.05)}$$

$$= \frac{(-\cos(-2\pi 0.05) - \cos(-2\pi 0.05)\cos(-4\pi 0.05)) - \sin(-2\pi 0.05)\sin(-4\pi 0.05)) + j\left(\underbrace{(-\sin(-2\pi 0.05) - \sin(-2\pi 0.05)\cos(-4\pi 0.05))}_{0.5590169944375} + \underbrace{\cos(-2\pi 0.05)\sin(-4\pi 0.05)}_{-0.5590169944375}\right)}{(1 + \cos(-4\pi 0.05))^2 + (\sin(-4\pi 0.05))^2}$$

bent fraction

yields a floating point error

$$= \frac{-1.902113032594 + j(-\sin \ldots}{3.618033988875}$$

(kept exact due to floating point error)

$$= -0.525731112119 + j(-\sin(-2\pi 0.05) - \sin(-2\pi 0.05)\cos(-4\pi 0.05)) + \cos(-2\pi 0.05)\sin(-4\pi 0.05))$$

one solution. Yes its possible

$$= -0.525731112119 + 0$$

(the imaginary part does cancel out)