Chapter 4: Techniques of Circuit Analysis

EEL 3112c – Circuits-II

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Terminology

- To discuss the more involved methods of circuit analysis, we must define a few basic terms first

TABLE 4.1 Terms for Describing Circuits

Name	Definition
node	A point where two or more circuit elements join
essential node	A node where three or more circuit elements join
path	A trace of adjoining basic elements with no elements included more than once
branch	A path that connects two nodes
essential branch	A path which connects two essential nodes without passing through an essential node
loop	A path whose last node is the same as the starting node
mesh	A loop that does not enclose any other loops
planar circuit	A circuit that can be drawn on a plane with no crossing branches

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Planar circuits: Those circuits that can be drawn on a plane with no crossing branches

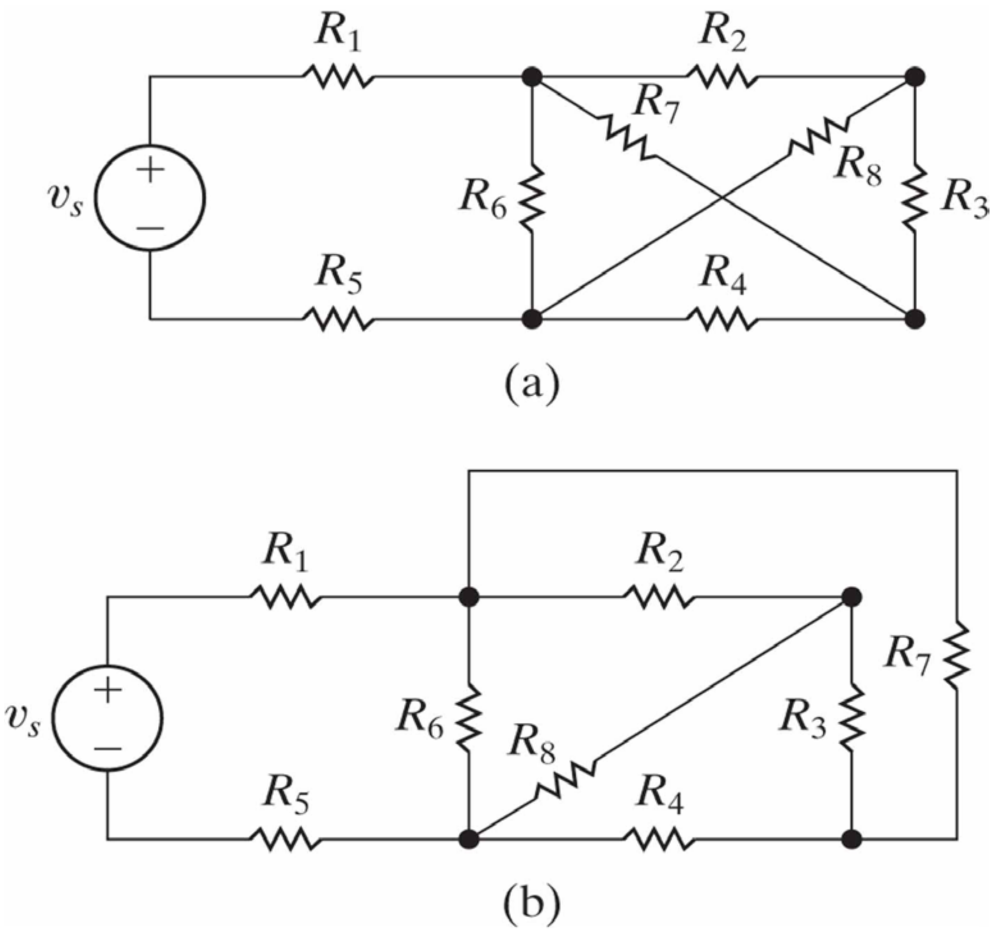

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Figure 4.1 (a) A planar circuit. (b) The same circuit redrawn to verify that it is planar.

Nonplanar circuits: Circuits that cannot be redrawn in such a way that all the node connections are maintained and no branches overlap

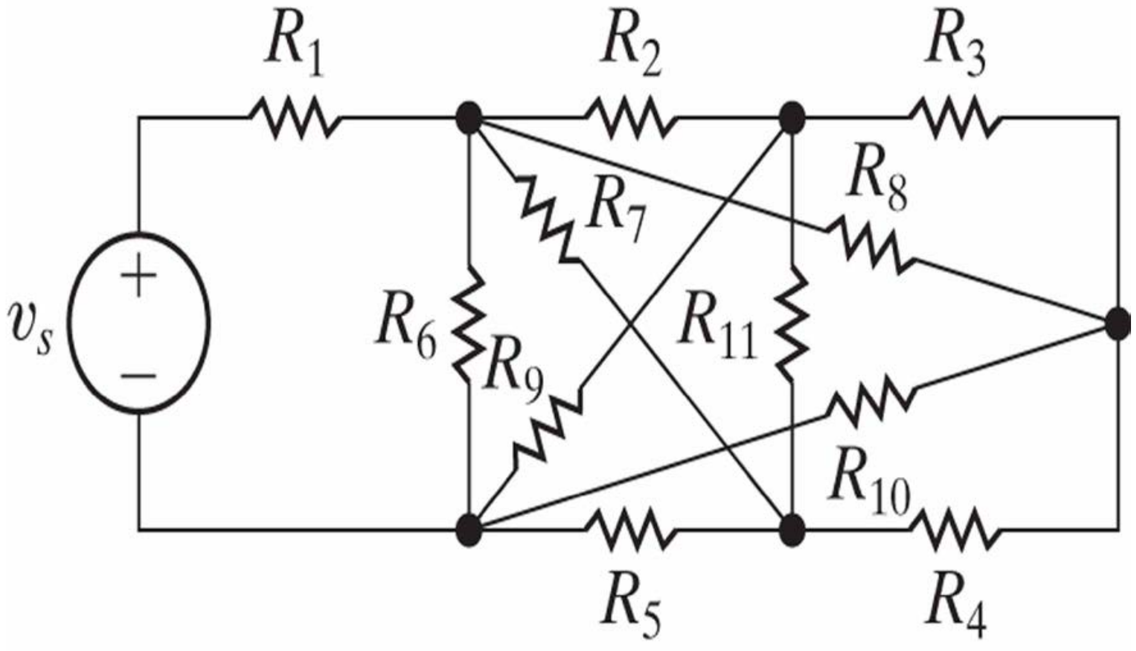

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Figure 4.2 A nonplanar circuit.

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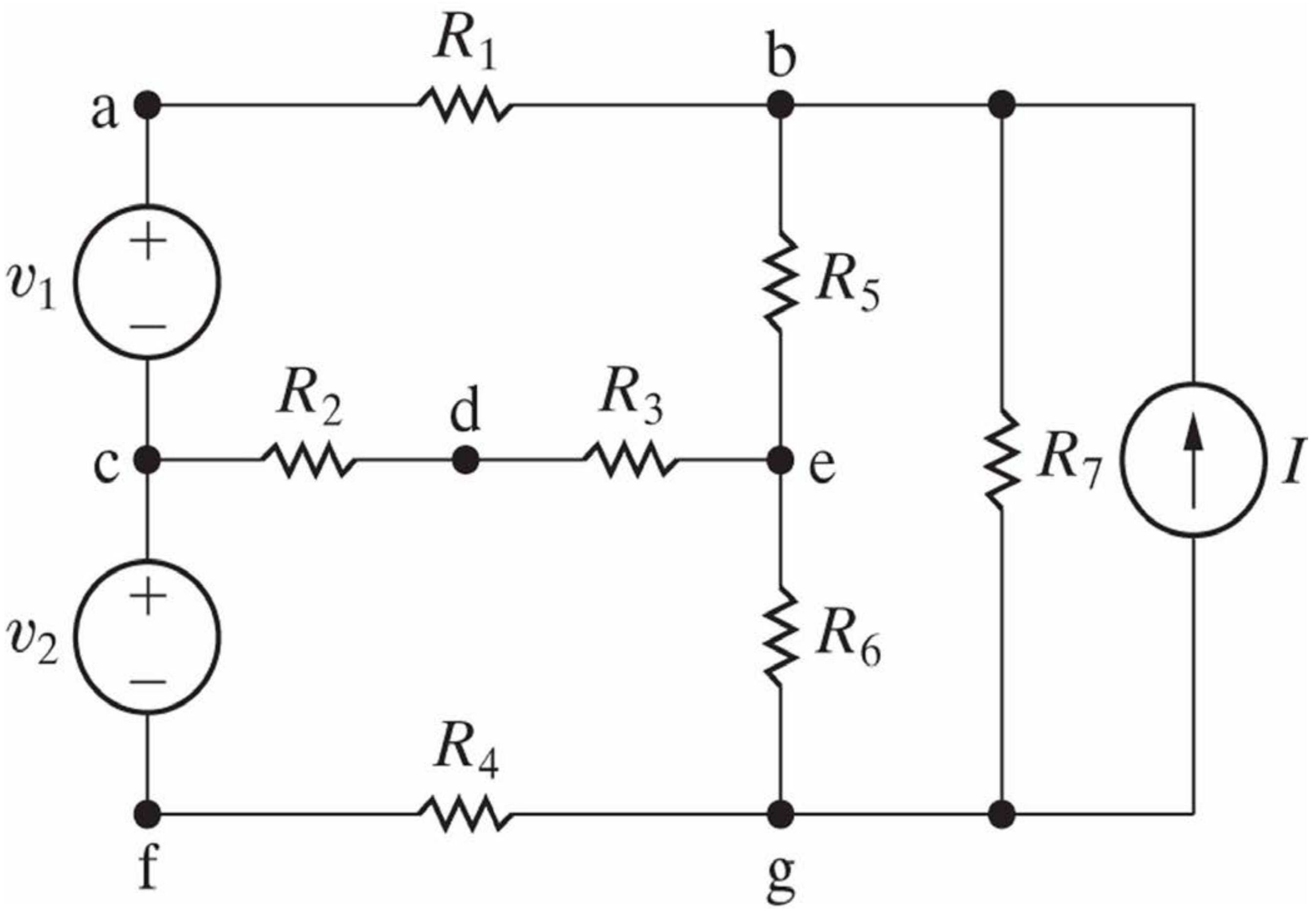
Example 4.1

- For the circuit shown here, identify
  - a) all nodes
  - b) all essential nodes
  - c) all branches
  - d) all essential branches
  - e) all meshes
  - f) two paths that are not loops or essential branches
  - g) two loops that are not meshes



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Example 4.1 – cont.

a) The nodes are a, b, c, d, e, f, and g.

b) The essential nodes are b, c, e, and g.

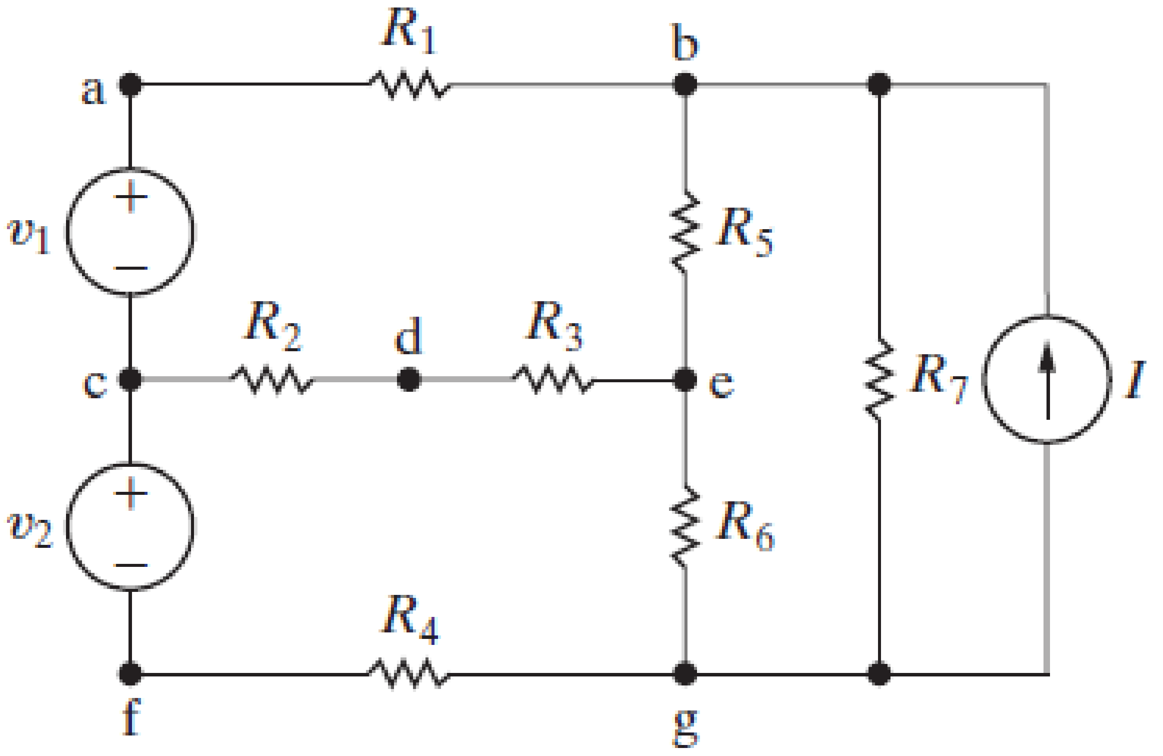
c) The branches are v_1, v_2, R_1, R_2, R_3, R_4, R_5, R_6, R_7, and I.

d) The essential branches are $v_1 - R_1$, $R_2 - R_3$, $v_2 - R_4$, R_5, R_6, R_7, and I.

e) The meshes are $v_1 - R_1 - R_5 - R_3 - R_2$, $v_2 - R_2 - R_3 - R_6 - R_4$, $R_5 - R_7 - R_6$, and $R_7 - I$.

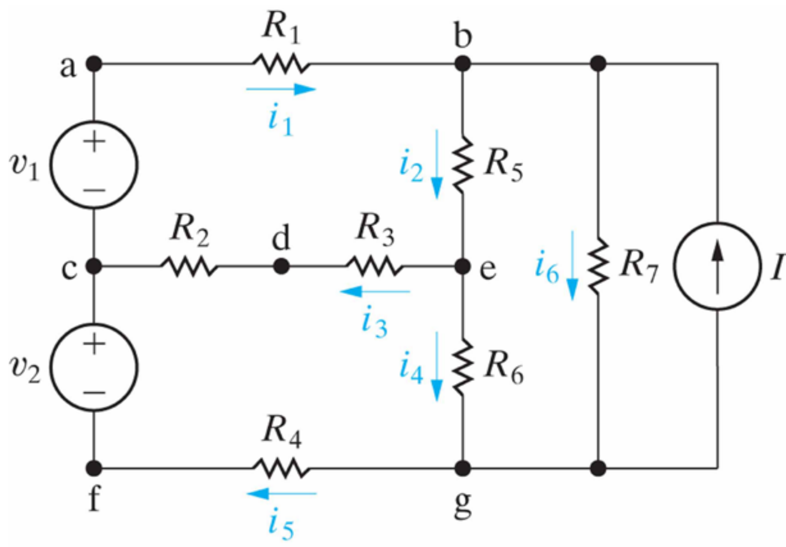
f) $R_1 - R_5 - R_6$ is a path, but it is not a loop (because it does not have the same starting and ending nodes), nor is it an essential branch (because it does not connect two essential nodes). $v_2 - R_2$ is also a path but is neither a loop nor an essential branch, for the same reasons.

g) $v_1 - R_1 - R_5 - R_6 - R_4 - v_2$ is a loop but is not a mesh, because there are two loops within it. $I - R_5 - R_6$ is also a loop but not a mesh.



Simultaneous Equations

- The number of unknown currents in a circuit equals the number of branches, *b*, where the current is not known
- In the circuit shown here, we have 6 branches in which the current is unknown
  - we must have *b* independent equations to solve a circuit with *b* unknown currents



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- If we let *n* represent the number of nodes in the circuit, we can derive *n-1* independent equations by applying KCL to any set of *n-1* nodes
  - We will see why in illustration example
- For finding remaining *b* equations: we must apply KVL to loops or meshes to obtain the remaining *b-(n-1)* equations

Basically we want to have a number of equations equal to the number of unknowns

Simultaneous Equations – cont.

- Thus by counting nodes, meshes, and branches where the current is unknown, we have established a systematic method for writing the necessary number of equations to solve a circuit
- The same observations also are valid in terms of essential nodes and essential branches:

b_e: number of essential branches

n_e: number of essential nodes

Kirchhoff's current law: n_e-1 independent equations

Kirchhoff's voltage law: b_e-(n_e-1) independent equations

Illustration of the Systematic Approach

- The circuit has 4 essential nodes (b, e, c, and g) and 6 essential branches, denoted $i_1 - i_6$, for which the current is unknown
- We derive three of the six simultaneous equations needed by applying Kirchhoff's current law to any three of the four essential nodes
- We use the nodes b, c, and e to get

Node b: $-i_1 + i_2 + i_6 - I = 0,$

Node c: $i_1 - i_3 - i_5 = 0,$

Node e: $i_3 + i_4 - i_2 = 0.$ (4.1)

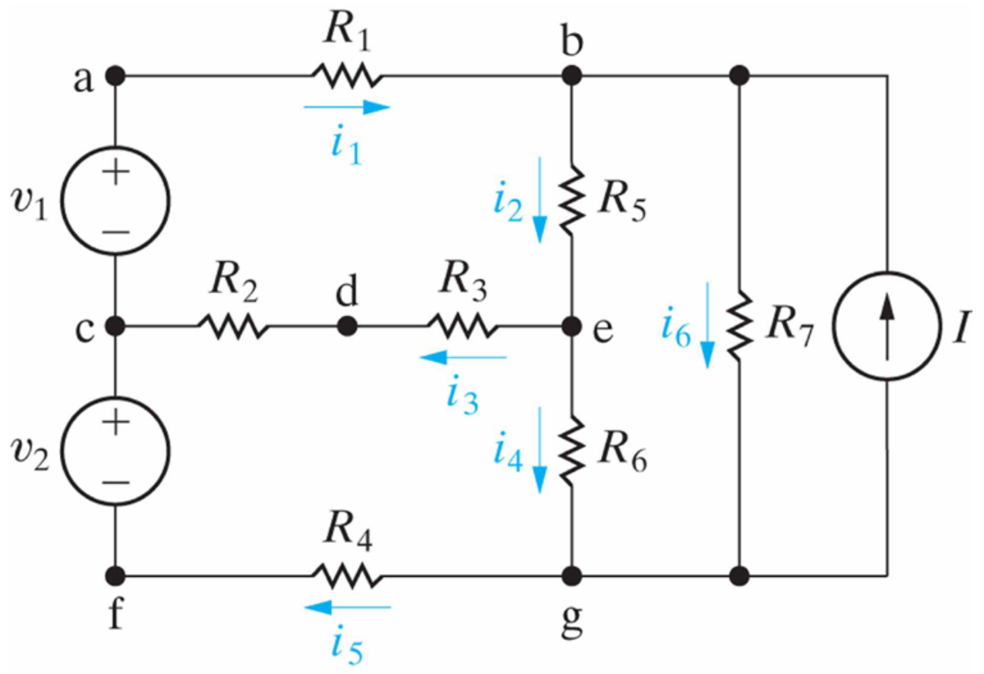

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Illustration of the Systematic Approach – cont.

- We derive the remaining three equations by applying Kirchhoff's voltage law around three meshes
  - Because the circuit has four meshes, we need to dismiss one mesh

Mesh 1: $R_1 i_1 + R_5 i_2 + i_3(R_2 + R_3) - v_1 = 0,$

Mesh 2: $-i_3(R_2 + R_3) + i_4 R_6 + i_5 R_4 - v_2 = 0,$

Mesh 3: $-i_2 R_5 + i_6 R_7 - i_4 R_6 = 0.$ (4.2)

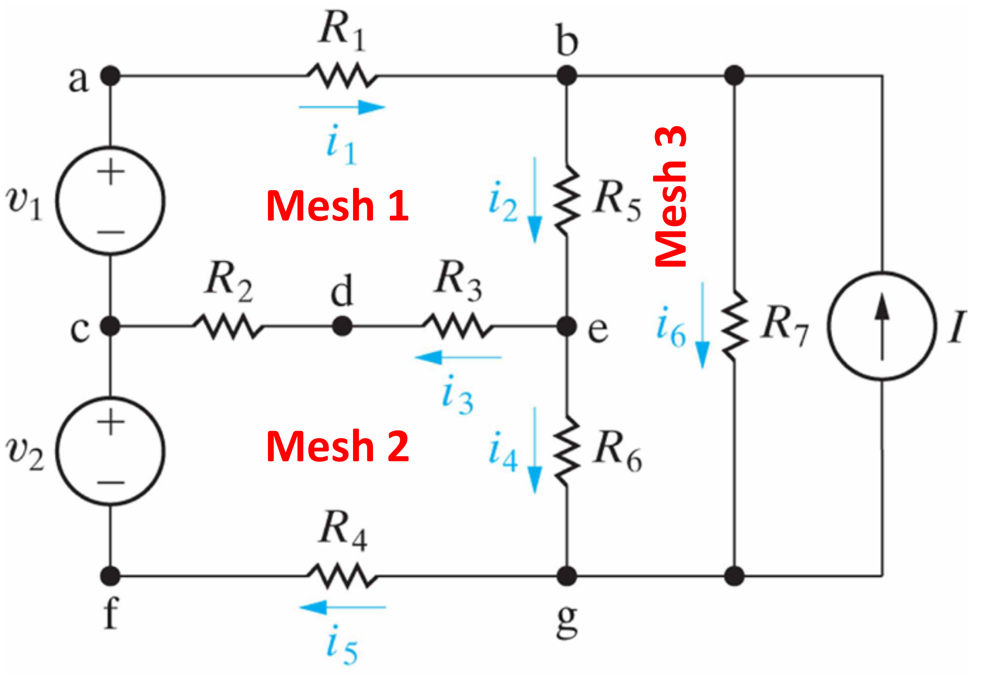

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Illustration of the Systematic Approach

- Rearranging Eqns. 4.1 and 4.2 to facilitate their solution yields the set

$$-i_1 + i_2 + 0i_3 + 0i_4 + 0i_5 + i_6 = I,$$

$$i_1 + 0i_2 - i_3 + 0i_4 - i_5 + 0i_6 = 0,$$

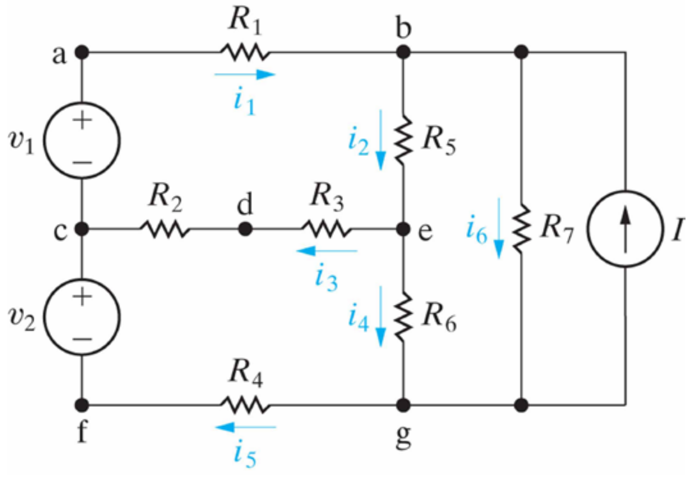
$$0i_1 - i_2 + i_3 + i_4 + 0i_5 + 0i_6 = 0,$$

$$R_1 i_1 + R_5 i_2 + (R_2 + R_3) i_3 + 0i_4 + 0i_5 + 0i_6 = v_1,$$

$$0i_1 + 0i_2 - (R_2 + R_3) i_3 + R_6 i_4 + R_4 i_5 + 0i_6 = v_2,$$

$$0i_1 - R_5 i_2 + 0i_3 - R_6 i_4 + 0i_5 + R_7 i_6 = 0. \qquad (4.3)$$

- Note that summing the current at the *n*th node (g in this example) gives $i_5 - i_4 - i_6 + I = 0$
- This equation is not independent
  - We can derive it by summing eqns. 4.1 and multiply them by -1
  - In other words, it doesn't add any extra information to help solving the system of equations



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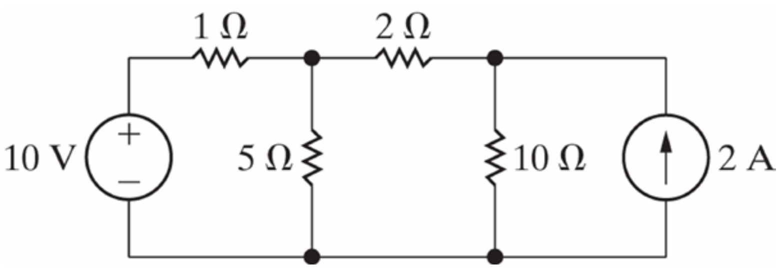
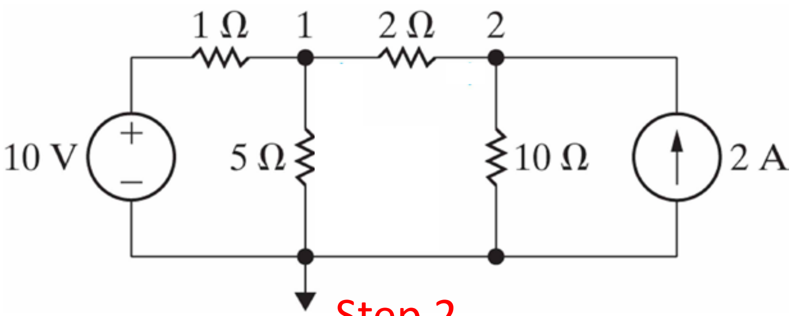
Now we have 6 equations and 6 unknowns, so we can solve this system of eqns.

Introduction to the Node-Voltage Method

- The node-voltage method uses the **essential nodes** of the circuit
- To perform circuit analysis using the node-voltage method we need to follow some steps
- The first step is to make a neat layout of the circuit so that no branches cross over and clearly mark the essential nodes
- The next step is to select one of the essential nodes as the reference node
  - Usually the node with the most branches is a good choice
    - We flag the chosen reference node with the symbol ▼



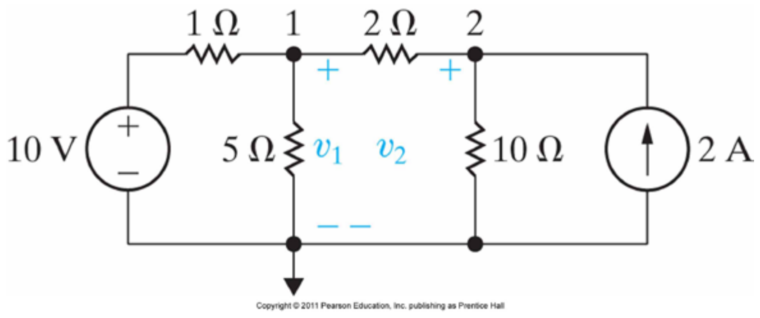
Step 1



Step 2

Introduction to the Node-Voltage Method – cont.

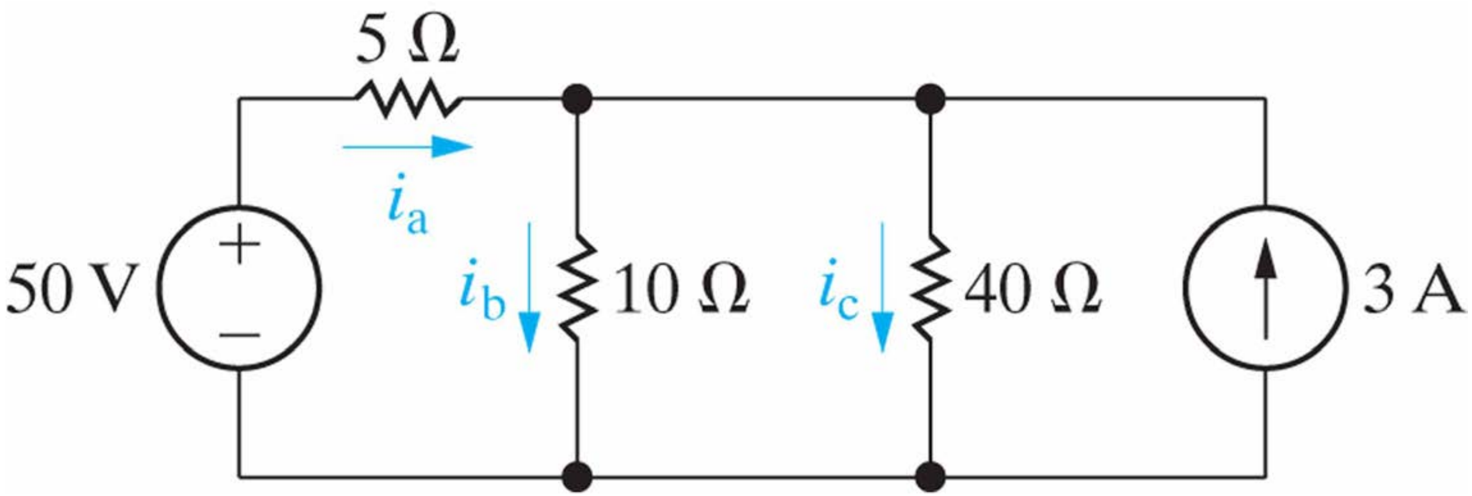
- After selecting the reference node, we define the node voltages on the circuit diagram
  - **Node voltage is the voltage between each <u>essential node</u> & the <u>reference node</u>**



- We are now ready to generate the node-voltage equations
  - We do so by writing the current in each branch connected to a non-reference node as a function of the node voltages
  - Then apply KCL to each essential node

Example 4.2

- Use the node-voltage method of circuit analysis to find the branch currents i_a, i_b , and i_c in the circuit shown here
- Find the power associated with each source, and state whether the source is delivering or absorbing power



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Example 4.2 – cont.

- a) We begin by noting that the circuit has two essential nodes
  - We select the lower node as the reference node
  - Thus we need to write a single node-voltage expression
    - Let us define the unknown node voltage as v_1
  - Figure below illustrates these decisions
  - Summing the currents away from node 1 generates the node-voltage equation

$$\frac{50 - v1}{5} - \frac{v1}{10} - \frac{v1}{40} + 3 = 0$$

currents entering → Positive
currents leaving → Negative

Solving for v_1 gives

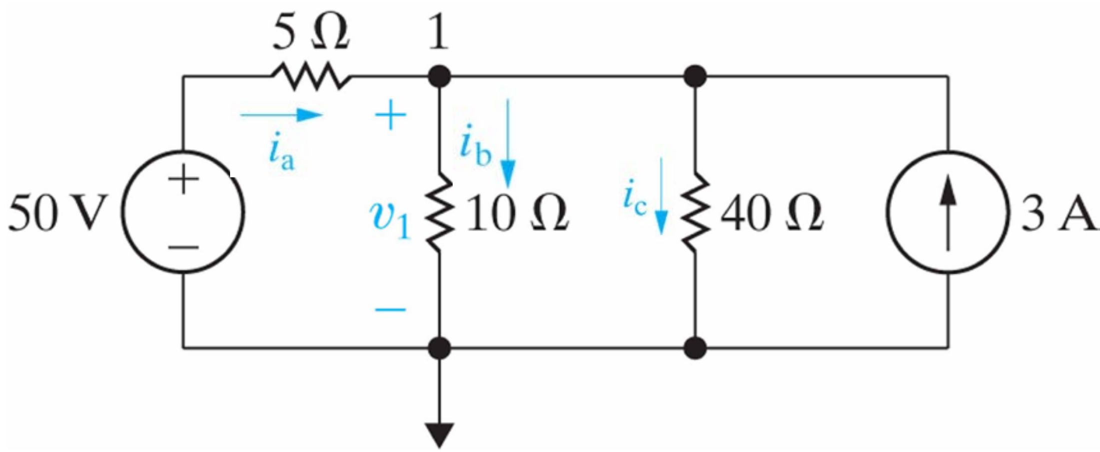
$$v_1 = 40 \text{ V}.$$

Hence

$$i_a = \frac{50 - 40}{5} = 2 \text{ A},$$

$$i_b = \frac{40}{10} = 4 \text{ A},$$

$$i_c = \frac{40}{40} = 1 \text{ A}.$$



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Example 4.2 – cont.

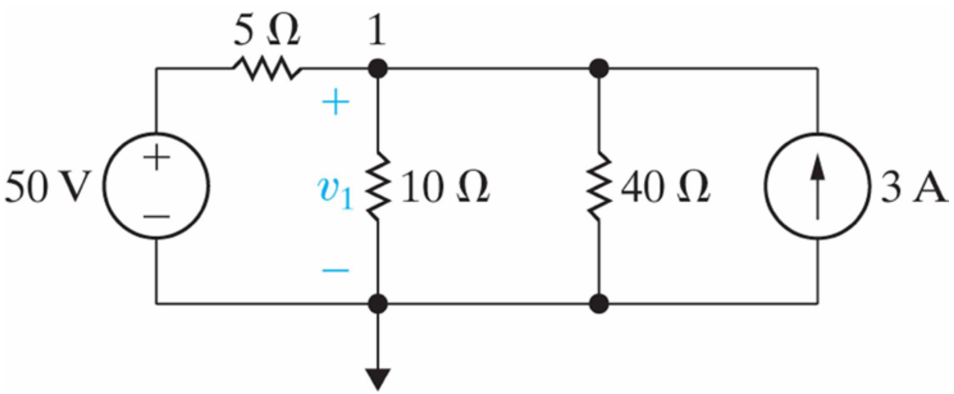
b) The power associated with the 50 V source is

$$p_{50V} = -50i_a = -100\text{ W (delivering)}.$$

The power associated with the 3 A source is

$$p_{3A} = -3v_1 = -3(40) = -120\text{ W (delivering)}.$$

We check these calculations by noting that the total delivered power is 220 W. The total power absorbed by the three resistors is 4(5) + 16(10) + 1(40), or 220 W, as we calculated and as it must be.



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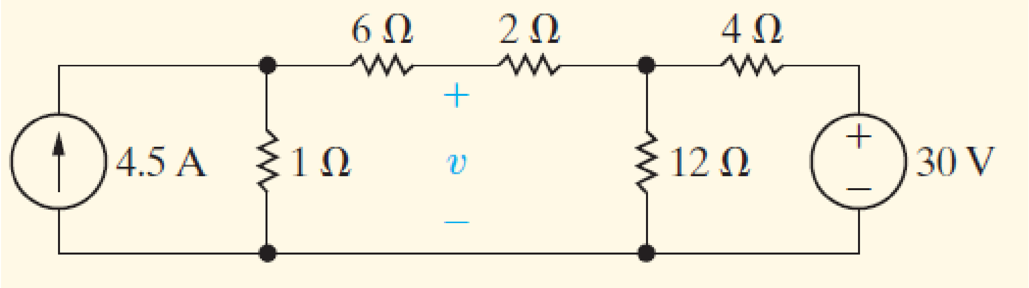
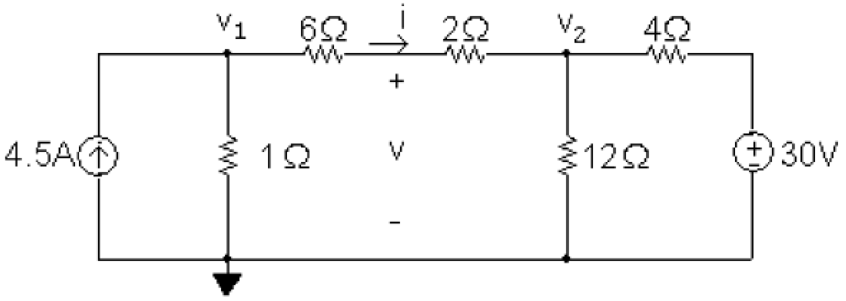
Assessment Problem 4.2

- Use the node-voltage method to find v in the circuit shown



Assessment Problem 4.2 – cont.

- Redraw the circuit, choosing the node voltages and reference node as shown:



The two node voltage equations are:

$$-4.5 + \frac{v_1}{1} + \frac{v_1 - v_2}{6+2} = 0$$
$$\frac{v_2}{12} + \frac{v_2 - v_1}{6+2} + \frac{v_2 - 30}{4} = 0$$

Place these equations in standard form:

$$v_1\left(1 + \frac{1}{8}\right) + v_2\left(-\frac{1}{8}\right) = 4.5$$
$$v_1\left(-\frac{1}{8}\right) + v_2\left(\frac{1}{12} + \frac{1}{8} + \frac{1}{4}\right) = 7.5$$

Here they decided to use the following assumption:
currents entering → -ve
currents leaving → +ve
You can assume whatever you want and you will get the same answer

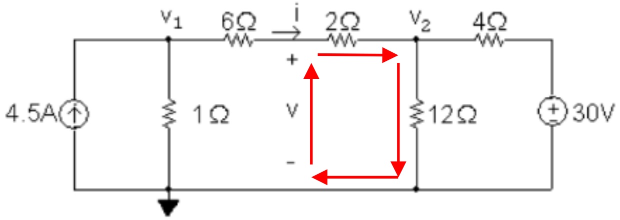
Solving, $v_1 = 6$ V $\qquad v_2 = 18$ V

To find the voltage v, first find the current i through the series-connected $6\,\Omega$ and $2\,\Omega$ resistors:

$$i = \frac{v_1 - v_2}{6+2} = \frac{6-18}{8} = -1.5\text{ A}$$

Using a KVL equation, calculate v:

$$v = 2i + v_2 = 2(-1.5) + 18 = 15\text{ V}$$



Introduction to the Node-Voltage Method

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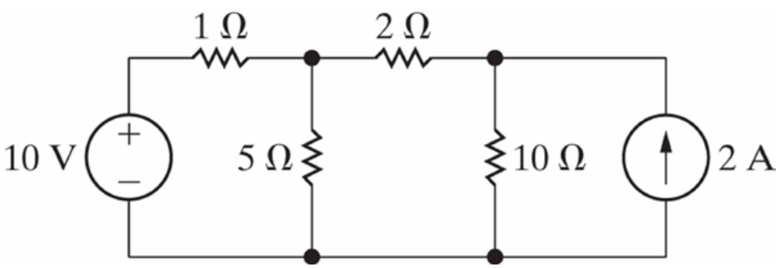
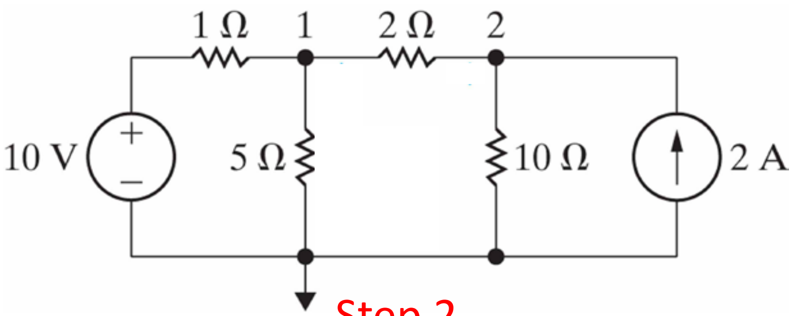


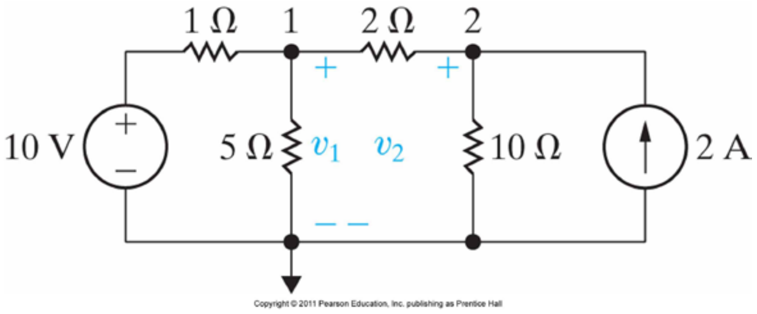
Step 1



Step 2

Introduction to the Node-Voltage Method – cont.

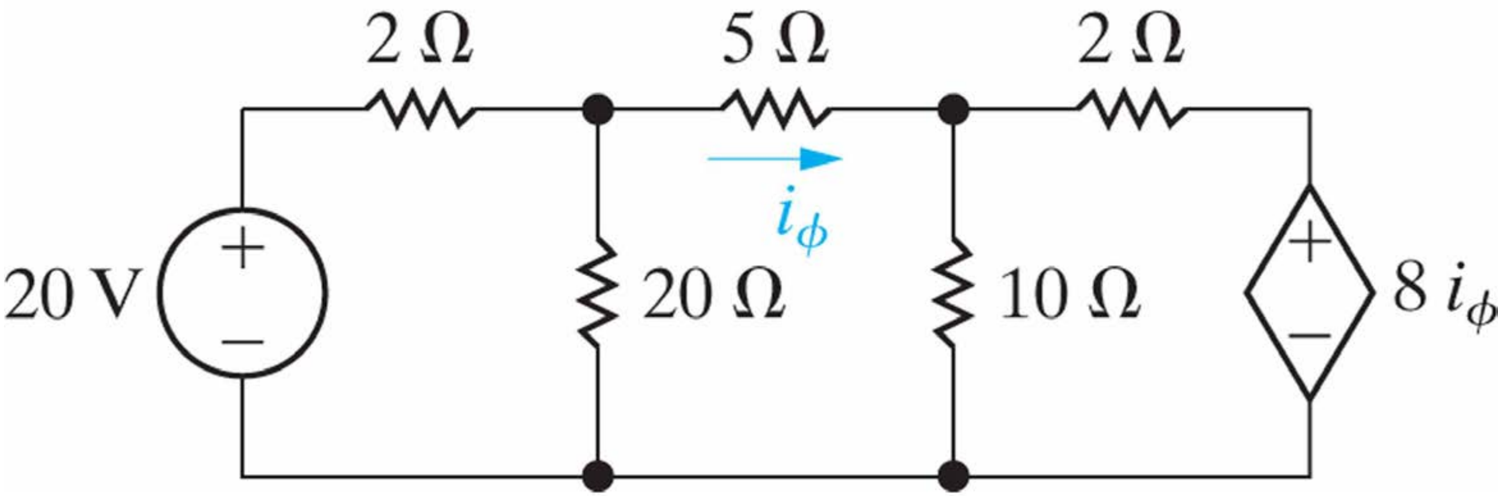
- After selecting the reference node, we define the node voltages on the circuit diagram
  - A node voltage is the voltage between each essential node and the reference node



- We are now ready to generate the node-voltage equations
  - We do so by writing the current leaving each branch connected to a non-reference node as a function of the node voltages
  - Then apply KCL to each essential node

Example 4.3

- Use the node-voltage method to find the power dissipated in the 5 Ω resistor in the circuit shown here



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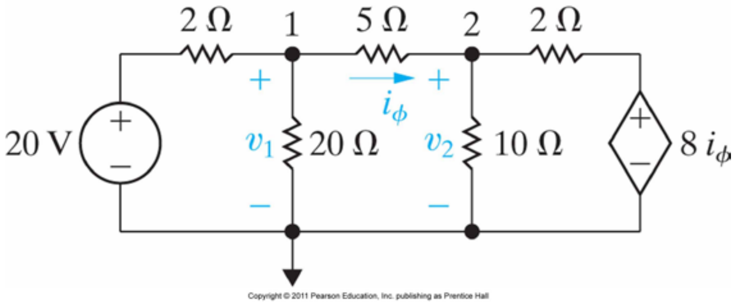
Example 4.3 – cont.

- We begin by noting that the circuit has three essential nodes
  - Hence we need two node-voltage equations to describe the circuit
  - Four branches terminate on the lower node, so we select it as the reference node
- The two unknown node voltages are defined on the circuit shown in below

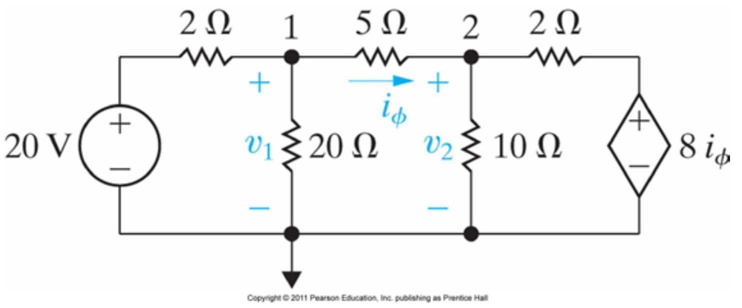


- Now we apply KCL on the essential nodes
  - Assume currents entering are –ve, and currents leaving are +ve
- For essential node 1:

$$-\frac{20 - v_1}{2} + \frac{v_1}{20} + \frac{(v_1 - v_2)}{5} = 0$$

In general, you can always assume that all currents are leaving the essential node you are working with to simplify the math

Example 4.3 – cont.



- For essential node 2:

$$\frac{v_2 - v_1}{5} + \frac{v_2}{10} + \frac{(v_2 - 8i_\phi)}{2} = 0$$

- As written, these two node-voltage equations contain three unknowns, namely, $v_1, v_2, and\ i_\phi$
- To eliminate i_ϕ, we must express this controlling current in terms of the node voltages

$$i_\phi = \frac{(v_1 - v_2)}{5}$$

- Substituting this relationship into the node 2 equation simplifies the two node-voltage equations to

$$0.75v_1 - 0.2v_2 = 10$$
$$-v_1 + 1.6v_2 = 0$$

Example 4.3 – cont.

- Solving for v_1 and v_2 gives $v_1 = 16V, and\ v_2 = 10V$

- Now, we can find i_ϕ as:

$$i_\phi = \frac{(16-10)}{5} = 1.2\ A$$

- The power dissipated by the 5Ω resistor is:

$$p_{5\Omega} = (1.2)^2(5) = (1.44)(5) = 7.2W$$

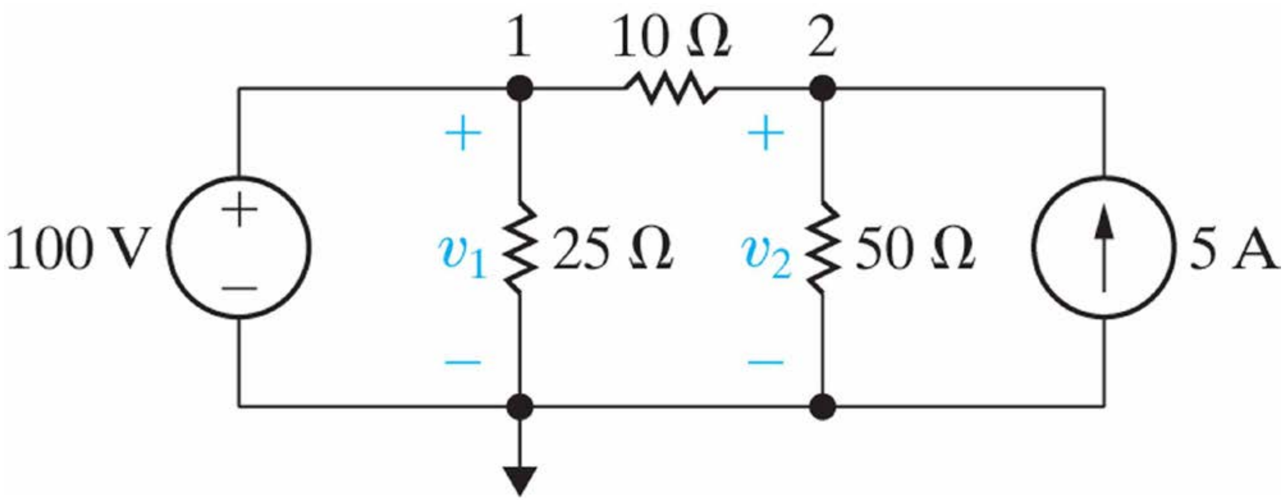
Node-Voltage Method: Special Cases

- When a **voltage source** is the **only element** between two essential nodes, the node-voltage method is simplified
- In the circuit below, there are three essential nodes in this circuit, which means that two simultaneous equations are needed
- From these three essential nodes, a reference node has been chosen and two other nodes have been labeled

- The 100 V source constrains the voltage between node 1 and the reference node to 100 V (parallel connection)
- This means that there is only one unknown node voltage (v_2)
- Thus, solution of this circuit involves only a single node- voltage equation at node 2

$$\frac{(v_2 - v_1)}{10} + \frac{v_2}{50} - 5 = 0$$

  - We know v_1 is 100V, thus v_2 is 125 V



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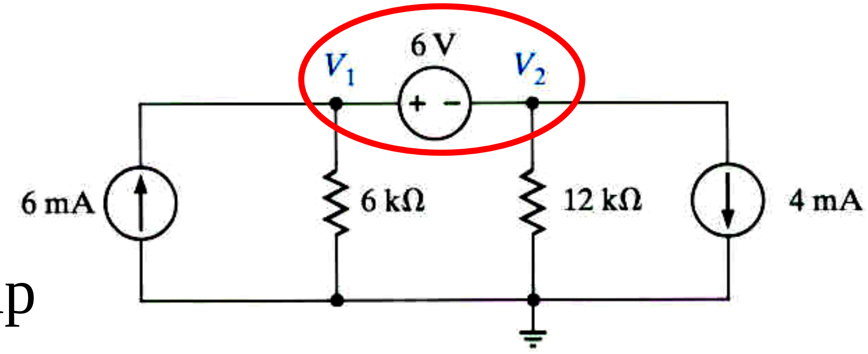
Node-Voltage Method: Special Cases

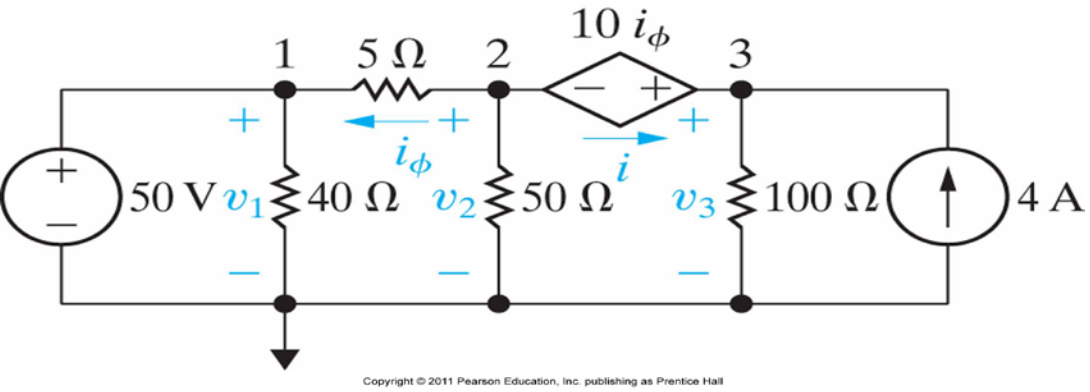
- In general, when you use the node-voltage method to solve circuits that have voltage sources connected directly between essential nodes, the number of unknown node voltages is reduced
- The reason is that, whenever a voltage source connects two essential nodes, it constrains the difference between the node voltages at these nodes to equal the voltage of the source
- Taking the time to see if you can reduce the number of unknowns in this way will simplify circuit analysis

Node-Voltage Method: The Supernode

- When a **voltage source** is the **only path** between two essential nodes, we can combine those nodes to form a **supernode**
  - Obviously, Kirchhoff's current law must hold for the supernode
- Steps
  1. First encircle the voltage source and the two connecting nodes to form the super node
  2. Write the equation that defines the voltage relationship between the two connecting nodes as a result of the presence of the voltage source
  3. Remove voltage source and write KCL equation for the supernode
  4. If the voltage source is dependent, then write the equation for the controlling quantity

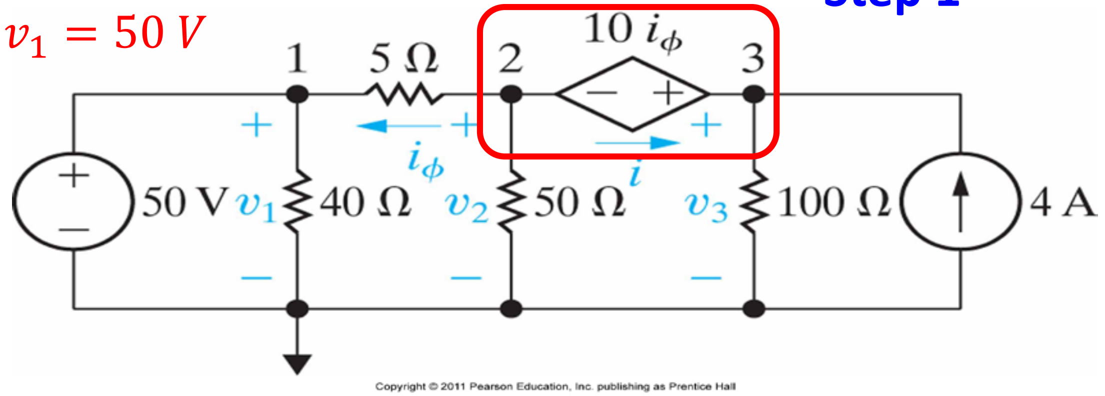


Node-Voltage Method: The Supernode (Dependent Source)



Note that $v_1 = 50\ V$

Step 1



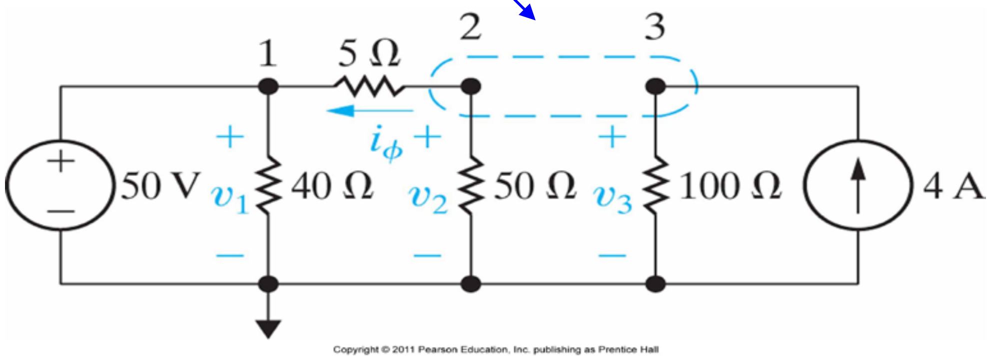
Step 2: $v_2 + 10i_\phi = v_3$

We consider nodes 2 and 3 to be a single node and simply apply KCL in terms of the node voltages

Step 4: $i_\phi = \dfrac{v_2 - v_1}{5} = \dfrac{v_2 - 50}{5}$

Step 3: write KCL equation for the super node after removing the voltage source

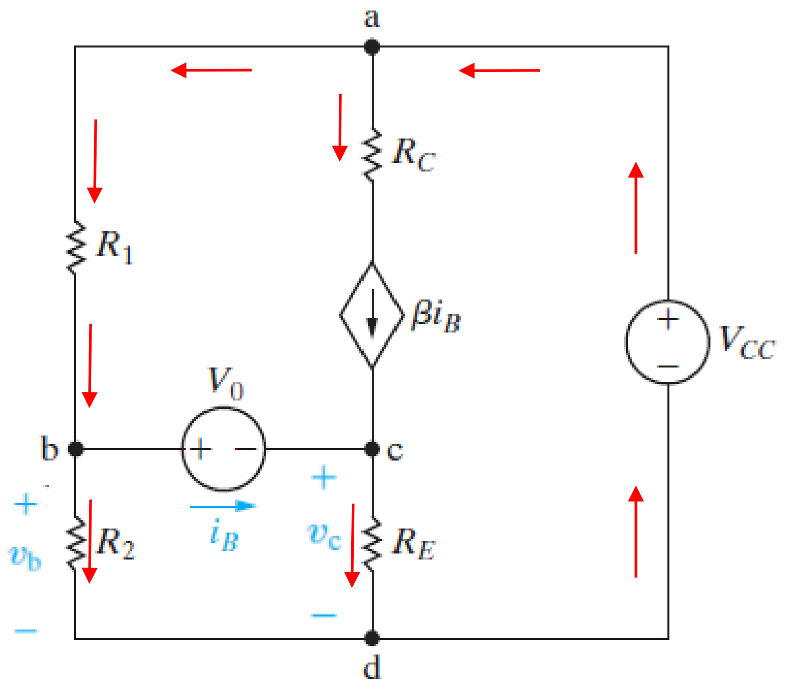
Super node: $\dfrac{v_2 - v_1}{5} + \dfrac{v_2}{50} + \dfrac{v_3}{100} - 4 = 0$



Entering → -ve
Leaving → +ve
Or for this step only just assume all are leaving

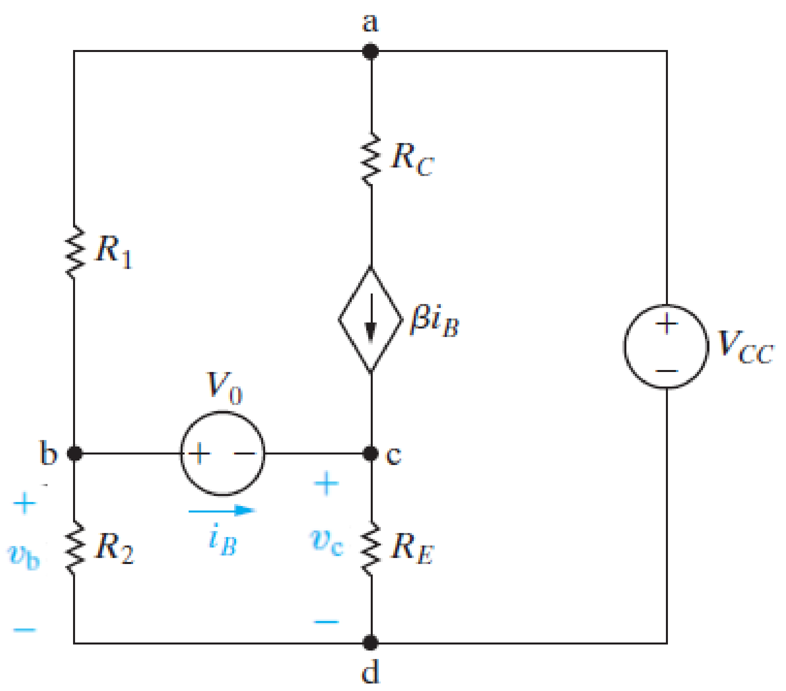
Node-Voltage Method: The Supernode – cont.

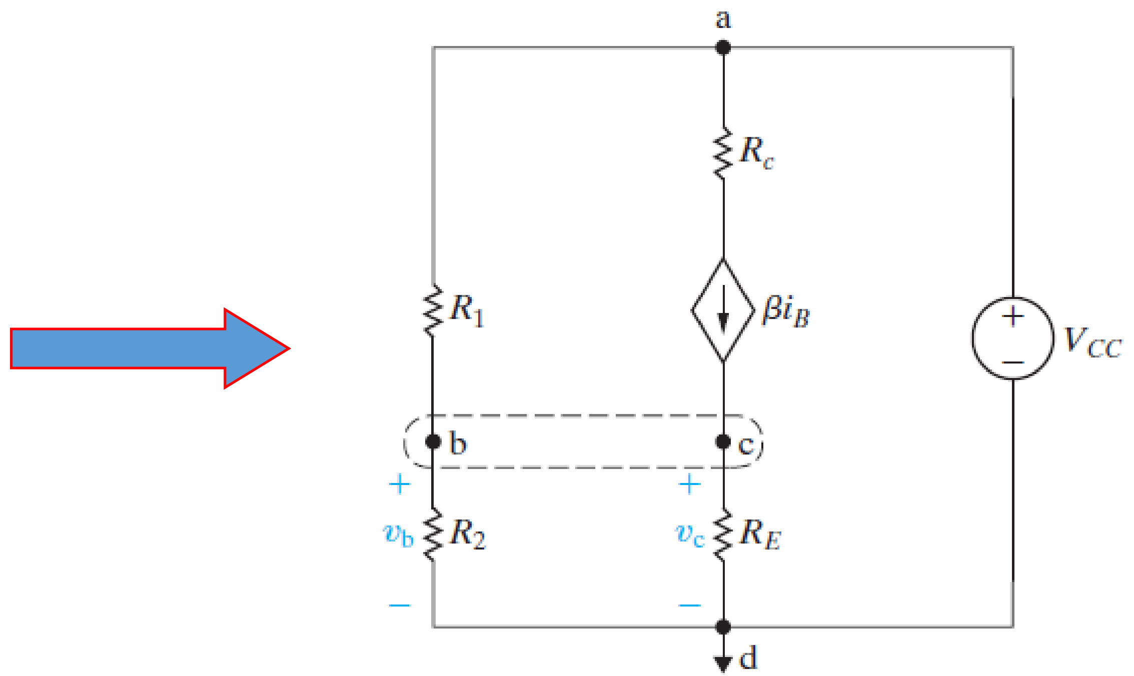
- Example:
- Consider the circuit shown in the figure below, find an expression for v_b using node-voltage method



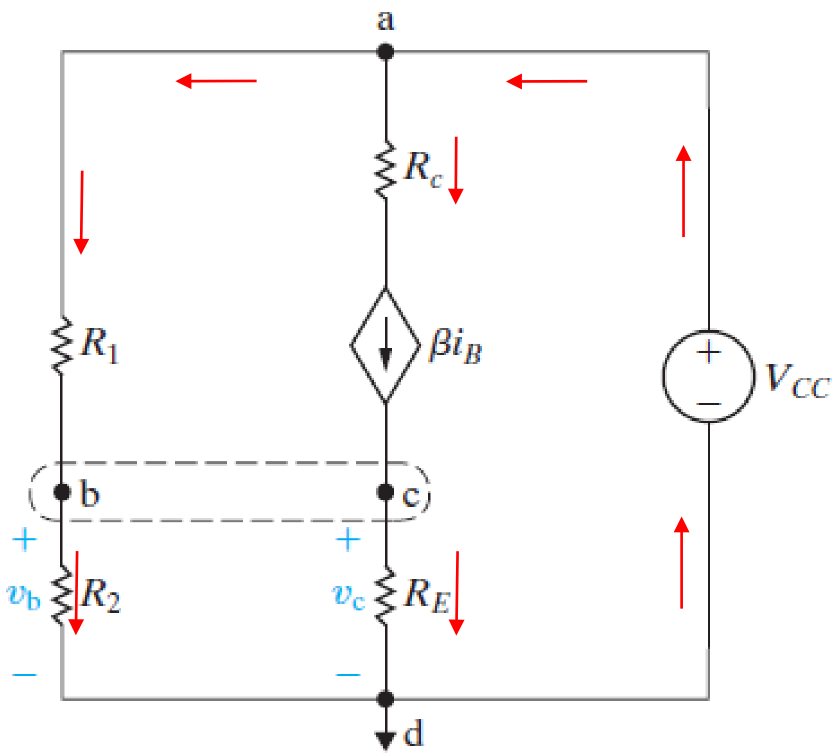
Node-Voltage Method: The Supernode – cont.

- Let us consider the circuit shown in the figure below
  - We have 4 essential nodes (a, b, c, d)
  - We can use essential node d as the reference node
  - We can combine essential nodes b and c into one supernode





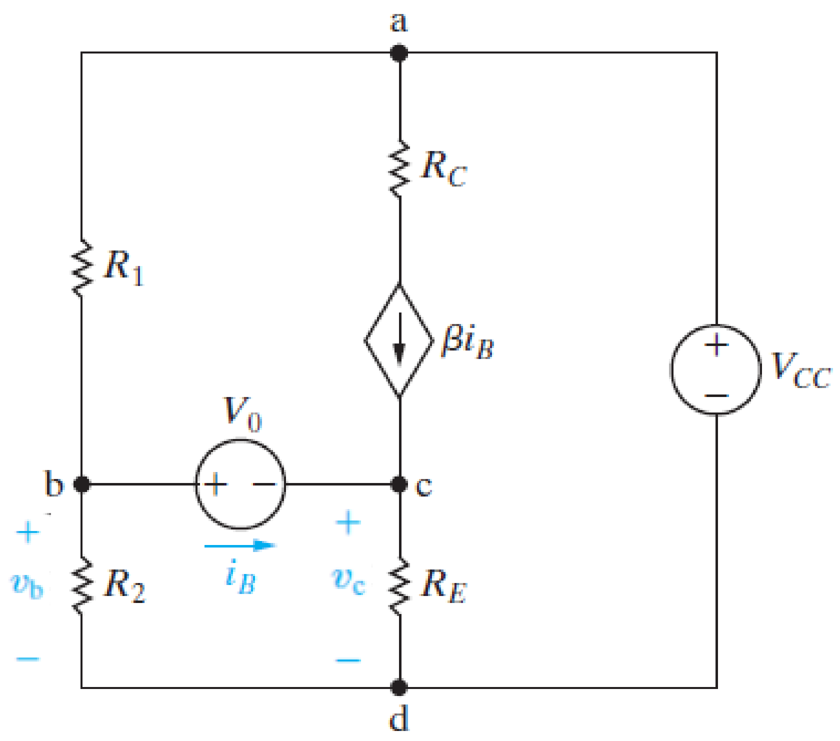
Node-Voltage Method: The Supernode – cont.



Assume currents entering are –ve, and currents leaving are +ve

KCL @ Supernode: $\frac{v_b}{R_2} + \frac{v_b - V_{CC}}{R_1} + \frac{v_c}{R_E} - \beta i_B = 0.$

Node-Voltage Method: The Supernode – cont.



$$v_c = (i_B + \beta i_B)R_E,$$

$$v_c = v_b - V_0.$$

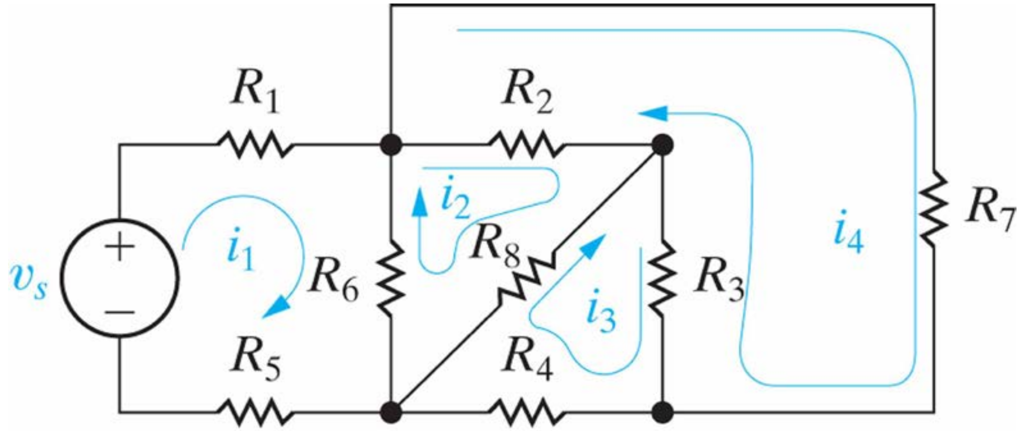
- Two currents are entering node c which are i_B and βi_B
- Thus, the current that passes through R_E is the sum of these two currents

$$v_b\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{(1+\beta)R_E}\right] = \frac{V_{CC}}{R_1} + \frac{V_0}{(1+\beta)R_E}.$$

$$v_b = \frac{V_{CC}R_2(1+\beta)R_E + V_0R_1R_2}{R_1R_2 + (1+\beta)R_E(R_1+R_2)}.$$

Introduction to the Mesh-Current Method

- Recall that a mesh is a loop with no other loops inside it
- A **mesh current** is the current that exists only in the boundary of a mesh
- On a circuit diagram it appears as either a closed solid line or an almost-closed solid line that follows the boundaries of the appropriate mesh
- An arrowhead on the solid line indicates the reference direction for the mesh current
- Note that by definition, mesh currents automatically satisfy Kirchhoff's current law



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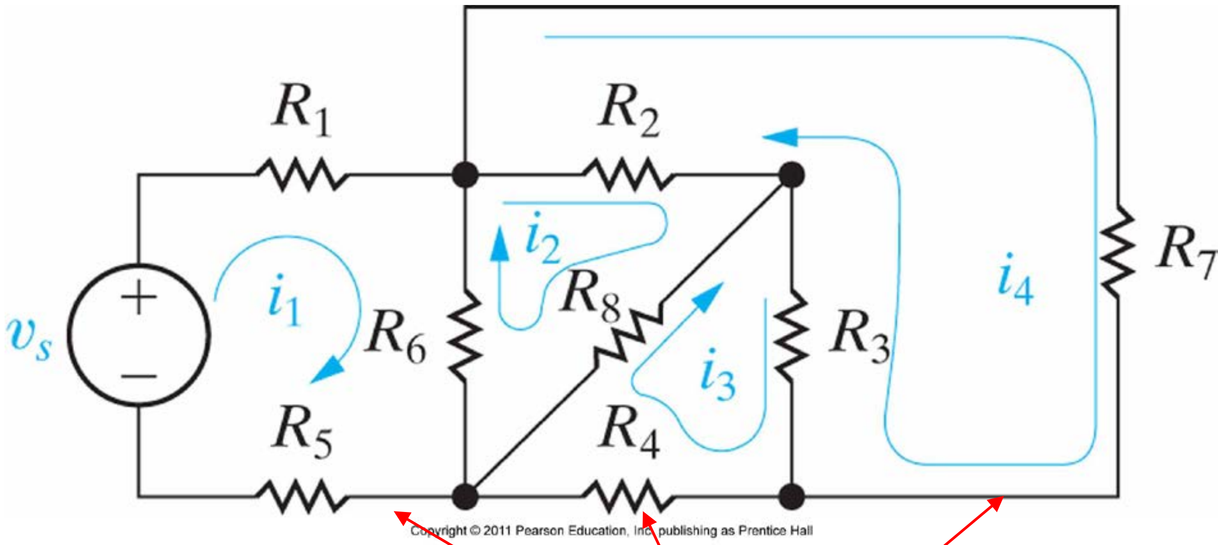
Introduction to the Mesh-Current Method – cont.

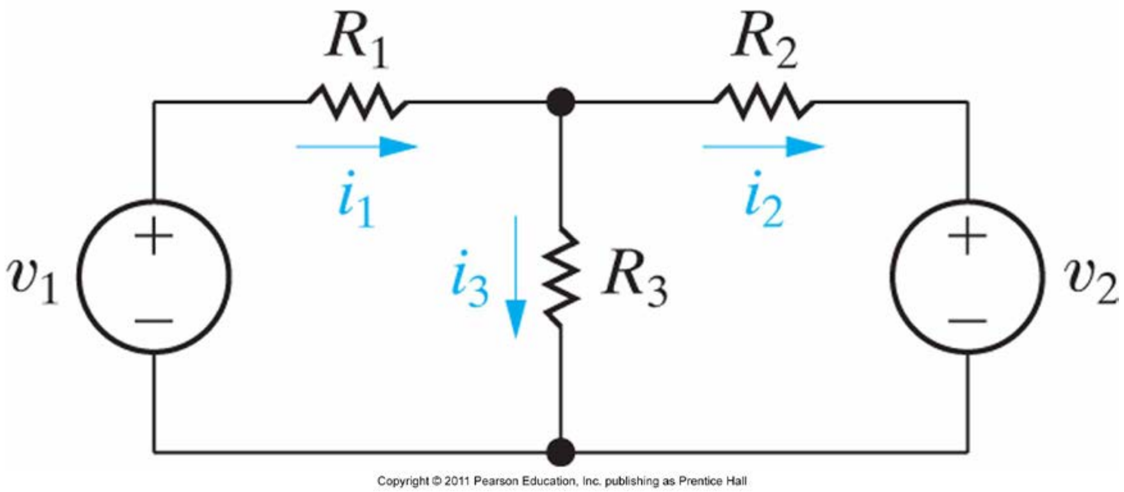
- The circuit in this figure also shows that identifying a mesh current in terms of a branch current is not always possible
- For example, the mesh current i_2 is not equal to any branch current, whereas mesh currents i_1, i_3 and i_4 can be identified with branch currents
- Thus **measuring** a mesh current is not always possible
  - Note that there is no place where an ammeter can be inserted to measure the mesh current i_2

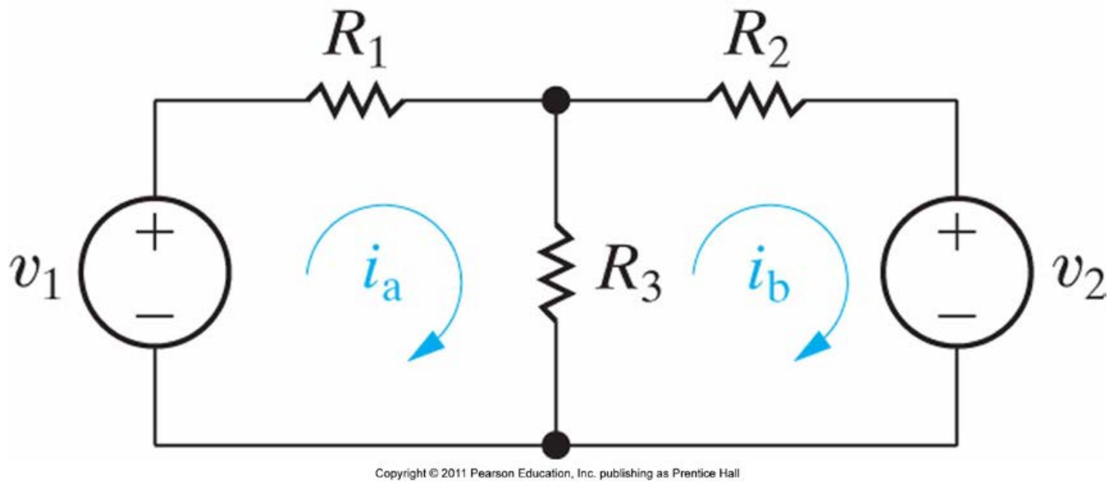


The branch current here is actually the mesh current

Introduction to the Mesh-Current Method – cont.

- Using these concepts of mesh and branch currents help developing what we call the mesh-current method of circuit analysis
- The main idea of the mesh-current method is to reduce the number of independent equations needed to do the circuit analysis
- This can be achieved by using mesh currents instead of branch currents





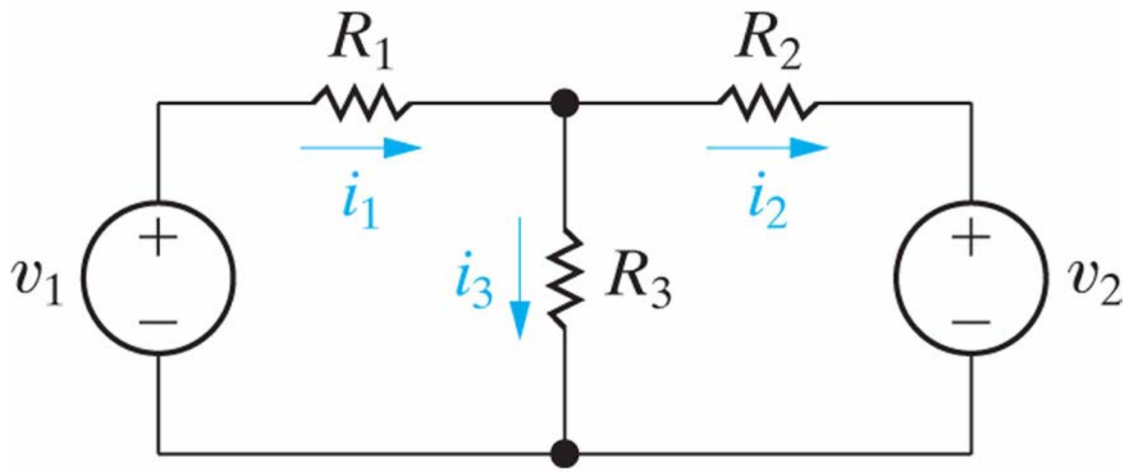
Introduction to the Mesh-Current Method – cont.

- Applying Kirchhoff's current law to the upper node and Kirchhoff's voltage law around the two meshes generates the following set of equations:

$$i_1 = i_2 + i_3,$$

$$v_1 = i_1 R_1 + i_3 R_3,$$

$$-v_2 = i_2 R_2 - i_3 R_3.$$



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Introduction to the Mesh-Current Method – cont.

- Apply Kirchhoff's voltage law around the two meshes, expressing all voltages across resistors in terms of the mesh currents, will lead to the following equations

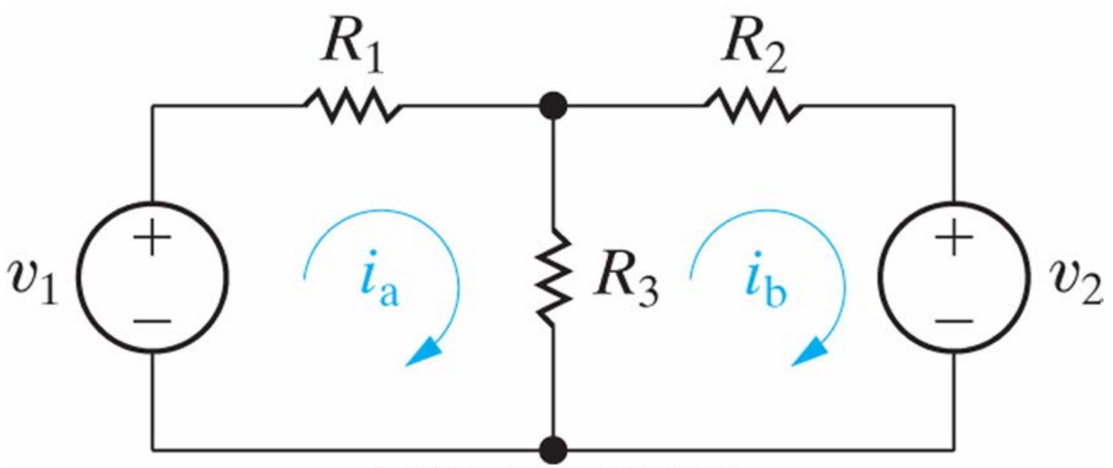
$$v_1 = i_a R_1 + (i_a - i_b) R_3,$$

$$-v_2 = (i_b - i_a) R_3 + i_b R_2.$$

Collecting the coefficients of i_a and i_b

$$v_1 = i_a (R_1 + R_3) - i_b R_3,$$

$$-v_2 = -i_a R_3 + i_b (R_2 + R_3)$$



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$$i_1 = i_a,$$

$$i_2 = i_b,$$

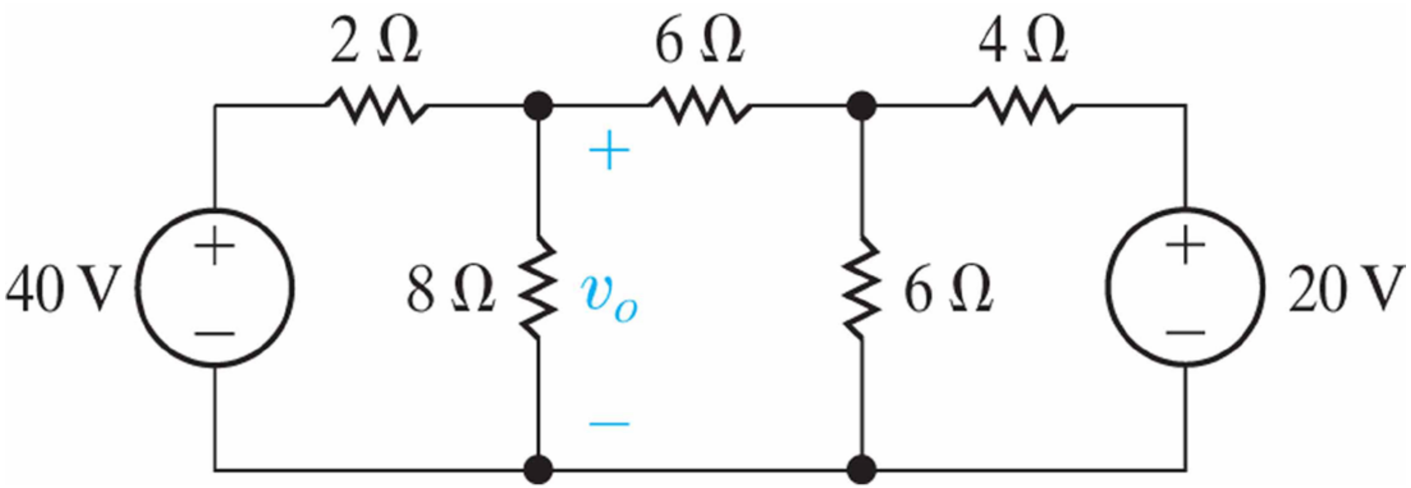
$$i_3 = i_a - i_b.$$

Once you know the mesh currents, you also know the branch currents. And once you know the branch currents, you can compute any voltages or powers of interest.

Example 4.4

a) Use the mesh-current method to determine the power associated with each voltage source in the circuit shown below

b) Calculate the voltage v_0 across the 8 Ω resistor.



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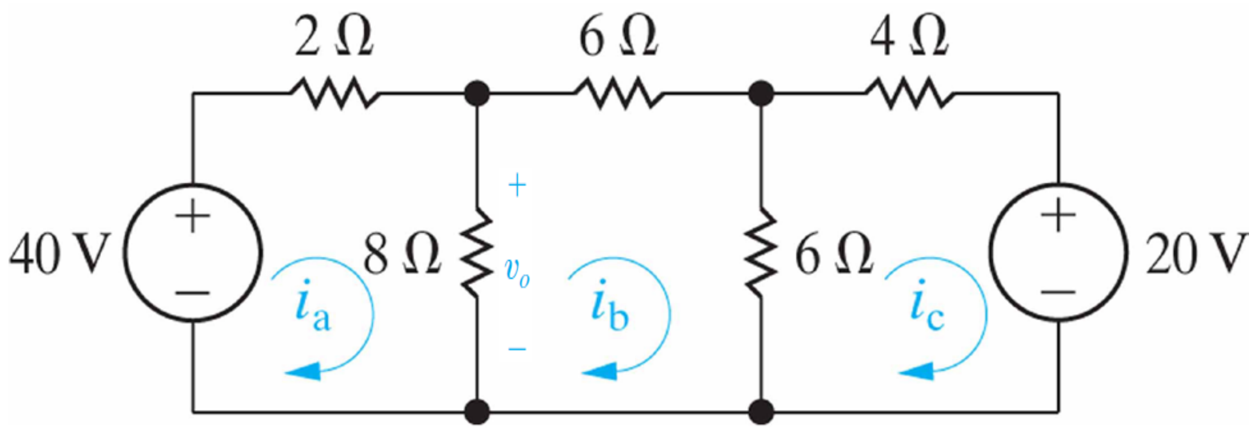
Example 4.4 – cont.

- To calculate the power associated with each source, we need to know the current in each source
- We want to use the mesh-current method to determine that
- We start by identifying the meshes of the circuit



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- The circuit indicates that these source currents will be identical to mesh currents.

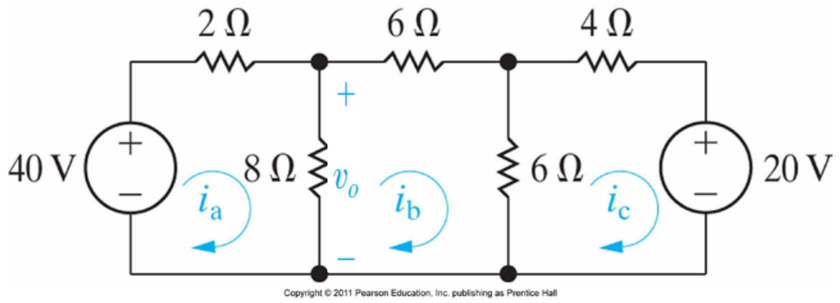
Example 4.4 – cont.

- Apply Kirchhoff's voltage law around the three meshes, expressing all voltages across resistors in terms of the mesh currents

$$-40 + 2i_a + 8(i_a - i_b) = 0,$$

$$8(i_b - i_a) + 6i_b + 6(i_b - i_c) = 0,$$

$$6(i_c - i_b) + 4i_c + 20 = 0.$$



Reorganizing the equations in anticipation of using any available tool to solve the equations will yield:

$$10i_a - 8i_b + 0i_c = 40;$$

$$-8i_a + 20i_b - 6i_c = 0;$$

$$0i_a - 6i_b + 10i_c = -20.$$

$$i_a = 5.6\text{ A},$$

$$i_b = 2.0\text{ A},$$

$$i_c = -0.80\text{ A}.$$

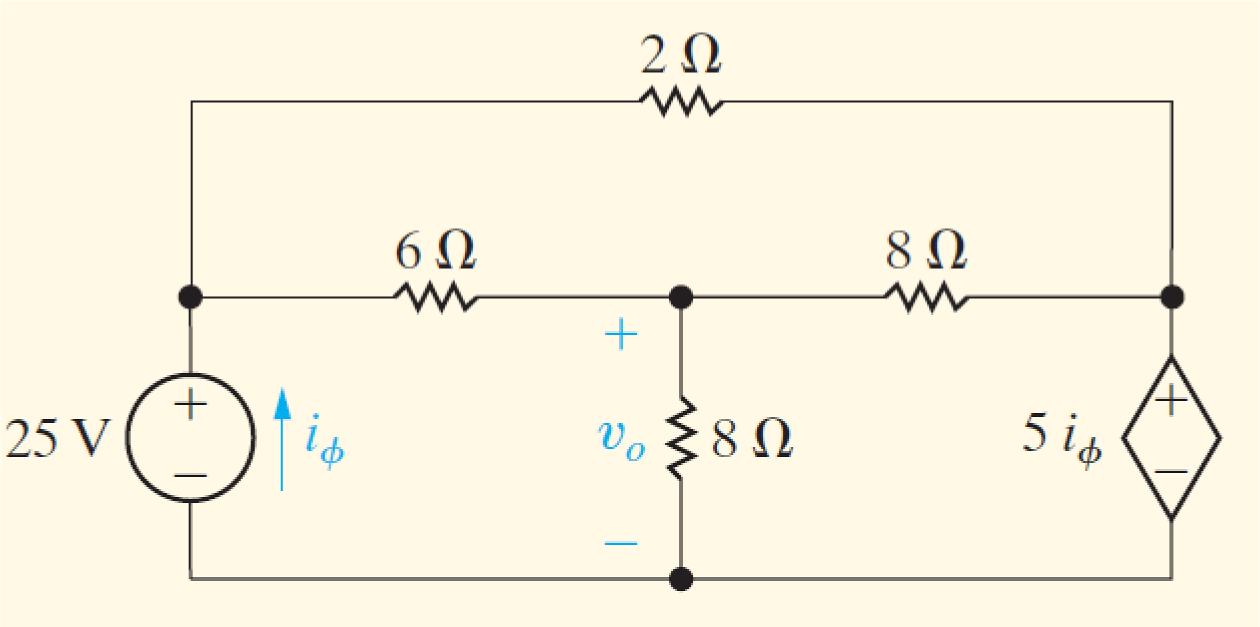
a) $p_{40\text{V}} = -40i_a = -224\text{ W}.$

$p_{20\text{V}} = 20i_c = -16\text{ W}.$

b) $v_o = 8(i_a - i_b) = 8(3.6) = 28.8\text{ V}.$

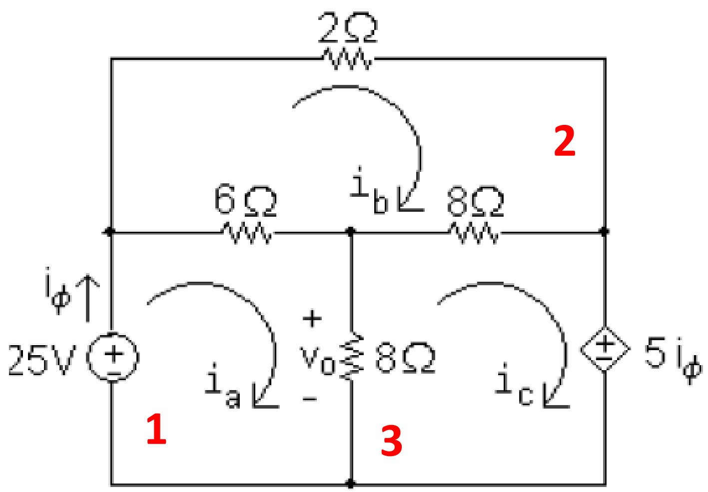
Assessment Problem 4.9

- Use the mesh-current method to find v_0 in the circuit shown below



Assessment Problem 4.9 – cont.

- Redraw the circuit identifying the three mesh currents
- The mesh current equations are:



$$-25 + 6(i_{\mathrm{a}} - i_{\mathrm{b}}) + 8(i_{\mathrm{a}} - i_{\mathrm{c}}) = 0$$ **Mesh 1**

$$2i_{\mathrm{b}} + 8(i_{\mathrm{b}} - i_{\mathrm{c}}) + 6(i_{\mathrm{b}} - i_{\mathrm{a}}) = 0$$ **Mesh 2**

$$5i_{\phi} + 8(i_{\mathrm{c}} - i_{\mathrm{a}}) + 8(i_{\mathrm{c}} - i_{\mathrm{b}}) = 0$$ **Mesh 3**

- Knowing that $i_\phi = i_a$, we can substitute this simple expression for i_ϕ into the third mesh equation

Mesh 1: $$14i_{\mathrm{a}} - 6i_{\mathrm{b}} - 8i_{\mathrm{c}} = 25$$

Mesh 2: $$-6i_{\mathrm{a}} + 16i_{\mathrm{b}} - 8i_{\mathrm{c}} = 0$$

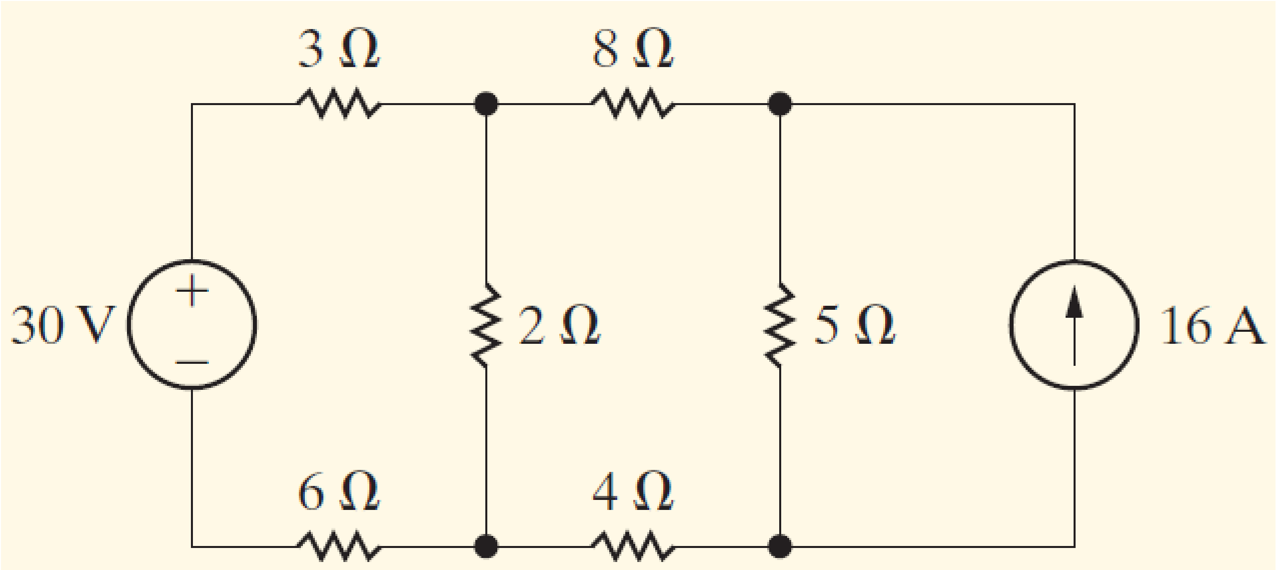
Mesh 3: $$-3i_{\mathrm{a}} - 8i_{\mathrm{b}} + 16i_{\mathrm{c}} = 0$$

$$i_{\mathrm{a}} = 4\,\mathrm{A}; \qquad i_{\mathrm{b}} = 2.5\,\mathrm{A}; \qquad i_{\mathrm{c}} = 2\,\mathrm{A}$$

$$v_o = 8(i_{\mathrm{a}} - i_{\mathrm{c}}) = 8(4 - 2) = 16\,\mathrm{V}$$

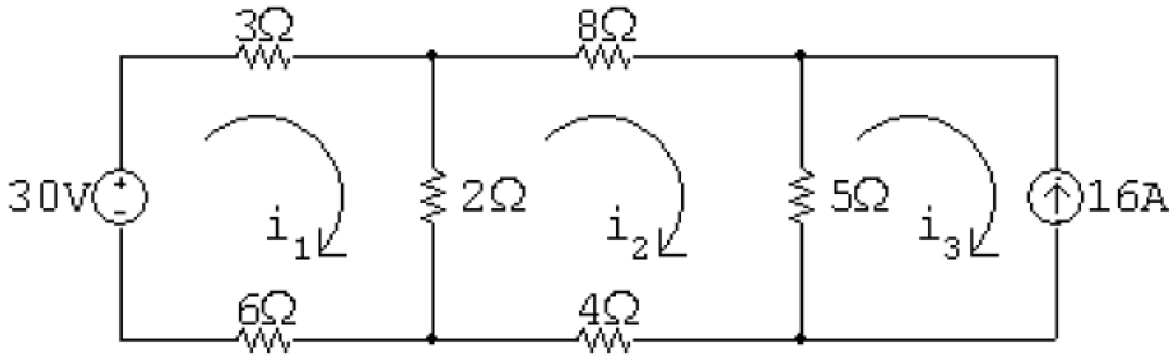
Assessment Problem 4.10

- Use the mesh-current method to find the power dissipated in the 2Ω resistor in the circuit shown below



Assessment Problem 4.10 – cont.

- Redraw the circuit identifying the mesh currents

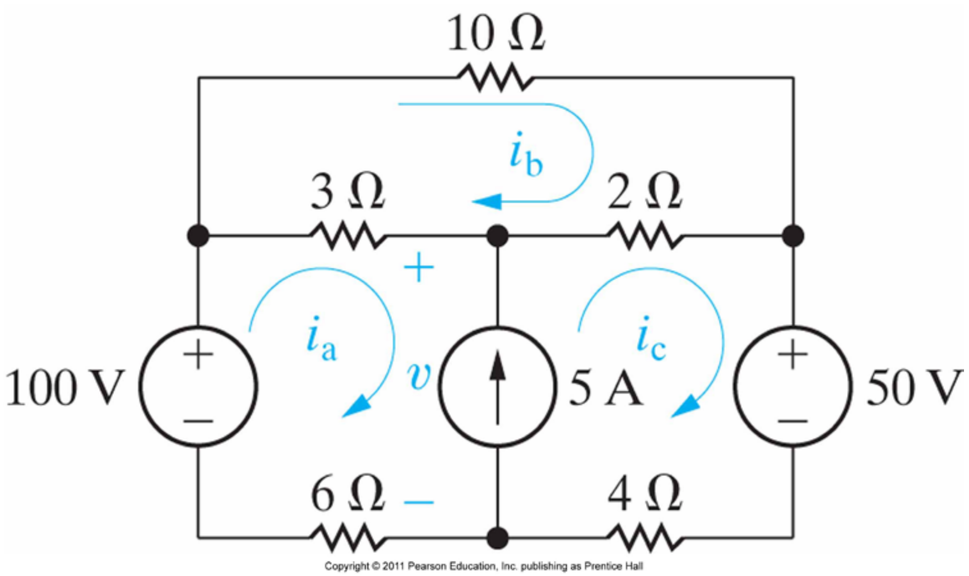


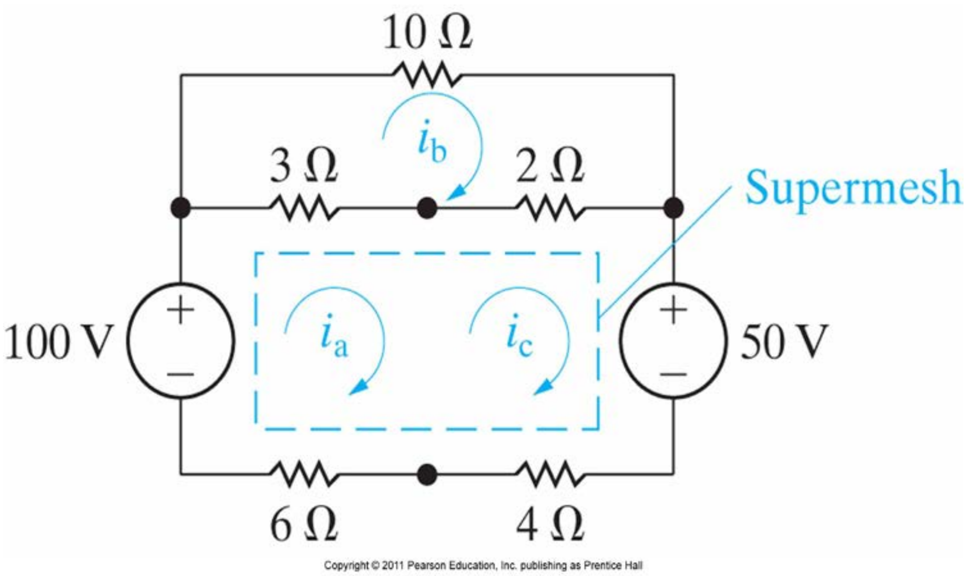
- Since there is a current source on the perimeter of the i_3 mesh, we know that $i_3 = -16\ A$
- The remaining two mesh equations are

$$\begin{aligned} -30 + 3i_1 + 2(i_1 - i_2) + 6i_1 &= 0 \\ 8i_2 + 5(i_2 + 16) + 4i_2 + 2(i_2 - i_1) &= 0 \end{aligned} \Rightarrow \begin{aligned} 11i_1 - 2i_2 &= 30 \\ -2i_1 + 19i_2 &= -80 \end{aligned} \Rightarrow \begin{aligned} i_1 &= 2\,\text{A}, \\ i_2 &= -4\,\text{A}, \\ i_3 &= -16\,\text{A} \end{aligned}$$

Special Cases

- When a branch includes a current source, the mesh-current method requires some additional manipulations
- This is done be using the concept of supermesh
  - To create a supermesh, we mentally remove the current source from the circuit by simply avoiding this branch when writing the mesh-current equations





Special Cases – cont.

- When we sum the voltages around the supermesh (denoted by the dashed line), we obtain the equation

$$-100 + 3(i_a - i_b) + 2(i_c - i_b) + 50 + 4i_c + 6i_a = 0,$$

which reduces to

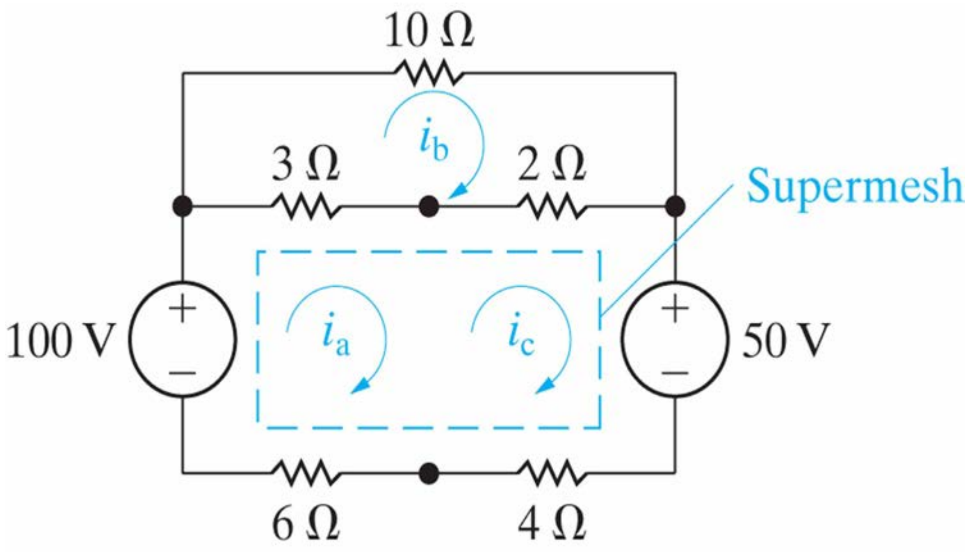
$$50 = 9i_a - 5i_b + 6i_c.$$

Then, we can write the mesh-current equation from the second mesh:

$$0 = 3(i_b - i_a) + 10i_b + 2(i_b - i_c).$$

Finally, we can reduce this system of equations to two equations and two unknowns by using the constraint that:

$$i_c - i_a = 5.$$



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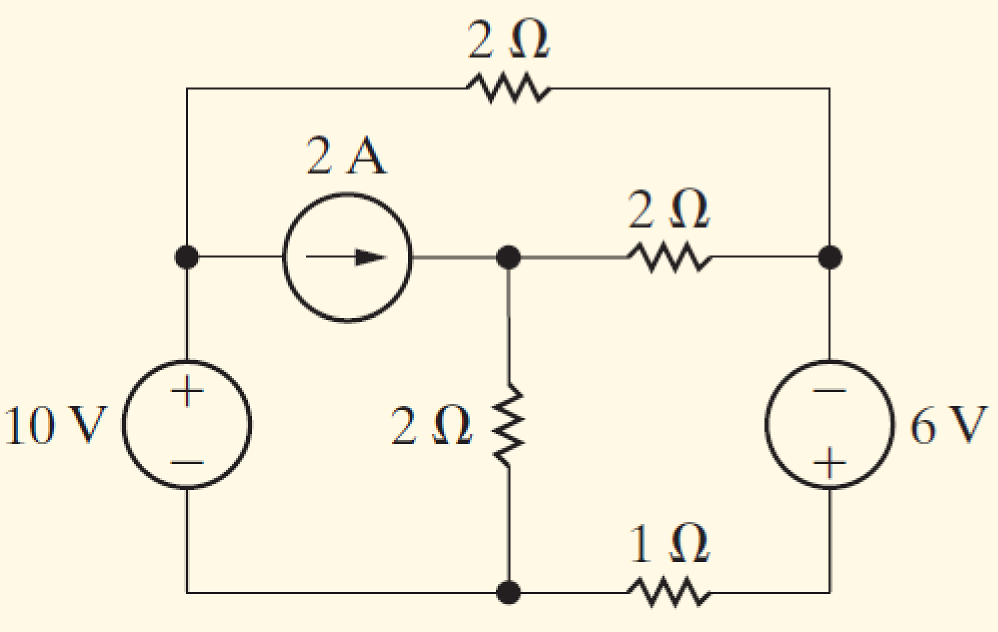
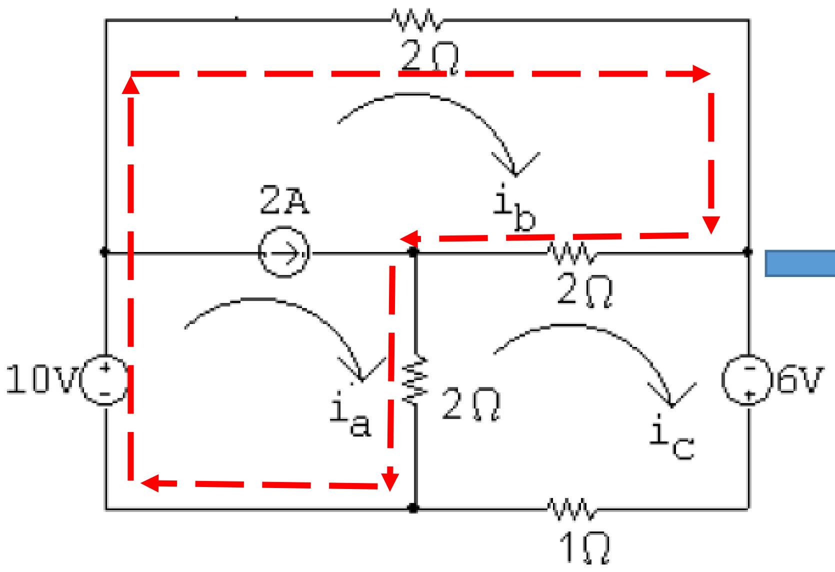
Assessment Problem 4.12

- Use the mesh-current method to find the power dissipated in the 1Ω resistor in the circuit shown below



Assessment Problem 4.12 – cont.

- Redraw the circuit and identify the mesh currents
- The 2 A current source is shared by the meshes i_a and i_b
  - Thus we combine these meshes to form a supermesh



$$-10 + 2i_b + 2(i_b - i_c) + 2(i_a - i_c) = 0$$

The other mesh current equation is

$$-6 + 1i_c + 2(i_c - i_a) + 2(i_c - i_b) = 0$$

The supermesh constraint equation is

$$i_a - i_b = 2$$

Place these three equations in standard form:

$$\begin{aligned} 2i_a + 4i_b - 4i_c &= 10 \\ -2i_a - 2i_b + 5i_c &= 6 \\ i_a - i_b + 0i_c &= 2 \end{aligned}$$

Solving, $i_a = 7\,\mathrm{A}$; $i_b = 5\,\mathrm{A}$; $i_c = 6\,\mathrm{A}$

Thus, $p_{1\,\Omega} = i_c^2(1) = (6)^2(1) = 36\,\mathrm{W}$

Cramer’s Rule

- In linear algebra, **Cramer's rule** is an explicit formula for the solution of a system of linear equations with as many equations as unknowns, valid whenever the system has a unique solution
- Please make sure to revise this rule (Appendix A in your textbook) or refer to any other resources

Node-Voltage Vs. Mesh-Current

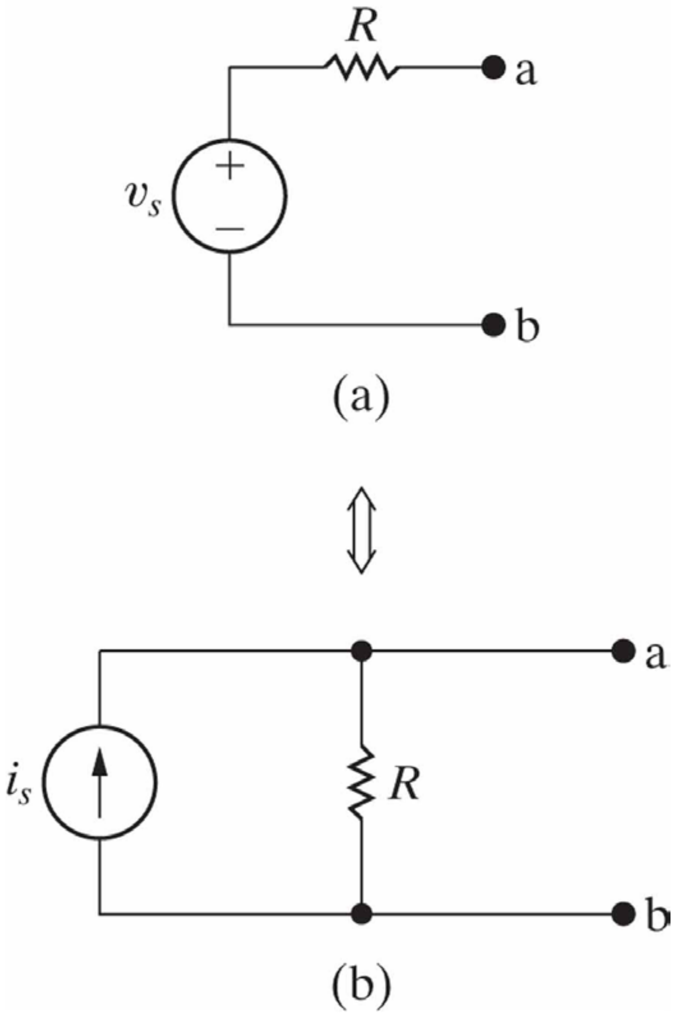
- The greatest advantage of both the node-voltage and mesh-current methods is that they reduce the number of simultaneous equations that must be manipulated
- You may ask, when is the node-voltage method preferred to the mesh-current method and vice versa?
  - As you might suspect, there is no clear-cut answer
- However, you can ask a number of questions help you identify the more efficient method before plunging into the solution process

Node-Voltage Vs. Mesh-Current – cont.

- These questions can be summarized as:
  - Does one of the methods result in fewer simultaneous equations to solve?
    - If so use the one that leads to fewer equations
  - Does the circuit contain super nodes?
    - If so, using the node-voltage method will permit you to reduce the number of equations to be solved
  - Does the circuit contain super meshes?
    - If so, using the mesh-current method will permit you to reduce the number of equations to be solved
  - Will solving some portion of the circuit give the requested solution?
    - If so, which method is most efficient for solving just the relevant portion of the circuit

Source Transformation

- Even though the node-voltage and mesh-current methods are powerful techniques for solving circuits, we are still interested in other methods that can be used to simplify circuits
- Series-parallel reductions and $\Delta - to - Y$ transformations are already on our list of simplifying techniques
- A **source transformation**, allows a voltage source in series with a resistor to be replaced by a current source in parallel with the same resistor or vice versa
- We need to find the relationship between v_s & i_s that guarantees the two configurations are equivalent with respect to nodes **a** and **b**



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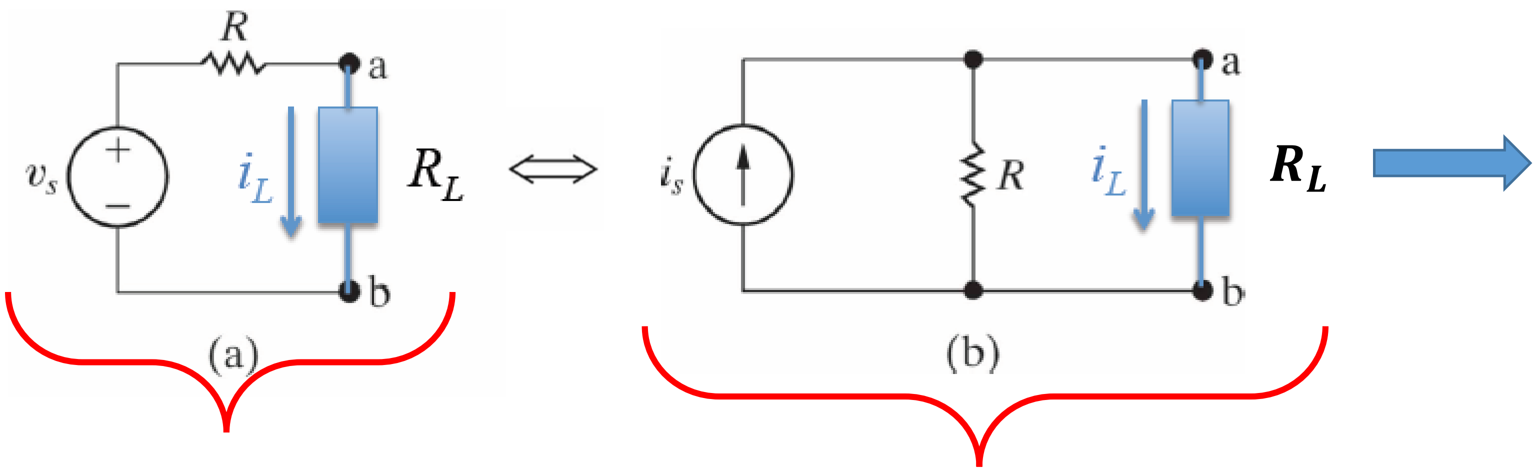
Source Transformation – cont.

- Equivalence is achieved if any resistor R_L experiences the same current flow, and thus the same voltage drop, whether connected between nodes a,b in Fig. 4.36(a) or Fig. 4.36(b).



$$i_L = \frac{v_s}{R + R_L}.$$

$$i_L = \frac{R}{R + R_L} i_s.$$

- If these two circuits are equivalent, these resistor currents (i_L) must be the same
- Equating the right-hand sides of the two i_L equations and simplify them will yield to :

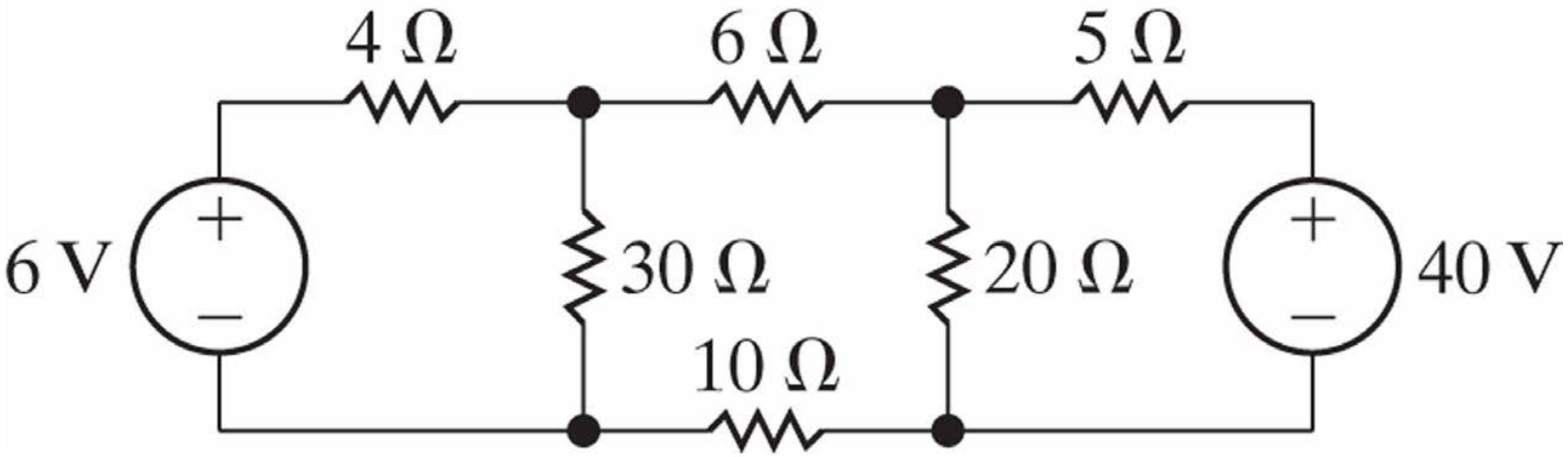
$$\boldsymbol{i_s = \frac{v_s}{R}}$$

When this equation is satisfied, the current in R_L is the same for both circuits, hence the voltage drop across the R_L resistor will be the same for both circuits

Example 4.8

a) For the circuit shown below, find the power associated with the 6 V source

b) State whether the 6 V source is absorbing or delivering the power calculated in (a)



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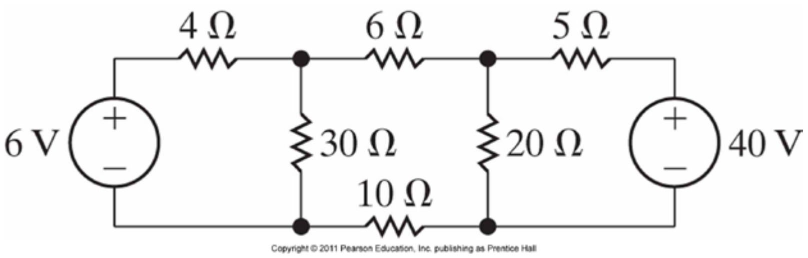
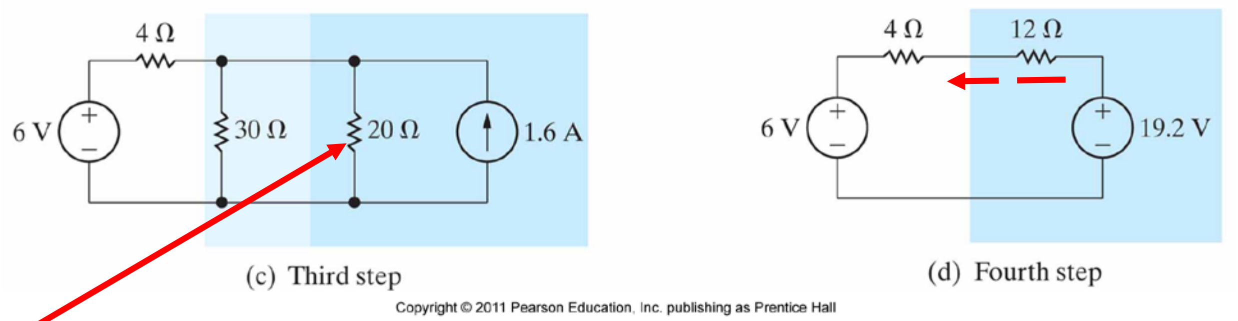
Example 4.8 –cont.

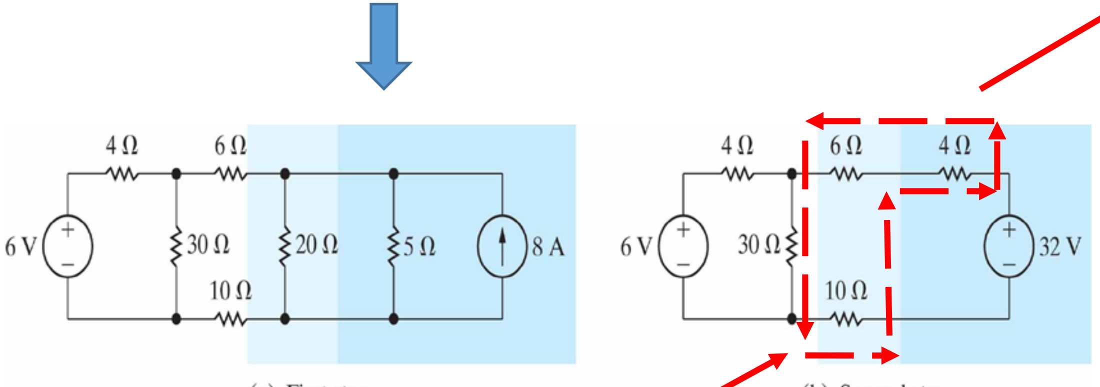
- We must reduce the circuit in a way that preserves the identity of the branch containing the 6 V source
  - We have no reason to preserve the identity of the branch containing the 40 V source





(c) Third step

(d) Fourth step



(a) First step

(b) Second step

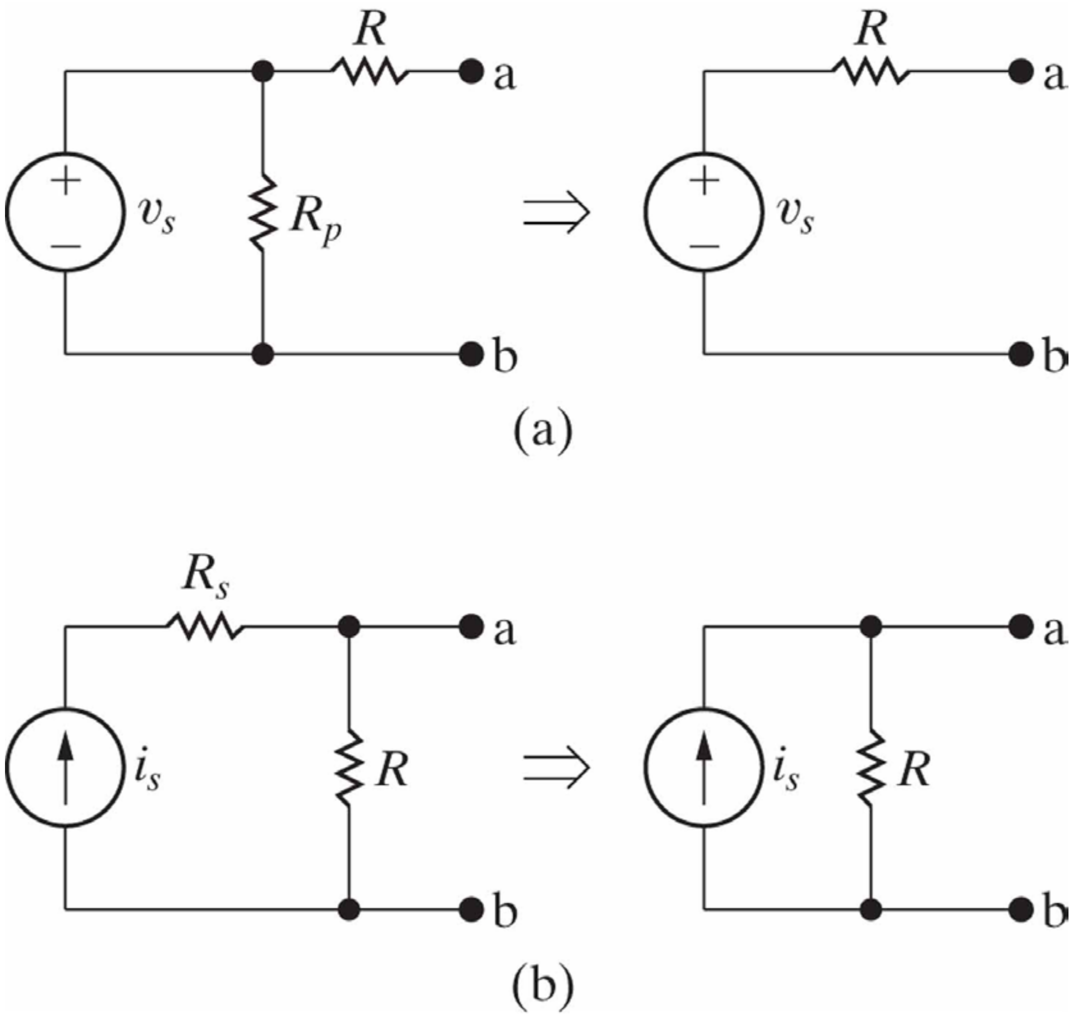
All the resistors that are connected in series with the voltage source

a) The current in the direction of the voltage drop across the 6 V source is $\frac{(19.2-6)}{12+4} = 0.825$ A. Therefore the power associated with the 6 V source is $p_{6V} = (0.825)(6) = 4.95\ W$

b) The voltage source is absorbing power (+ve power)

Special Case

- The two circuits depicted in the figures shown here are equivalent with respect to terminals a and b
- Fig(a) circuits are equivalent with respect to terminals a,b
  - They produce the same voltage and current in any resistor inserted between nodes a,b
- The same can be said for the circuits shown in Fig(b) with respect to terminals a,b
  - The same current is going to pass through R regardless R_s



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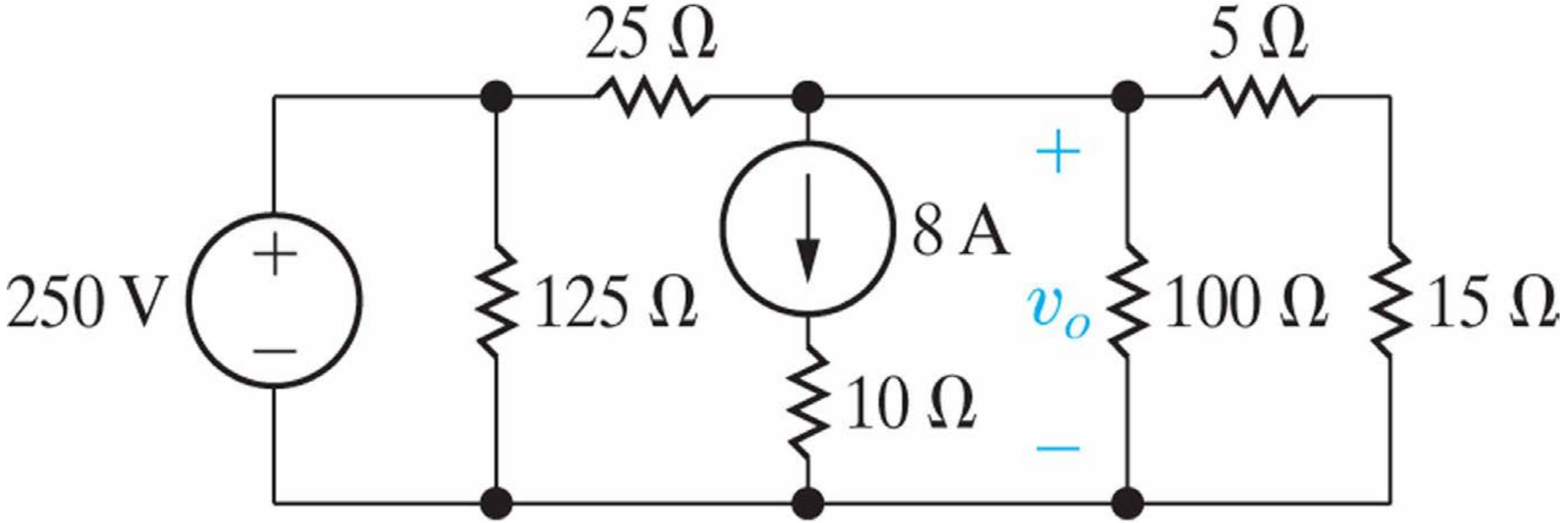
Example 4.9

Use source transformations to find the voltage v_0 in the circuit shown in figure below (Hint: Use special cases from previous slide)



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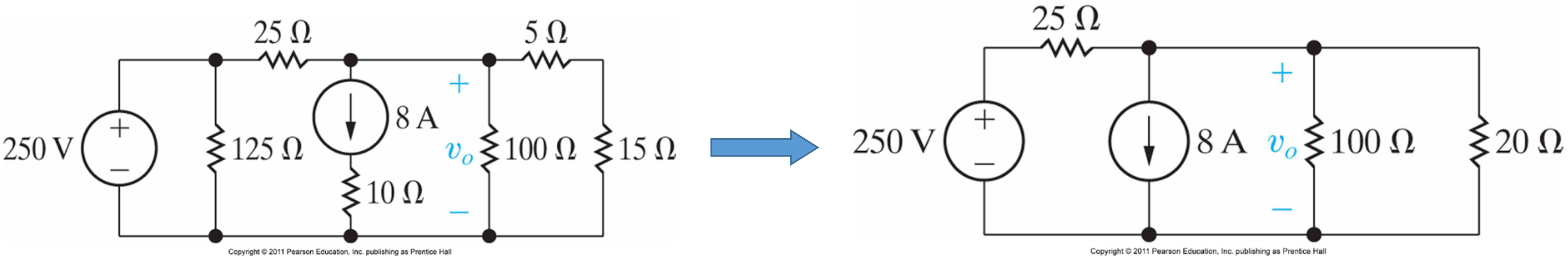
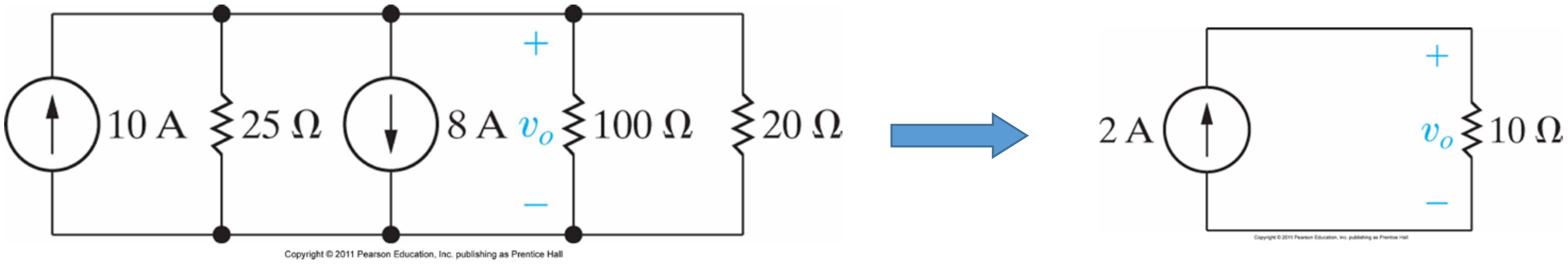
Example 4.9 – cont.

- We begin by removing the 125 Ω and 10 Ω resistors, because the 125 Ω resistor is connected across the 250 V voltage source and the 10 Ω resistor is connected in series with the 8 A current source
- We also combine the series-connected resistors into a single resistance of 20Ω



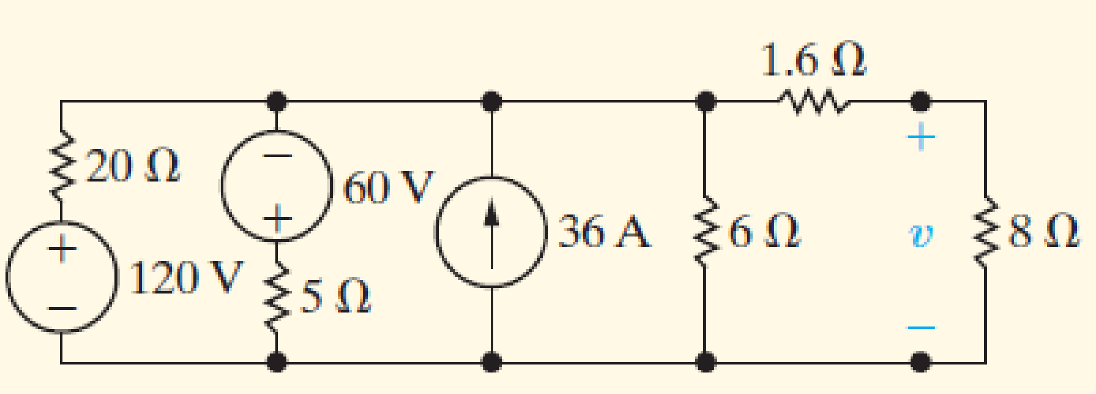
Example 4.9 – cont.

- We now use a source transformation to replace the 250 V source and 25 Ω resistor with a 10 A source in parallel with the 25 Ω resistor, as shown in figure
- We can now simplify this circuit by using KCL to combine the parallel current sources into a single source
- The parallel resistors combine into a single resistor
- Hence, $v_0 = 20V$



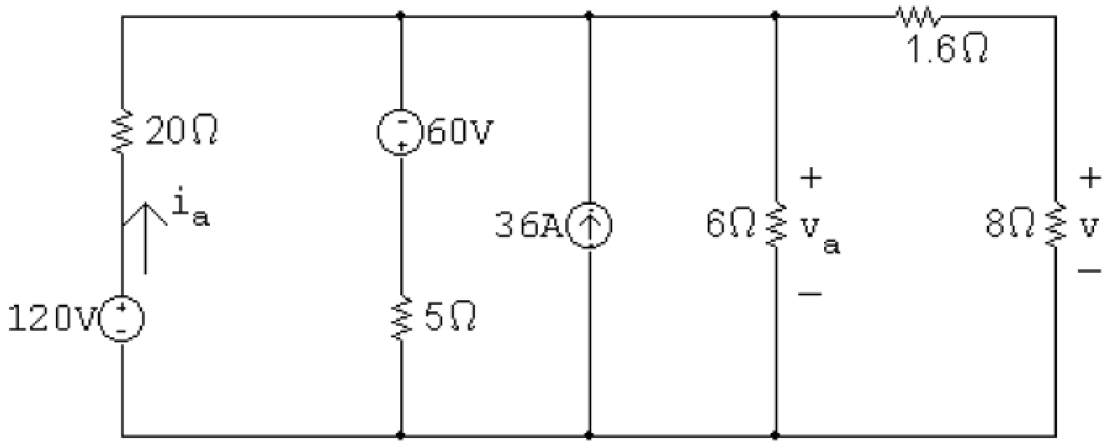
Assessment Problem 4.15

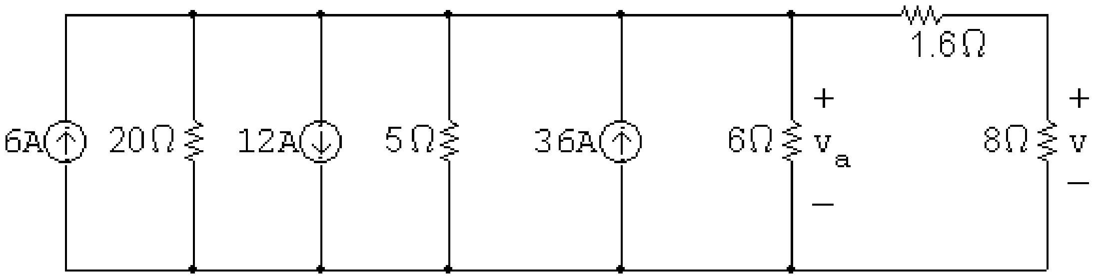
- Use a series of source transformations to find the voltage v in the circuit shown below



Assessment Problem 4.15 – cont.

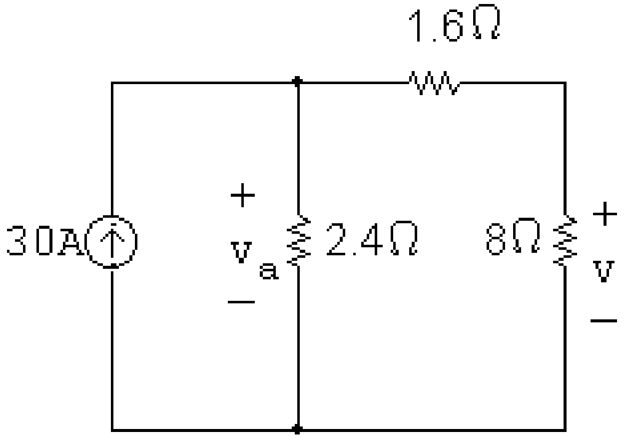
- Redraw the circuit with more voltages and currents labeled
- Transform the 120 V source in series with the 20 Ω resistor into a 6 A source in parallel with the 20 Ω resistor
- Also transform the −60 V source in series with the 5Ω resistor into a −12 A source in parallel with the 5 Ω resistor
- The result is the following circuit:



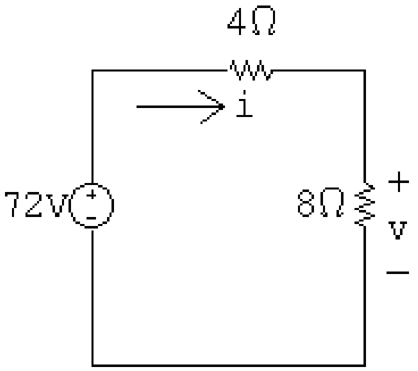


Assessment Problem 4.15 – cont.

- Combine the three current sources into a single current source, using KCL, and combine the 20Ω, 5Ω, and 6Ω resistors in parallel



- To simplify the circuit further, transform the resulting 30 A source in parallel with the 2.4 Ω resistor into a 72 V source in series with the 2.4Ω resistor



Use voltage division in the circuit on the right to calculate v as follows:

$$v = \frac{8}{12}(72) = 48\,V$$

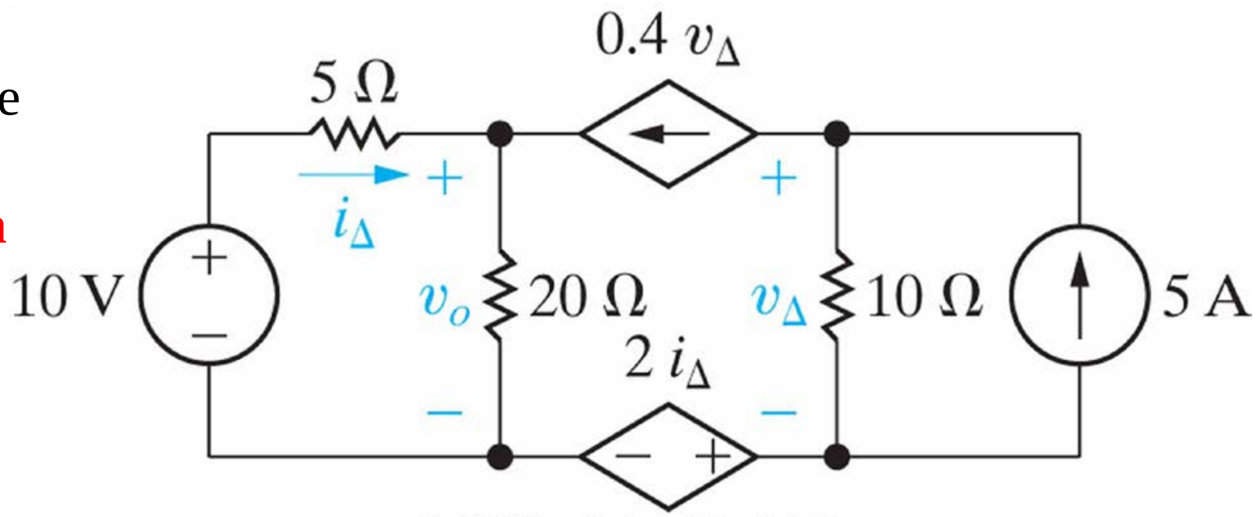
Superposition

- A linear system obeys the principle of **superposition**, which states that whenever a linear system is excited, or driven, by more than one independent source of energy, the total response is the sum of the individual responses
  - An individual response is the result of an independent source acting alone
- Superposition is required only if the independent sources in a circuit are fundamentally different
  - In this course, all independent sources are dc sources, so superposition is not required
  - We introduce superposition here in anticipation of later chapters in which circuits will require it

Example 4.13

- Use the principle of superposition to find v_0 in the circuit shown below

- Superposition means that we need to study the effect of independent sources individually
  - One at a time
- To do that we need to deactivate other sources while analyzing the one of interest
- **To deactivate a *current source*, we replace it with an *open circuit***
- **To deactivate a *voltage source*, we replace it with a *short circuit***
- When applying superposition to linear circuits containing both independent and dependent sources, you must keep in mind that the dependent sources are never deactivated



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Example 4.13 – cont.

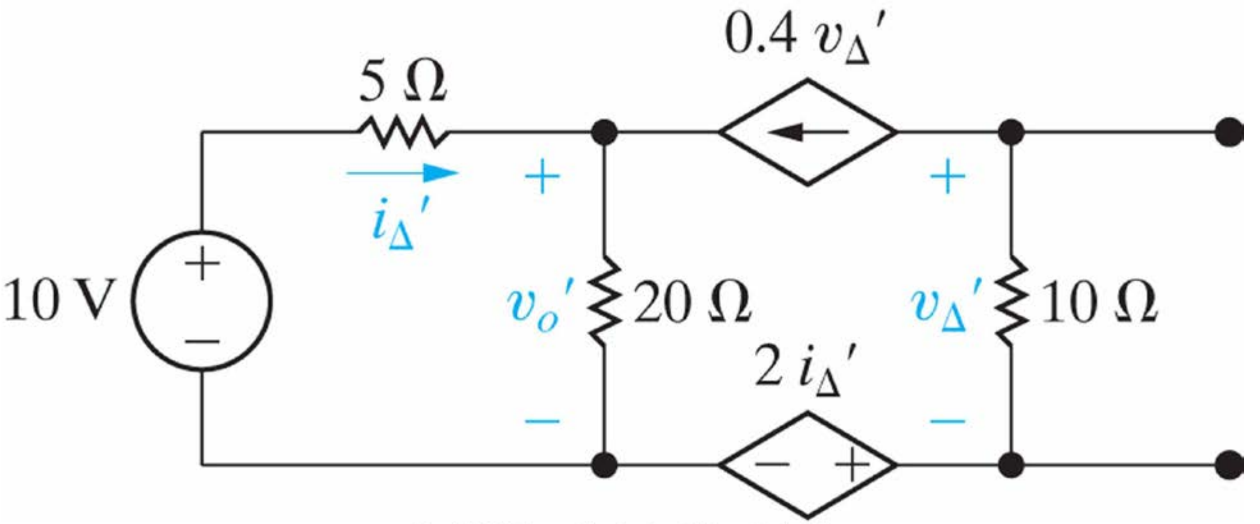
- We begin by finding the component of v_0 resulting from the 10 V source. The figure below shows the circuit with the 5 A source deactivated

- With the 5 A source deactivated, v'_Δ must equal:

$$v'_\Delta = (-0.4v'_\Delta)(10)$$
$$v'_\Delta = -4v'_\Delta$$
$$5v'_\Delta = 0 \;\rightarrow v'_\Delta = 0$$

- This means that the branch containing the two dependent sources is open
- Thus, v'_0 can be calculated as:

$$v'_0 = \frac{20}{25}(10) = 8V$$



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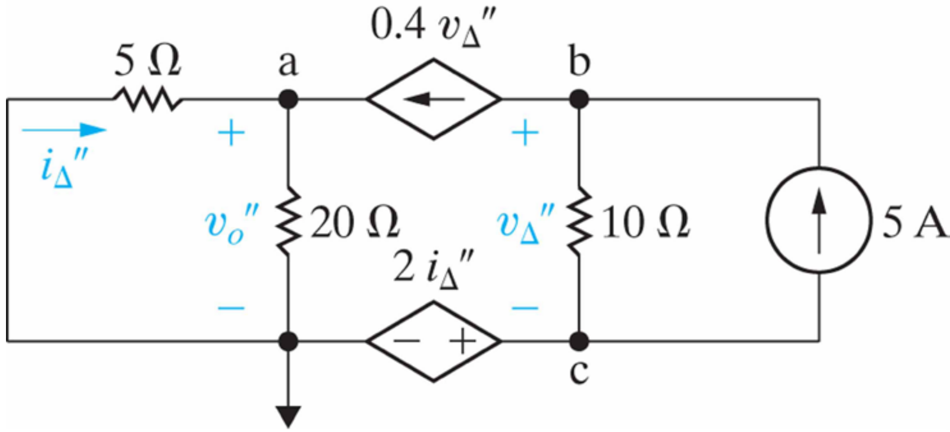
Example 4.13 – cont.

- Next, we find the component of v_0 resulting from the 5 A source. The figure below shows the circuit with the 10 V source deactivated
- We have added a reference node and the node designations a, b, and c to aid the discussion



- Summing the currents entering & leaving node **b** yields

$$0.4v''_\Delta+\frac{v_\Delta''}{10}-5=0 \rightarrow 4v_\Delta''+v_\Delta''-50=0 \rightarrow v_\Delta''=10\text{ V}$$

- v_0'' is the voltage across R_{eq} from the 5 Ω *and the* 20 Ω

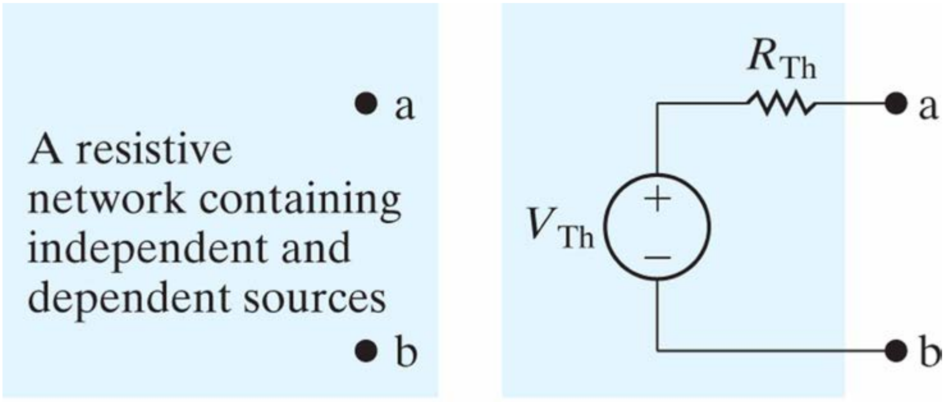
$$R_{eq}=\frac{5*20}{25}=\frac{100}{25}=4$$

- v_0'' is the current from the dependent current source ($0.4v_\Delta''=4A$) → $v_0''=4*4=16\ V$
- The final value of v_0 is the sum of v_0\` and v_0'' → $v_0=16+8=24\ V$

Thévenin Equivalent

- At times in circuit analysis, we want to concentrate on what happens at a specific pair of terminals
- Thévenin and Norton equivalents are circuit simplification techniques that focus on terminal behavior and thus are extremely valuable in analysis
- We can best describe a Thévenin equivalent circuit by reference to the circuit shown below. It represents any circuit made up of sources (both independent and dependent) and resistors
- The letters **a** and **b** denote the pair of

  terminals of interest



(a) (b)

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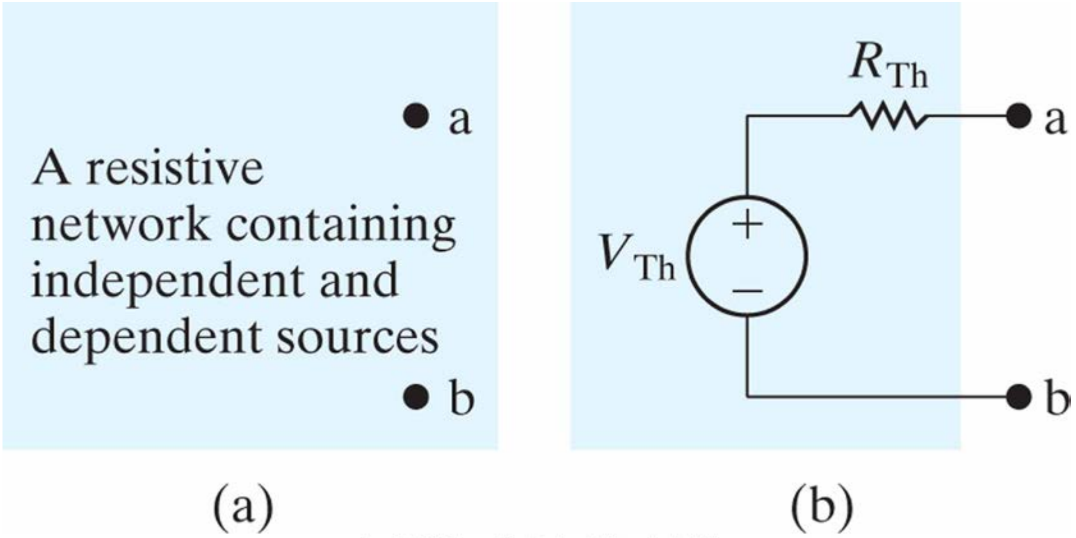
Thévenin Equivalent – cont.

- **Thévenin equivalent circuit** is an independent voltage source V_{Th} in series with a resistor R_{Th} which replaces an interconnection of sources and resistors
- This series combination of V_{Th} and R_{Th} is equivalent to the original circuit in the sense that, if we connect the same load across the terminals **a,b** of each circuit, we get the same voltage and current at the terminals of the load
  - This equivalence holds for all possible values of load resistance



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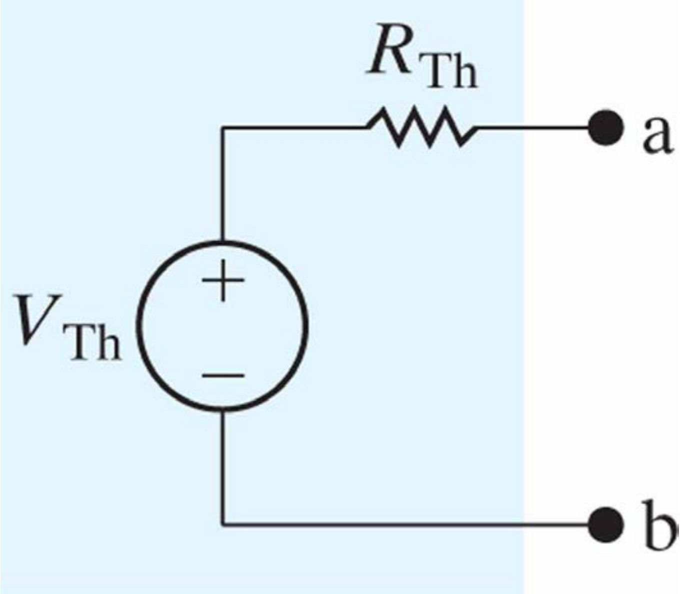
Thévenin Equivalent – cont.

- To represent the original circuit by its Thévenin equivalent, we must be able to determine the Thévenin voltage V_{Th} and the Thévenin resistance R_{Th}
- The open-circuit voltage at the terminals **a, b** in the circuit shown here is V_{Th}
  - Therefore, to calculate the Thévenin voltage V_{Th} we simply calculate the open-circuit voltage in the original circuit



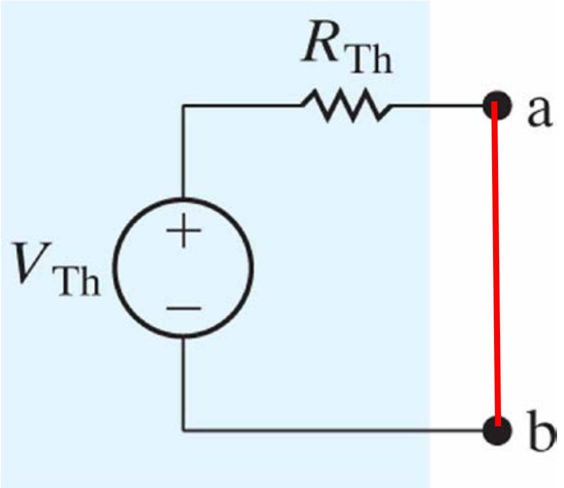
Thévenin Equivalent – cont.

- Now, if we place a short circuit across the terminals **a,b** of the Thévenin equivalent circuit, the short-circuit current directed from **a** to **b** is

$$i_{sc} = \frac{V_{Th}}{R_{Th}}$$

- Thévenin equivalent hypothesizes that this short circuit current i_{sc} is identical to the short-circuit current that exists if a short circuit is placed across the terminals **a,b** of the original network
- Thus,

$$R_{Th} = \frac{V_{Th}}{i_{sc}}$$

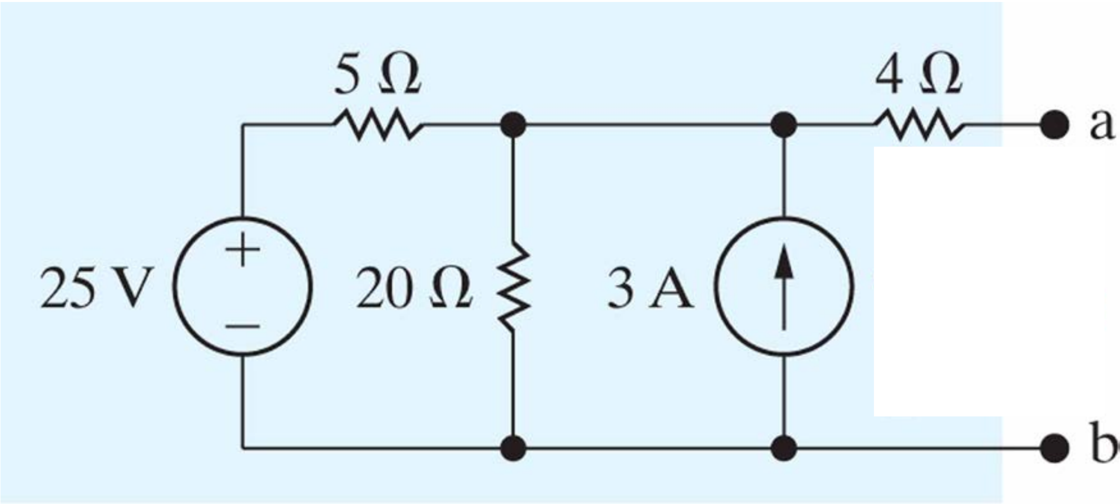


R_{Th} is the ratio of the open-circuit voltage to the short-circuit current

Thévenin Equivalent – Demonstration Example

- We need to find the Thévenin equivalent of the circuit shown below

- To do that:
  - First we find the open-circuit voltage between terminals **a** and **b**
  - Second, we find the short circuit current between **a** and **b**
  - Finally, we calculate R_{Th} as the ratio of these two



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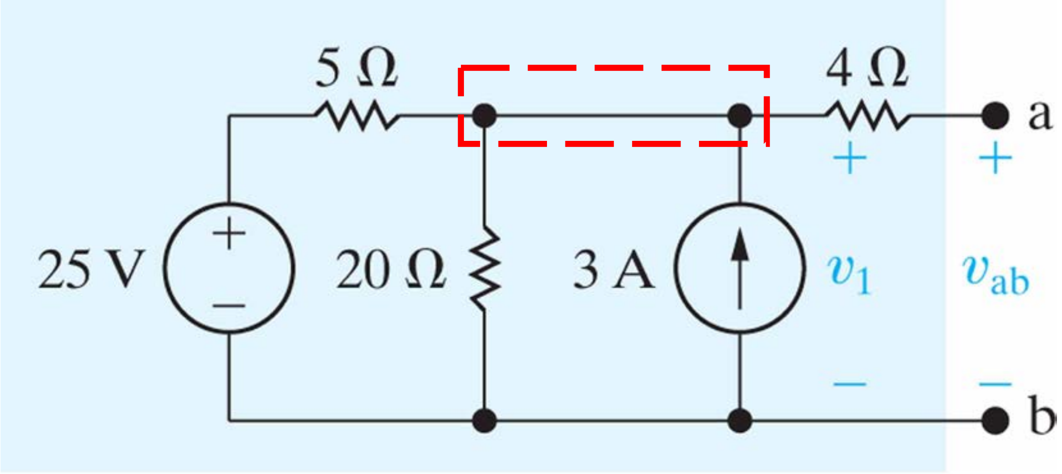
Thévenin Equivalent – Demo Example cont.

- Let us find the open circuit voltage (V_{Th}) between terminals **a** and **b**
  - We find the voltage by solving a single node-voltage equation, choosing the lower node as the reference node
  - Note that when the terminals **a,b** are open, there is no current in the 4Ω resistor

- Apply node-voltage analysis on this node
- Current entering the node is –ve
- Current leaving the node is +ve

$$-\frac{25-v_1}{5} - 3 + \frac{v_1}{20} = 0 \;\rightarrow v_1 = 32\,V = V_{Th}$$



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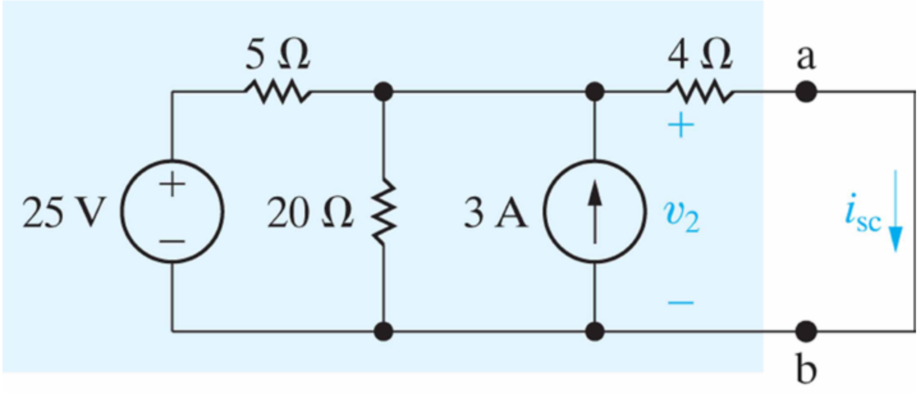
Thévenin Equivalent – Demo Example cont.

- The next step is to find the short circuit current (i_{sc}) between **a** and **b**
  - The short-circuit current (i_{sc}) is found easily once v_2 is known
  - Therefore, the problem reduces to finding v_2 with the short in place
  - Again, if we use the lower node as the reference node, the equation for v_2 becomes

$$-\frac{25 - v_2}{5} - 3 + \frac{v_2}{20} + \frac{v_2}{4} = 0 \ \rightarrow v_2 = 16\,V$$



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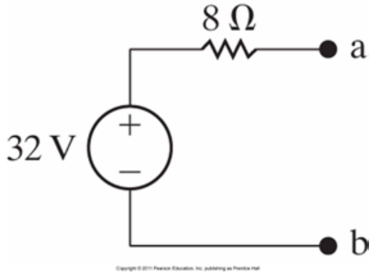
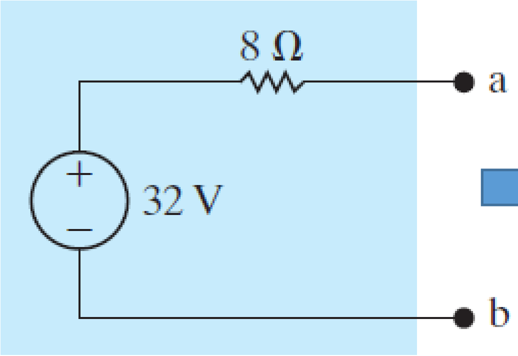
  - Hence, the short-circuit current: $i_{sc} = \frac{16}{4} = 4A$
  - We now find the Thévenin resistance:
  - $R_{th} = \frac{V_{Th}}{i_{sc}} = \frac{32}{4} = 8\Omega$

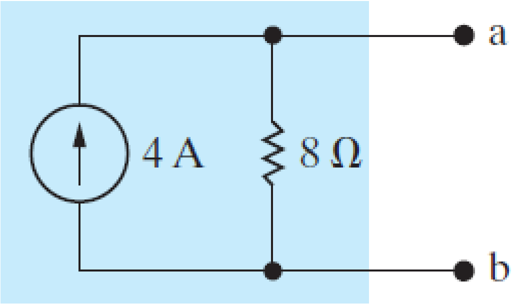


The Norton Equivalent

- A **Norton equivalent circuit** consists of an independent current source in parallel with the Norton equivalent resistance
- We can derive it from a Thévenin equivalent circuit simply by making a source transformation
- Thus the Norton current equals the short-circuit current at the terminals of interest, and the Norton resistance is identical to the Thévenin resistance



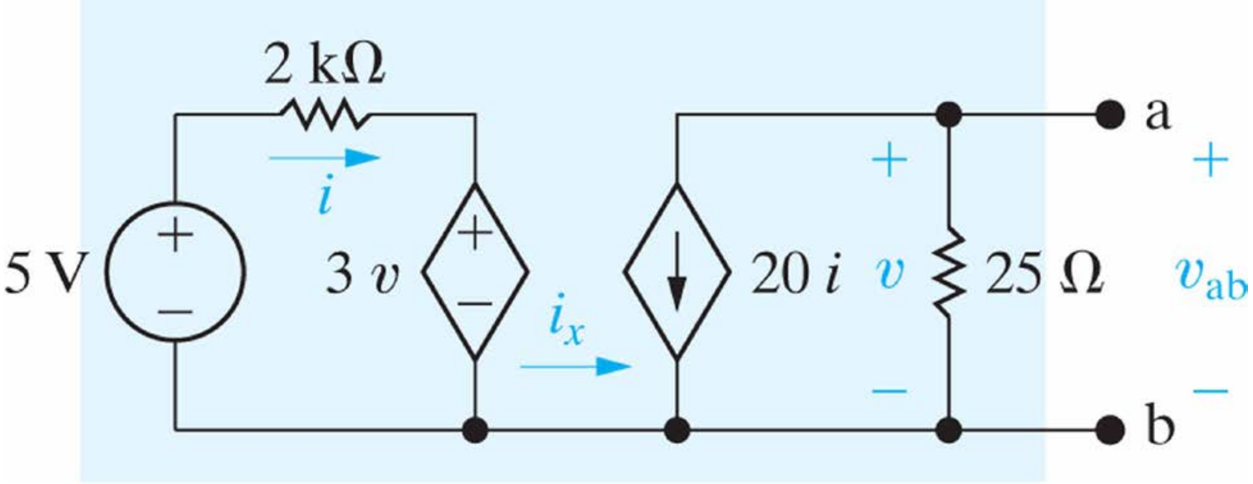
Thevenin Equivalent



Norton Equivalent

Example 4.10

- Find the Thévenin equivalent for the circuit containing dependent sources shown below



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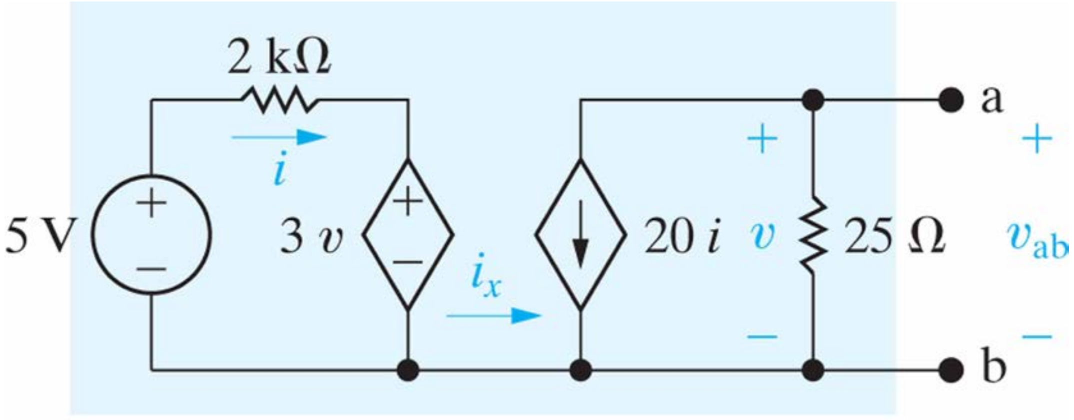
Example 4.10 – cont.

- The first step in analyzing this circuit is to recognize that the current labeled i_x must be zero
  - Note the absence of a return path for i_x to enter the left-hand portion of the circuit
- The open-circuit, or Thévenin, voltage will be the voltage across the 25Ω resistor with $i_x = 0$

$$V_{Th} = v_{ab} = (-20i)(25) = -500i$$

- The current i is

$$i = \frac{5 - 3v}{2000} = \frac{(5 - 3V_{Th})}{2000}$$



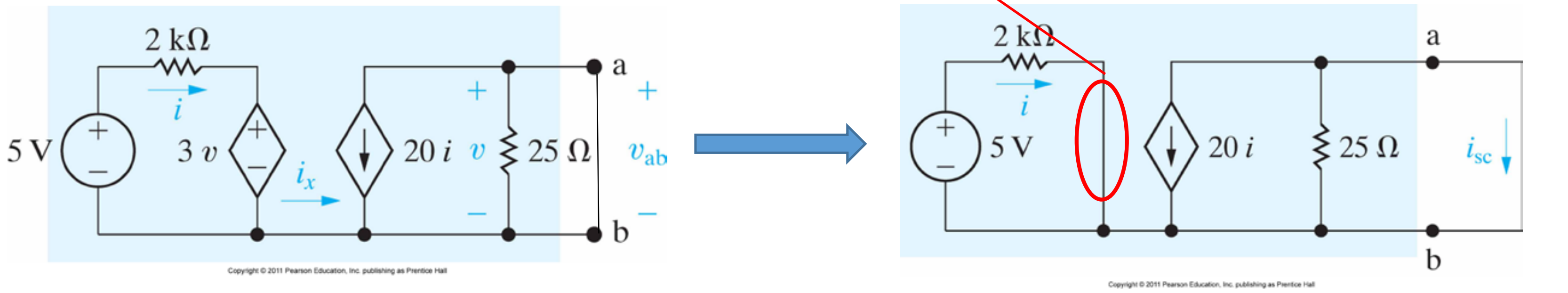
Example 4.10 – cont.

- From the previous two equations we obtain $V_{Th} = -5V$
- To calculate the short-circuit current i_{sc} we place a short circuit across **a,b**
- With the short circuit shunting the 25 Ω resistor, all the current from the dependent current source appears in the short, so:

$$i_{sc} = -20i$$

- The short circuit shunting the 25 Ω resistor will also lead to have the controlling voltage $v = 0$ which reduces the dependent voltage source to zero

$$i = \frac{5}{2000} = 2.5\ mA$$
$$i_{sc} = (-20)(2.5\ mA) = -50\ mA$$

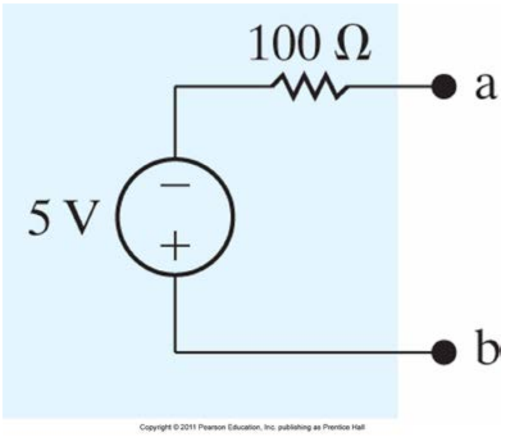


Example 4.10 – cont.

- From i_{sc} and V_{Th} we can get R_{Th}:

$$R_{Th} = \frac{V_{Th}}{i_{sc}} = -\frac{5}{50 \times 10^{-3}} = 100\ \Omega$$

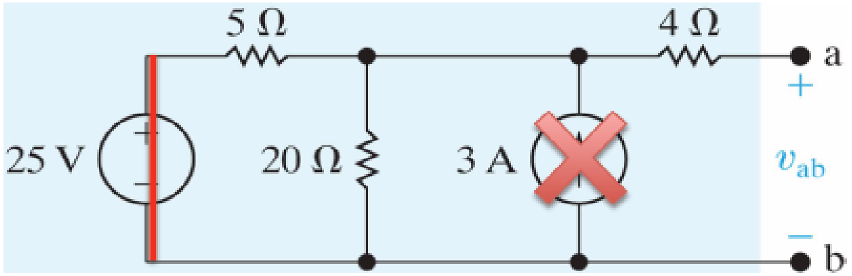
- And the Thévenin equivalent can be illustrated as



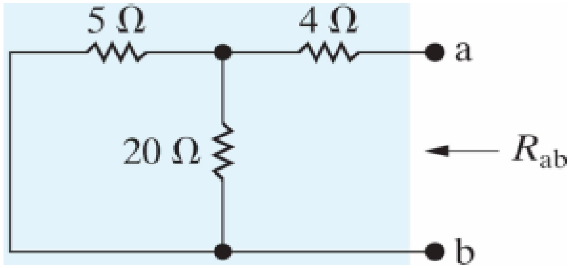
Using source transformation, we can change that to a current source with a parallel resistor. This will turn this circuit into the Norton equivalent

Alternative Methods

- The technique we discussed to determine R_{Th} is not always the easiest method available
- Other methods are available for us to use to find R_{Th}
- If the network **contains only independent sources**, we can do the following



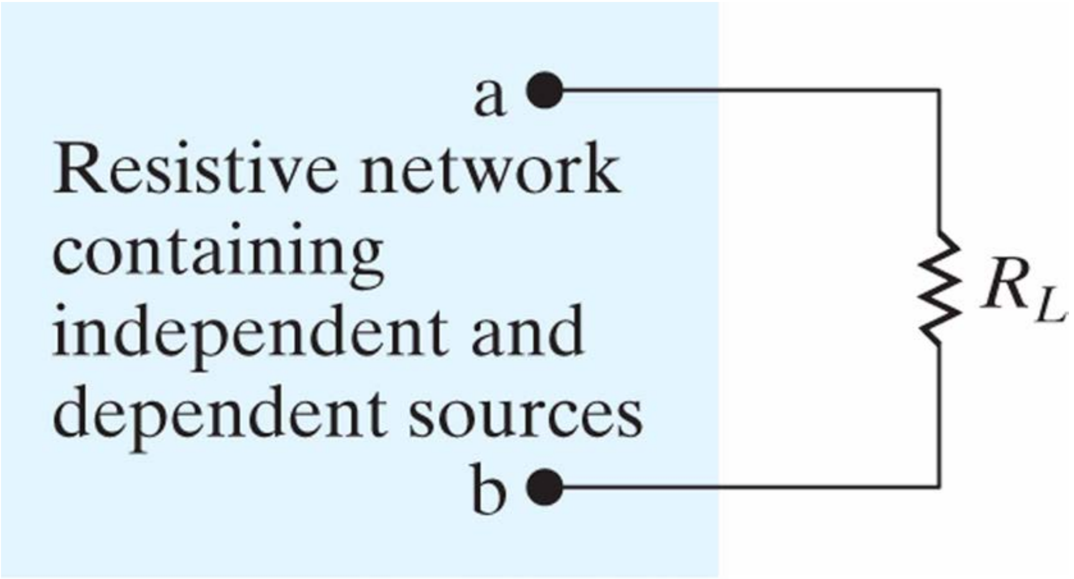
A voltage source is deactivated by replacing it with a short circuit.
A current source is deactivated by replacing it with an open circuit.



$$R_{ab} = R_{Th} = 4 + \frac{5 \times 20}{25} = 8\ \Omega$$

Maximum Power Transfer

- Maximum power transfer can be best described with the aid of the circuit shown below
  - We assume a resistive network containing independent and dependent sources and a designated pair of terminals, a,b, to which a load, R_L, is to be connected
- The problem is to determine the value of R_L that permits maximum power delivery to R_L



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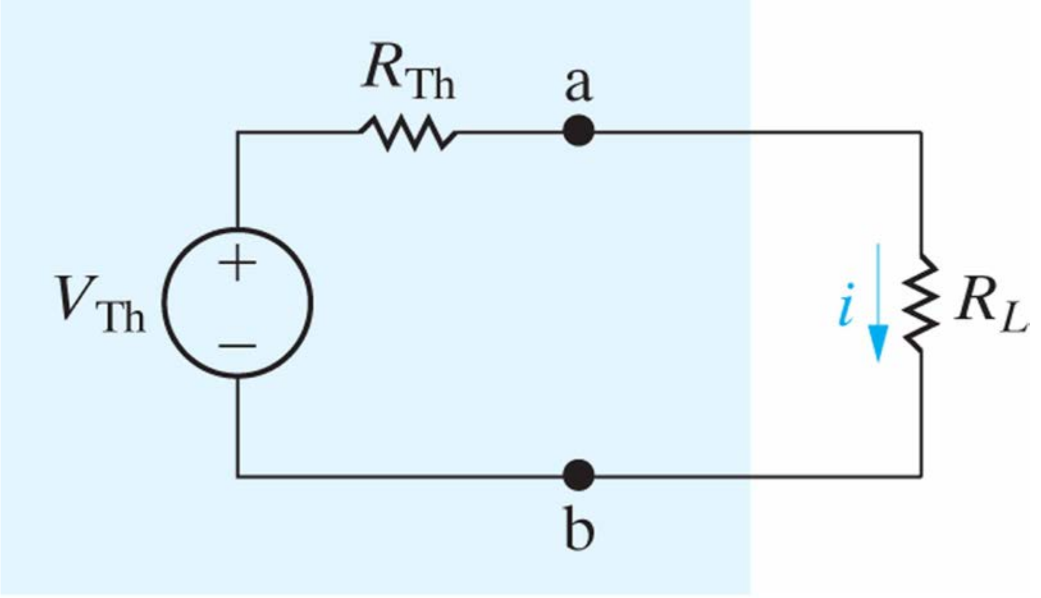
Maximum Power Transfer – cont.

- The first step in solving the problem is to recognize that a resistive network can always be replaced by its Thévenin equivalent
- Therefore, we redraw the circuit shown in previous slide as the one shown here
  - Replacing the original network by its Thévenin equivalent greatly simplifies the task of finding R_L
- The power dissipate in R_L:

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

For a given circuit, V_{Th} *and* R_{Th} are fixed. Therefore, the power dissipated is a function of R_L



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Maximum Power Transfer – cont.

- To find the value of R_L that maximizes the power, we use calculus and find the first derivative of the power equation with respect to R_L

$$\frac{dp}{dR_L} = V_{\mathrm{Th}}^2 \left[\frac{(R_{\mathrm{Th}} + R_L)^2 - R_L \cdot 2(R_{\mathrm{Th}} + R_L)}{(R_{\mathrm{Th}} + R_L)^4} \right].$$

- If we equate the derivative with zero, then the power is maximized when

$$(R_{\mathrm{Th}} + R_L)^2 = 2R_L(R_{\mathrm{Th}} + R_L).$$

- **Solving this equation yields that the maximum power transfer occurs when the load resistance R_L equals R_{Th}**

Maximum Power Transfer – cont.

- To find the maximum power delivered to R_L in a circuit, we simply substitute $R_L = R_{Th}$ in the power equation

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

- This yields,

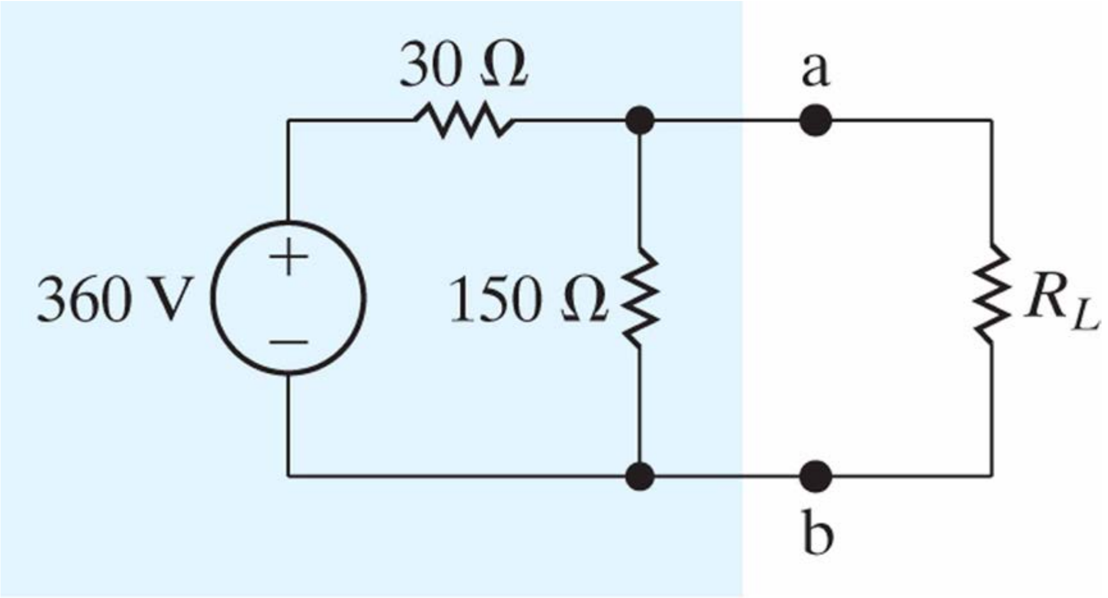
$$p_{\max} = \frac{V_{\mathrm{Th}}^2 R_L}{(2R_L)^2} = \frac{V_{\mathrm{Th}}^2}{4R_L}.$$

Example 4.12

a) For the circuit shown below, find the value of R_L that results in maximum power being transferred to R_L

b) Calculate the maximum power that can be delivered to R_L

c) When R_L is adjusted for maximum power transfer, what percentage of the power delivered by the 360 V source reaches R_L?



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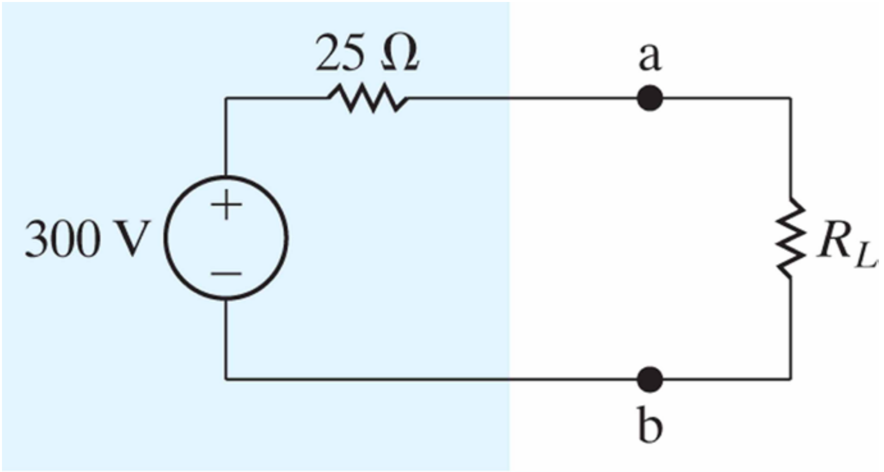
Example 4.12 – cont.

a) The Thévenin voltage for the circuit to the left of the terminals a,b is:

$$V_{\text{Th}} = \frac{150}{180}(360) = 300\ \text{V}.$$

The Thévenin resistance is

$$R_{\text{Th}} = \frac{(150)(30)}{180} = 25\ \Omega.$$



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- This indicates that R_L must equal 25 Ω for maximum power transfer

Example 4.12 – cont.

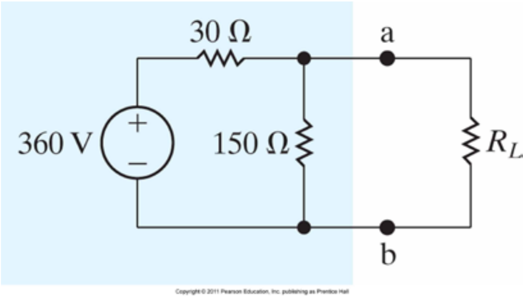
b) The maximum power that can be delivered to R_L is $p_{\max} = \left(\frac{300}{50}\right)^2(25) = 900 \text{ W}$

$R_{total} = R_{Th} + R_L = 25 + 25 = 50\Omega$

c) When R_L equals 25 Ω, the voltage v_{ab} is: $v_{ab} = \left(\frac{300}{50}\right)(25) = 150 \text{ V}$

- The current of the voltage source in the direction of the voltage:

$$i_s = \frac{360 - 150}{30} = \frac{210}{30} = 7 \text{ A}.$$



- Therefore, the power delivered by the source to the circuit is:

$$p_s = -i_s(360) = -2520 \text{ W}.$$

- The percentage of the source power delivered to the load is $\frac{900}{2520} \times 100 = 35.71\%$.