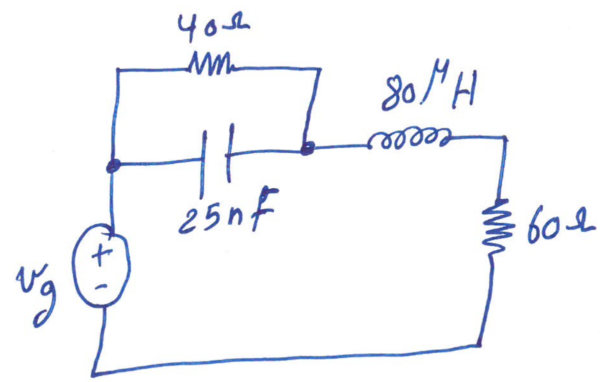
Example ch10 3 Power Calculation ①

⊛ For the circuit shown below, we need to find the following knowing that : $v_g = 40\cos(10^6 t)$.

a) The average power (P) supplied by the voltage source.

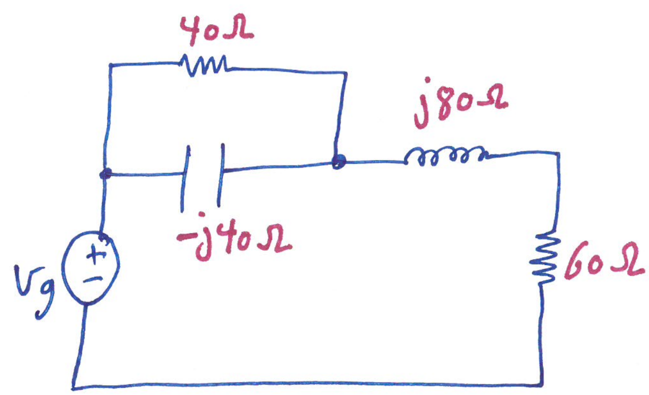
b) The reactive power (Q) supplied by the voltage source.

c) The apparent power (|S|) supplied by the voltage source.



Solution:

⊛ The first thing we can do is to simplify the circuit by finding Z_{eq}.

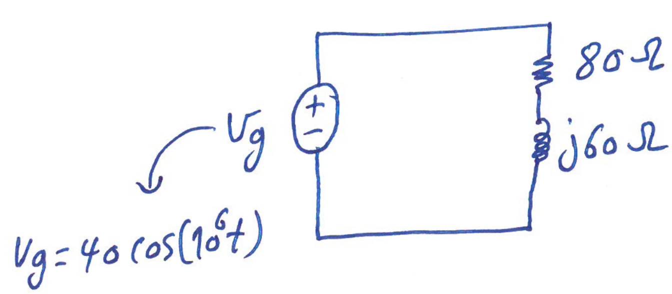


$$\Rightarrow (40\Omega \,||\, 25nF) + 80\mu H + 60\Omega$$

$$\Rightarrow Z_{eq} = (40\Omega \,||\, -j40\Omega) + j80 + 60$$

$$\boxed{Z_{eq} = 80 + j60\ \Omega}$$

⊛ Thus, the circuit can be simplified to:



Solution:- cont.

a) we need to find the average Power (P).

⇒ We can have more than one way to find "P".

⇒ let us find the current of the circuit.

$$\Rightarrow i = \frac{V_g}{80+j60} = \frac{40\angle 0^\circ}{80+j60} = \frac{40\angle 0^\circ}{100\angle 36.87^\circ} = 0.4\angle -36.87^\circ \text{ Amps.}$$

⇒ one way to find "P" is to use the following equation:

$$P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) = \frac{40 \cdot 0.4}{2}\cos(0-(-36.87)) = 8\cos(36.87)$$
$$= 8(0.8) = 6.4\,W.$$

⇒ Another way to calculate "P" is to use the following equation:

$$P = I_{rms}^2 (R) = \left(\frac{0.4}{\sqrt{2}}\right)^2 (80) = 0.08(80) = 6.4\,W.$$

⇒ note that we only used the real part of the impedance only. The real part is where average power is dissipated.

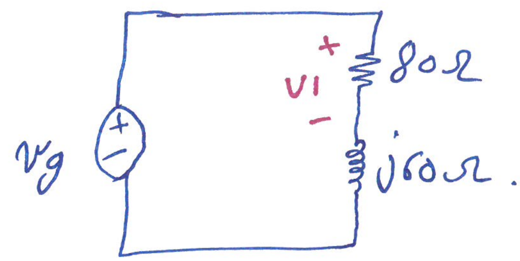
⇒ One more way we can use to calculate "P":

$P = \frac{V_{rms}^2}{R}$, however we need to be careful which voltage we need to use. Note that we are interested in the real part of the impedance ($80\,\Omega$), so we need to use the voltage drop across the $80\,\Omega$ resistor only.

⇒

* Solution: - cont.

$$\Rightarrow V_1 = V_g * \frac{80}{80+j60}$$ ← voltage divider.



$$\Rightarrow V_1 = \frac{(40\angle 0^\circ)(80)}{100\angle 36.87^\circ} = \frac{3200\angle 0^\circ}{100\angle 36.87^\circ}$$

$$= 32\angle -36.87 \text{ Volts.}$$

* Now, we can use this voltage to calculate power "P".

$$P = \frac{V_{rms}^2}{R} = \frac{(32/\sqrt{2})^2}{80} = \frac{1024/2}{80} = 6.4\text{ W.}$$

* because of that, I recommend using the current instead of the voltage to calculate power. The current will be the same across all circuit components in this example, so we will not need to worry about voltage drops across each component.

b) To calculate "Q", we also have more than one way:

$$Q = \frac{V_m I_m}{2} \sin(\theta_v - \theta_i) = \frac{40\,(0.4)}{2} \sin(36.87) = 4.8\text{ VARs.}$$

Or:

$$Q = I_{rms}^2 \underbrace{(X)}_{\text{reactance.}} = (0.4/\sqrt{2})^2 (60) = 4.8\text{ VARs.}$$

* Solution: - Cont.

c) to calculate S, we can simply use the result from "a" & "b".

$$\Rightarrow S = P + jQ = -6.4 - j\,4.8 \text{ VA.}$$

we add the "-ve" sign, because this power is delivered by the source.

$$\Rightarrow |S| = \sqrt{P^2 + Q^2} = 8 \text{ VA.}$$

* another way we can use to calculate "S" directly is:

$$S = -\tfrac{1}{2} V_g I_g^* = -\tfrac{1}{2} \underbrace{(40)}_{V_g} \underbrace{(0.32 + j0.24)}_{I_g^*} = -6.4 - j4.8 \text{ VA.}$$

* I recommend this way, it is easier, & you can ~~calculate~~ tell P & Q immediately from the expression we have for "S". $S = P + jQ$.