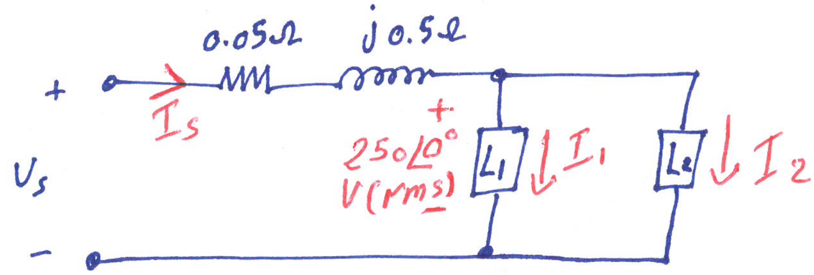
Example 10.6

* We have the following circuit:



→ Load 1 (L_1) absorbs an average power (P) of 8 kW @ a leading power factor of 0.8

- Leading power factor means that Q is negative.

→ Load 2 (L_2) absorbs 20kVA @ a lagging power factor of 0.6.

- lagging power factor means that Q is positive.
- From the unit of the absorbed power (VA), we can tell that this is "S", the complex power.

* Solution:

a) We need to determine the power factor of the two loads in parallel.

→ Let us start with the information we have:

$$S = S_1 + S_2$$

a) cont.:

$$S_1 = P_1 + jQ_1 \quad , P_1 = 8000\,W, \ \cos(\theta) = 0.8$$

$$S_2 = P_2 + jQ_2 \quad , S_2 = 20{,}000\ VA, \ \cos(\theta) = 0.6$$

*

$$\Rightarrow P_1 = |S_1| \cos(\theta) \Rightarrow 8000 = |S| \cos(\theta) = |S|(0.8)$$

$$\Rightarrow |S_1| = \frac{8000}{0.8} = 10{,}000\ VA$$

$$\Rightarrow Q_1 = |S_1| \sin(\theta)$$

$\cos(\theta) = 0.8 \Rightarrow \theta = 36.87^\circ$
$\sin(36.87) = 0.6$

$$Q_1 = 10{,}000 * (0.6) = 6{,}000\ VAR$$

→ However, since L_1 has a leading power factor, then "Q" is -ve $\Rightarrow Q_1 = -6000\ VAR$.

*

$$\Rightarrow P_2 = |S_2| \cos(\theta) = 20{,}000 * (0.6) = 12{,}000\ W.$$

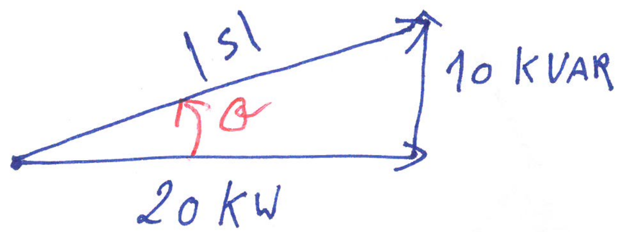
$$Q_2 = |S_2| \sin(\theta) = 20{,}000 * (0.8) = 16{,}000\ VAR.$$

→ since L_2 has a Lagging power factor, the "Q" is +ve $\Rightarrow Q_2 = 16{,}000\ VAR$.

$$\left.\begin{aligned} \Rightarrow S_1 &= 8000 - j6000\ VA \\ S_2 &= 12000 + j16000\ VA \end{aligned}\right\} \Rightarrow \boxed{\begin{aligned} S &= S_1 + S_2 \\ &= 20000 + j10000\ VA \end{aligned}}$$

a) Cont.:

* To find the over all power factor, we can use the power triangle:



$$\theta = \tan^{-1}\left(\frac{10,000}{20,000}\right) = \tan^{-1}(0.5) = 26.565^\circ$$

⇒ Thus, power factor = $\cos(26.565) = 0.8944$

- Since the over all reactive power is positive, $Q = 10,000$ VAR, then this is a Lagging power factor.

⇒ $\boxed{pf = 0.8944 \text{ lagging.}}$

b) We need to find the apparent power required to supply the loads, the magnitude of the current I_s, & the average power loss in transmission line.

* $|S| = |20,000 + j\,10,000| = \sqrt{(20k)^2 + (10k)^2}$

$= \boxed{22.36 \text{ KVA.}}$

b) Cont.:

* To find the current I_s, we can use the following equation:

$$S = S_1 + S_2 = (250) I_1^* + (250) I_2^*$$

$$= 250 \underbrace{(I_1^* + I_2^*)}_{\text{This summation is } I_s^*}$$

$$\Rightarrow I_s^* = \frac{S}{250\angle 0^\circ} = \frac{20{,}000 + j\,10{,}000}{250\angle 0^\circ}$$

$$\Rightarrow I_s^* = 80 + j40 \text{ A}$$

$$\Rightarrow \boxed{I_s = 80 - j40 \text{ A}}$$

* To find average power loss in the transmission line:

$$P = |I_s|^2 R$$

We use "R" only because we want the average power (real power)

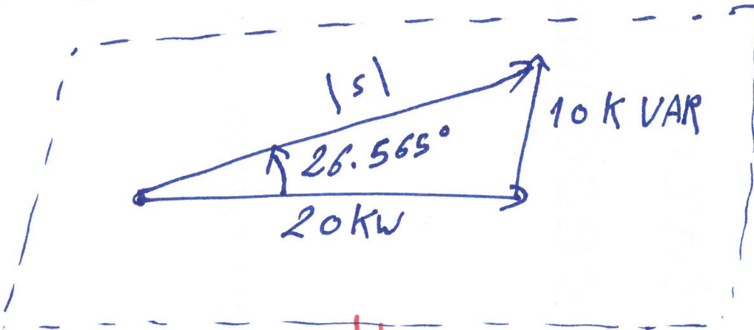
$$|I_s| = |80 - j40| = \sqrt{80^2 + 40^2} = 89.44 \text{ A}$$

$$\Rightarrow \boxed{P = (89.44)^2 (0.05) = 400 \text{ W.}}$$

c) We are asked to find the value of a capacitor that if added in parallel to the loads L_1 & L_2, will correct the power factor to "1".

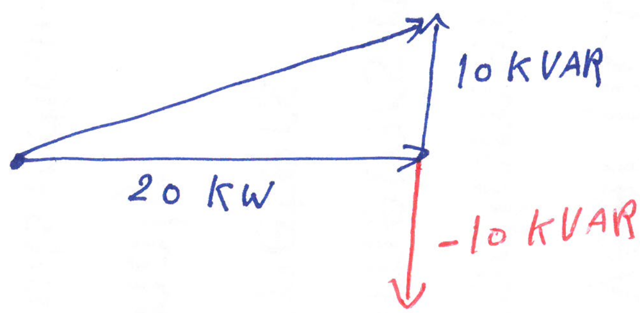
$$pf = 1 = \cos(\theta) \Rightarrow \theta = 0^\circ.$$

$\Rightarrow$ Thus, we want the angle θ in the power triangle to be 0°.



← This is the original power triangle befor adding the capacitor.

⊛ To change "θ" from 26.565° to "0", then the capacitor must provide −10 K VAR



This will make $\theta = 0^\circ$.

$\Rightarrow$ Thus, Q for the capacitor = −10 K VAR.

$$Q = \frac{|V_{eff}|^2}{\underbrace{X}_{\text{Reactance}}} \Rightarrow X = \frac{|V_{eff}|^2}{Q} = \frac{(250)^2}{-10K} = -6.25\ \Omega$$

c) Cont.:

* We are given that the frequency = 60 Hz

$$\Rightarrow \omega = 2\pi f = 2\pi(60) = 376.99 \text{ rad/sec.}$$

$$X = \frac{-1}{\omega C} \Rightarrow C = \frac{-1}{\omega X} = \frac{-1}{(376.99)(-6.25)}$$

$$\Rightarrow C = 424.4 \ \mu F$$

$$\boxed{= 424.2 \times 10^{-6} \ F}$$