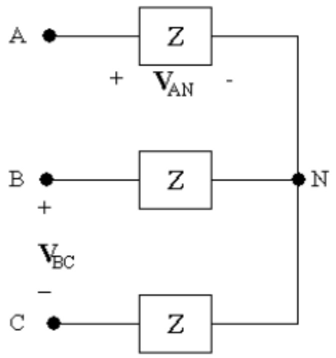
Circuits II

Ch11 Additional Problems Solution

AP 11.1 Make a sketch:



We know $\mathbf{V}_{AN}$ and wish to find $\mathbf{V}_{BC}$. To do this, write a KVL equation to find $\mathbf{V}_{AB}$, and use the known phase angle relationship between $\mathbf{V}_{AB}$ and $\mathbf{V}_{BC}$ to find $\mathbf{V}_{BC}$.

$$\mathbf{V}_{AB} = \mathbf{V}_{AN} + \mathbf{V}_{NB} = \mathbf{V}_{AN} - \mathbf{V}_{BN}$$

Since $\mathbf{V}_{AN}$, $\mathbf{V}_{BN}$, and $\mathbf{V}_{CN}$ form a balanced set, and $\mathbf{V}_{AN} = 240\angle -30^\circ$V, and the phase sequence is positive,

$$\mathbf{V}_{BN} = |\mathbf{V}_{AN}|\angle\left(\angle\mathbf{V}_{AN} - 120^\circ\right) = 240\angle\left(-30^\circ - 120^\circ\right) = 240\angle -150^\circ \text{ V}$$

Then,

$$\mathbf{V}_{AB} = \mathbf{V}_{AN} - \mathbf{V}_{BN} = (240\angle -30^\circ) - (240\angle -150^\circ) = 415.46\angle 0^\circ \text{ V}$$

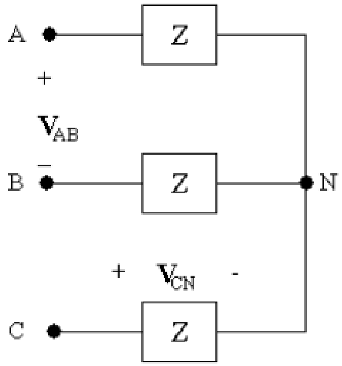
Since $\mathbf{V}_{AB}$, $\mathbf{V}_{BC}$, and $\mathbf{V}_{CA}$ form a balanced set with a positive phase sequence, we can find $\mathbf{V}_{BC}$ from $\mathbf{V}_{AB}$:

$$\mathbf{V}_{BC} = |\mathbf{V}_{AB}|\angle\left(\angle\mathbf{V}_{AB} - 120^\circ\right) = 415.69\angle\left(0^\circ - 120^\circ\right) = 415.69\angle -120^\circ \text{ V}$$

Thus,

$$\mathbf{V}_{BC} = 415.69\angle -120^\circ \text{ V}$$

AP 11.2 Make a sketch:



We know $\mathbf{V}_{\rm CN}$ and wish to find $\mathbf{V}_{\rm AB}$. To do this, write a KVL equation to find $\mathbf{V}_{\rm BC}$, and use the known phase angle relationship between $\mathbf{V}_{\rm AB}$ and $\mathbf{V}_{\rm BC}$ to find $\mathbf{V}_{\rm AB}$.

$$\mathbf{V}_{\rm BC} = \mathbf{V}_{\rm BN} + \mathbf{V}_{\rm NC} = \mathbf{V}_{\rm BN} - \mathbf{V}_{\rm CN}$$

Since $\mathbf{V}_{\rm AN}$, $\mathbf{V}_{\rm BN}$, and $\mathbf{V}_{\rm CN}$ form a balanced set, and $\mathbf{V}_{\rm CN} = 450\underline{/-25^\circ}$ V, and the phase sequence is negative,

$$\mathbf{V}_{\rm BN} = |\mathbf{V}_{\rm CN}|\underline{/\angle\mathbf{V}_{\rm CN} - 120^\circ} = 450\underline{/-23^\circ - 120^\circ} = 450\underline{/-145^\circ}\text{ V}$$

Then,

$$\mathbf{V}_{\rm BC} = \mathbf{V}_{\rm BN} - \mathbf{V}_{\rm CN} = (450\underline{/-145^\circ}) - (450\underline{/-25^\circ}) = 779.42\underline{/-175^\circ}\text{ V}$$

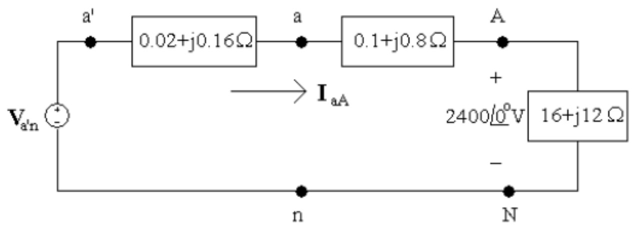
Since $\mathbf{V}_{\rm AB}$, $\mathbf{V}_{\rm BC}$, and $\mathbf{V}_{\rm CA}$ form a balanced set with a negative phase sequence, we can find $\mathbf{V}_{\rm AB}$ from $\mathbf{V}_{\rm BC}$:

$$\mathbf{V}_{\rm AB} = |\mathbf{V}_{\rm BC}|\underline{/\angle\mathbf{V}_{\rm BC} - 120^\circ} = 779.42\underline{/-295^\circ}\text{ V}$$

But we normally want phase angle values between $+180^\circ$ and -180°. We add 360° to the phase angle computed above. Thus,

$$\mathbf{V}_{\rm AB} = 779.42\underline{/65^\circ}\text{ V}$$

AP 11.3 Sketch the a-phase circuit:



[a] We can find the line current using Ohm's law, since the a-phase line current is the current in the a-phase load. Then we can use the fact that $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{cC}$ form a balanced set to find the remaining line currents. Note that since we were not given any phase angles in the problem statement, we can assume that the phase voltage given, $\mathbf{V}_{AN}$, has a phase angle of 0°.

$$2400\underline{/0^\circ} = \mathbf{I}_{aA}(16 + j12)$$

so

$$\mathbf{I}_{aA} = \frac{2400\underline{/0^\circ}}{16 + j12} = 96 - j72 = 120\underline{/-36.87^\circ}\,\text{A}$$

With an acb phase sequence,

$$\underline{/\mathbf{I}_{bB}} = \underline{/\mathbf{I}_{aA}} + 120^\circ \quad \text{and} \quad \underline{/\mathbf{I}_{cC}} = \underline{/\mathbf{I}_{aA}} - 120^\circ$$

so

$$\mathbf{I}_{aA} = 120\underline{/-36.87^\circ}\,\text{A}$$

$$\mathbf{I}_{bB} = 120\underline{/83.13^\circ}\,\text{A}$$

$$\mathbf{I}_{cC} = 120\underline{/-156.87^\circ}\,\text{A}$$

[b] The line voltages at the source are $\mathbf{V}_{ab}$ $\mathbf{V}_{bc}$, and $\mathbf{V}_{ca}$. They form a balanced set. To find $\mathbf{V}_{ab}$, use the a-phase circuit to find $\mathbf{V}_{AN}$, and use the relationship between phase voltages and line voltages for a y-connection (see Fig. 11.9[b]). From the a-phase circuit, use KVL:

$$\mathbf{V}_{an} = \mathbf{V}_{aA} + \mathbf{V}_{AN} = (0.1 + j0.8)\mathbf{I}_{aA} + 2400\underline{/0^\circ}$$

$$= (0.1 + j0.8)(96 - j72) + 2400\underline{/0^\circ} = 2467.2 + j69.6$$

$$2468.18\underline{/1.62^\circ}\text{ V}$$

From Fig. 11.9(b),

$$\mathbf{V}_{ab} = \mathbf{V}_{an}(\sqrt{3}\underline{/-30^\circ}) = 4275.02\underline{/-28.38^\circ}\text{ V}$$

With an acb phase sequence,

$$\underline{/\mathbf{V}_{bc}} = \underline{/\mathbf{V}_{ab}} + 120^\circ \quad \text{and} \quad \underline{/\mathbf{V}_{ca}} = \underline{/\mathbf{V}_{ab}} - 120^\circ$$

so

$$\mathbf{V}_{ab} = 4275.02\underline{/-28.38^\circ}\text{ V}$$

$$\mathbf{V}_{bc} = 4275.02\underline{/91.62^\circ}\text{ V}$$

$$\mathbf{V}_{ca} = 4275.02\underline{/-148.38^\circ}\text{ V}$$

[c] Using KVL on the a-phase circuit

$$\mathbf{V}_{a'n} = \mathbf{V}_{a'a} + \mathbf{V}_{an} = (0.2 + j0.16)\mathbf{I}_{aA} + \mathbf{V}_{an}$$

$$= (0.02 + j0.16)(96 - j72) + (2467.2 + j69.9)$$

$$= 2480.64 + j83.52 = 2482.05\underline{/1.93^\circ}\text{ V}$$

With an acb phase sequence,

$$\underline{/\mathbf{V}_{b'n}} = \underline{/\mathbf{V}_{a'n}} + 120^\circ \quad \text{and} \quad \underline{/\mathbf{V}_{c'n}} = \underline{/\mathbf{V}_{a'n}} - 120^\circ$$

so

$$\mathbf{V}_{a'n} = 2482.05\underline{/1.93^\circ}\text{ V}$$

$$\mathbf{V}_{b'n} = 2482.05\underline{/121.93^\circ}\text{ V}$$

$$\mathbf{V}_{c'n} = 2482.05\underline{/-118.07^\circ}\text{ V}$$

AP 11.4

$$\mathbf{I}_{\mathrm{cC}} = (\sqrt{3}\underline{/-30^\circ})\mathbf{I}_{\mathrm{CA}} = (\sqrt{3}\underline{/-30^\circ}) \cdot 8\underline{/-15^\circ} = 13.86\underline{/-45^\circ}\,\mathrm{A}$$

AP 11.5

$$\mathbf{I}_{\mathrm{aA}} = 12\underline{/(65^\circ - 120^\circ)} = 12\underline{/-55^\circ}$$

$$\mathbf{I}_{\mathrm{AB}} = \left[\left(\frac{1}{\sqrt{3}}\right)\underline{/-30^\circ}\right]\mathbf{I}_{\mathrm{aA}} = \left(\frac{\underline{/-30^\circ}}{\sqrt{3}}\right) \cdot 12\underline{/-55^\circ}$$

$$= 6.93\underline{/-85^\circ}\,\mathrm{A}$$

AP 11.7

$$\mathbf{I}_\phi = \frac{110}{3.667} + \frac{110}{j2.75} = 30 - j40 = 50\underline{/-53.13^\circ}\,\mathrm{A}$$

Therefore $|\mathbf{I}_{\mathrm{aA}}| = \sqrt{3}\mathbf{I}_\phi = \sqrt{3}(50) = 86.60\,\mathrm{A}$

AP 11.9 [a] $\mathbf{V}_{\mathrm{AN}} = \left(\frac{2450}{\sqrt{3}}\right)\underline{/0^\circ}\,\mathrm{V};\qquad \mathbf{V}_{\mathrm{AN}}\mathbf{I}_{\mathrm{aA}}^* = S_\phi = 144 + j192\,\mathrm{kVA}$

Therefore

$$\mathbf{I}_{\mathrm{aA}}^* = \frac{(144 + j192)1000}{2450/\sqrt{3}} = (101.8 + j135.7)\,\mathrm{A}$$

$$\mathbf{I}_{\mathrm{aA}} = 101.8 - j135.7 = 169.67\underline{/-53.13^\circ}\,\mathrm{A}$$

$$|\mathbf{I}_{\mathrm{aA}}| = 169.67\,\mathrm{A}$$

[b] $P = \frac{(2450)^2}{R};\qquad \text{therefore}\quad R = \frac{(2450)^2}{144{,}000} = 41.68\,\Omega$

$$Q = \frac{(2450)^2}{X};\qquad \text{therefore}\quad X = \frac{(2450)^2}{192{,}000} = 31.26\,\Omega$$

[c] $Z_\phi = \frac{\mathbf{V}_{\mathrm{AN}}}{\mathbf{I}_{\mathrm{aA}}} = \frac{2450/\sqrt{3}}{169.67\underline{/-53.13^\circ}} = 8.34\underline{/53.13^\circ} = (5 + j6.67)\,\Omega$

$$\therefore\quad R = 5\,\Omega,\qquad X = 6.67\,\Omega$$