Underdamped Response

Parallel RLC Underdamped Natural Response

$$v(t) = B_1 e^{-\alpha t} \cos(w_d t) + B_2 e^{-\alpha t} \sin(w_d t)$$

* To determine B_1 & B_2, we use the initial conditions for the circuit.

1) $v(0^+) = B_1 e^{-\alpha(0)} \cos(w_d(0)) + B e^{-\alpha(0)} \sin(w_d(0))$

$\to 1$, $\to 1$, $\to 1$, $\to \emptyset$

$$\Rightarrow \boxed{v(0^+) = B_1}$$

2) $$\frac{dv(0^+)}{dt} = B_1\left[e^{-\alpha t}(-w_d \sin(w_d t)) + \cos(w_d t)(-\alpha)e^{-\alpha t}\right] + B_2\left[e^{-\alpha t}(w_d \cos(w_d t)) + \sin(w_d t)(-\alpha)e^{-\alpha t}\right]$$

* using the product & chain rule;

$\Rightarrow (\cos(ax))' = -a\sin(ax)$

$\Rightarrow (\sin(ax))' = a\cos(ax)$

$\Rightarrow (x \cdot y)' = x'y + y'x$

$$\Rightarrow \frac{dv(0^+)}{dt} = B_1\left[\cancelto{1}{e^{(0)}}(-w_d \cancelto{\emptyset}{\sin(0)}) + \cancelto{1}{\cos(0)}(-\alpha)\cancelto{1}{e^{(0)}}\right] + B_2\left[\cancelto{1}{e^{(0)}}(w_d \cancelto{1}{\cos(0)}) + \cancelto{\emptyset}{\sin(0)}(-\alpha)\cancelto{1}{e^{0}}\right] = \boxed{-\alpha B_1 + w_d B_2}$$