Name: Program: EE / CE 10/02/2019

EEL 3112C – Circuits-II

Fall 2019 First Exam

Important notes:

- Write your **Full Name** on all pages
- Time limit is 50 minutes
- The exam is closed book and closed notes
- You are allowed to use a calculator
- **Always JUSTIFY your final answer and show relevant intermediate steps**
- Your answers must be organized and easy to follow
- If I cannot read or follow your answer, I will not grade it
- When answering different parts of a problem, please label them
- Always mention the unit for any physical quantity you find

Problem Number	Grade	Full Grade
Problem #1		40
Problem #2		30
Problem #3		30
Total		**100**

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Problem 1: (40 points)

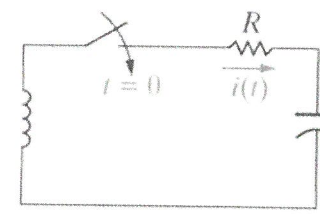
For the circuit shown below, the initial voltage across the 500 mH inductor is 15 V. The capacitor (C) value is 40 nF, and the resistor (R) equals 10 $k\Omega$. Find the following:

a) Find the roots of the characteristics equation that describes the natural behavior of the current i (i.e. find s_1 & s_2)? (10 points)

b) What type of damping describes the system and why? (10 points)

c) Find the numerical values for $i(0)$ and $di(0)/dt$ immediately after the switch has been closed. (10 points)

d) $i(t)$ for $t \geq 0$ (10 points)



Problem 1:

For the series R, L, C circuit, we need to find:

a) S_1 & S_2:

$$S_1 = -\alpha + \sqrt{\alpha^2 - W_0^2} \Rightarrow \alpha = \frac{R}{2L} = \frac{10\times10^3}{0.5} = 20{,}000 \text{ rad/s.}$$

$$S_2 = -\alpha - \sqrt{\alpha^2 - W_0^2}$$

$$W_0^2 = \frac{1}{LC} = \frac{1}{0.5\,(40\times10^{-9})} = 50\times10^6$$

$$\Rightarrow S_1 = -20{,}000 + \sqrt{(20{,}000)^2 - (50\times10^6)} = -1291.7$$

$$\Rightarrow S_2 = -20{,}000 - \sqrt{(20{,}000)^2 - (50\times10^6)} = -38708.3$$

b) $W_0^2 < \alpha^2 \Rightarrow$ The system is overdamped.

c) $i(0) = \emptyset.$

$$\frac{di(0)}{dt} = \frac{V_c(0)}{L} \longrightarrow$$ @ $t=\emptyset$, no current is passing throug the circuit, & $V_c(0) = V_L(0) = 15$

$$\Rightarrow \frac{di(0)}{dt} = \frac{15}{0.5} = 30 \text{ A/s.}$$

d) $$i_1(t) = A_1 e^{-1291.7t} + A_2 e^{-38708.3t}$$

$$i(0) = A_1 + A_2 = \emptyset, \quad \frac{di(0)}{dt} = -1291.7A_1 - 38708.3A_2 = 30$$

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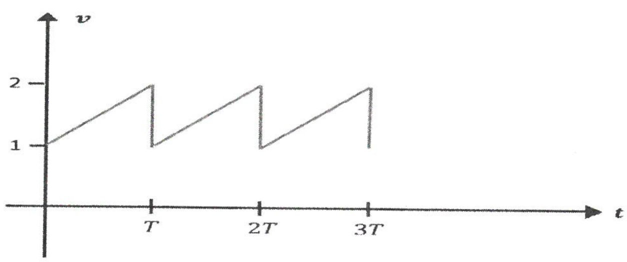
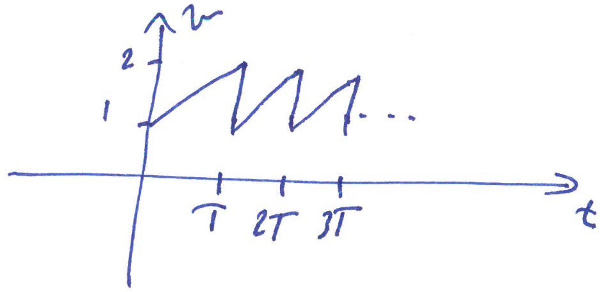
Problem 2: (30 points)

Derive the expression for the RMS value of the following signal:



⊛ Problem 2:

We need to find the RMS value of the signal given in the plot.



⊛ The period of this signal is T.

⊛ The slope of this signal is : $m = \frac{2-1}{T-0} = \frac{1}{T}$

⊛ The equation of the line between 0 & T is: $v = \frac{1}{T}t + 1$

$\cancel{y} = mx + b$

⊛ Now, we have all what we need to find the RMS:

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T \underbrace{\left(\frac{1}{T}t+1\right)^2}\, dt}$$

$$v^2(t) = \left(\frac{1}{T}t+1\right)^2 = \frac{1}{T^2}t^2 + \frac{2t}{T} + 1$$

$$\Rightarrow \int_0^T v^2(t)\,dt = \int_0^T \frac{1}{T^2}t^2 dt + \int_0^T \frac{2}{T}t\,dt + \int_0^T 1\,dt$$

$$= \frac{1}{T^2}\left[\frac{T^3}{3}\right] + \frac{2}{T}\frac{T^2}{2} + T$$

$$= \frac{T}{3} + T + T = \frac{7T}{3}$$

$$\Rightarrow V_{rms} = \sqrt{\frac{1}{\cancel{T}}\left[\frac{7\cancel{T}}{3}\right]} = \boxed{\sqrt{\frac{7}{3}}} \ \#.$$

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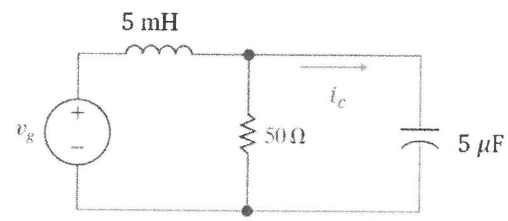
Problem 3: (30 points)

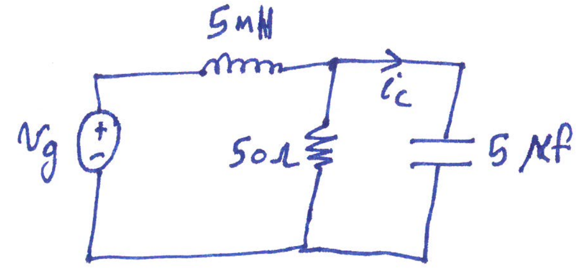
The circuit shown below is operating in the sinusoidal steady state. Find the steady-state expression for $i_c(t)$ if $v_g = 50\sin(2000t)\ V$.



Problem 3:

- We need to find $i_c(t)$ for the given circuit given that

$v_g(t) = 50 \sin(2000t)$ V.



- First, we need to construct the frequency domain equivalent model:

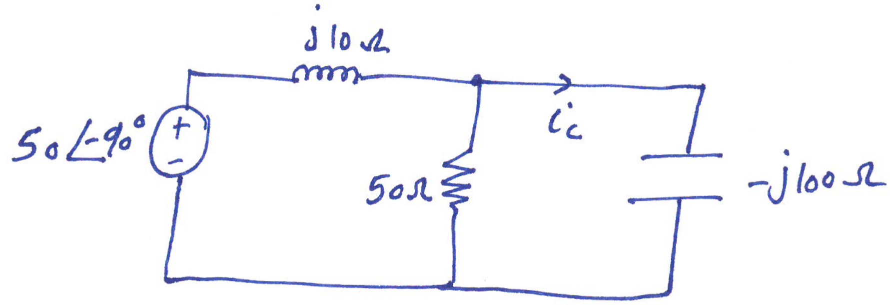
- For the capacitor: $\dfrac{1}{j\omega C} = \dfrac{1}{j(2000)(5\times10^{-6})} = \dfrac{1}{j(10^{-2})} = -j100$

- For the inductor: $j\omega L = j(2000)(5\times10^{-3}) = j10$

- $V_g = 50\angle{-90^\circ}$ V. ← note that we had to change the expression for $v_g(t)$ from sin to cos

$\Rightarrow v_g(t) = 50 \cos(2000t - 90)$ V.

$\Rightarrow$

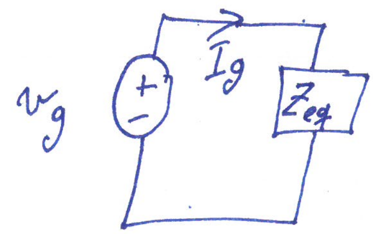


- To find i_c, we need to find the main current in the circuit & then use current divider.

Problem 3 - Cont.:

- To find the main current, we can simplefy the circuit as follows:

$$Z_{eq} = j10 + (50 \,||\, -j100)$$



$$= j10 + \left(\frac{-j5000}{50 - j100}\right) = j10 + \left(\frac{5000\angle -90}{111.8\angle -63.4}\right)$$

$$= j10 + 44.7\angle -26.6$$

$$\Rightarrow Z_{eq} = j10 + 39.97 - j20 = 39.97 - j10$$

$$\Rightarrow I_g = \frac{V_g}{Z_{eq}} = \frac{50\angle -90}{41.2\angle -14.04} = 1.21\angle -75.96 \text{ A.}$$

- now, to find i_c we can use current divider:

$$I_c = I_g \times \left(\frac{50}{50 - j100}\right) = I_g \times \left[\frac{50\angle 0^\circ}{111.8\angle -63.4}\right]$$

$$I_c = I_g \left[0.4472\angle 63.4\right] = (1.21\angle -75.96)(0.4472\angle 63.4)$$

$$\Rightarrow I_c = 0.5411\angle -12.56 \Rightarrow \boxed{i_c(t) = 0.5411 \cos(2000t - 12.56) \text{ A.}}$$

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