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COSC 527
Project 2

For this project, a two-dimensional activation-inhibition cellular automaton (AICA) was coded in Python 3 (code can be in html and ipynb files included in the tar-ball with this report). The AICA only depends on five parameters: activation coefficient ($J1$), inhibition coefficient ($J2$), activation radius ($R1$), outer inhibition radius ($R2$), and a systemic bias (h). The goal of this project is too understand the impact these parameters have on the AICA behavior in a systematic way. To help with this, statistical properties, such as the spatial correlation and mutual information, are used. To cancel out noise of using random initial states, all the relevant quantities are averaged over three independent runs for the same parameters.

Experiment 3
Experiment 3 analyzes the behavior of the AICA for $J1 = 1.0, J2 = -0.1$. Figures 1, 2, and 3 illustrate how the mutual information and spatial correlation depend on increasing a negative bias. Consistently, increasing the bias shifts the curves upwards, meaning a bias closer to zero (from the negative side) results in more mutual information and spatial correlation at a given separation between cells. Looking at Figure 4, it's apparent that this translates to the white structures becoming bigger and more connected, as follows intuitively by increasing the correlations. Moreover, Figure 6 includes positive bias values in the experiment 3 parameter set, illustrating that it is actually the magnitude of bias that is important. Though a positive and negative values are not symmetric, this is expected because J1 and J2 (as well as the areas the cover) are not exactly symmetric.

Comparing Figures 1 and 2 directly (and correspondingly figures 4 and 5) provides insight into the differences due to changing $R1$ only (from 1 to 3). Increasing the activation radius for a given bias and outer radius shifts the peaks mutual information and spatial correlation curves to higher length. The first minimum for the mutual information is directly dependent on R1 (i.e., the mutual information decreases until $l = R1$). However, it's clear that there is a non-linear dependence here, where the degree to which changing the bias/activation radius depends on some ratio between R1, R2, and h.

Finally, figure 7 looks at a fixed R1 and bias with variable R2. The mutual information and spatial correlation (to bit of a lesser extent) show little sensitivity at low l to $R2$, reinforcing the findings of the previous paragraph that the qualitative behavior for lengths below and near $R1$ is (intuitively) determined by $R1$. Similarly, figure 7 confirms the intuition that $R2$ most significantly impacts the behavior at long distances. This is perhaps further supported by the characteristic correlation length (see Table 1 and 2) as λ doesn't change too much for chaning R2 (meaning similar initial decay phase).

The correlation length, λ, may also help illustrate the compound dependences between bias and radii. According to table 1, λ can vary significantly from 1.33 to 3.0 and gives evidence that changes due to increasing bias are due to the value of the bias itself (and perhaps also relative to the R1 and R2). Moreover, the change in lambda is not symmetric about $h = 0.0$ in accordance with above. While the exponential model for spatial correlation length is obviously flawed, doing a more rigorous evaluation in the same vein may be beneficial.

Experiments 2 and 1

Experiment 2 sets $J1 = 0.0, J2 = -0.1$ and right away shows some qualitative differences from experiment 3, possessing a lot of regularity. Figures 9, 10, 11, 12 illustrate how turning off the bias for these runs drives the AICA into a periodic structure - losing the "organic" qualities seen in the structures of experiment 3. The mutual information and spatial correlation reflect this periodicity quite nicely (particularly the average mutual information in Figure 9). Figure 13 and Figure 14 illustrate that this periodicity seems to be quite general for experiment 2 (note figure 5 in experiment 3 had a periodic AICA so it's not limited to experiment 2).

Experiment 1, on the other hand, couldn't produce anything interesting in the 11 parameter sets I ran. At best, the AICA would just be divided into a couple of solid rectangles, though most of the time the AICA was solid black. This does facilitate a describing the experiments in terms of Wolfram Classes. Experiment 3 has potential to have class IV behavior (and II and III), where as Experiment 2 is dominated by class II behavior and experiment 1 is just class I. Hence, in this project we have done a lot to map out the phases transitions of the AICA, regarding Wolfram classes.

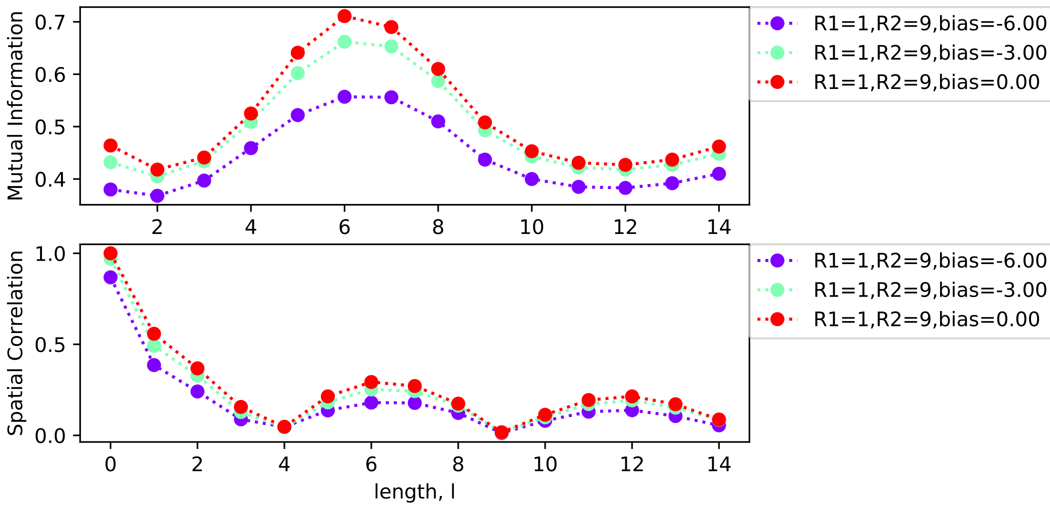

Figure 1: $J1 = 1.0, J2 = -0.1$. Comparing Mutual information and Spatial Correlation for fixed $R1, R2$

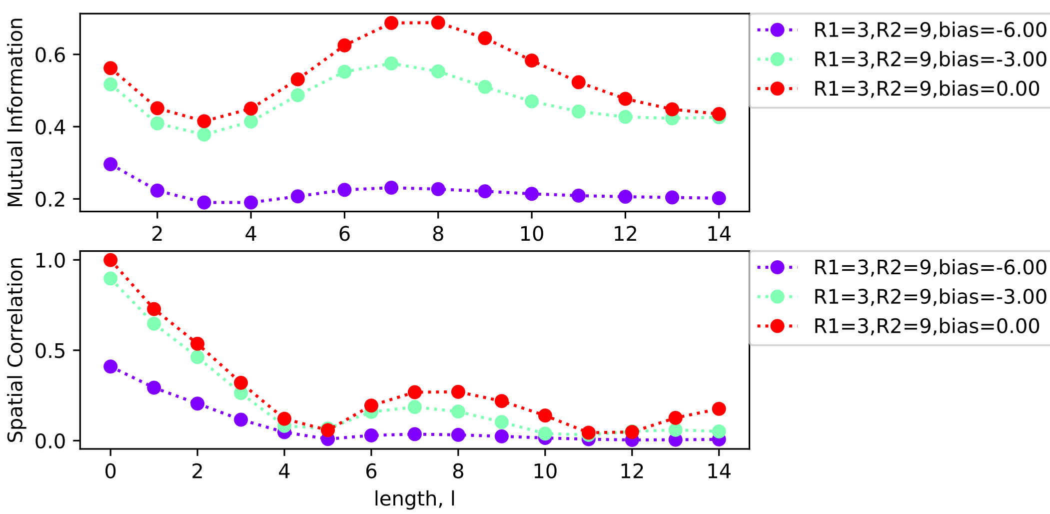

Figure 2: $J1 = 1.0, J2 = -0.1$. Comparing Mutual information and Spatial Correlation for fixed $R1, R2$

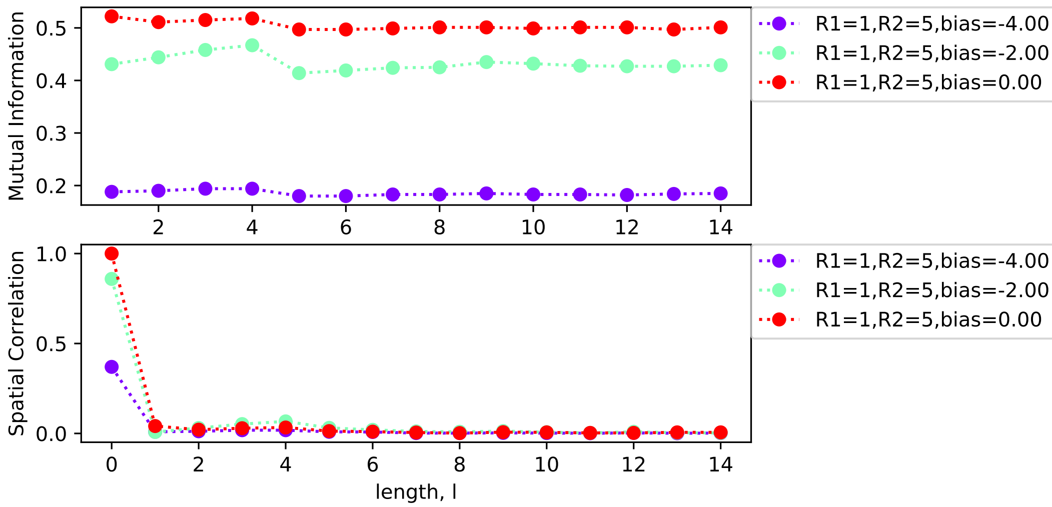

Figure 3: $J1 = 1.0, J2 = -0.1$. Comparing Mutual information and Spatial Correlation for fixed $R1, R2$

Bias	-6.0	-3.0	0.0	3.0	6.0
λ	2.67	3.0	3.0	2.0	1.33

Table 1: Correlation length varies with bias. For $R1 = 3, R2 = 9$

R2	3	9	14
λ	3.67	3.0	3.33

Table 2: Correlation length corresponding to different R2 values and $R1 = 3, h = 0.0$. See figure 7

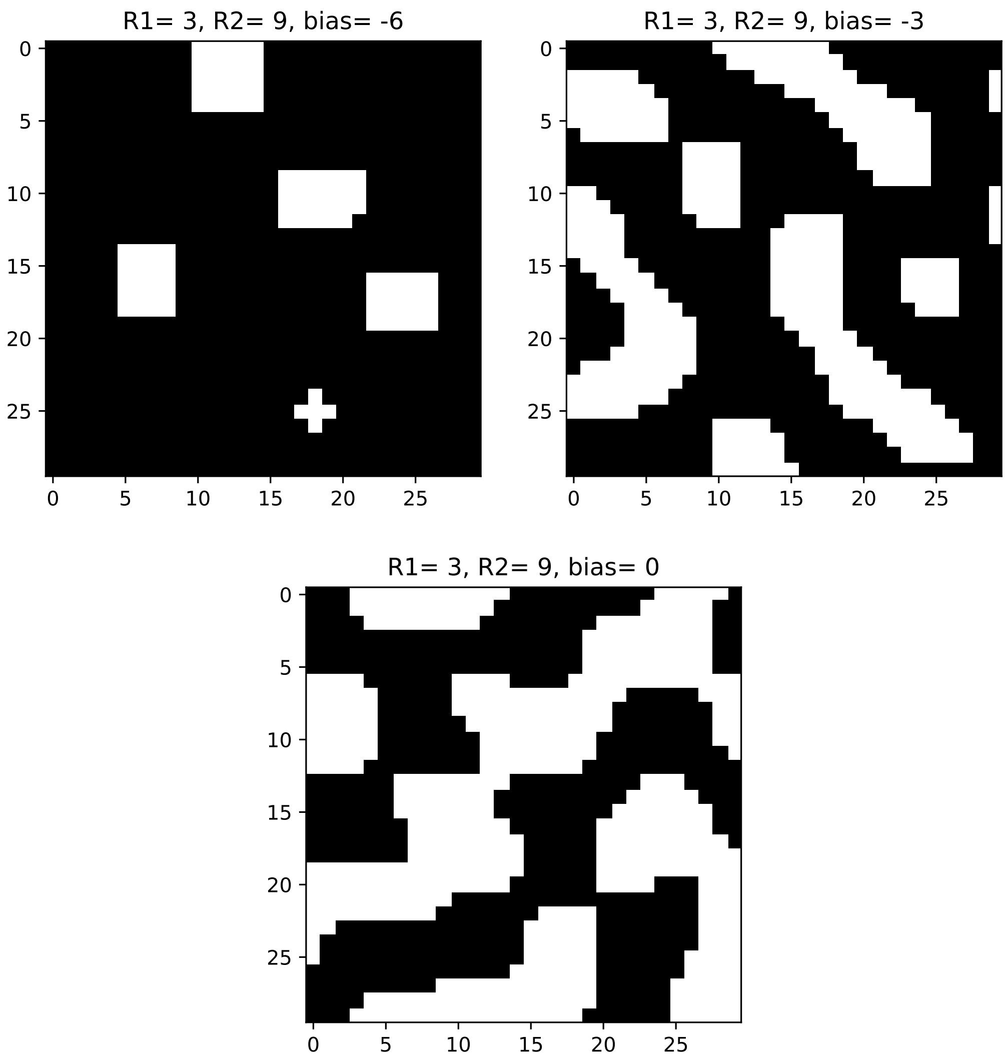

Figure 4: AICA for increasing bias, matches Figure 2 above

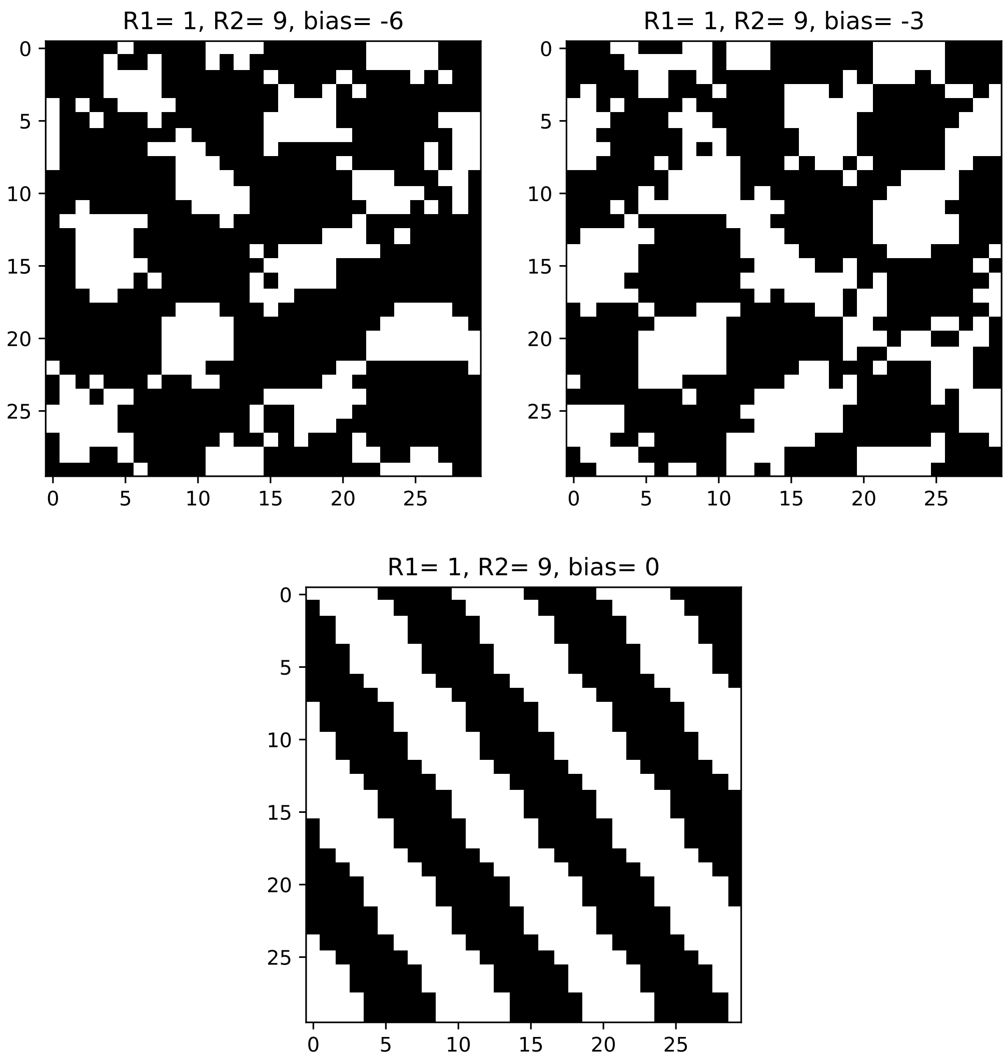

Figure 5: AICA for increasing bias, matches Figure 1 above

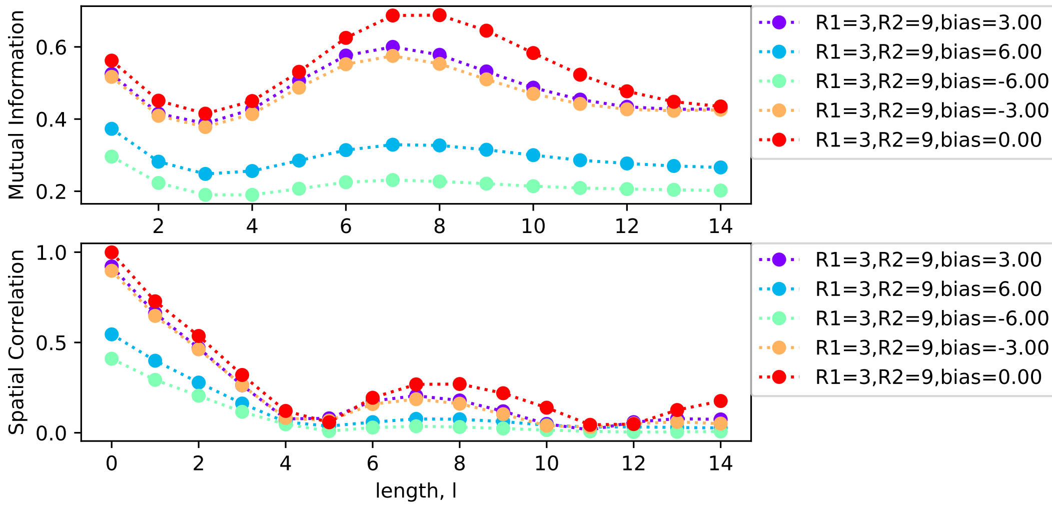

Figure 6: $J1 = 1.0, J2 = -0.1$. including positive values for bias for fixed $R1, R2$

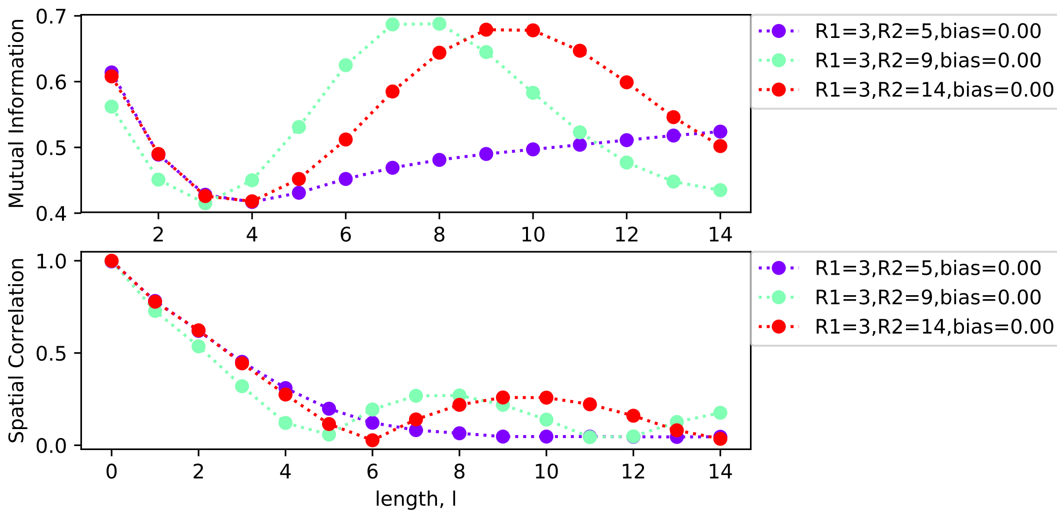

Figure 7: Looking at the impact R2 has on structure of AICA

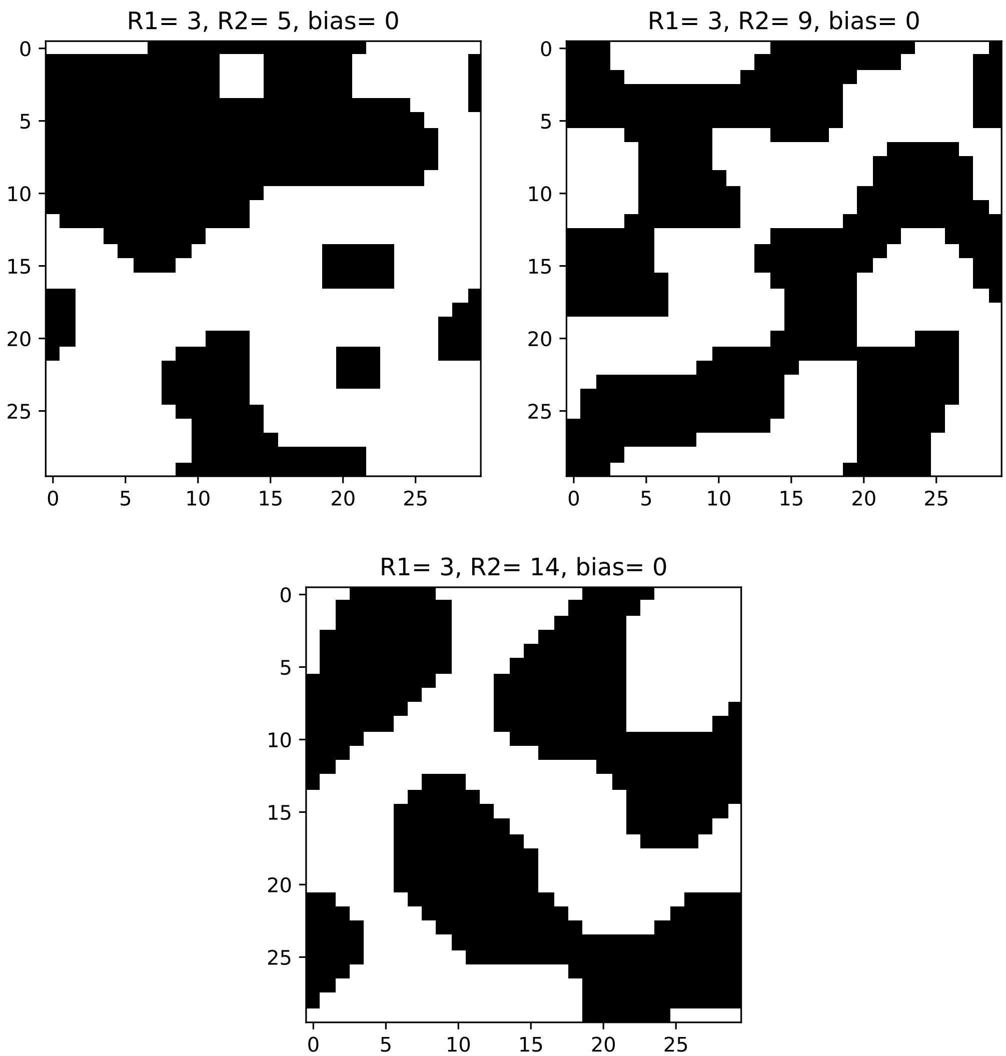

Figure 8: AICA for increasing R2, matches Figure 7 above

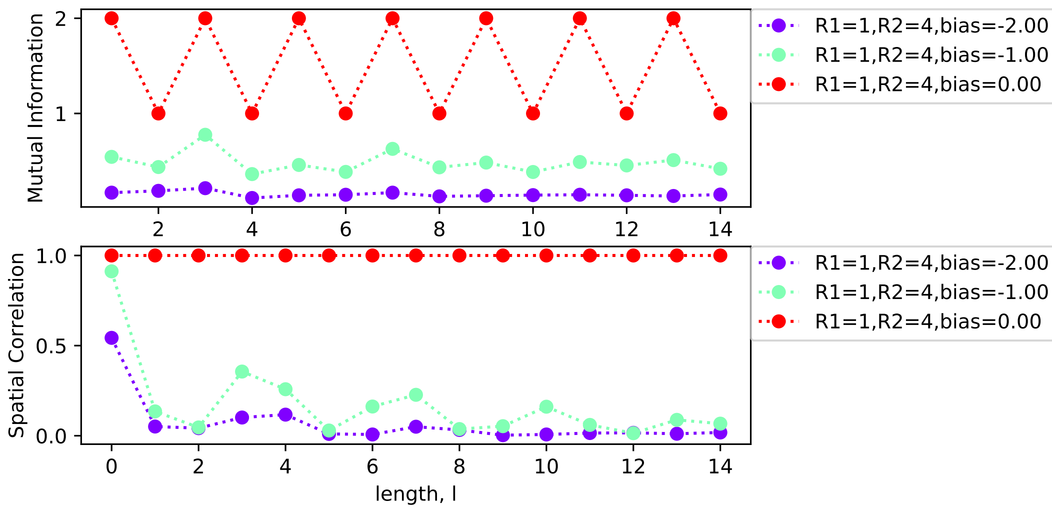

Figure 9: Changing bias for experiment 2

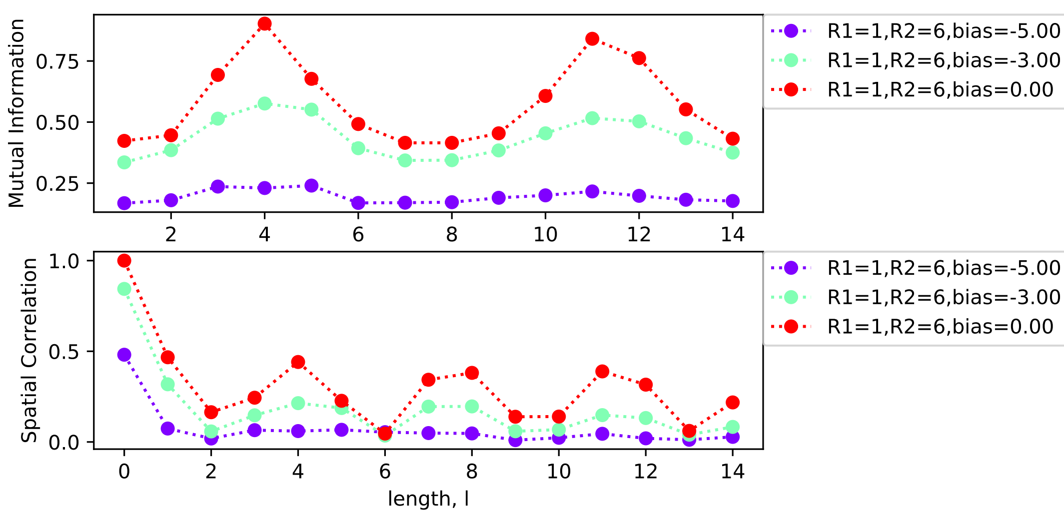

Figure 10: changing bias for experiment 2

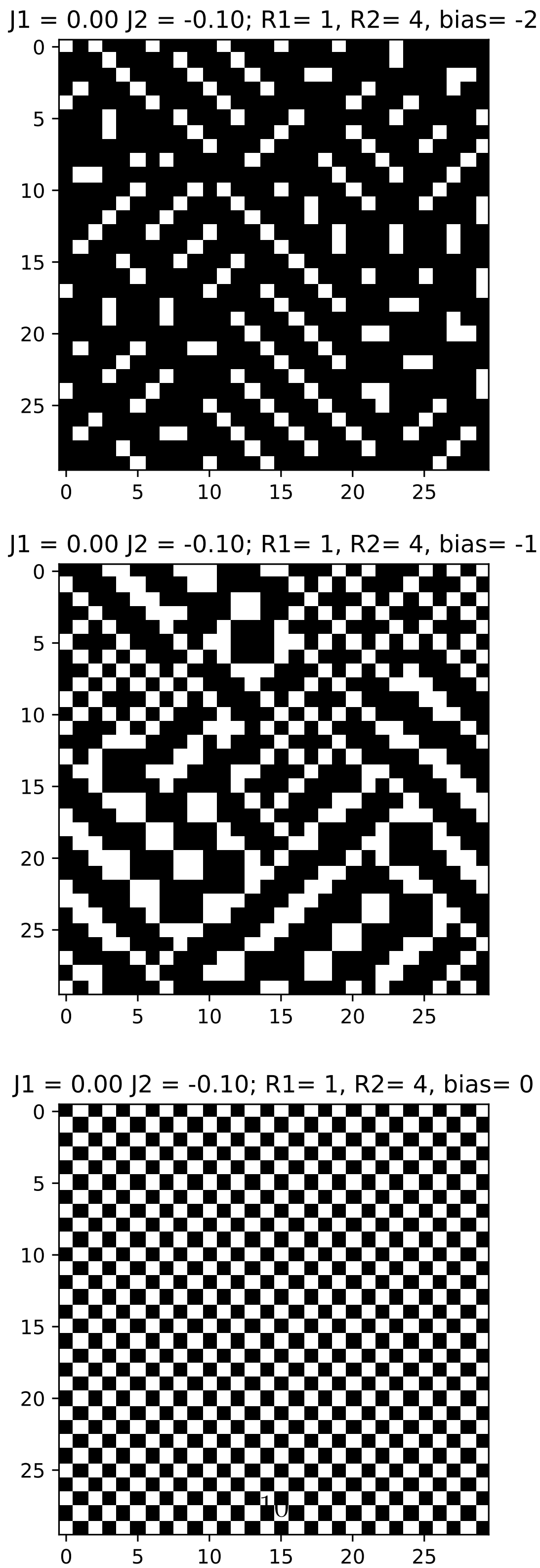

Figure 11: AICA from experiment 2, matches with Figure 9

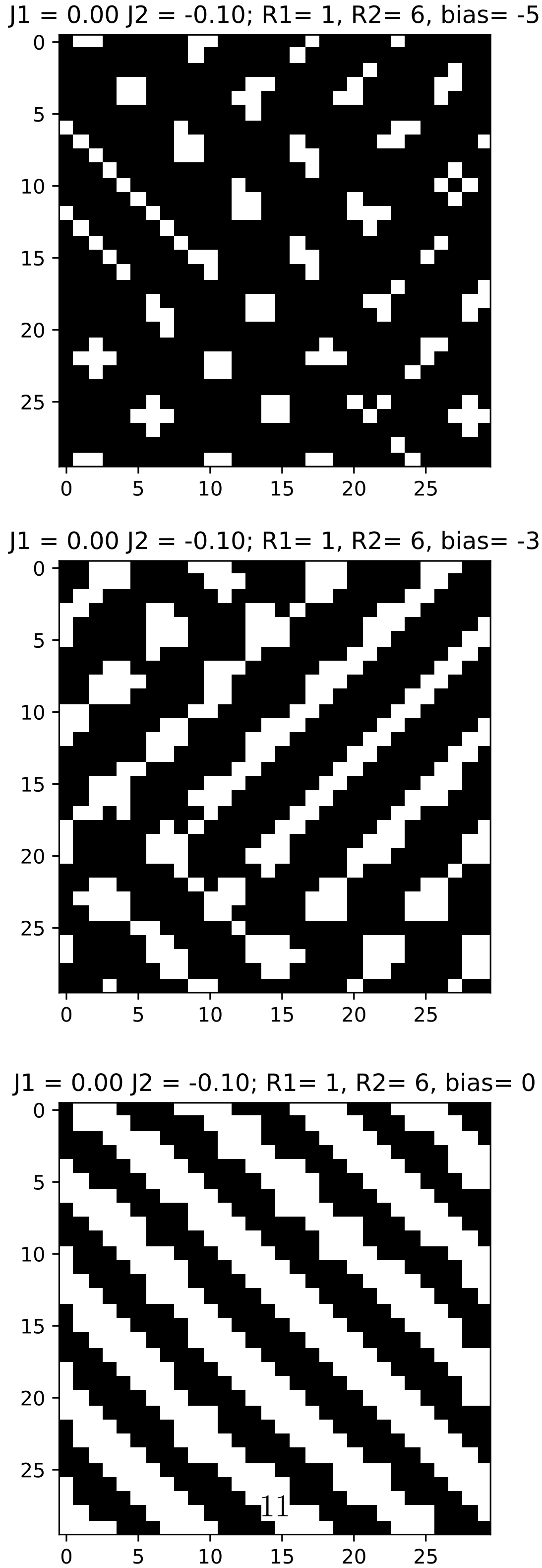

Figure 12: AICA from experiment 2, matches with Figure 10

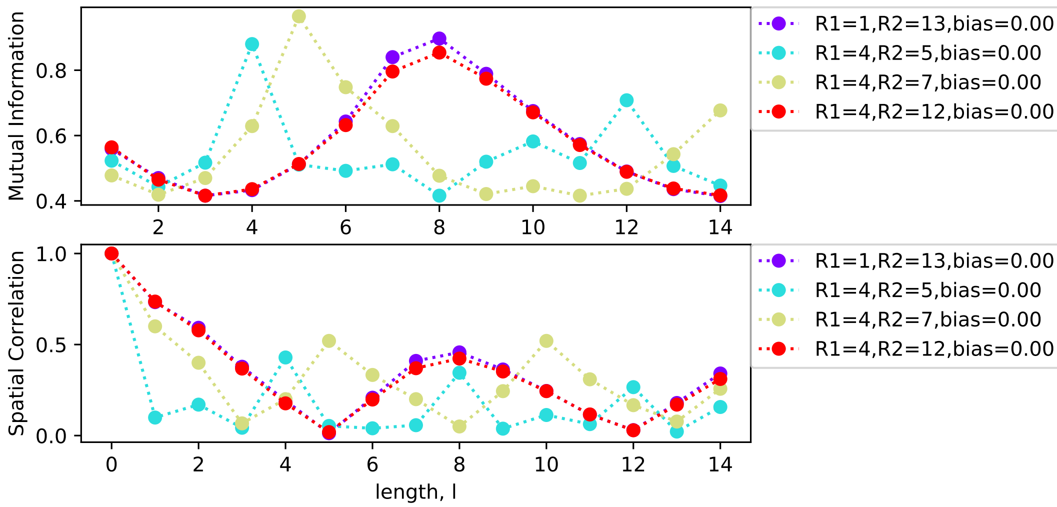

Figure 13: Other 0 bias runs

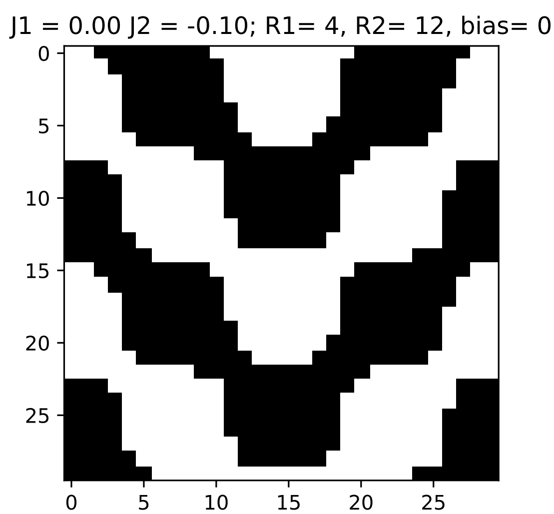

Figure 14: AICA from experiment 2, matches with Figure 13