

# **Bank Capitalization Heterogeneity and Monetary Policy \***

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### **Abstract**

This paper shows that heterogeneity in bank capitalization rates plays a crucial role in the transmission of monetary policy to bank lending. First, I offer new empirical evidence on the dependence of bank lending responses to monetary policy shocks on their capitalization rates. Highly-capitalized banks reduce their lending more after a monetary tightening, even after controlling for bank liquidity, size, and market power in the deposit market. I also document that highly capitalized banks have a riskier portfolio, as measured by loan charge-off rates, and default rates on their loans increase relatively more after a tightening in monetary policy. I then construct a dynamic macroeconomic model that rationalizes the empirical evidence through the interaction of heterogeneous recovery technologies of banks facing a risk-weighted capital constraint. In particular, after an increase in the policy rate, the model predicts that loan rates and default probabilities increase in both sectors. Higher-capitalized banks with a riskier portfolio are more sensitive because the risk-weighted capital constraint affects them more, so they contract lending more. In a counterfactual analysis, I find that higher capital requirements amplify the effects of monetary policy.

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## 1. Introduction

Many economic consequences of the Global Financial Crisis are associated with the worsening of banks' balance sheets. This reinvigorated the literature that stresses the central role of financial intermediaries in macroeconomic models. Apart from aggregate implications, the literature uncovered broad heterogeneity in the responses of financial institutions to changes in economic conditions. This paper contributes to this literature by studying the role of the bank capitalization rate in shaping the pass-through of monetary policy shocks to bank lending.

This is not the first paper that studies the cross-sectional implications of changes in interest rate for the lending of the banking sector. For example, [Kashyap and Stein \(1994\)](#) stress the importance of bank liquidity, while [Drechsler et al. \(2017\)](#) emphasize the role of bank market power in the deposit market. Closest to this paper in its aim is the work of [Van den Heuvel \(2012\)](#), which also links bank capitalization to sensitivity of bank lending. This papers study the relationship between the level of interest rates and the cross-sectional implications of bank lending; instead, I revisit these views with better data and careful identification of monetary policy shocks.

Specifically, I first provide three new empirical facts related to the response of bank lending to monetary policy shocks across banks with different capitalization rates, the response of default rates on loans with different riskiness to monetary policy shocks, and the portfolio composition of banks with different capitalization rates. I then rationalize these cross-sectional facts in a dynamic macroeconomic model with heterogeneous banks that face a risk-weighted asset (RWA) constraint. The model emphasizes the role the RWA constraint plays in shaping banks' portfolios and capitalization rates, and their response to the monetary policy shock.

In the empirical part of the paper, I combine data on monetary shocks, measured using the high-frequency event study approach, as proposed by [Gurkaynak \(2005\)](#) and [Gorodnichenko and Weber \(2016\)](#), with cross-sectional U.S. banking data sets known as "call reports". I also test other factors that previous empir-

ical papers determined to be important for the propagation of monetary policy through the banking sector. Here, are use the Herfindahl index of geographical concentration to measure bank market power in deposits, as explained by Drechsler et al. (2017).

First, I document that banks with higher capitalization rates reduce their lending more than less capitalized banks in response to monetary policy tightening. In particular, a bank with a capitalization rate one standard deviation above the mean of the capitalization rate distribution reduces lending by 0.75 percentage points more than a bank with that lies at the mean of the capitalization rate distribution. These results are robust to controlling for size, liquidity, and market power on deposits. The results are consistent across all types of loans (Commercial and Industrial (C&I), Real Estate, and Personal loans). The contraction in credit is not substituted with investment in other assets—I show that in response to a monetary tightening, better capitalized banks reduce their overall balance sheets more than their less capitalized counterparts.

In addition, I provide two pieces of empirical evidence that play a crucial role in supporting the economic mechanism underlying the above result. Specifically, I show that loan default rates, proxied with delinquency rates and charge-off rates, increase after a monetary policy shock. This result is also consistent across all types of loans. I also document the heterogeneity in the composition of bank loan portfolios. Portfolios of highly capitalized banks portfolios are more oriented toward C&I and personal loans, which are riskier, as measured by charge-off rates, compared with real estate loans.

In the second part of the paper I propose a theoretical mechanism consistent with empirical evidence described above. Consider banks which differ in their ability to recover debtors' assets after a loan default, and face a RWA constraint where the risk weights reflect the default risk but not the bank-specific recovery rates. Banks with better recovery technologies have a comparative advantage in lending to riskier borrowers and hence hold riskier loan portfolios. The RWA constraint then forces them to hold more capital against this loan risk.

A monetary policy tightening translates into increases in loan rates across dif-

ferent sectors. Because of this effect on rates as well as due to other general equilibrium effects, the default rate of each type of loan increases. This effect is stronger for riskier loans, which in the data correspond to C&I lending and personal loans, as opposed to safer real estate loans. Banks will then seek to reduce their exposure to riskier assets. Banks with better recovery technology, which are better capitalized in equilibrium, have their portfolios more heavily tilted toward riskier loans, and the RWA constraint forces them to contract lending more than their counterparts with worse recovery technology.

In the model, I treat the heterogeneity in banks' ability to recover assets from defaulting debtors as a bank-specific technological primitive. The RWA constraint is a policy primitive. In the main text, I present a stripped-down version of the model that highlights the theoretical mechanism, while preserving relevant quantitative aspects. The central bank directly controls the real rate at which deposits are supplied to the banking sector. There are two types of banks which differ in their recovery technologies, and these banks lend to two types of firms that differ in their riskiness.<sup>1</sup> Banks with the better recovery technology tilt their portfolios toward lending to riskier firms, and endogenously choose a higher capitalization rate due to the presence of the RWA constraint.

The model generates all three empirical relationships that I documented in the first part of the paper. The RWA constraint in the presence of differences in recovery technologies generates the positive association between bank capitalization and riskiness of banks' portfolios. After monetary policy tightening, loan default rates increase, the better capitalized banks holding riskier portfolios contract lending more in response. To assess the quantitative performance of the calibrated model, I study the model-implied bank-specific lending responses to a monetary policy shock as a function of bank's capitalization rate. In my baseline calibration I find that the model generates sensitivity in the lending response to the capitalization rate that is very close to the data, but not enough cross-sectional differences

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<sup>1</sup>A full general equilibrium model that features a riskier corporate and a safer mortgage sector, as well as a New-Keynesian structure linking a nominal policy rate to the real economy, is in the appendix.

in capitalization rates. This implies that not all the heterogeneity in capitalization rate is due to recoveries technologies because banks are heterogeneous in many different aspects which can generate differences in capitalization rate, I only focus in one dimension.

In addition, I use the model to conduct a policy experiment that analyzes the implications of bank regulation for the bank lending channel of monetary policy. The question is: what is the effect of higher capital requirements on the effectiveness of monetary policy? I find that an economy with higher capital requirements, the monetary policy shock has more adverse effects. Therefore, a monetary policy shock generates a higher reaction of the main economic variables.

**Literature.** This paper adds to three strands of literature. First, I contribute to the literature on how the effect of monetary policy varies across banks by showing that banks with higher capitalization rates contract their lending more than lower-capitalized banks after a monetary policy tightening. Studies such as [Kashyap and Stein \(1994\)](#), [Bernanke and Gertler \(1995\)](#), and [Kashyap and Stein \(2000\)](#), argue that banks with low liquidity in their balance sheets are more responsive to monetary policy, a mechanism that I denote the “liquidity view”. [Van den Heuvel \(2012\)](#) advocates a “bank capital view” of the transmission of monetary policy. He uses state-level data and argues that the effect of monetary policy is stronger in states where banks have a low capital-asset ratio. He finds that bank liquidity measures are not associated with variation in the impact of monetary policy on output at the state level monetary policy.<sup>2</sup> However, I finds the opposite association between capitalization rates and sensitivity of bank lending.

Recent empirical studies focus on how the transmission of monetary policy to households and the real economy depends on banks’ market power. A number of papers, including [Drechsler et al. \(2017\)](#), find empirical evidence of market power in the deposit market and show that monetary policy has a powerful impact on the price and quantity of deposits supplied by the banking system. Additionally, [Scharfstein and Sunderam \(2016\)](#) find evidence of market power in the loan mar-

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<sup>2</sup>This result implies that the bank lending channel is not operational; and uses a panel of state-level data to assess the bank capital channel.

ket, where higher market power leads to lower pass-through of secondary market rates to households, and lower refinancing activity in response to declining interest rates.

In this paper, I find a key role for heterogeneity in capitalization rates. After monetary tightening better capitalized banks reduce lending more, in contrast to [Van den Heuvel \(2012\)](#)'s results. When I simultaneously allow for different channels, I do not find the market power view to be statistically significant, and the liquidity channel is substantially weakened, while the capitalization rate continues to play an important role.

My results do not necessarily contradict these studies; I simply study different features of the data with a better identification of monetary policy shocks and better econometric specification in which I control for bank's characteristics, for bank fixed effects to capture permanent differences across banks, and for state-by-quarter effects to capture differences in how states respond to aggregate shocks. I promote to a more detailed discussion of the differences in my empirical approach where I reproduce the empirical specification.

Second, on the theoretical front, I contribute to the literature on how micro-level heterogeneity affects the understanding of monetary policy relative to traditional representative agent models in a real model and a New-Keynesian model. A growing strand of literature focuses on how household-level heterogeneity affects the consumption channel of monetary policy; see for example [Auclert \(2019\)](#), [Wong \(2019\)](#), [Kaplan et al. \(2018\)](#), and [McKay et al. \(2016\)](#). Another strand of the literature analyzes the role of firm-level heterogeneity in determining the investment channel of monetary policy; see for example [Ottonezzo and Winberry \(2020\)](#), and [Jeenas \(2019\)](#). In contrast, my paper analyzes the role of bank-level heterogeneity in determining the lending channel of monetary policy and explores bank heterogeneity in recovery rates on defaulting loans as a theoretical mechanism that affects the lending channel.

Finally, I contribute to the literature that embeds the banking sector in a general equilibrium macroeconomic model. To date, papers such as [Gertler and Kiyotaki \(2010\)](#), [Gertler and Karadi \(2011\)](#), [Wang \(2018\)](#), and [Arce et al. \(2019\)](#) assume

a representative bank in a standard New Keynesian DSGE to assess unconventional monetary policy. Other papers, such as [Balloch and Koby \(2019\)](#) or [Coimbra and Rey \(2020\)](#), develop a heterogeneous banking sector. [Coimbra and Rey \(2020\)](#) present a flexible-price model that introduces heterogeneity in the Value-at-Risk of financial intermediaries and assess monetary and financial stability jointly. [Balloch and Koby \(2019\)](#) focus on how low nominal rate environments affect bank credit supply. Their model assumes that banks have market power on deposits and there is a leverage constraint that limits lending. My heterogeneous bank model is based on the work by [Elenev et al. \(2020\)](#), who study the effect of tighter bank capital requirements in response to the Global Financial Crisis. I contribute to this literature in three ways. First, I incorporate heterogeneity in the banking sector, specifically for commercial banks. Second, I incorporate borrower heterogeneity across sectors, with a high-risk and a low-risk sector that differ in their equilibrium default rates due to differences in the volatility of their productivity shocks. Third, I use my framework to study how bank heterogeneity affects the transmission of monetary policy shocks.

**Outline.** The remainder of the paper is structured as follows. Section 1 presents data and empirical analysis. Section 2 builds the baseline dynamics equilibrium model. Section 3 lays out the qualitative analysis of the model and counterfactual. The last section concludes and explains ongoing work. Additional details can be found in the Appendix.

## 2. Empirical work: Data, methodology, and empirical results

In this section, I summarize the main data sources, focusing on the U.S economy. Detailed descriptions can be found in the [Appendix A](#).

First, I use bank-level variables from the Consolidated Reports of Condition and Income (known as "Call Reports") filed quarterly by all banks. I use quarterly income and balance sheet data for all U.S. publicly commercial banks (only com-

mercial banks are indicated by the SIC Codes 60, 61, and 6712 and charter type equal to 200, which means only commercial banks). This a panel sample for 1990-2007. I end the sample before the GFC because the latter was followed by a period of unconventional monetary policy and effective lower bound on interest rates, for example, after 2008 monetary policy is not based on the interest rate, but on unconventional monetary policy like QE and forward guidance. Therefore, using the interaction with Fed Funds rate changes could yield misleading or biased results, because it was not the main monetary policy tool after 2008. Table 1 shows the cross-sectional average for top 10% banks and bottom 90% in the sample about the components of the balance sheet. Basically, the table shows that deposits and loans are the most important elements of the balance sheet. In addition, I follow Drechsler et al. (2017) to get a measure of bank market power in the deposit market, this is measured as the weighted average HHI across all of a bank's branches, using branch deposits as weights.

Table 1: Bank balance sheet statistics

Fraction total assets (\%)	All sample: 1990-2007	
	top 10 %	bottom 90%
Cash / fed fund repo	9	11
Securities	23	28
Loans	63	57.5
Deposits	79	86
Other borrowing, fed fund repo	12.2	3
Equity	8.8	11

(Top 10 % and bottom 90% refers to total assets)

Second, I use a measure of monetary policy shocks based on high-frequency identification. These monetary policy shocks must be understood as surprises or unanticipated economic forces uncorrelated with other structural shocks im-

plied by the Fed Funds rate. The strategy for measuring monetary policy shocks based on high-frequency identification builds on the series used by [Gurkaynak et al. \(2004\)](#) and [Gorodnichenko and Weber \(2016\)](#). The idea is to isolate the unexpected (surprise) policy change that can generate market response. These series are constructed by measuring the reaction of the implied Fed Funds rate from a current-month Federal Funds future contract during the window from fifteen minutes before to forty-five minutes after the release of the announcement of FOMC meetings. Further details can be found in Appendix section A.2.

Given the bank data characteristics, my empirical strategy is based on panel data regression and a local forecasting method proposed by [Jordà \(2005\)](#) to estimate impulse responses. Second, given the results (i.e., that the response depends of the capitalization rate), I tested the traditional and modern view of the bank lending channel mentioned in the motivation part by including an additional interaction between bank size, liquidity, market power on deposits, and monetary policy. Third, I decompose total loans and analyze the response of different types of loans instead of overall loan growth. I find that higher-capitalization banks react more across different types of loans than lower-capitalization banks after a monetary policy tightening. Fourth, I analyze how loan portfolio composition and riskiness is conditional on bank capitalization. Fifth, I analyze the relationship between bank capitalization and default rates for different types of loans over the business cycle, and find no evidence of significant differences in cyclical of customers with different capitalization rates. Finally, I propose a mechanism that explains my findings and is consistent with how the overall components of bank's balance sheet move after a tightening. The following subsections describe these results.

## 2.1 Fact 1

### 2.1.1 Dynamic response: heterogeneous responses to monetary policy shock

This section documents the heterogeneity impact of monetary policy shocks on bank lending. First, I answer my main question with a linear specification by focusing on the estimation of the interaction coefficient between the capitalization rate and a monetary policy shock on bank lending. Second, I study the dynamic version of my linear specification in order not only to assess the moment of the policy shock but also the dynamic behavior of the interaction coefficient at some horizon in the future in response to a change in policy today.

**Linear specification:** I begin by estimating the following specification:

$$\Delta \text{logloan}_{i,t} = \alpha_i + \alpha_{st} + \delta_1 \text{MPShock}_t + \delta_2 X_{i,t-1} + \beta (\text{MPShock}_t * X_{i,t-1}) + \Gamma'_1 \text{macro}_t + \Gamma'_2 Y_{i,t-1} + \epsilon_{i,t} \quad (1)$$

where  $\alpha_i$  is a bank's  $i$  fixed effect<sup>3</sup>,  $\alpha_{st}$  is a state  $s$  by quarter  $t$  fixed effect<sup>4</sup>,  $\text{MPShock}_t$  is the monetary policy shock,  $X_{i,t-1}$  represents a set of explanatory variables under consideration for a given specification, such as bank's capitalization, liquidity, and market power.  $Y_{i,t-1}$  is a vector of bank-level controls such as age, size, liquidity, capitalization, loan loss, deposit over liabilities, wholesale funding over liabilities.  $\gamma_1, \gamma_2, \Gamma_1, \Gamma_2$  are regression coefficients. The main coefficient of interest in the regression (1) is  $\beta$  which measures the semi-elasticity of loans with respect to a monetary policy shock depending on a bank's capitalization rate<sup>5</sup>. Note that we use the lag of the explanatory and control variables to ensure they are predetermined at the time of the monetary policy shock<sup>6</sup>. I cluster standard errors at bank and time level.  $\beta < 0$  implies that banks with a higher capitalization rate reduce their lending more than bank with a lower capitalization rate after a positive

<sup>3</sup>Captures permanent differences in lending behaviour across banks

<sup>4</sup>State by quarter fixed effects capture differences in how broad states are exposed to aggregate shocks

<sup>5</sup>Alternately,  $\beta$  measures the importance of variable  $X_{i,t}$  on predicting heterogeneity in bank lending response.

<sup>6</sup>Note that a positive monetary policy shock represents a Fed Funds rate increase, and a negative  $\delta$  (interaction coefficient) reflects that banks with a greater explanatory variable ( $X_{i,t}$ ) prior to the shock experience smaller loan growth (or a larger contraction) after a contractionary shock

monetary policy surprise.

Table 2: Heterogeneous Effects of Monetary Policy on Bank's lending

	(1)	(2)	(3)	(4)
	<b>Loan Growth</b>			
Capitalization × MPshock	-0.758*** (0.27)	-0.769*** (0.26)	-0.936*** (0.26)	-0.825*** (0.26)
MPshock			0.607 (0.46)	0.925** (0.39)
Observations	642311	642303	642303	642303
R <sup>2</sup>	0.281	0.295	0.275	0.278
Bank controls	no	yes	yes	yes
Time sector FE	yes	yes	no	no
Macro control	no	no	no	yes
Bank, Time clustering	yes	yes	yes	yes

Robust standard errors in parenthesis

\*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$

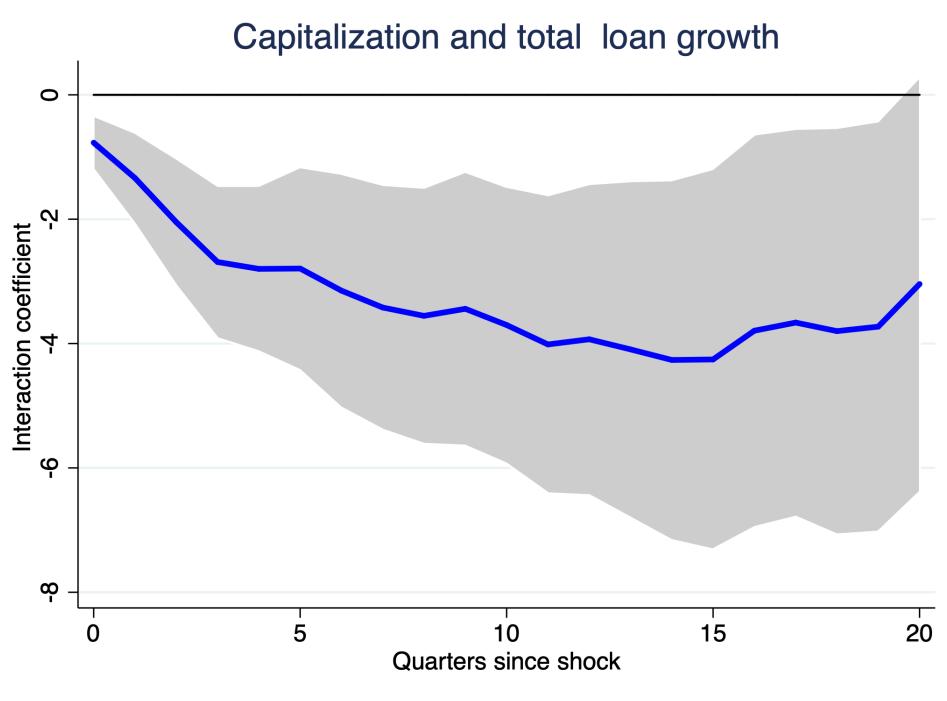
Table 2 shows the results from the estimation of equation (1). The four columns in the table 2 show a negative coefficient  $\beta < 0$ , which implies that higher capitalized banks reduce their lending more than lower capitalized banks after a positive monetary policy surprise. Column (1) reflects that banks with one standard deviation above the mean in the capitalization rate distribution react on average by 0.8 percentage points more than a bank located at the mean of the capitalization rate distribution. Columns (3) and (4) drop the time-fixed effect, so I can estimate the average effect of monetary policy. This coefficient in column (4), which is statistically significant, indicates that a one percent increase in the policy rate increases loan growth by around 0.9 percent.

In order to estimate the **dynamic** response across banks, I estimate the Jordà (2005) local projection specification:

$$\Delta \log \text{loan}_{i,t+h} = \alpha_i^h + \alpha_{st}^h + \delta_1^h \text{MPShock}_t + \delta_2^h X_{i,t-1} + \beta^h (X_{i,t-1} \cdot \text{MPShock}_t) + \Gamma'^h Y_{i,t-1} + \Gamma_2^h \text{macro}_{t-1} + \epsilon_{i,t+h} \quad (2)$$

where  $h \geq 0$  is the forecast horizon. Now  $\beta^h$  indicates the cumulative response of lending in quarter  $t + h$  to a monetary policy shock in quarter  $t$ , which depends on the bank capitalization rate.

Figure 1: Dynamics of Differential Response to Monetary Shocks: Capitalization



Note: dynamics of the interaction coefficient between the capitalization rate and monetary shocks over time

Figure 1 shows the dynamic response. The estimated interaction coefficient  $\beta^h < 0$  implies that higher-capitalized banks are more responsive to monetary policy shocks at the time of a contractionary monetary policy shock over horizon  $h$ . The point estimate is negative and statistically significant over the horizon until quarter 20<sup>7</sup>. This result is in contrast with the capital approach proposed by [Van den Heuvel et al. \(2002\)](#) in the sense that lower capitalized banks are more responsive to a monetary policy shock.

<sup>7</sup>This result is robust if we use the tier1 capital-to-asset ratio or the tier 1 capital-to-risk weighted assets ratio.

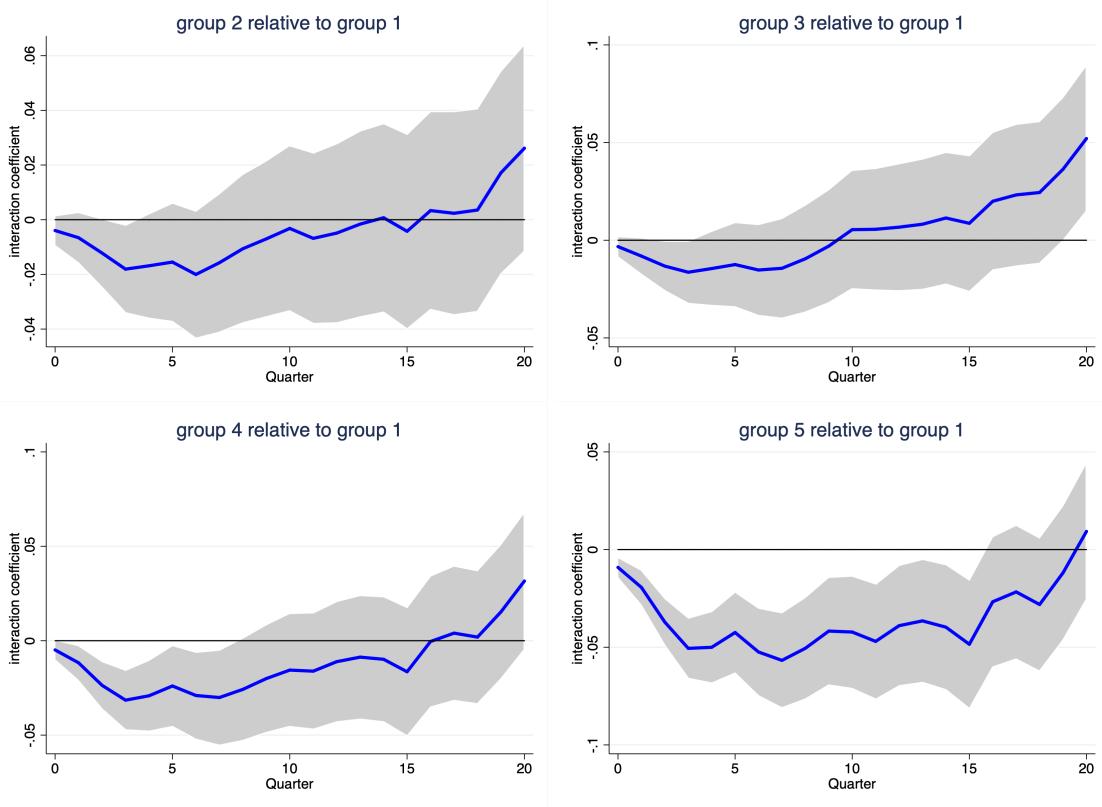
**Non-Linear model specification:** As a robustness check, I use a non-linear specification as follows:

$$X_{i,t+h} - X_{i,t-1} = \alpha_i^h + \alpha_t^h + \sum_{g=1}^{G-1} \alpha_g^h \times D_{gi,t}^h + \sum_{g=1}^{G-1} \beta_g^h \times D_{gi,t}^h \times \text{MPshock} + \delta^h \text{MPshock} + \Gamma^h Y_{i,t-1} + \epsilon_{i,t+h} \quad (3)$$

where  $X$  is the endogenous variable of interest, bank total lending,  $\alpha_i^h$ ,  $\alpha_t^h$  banks fixed effects and time fixed effects,  $D_g$  is dummy for a group of capitalization rate in the previous quarter. I divide the sample in quintiles where banks are ranked by capitalization rate and each group represent 20% of total assets in the sample,  $Y_{i,t-1}$  is bank's control which are the same in the previous specification, MPshock is the monetary policy shock at time  $t$ . Again, the coefficient of interest is  $\beta_g^h$ , which is the impulse response for a group  $g$  at forecast horizon  $h$ . Finally, the standard errors are clustered by banks. The figure 2 show the results of the non-lineal specification. The first group, the lowest capitalization rate quantile, is omitted. Therefore, the coefficient of interest  $\beta_g^h$  is interpreted as the response relative to group 1. Figure 2 shows that the response of group 5 (higher capitalization rate) relative to group 1 (lower capitalization) is negative and statistically significant on impact and over some horizon going forward.

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Figure 2: No-Linear Response



### 2.1.2 Testing different channels

In this section, I tested the other approaches found in the empirical literature about the response of bank lending to monetary policy. So far, there are two main works in the literature; i.e., the traditional and modern approaches to the bank lending channel mentioned in the motivation part. The idea is to include an additional interaction between bank size, liquidity, market power on deposits, and monetary policy in my main specification. The specification is as follows:

$$\Delta \text{logloan}_{i,t+h} = \alpha_i^h + \alpha_{st}^h + \delta_1^h \text{MPShock}_t + \delta_2^h X_{i,t-1} + \beta^h (X_{i,t-1} \cdot \text{MPShock}_t) + \Gamma'^h Y_{i,t-1} + \epsilon_{i,t+h} \quad (4)$$

where  $X^1 = \{\text{capitalization}, \text{size}\}$ ,  $X^2 = \{\text{capitalization}, \text{Liquidity}\}$ ,  $X^3 = \{\text{capitalization}, \text{Market Power}\}$ .

This specification will allow me to answer a sub-question: does my result sur-

vives controlling by the interaction with bank size (or liquidity, or market power) with monetary policy shock? I find that the capitalization rate is still significant when I test the other channel at the same time.

First, the traditional channel of bank lending proposed by Kashyap and Stein (2000) suggests that a tightening of monetary policy reduces lending more in less liquid banks, since they cannot sell assets to meet reserve requirements. Additionally, they claim that the sensitivity of the contraction to liquidity is stronger for small banks. Figure 3 shows the results of the dynamic response by controlling the double interaction with the size of banks. Figure 4 shows the result by controlling the double interaction with liquidity. Both figures show that the effect of the capitalization rate is negative and statistically significant. The effect of liquidity is there, but becomes less important going forward.

Second, the modern bank lending channel proposed by Drechsler et al. (2017) suggests that banks with more market power are more responsive to a monetary policy tightening. They can keep interest rates on deposits low when monetary policy tightens, thus increasing spreads. Figure 2.1.2 shows the result by controlling the double interaction with bank market power on deposit and a monetary policy shock.

Table 8 summarizes the main differences concerning the main empirical literature on the heterogeneous response across banks with different capitalization rates, market power on deposits, and liquidity in the U.S. economy (see appendix B for more details). I view these findings as reflecting that once I also allow for these different channels jointly I do not find the market power view to be statistically significant and liquidity channel is there, but is less important. Therefore, heterogeneity in bank capitalization rates plays a crucial role in the transmission of monetary policy to bank lending.

Figure 3: Dynamics: joint regression capitalization rate and real size

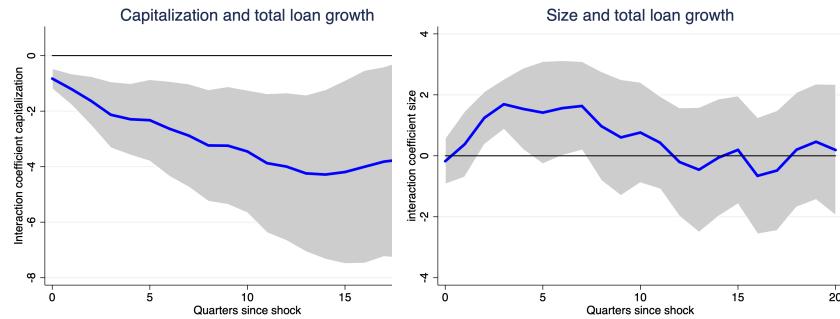


Figure 4: Dynamics: joint regression capitalization and liquidity

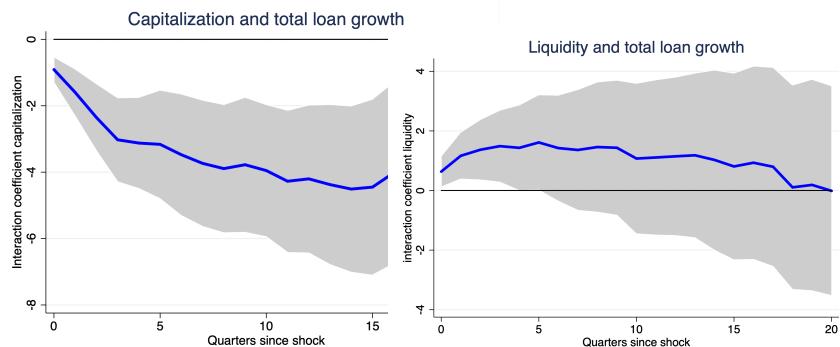


Figure 5: Dynamics: joint regression capitalization and market power

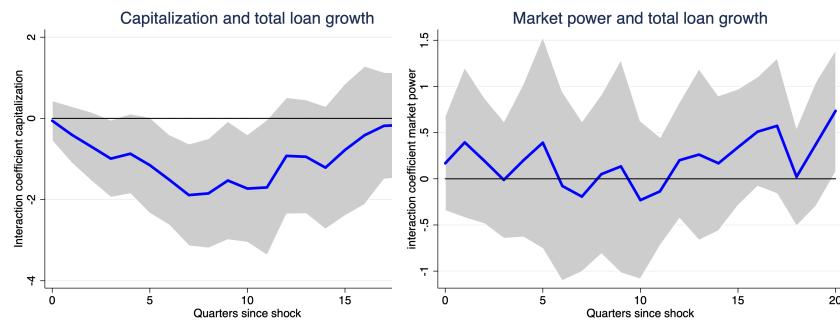


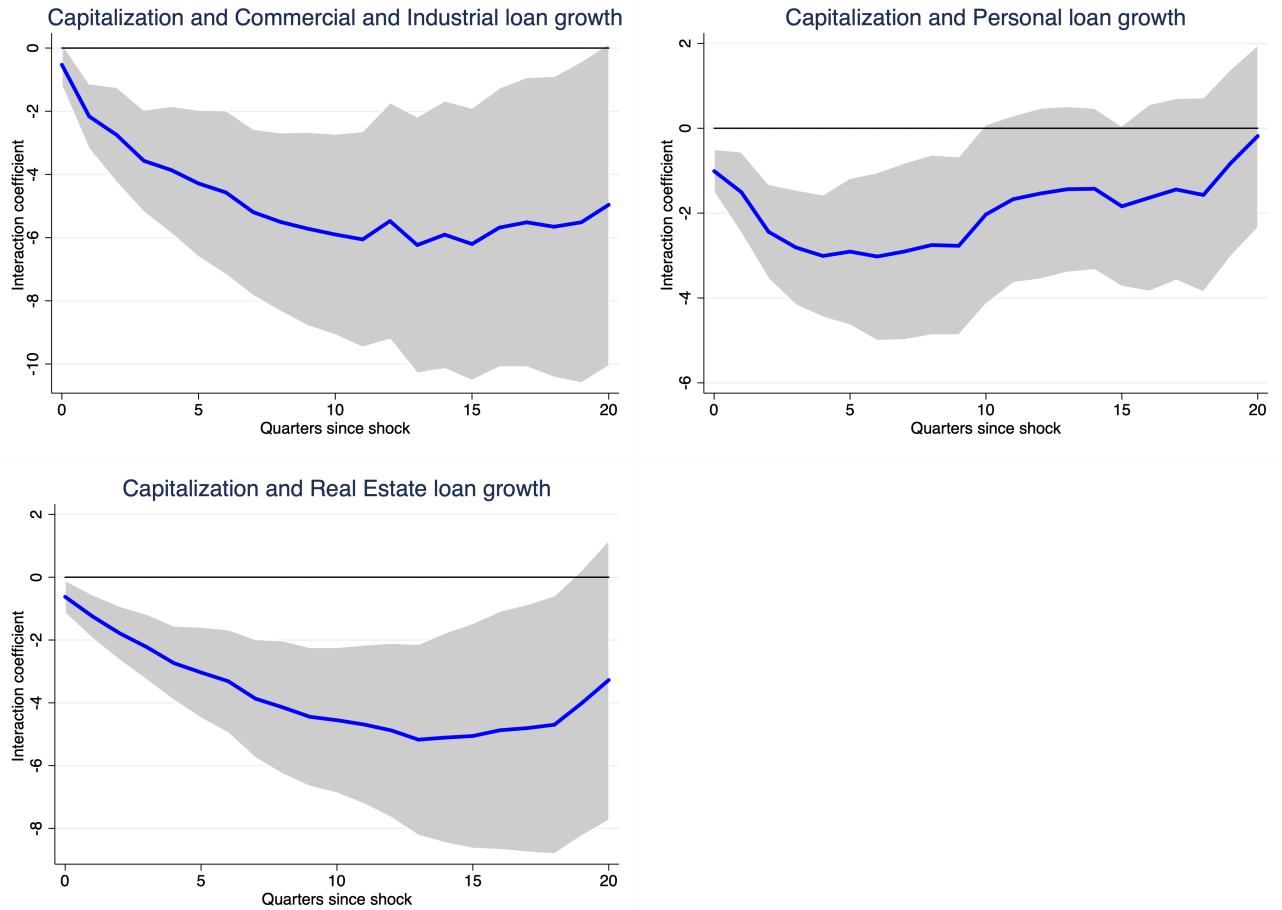
Table 3: Comparison to main existing empirical literature

	<b>Monetary Policy Measure</b>	<b>Sample Period and frequency</b>	<b>Individual Analysis</b>	<b>Econometric Specification</b>
Paz (2020)	High frequency identification	1990-2007 quarterly	Bank-level	-Linear regression with bank controls, interaction term, bank fixed effect, state X times fixed effects, standard errors are clustered at bank and time level, macro controls. -Dynamic: Local projection Method -Robustness: Non-Linear regression.
Drechsler, I., Savov, A., and Schnabl, P. (2017, QJE)	Change in Fed Funds	1994-2013 quarterly	Bank-level	Linear regression with interaction term, bank fixed effect, quarter fixed effects, standard error are clustered by bank
Van den Heuvel, (2012, BEJM)	Change in Fed Funds, Bernanke-Mihov indicator	1969-1995 annual	State-level	Linear Regression with interaction term, with state fixed effects
Kashyap, A. K. and Stein, J. C. (2000, AER)	Change in Fed Funds, Bernanke-Mihov indicator	1973-1996 quarterly	Bank-level	Two-Step Regression for different size class

### 2.1.3 Dynamic response for types of lending

This section documents the response of lending across different types of loans instead of overall loan growth. The specification is the same as in equation (2), but the dependent or endogenous variables are loan growth rates for different types of loans: commercial and industrial (C&I), real estate, and personal loans. I find that higher capitalization banks react more in reducing their loans across different types of loans than lower capitalization banks after a monetary shock. The effect is negative, statistically significant, and lasts several quarters after the shock. Figure 6 shows the results for each type of loan. I find that C&I loans are more sensitive on average than real estate loans to monetary shocks. Therefore, my main result holds for all types of loans. These results allow me to conclude that there is not a sectoral-driven or sectoral-risk history; i.e., it is not the type of loan that matters, because my results hold across different types of loans.

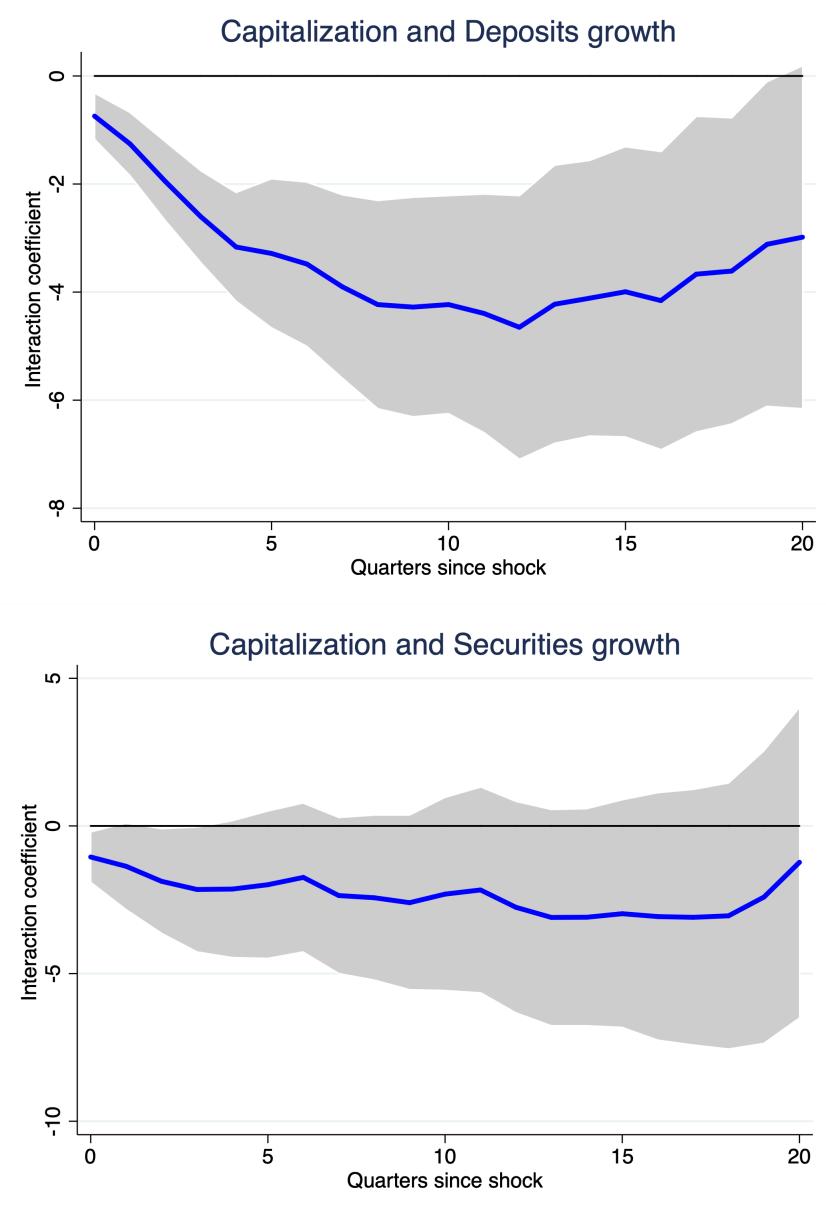
Figure 6: Dynamic responds on Banks' loan portfolio by type



#### 2.1.4 Bank balance sheets and monetary policy: deposits and securities

This section documents the response of other banks' balance sheet variables, such as deposits and securities, after a monetary shock. First, the top part of the figure 7 shows bank deposits' response to a monetary shock. I find that higher capitalized banks reduce more their deposits than lower capitalized bank. Second, the bottom part of the figure 7 shows bank securities' response a monetary shock. I find that securities' response on average (blue line) is systematically below zero and the gray band is wide, meaning that highly capitalized banks also reduce security holdings. In sum, highly capitalized banks reduce deposits, securities, and loans; i.e., overall balance sheet shrinks.

Figure 7: Bank balance sheets and monetary policy: deposits and securities

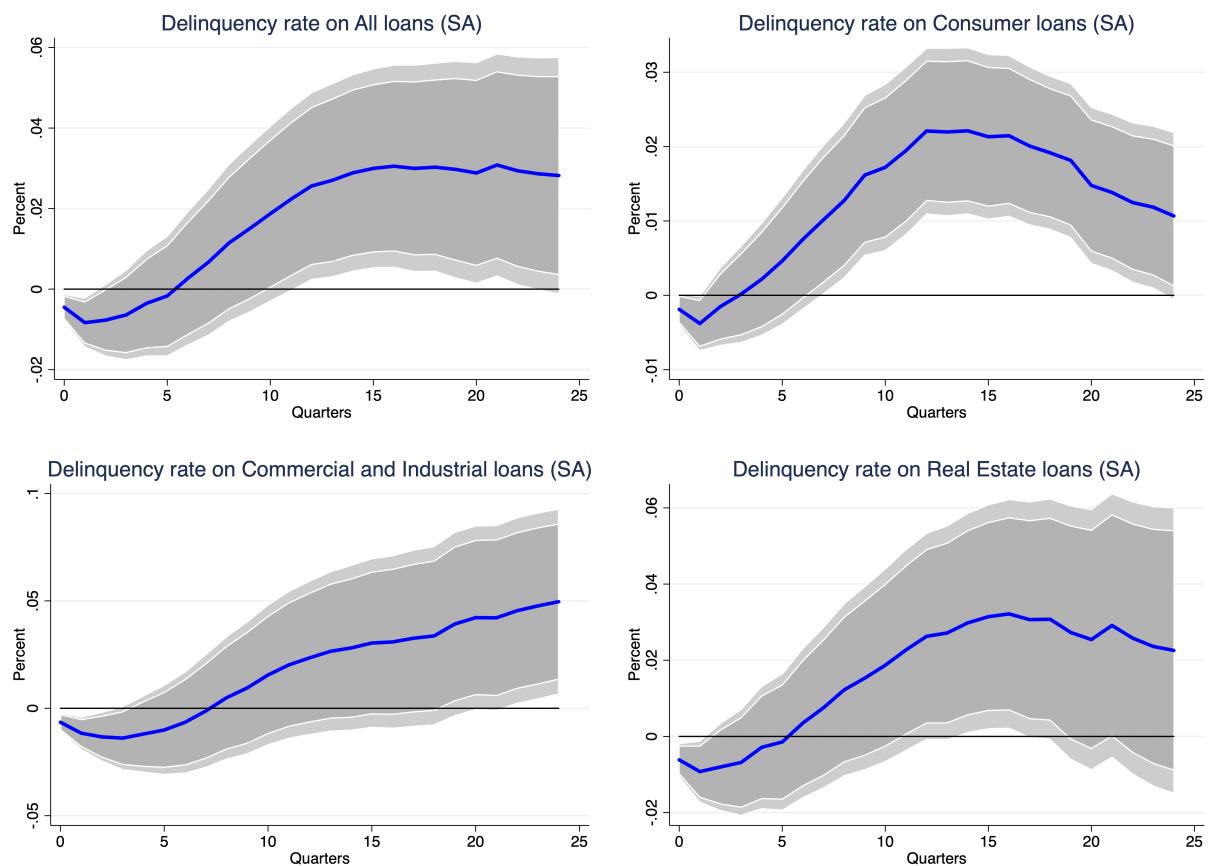


## 2.2 Fact 2

### 2.2.1 Default rates, bank capitalization rate, and monetary policy

This section documents the relation between default rates and a monetary shock. Given the aggregate data in delinquency rates (charge-off rates for each category of loans), I document the response of a proxy of default rates to a monetary shock. Figure 8 shows the response of delinquency rates to a monetary shock for each type of loan (main fact 2). I find that delinquency (proxy of default) goes up for all types of loans. In particular, default rates increase over two years after a monetary tightening (see Appendix C for the charge-off responses).

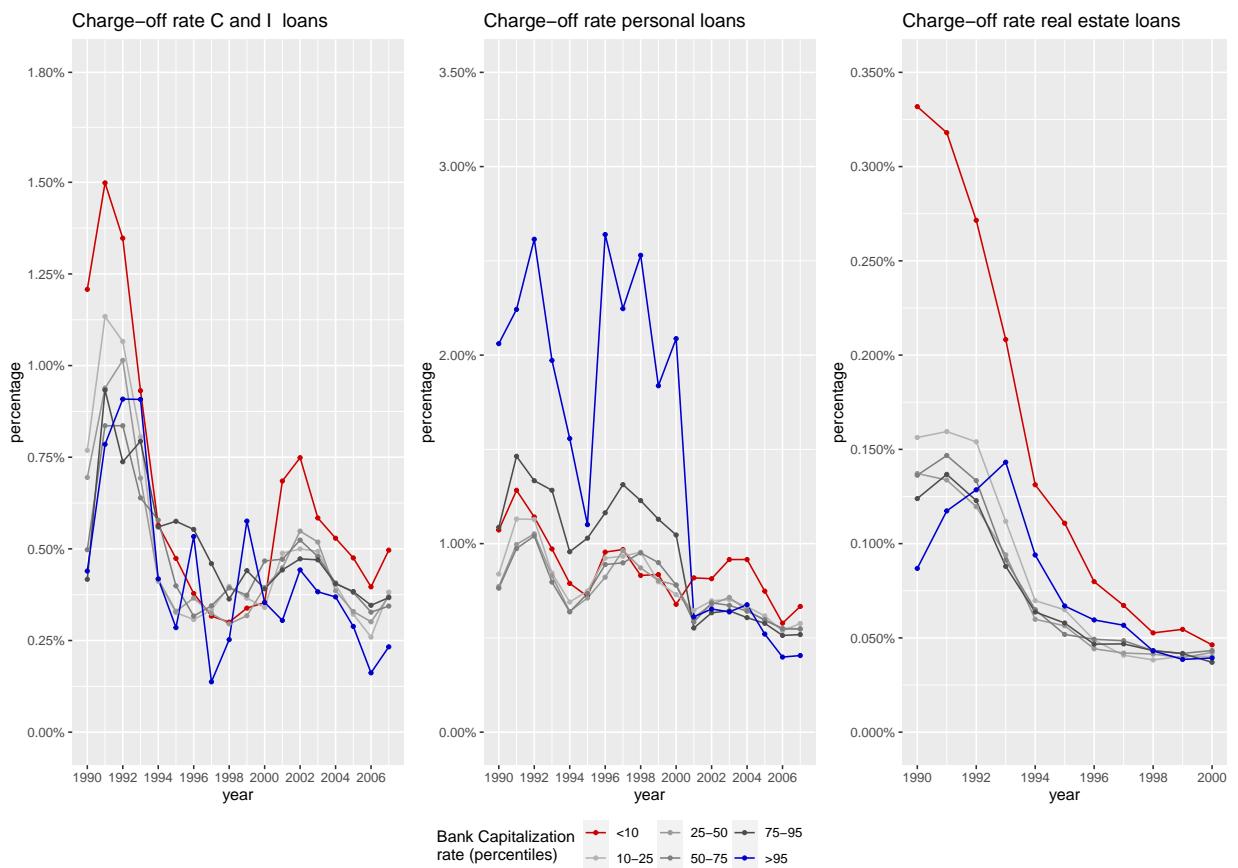
Figure 8: Aggregate: Delinquency responses to monetary policy shock



This evidence suggests that it is not about loans becoming riskier (they are intrinsically risky). In addition, it is important to notice that central banks tighten monetary conditions when the economy is doing well (a context where there should be few defaults). However, after tightening occurs, there will be more defaults, so the effect of tightening on the cost of financing these types of sectors matters. Therefore, there is a first-order effect that leads to an increase in default rates from the monetary tightening.

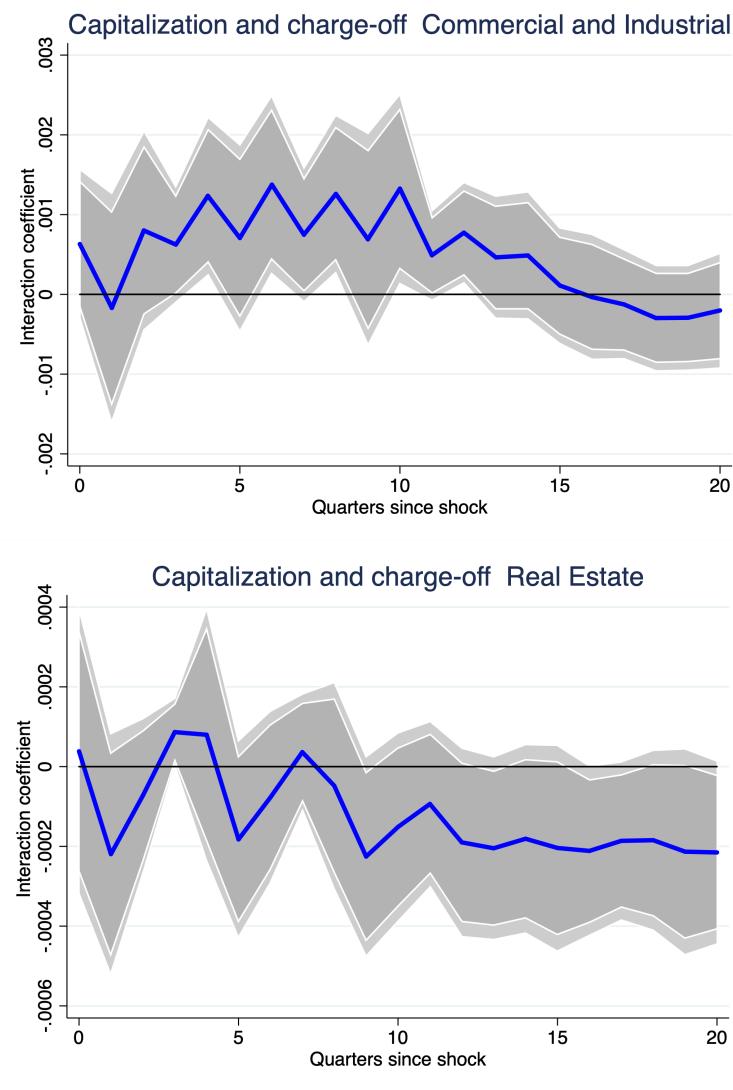
In addition, I use cross-sectional bank-level data to show the charge-off rate for each category of loans across percentiles of bank capitalization. Figure 9 shows that there is no clear pattern in default rates across capitalization rates.

Figure 9: Cross-sectional: Charge-off rates for loan types across bank capitalization rates



I analyze (conditional on loan types) whether the response of default rates to a monetary shock depends on capitalization rates. Figure 10 shows that within a given sector (e.g., real estate), high- and low-cap banks have the same defaults rates. A likely interpretation is that high- and low-cap banks lend to have similar

Figure 10: Cross-sectional: Charge-off responses to a monetary shock across capitalization rates



borrowers; i.e., credit risk is similar for both. In addition, Appendix D documents

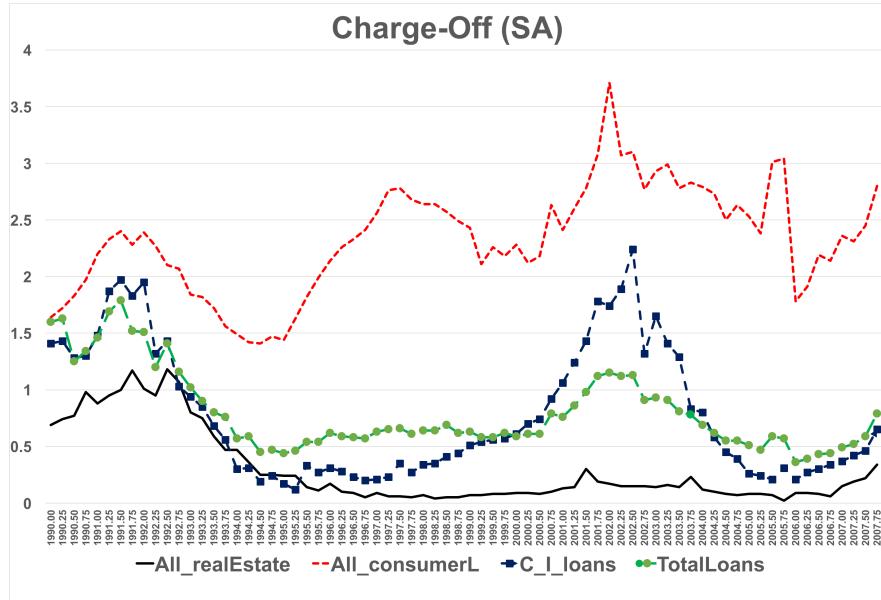
the relationship between bank capitalization rates, default rates, and business cycles. I find that there is a negative relation between default rates and GDP growth, but the effect across banks for each type of loan is not statistically different. Therefore, it is possible to rule out the demand-driven history that one bank type lends more cyclically than the other.

## 2.3 Fact 3

### 2.3.1 Riskiness of types of loan:

This section analyzes which types of loans are riskier. I define riskiness as a higher frequency of default. I consider charge-off rates for each loan category as proxy for default rates. Figure 11 shows the evolution of the aggregate data for charge-off and delinquency rates (proxy of default) for total loans and each loan category in all U.S. commercial banks. I show that, in the period of analysis, the charge-off rates are lower for real estate loans.

Figure 11: Aggregate Charge-Off Rates



Second, I use cross-sectional data to study the relative effect of risk between loan types using a charge-off rate for each category. The empirical strategy is regressing the charge-off rate for each bank against a charge-off indicator.

$$y_{ikt} = \alpha_i + \beta^p \times \mathbb{1}_{\{k=p\}} + \beta^{ci} \times \mathbb{1}_{\{k=ci\}} + \beta^{ag} \times \mathbb{1}_{\{k=ag\}} + \gamma' x_{i,t} + \epsilon_{i,t} \quad (5)$$

where:  $y_{ikt}$  : is the charge-off rate (proxy of default) of bank  $i$  with loan type  $k$  at time  $t$ ,  $x_{i,t}$  : bank control variables,  $\mathbb{1}_{\{k=\tau\}}$  is an indicator for the charge-off rate,  $\tau = \{p, ci, ag\}$ , where  $\mathbb{1}_{\{k=re\}}$  serves as the omitted category, and  $\beta^k$ : how risky is loan type  $k$  relative to real estate loans. Table 4 shows the result of the regression. The coefficient  $\beta$  reflects how risky loan type  $k$  is relative to real estate loans, where  $k = \{\text{C\&I, personal}\}$ . I find that personal and C&I loans are riskier than real estate loans.

Table 4

VARIABLES	(1) Charge Off rate	(2) Charge Off rate
$\beta^{ci}$	0.341*** [0.006]	0.346*** [0.006]
$\beta^p$	0.530*** [0.006]	0.535*** [0.006]
$\beta^{ag}$	0.014** [0.006]	0.012** [0.006]
constant	0.105*** [0.004]	0.022* [0.013]
Bank fe	Y	Y
Bank controls	N	Y
Obs	1,205,998	1,205,998
$R^2$	0.0874	0.091

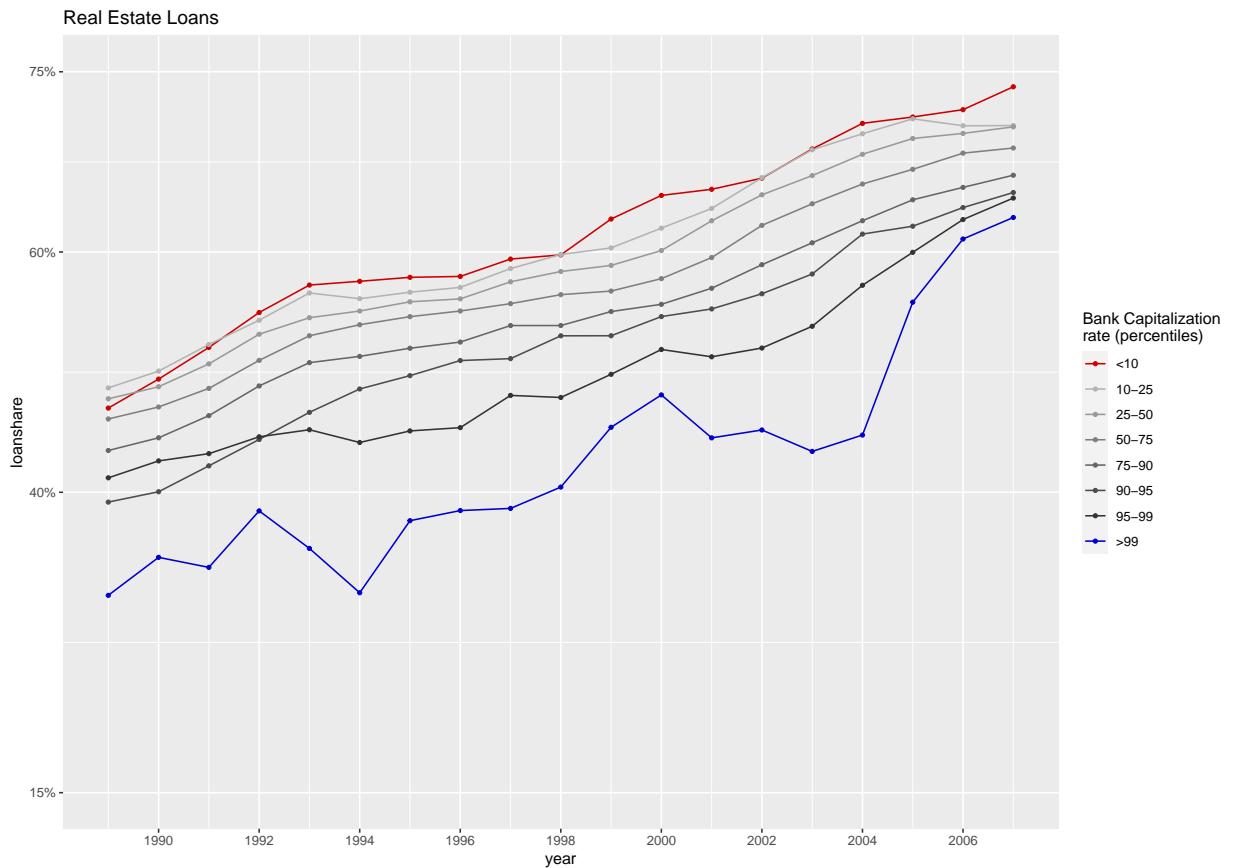
SE in brackets

### 2.3.2 Banks' loan portfolio composition:

In this section, given that the response is different across loan types, I document the loan portfolio composition for banks with different capitalization rates. First,

Figure 32 shows the average loan portfolio across bank capitalization rate percentiles for real estate loans. I find that higher capitalized banks have a lower share of real estate loans than lower capitalized banks.

Figure 12: Average portfolio share for real estate loan across bank capitalization percentiles



Second, while the evidence of figure 32 suggests that the portfolio composition of banks with higher capitalization rates is less oriented to real estate loans; see Appendix E for the other types of loans. It might be the case that bank size, or the effect on fixed effect on states, is the driving force. I calculate the trend in average bank capitalization after controlling for bank size, state, and size-state interactions. The empirical strategy is regressing the portfolio share associated with

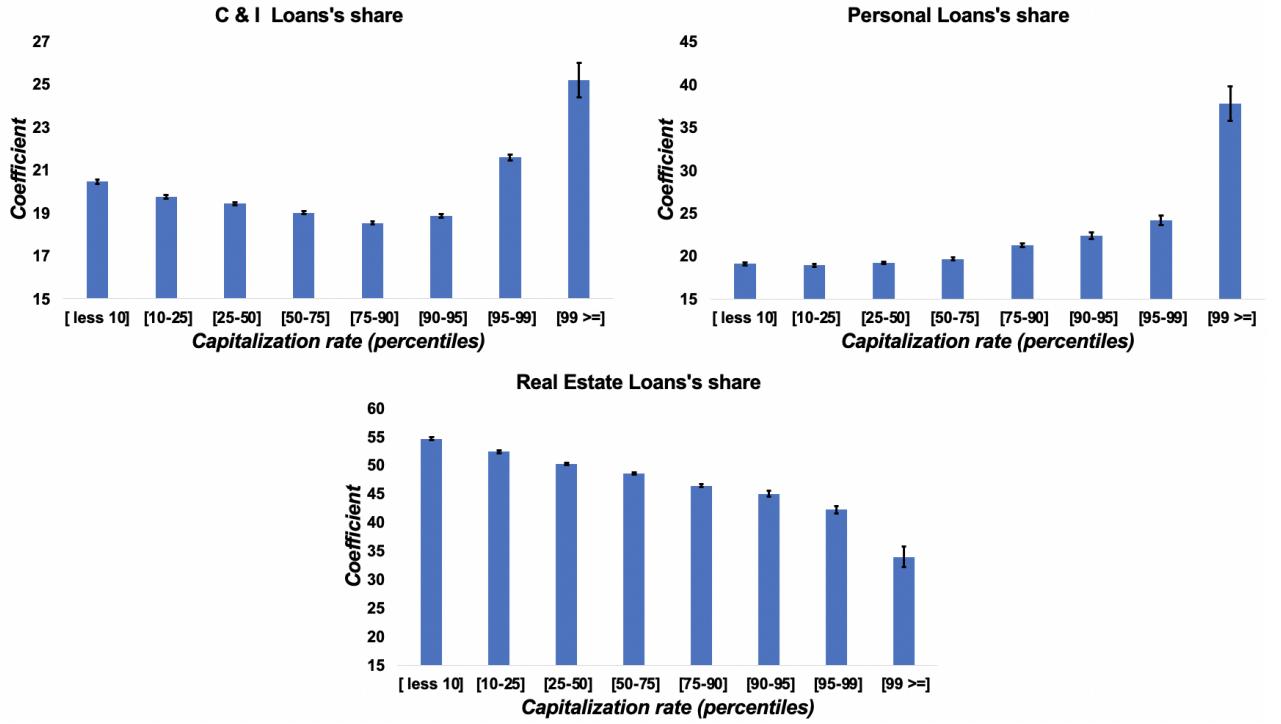
each category against different percentiles of bank capitalization rates.

$$y_{jbt} = \sum_{i \in I} \beta_i^j \mathbb{1}_{\{bt \in i\}} + \Gamma^j Z_t + \delta_t + \delta_{\text{state}} + \epsilon_{jbt} \quad (6)$$

where  $y_{jt}$  is the bank's loan type share,  $j$ : loan type: {C&I, personal, real estate},  $I$ : percentiles groups  $i$ ,  $Z_t$ : bank control: size. Therefore, the coefficient of interest is  $\beta_{cap}$ . I find that, on average, higher capitalized banks have a higher average share on C&I and personal loans, and lower capitalized banks have more real estate loans.

Table 13 shows the results for personal and real estate loans on the left and right hand side, respectively. I find that higher capitalization banks have a higher portfolio of personal loans (the same for C&I loans). In contrast, lower capitalization banks have a higher portfolio of real estate loans.

Figure 13: Estimation: Average Portfolio Share Parameters  $\beta_{cap}$



## 2.4 Inspecting the mechanism:

Against this backdrop, I set out to explore the mechanisms underpinning my findings. A framework intending to study the heterogeneous transmission of monetary policy to the economy through the banking sector should include several features absent in conventional macro-finance models. The main facts about banks' loan portfolios and the response of bank lending to a contractionary monetary shock (positive monetary policy surprise) are the following:

1. Portfolio composition and loan risk: Higher capitalized banks have a higher share of C&I and personal loans. These types of loans are riskier compared to real estate loans, as measured by charge-off rates.
2. Response to monetary tightening: default rates increase.
3. Response to monetary tightening: higher capitalized banks reduce lending more. This holds across all types of loans (C&I, personal, and real estate). In addition, they contract their balance sheet more (i.e., deposits and securities fall).

A possible mechanism is the following: an unanticipated increase in the Fed Funds rate increases the probability of loan defaults. Therefore, banks reduce their exposure to all risky assets. In particular, in terms of portfolio composition and riskiness, higher capitalized banks have a higher share of risky loans than lower capitalized banks; and since they have a risk-sensitive capital requirement, they reduce loans even more relative to lower capitalized banks. In consequence, higher capitalized banks reduce their overall loans more than lower capitalized banks. This effect on lending will have a negative impact on economic activity.

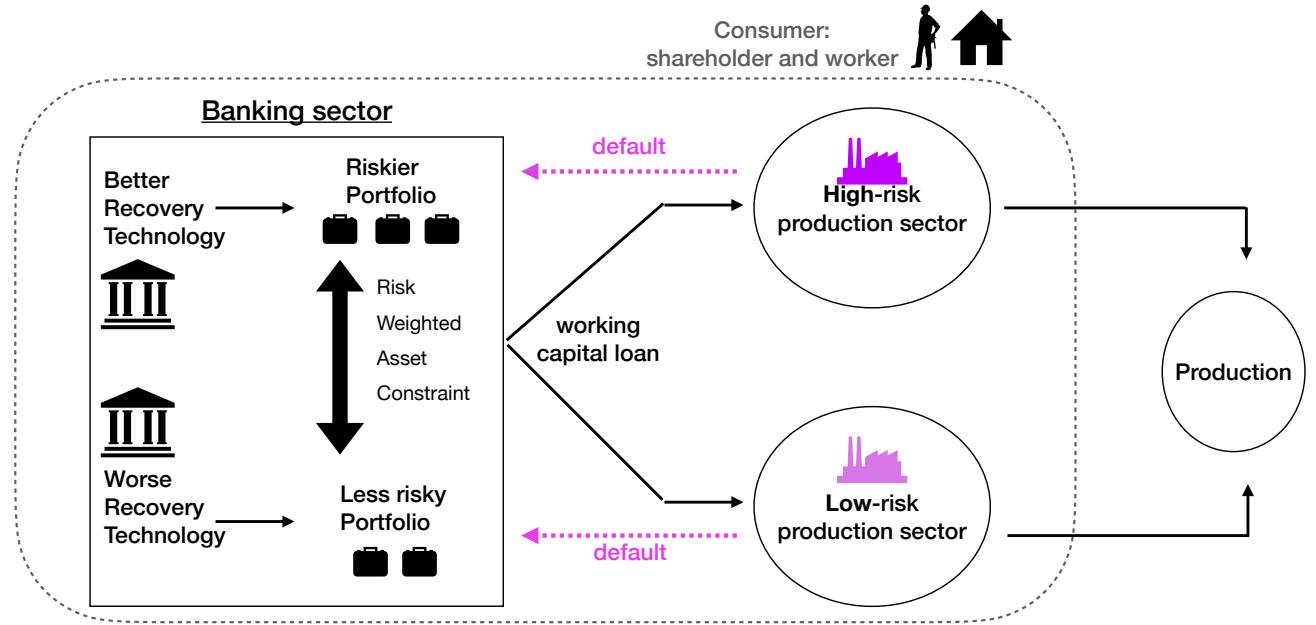
## 3. Baseline model

In the second half of the paper, I develop a heterogeneous bank model that considers risk-sensitive capital requirements to rationalize the empirical facts. This

dynamic stochastic general equilibrium model is based on the [Elenev et al. \(2020\)](#) framework. The proposed model has the following key elements: (1) Two banks that are heterogeneous in the recovery rates on defaulting loans and face capital regulation with a risk-weighted asset constraint. This implies an endogenous difference in capitalization rates and portfolio composition. (2) Two risky production sectors, with heterogeneous volatility in idiosyncratic productivity shocks on each sector. Additionally, They have a CES demand for loans. This implies differences in steady-state default rates and in lending responses to monetary shocks. (3) The aggregate fluctuations are driven by the monetary shock, where the deposit rate is given and follows a standard order-1 autoregressive process.

Figure 14 provides an overview of the model. The banking sector is composed by two banks. Additionally, there are two productive sectors. One of them has higher idiosyncratic volatility than the other (high- and low-risk sectors). Banks are heterogeneous in the the ability to recover losses from loans; face a regulatory constraint (a Basel I capital requirement with risk-weighted assets); and maximize the present-value dividends paid to their shareholders. They take the interest rate as given and can issue equity from consumers and extend loans to both production (non-financial) sectors. Banks cannot default. Importantly, banks extend high-risk lending to the firms in the high-risk productive sector and less risky lending to the firms in the low-risk sector. The bank lending is mostly in the form of working capital loans. Both productive sectors can default their loans to the banks. Producers maximize profits and operate a production technology using labor and capital. They are funded by working capital loans from banks. They also buy capital from consumers. Finally, consumers maximize inter-temporal expected utility, work for the firms (the labor supply is inelastic), and own firms and banks.

Figure 14: Overview of the model



### 3.1 Environment

The model is formulated in discrete time over an infinite horizon. There are three agents: consumers, firms, and banks. I develop a heterogeneous bank model in order to interpret the cross-sectional empirical evidence and understand monetary policy transmission to bank lending considering the heterogeneity in bank capitalization rates. I describe the model in three blocks: (1) sectoral firm block, which captures the difference in default rates; (2) banking block, which generates the differences in capitalization rates, portfolio composition, and lending responses to a monetary shock; and (3) a representative consumer or household, which closes the model.

### 3.1.1 Two Risky Production Sector Block

There are two types of firms  $j \in \{H, L\}$  with heterogeneous risk. In each sector there is a continuum of firms facing an idiosyncratic productivity shock. I assume there is perfect risk-sharing to have a representative firm in each sector with a default rate in equilibrium. Each risky productive sector uses a Cobb-Douglas production function with capital and labor  $\ell$ .

$$Y_{t,j} = \omega_{t,j} K_{t,j}^{1-\alpha} \ell_j^\alpha$$

where  $\omega_{t,j}$  is drawn i.i.d. from c.d.f. gamma distribution,  $E[\omega_{t,j}] = 1$ , and  $\sigma_{\omega_H} > \sigma_{\omega_L}$ . Firm  $j$  issues debt to finance working capital to bank  $i$ , at interest rate  $R_{t,j}^i = 1/q_{t,j}^i$ . The firm's problem in each sector can be explained in two stages.

**Stage I:** Given the interest rates, firms determine what fraction of loans to borrow from each bank. I assume that the representative firm has a preference for a variety for loans (multiple relationship). This assumption has an empirical counterpart; e.g., for emerging markets, there is empirical evidence from [Khwaja and Mian \(2008\)](#) in the case of Pakistan that 60% of firms borrow from multiple banks; and 56% of lending is in the form of working capital. In an example for developed countries, in this case Japan, [Amiti and Weinstein \(2018\)](#) show that the median firm borrows from seven banks, and 97% of the firms in their sample borrowed from more than one bank.

Formally, the firm will solve a standard problem and I assume loans are differentiated by sector according to a CES functional form<sup>8</sup>:

$$\max_{\{L_j^1, L_j^2\}} \hat{WC}_{t,j} = \left( \sum_{i=1}^2 (\nu_j)^{\frac{1-\sigma}{\sigma}} (L_j^i)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \quad s.t. \quad \left( \frac{1}{q_j^1} \right) L_j^1 + \left( \frac{1}{q_j^2} \right) L_j^2 = \left( \frac{1}{Q_j} \right) \hat{WC}_j$$

where  $\nu_j$  is a weight parameter,  $\sigma$  is the elasticity of substitution between the two types of loans,  $L_j^i$  denotes bank  $i \in \{1, 2\}$  loans in sector  $j \in \{H, L\}$ ,  $\hat{WC}_j$  is the amount of working capital needed for firm  $j$ , and  $Q_j$  is the aggregate loan price index of both banks' loans prices for firm  $j$

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<sup>8</sup>This is very common in international trade literature

The solution to this problem provides the demand for loans as:

$$L_{t,j}^i = \overbrace{\left( \frac{\frac{1}{Q_{t,j}}}{\frac{1}{q_{t,j}^i}} \right)^\sigma (\nu_j)^{1-\sigma} \times \underbrace{\phi^j w_{t,j} \bar{l}_{t,j}}_{\text{working capital } (\bar{W}C_{t,j})}}^{\text{Loan's firm j (sector) demand for bank } j}$$

Fundamentally, this allows me to endogenously determine what fraction of working capital is provided by each bank. This fraction will depend on the interest rate, which in turn will depend on the recovery value of each bank, which is a technology parameter. Additionally, in equilibrium banks coexist with an interior solution of portfolio composition.

**Stage II:** Given borrowing decisions, firms hire labor and buy capital at price  $p_{jt}^K$  to maximize the present discounted value of dividends paid to shareholders and produce final goods using the Cobb-Douglas production function. Failed producer are replaced by new producers.

The flow of profit for the firm is:

$$\underbrace{\omega_j k_j^{1-\alpha} l^\alpha - (1-\phi) w_j l - \frac{1}{Q_j} \mathbf{w} \mathbf{c}_j}_{\text{profit flow}} \quad (7)$$

Producers with a negative profit flow are in default and shut down. Alternately, a firm defaults if its sales do produce enough cash to pay back working capital loans. This implies a default threshold

$$\omega_j^* = \frac{(1 + \phi^j (\frac{1}{Q_j} - 1)) w_j \bar{l}_j}{y_j} \quad (8)$$

Note that firms with low idiosyncratic shock  $\omega_{t,j} < \omega_{t,j}^*$  default.

The firm's recursive problem is:

$$V_j(n_j) = \max_{k'} \text{div}_j + \mathbb{E}_t[m_{t,t+1} \tilde{V}_j(k'_j)] \quad (9)$$

$$\overbrace{n_j - p^{k_j} k'_j + \underbrace{\mathbf{wc}_j}_{\text{new debt}}}_{\text{div}^j_{ft}} \geq 0 \quad (10)$$

$$n_j = \underbrace{\omega_j k_j^{1-\alpha} l^\alpha - (1-\phi) w_j l - \frac{1}{Q_j} \mathbf{wc}_j + p^{k_j} (1-\delta^{k_j}) k_j}_{\text{profit flow}} \quad (11)$$

where  $m_{t,t+1}$  is the stochastic discount factor for the firm, and

$$\tilde{V}_j(k_j) = \max_{l_j} [\Omega(\omega_j^*) \mathbb{E}_t(V_j(n_j) | \omega_j > \omega_j^*)] \quad (12)$$

Note that a firm hires labor before the idiosyncratic shock occurs. This implies that the firm chooses labor with the expected value of the firm's idiosyncratic productivity conditional on not defaulting. The complete solution of the firm problem is in Appendix E.1.

### 3.1.2 Banking sector Block

The banking sector block consist of two banks  $i \in \{1, 2\}$  which are intermediaries and grant loans to both sectors (high and low risk). The supply of deposits is perfectly elastic at the policy rate. These banks are owned by consumers and face equity issuance costs. These two banks are heterogeneous in their default recovery rates  $(1 - \zeta_j^i)$ . They will receive a coupon payment on performing loans  $\Omega(\omega_{t,j}^*) L_{t,L}^i$ , and firms that default go into liquidation and banks repossess them, sell the current period's output, pay the current period's wage, and sell off the assets. Therefore, the total payoff per loan type unit  $j$  is:

$$\tilde{M}_{t,j}^i = \underbrace{\Omega(\omega_{t,j}^*)}_{\text{No default}} + \underbrace{\frac{(1 - \Omega(\omega_{t,j}^*))}{L_{t,j}^i / q_{t,j}^i} \left[ \varpi_{t,j}^i (1 - \zeta_j^i) \left( \mathbb{E}_\omega[\omega < \omega^*] Y_t + ((1 - \delta_j^k) p_t^{K_j}) K_{t,j} \right) - \varpi_{t,j}^i w_{t,j} \bar{l}_j \right]}_{\text{default (recovery value)}} \quad (13)$$

where  $\zeta$  is the fraction of firm assets and output lost to banks in bankruptcy.

The bank portfolio consists of choosing the loan interest rate for each type of

firm, subject to bank capital regulation; i.e., a risk-weighted capital constraint.

$$\text{Networth}^i \geq \theta \underbrace{(\varpi_H L_{H,t}^i + \varpi_L L_{L,t}^i)}_{\text{risk weighted assets}}$$

where  $\varpi_H, \varpi_L$ , are the risk weights for each type of loan.

The bank problem is:

$$V^i(N_t^i) = \max_{q_{Ai,t}, D_t^i, e_t^i} \underbrace{div_{bt}^i - e_t^i}_{\text{Netdiv}_{bt}^i} + E_t[M_{t+1,t}^B V^i(N_{t+1}^i)] \quad (14)$$

$$N_{t-1}^i + D_t^i + e_t^i = L_{t,H}^i + L_{t,L}^i + div_t^i + \Psi^i(e_t^i) \quad (\text{budget constraint}) \quad (15)$$

$$D_t^i \leq \xi_H L_{t,H}^i + \xi_L L_{t,L}^i \quad (\text{leverage constraint}) \quad (16)$$

$$\pi_t^i = \left( \frac{\tilde{M}_{t,j}^i}{q_{t,H}^i} - 1 \right) L_{t,H}^i + \left( \frac{\tilde{M}_{t,j}^i}{q_{t,L}^i} - 1 \right) L_{t,L}^i - (R_t - 1) D_t^i \quad (\text{profit flow}) \quad (17)$$

$$N_t^i = N_{t-1}^i + \underbrace{\pi_t^i - div_t^i + e_t^i - \Psi^i(e_t^i)}_{\text{retaining earnings + equity injections}} \quad (\text{Law of motion of networth}) \quad (18)$$

The complete solution of the bank problem is in Appendix E2.

### 3.1.3 Representative Household

There is a representative household with log-utility preferences over consumption represented by an expected utility function.

$$\mathbb{E}_t \left[ \sum_{t=0}^{\infty} \beta^t [\log(C_t)] \right] \quad (19)$$

where  $\beta$  is the discount factor. Households are the owners of firms and banks. They provide labor in fixed supply and choose consumption and investment in

both sectors subject to a budget constraint. The consumer problem is:

$$\max_{C_t, X_t^A, X_t^M} \mathbb{E}_t \left[ \sum_{t=0}^{\infty} \beta^t [\log(C_t)] \right] \quad (20)$$

s.t.

$$C_t + \sum_{i=1}^2 (X_t^j + \Psi(X_t^j, K_t^j)) \leq w^j \bar{L} + \sum_{j=1}^2 \text{div}_t^j + \sum_{i=1}^2 \text{Netdiv}_t^i + \sum_{j=1}^2 p_t^{K^j} X_t^j \quad (21)$$

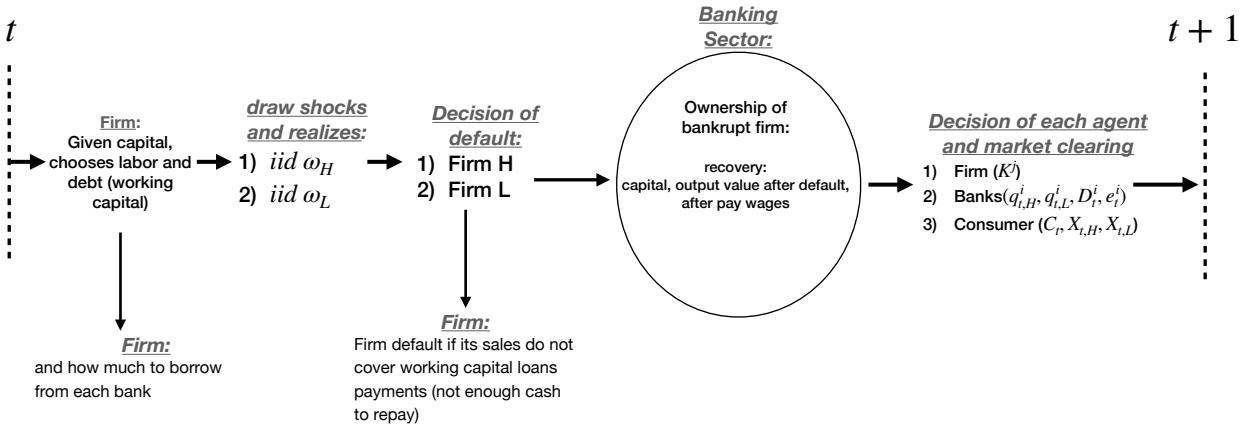
$$K_{t+1}^j = (1 - \delta_K) K_t^j + X_t^j \quad (22)$$

The complete solution of the consumer problem is in Appendix G.

### 3.1.4 Timing:

Figure 15 summarizes the timing of the model.

Figure 15: Timing



At the beginning of period  $t$ , given capital, firms choose labor and working capital. Firms also decide how much to borrow from each bank. Idiosyncratic productivity shocks for intermediate good producers are realized in each sector and then their production occurs. Firms default if their sales do not cover working capital loans payments or there is not enough cash to repay. Banks assume own-

ership of bankrupt firms. Firms decide how much of the capital to take. All agents solve their consumption and portfolio choice problems. The market clears and all agents consume.

### 3.1.5 Competitive Equilibrium

Equilibrium is defined in the standard way. Competitive equilibrium is a sequence of monetary policy shocks  $\{\epsilon_t^{MP}\}$ , an idiosyncratic productivity shock  $\{\omega_{t,j}\}$  for each sector  $j \in \{H, L\}$ , and competitive equilibrium is defined as an allocation of:

- $\{C_t, X_{t,H}, X_{t,L}\}$  for consumers
- $\{K_{t,j}\}$  for firms  $j \in \{H, L\}$
- $\{D_{t,i}, e_{t,i}, q_{t,i}^j\}$  for banks  $i$  in  $\{1, 2\}$
- A set of prices
- Such that given prices:
  1. Consumers maximize life-time utility subject to their constraint.
  2. Producers in each sector maximize dividends subject their constraints.
  3. Banks maximize net dividends subject to their constraints.
  4. The market clears.

## 3.2 Mechanism

This section explains how the primitive model delivers the qualitative results that I show in the empirical evidence section.

First, I explain the relationship between recovery rates and risky portfolio share. Higher-recovery banks wish to lend more. In particular, they allocate a higher share of their portfolio to the riskier sector. Additionally, as these banks grant

more loans, they need more funds to provide more loans (both deposit and equity issuance). The underlying idea is comparative advantage. The easy way to approach the matter is that if a bank has a 100 percent recovery (i.e., it is good at risky business recovery), then it will specialize in those kinds of loans. In contrast, if the other bank has a lower recovery performance (i.e., cannot recover 100 percent), it will try to specialize in relatively less risky (safer) business.

Second, I explain the recovery and capitalization rates. The financing constraint is a function of risk-weighted assets. The regulator does not understand that one bank has a better technology than the other (i.e., they do not know what are the recovery rates for each bank) imposes the same risk-weighted constraint on both banks. Therefore, banks with better skills or with higher recovery rates are able to invest in risky firms (or risky sectors), but they might not have the necessary capital to do so. Therefore, they need to be better capitalized. Note that, in my model, the recovery rate is not a property of the asset: it is a property of the bank, its technology.

The following example can illustrate the situation: there are two banks, one of them with a comparative advantage in asset management, but it is mandated to deploy the same amount of capital as a less efficient bank. In a two-sector economy, the better bank is able to manage the riskier sector better; i.e., is willing to invest more in the risky sector because it has a comparative advantage, which makes its portfolio riskier. The regulator mandates a higher capital requirement on this bank compared with the bank that invests in safer assets (less risky portfolio). Providing the capital is costly for the more efficient bank, so imposing the risk-weighted constraint actually pushes it away from the risky sector. This affects the better bank more than proportionally, as it is investing more in the risky sector. Therefore, the riskier will tend to withdraw more intensively away from the riskier sector, but to the extent that it equalizes its portfolio composition with the worse bank. The better bank will actually push the portfolio composition to the same structure as the worse bank. Therefore, now they will face the same collateral constraint (or regulatory constraint), as they have the same asset composition. However, the better bank has the same constraint, but the advantage of managing

the riskier sector; in consequence, it would still be willing to invest more in the riskier sector, but for that it would need to be better capitalized.

Note that, with these two relationships, banks with higher recovery rates will have a riskier portfolio share and a higher capitalization rate to meet regulations.

Third, it is important to consider the sensitivity of well-capitalized banks to monetary policy shocks. The sequence is: increase in the policy interest rate, increase in loan rates, and increase in the default probability. Note that the default rate increases more in the risky sector; therefore, the highly capitalized bank with a higher-risk portfolio is more sensitive: as it faces a higher default rate, it contracts more.

### 3.3 Parameterization and Results

In this section, I use the model to analyze in detail the novel channel of heterogeneous transmission of monetary policy shocks through differences in capitalization rates. I start by calibrating the model under assuming bank heterogeneity on recovery defaulting loans, as primitive parameter, that face risk-weighted capital requirement. Then, I computed for the deterministic steady-state, and I shock the economy with a positive monetary policy shock to verify the model performance in terms of my key features of the micro data.

#### 3.3.1 Calibration

**Household preferences and production function.** For simplicity, I assume standard preferences for the consumer  $u(C) = \log C_t$  or I set the intertemporal elasticity of substitution (IES) to 1. The consumer's discount factor  $\beta$  is set to 0.85. On the production side, the labor share  $\alpha$  in the final good is set to 0.71, which is a standard value in the business cycle literature. For the investment sector, I also assume standard quadratic specification for investment adjustment cost and I set the marginal adjustment cost parameter  $\Psi$  to 2 in order to match the adjustment cost and its first derivative are zero in the steady-state. **I set an annual depreciation of capital  $\delta_K$  of 8.25 to match the investment-to output ratio observed in the data.** For the working capital loan, it is well-known in corporate finance literature that a firm requires to cover the cash flow mismatch between the payments made at the beginning of the period and the realization of revenues [Mahmoudzadeh et al. \(2018\)](#). I set the working capital parameters to 0.8, which is in line with [Galindo Gil \(2020\)](#) and [Christiano et al. \(2010\)](#). For the CES functional form to the firm in which loans are differentiated by sector, I set an elasticity of substitution between loan's banks  $\sigma = 7$ , implying a standard elasticity between these banks typically used between monopolistically competitive goods. Also, I set the weighting parameters  $\nu_1 = 1.12$  and  $\nu_2 = (1 - \nu_1^{1-\sigma})^{\frac{1}{(1-\sigma)}}$  to both banks hold 50 percent of the loans in equilibrium when there is not heterogeneity in recovery rates.

**Idiosyncratic Productivity.** Idiosyncratic shock are assumed to be Gamma distributed with parameters  $\mu_\omega$  and  $\sigma_\omega$ . I normalize the mean of idiosyncratic productivity at  $\mu_\omega = 1$  for both sector  $j \in \{H, L\}$ . In the case of low risk productive sector, the cross-sectional standard deviation of the idiosyncratic productivity  $\sigma_{\omega,L}$  targets the unconditional mean of default rate. The model-implied average default rate of 2% is similar to the data corresponding to average delinquency rate of 2% for the residential real estate loans. In the case of high risk productive sector, the cross-sectional standard deviation of the idiosyncratic productivity  $\sigma_{\omega,H}$  targets the unconditional mean of default rate. The model-implied average default rate of 3% is similar to the data corresponding to average delinquency rate of 3% for the commercial and industrial loans<sup>9</sup>.

**Banking sector.** The intermediaries face the risk weighted capital constraint. The capital requirement or minimum regulatory equity capital requirement  $\theta$  is set to 8% of risk-weighted assets, risk weights to riskier type of loan is set  $\varpi_H$  set to 1, and risk weight to less risky type of loan is set  $\varpi_L$  set to 0.8, consistent with the general requirement for banks under Basil I regulatory framework ([BCBS \(1998\)](#)). The dividend target of banks  $\phi_0$  and the marginal bank equity issuance cost  $\phi_1$  is set 0.096 and 7, respectively, as [Elenev et al. \(2020\)](#). Two parameters drive the heterogeneity in the banking sector which are the recovery rates on defaulting loans. I construct a proxy of recovery rates using the bank-level data. Over my period of analysis, top 90 percentile of bank capitalization rate have on average .4 of recovery rates in personal loans and have on average .32 of recovery rates in real estate loans. In addition, bottom 10 percentile of bank capitalization rate have on average .33 of recovery rates in personal loans and have on average .21 of recovery rates in real estate loans.

I construct a proxy of recovery rates as a ratio of recoveries on allowances for loan and lease losses to charge-offs on allowances for loan and lease losses using the bank-level data<sup>10</sup>. Over my period of analysis, top 75 percentile of bank's recov-

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<sup>9</sup>From the Federal Reserve Board of Governors, I got delinquency rates on Residential Real Estate, and Commercial and Industrial loans by U.S. Commercial Banks for the period 1990-2007.

<sup>10</sup>Recoveries on allowance for loan and lease losses(*RIAD4605*) and charge-offs on allowance for loan and lease losses(*RIAD4265*) on the "Call Report" data base. For further references see [The Fed](#)

ery rates in *C&I* loans and real estate loans have on average .8 and 0.5, respectively, over the sample. Then, I assume that bank 1, have a higher recovery rate in both sector equal to 0.8, and bank 2 have a lower recovery rate of 0.2 in the high risk sector and 0.5 in the less risk sector. Finally, [Christiano et al. \(2010\)](#) suggest that the persistence and the standard deviation of the interest rate shock in terms of monetary policy are 0.87 and 0.51 respectively. However, no all the volatility of the monetary shock is transmitted to the interest rate of loans. As a result, I assume that the relevant volatility of the interest rate shock for the firm is one-fifth of the corresponding to monetary policy  $\sigma_R = 0.01$  and less the persistence  $\rho_R = 0.7$ .

Table 5: Parameters of the model

Parameter	Name	Value	Target/Sources
Preferences			
$\beta$	Discount factor	0.85	See text
$\eta$	IES	1	Log Utility
Technology $j \in \{H, L\}$			
$\alpha$	labor share	0.71	Standard
$\Psi$	capital adjustment cost	2	Standard
$\delta_K$	depreciation rate	8.25%	Standard
$\phi_K$	working capital parameter	0.8	<a href="#">Christiano et al. (2010)</a>
$\sigma$	elasticity of substitution	7	See text
$\nu$	weighting parameter	1.12	See text
Banking: Banks $i \in \{1, 2\}$			
$[1 - \zeta_H^1, 1 - \zeta_L^1]$	bank 1 recovery rates on defaulting loans	[0.8,0.8]	See text
$[1 - \zeta_H^2, 1 - \zeta_L^2]$	bank 2 recovery rates on defaulting loans	[0.2,0.5]	<a href="#">Elenev et al. (2020)</a>
$\phi_0$	target bank dividend	0.096	<a href="#">Elenev et al. (2020)</a>
$\phi_1$	bank equity issuance cost	7	<a href="#">Elenev et al. (2020)</a>
$\theta$	regulatory constraint	0.08	Basilea I
$[\varpi_H, \varpi_L]$	risk weights to each type loan	[1, 0.8]	Basilea I
Shock parameters or shock structure			
$\rho_R$	Persistence of policy rate	0.7	Standard
$\sigma_R$	volatility of policy rate	0.01	Standard
$\sigma_{\omega_L}$	volatility idiosyncratic low risk sector	0.03	Default rate 2%
$\sigma_{\omega_H}$	volatility idiosyncratic high risk sector	0.05	Default rate rate 3%

### 3.3.2 Steady State and Monetary Policy Analysis

Table 6 shows the portfolio composition in the steady state. I find that higher capitalized banks have a higher portfolio share of risky assets compared with lower capitalized banks. This proves a qualitative result that I find in my empirical exercise. Figure 16, 17 shows the response of variables like deposits, loans, and default rates after a 1-percentage-point increase in the interest rate.

Table 6: Portfolio composition, **Fact 3**

Steady State		
Portfolio Composition		
	High risk sector	Less risk sector
High-cap Bank	53%	47%
Low-cap Bank	45%	55%

Figure 16: Experiment: Monetary policy shock and Fact 1

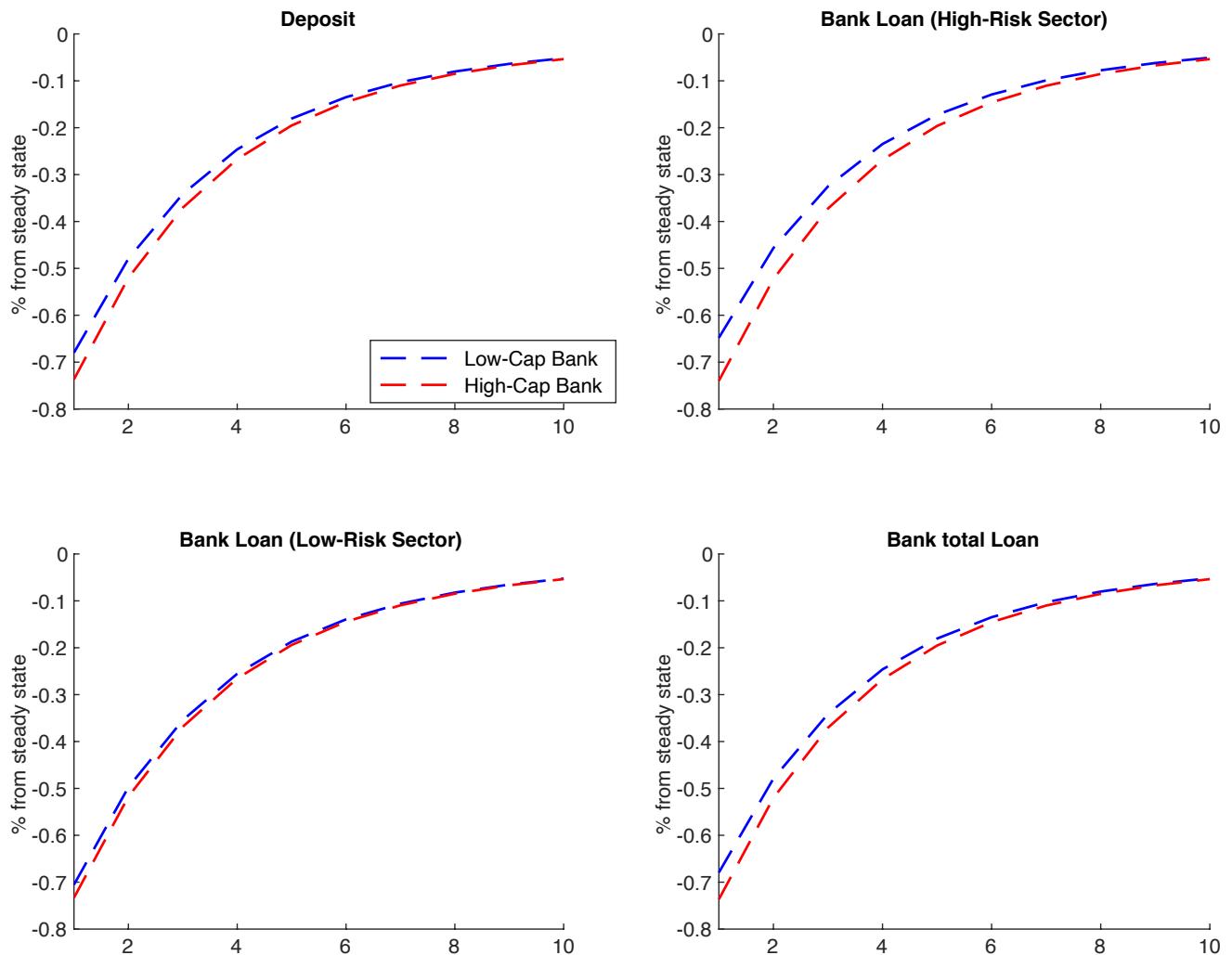
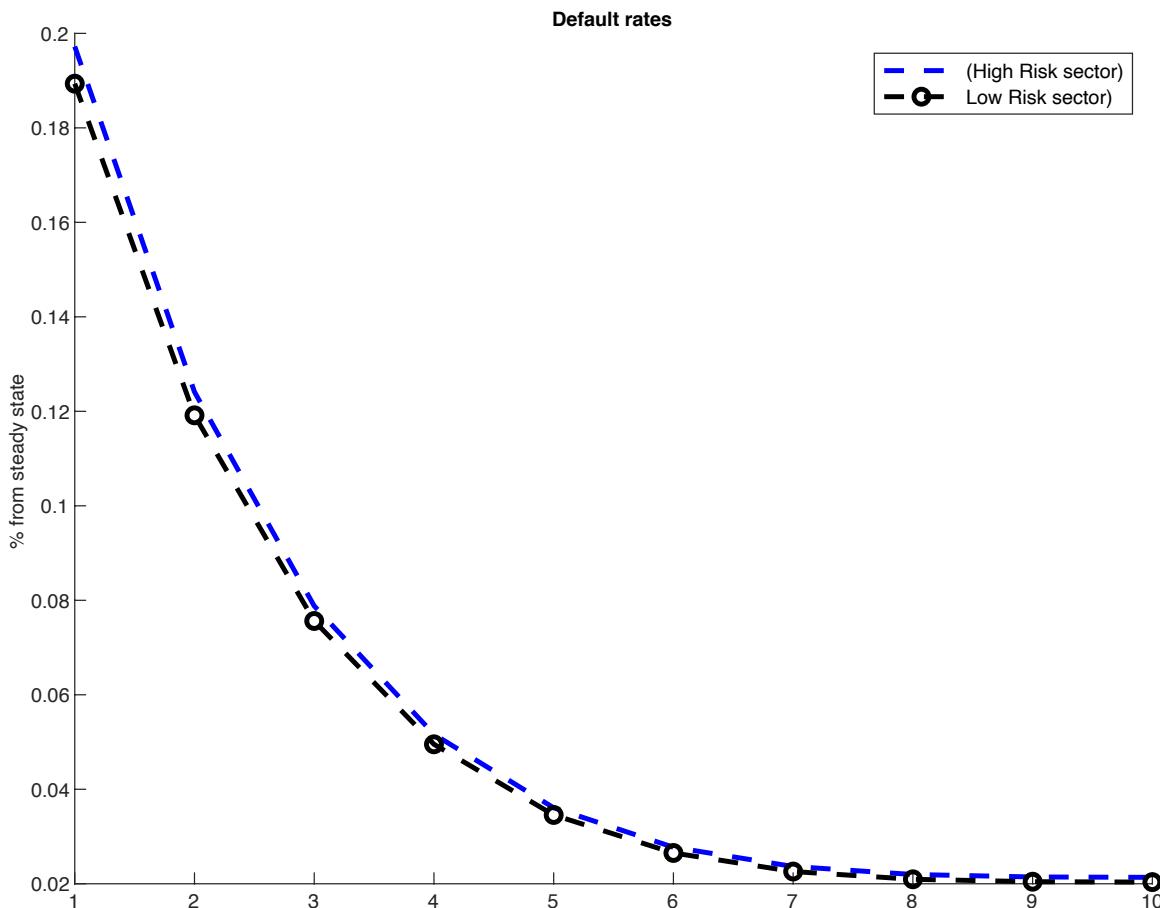


Figure 17: Experiment: Monetary policy shock and Fact 2



## 3.4 Model vs. Data

This section compare the empirical regression on impact to the model, and show some evidence on the link between capitalization rate and recovery rates.

### 3.4.1 Banks' lending response vs. cross-sectional interaction coefficients

This section discusses how the model captures the interaction coefficient from the empirical evidence in terms of the sensitivity of the response to a monetary shock as a function of the capitalization rate. First note that from the data, a standard deviation of the bank capitalization rate is 4.5 percentage points, and banks with a capitalization rate one standard deviation above the mean reduce lending by  $\beta^{\text{micro}} = -0.76$  percentage points.

In the model, the steady state difference between high and low bank capitalization rates is  $\Delta_{HL}^{\text{model}} = 0.2$  percentage points. At this point I perform an exercise to compare the lending response of banks whose capitalization rates differ by as much as the cap-rate in the model. First, the high-cap relative response of lending, normalized  $= \frac{\beta^{\text{micro}}}{SD_{\text{data}}} \times \Delta_{HL}^{\text{model}} = -0.033$  percentage points. Second, the model's high-cap relative response of lending: a 100 bp increase in the interest rate leads to a high-cap relative response of  $-0.0565$  percentage points. Therefore, the model generates enough sensitivity in the response to capitalization rates as the data, but not enough difference in capitalization rates.

### 3.4.2 Proxy of recovery rates and capitalization rates in the data

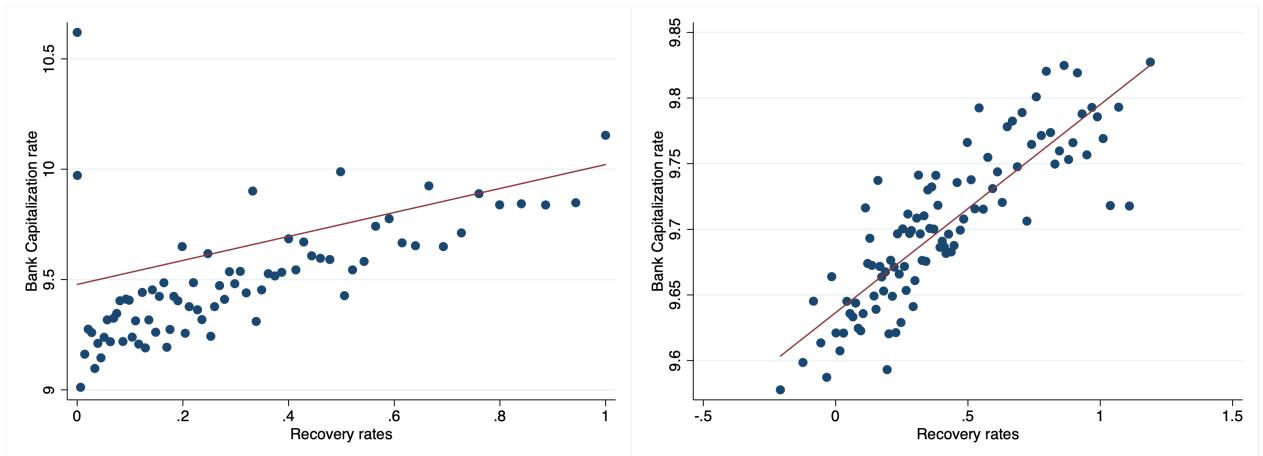
In this subsection I provide direct evidence of the relation between recovery rates and bank capitalization rates. In the model I assume that recovery rates on defaulting loans generate heterogeneity in capitalization rates. From the data, I construct a proxy of banks' recovery rates as a ratio of recoveries on allowances for loan and lease losses to charge-offs on allowances for loan and lease losses.<sup>11</sup>. Figure 18 on

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<sup>11</sup>Recoveries on allowance for loan and lease losses(RIAD4605) and charge-offs on allowance for loan ans lease losses(RIAD4265) on the "Call Report" data base. For further references see [The Fed-Micro Data Reference Manual](#)

the left panel presents bin-scatter plots of the bank capitalization rate against my proxy of bank recovery rates. It shows a positive relation between bank recovery rates and bank capitalization rates. This result is in line with the prediction of my model. Therefore, banks that are better at recovery tend to have a higher capitalization rate. The right panel is the same bin-scatter plot including bank fixed effects. This relation is strongly positive. This evidence strengthens the rationale of the proposed mechanism. See Appendix I for further details and the same analysis by type of loan.

Figure 18: Proxy of recovery rates and capitalization rate



Alternatively, I construct another proxy of recovery rates based on the recovery rates as a fraction of non-performing loans (past due 90 plus non-accrual). This information is available for my period of analysis at the "Call reports", but not for the full period in the case of type of loans. See Appendix J for further details. Figure 19 show a bar figure of the average of the recovery rates on non-performing loans ratio by each capitalization rate percentile group in the all sample. I evidence show a positive relation between recovery rates and bank capitalization rate. Higher capitalization rate banks have on average a higher recovery rates on non-performing loans. In addition, 20 presents bin-scatter plots of the bank capitalization rate against my proxy of bank's recovery rates on non-performing loans.

Figure 19: Recovery rates and capitalization rate

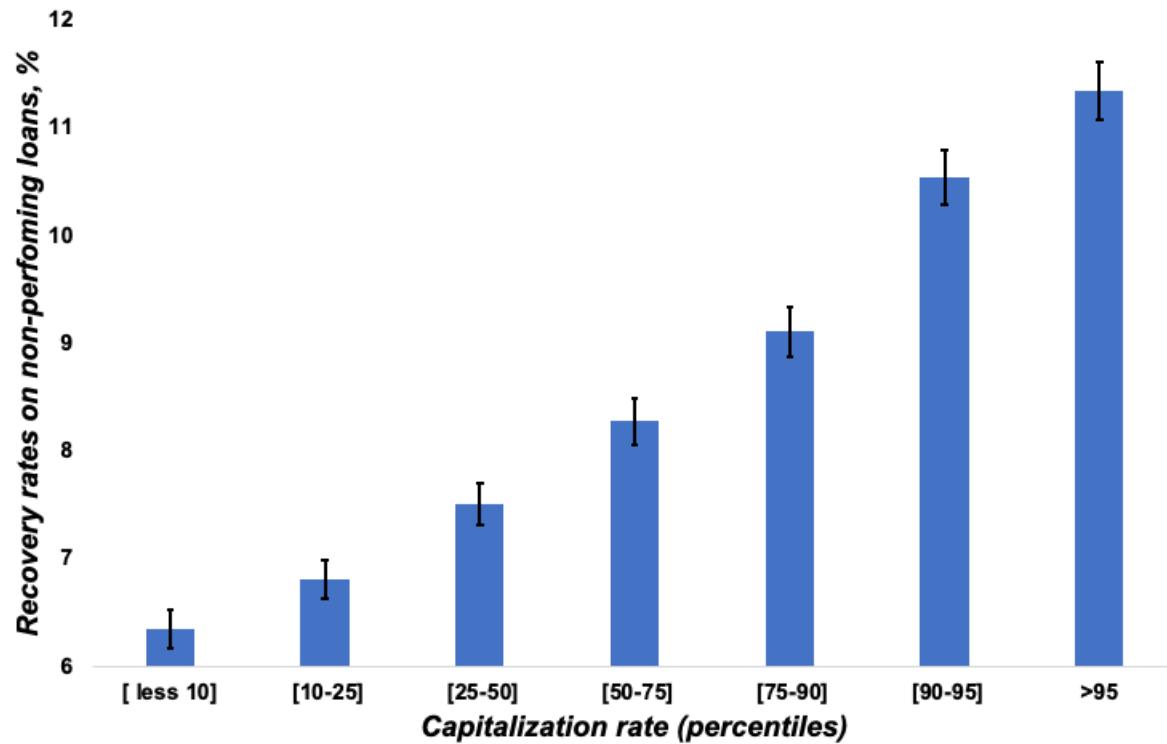
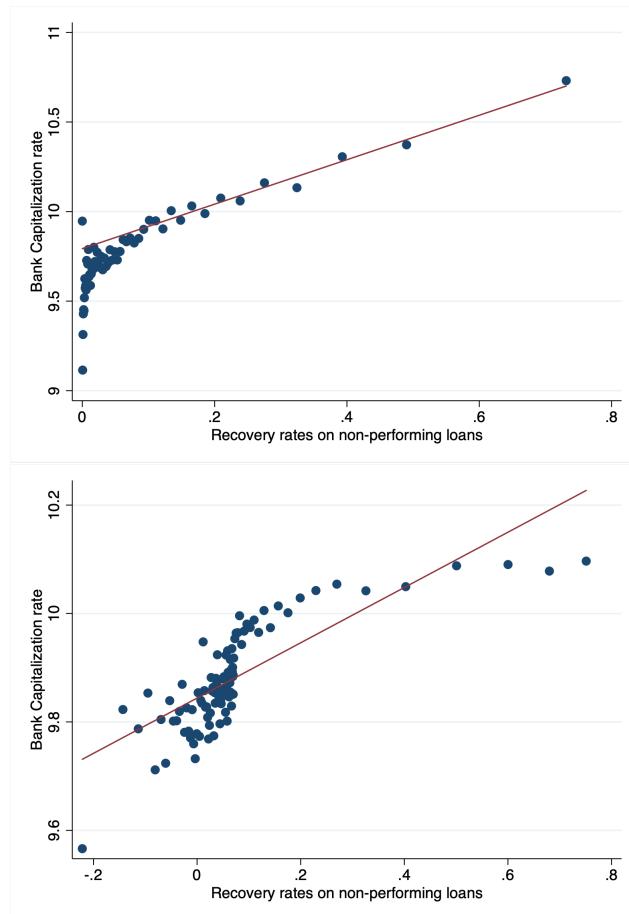


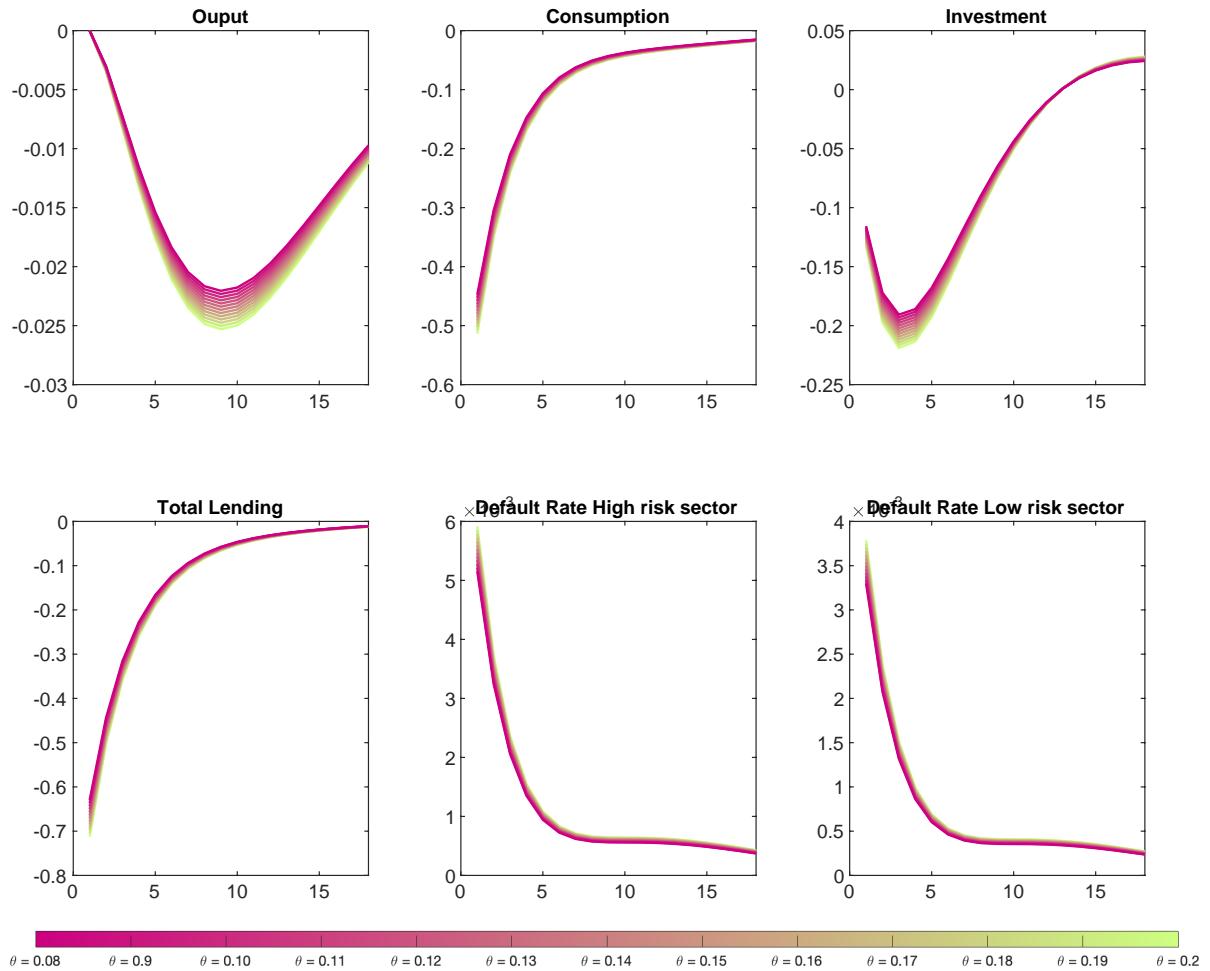
Figure 20: Recovery rates and capitalization rate



### 3.5 Counterfactual Analysis

This section discusses the effect of changing capital requirements. Figure 21 shows the effect on aggregate economic variables like consumption, output, investment, total lending, and default rates. The dark red line represents the baseline case when the capital requirement is 0.08 and the green line represents 0.2 of the minimum capital requirement. I find that, as the capital requirement increases, the effects of a monetary shock are more adverse; i.e., higher capital requirements amplify the effects of a monetary policy shock.

Figure 21: Aggregate: Delinquency responses to monetary policy shock



## 4. Conclusion

In this paper I assess the role of heterogeneity in bank capitalization in the pass-through of monetary policy to bank lending. It provides new empirical evidence using bank-level data, where I find that the capitalization rate plays a crucial role in the transmission of monetary policy to bank lending. Highly capitalized banks have a higher share of commercial and industrial loans and personal loans, which are riskier compared with real estate loans. Highly capitalized banks contract more after a monetary policy tightening, in contrast with the capital view<sup>a</sup> [Van den Heuvel \(2002\)](#). I also propose a theoretical mechanism to support the empirical evidence, based on the default channel and the risk composition of banks' portfolios. In addition, I develop a dynamic macro model with a novel bank heterogeneity feature in the recovery rates for defaulting loans; i.e., and interaction with a risk-weighted asset constraint. Finally, I show in a counterfactual exercise that higher a capital requirement amplifies the effects of monetary policy.

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## A. Empirical Appendix

### A.1 Capitalization rate distribution

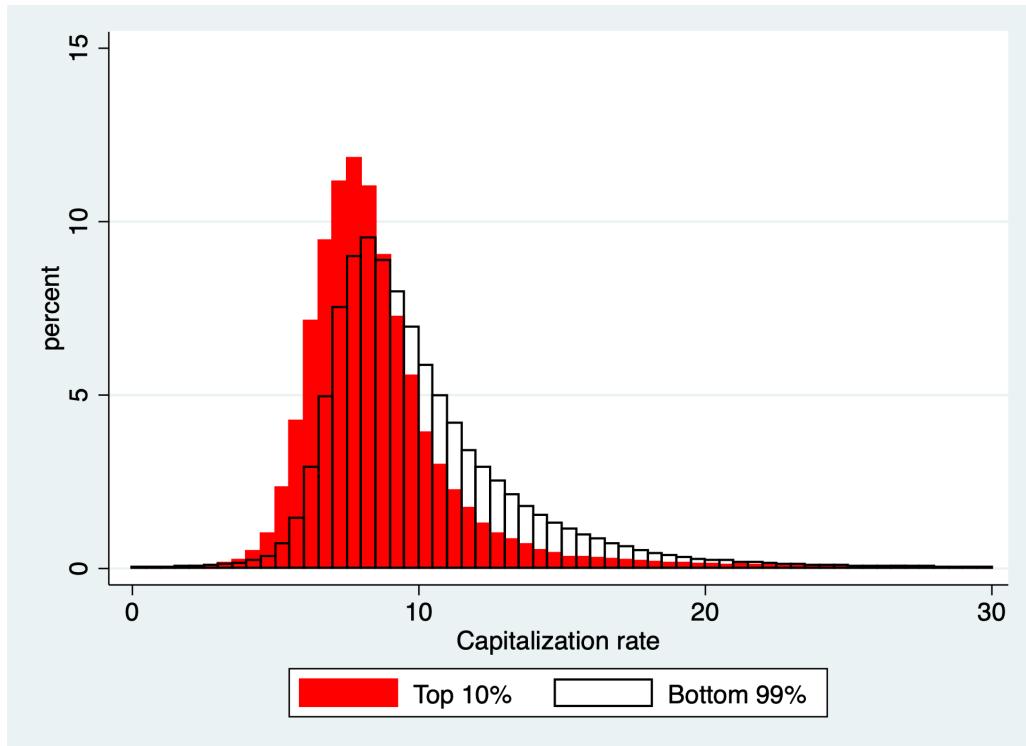


Figure 22: Summary Statistics:

fraction total assets (%)	1990		2000		2007	
	top 10 %	bottom 90%	top 10 %	bottom 90%	top 10 %	bottom 90%
Cash/ fed fund repo	12.3	13.9	7.8	9.2	5.8	9.7
Securities	20.7	30.2	23.2	26.0	17.6	20.9
Loans	63.8	52.8	64.7	61.3	70.2	65.1
Deposits	81.1	88.4	73.9	83.4	74.3	81.8
Other borrowing, fed funds repo	8.0	0.9	13.2	4.1	12.2	4.8
Equity	7.2	9.4	9.1	11.4	10.6	12.5

(Top 10% and bottom 90% refers to total assets)

This table shows that deposits and loans are the most important parts of the balance sheet.

## A.2 Monetary policy shock

The measure of monetary shocks is using the high-frequency movements in the Federal Funds rate in a short window of time around the FOMC announcements or policy meeting (known as event study approach). Following [Gurkaynak \(2005\)](#), [Gorodnichenko and Weber \(2016\)](#), [Wong \(2019\)](#), [Ottonegro and Winberry \(2020\)](#). The monetary policy shock is constructed as

$$\epsilon_t^{MP} = \frac{M}{M-t} (r_{t+\Delta^+}^{FFR} - r_{t-\Delta^-}^{FFR})$$

where  $M$  is the number of days in a month,  $t$  is the time of the monetary announcement,  $r_t^{FFR}$  is the average Fed Funds rate in the month based on Fed Funds futures contract rate up to time  $t^{12}$ ,  $\Delta^-$  is fifteen minutes before the policy announcement and  $\Delta^+$  is forty five minutes after the announcement. The shock series begin in 1990 and ends 2007 in order to focus in conventional monetary policy. Table 7 show some moments of the shocks. First, the raw data have 164 shocks with a mean approximately to zero and standard deviation of 9 basis points. Second, the second column of the table 7 show the statistics of the monetary policy shock smoothed , as for example done by [Ottonegro and Winberry \(2020\)](#). I construct a moving average of the raw shocks weighted by the number of days in the quarter after the shock occurs. Third, show the statistics of monetary policy shocks by simply summing all the shocks occurs within a quarter, as for example done [Wong \(2019\)](#). Figure 23 shows a time series graph of the monetary policy shocks for the different time aggregation. All my results is based on the monetary policy shocks using the time aggregation of simply summing all the shocks within any quarter. For robustness, I also use the alternative time aggregation of monetary policy shock smoothed. I find that my results using these alternative shocks do not significantly differs.

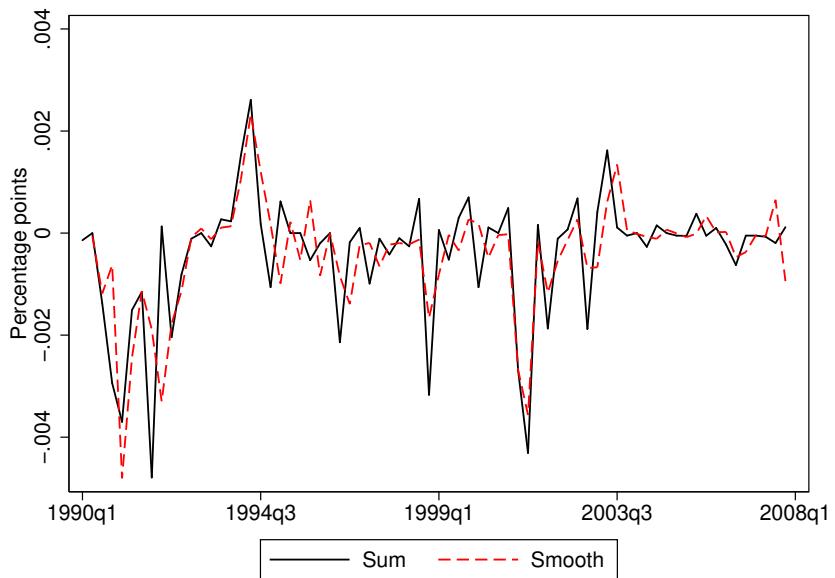
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<sup>12</sup>Fed Funds futures have been traded on Chicago Board of Trade since 1988. The contract for a particular month that pays the average of effective Federal Funds rate over the month.

Table 7: Summary statistics of monetary policy shock

	high frequency	smoothed (quarterly)	Sum (quarterly)
mean	-0.019	-0.043	-0.042
median	0	-0.0127	-0.0051
std	0.086	0.108	0.124
min	-0.463	-0.480	-0.479
max	0.152	0.233	0.261
num	164	71	72

Figure 23: Monetary policy shocks



## B. Comparison to Existing Empirical Literature

This subsection relates my findings to empirical studies documenting heterogeneous responses across banks with different market power on deposits. Subsection B.1 replicates the results of Drechsler et al. (2017) with my sample and shows that including their measure of market power does not affect my results. Subsection B.2 reconciles the empirical evidence of Van den Heuvel (2002) regarding the capitalization rate. Subsection B.3 reconciles the empirical evidence of Kashyap and Stein (2000) regarding the liquidity variable. I wish to make a point that my results differ from the above mentioned due to three main characteristics: (1) I use an identified monetary policy shock instead of changes on the Fed funds rate<sup>13</sup>; (2) I use a different sample period; and (3) the econometric specification. Table 8 summarizes the main differences concerning the main empirical literature that studies the heterogeneous response across banks with different capitalization rates, market power on deposits, and liquidity in the U.S. economy.

Table 8: Comparison to main existing empirical literature

	Monetary Policy Measure	Sample Period and frequency	Individual Analysis	Econometric Specification
Paz (2020)	High frequency identification	1990-2007 quarterly	Bank-level	-Linear regression with bank controls, interaction term, bank fixed effect, state X times fixed effects, standard errors are clustered at bank and time level, macro controls. -Dynamic: Local projection Method -Robustness: Non-Linear regression.
Drechsler, I., Savov, A., and Schnabl, P. (2017, QJE)	Change in Fed Funds	1994-2013 quarterly	Bank-level	Linear regression with interaction term, bank fixed effect, quarter fixed effects, standard error are clustered by bank
Van den Heuvel, (2012, BEJM)	Change in Fed Funds, Bernanke-Mihov indicator	1969-1995 annual	State-level	Linear Regression with interaction term, with state fixed effects
Kashyap, A. K. and Stein, J. C. (2000, AER)	Change in Fed Funds, Bernanke-Mihov indicator	1973-1996 quarterly	Bank-level	Two-Step Regression for different size class

<sup>13</sup>So there is a well known endogeneity problem that the Fed's action creates, which is in response to changes in economic conditions.

## B.1 Relation to Drechsler et al. (2017) and market power on deposit

Drechsler et al. (2017) showed that banks with more market power are more sensitive to changes in the Fed funds rate. First, I replicate their result using my bank-level data, their measure of market power on deposits, and their specification. Table 9 shows my results, which are consistent with the Table VIII in Drechsler et al. (2017). Note that the data are at the bank-quarter level and cover all commercial banks from January 1994 to December 2013. I find that my estimates are consistent with their paper. Second, I replicate the same table, but I consider standard errors clustered at the time and bank levels on the regression<sup>14</sup>. The table 10 shows the results where I consider standard errors clustered at the time and bank levels. I find that the results on the deposit side are still negative and significant. Still, the result on the asset side, particularly for total loans and real estate loans, is not significant. Third, I want to be able to compare my results with their table. Therefore, I replicate the same table, but considering my sample period until 2007. I do this because I focus only on conventional monetary policy and end the sample before the GFC. Additionally, after 2008 monetary policy is not based on the interest rate, but on unconventional monetary policy like QE and forward guidance. Therefore, using the interaction with the Fed Funds rate changes could yield misleading results, because it was not the main monetary policy tool after 2008. Table 11 and 12 show the results considering the pre-crisis period for the case standard errors clustered at the bank level and the case standard errors at the time and bank levels, respectively. In the case of deposits, the interaction coefficient is negative and statistically significant. Banks with higher market power are more sensitive to monetary policy tightening measures by changes in the Fed Funds rate. In addition, in the case of loans, the interaction coefficient is positive and not statistically significant. I will use this coefficient interaction to compare with my result at im-

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<sup>14</sup>Clustering at the bank level allows for fully flexible dependence in the error terms across time within each bank, thereby affecting the estimated standard error. To provide the most conservative confidence intervals, I also cluster at the time level. Without doing this, any confidence intervals on estimates presented tend to be considerably narrower

pact response and the dynamic response.

Table 9: Bank-Level Results replication of Deposit Channel - Bank Liabilities and Lending, 1994-2013

VARIABLES	$\Delta$ Total deposit (1)	$\Delta$ Deposit Spreads (2)	$\Delta$ Savdep (3)	$\Delta$ Time deposit (4)	$\Delta$ wholesale (5)	$\Delta$ Tot Liab (6)
$\Delta FF \times$ bank HHI	-1.493*** [0.145]	0.063*** [0.009]	-1.212*** [0.244]	-2.181*** [0.213]	2.403** [0.947]	-1.296*** [0.139]
Bank f.e.	Y	Y	Y	Y	Y	Y
Quarter f.e.	Y	Y	Y	Y	Y	Y
Cluster Bank Level	Y	Y	Y	Y	Y	Y
Cluster Time Level	N	N	N	N	N	N
Observation	565,341	565,341	565,341	565,341	565,341	565,341
$R^2$	0.160	0.364	0.078	0.166	0.033	0.172
VARIABLES	$\Delta$ Total assets (1)	$\Delta$ cash (2)	$\Delta$ Securities (3)	$\Delta$ Total loans (4)	$\Delta$ Real estate loans (5)	$\Delta$ C&I loans (6)
$\Delta FF \times$ bank HHI	-1.215*** [0.124]	-2.393*** [0.664]	-0.948*** [0.337]	-0.491*** [0.152]	-0.878*** [0.200]	-0.973*** [0.353]
Bank f.e.	Y	Y	Y	Y	Y	Y
Quarter f.e.	Y	Y	Y	Y	Y	Y
Cluster Bank Level	Y	Y	Y	Y	Y	Y
Cluster Time Level	N	N	N	N	N	N
Observation	565,341	565,341	565,341	565,341	565,341	565,341
$R^2$	0.173	0.050	0.062	0.219	0.172	0.060

\*\*\* p<0.01, \*\* p<0.05, \* p<0.1

Table 10: Bank-Level Results replication of Deposit Channel - Bank Liabilities and Lending, 1994-2013

VARIABLES	$\Delta$ Total deposit (1)	$\Delta$ Deposit Spreads (2)	$\Delta$ Savdep (3)	$\Delta$ Time deposit (4)	$\Delta$ wholesale (5)	$\Delta$ Tot Liab (6)
$\Delta FF \times$ bank HHI	-1.493*** [0.506]	0.063*** [0.020]	-1.212 [0.939]	-2.181*** [0.447]	2.403 [2.822]	-1.296*** [0.460]
Bank f.e.	Y	Y	Y	Y	Y	Y
Quarter f.e.	Y	Y	Y	Y	Y	Y
Cluster Bank Level	Y	Y	Y	Y	Y	Y
Cluster Time Level	Y	Y	Y	Y	Y	Y
Observation	565,341	565,341	565,341	565,341	565,341	565,341
R <sup>2</sup>	0.160	0.364	0.078	0.166	0.033	0.172
VARIABLES	$\Delta$ Total assets (1)	$\Delta$ cash (2)	$\Delta$ Securities (3)	$\Delta$ Total loans (4)	$\Delta$ Real estate loans (5)	$\Delta$ C&I loans (6)
$\Delta FF \times$ bank HHI	-1.215*** [0.408]	-2.393** [1.072]	-0.948 [0.738]	-0.491 [0.502]	-0.878 [0.549]	-0.973** [0.462]
Bank f.e.	Y	Y	Y	Y	Y	Y
Quarter f.e.	Y	Y	Y	Y	Y	Y
Cluster Bank Level	Y	Y	Y	Y	Y	Y
Cluster Time Level	Y	Y	Y	Y	Y	Y
Observation	565,341	565,341	565,341	565,341	565,341	565,341
R <sup>2</sup>	0.173	0.050	0.062	0.219	0.172	0.060

\*\*\* p<0.01, \*\* p<0.05, \* p<0.1

Table 11: Bank-Level Results replication of Deposit Channel - Bank Liabilities and Lending, 1994-2007

VARIABLES	$\Delta$ Total deposit (1)	$\Delta$ Deposit Spreads (2)	$\Delta$ Savdep (3)	$\Delta$ Time deposit (4)	$\Delta$ wholesale (5)	$\Delta$ Tot Liab (6)
$\Delta FF \times$ bank HHI	-0.676*** [0.156]	0.087*** [0.011]	0.421 [0.272]	-2.112*** [0.257]	4.656*** [1.118]	-0.475*** [0.152]
Bank f.e.	Y	Y	Y	Y	Y	Y
Quarter f.e.	Y	Y	Y	Y	Y	Y
Cluster Bank Level	Y	Y	Y	Y	Y	Y
Cluster Time Level	N	N	N	N	N	N
Observation	416,901	416,901	416,901	416,901	416,901	416,901
R <sup>2</sup>	0.159	0.309	0.079	0.146	0.028	0.169
VARIABLES	$\Delta$ Total assets (1)	$\Delta$ cash (2)	$\Delta$ Securities (3)	$\Delta$ Total loans (4)	$\Delta$ Real estate loans (5)	$\Delta$ C&I loans (6)
$\Delta FF \times$ bank HHI	-0.465*** [0.135]	-3.079*** [0.644]	0.113 [0.380]	0.195 [0.195]	0.143 [0.255]	-0.148 [0.428]
Bank f.e.	Y	Y	Y	Y	Y	Y
Quarter f.e.	Y	Y	Y	Y	Y	Y
Cluster Bank Level	Y	Y	Y	Y	Y	Y
Cluster Time Level	N	N	N	N	N	N
Observation	416,901	416,901	416,901	416,901	416,901	416,901
R <sup>2</sup>	0.170	0.057	0.058	0.199	0.150	0.050

\*\*\* p<0.01, \*\* p<0.05, \* p<0.1

**Table 12: Bank-Level Results replication of Deposit Channel - Bank Liabilities and Lending, 1994-2007**

VARIABLES	$\Delta$ Total deposit (1)	$\Delta$ Deposit Spreads (2)	$\Delta$ Savdep (3)	$\Delta$ Time deposit (4)	$\Delta$ wholesale (5)	$\Delta$ Tot Liab (6)
$\Delta FF \times$ bank HHI	-0.676 [0.509]	0.087*** [0.015]	0.421 [0.884]	-2.112*** [0.550]	4.656 [3.760]	-0.475 [0.408]
Bank f.e.	Y	Y	Y	Y	Y	Y
Quarter f.e.	Y	Y	Y	Y	Y	Y
Cluster Bank Level	Y	Y	Y	Y	Y	Y
Cluster Time Level	Y	Y	Y	Y	Y	Y
Observation	416,901	416,901	416,901	416,901	416,901	416,901
R2	0.159	0.309	0.079	0.146	0.028	0.169
VARIABLES	$\Delta$ Total assets (1)	$\Delta$ cash (2)	$\Delta$ Securities (3)	$\Delta$ Total loans (4)	$\Delta$ Real estate loans (5)	$\Delta$ C&I loans (6)
$\Delta FF \times$ bank HHI	-0.465 [0.334]	-3.079** [1.520]	0.113 [0.794]	0.195 [0.618]	0.143 [0.477]	-0.148 [0.532]
Bank f.e.	Y	Y	Y	Y	Y	Y
Quarter f.e.	Y	Y	Y	Y	Y	Y
Cluster Bank Level	Y	Y	Y	Y	Y	Y
Cluster Time Level	Y	Y	Y	Y	Y	Y
Observation	416,901	416,901	416,901	416,901	416,901	416,901
R2	0.170	0.057	0.058	0.199	0.150	0.050

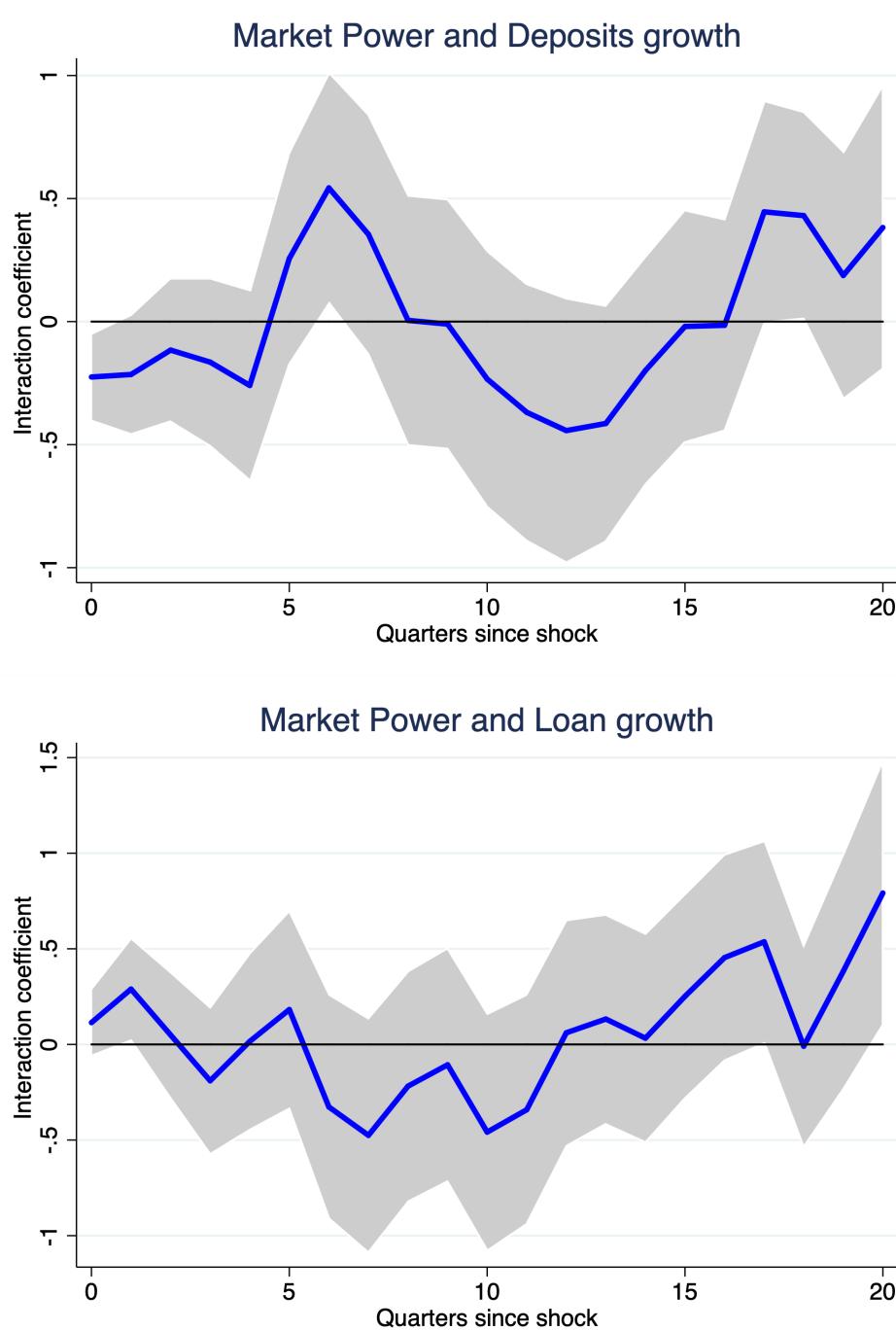
\*\*\* p<0.01, \*\* p<0.05, \* p<0.1

This section, I show my results using the main econometric specification used in the paper, with my measure of monetary policy shocks for the dependent variable deposit growth and loan growth. First, for the case of deposit growth, the top part of the figure 24 shows the response of deposit growth to monetary policy shock considering the interaction with market power. I find that banks with higher market power reduce more their deposit on impact than bank with lower market power. I conclude this is consistent with the deposit channel's replication table 11, which considers the sample until 2007. Second, for the dependent variable loan growth, I find the loan response is positive and not significant on impact. Again, I conclude this is consistent with the deposit channel's replication table 11<sup>15</sup>.

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<sup>15</sup>This result shows that my specification and my measure of monetary policy shock are consistent with the effect on impact on the QJE's paper for deposit growth and loan growth

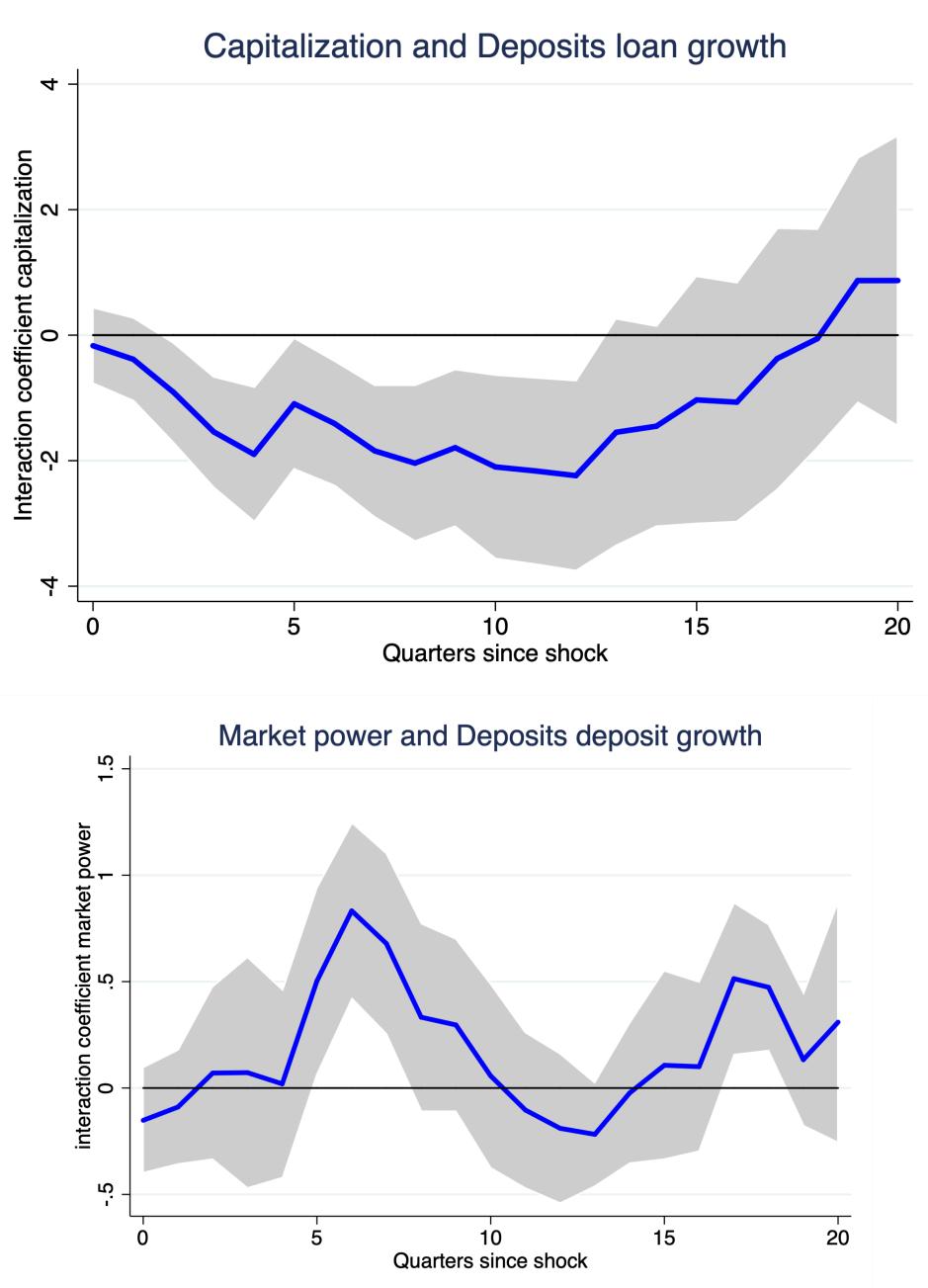
Figure 24: Dynamics of Differential Response to Monetary Shocks: Market Power



*Notes:* dynamics of the interaction coefficient between capitalization rate and monetary shocks over time.

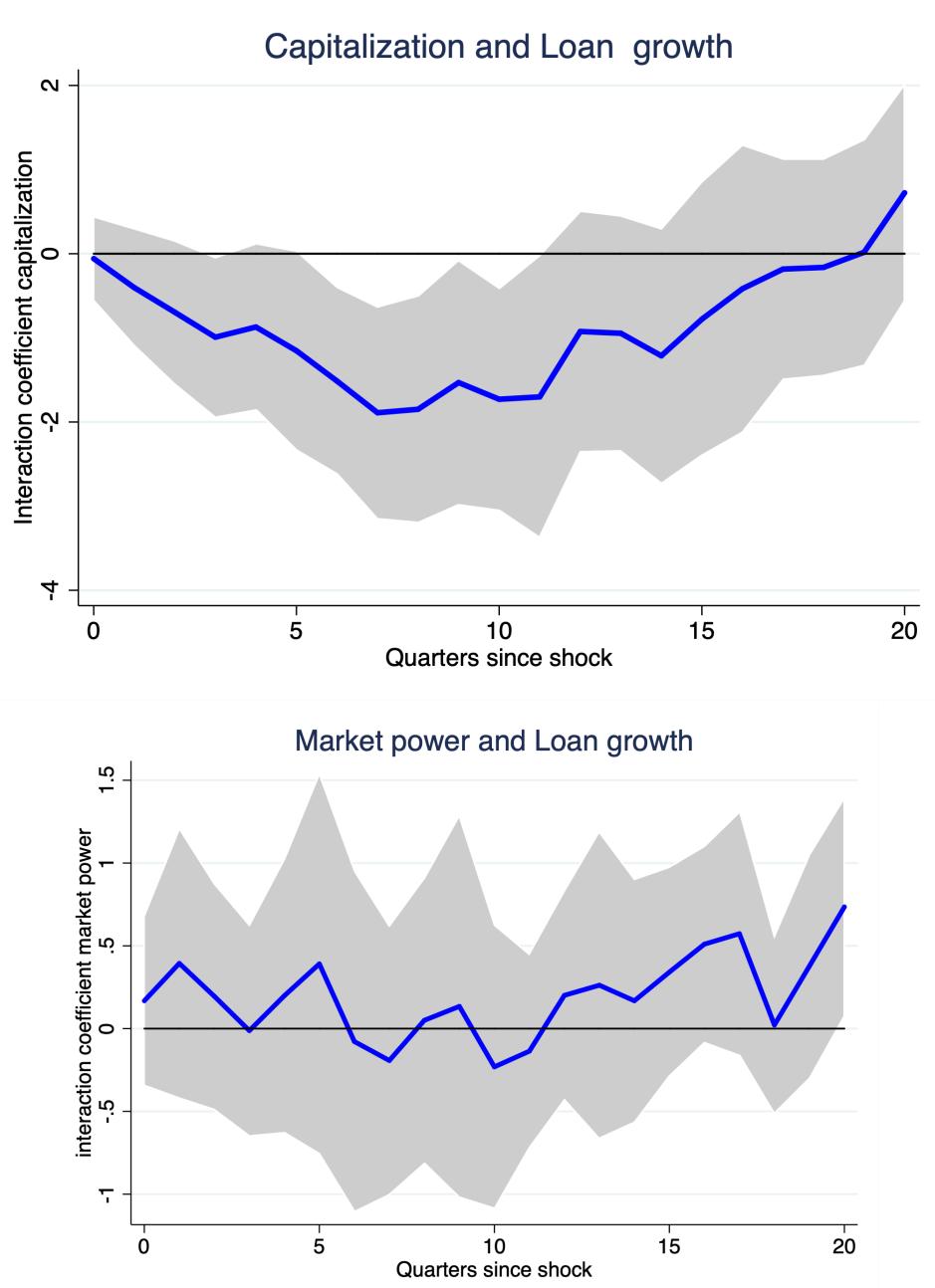
Now, I show if the above result survives considering the capitalization rate jointly with market power. First, in the case of deposit growth, the top part of the figure 25 shows the coefficient of interaction associated with capitalization rate and monetary policy shock, the bottom part of the figure 25 shows the coefficient of interaction associated with market power and monetary policy shock. I find that market power's effect on deposit growth disappears or is not statistically significant on impact. Also, the effect of the capitalization rate is negative on impact, then persistent negative and statistically significant going forward. Therefore the effect of the capitalization rate is important. Second, in the case of loan growth, the top part of figure 26 shows the coefficient of interaction associated with capitalization rate and monetary policy shock, the bottom part of the figure 26 shows the coefficient of interaction associated with market power and monetary policy shock. I find that the effect on loan growth of market power disappears or is not statistically significant on impact and going forward. Also, the effect of capitalization rate is on average negative and persistence negative going forward too. Therefore the effect of the capitalization rate is essential.

Figure 25: Dynamics of Differential Response to Monetary Shocks: Market Power



*Notes:* dynamics of the interaction coefficient between capitalization rate and monetary shocks over time.

Figure 26: Dynamics of Differential Response to Monetary Shocks: Market Power

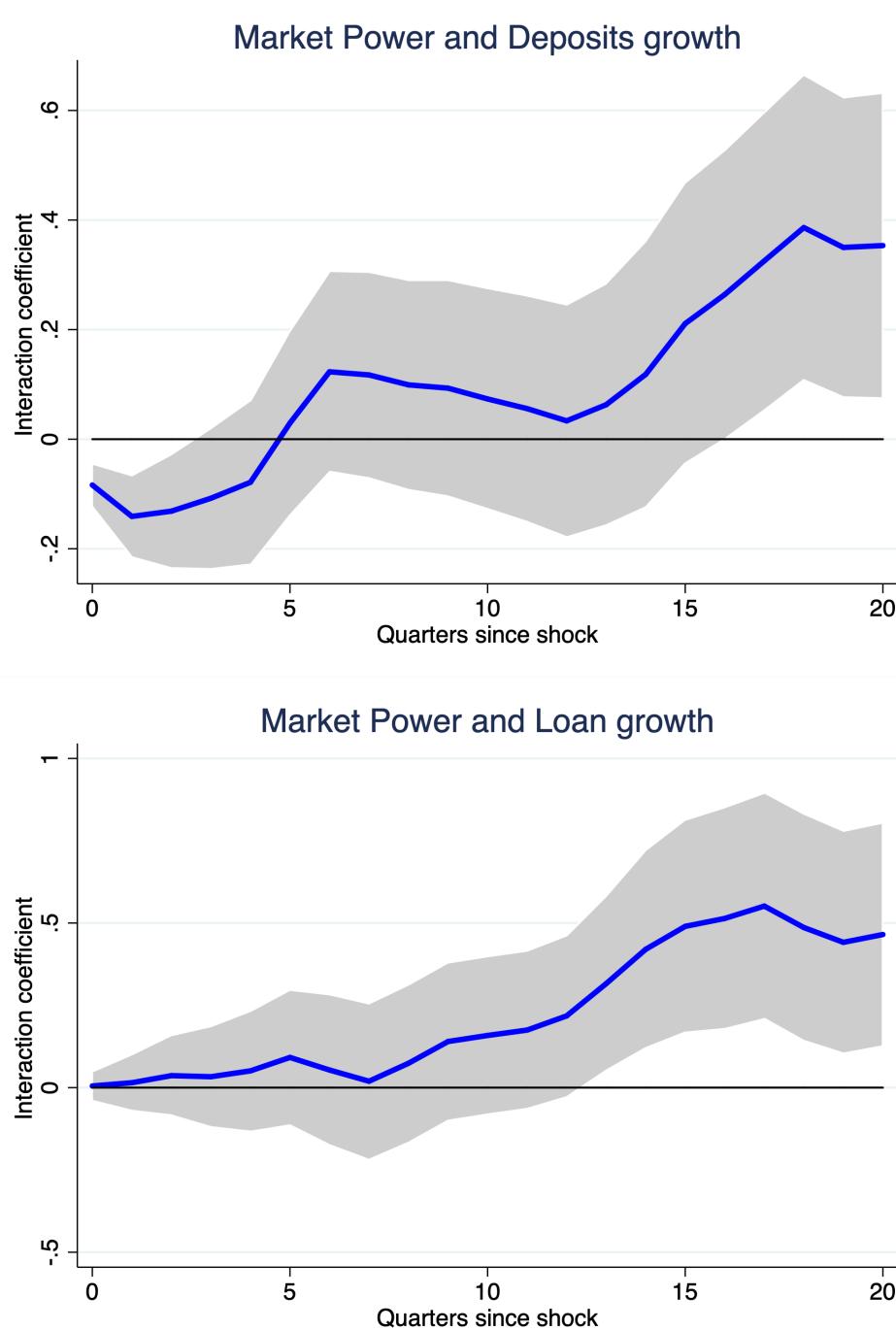


*Notes:* dynamics of the interaction coefficient between capitalization rate and monetary shocks over time.

Finally, I reproduce the same econometric specification for the dynamic response but using Fed Funds Rate's change as a measure of monetary policy shock, following Drechsler et al. (2017). Figure 27 shows the result considering only market power for the dependent variable deposits and loan growth. Figure 28 considers the double interaction of market power and capitalization rate for the dependent variable deposits growth. Similarly, I find that market power's effect on deposit growth disappears or is not statistically significant on impact, and the effect of the bank capitalization rate is negative and statistically significant and persistent negative going forward. Therefore again the effect of the capitalization rate is important. The figure 29 considers the double interaction of market power and capitalization rate for the dependent variable loan growth. I find my results hold qualitative on impact, but the dynamic results are different using their monetary policy tightening measure.

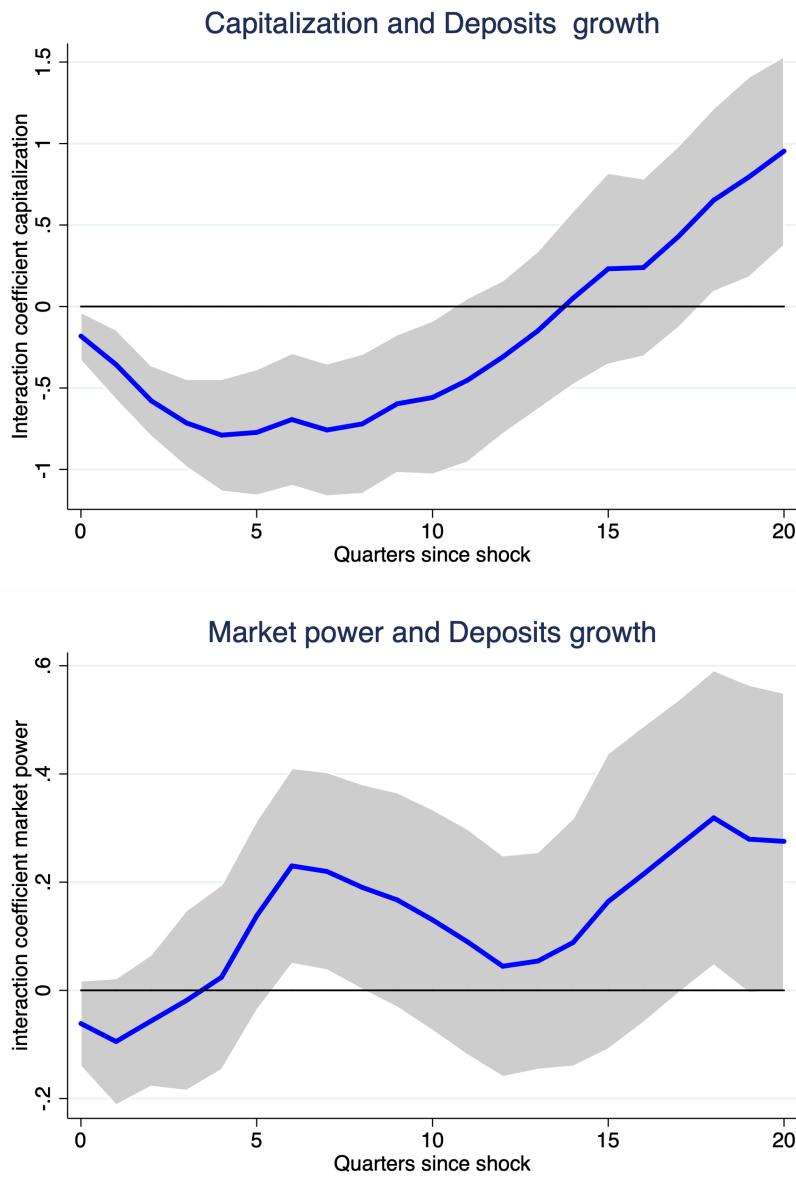
I view these findings as reflecting that the market power mechanism loses power or disappears when I consider bank capitalization in the regressions, which comes.

Figure 27: Dynamics of Differential Response to Fed Funds Rate: Market Power



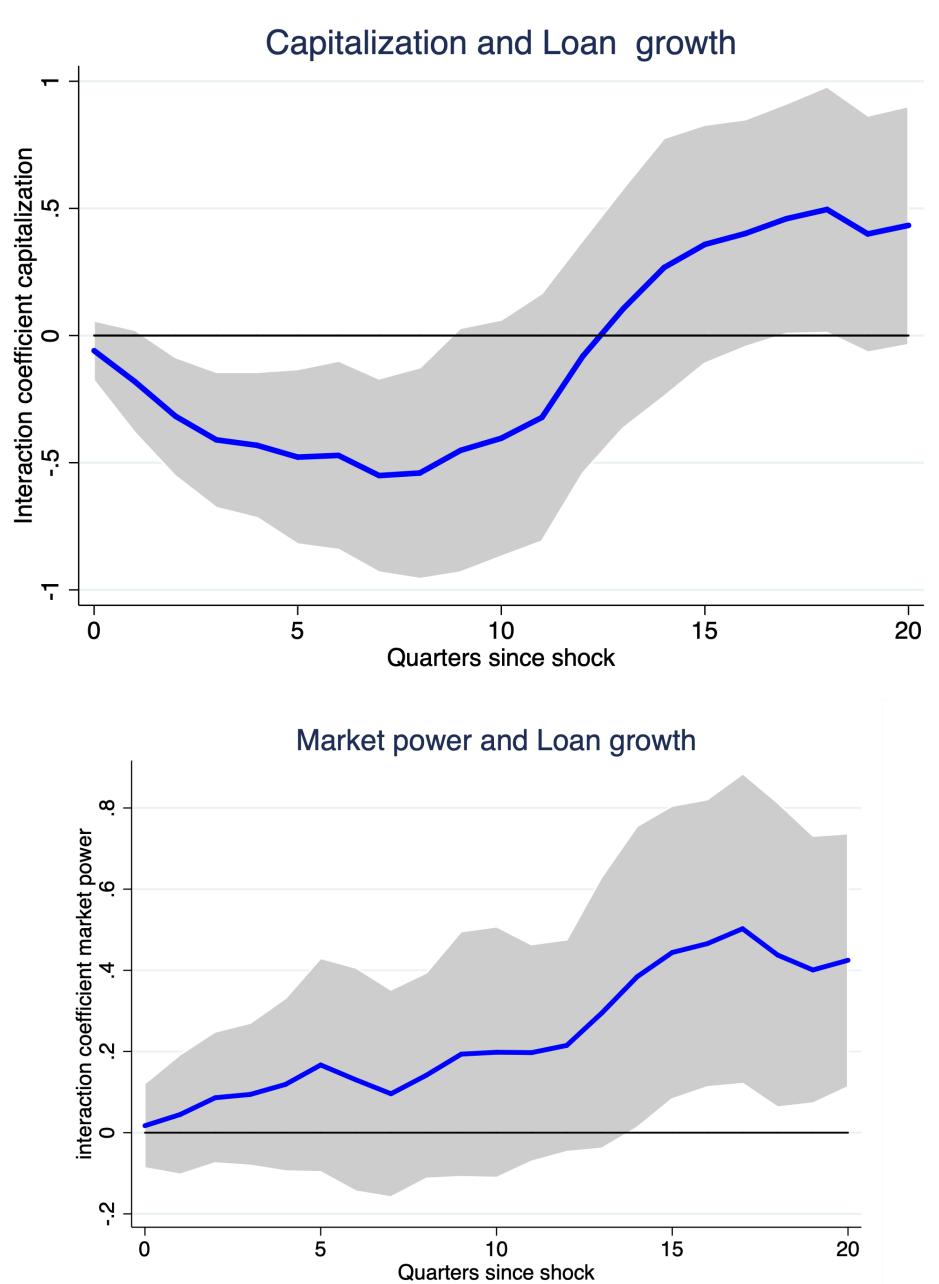
*Notes:* dynamics of the interaction coefficient between capitalization rate and monetary shocks over time.

Figure 28: Dynamics of Differential Response to Fed Funds Rate: Capitalization rate and Market Power



*Notes:* dynamics of the interaction coefficient between capitalization rate and monetary shocks over time.

Figure 29: Dynamics of Differential Response to Fed Funds Rate: Capitalization rate and Market Power



*Notes:* dynamics of the interaction coefficient between capitalization rate and monetary shocks over time.

## B.2 Relation to Van den Heuvel (2002)

In this subsection, I explain the main differences with the [Van den Heuvel \(2002\)](#). [Van den Heuvel \(2002\)](#) shows that lower capitalized states are more sensitive to monetary policy. First, the main difference is the data limitation, specifically the type of data. His data set is at the state-level and not the individual bank-level. Second, his econometric specification is different. I could say that there is an aggregation problem. All his analysis is at the state level; therefore, the analysis of the heterogeneity at the bank level is lost, and the relation can be misleading. For example, for the aggregate capitalization rate in a given state, there is no information about the capitalization rate distribution across banks in that state. It could be the case that a state may have a higher capitalization rate because one bank could have a high capitalization rate. Still, the other banks in the same state could have lower capitalization rates, but the banks with lower capitalization rates can account for more loans. Therefore, there could be a misleading relationship, which could be the case of a state with a higher capitalization bank responding less to their lending. Thus, all the changes in loans come from the banks that have a lower capitalization rate, but the higher-capitalized bank moves the main variable of interest capitalization rate at the state level. Then it could be the case that the state that is highly capitalized does not respond to monetary policy, but this is not the same as higher-capitalized banks not responding to monetary policy because it comes only due to the aggregation at the state level. I view these findings as reflecting the fact that I analyze a different dimension of the data. The main differences are individual analysis, monetary policy shock, the period of the sample, and econometric specification.

## B.3 Relation to Kashyap and Stein (2000)

In this subsection I explain the main differences with the [Kashyap and Stein \(2000\)](#). [Kashyap and Stein \(2000\)](#) shows bank lending contracts when monetary policy tightens, the contraction is stronger for less liquid banks, and the sensitivity of the

contraction to liquidity is stronger for small banks. This traditional result comes from a close connection between reserves and deposits, and the idea is that a bank has a reserve requirement. A contractionary monetary policy reduces the amount of reserves. This then has an impact on deposits, unless banks have sufficient liquidity or sufficient capacity to replace deposits with other types of funds. Therefore, if a bank is less liquid, then it contracts its lending more. The main differences in my paper are the following: First, I used an identified monetary policy shock using high-frequency data. They instead use the Fed Funds rate and other two different monetary measures, such as the Bernanke and Mihov, and Boschen-Mills indexes. Second, I used bank-level quarterly data for the period 1990-2007, focusing on all commercial banks in the sample. They also use bank-level data, but the period of the analysis is 1973-1996 quarterly, and they split banks into three size groups ( $< 95$ th percentile,  $95-99$  percentile,  $> 99$  percentile) and the measure of liquidity is the ratio of securities and Fed Funds contracts sold to total assets.

Third, my econometric specification is more general and considers a better identification strategy of the measure of monetary policy shock. My baseline dynamic model are robust to bank controls, banked fixed effects, state times time fixed effect, risk weighted regression, and cluster at bank and time level. They have a different econometric specification, which consists in running a two-part regression. First, for all  $t$  in they sample, for each size group  $g$ , they individually estimate:

$$\Delta \log(L_{it}) = \sum_{j=1}^4 \alpha_{gtj} \Delta \log(L_{i,t-j}) + \beta_{gt} B_{it-1} + \sum_{k=1}^{12} \Psi_{gkt} FRB_{ik} + \epsilon_{it}$$

where  $\beta_{g,t}$  captures the sensitivity of lending changes to liquidity for size group  $g$  in period  $t$ ,  $L_{it}$  is total lending,  $B_{it-1}$  liquidity . Then, they estimate

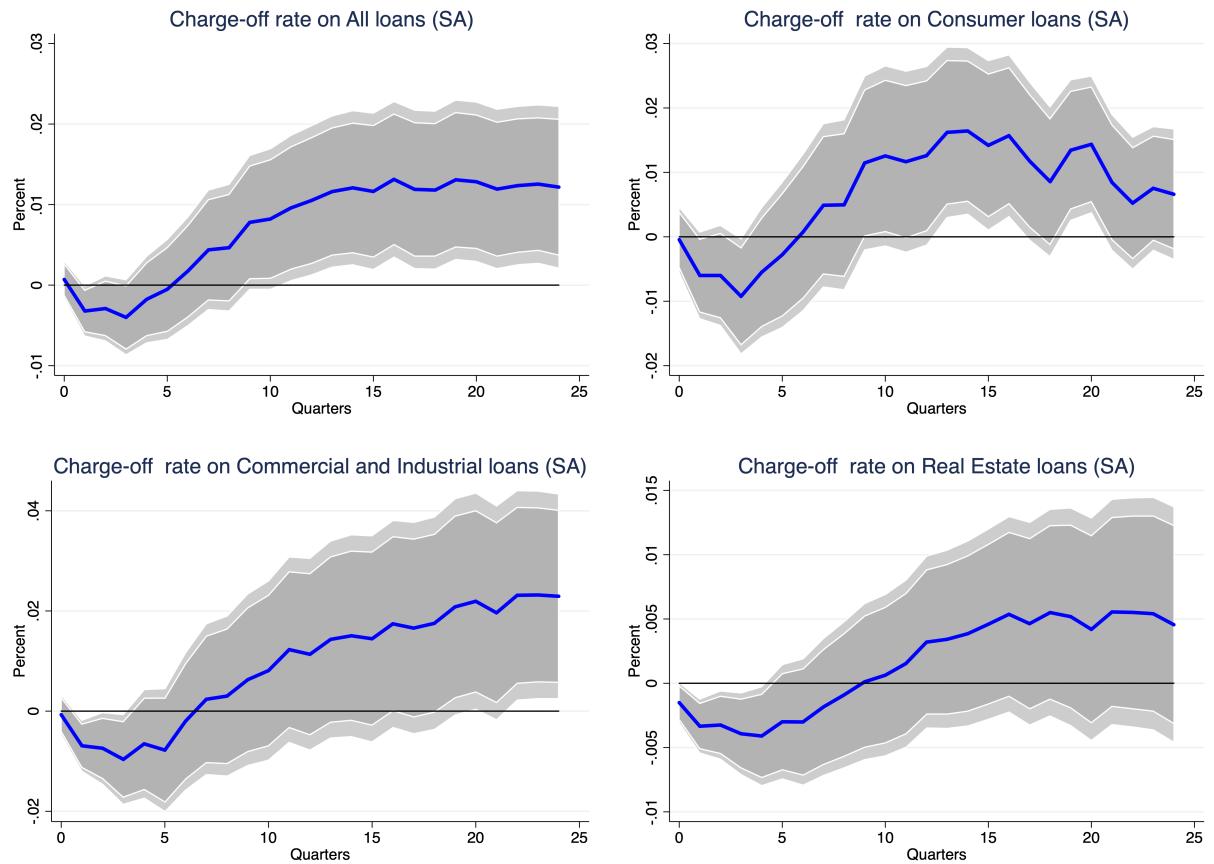
$$\hat{\beta}_{g,t} = \eta_g + \sum_{j=0}^4 \phi_{j,g} \Delta M_{t-j} + \delta_g t + u_{g,t}$$

where  $M_t$  is the measure of monetary policy , $\sum_{j=0}^4 \phi_{j,g}$  captures the correlation be-

tween lagged monetary policy and lending sensitivity to liquidity for size group  $g$ . They also try a 'bivariate' regression, where they add a four-quarter flexible form distributed lag function on GDP growth. This technique is a precursor of modern empirical macro literature. The paper does not meet the standard for identification today.

## C. Default rates and monetary policy

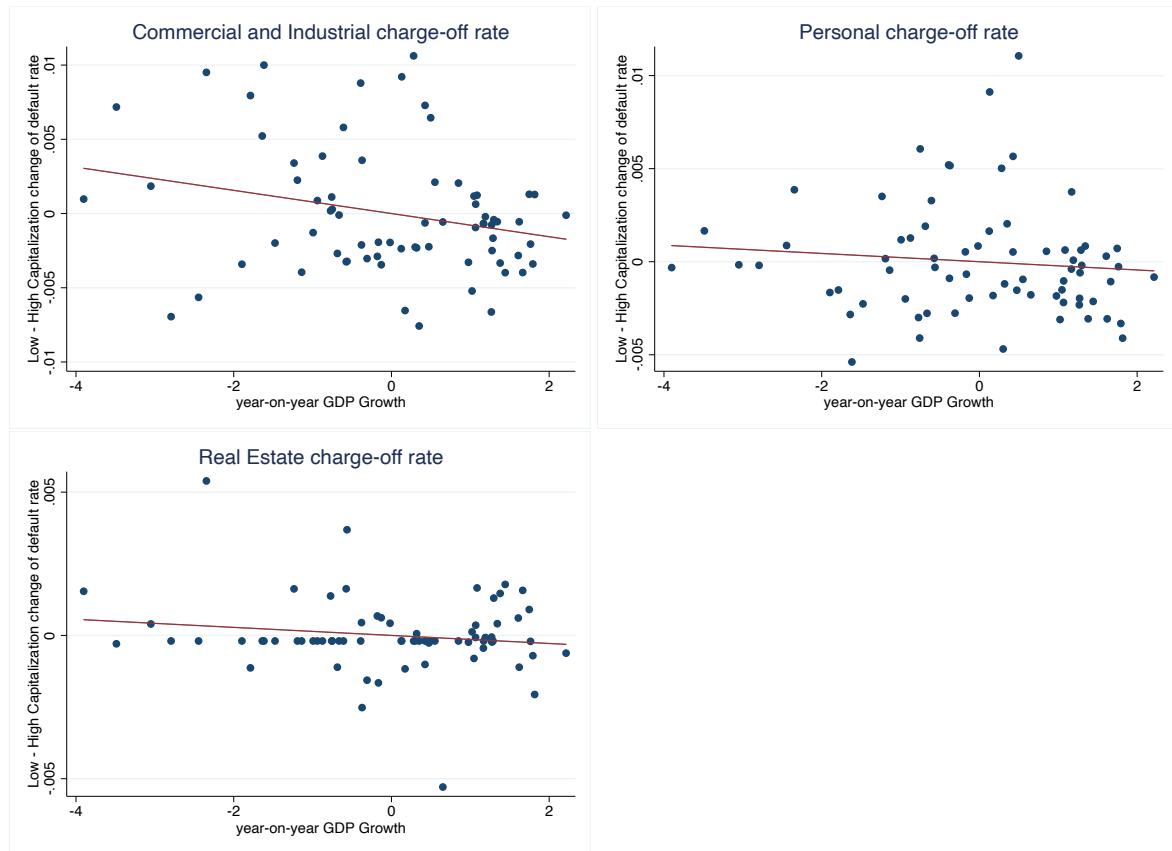
Figure 30: Aggregate: Charge-off responses to monetary policy shock



## D. Relationship bank capitalization, default rates and business cycles

This section documents the relation between relationship bank capitalization rate, default rates and business cycles. First, I study how the default rate of lower capitalization bank minus higher capitalization bank is affected by the GDP growth. I find that there is a negative relation between default rate and GDP growth but the effect across banks is not statistically different.

Figure 31: Difference between low and high capitalization of charge-off rates and year-on-year GDP growth



Second, I study the same question but now with the following specification that allow me control for banks fixed effects and state fixed effects. The empirical

model is as follow:

$$y_{i,t} = \sum_{j \in J} (\beta_j + \alpha_j \Delta GDP_t) \mathbb{1}_{\{i=I\}} + \sum_{s \in S} (\gamma_s + \delta_s \Delta GDP_t) \mathbb{1}_{\{i=S\}} + \epsilon_{i,t} \quad (23)$$

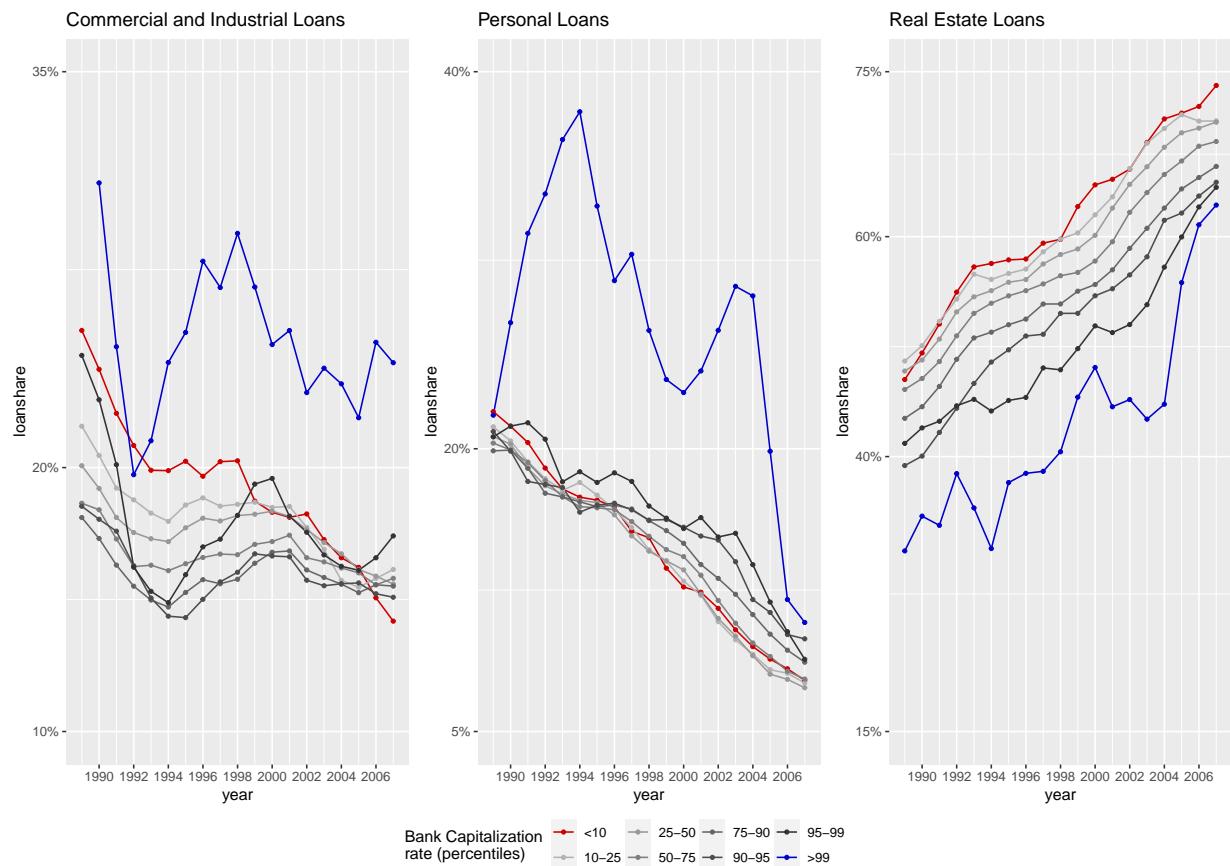
where: where  $i$  identified a bank, and  $t$  a quarter. The depend variable  $y_{i,t}$ , is the year -on-year change in charge-off rate. The set  $J$  define a capitalization rate group, I define 5 groups and each group have a 20 percent of assets. Moreover,  $\Delta GDP_t = \log(\frac{GDP_t}{GDP_{t-4}})$  is the year-on-year growth rate of GDP, and  $S$  is a set of US's states. The table 13 show the GDPgrowth does not have effects on the default rates across capitalization rate and across types of loan. There is only significant for lower capitalization bank in the bottom of the distribution.

Table 13: Regression of charge-off rates on GDP growth for banks

	(1)	(2)	(3)
[10-25]x GDP growth	0.035*** (0.01)	0.004 (0.00)	0.012 (0.01)
[25-50]x GDP growth	0.034** (0.01)	0.003 (0.00)	0.015* (0.01)
[50-75]x GDP growth	0.036** (0.01)	0.007** (0.00)	0.009 (0.01)
[75-90]x GDP growth	0.061*** (0.02)	0.002 (0.00)	0.011 (0.01)
[90-95]x GDP growth	0.033 (0.02)	0.005 (0.00)	0.043** (0.02)
[>95,100]x GDP growth	0.010 (0.04)	0.003 (0.00)	0.039 (0.03)
GDPgrowth	-0.114*** (0.03)	-0.006 (0.00)	-0.042** (0.02)
Observations	216108	392147	216879
R <sup>2</sup>	0.041	0.043	0.056
State controls	yes	yes	yes
Bank fixed effects	yes	yes	yes
Quarter fixed effects	yes	yes	yes
Bank Time clustering	yes	yes	yes

## E. Loan's portfolio composition of banks:

Figure 32: Average portfolio share for real estate loan across bank capitalization percentiles



## F. Baseline model

### F.1 Firm problem

$$\begin{aligned}
 V(n; q^a) &= \max_{k'} \text{div} + \mathbb{E}_t[\mathcal{M}_{t,t+1}^B \tilde{V}(k'; q^{a'})] \\
 \text{div}(k, k', l; q^a, \omega) &= \underbrace{\omega k^{1-\alpha} l^\alpha}_{\text{Revenues}} - \underbrace{(k' - (1 - \delta_k)k)}_{\text{investment}} - [(1 - \phi)wl + a(\frac{1}{q^a})] \\
 a &= \phi wl \\
 \omega^* &= \frac{(1 + \phi(\frac{1}{q_t^a} - 1))w_t l_t}{k^{1-\alpha} l^\alpha} \\
 \tilde{V}(k; q^a) &= \max_{l_t} [\Omega_A(\omega_t^*) \mathbb{E}_t(V(n; q^a) | \omega_t > \omega_t^*)]
 \end{aligned}$$

Solution:

$$\begin{aligned}
 V(n; q^a) &= \max_{k'} \text{div} + \mathbb{E}_t[\mathcal{M}_{t,t+1}^B \tilde{V}(k'; q^{a'})] \\
 \text{div} &= \omega k^{1-\alpha} l^\alpha - [1 + \phi(\frac{1}{q^a} - 1)]wl + (1 - \delta_k)k = n - k' + a \\
 \tilde{V}(k; q^a) &= \max_{l_t} [\Omega_A(\omega_t^*) \mathbb{E}_t(V(n; q^a) | \omega_t > \omega_t^*)] \\
 \omega^* &= \frac{(1 + \phi(\frac{1}{q_t^a} - 1))w_t l_t}{k^{1-\alpha} l^\alpha}
 \end{aligned}$$

Step 1:

$$\tilde{V}(k; q^a) = \max_l [\Omega_A(\omega_t^*) \mathbb{E}_t(V(n; q^a) | \omega_t > \omega_t^*)] = \max_{l_t} [\Omega_A(\omega_t^*) v(q^a) \mathbb{E}_t(n | \omega > \omega^*)]$$

$\Omega_A(\omega^*) = 1 - F(\omega^*)$  where  $F$  is the probability of default.

$$n = \omega k^{1-\alpha} l^\alpha - (1 - \phi)wl - (\frac{1}{q^a})a + (1 - \delta_k)k \Rightarrow \mathbb{E}_t(n_t^P | \omega > \omega^*) = (1 - \delta_k)k$$

Note that:

$$\mathbb{E}_t(n|\omega > \omega^*) = \mathbb{E}_t\left(\left[\omega k^{1-\alpha}l^\alpha - (1-\phi)wl - \left(\frac{1}{q^a}\right)a + (1-\delta_k)k\right]|\omega_t > \omega_t^*\right)$$

If  $E[]$  is over  $\omega$ , check:

$$\begin{aligned} & E([\omega k^{1-\alpha}l^\alpha - (1-\phi)wl - \left(\frac{1}{q^a}\right)a + (1-\delta_k)k]|\omega > \omega^*) \\ & E([\omega|\omega > \omega^*])k^{1-\alpha}l^\alpha - (1-\phi)wl - \left(\frac{1}{q^a}\right)a + (1-\delta_k)k \\ & \omega^+k^{1-\alpha}l^\alpha - (1-\phi)wl - \left(\frac{1}{q^a}\right)(\phi wl) + (1-\delta_k)k \\ & \omega^+y - (1-\phi + \frac{1}{q^a}\phi)wl + (1-\delta_k)k \\ & y\left(\omega^+ - (1-\phi + \frac{1}{q^a}\phi)\frac{wl}{y} + (1-\delta_k)\frac{k}{y}\right) \\ & y\left(\omega^+ - \omega^* + (1-\delta_k)\frac{k}{y}\right) \\ & y(\omega^+ - \omega^*) + (1-\delta_k)k \end{aligned}$$

$$\mathbb{E}_t(n_t|\omega_t > \omega_t^*) = y(\omega^+ - \omega^*) + (1-\delta_k)k$$

$$\begin{aligned} V(n; q^a) &= \max_{k'} \text{div} + \mathbb{E}_t[m_{t,t+1}^B \tilde{V}(k'; q^{a'})] \\ V(n; q^a) &= \max_{k'} \omega k^{1-\alpha}l^\alpha - [1 + \phi(\frac{1}{q^a} - 1)]wl + (1-\delta_k)k + \mathbb{E}_t[m_{t,t+1}^B \tilde{V}(k'; q^{a'})] \\ V(n; q^a) &= n - k' + a + \mathbb{E}_t[m_{t,t+1}^B \tilde{V}(k'; q^{a'})] \\ \tilde{V}(k; q^a) &= \max_{l_t} [\Omega_A(\omega_t^*) \mathbb{E}_t(V(n; q^a) | \omega_t > \omega_t^*)] \\ \omega^* &= \frac{(1 + \phi(\frac{1}{q_t^a} - 1))w_t l_t}{k^{1-\alpha}l^\alpha} \end{aligned}$$

So clearly,  $V(n)$  is homogenous of degree 1 in n.

$l_t$ :

$$\begin{aligned}\frac{\partial[\Omega_A(\omega_t^*)v(q^a)\mathbb{E}_t(n|\omega > \omega^*)]}{\partial l} &= 0 \\ \frac{\partial[\Omega_A(\omega_t^*)]\mathbb{E}_t(n|\omega > \omega^*) + \Omega_A(\omega_t^*)\frac{\partial[\mathbb{E}_t(n|\omega > \omega^*)]}{\partial l}}{\partial l} &= 0 \\ (-f_{\omega^*})\frac{\partial\omega_t^*}{\partial l_t}\mathbb{E}_t(n_t^P|\omega_t > \omega_t^*) + \Omega_A(\omega_t^*)\frac{\partial[\mathbb{E}_t(n|\omega > \omega^*)]}{\partial l} &= 0\end{aligned}$$

observation:

$$\frac{\partial[\mathbb{E}_t(n|\omega > \omega^*)]}{\partial l} = (\omega_t^+ \text{MPL} - [(1 + \phi^i(\frac{1}{q^a} - 1))w_t])$$

where:  $\omega_t^+ = E(\omega|\omega_t > \omega_t^*)$

$$(-f_{\omega^*})\frac{\partial\omega_t^*}{\partial l_t}[y(\omega^+ - \omega^*) + (1 - \delta_k)k] + \Omega_A(\omega_t^*)(\omega_t^+ \text{MPL} - [(1 + \phi^i(\frac{1}{q^a} - 1))w_t]) = 0$$

Note:

$$\frac{\partial\omega_t^*}{\partial l_t} = \frac{1}{y} \left( (1 + \phi(\frac{1}{q_t^a} - 1))w_t - \text{MPL}_t \omega^* \right)$$

$$\begin{aligned}\Omega_A(\omega_t^*)(\omega_t^+ \text{MPL}) &= f_\omega^* \frac{[y(\omega^+ - \omega^*) + (1 - \delta_k)k]}{y} \left( (1 + \phi(\frac{1}{q_t^a} - 1))w_t - \text{MPL}_t \omega^* \right) + [1 + \phi^i(\frac{1}{q_t^a} - 1)]w_t \Omega_A(\omega_t^*) \\ \text{MPL} &= w_t \left( \frac{\left[ \Omega_A(\omega_t^*) + \frac{f_\omega^*[y(\omega^+ - \omega^*) + (1 - \delta_k)k]}{y} \right]}{\left[ \Omega_A(\omega_t^*)(\omega_t^+) + f_\omega^* \frac{\omega^*[y(\omega^+ - \omega^*) + (1 - \delta_k)k]}{y} \right]} \left( 1 + \phi(\frac{1}{q_t^a} - 1) \right) \right)\end{aligned}$$

now: FOC  $k'$ :

$$\begin{aligned}
1 - E_t[m^B \frac{\partial \tilde{V}(k'; q^{a'})}{\partial k'}] &= 0 \\
1 - E_t[m^B \frac{\partial [\Omega_A(\omega^{*'}) \mathbb{E}_t(n' | \omega' > \omega^{*'})]}{\partial k'}] &= 0 \\
1 - E_t[m^B v(q^{a'}) \left( \frac{\partial [\Omega_A(\omega^{*'})]}{\partial k'} \mathbb{E}_t(n' | \omega' > \omega^{*'}) + \Omega_A(\omega^{*'}) \frac{\partial [\mathbb{E}_t(n' | \omega' > \omega^{*'})]}{\partial k'} \right)] &= 0 \\
1 - E_t[m^B v(q^{a'}) \left( (-f_{\omega^*}) \frac{\partial \omega_t^*}{\partial k'} \mathbb{E}_t(n' | \omega' > \omega^{*'}) + \Omega_A(\omega_t^*) \frac{\partial [\mathbb{E}_t(n' | \omega > \omega^{*'})]}{\partial k'} \right)] &= 0 \\
1 - E_t[m^B v(q^{a'}) \left( (-f_{\omega^{*'}}) \frac{\partial \omega^{*'}}{\partial k'} [y' (\omega_{t+1}^+ - \omega_{t+1}^*) + (1 - \delta_k) p_{t+1}^K k'] + \Omega_A(\omega^{*'}) \frac{\partial [\mathbb{E}_t(n' | \omega' > \omega^{*'})]}{\partial k'} \right)] &= 0
\end{aligned}$$

observation:

$$\begin{aligned}
\frac{\partial [\mathbb{E}_t(n | \omega > \omega^*)]}{\partial k'} &= \omega^{*'} \text{MPK}' + (1 - \delta_K) \\
\frac{\partial \omega^{*'}}{\partial k'} &= \frac{1}{y} (-\text{MPK}' \omega^{*'})
\end{aligned}$$

$$\begin{aligned}
1 - E_t[m^B v(q^{a'}) \left( (-f_{\omega^{*'}}) \left( \frac{1}{y} (-\text{MPK}' \omega^{*'}) \right) [y' (\omega_{t+1}^+ - \omega_{t+1}^*) + (1 - \delta_k) p_{t+1}^K k'] + \Omega_A(\omega^{*'}) (\omega^{*'} \text{MPK}' + (1 - \delta_K)) \right)] &= 0 \\
1 = E_t[m^B v(q^{a'}) \left( f_{\omega^{*'}} \left( \frac{\text{MPK}' \omega^{*'}}{y} \right) [y' (\omega_{t+1}^+ - \omega_{t+1}^*) + (1 - \delta_k) p_{t+1}^K k'] + \Omega_A(\omega^{*'}) (\omega^{*'} \text{MPK}' + (1 - \delta_K)) \right)] &= 0
\end{aligned}$$

Define  $m^P = m^B v(q^{a'})$

$$1 = E_t[m^P \left( \Omega_A(\omega^{*'}) (\omega^{*'} \text{MPK}' + (1 - \delta_K)) + f_{\omega^{*'}} \left( \frac{\text{MPK}' \omega^{*'}}{y'} \right) [y' (\omega_{t+1}^+ - \omega_{t+1}^*) + (1 - \delta_k) p_{t+1}^K k'] \right)]$$

In the standard RBC without adjustment cost, the optimal investment:

$$1 = \beta E(\text{MPK}' + (1 - \delta_K))$$

Stage I:

CES:

$$A_t^{P_A} = \left( (\nu_A^F)^{\frac{1-\sigma_A^F}{\sigma_A^F}} A_{1t}^{\frac{\sigma_A^F-1}{\sigma_A^F}} + (1-\nu_A^F)^{\frac{1-\sigma_A^F}{\sigma_A^F}} A_{2t}^{\frac{\sigma_A^F-1}{\sigma_A^F}} \right)^{\frac{\sigma_A^F}{\sigma_A^F-1}}$$

Loan's firm(sector) A demand for each bank:

$$\begin{aligned} A_{1t} &= \left( \frac{\frac{1}{Q_t^a}}{\frac{1}{q_{1t}^a}} \right)^{\sigma_A^F} (\nu_A^F)^{1-\sigma_A^F} A_t^{P_A} \\ A_{2t} &= \left( \frac{\frac{1}{Q_t^a}}{\frac{1}{q_{2t}^a}} \right)^{\sigma_A^F} (1-\nu_A^F)^{1-\sigma_A^F} A_t^{P_A} \end{aligned}$$

where:

$$\begin{aligned} Q_t^a &= 1 / \left( \left( \frac{\nu_A^F}{q_{1t}^a} \right)^{1-\sigma_A^F} + \left( \frac{1-\nu_A^F}{q_{2t}^a} \right)^{1-\sigma_A^F} \right)^{\frac{1}{1-\sigma_A^F}} \\ A_t^{P_A} &= \phi^A w_t^A L_t^A \end{aligned}$$

$$V_j(n_j) = \max_{k'} \text{div}_j + \mathbb{E}_t[m_{t,t+1} \tilde{V}_j(k'_j)]$$

$$\overbrace{n_j - p^{k_j} k'_j + \underbrace{\mathbf{wc}_j}_{\text{new debt}}}_{\text{div}_{ft}^j} \geq 0$$

$$n_j = \underbrace{\omega_j k_j^{1-\alpha} l^\alpha - (1-\phi) w_j l - \frac{1}{Q_j} \mathbf{wc}_j}_{\text{profit flow}} + p^{k_j} (1 - \delta^{k_j}) k_j$$

$$\omega_j^* = \frac{(1 + \phi^j (\frac{1}{Q_j} - 1)) w_j \bar{l}_j}{y_j} \quad , \quad \tilde{V}_j(k_j) = \max_{l_j} [\Omega(\omega_j^*) \mathbb{E}_t(V_j(n_j) | \omega_j > \omega_j^*)]$$

## F.2 Bank problem

The bank problem is the following:

$$V^i(N_t^i, \mathcal{S}_t) = \max_{q_{Ai,t}, D_t^i, e_t^i} div_t^i - e_t^i + E_t[m_{t+1,t}^B V^i(N_{t+1}^i)]$$

s.t

$$N_t^i + D_t^i + e_t \geq L_{At}^i + div_t^i + \Psi^i(e_t^i)$$

$$D_t^i \leq \xi_A L_{At}^i$$

$$N_{t+1}^i = (\frac{\tilde{M}_{t+1}^A}{q_{Ai,t}}) L_{At}^i - (\textcolor{blue}{R}_t) D_t^i$$

where:

$$\begin{aligned} div_t^i &= \phi_0 N_t^i \quad \Psi^i(e_t^i) = \frac{\phi_1^i}{2} (e_t^i)^2 \\ L_{At}^i &= \left( \frac{\frac{1}{Q_A}}{\frac{1}{q_{Ai}}} \right)^\sigma (\nu)^{1-\sigma} \bar{A} \\ \tilde{M}_t &= \underbrace{\Omega(\omega_t^*)}_{\text{No default}} + \underbrace{\frac{(1 - \Omega(\omega_t^*))}{L_t^i / q_{Ai,t}} \left[ \varpi^i (1 - \zeta^i) (\mathbb{E}_{\omega,t}[\omega < \omega^*] Y_t + ((1 - \delta^k) p_t^K) K_t) - \varpi^i w_t \bar{L} \right]}_{\text{default (recovery value)}} \end{aligned}$$

Assumptions:

- Everything that has a  $t$  subscript is known at time  $t$ , and everything that has a  $t + 1$  subscript is not known at time  $t$
- In the law of motion of networth, at time  $t$  the bank decides how much charge  $q_{Ai,t}$ , but the return on this is uncertain, as it depends of the firm default or not. For this reason,  $\tilde{M}_{t+1}^A$  has a  $t + 1$  subscript. On the other hand, the payment on  $D_t$  is known at time  $t$ , so we have  $R_t$  not  $R_{t+1}$  there.
- In the equation for  $N_{t+1}^i$ ,  $N_{t+1}^i$  would be banks  $t + 1$  networth  $N_{t+1}^i$ , that would be used for next period lending. Now the action today affects tomorrow state  $N_{t+1}^i$ .

Note that  $L_{At}^i = \left( \frac{\frac{1}{Q_A}}{\frac{1}{q_{A1}}} \right)^\sigma (\nu)^{1-\sigma} \bar{A}$  and  $\varpi^i = \frac{\frac{1}{q_{Ai}} L_{At}^i}{\frac{1}{Q_A} \bar{A}_t}$

STEP 1:

$$V^i(N_t^i) = \phi_0 N_t^i - e_t^i + E_t[m_{t+1,t}^B V^i(N_{t+1}^i, \mathcal{S}_t)] + \lambda_t[(1-\phi)N_t - e_t - \Psi(e_t) - L_{Ai}^i - D_t] + \mu_t[\xi_A L_{At}^i - D_t]$$

where:

$$\begin{aligned} N_t^i &= \left( \frac{\tilde{M}_t^A}{q_{Ai,t}} \right) L_{At}^i - (R_t) D_t \\ L_{At}^i &= \left( \frac{q_{Ai}}{Q_A} \right)^2 \bar{A}_t \\ \Psi^i(e_t^i) &= \frac{\phi_1^i}{2} (e_t^i)^2 \\ \tilde{M}_t &= \underbrace{\Omega(\omega_t^*)}_{\text{No default}} + \underbrace{\frac{(1 - \Omega(\omega_t^*))}{L_t^i / q_{Ai,t}} [\varpi^i (1 - \zeta^i) (\mathbb{E}_{\omega,t}[\omega < \omega^*] Y_t + ((1 - \delta^k) p_t^K) K_t) - \varpi^i w_t \bar{L}]}_{\text{default (recovery value)}} \\ \varpi^i &= \frac{\frac{1}{q_{Ai}} L_{At}^i}{\frac{1}{Q_A} \bar{A}_t} \end{aligned}$$

FOC of  $q_{Ai}$ :

$$\begin{aligned} \frac{1}{q_{Ai}} &= \left( \frac{\frac{\partial L_{Ai}}{\partial q_{Ai}} \frac{q_{Ai}}{L_{Ai}}}{\frac{\partial L_{Ai}}{\partial q_{Ai}} \frac{q_{Ai}}{L_{Ai}} - 1} \right) \left( \frac{1 - \tilde{\mu} \xi_A}{\frac{m_t^I}{m_t^B}} \right) \frac{1}{\left( \Omega(\omega_t^*) + (1 - \Omega(\omega_t^*)) \frac{(X_t - Z_t)}{\frac{1}{Q_A} \bar{A}} \right)} \\ \frac{1}{q_{Ai}} &= \left( \frac{\sigma}{\sigma - 1} \right) \left( \frac{1 - \tilde{\mu} \xi_A}{\frac{m_t^I}{m_t^B}} \right) \frac{1}{\left( \Omega(\omega_t^*) + (1 - \Omega(\omega_t^*)) \frac{(X_t - Z_t)}{\frac{1}{Q_A} \bar{A}} \right)} \end{aligned}$$

where:  $X_t = (1 - \zeta^i) (\mathbb{E}_{\omega,t}[\omega < \omega^*] Y_t + ((1 - \delta^k) p_t^K) K_t)$

$$Z_t = w_t \bar{L}$$

**FOC of  $e_t$ :**

$$-1 + \lambda_t(1 - \Psi_e(e)) = 0 \Rightarrow \lambda_t = \frac{1}{1 - \Psi_e(e)}$$

**FOC of  $N_t$ :**

$$\frac{\partial V}{\partial N} = \phi_0 + (1 - \phi_0)\lambda_t \Rightarrow \frac{\partial V_{t+1}}{\partial N_{t+1}} = \phi_0 + (1 - \phi_0)\lambda_{t+1}$$

Define:  $m_{t+1,t}^I \equiv m_{t+1,t}^B \frac{\partial V_{t+1}}{\partial N_{t+1}} \frac{1}{\lambda_t}$  and define  $\tilde{\mu} = \frac{\mu}{\lambda}$  to simplify the notation that i will use below:

**FOC of  $D_{it}$ :**

$$\begin{aligned} E_t[m_{t+1,t}^B \frac{\partial V^i(N_{t+1}^i)}{\partial N_{t+1}} \frac{\partial N_{t+1}}{\partial D_{it}}] - \lambda_t - \mu_t &= 0 \\ E_t[m_{t+1,t}^B \frac{\partial V^i(N_{t+1}^i)}{\partial N_{t+1}} (-R_t)] + \lambda_t - \mu_t &= 0 \\ 1 = \tilde{\mu}_t + E_t[m_{t+1,t}^I]R_t & \end{aligned}$$

## G. Consumer problem

$$\max_{C_t, X_t^A, X_t^M} \mathbb{E}_t \left[ \sum_{t=0}^{\infty} \beta^t [\log(C_t)] \right]$$

s.t.

$$\begin{aligned} C_t + \sum_{i=1}^2 (X_t^j + \Psi(X_t^j, K_t^j)) &\leq w^j \bar{L} + \sum_{j=1}^2 \text{div}_t^j + \sum_{i=1}^2 \text{Netdiv}_t^i + \sum_{j=1}^2 p_t^{K^j} X_t^j \\ K_{t+1}^j &= (1 - \delta_K) K_t^j + X_t^j \end{aligned}$$

## H. Resource constraint derivation

**Resource constraint derivation:**

$$\begin{aligned}
 C^B &= w^B \bar{L}^B + Div_t^P + NetDiv_t^1 + NetDiv_t^2 + pX_t - X_t - \Psi(X, K) \quad \text{consumer's B.C} \\
 A_{1,t}^I - D_{1t}^I &= N^{I_1} - \phi_0^I N^{I_1} + e^{I_1} - \Psi^{I_1}(.) \quad \text{Bank 1's Budget constraint} \\
 A_{2,t}^I - D_{2t}^I &= N^{I_2} - \phi_0^I N^{I_2} + e^{I_2} - \Psi^{I_2}(.) \quad \text{Bank 2's Budget constraint}
 \end{aligned}$$

This implies:

$$\begin{aligned}
 C^B + (A_{1,t}^I + A_{2,t}^I) - (D_{1t}^I + D_{2t}^I) &= w^B \bar{L}^B + Div_t^P + NetDiv_t^1 + NetDiv_t^2 + pX_t - X_t - \Psi(X, K) \\
 &\quad + N^{I_1} - \phi_0^I N^{I_1} + e^{I_1} - \Psi^{I_1}(.) \\
 &\quad + N^{I_2} - \phi_0^I N^{I_2} + e^{I_2} - \Psi^{I_2}(.)
 \end{aligned}$$

Using:

$$\begin{aligned}
 A^P &= \phi w L \\
 A_{1,t}^I &= \left( \frac{\frac{1}{Q_A}}{\frac{1}{q_{A1}}} \right)^\sigma (\nu)^{1-\sigma} A^P \\
 A_{2,t}^I &= \left( \frac{\frac{1}{Q_A}}{\frac{1}{q_{A2}}} \right)^\sigma (\nu)^{1-\sigma} A^P
 \end{aligned}$$

$$Div_t^P = N^P - (p_t K_{t+1}) + A_t^P - F(\omega^*) n^0$$

$$NetDiv_{1t}^I = N_1^I \phi_0^I - e_1^I$$

$$NetDiv_{2t}^I = N_2^I \phi_0^I - e_2^I$$

$$K_{t+1} = (1 - \delta_K) K_t + X_t$$

$$\begin{aligned} C^B + X_t + \Psi(X, K) + \Psi^{I_2}(.) + \Psi^{I_1}(.) &= w^B \bar{L}^B + N^P - (1 - \delta_K) p_t K_t + (A_t^P - (A_{1,t}^I + A_{2,t}^I)) \\ &\quad - (1 - \Omega(\omega^*)) n^0 + N^{I_1} + N^{I_2} + (D_{1t}^I + D_{2t}^I) \end{aligned}$$

Using:

$$\begin{aligned} N^P &= \left[ \Omega(\omega^*) E(\omega | \omega > \omega^*) Y - \Omega(\omega^*) w^B \bar{L}^B - \Omega(\omega^*) \left( \frac{1}{Q_A} \right) A^P \right] \\ &\quad + (\Omega(\omega^*)) (1 + \delta_K) p_t K_t + (1 - \Omega(\omega^*)) n^0 \\ N_t^{I_1} &= \left( \frac{\tilde{M}_t^{A1}}{q_{A1,t}} \right) L_{At}^1 - (R_t) D_t^1 \\ N_t^{I_2} &= \left( \frac{\tilde{M}_t^{A2}}{q_{A2,t}} \right) L_{At}^2 - (R_t) D_t^2 \end{aligned}$$

$n^0$  is an initial networth.

$$\begin{aligned} \tilde{M}_t^{A1} &= \underbrace{\Omega(\omega_t^*)}_{\text{No default}} + \underbrace{\frac{(1 - \Omega(\omega_t^*))}{L_t^1/q_{A1,t}} \left[ \varpi^1 (1 - \zeta^1) (\mathbb{E}_{\omega,t} [\omega < \omega^*] Y_t + ((1 - \delta^k) p_t^K) K_t) - \varpi^1 w_t \bar{L} \right]}_{\text{default (recovery value)}} \\ \tilde{M}_t^{A2} &= \underbrace{\Omega(\omega_t^*)}_{\text{No default}} + \underbrace{\frac{(1 - \Omega(\omega_t^*))}{L_t^2/q_{A2,t}} \left[ \varpi^2 (1 - \zeta^2) (\mathbb{E}_{\omega,t} [\omega < \omega^*] Y_t + ((1 - \delta^k) p_t^K) K_t) - \varpi^2 w_t \bar{L} \right]}_{\text{default (recovery value)}} \end{aligned}$$

Define:

$$\begin{aligned} \Sigma(\zeta^1) &= [(1 - \zeta^1) (\mathbb{E}_{\omega,t} [\omega < \omega^*] Y_t + ((1 - \delta^k) p_t^K) K_t) - w_t \bar{L}] \\ \Sigma(\zeta^2) &= [(1 - \zeta^2) (\mathbb{E}_{\omega,t} [\omega < \omega^*] Y_t + ((1 - \delta^k) p_t^K) K_t) - w_t \bar{L}] \end{aligned}$$

$$\begin{aligned} N_t^{I_1} &= \Omega(\omega_t^*) \frac{L_{At}^1}{q_{A1,t}} + (1 - \Omega(\omega_t^*)) \varpi^1 \Sigma(\zeta^1) - (R_t) D_t^1 \\ N_t^{I_2} &= \Omega(\omega_t^*) \frac{L_{At}^2}{q_{A2,t}} + (1 - \Omega(\omega_t^*)) \varpi^2 \Sigma(\zeta^2) - (R_t) D_t^1 \end{aligned}$$

$$D \equiv D_{1t}^I + D_{2t}^I$$

The resource constraint becomes:

$$\begin{aligned}
C^B + X_t + \Psi(X, K) + \Psi^{I_2}(\cdot) + \Psi^{I_1}(\cdot) + DWL + D(R - 1) &= Y \\
+ A_t^P(1 - \frac{\Omega(\omega^*)}{Q_A}) - A_{1,t}^I(1 - \frac{\Omega(\omega_t^*)}{q_{A1,t}}) - A_{2,t}^I(1 - \frac{\Omega(\omega_t^*)}{q_{A2,t}})
\end{aligned}$$

where:

$$DWL = (1 - \Omega(\omega_t^*)) \left[ \mathbb{E}_{\omega,t}[\omega < \omega^*]Y_t + ((1 - \delta^k)\textcolor{red}{p}_t^K)K_t \right] (\varpi^1\zeta^1 + \varpi^2\zeta^2)$$

## I. Recovery rates and capitalization rates in the data

This subsection describes the bank-level variables used to calculate the relation between recovery rates for each type of loan and bank capitalization rates, based on "Call reports". First, I construct a proxy of banks' recovery rates using the variable recoveries on allowance for loan and lease losses. First, In the case of recoveries on commercial loans, recoveries on loans to individual for households, recoveries on real estate loans I use *riad4608,riad4609,riad4257* respectively. Second, charge-offs on allowance for loan and lease losses for commercial loans, individual for households, real estate loans are *riad4638,riad4639,riad4256* respectively. I divide t. Then I winsorize the observation for the proxy of bank's recovery rates to have recovery rate on the interval [0,1]. Figure 18 presents bin-scatter plots of the bank capitalization rate against my proxy of bank's recovery rates for each type of loans.

Figure 33: Recovery rates on personal loans and capitalization rate

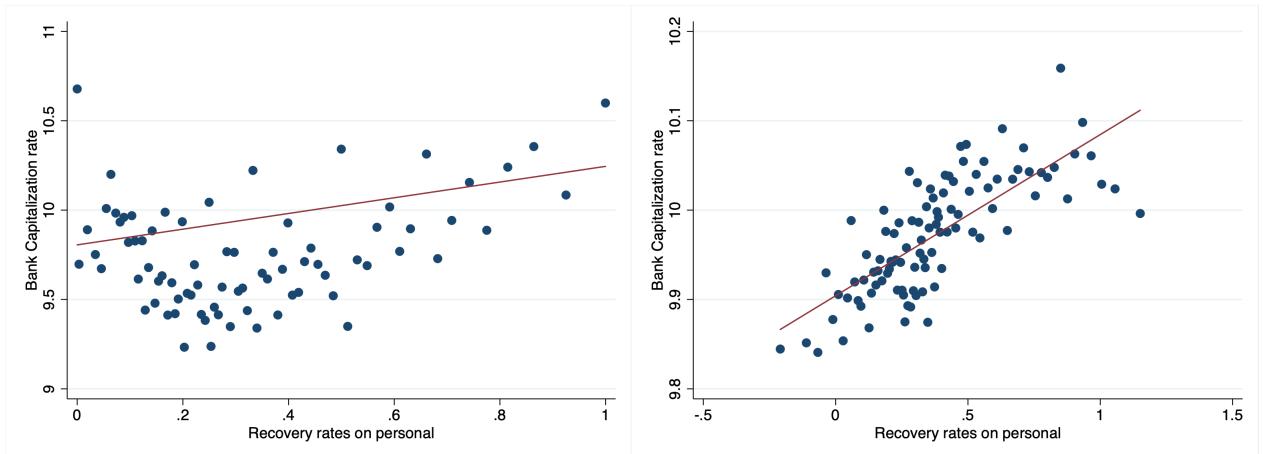


Figure 34: Recovery rates on real estate loans and capitalization rate

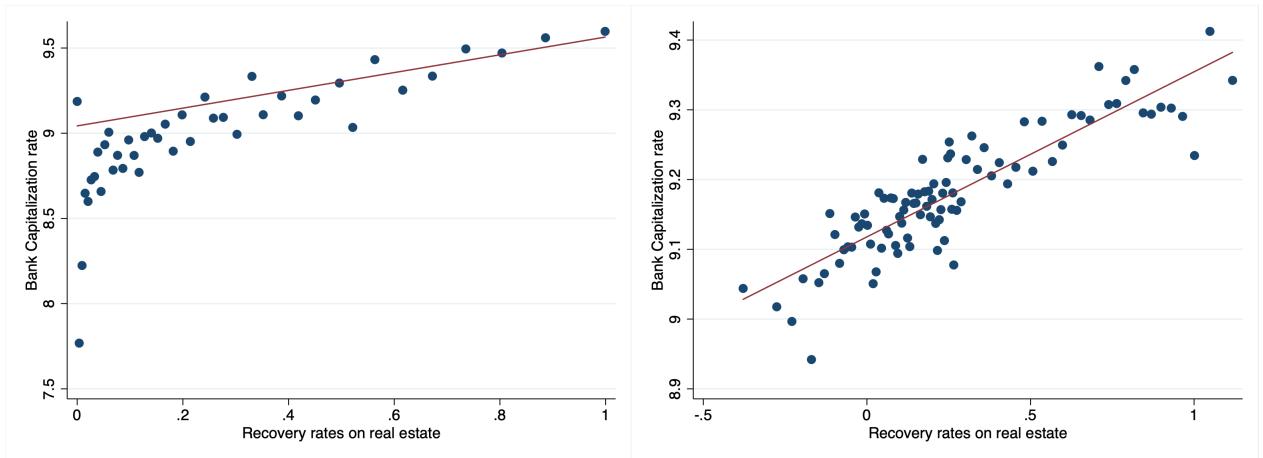
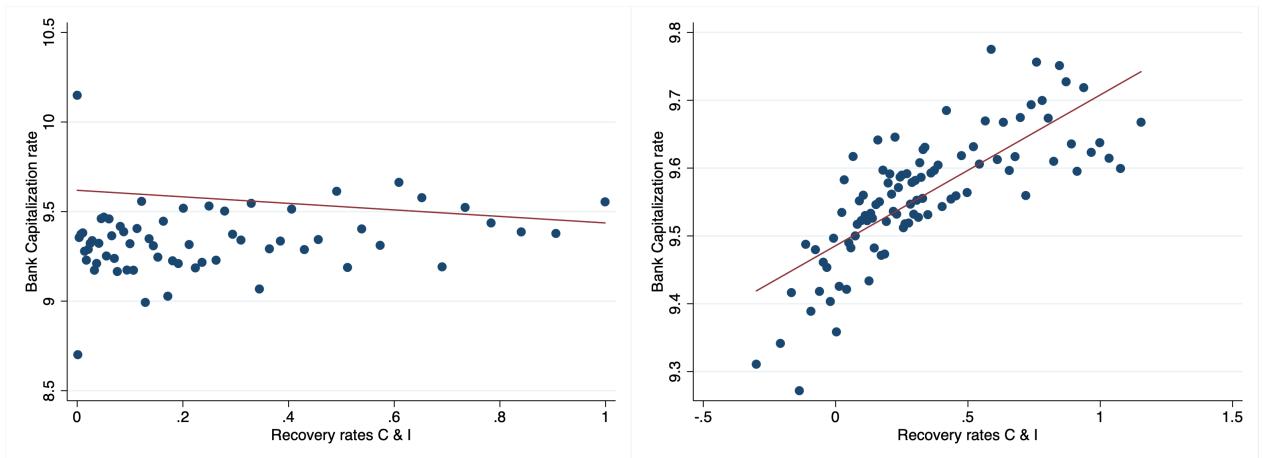


Figure 35: Recovery rates on commercial and industrial loans and capitalization rate



## J. Alternative measure of recovery rates

First, in the case of non-accrual on total loans and lease I use *rcfd1403*. Second, total loans and lease past 90 or more and still accruing I use *rcfd1407*. sum them

and define as non-performing loans. Then I construct a proxy of recovery rate by summing the recovery of each type of loans. Finally , I divide them. Then, I winsorize the observation for the proxy of bank's recovery rates to have recovery rate on the interval [0,1]. Figure 18 presents bin-scatter plots of the bank capitalization rate against my proxy of bank's recovery rates for each type of loans. Appendix:

This subsection describes the bank-level variables used to calculate the relation between recovery rates and bank capitalization rates, based on "Call reports". First, In the case of nonaccrual on C&I loans, nonaccrual loans to individual for households, nonaccrual loans secured by real estate I use *rcfd1608, rcf1981, rcf1423* respectively. Second, loans past 90 or more and still accruing on C&I loans, nonaccrual loans to individual for households, nonaccrual loans secured by real estate I use *rcfd1607, rcf1979, rcf1422* respectively. I sum them and define as non-performing loans. Then I use of recovery rate for each type of loan for a given bank, I divide them. Then, I winsorize the observation for the proxy of bank's recovery rates to have recovery rate on the interval [0,1]. Figure 18 presents bin-scatter plots of the bank capitalization rate against my proxy of bank's recovery rates for each type of loans.

<sup>16</sup>. Then I winsorize the observation of bank's recovery rates to have recovery rate on the interval [0,1]. Figure 18 presents bin-scatter plots of the bank capitalization rate against my proxy of bank's recovery rates for each type of loans.

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<sup>16</sup>For further references see [The Fed- Micro Data Reference Manual](#)

Figure 36: Recovery rates on commercial and industrial loans and capitalization rate

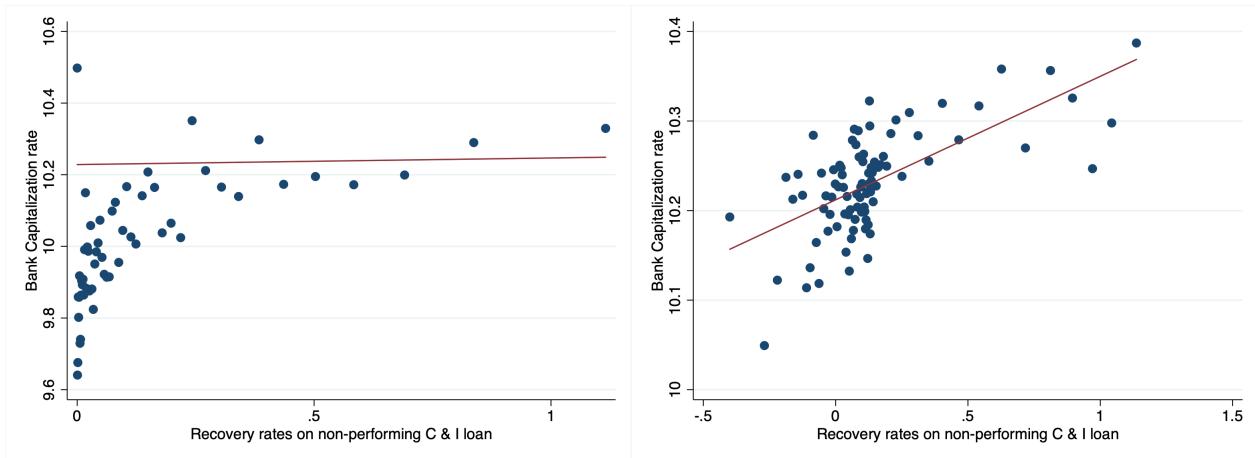


Figure 37: Recovery rates on personal loans and capitalization rate

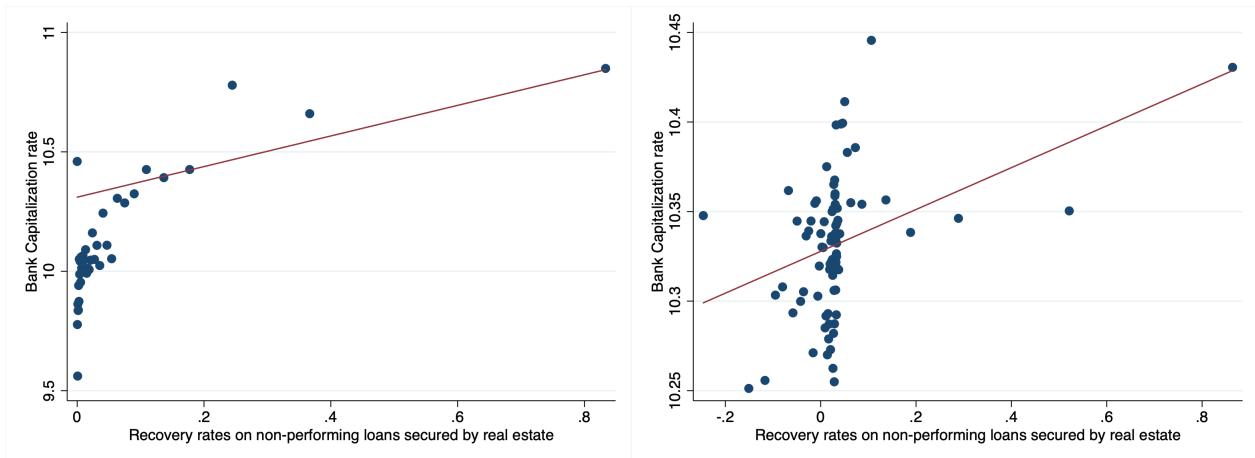
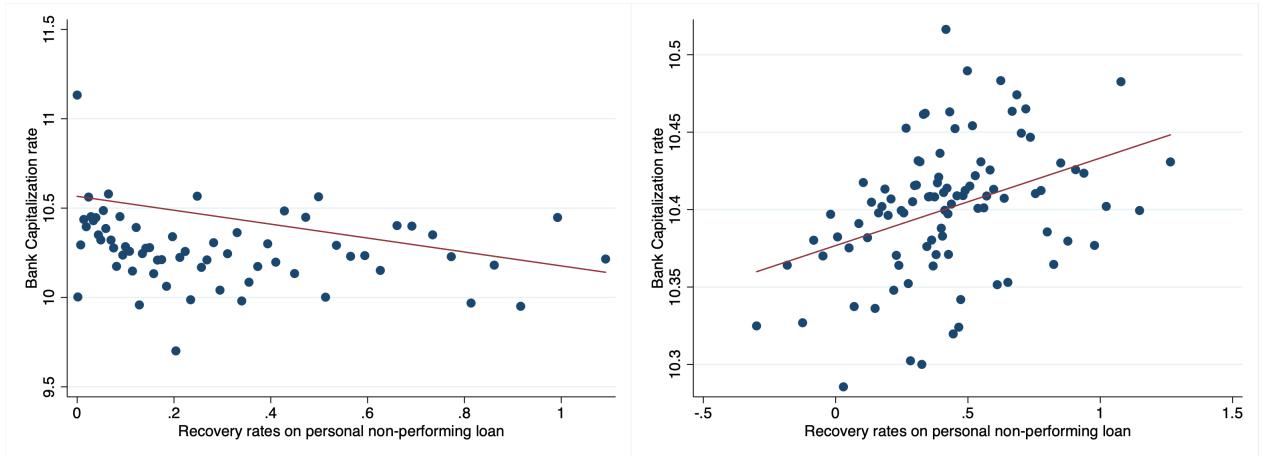


Figure 38: Recovery rates on personal loans and capitalization rate

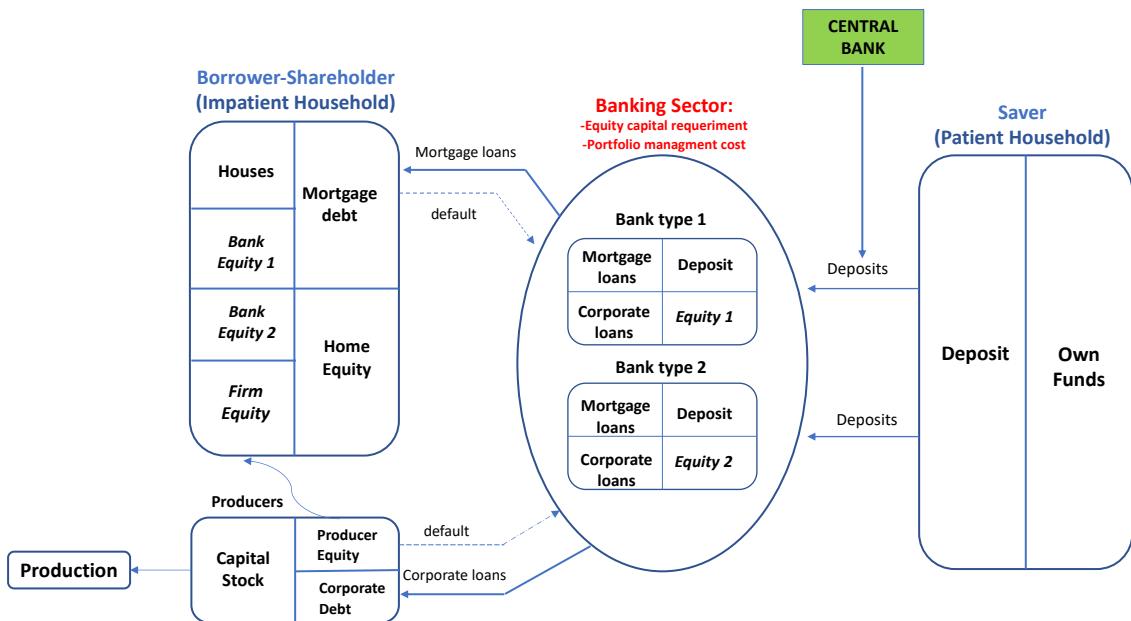


## K. Extension of the baseline model

In the second half of the paper, I develop a heterogenous bank New Keynesian model in order to interpreted the cross sectional evidence in Section 2 and get some aggregate implications. This dynamic stochastic general equilibrium model is based on [Elenov et al. \(2020\)](#) framework. The proposed model capture several realistic features of the types of loan and banking heterogeneity, therefore my approach is build a general equilibrium (GE) model with two key parts: (1) two banks with different level of capitalization rate, (2) two loans (assets) with different riskiness which can be interpreted as defaultable debt. The figure 39 depicts the connection between the balance sheet of different agents in the economy. The *representative saver* takes consumption and saving decision to maximize inter-temporal expected utility. She inelastically supplies labor and can invest its saving in bank deposits. *Banks* maximize the present discounted dividends paid to their shareholder. They borrow from savers and issue equity from the borrower-shareholder, and they extend loans to borrower-shareholders and producer(non-financial firms) as mortgage loan and corporate loans respectively. Banks are dif-

ferent in the tightening of the capital requirements constraint and in the portfolio management cost. The *Borrower-shareholder* (can be thinking as capitalist or entrepreneur) maximize inter-temporal expected utility. She also inelastically supplies labor and have housing, then she is funded by long-term defaultable mortgage debt that she issue to banks and have home equity. *Producers or intermediate good producer* maximize profit and operate a production technology using labor and capital. They are funded by long-term defaultable corporate debt that they issue to banks and by equity issued by the household shareholders. Also they buy capital from borrower and sell their intermediate output to retailers, which this retailer have a monopolistic competition and face a quadratic price adjustment cost. Finally, to close the model, there is a representative final good producer combines retailer good into final goods and the monetary authority follows a standard taylor rule.

Figure 39: Overview of the model



## K.1 Environment

The model is formulated in discrete time with an infinite horizon.

**Demographics** There are two groups of households: borrower-shareholder (which can be think as entrepreneurs or capitalists), and savers. I assume savers are more patient than borrower-shareholders. Also, there are other agents in the model which are two banks, an intermediary firm producer, retailers, final good producer and a monetary authority.

**Preferences** The households have logarithm preferences over consumption and housing services.

$$U_t^j = \log(C_t^j) + \xi^j \log(H_t^j) \quad j \in \{S, B\}$$

I assume there is segmentation in the housing market so that saver do not consume housing ( $\xi^S = 0$ ) this is for simplicity.

**Technology and Housing** Each intermediate good producer use a Cobb-Douglas production function using capital and labor. The level of house is fixed supply.

**Financial assets** There are three assets in the economy. The first is deposits, which is a one-period short term bond. The second is a mortgage debt, which aggregate the mortgage loan made to all borrower-entrepreneurs households. The third is corporate bond, which aggregate to all firms. These both types of loans have two characteristics: (1) long term model as a perpetuity with coupon payments decay geometrically ( $\delta^M, \delta^A$ ) and face value of individual bonds ( $F^M, F^A$ ), respectively for mortgage and corporate loan, and (2) defaultable debt. First, in the case of mortgage loan, borrowers experience receive an idiosyncratic house valuation shock  $\omega_{i,t}^H \stackrel{\text{iid}}{\sim} F_{\omega^H,t}$  then the value of the house after shock is  $\omega_i^H q_t^H H_t^B$ , then borrower-shareholder optimally choose which member default  $\Rightarrow$  threshold  $\omega_t^{H,*}$  s.t. default

for all  $\omega_i < \omega_t^{H,*}$ , finally bank seize housing capital and erase debt of defaulting borrowers. Second, in the case of corporate loan, each firm-producer receives an idiosyncratic productivity shocks  $\omega_{i,t} \stackrel{\text{iid}}{\sim} F_{\omega,t}$ , each firm-producer defaults on debt if flow of profit  $\pi(\omega_{i,t}) < 0$ . This implies  $\omega_t^*$  threshold. Alternately, producer with low productivity  $\omega_t < \omega_t^*$  default. therefore bank seizes bankrupt firm and unwinds it.

**Timing:** The summary of the timing is the following. At the beginning of period t:

1. Intermediate good producers choose labor inputs and pay a fixed cost of production, and borrower-shareholder enter with housing, and mortgage debt.
2. Idiosyncratic housing valuation shock for borrower-shareholder are realized. Also, idiosyncratic productivity shock for intermediate good producers are realized and then their production occurs.
3. Borrowers decide on mortgage default, and firms with negative profits defaults. Banks repossess the house in the case for borrower. In the case of firm, banks assume ownership of bankrupt firms..
4. Borrowers choose how much of the remanning mortgage balance to refinance. Firms decide how much of the capital and corporate loan to take. All agents, solve their consumption and portfolio choice problems. Market clear. All agent consume.

I describe the model in four block: the banking sector block, which captures the heterogenous banks; intermediate good production and borrower block, which capture the corporate and mortgage sector in the model, a New Keynesian block, which generate a New Keynesian Phillips Curve, and representative saver households, which represente the depositors.

### K.1.1 Banking sector Block

My banking sector block consist in two banks that intermediates between saver and borrower-shareholders and firm-producers. These banks are owned by borrower-shareholder and pay them dividends subject to convex adjustment cost. These two banks are heterogenous in: (i) Equity capital requirement, (ii) portfolio management cost. The bank portfolio consist on choose: (i) how many new corporate loans to make  $A_t^i$  with price  $q_t^a$ , then they will receive a coupon payment on performing loans  $\Omega_A(\omega_t^*)A_t^i$ , and firms that default go into liquidation and the recovery is:

$$\underbrace{[1 - \zeta]}_{\text{not lost}} \underbrace{[(1 - \Omega_A(\omega_t^*)) ((1 - \delta_K)p_t - \zeta)K]}_{\text{frac. of default}} + \underbrace{(1 - \Omega_A(\omega_t^*)) E_{\omega,t}[\omega | \omega < \omega^*] MC_t Y_t}_{\text{sell off capital}} - \underbrace{(1 - \Omega_A(\omega_t^*)) \sum_j w^j L^j}_{\text{average product. sell output}} \underbrace{\sum_j w^j L^j}_{\text{fraction of default, pay wages}}$$

where  $\zeta$  is the fraction of firm assets and output lost to banks in bankruptcy; (ii) how many mortgage loans  $M_t^i$  with price  $q_t^m$ , then they receive coupon payment on performing loans  $\Omega_M(\omega_t^{h*})M_t^i$ , and borrower-household that default, mortgage go foreclosure and the recovery is:

$$\underbrace{[1 - \zeta^h]}_{\text{not lost}} \times \underbrace{(\mu_{\omega^h} - \Omega_H(\omega_t^{h*})) q_t^h H_t^{B^h}}_{\text{value of home after after default decisions have been made}} \quad \text{where } \zeta^h \text{ is the fraction of home value destroyed or lost in a foreclosure (measures the foreclosure costs); (iii)how many deposits } (D^i) \text{ for next period to borrow with price } q^f. \text{ I assume that savers are indifferent to taking deposits in either type (see the appendix L.4 for the complete bank's problem description).}$$

### K.1.2 Borrower-shareholders and Intermediate good producers firm Block

#### Borrower-shareholders (denoted by B)

The borrower-shareholder owns the firm, and each type of bank. then, she receive dividend payments. Also, he build new capital goods  $X_t$ , requires  $X_t + \Phi(X_t/K_t)K_t$  where the adjustment cost  $\Phi$  satisfies  $\Phi(\delta_K) = \Phi'(\delta_K) = 0$  and  $\Phi''(\delta_K) > 0$ . Let  $\iota(\omega^h) : [0, \infty) \rightarrow \{0, 1\}$  indicate the borrower's family decision to default on a house quality  $\omega$ . The default decision is characterized by a threshold level  $\omega^{h*}$ . Given this

threshold, define

$$Z_M(\omega_t^{h*}) = \int_0^\infty (1 - \iota(\omega^h)) dF_{\omega^h, t}(\omega_t^h) = Pr[\omega_{i,t}^h \geq \omega_t^{h*}]$$

$$Z_H(\omega^{h*}) = \int_0^\infty (1 - \iota(\omega^h)) \omega^h dF_{\omega^h, t}(\omega_t^h) = Pr[\omega_{i,t}^h \geq \omega_t^{h*}] \times E[\omega_{i,t}^h | \omega_{i,t}^h \geq \omega_t^{h*}]$$

$Z_M(\omega^{h*})$  fraction of debt repaid to banks, and  $Z_H(\omega^{h*})$  value of the borrower of the residual (non-defaulted) housing stock after default decision have been made. After making a coupon payment of 1 per unit of outstanding debt, the amount of outstanding debt declines to  $\delta Z_M(\omega^{h*}) M_t$ . Borrowing constraint on shareholder leverage with max LTV  $\Phi^B$ .

$$F^H M_{t+1+j}^B \leq \Phi^B q_{t+j}^h H_{t+1+j}$$

Borrower-shareholder produces new capital and sells it to the final firm. Inelastically supply their unit of labor  $\bar{L}^B$  and earn wage  $w_t^B$ . The budget constraint:

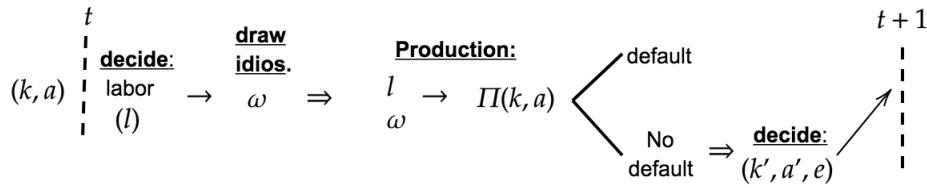
$$C_{t+j}^B + X_{t+j} + \Psi(X_{t+j}, K_{t+j}) K_{t+j} + Z_M(\omega_{t+j}^{h*}) M_{t+j}^B (1 + \delta_m q_{t+j}^m) + q_{t+j}^h H_{t+1+j} \\ \leq w_{t+j}^B \bar{L}^B + p_{t+j} X_{t+j} + q_{t+j}^m M_{t+1+j}^B + Z_H(\omega_{t+j}^{h*}) q_{t+j}^h H_{t+j} + Div_{t+j}^P + Div_{1,t+j}^I + Div_{2,t+j}^I$$

### Intermediate good producers (denoted by P):

The firm maximize the present discount of dividends paid to their shareholders. The firm combines capital  $k$  and labor  $l$  to produce final good using the Cobb-Douglas production function and face an idiosyncratic productivity shock  $\omega$ . Also, she buys capital at price  $p_t$  in a competitive market, and issue long-term debt defaultable bonds  $a$  at price  $q_t^a$ . I assume that he faces a borrowing constraint on firm leverage with maximum loan to value (LTV)  $\Phi^P$  and capital depreciates at rate  $\delta_K$  (see the appendix L.2 for the complete firm's problem description). Figure 7 shows

the timing of the firm within period: given capital and outstanding debt, producer choose labor inputs and pay a fix operational cost, then idiosyncratic shock are realized, production occurs, then some firms that do not default pay dividends, issue new debt and new capital for next period

Figure 40: Timing of events within period for the firm is as follows



### K.1.3 New Keynesian Block

The New Keynesian block of model is designed to generate a New Keynesian Phillips curve relation nominal variables to the real economy.

#### Retailers good producers:

I assume that there are monopolistically competitive **retailers** (fixed mass of retailer  $i \in [0, 1]$ ) at the retail level. They buy inputs from the intermediate good firms, they have a technology that can transform them one-for-one into retail good varieties ( $\tilde{Y}_{it} = Y_{it}$ ), and sell these to final good producers. Each retailer set  $P_{it}$  and Rotemberg sticky prices given demand curve  $Y_t^d(P_{i,t})$  and price of intermediate good  $P_t^y$ , implies marginal cost  $MC_t = \frac{P_t^y}{P_t}$ . New Keynesian Phillips Curve

$$\frac{\Pi_t}{\bar{\Pi}} \left( \frac{\Pi_t}{\bar{\Pi}} - 1 \right) = \frac{1}{\kappa} (1 - \eta + \eta MC_t) + \mathbb{E}_t \left[ m_{t,t+1}^S \left( \frac{\Pi_{t+1}}{\bar{\Pi}} - 1 \right) \frac{\Pi_{t+1}}{\bar{\Pi}_t} \frac{Y_{t+1}}{Y_t} \right]$$

### **Final Good Producers:**

I assume a competitive representative final good producer that combines the continuum of differentiated retailer goods indexed by  $i \in [0, 1]$  into final output using a Dixit-Stiglitz technology,  $Y_t = \left( \int Y_{it}^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}}$ , where  $\gamma$  is the elasticity of substitution across retail goods.

### **Monetary authority**

Monetary authority follows Taylor rule:

$$i_t = \left( \frac{1}{\beta_S} - 1 \right) + (1 - \rho_i) \left[ \gamma_Y (Y_t - \bar{Y}) + \gamma_\Pi \left( \frac{\Pi_t}{\bar{\Pi}} - 1 \right) \right] + \rho_i \left[ i_{t-1} - \left( \frac{1}{\beta^S} - 1 \right) \right] + \epsilon_t^{mp}$$

#### **K.1.4 Representative saver Block (denoted by S)**

The saver household is infinitely lived, and each period  $t$ , obtains utility from consumption of non-durable good  $C_t^S$ , where  $U(\cdot)$  is a standard concave, twice continuously differentiable function. He inelastically supplies labor  $\bar{L}^S$  remunerated with a wage  $w_t^S$ . The problem of the saver involves choosing consumption  $C^S$ , deposits  $D_{t+1}$  with price  $q_t^f$  to maximize its expected discounted lifetime utility (see the appendix L.3 for the complete saver's problem description). I assume that the saver can invest in government bonds (zero net supply) that pay nominal interest rate  $i_t$ . Then by arbitrage condition and representative saver, there will no trade in bonds, then  $B = 0$  in equilibrium. Allow to obtain a relation between deposit and nominal rate:

$$\frac{1}{q_t^f} = \frac{1 + i_{t-1}}{\Pi_t}$$

### K.1.5 Equilibrium and market clearing conditions

**Definition:** Given a sequence of monetary policy shock  $\{\epsilon_t^{mp}\}$ , an idiosyncratic housing quality shocks  $\{\omega_t^h\}$ , and an idiosyncratic productivity shocks  $\{\omega_t\}$ , a **competitive equilibrium** is an allocation  $\{C_t^B, H_t^B, M_t^B, \omega_t^{h*}, X_t\}$  for borrower-shareholder;  $\{K_t^P, A_t^P, e_t^P\}$  for producer;  $\{M_t^{I_i}, A_t^{I_i}, D_t^{I_i}, e_t^{I_i}\}$  for banks type  $i \in \{1, 2\}$ ;  $\{C_t^S, D_t^S\}$  for savers (depositors);  $\{\{Y_t(i)\}_{i \in [0,1]}\}$  for retail firms and a set of prices  $\{q_t^f, q_t^m, q_t^a, q_t^h, p_t^K, m_{ct}, w_t^B, w_t^S, \Pi_t\}$  such that given prices:

- Borrower-shareholder and saver maximize life-time utility subject their constraints.
- Producer and each type of banks maximize dividends subject their constraints.
- The nominal interest rate is given by Taylor rule.
- Markets clears:

– **Corporate loan**

$$A_t^P = A_t^{I_1} + A_t^{I_2}$$

– **Mortgage loan**

$$M_t^B = M_t^{I_1} + M_t^{I_2}$$

– **Deposits**

$$D_t^S = D_t^{I_1} + D_t^{I_2}$$

– **Capital**

$$K_t = (1 - \delta_K)K_{t-1} + X_t$$

– **Housing**

$$\bar{H} = H_t^B$$

– **Labor**

$$L_t^j = \bar{L}^j \quad \text{for } j = B, S$$

– **Final goods**

$$Y_t = C^S + C^B + X_t + \Psi(X_t, K_t) + \sum_{i=1}^2 (\Psi^{I_i}(e_t^{I_i}) + \Psi^{I_i}(A_t^{I_i}) + \Psi^{I_i}(M_t^{I_i})) + \Psi(e_t^P, N_t^P) + \sum_{i=h,k}^2 DWL^i$$

With the model in hand, I intend to explore a better interpretation of my proposed mechanism and do some policy exercises to evaluate the interplay between monetary policy and financial stability.

## K.2 Inspecting the mechanism or channel of monetary transmission

This section illustrate the mechanism of how policy rate affect the economy through the banking sector. I study the effect of an unexpected innovation to the Taylor rule followed by a perfect foresight transition back to the steady state. I focus on the financial intermediaries or bank, the optimal choice of  $A^i$  and  $M^i$ ,  $D^i$  satisfy the following conditions:

$$\begin{aligned} q_t^a(1 - \xi_A^i \tilde{\nu}_t^i) + \varphi_{A1}^i \left( \frac{A_t^i}{\varphi_{A0}^i} - 1 \right) &= \mathbb{E}_t \left[ \frac{m_{t,t+1}^i}{\Pi_{t+1}} \left[ \tilde{M}_{t+1}^A + \Omega_A(\omega_{t+1}^*) \delta_A q_{t+1}^a \right] \right] \text{ for bank } i \in \{1, 2\} \\ q_t^m(1 - \xi_M^i \tilde{\nu}_t^i) + \varphi_{M1}^i \left( \frac{M_t^i}{\varphi_{M0}^i} - 1 \right) &= \mathbb{E}_t \left[ \frac{m_{t,t+1}^i}{\Pi_{t+1}} \left( \tilde{M}_{t+1}^H + \Omega_M(\omega_{t+1}^{h*}) \delta_M q_{t+1}^m \right) \right] \text{ for bank } i \in \{1, 2\} \\ q_t^f(1 - \tilde{\nu}_t^i) &= \frac{\mathbb{E}_t[m_{t,t+1}^i]}{\Pi_{t+1}} \text{ for bank } i \in \{1, 2\} \\ \frac{1}{q_t^f} &= \frac{1 + i_{t-1}}{\Pi_t} \end{aligned}$$

Where the payoff per unit of long term debt for corporate loan and mortgage loan is in the appendix L.4. In addition the default thresholds of corporate and mortgage loan are:

$$\omega_t^* = \frac{\frac{A_{t-1}^P}{\Pi_t} + \varsigma \frac{K_{t-1}}{\Pi_t} + (\sum_j w_t^j \bar{L}^j)}{MC_t Y_t} \quad \omega^{*h} = \frac{(1 + \delta_m q_t^m) \frac{M_{t-1}^B}{\Pi_t}}{q_t^h H_{t-1}}$$

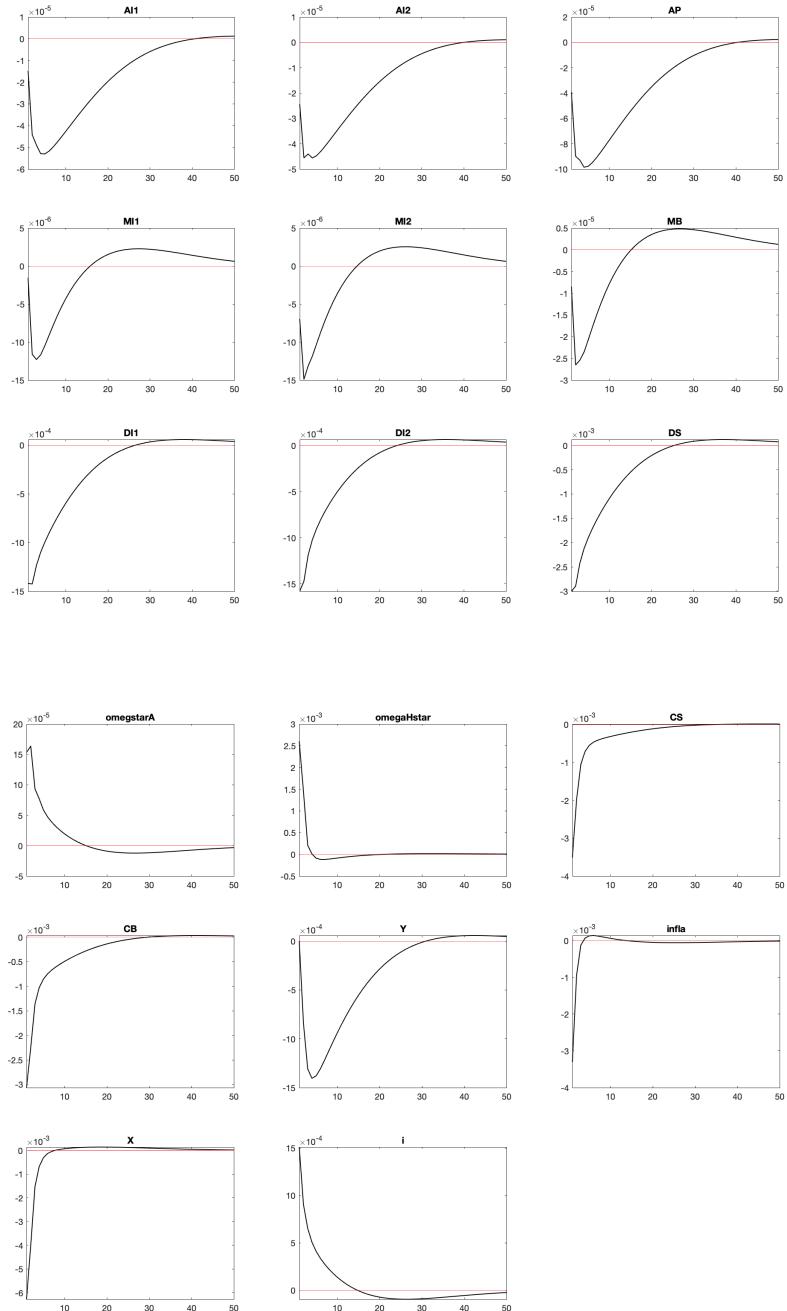
A key parameter in the FOC of banks is  $\xi_A^i$  and  $\xi_M^{I_i}$  which are related with the regulatory capital constraint, for example  $\xi_A^{I_i} = 1 - \Theta^i \varpi_A$ , which implies higher

$\Theta^{I_i}$ , higher capitalization rate.

### K.3 Calibration and monetary Policy shocks

This section present some preliminar results. First, I will show what is the calibrations and then the results. The calibration strategy for this version is based on [Elenov et al. \(2020\)](#). One difference is that corporate sector have a higher default rate than mortgage sector. This implies that corporate sector are riskier than mortgage sector to match my empirical evidence. Figure 41 show the response to a contractive monetary policy shock under the case of bank 1 have a higher capitalization rate than bank 2 and bank 1 have a higher share of corporate loan than bank 2 at the SS. This preliminary result are only qualitative and I show some qualitative result of an unexpected increase in interest rate which have higher default rate in both sector, this can be showed by the increase in default thresholds in both sector  $\omega^*, \omega^{h,*}$ , this implies higher defaults rate. Also, both banks decrease their loans in both sector too.

Figure 41: Response to a contractive monetary policy shock



## L. Extension of the baseline model

### L.1 Borrower-shareholder problem:

$$\begin{aligned}
& \max_{C_t^B, X_t, M_t, \omega_t^{h*}, H_t} \mathbb{E}_t \left[ \sum_{t=0}^{\infty} \beta_B^t \log(C_t^B) + \xi^B \log(H_t^B) \right] \\
& s.t \\
& C_t^B + X_t + \Psi(X_t, K_{t-1}) + \underbrace{Z_M(\omega_t^{h*}) \frac{M_{t-1}^B}{\Pi_t}}_{\text{repayment to bank on outstanding debt net of defaults}} + q_t^h H_t \\
& \leq w_t^B \bar{L}^B + Div_t^P + Div_{1,t}^I + Div_{2,t}^I + p_t X_t + q_t^h H_{t-1} Z_H(\omega_{t+j}^{h*}) + \underbrace{q_t^m * NM_t^B}_{\text{new bonds issued at price } q^m} \\
& q_t^m M_t^B \leq \Phi^B q_t^h H_t \quad NM_t^B = M_t^B - \underbrace{\delta_m Z_M(\omega_t^{h*}) \frac{M_{t-1}^B}{\Pi_t}}_{\text{remaining mortgage debt after default}}
\end{aligned}$$

OBS: payment per unit of mortgage bond is 1 in the current period,  $M_t^B$  is mortgage payment and the number of outstanding units of the mortgage bond .  $\Pi_t$  : inflation.

## L.2 Firm producer's problem

$$V^P(a_{t-1}^P, k_{t-1}, \mathcal{S}_t) = \max_{e_t^P, k_t, a_t^P} \text{div}_t^P - e_t^P + \mathbb{E}_t[\mathcal{M}_{t,t+1}^B V^{P,+}(a_{t+1}^P, k_{t+1}, \mathcal{S}_{t+1})]$$

subject to

$$\begin{aligned} \text{div}_t^P &= \underbrace{\pi_t + e_t^P - \Psi(e_t^P, n_t^P)}_{\text{profit net of equity issuance}} - \underbrace{\delta q_t^a(a_{t-1}^P/\Pi_t)}_{\text{debt repay}} + \underbrace{(1 - \delta_K)p_t^K(k_{t-1}/\Pi_t)}_{\text{depreciated k}} - \underbrace{p_t^K k_t}_{\text{buy new k}} + \underbrace{q_t^a a_t^P}_{\text{new debt}} \\ F a_t &\leq \Phi^P p_t^K k_t \quad n_t^P = \pi_t + (1 - \delta_K)p_t^K(k_{t-1}/\Pi_t) - \delta q_t^a(a_{t-1}^P/\Pi_t) \\ \pi_t &= \max_l \omega \frac{P_t^y}{P_t} k_{t-1}^{1-\alpha} l_t^\alpha - \sum_j w_t^j l_t^j - \frac{a_{t-1}^P}{\Pi_t} - \varsigma \frac{k_{t-1}}{\Pi_t} \end{aligned}$$

$$V^{P,+}(k_t, a_t, \mathcal{S}_t) = \max_{l_t^j} (1 - F_{\omega, t}(\omega_t^*)) \mathbb{E}_{\omega, t}[V^P(n_t^P, \mathcal{S}_t) | \omega_t > \omega^*]$$

$\mathcal{S}_t$  : represent aggregate state variable.

1. Given  $k, a^P$ ; producer choose  $l$ , pay fix cost of production  $\varsigma \Rightarrow \omega^* = \frac{a^P + \varsigma k + \sum w^j l^j}{k^{1-\alpha} l^\alpha}$
2.  $\omega_{i,t}$  is realized, production occurs, default occurs.
3. Failed firm are replaced by new firm to keep constant mass. Firm chose  $k', (a^P)', e^P$

## L.3 Saver's problem

$$\begin{aligned} \max_{C_t^S, D_t^S} \mathbb{E}_t &[ \sum_{t=0}^{\infty} \beta_S^t \log(C_t^S) + \xi^S \log(H_t^S) ] \\ \text{s.t.} \\ C_t^S + q_t^f D_t^S &\leq w_t^S \bar{L}^S + \frac{D_{t-1}^S}{\Pi_t} \end{aligned}$$

assume:  $\xi^S = 0$ , saver do not consume housing.

## L.4 Banks's problem

$$V^g(N_{t-1}^g, \mathcal{S}_t) = \max_{A_t^g, M_t^g, D_t^g, e_t^g} \underbrace{\phi_0^g N_t^g}_{\text{dividend}} - e_t^g + E_t[M_{t,t+1}^B V^g(N_{t+1}^g, \mathcal{S}_{t+1})]$$

subject to:

$$N_t^g - \phi_0^g N_t^g + e_t^g - \Psi^g(e_t^g) \geq q_t^a A_t^g + q_t^m M_t^g - q_t^f D_t^g + \Psi^{I_i}(A_t^g) + \Psi^{I_i}(M_t^g) \quad (\text{budget constraint})$$

$$q_t^f D_t^g \leq (1 - \Theta^g \varpi^a) q_t^a A_t^g + (1 - \Theta^g \varpi^m) q_t^m M_t^g \quad (\text{leverage constraint})$$

$$N_t^g = [\tilde{M}_t^A + \Omega_{\omega,t}(\omega_t^*) \delta_A q_{t-1}^a] \frac{A_{t-1}^g}{\Pi_{t-1}} + [\tilde{M}_t^M + \Omega_{\omega^h,t}(\omega_t^{h*}) \delta_M q_{t-1}^m] \frac{M_{t-1}^{I_i}}{\Pi_{t-1}} - \frac{D_{t-1}^{I_i}}{\Pi_{t-1}} \quad (\text{LoM bank's wealth})$$

The payoff per unit of long term debt (corporate loan):

$$\begin{aligned} \tilde{M}_t^A &= \underbrace{\Omega_A(\omega_t^*)}_{\text{No default}} + \underbrace{\frac{(1 - \Omega_A(\omega_t^*))}{A_{t-1}^P / \Pi_t} \left[ (1 - \zeta) (\mathbb{E}_{\omega,t}[\omega < \omega^*] MC_t Y_t + ((1 - \delta_K) p_t^K - \varsigma) K_t) - \sum_j w_t^j \bar{L}^j \right]}_{\text{default (recovery value)}} \\ \tilde{M}_t^M &= \Omega_M(\omega_t^{h*}) + (1 - \zeta^h) \left( \mu_\omega^h - Z_H(\omega_t^{h,*}) q_t^h H_{t-1} \right) \end{aligned}$$