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Passive Filters

Introduction

In the last couple of lessons, we studied the effect of complex impedances on performing AC circuit analysis. More specifically, we examined the phase difference between current and voltage associated with reactive components such as capacitors and inductors. In this lesson, we will learn how to use complex math and phasor addition to determine the response of simple passive filter circuits.

Discussion Overview

Processing audio signals, often times, involves separating the signal into low, mid and high frequencies. This is done by means of filtering. Although most modern audio processing equipment use complex and sophisticated active filters, there are simple passive filters that can be used in simple inexpensive hobby circuits that provide the basis for more complex filters. The circuit in Figure 1 is a simple "crossover" filter that separates an incoming audio signal into bass and treble components.

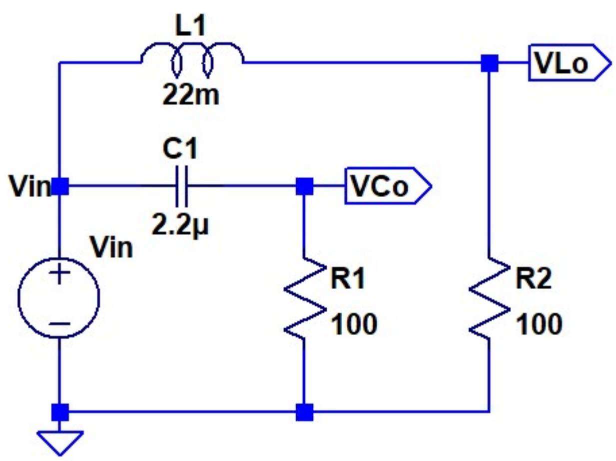

Figure 1 – Simple Crossover Circuit

Human ears can detect audio signals in the range of ~20Hz-20KHz. The circuit above places a crossover point at ~650Hz. For the purpose of our experiment in this lesson, we consider the signals lower than 650Hz as bass and the ones above as the treble components.

In the following sections, we will derive the expressions for the output voltages at V_{Lo} and V_{Co} separately.

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Bass Output

We know that at low frequencies an inductor acts as a short while at high frequencies, it acts as an open circuit. Therefore, we can expect that the voltage at V_{Lo} to be close to V_{in} at low frequencies and close to 0 at high frequencies. V_{Lo}, therefore, is our bass output.

To analytically examine the signal at V_{Lo}, we use the voltage divider equation. However, we need to be cognizant of the complex nature of an inductor's impedance and perform all the math operations in complex domain.

$$V_{Lo} = V_{in}\frac{R_2}{R_2 + j\omega L} \; or \; \frac{V_{Lo}}{V_{in}} = \frac{R_2}{R_2 + j\omega L}$$

Factoring out R_2 in the denominator, we have

$$\frac{V_{Lo}}{V_{in}} = \frac{R_2}{R_2\left(1 + j\omega\frac{L}{R_2}\right)} = \frac{1}{1 + j\omega\frac{L}{R_2}} \qquad \textbf{Eq. 1}$$

We are interested in the magnitude of the voltage ratio on the left hand side of the equation above. This can be found by dividing the magnitude of the numerator by the magnitude of the denominator.

$$\left|\frac{V_{Lo}}{V_{in}}\right| = \frac{1}{\sqrt{1 + \left(\omega\frac{L}{R_2}\right)^2}} \qquad \textbf{Eq. 2}$$

The ratio of the output voltage to the input voltage in Eq. 2 is the gain of the circuit, and since it is a function of frequency, it is also the frequency response of the circuit. To examine this gain, we first look at its value at $\omega_c = \frac{R_2}{L}$ called the corner frequency. At $\omega_c = \frac{R_2}{L}$,

$$\left|\frac{V_{Lo}}{V_{in}}\right|_{\omega=\frac{R_2}{L}} = \frac{1}{\sqrt{1 + \left(\frac{R_2}{L}\frac{L}{R_2}\right)^2}} = \frac{1}{\sqrt{2}}$$

As seen in the equation above, at the corner frequency, the gain is $\frac{1}{\sqrt{2}}$, or V_{Lo} is 3dB lower than V_{in} since

$$20\log\left(\frac{1}{\sqrt{2}}\right) = -3dB$$

At frequencies much lower than $\omega_c = \frac{R_2}{L}$, the denominator of the ratio in Eq. 2 is approximately 1; and therefore, the ratio is approximately 1.

$$\left|\frac{V_{Lo}}{V_{in}}\right|_{\omega\ll\omega_c} \approx \frac{1}{\sqrt{1+0^2}} = 1$$

At frequencies much larger than the corner frequency, the denominator is dominated by the second term; and therefore,

$$\left|\frac{V_{Lo}}{V_{in}}\right|_{\omega \gg \omega_c} \approx \frac{1}{\sqrt{\left(\omega\frac{L}{R_2}\right)^2}} = \frac{1}{\omega\frac{L}{R_2}}$$

In dB domain,

$$20\log\left(\left|\frac{V_{Lo}}{V_{in}}\right|_{\omega \gg \omega_c}\right) \approx 20\log\left(\frac{1}{\omega\frac{L}{R_2}}\right) = -20\log\left(\omega\frac{L}{R_2}\right) = -20\log\omega - 20\log\left(\frac{L}{R_2}\right)$$

which is a line with a slope of -20dB.

The frequency response of the circuit given by Eq. 2 is shown below.

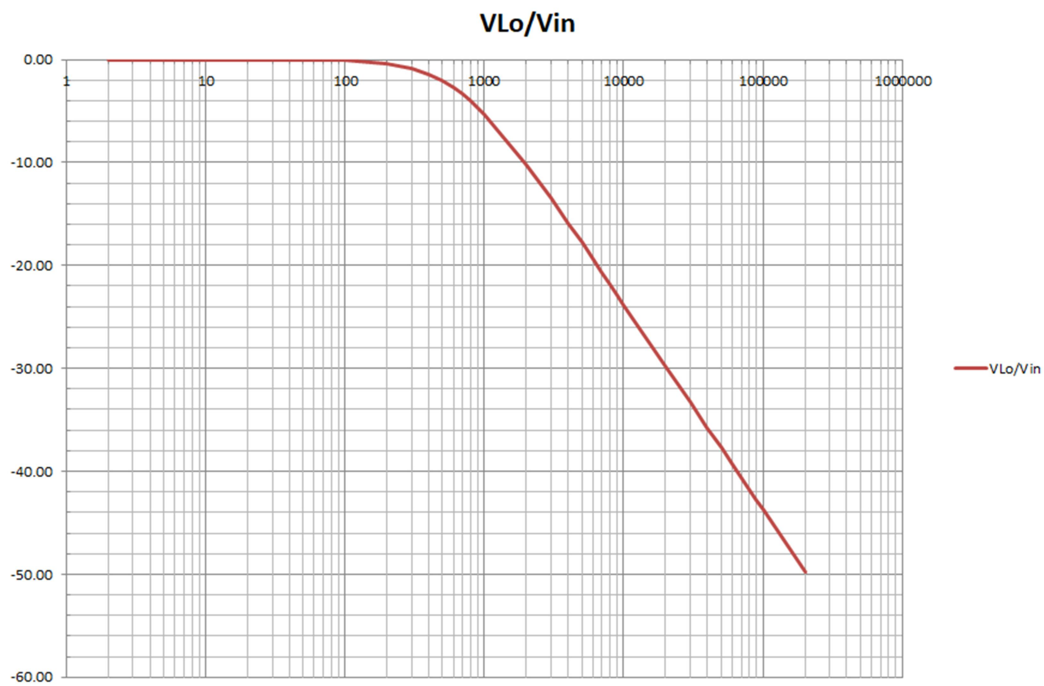

Figure 2 - Frequency Response of the Bass Output

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Treble Output

A capacitor, at low frequencies, acts as an open circuit while at high frequencies, it acts as a short. Therefore, we can expect that the voltage at V_{Co} to be close to V_{in} at high frequencies and close to 0 at low frequencies. V_{Co}, therefore, is our treble output.

The analytical examination of the signal at V_{Co}, follows that of V_{Lo} very closely.

$$V_{Co} = V_{in}\frac{R_1}{R_1 + \frac{1}{j\omega C}} \quad or \quad \frac{V_{Co}}{V_{in}} = \frac{j\omega R_1 C}{j\omega R_1 C + 1}$$

We are interested in the magnitude of the ratio above which can be found by dividing the magnitude of the numerator by the magnitude of the denominator.

$$\left|\frac{V_{Co}}{V_{in}}\right| = \frac{\omega R_1 C}{\sqrt{1 + (\omega R_1 C)^2}} \qquad \textbf{Eq. 3}$$

To examine the gain or the frequency response of this circuit, we again first look at its value at its corner frequency $\omega_c = \frac{1}{R_1 C}$. At the corner frequency,

$$\left|\frac{V_{Co}}{V_{in}}\right|_{\omega=\frac{1}{R_1 C}} = \frac{1}{\sqrt{1 + \left(\frac{1}{R_1 C}R_1 C\right)^2}} = \frac{1}{\sqrt{2}}$$

Again as seen above, at the corner frequency, the gain is $\frac{1}{\sqrt{2}}$, or V_{Co} is 3dB lower than V_{in} since

$$20\log\left(\frac{1}{\sqrt{2}}\right) = -3dB$$

At frequencies much larger than the corner frequency, the denominator is dominated by the second term; and therefore,

$$\left|\frac{V_{Co}}{V_{in}}\right|_{\omega \gg \omega_c} \approx \frac{\omega R_1 C}{\sqrt{(\omega R_1 C)^2}} = 1$$

At frequencies much lower than $\omega_c = \frac{1}{R_1 C}$, the denominator of the ratio in Eq. 3 is approximately 1; and therefore, the ratio is approximately

$$\left|\frac{V_{Co}}{V_{in}}\right|_{\omega \ll \omega_c} \approx \frac{\omega R_1 C}{\sqrt{1 + 0^2}} = \omega R_1 C$$

In dB domain,

$$20\log\left(\left|\frac{V_{Co}}{V_{in}}\right|_{\omega \ll \omega_c}\right) \approx 20\log(\omega R_1 C) = 20\log\omega + 20\log(R_1 C)$$

which is again a line with a slope of -20dB.

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The frequency response of the circuit given by Eq. 3 is shown below.

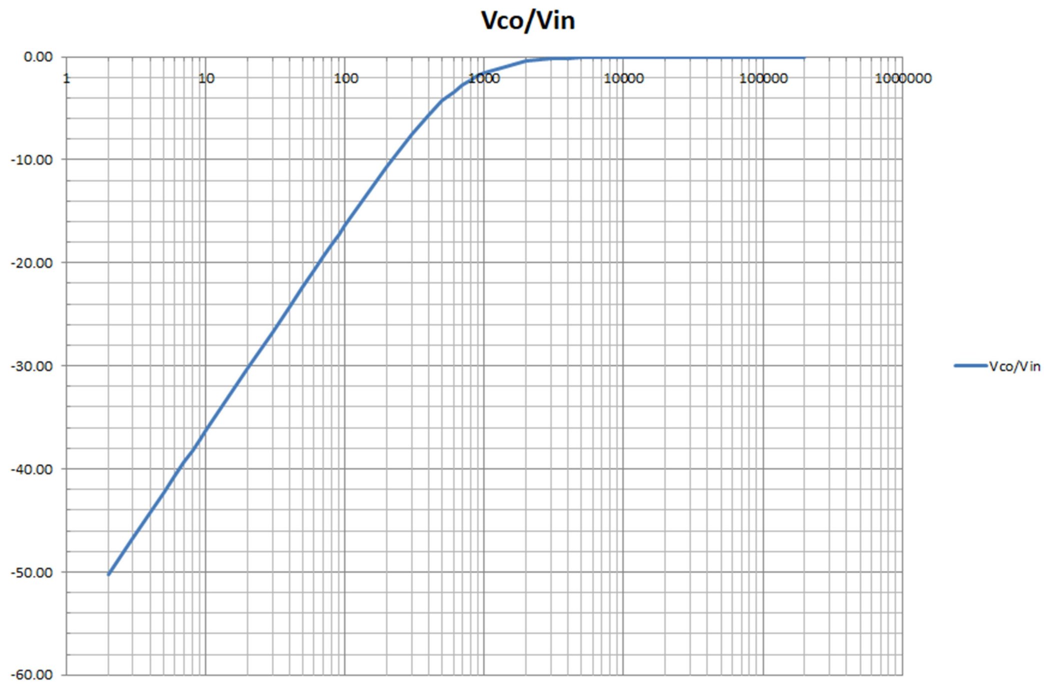

Figure 3 - Frequency Response of the Treble Output

The two frequency responses super imposed are shown in below.

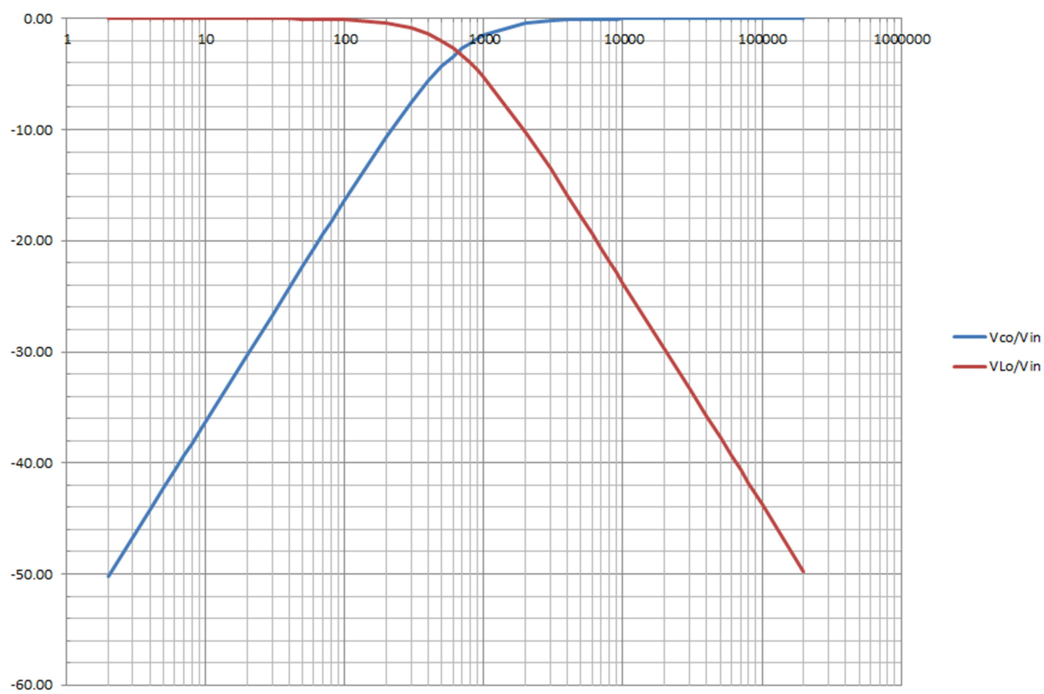

Figure 4 - Freqency Respone of the Crossover Filter

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Procedure

SPICE Simulation

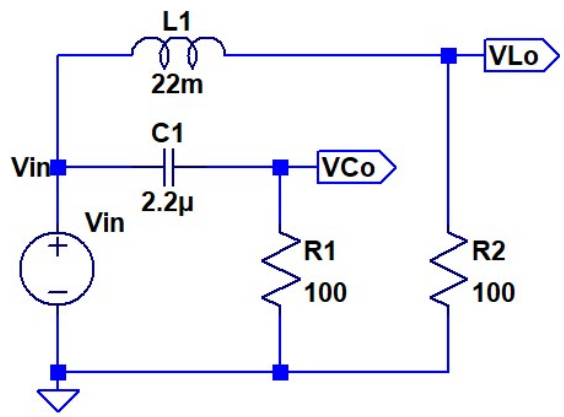

Figure 5 - Inductor and Current Source

A. Build the crossover filter circuit above in SPICE.
    a. Make sure the "Series Resistance" of the inductor is set to 0.
B. Configure the voltage source as a sine wave and set its AC amplitude to 3V.
C. Setup the model to run an "AC" simulation with the following parameters
    a. Steps set to decade,
    b. 100 points per decade,
    c. Starting frequency at 2Hz,
    d. Ending frequency at 200KHz

    Below is the syntax for your reference:

    .ac <oct, dec, lin> <Npoints> <StartFreq> <EndFreq]

D. Run the simulation and display the waveforms for the ratios $\frac{V_{Co}}{V_{in}}$ and $\frac{V_{Lo}}{V_{in}}$.
    a. What is the corner frequency for each ratio? *(Note that at the corner frequency, the ratio of the output voltage to the input voltage has dropped 3dB below its max.)*

f_{c_bass} = ______________________ Hz

f_{c_treble} = ______________________ Hz

b. What is the phase of the bass output for frequencies much lower than the corner frequency?

c. What is the phase of the bass output for the frequencies much higher than the corner frequency?

d. What is the phase of the treble output for frequencies much lower than the corner frequency?

e. What is the phase of the treble output for frequencies much higher than the corner frequency?

E. Select an inductor for L1 from the list of available inductors.
   a. Record the "Series Resistance" of the inductor below.

   $R_L =$ ______________________Ω

F. Run the simulations again and examine the waveforms for the ratios $\frac{V_{Co}}{V_{in}}$ and $\frac{V_{Lo}}{V_{in}}$.
   a. What changes do you see in the frequency response of the two ratios?

b. Did the location of the bass corner frequency change? If yes, what is the new value?

$f_{c\_bass} =$ ______________________ Hz

c. What is the value of the magnitude of the bass frequency response at the corner frequency?

$\left(\frac{V_{Lo}}{V_{in}}\right)_{dB} =$ ______________________ dB

Lab Measurements

A. Build the circuit shown in Figure 5. Make sure you measure the resistance of the inductor first and record it here.

$R_L =$ ____________________ Ω

B. Use the function generator for V_{in} and configure it as a sine wave with the following settings:
   a. Amplitude = 3V ($V_{pk-pk} = 6V$)
   b. Offset = 0V
   c. Frequency = 50Hz

C. Use one oscilloscope to measure the input voltage and the treble output voltage V_{Co}.
   a. Connect one probe to the input voltage and another probe to V_{Co}.
   b. Record the magnitude (max) of the output voltage at V_{Co} in column B of the table below for the frequencies listed in column A.
   c. Measure the time difference between the max of the output voltage at V_{Co} and the max input voltage and record it in column H in the table below for the frequencies listed.

D. Use another oscilloscope to measure the input voltage and the bass output voltage V_{Lo}.
   a. Connect one probe to the input voltage and another probe to V_{Lo}.
   b. Record the magnitude (max) of the output voltage at V_{Lo} in column C of the table below for the frequencies listed in column A.
   c. Measure the time difference between the max of the output voltage at V_{Lo} and the max input voltage and record it in column I in the table below for the frequencies listed.

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E. Perform the calculations in columns D-G and J-K in Google Sheets and plot the values for columns F-G and J-K vs. frequency in column A.

Table 1 - Frequency Response Measurements

A	B	C	D	E	F	G	H	I	J	K
Frequency	$\lvert V_{Co}\rvert$	$\lvert V_{Lo}\rvert$	$\left\lvert\frac{V_{Co}}{V_{in}}\right\rvert$	$\left\lvert\frac{V_{Lo}}{V_{in}}\right\rvert$	$20\log\left(\left\lvert\frac{V_{Co}}{V_{in}}\right\rvert\right)$	$20\log\left(\left\lvert\frac{V_{Lo}}{V_{in}}\right\rvert\right)$	$T_{V_{Co}} - T_{V_{in}}$	$T_{V_{Lo}} - T_{V_{in}}$	$\frac{T_{V_{Co}} - T_{V_{in}}}{T}$	$\frac{T_{V_{Lo}} - T_{V_{in}}}{T}$
50										
60										
70										
80										
90										
100										
200										
300										
400										
500										
600										
700										
800										
900										

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A	B	C	D	E	F	G	H	I	J	K
Frequency	$\lvert V_{Co}\rvert$	$\lvert V_{Lo}\rvert$	$\left\lvert\frac{V_{Co}}{V_{in}}\right\rvert$	$\left\lvert\frac{V_{Lo}}{V_{in}}\right\rvert$	$20\log\left(\left\lvert\frac{V_{Co}}{V_{in}}\right\rvert\right)$	$20\log\left(\left\lvert\frac{V_{Lo}}{V_{in}}\right\rvert\right)$	$T_{V_{Co}} - T_{V_{in}}$	$T_{V_{Lo}} - T_{V_{in}}$	$\frac{T_{V_{Co}} - T_{V_{in}}}{T}$	$\frac{T_{V_{Lo}} - T_{V_{in}}}{T}$
1000										
2000										
3000										
4000										
5000										
6000										
7000										
8000										
9000										
10000										

A. What is the corner frequency for each output? *(Note that at the corner frequency, the ratio of the output voltage to the input voltage has dropped 3dB below its max.)*

$f_{c\_bass} =$ ______________________ Hz

$f_{c\_treble} =$ ______________________ Hz

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B. What is the phase of the bass output for frequencies much lower than the corner frequency?

C. What is the phase of the bass output for the frequencies much higher than the corner frequency?

D. What is the phase of the treble output for frequencies much lower than the corner frequency?

E. What is the phase of the treble output for frequencies much higher than the corner frequency?

F. How do your results compare to those obtained from simulations?

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