CHAPTER 14
DECISION ANALYSIS

1. Introduction

. Decision-making is a process of selecting the best course of action. To make sound decisions, one has to use systematic, rational approaches to analyze relevant information. The current chapter reviews some of such approaches.

. Glossary:

(1) Action (or decision): An alternative available to a decision-maker.

(2) State of nature (or state of the world): An outcome over which the decision maker has no control.

(3) Payoff (or reward): The result from taking an action (i.e., making a decision) under a state of nature.

(4) Payoff matrix (or payoff table): A tabular format summarizing a finite number of alternatives, states of nature, and the corresponding payoffs.

(5) Expected payoff: A weighted value for taking an action (i.e., making a decision) determined by multiplying each payoff by the probability of the corresponding state of nature and then summing the results.

(6) Sample information: The partial information that results from a sampling study of the decision problem.

(7) Perfect information: The complete information that indicates which state of nature will occur with certainty.

. There are two general types of environment in which managerial decisions are made:

(1) Decision-making under certainty (DMUC): Two or more states of nature exist, and it is known exactly which state of nature will occur.

(2) Decision-making under uncertainty (DMUU):

(a) Probabilistic case - Two or more states of nature exist, and the decision-maker is able or willing to assign a probability of occurrence to each of them.

(b) Non-probabilistic case - Two or more states of nature exist, but the decision-maker is unable or unwilling to assess how likely each state of nature will occur.

2. Decision-Making under Certainty

. If it is known which state of nature will occur, one simply selects the alternative leading to the best payoff under that particular state of nature.

. **Example 14.1:** Allen Young must decide whether to invest $10,000 in the stock market (SM) or in a six-month certificate of deposit (CD) at an interest rate of 8.5 percent. If he invests in the stock market and the market is good (G), he could get a 14 percent return on his money. If the market is fair (F), he expects to get an 8 percent return. If the market is bad (B), he will get no return at all.

(1) Identify the (a) actions (or decisions), (b) states of nature, and (c) payoffs in the decision problem.

(2) Set up a payoff matrix for the problem.

(3) If the stock market will be fair with certainty, what should Allen do?

[Solution] (1) (a) SM, CD.

(b) G, F, B.

(c) 1,400 (= 10,000 x 14%), 800 (10,000 x 8%), 0 (= 10,000 x 0%), 850 (= 10,000 x 8.5%).

(2)

	G	F	B
SM	1,400	800	0
CD	850	850	850

(3) He should invest the money in the six-month certificate of deposit since 850 > 800.

3. Decision-Making under Uncertainty - Non-probabilistic Case

. The following three criteria are widely used for making decisions under uncertainty when the decision-maker is unable or unwilling to assign a probability of occurrence to each of the states of nature:

(1) Maxi-min (or Pessimistic or Wald): Determine the worst possible payoff for each alternative and then choose the alternative leading to the best of the worsts.

(2) Maxi-max (or Optimistic): Determine the best possible payoff for each alternative and then choose the alternative leading to the best of the bests.

(3) Mini-max regret: Set up the regret matrix (or matrix of opportunity losses) and determine the largest regret for each alternative. Then choose the alternative leading to the smallest largest.

. **Example 14.2:** PSH Corporation, a toy manufacturer in Urbana, IL, must choose among four prototype designs for Tippi-Toes, a dancing ballerina doll that does pirouettes and jetés. Each prototype represents a different technology for the moving parts, all powered by small, battery-operated motors. The dolls are all identical in all functional aspects.

The choice of the movement deign will be based solely on a comparison of the contributions to profits (in dollars) by the four prototypes. The demand for Tippi-Toes is uncertain, but management feels that one of the following events will occur: low demand, moderate demand, and high demand. The payoff table is shown below:

	Low (L)	Moderate (M)	High (H)
Gears and levers (G)	-105,000	500,000	640,000
Spring action (S)	-60,000	430,000	740,000
Weights and pulleys (W)	-125,000	445,000	820,000
Pneumatic control (P)	-90,000	370,000	830,000

Determine PSH's best choice of prototype design using each of the following three criteria: (1) maxi-min, (2) maxi-max, and (3) mini-max regret.

[Solution] In what follows, all the payoffs are in thousands of dollars:

	L	M	H	Maxi-min	Maxi-max
G	-105	500	640	-105	640
S	-60	430	740	<u>-60</u>	740
W	-125	445	820	-125	820
P	-90	370	830	-90	<u>830</u>

(1) PSH should choose spring action (S) since -60 > -90 > -105 > -125.

(2) PSH should choose pneumatic control (P) since 830 > 820 > 740 > 640.

(3) It is seen from the regret matrix presented below that PSH should choose weights and pulleys (W) since 65 < 90 < 130 < 190.

	L	M	H	Mini-max regret
G	45	0	190	190
S	0	70	90	90
W	65	55	10	<u>65</u>
P	30	130	0	130

4. Decision-Making under Uncertainty - Probabilistic Case

. All of the modal outcome (MO) criterion, the expected value (EV) criterion, the expected regret (ER) criterion are commonly used in decision making under uncertainty when the decision-maker is able or willing to assign a probability of occurrence to each of the states of nature:

(1) MO: Based on this criterion, the action leading to the best payoff under the state of nature with the highest probability of occurrence should be chosen.

(2) EV: Based on this criterion, the action leading to the best expected payoff should be chosen.

(3) ER: Based on this criterion, the action leading to the smallest expected regret should be chosen.

. **Example 14.3:** Suppose in Example 14.1 that the probabilities of a good market, a fair market, and a bad market are 0.3, 0.5, and 0.2, respectively. What should Allen do based on each of the following criteria: (1) MO, (2) EV, or (3) ER?

[Solution] The payoff matrix along with probabilities of occurrence for the states of nature is shown below:

	G	F	B
SM	1,400	800	0
CD	850	850	850
	0.3	0.5	0.2

(1) Since 0.5 > 0.3 > 0.2, the state of nature with the highest probability of occurrence is fair (F). Allen should then invest in the six-month certificate of deposit because 850 > 800 under the state of nature F.

(2) SM: EV = 0.3(1,400) + 0.5(800) + 0.2(0) = 820
CD: EV = 0.3(850) + 0.5(850) + 0.2(850) = 850

Since 850 > 820, Allen should invest the money in the six-month certificate of deposit.

(3) The regret matrix along with probabilities of occurrence for the states of nature is given below:

	G	F	B
SM	0	50	850
CD	550	0	0
	0.3	0.5	0.2

SM: ER = 0.3(0) + 0.5(50) + 0.2(850) = 195
CD: ER = 0.3(550) + 0.5(0) + 0.2(0) = 165

Since 165 < 195, Allen should invest the money in the six-month certificate of deposit.

. Remark: The EV criterion and the ER criterion always lead to the same conclusion.

. Expected value of perfect information (EVPI): EVPI is an important concept in decision-making under uncertainty when the decision-maker is able or willing to assign a probability of occurrence to each of the states of nature. It places an upper bound on what the decision-maker would be willing to pay for perfect information. By definition,

EVPI = |Expected payoff with perfect information - Best expected payoff based on the EV criterion|

Alternatively, it can be shown that EVPI is equal to the best expected regret (i.e., EVPI = Best ER).

. **Example 14.4:** Refer to Example 14.3. Suppose Allen is thinking of paying for a stock market newsletter which always predicts accurately whether the market will be good, fair, or bad.

(1) What is the EVPI based on the first approach?

(2) What is the EVPI based on the second approach?

(3) Are the results in (1) and (2) above consistent?

(4) Should Allen pay for the newsletter if it will cost him $180?

[Solution] The payoff matrix is reproduced below along with the known probabilities of the states of nature:

	G	F	B
SM	1,400	800	0
CD	850	850	850
	0.3	0.5	0.2

(1) The expected payoff with perfect information is 0.3(1,400) + 0.5(850) + 0.2(850) = 1,015. However, as shown in the solutions to Part (2) of Example 4.3, the best expected payoff based on the EV criterion is 850. It follows that the EVPI = |1,015 – 850| = |165| = 165.

(2) Based on the solutions to Part (3) of Example 4.3, the best ER is 165. So EVPI = 165.

(3) Yes, they are.

(4) Since EVPI = 165 < 180, Allen should not pay for the newsletter.

- **Example 14.5:** Wendy has three major routes to take to work. She can take Main Street the entire way, she can take several back streets, or she can use the expressway. Over the past several months, Wendy has tried each route several times under different traffic conditions. The information is summarized in minutes of travel time in the following table:

	No traffic congestion	Mild traffic congestion	Severe traffic congestion
Main Street	18	33	46
Back streets	25	27	31
Expressway	30	34	37

(1) Determine which route Wendy should take based on each of the following criteria: (a) maxi-min; (b) maxi-max.

(2) In the last fifty days, Wendy encountered severe congestion 10 days and mild congestion 20 days. Assume that the last 50 days are typical of traffic conditions. Which route should she take based on the EV criterion?

(3) Suppose Wendy is about to buy a radio for her car that would tell her the exact traffic conditions before she starts to work every morning. How much time in minutes would Wendy save, on the average, by buying the radio? (Hint: Use the concept of EVPI.)

[Solution] (1)

	N	M	S	Maxi-min	Maxi-max
M	18	33	46	46	<u>18</u>
B	25	27	31	<u>31</u>	25
E	30	34	37	37	30

(a) Wendy should take the back streets since 31 < 37 < 46.

(b) Wendy should take Main Street since 18 < 25 <30.

(2) The probabilities of occurrence for N, M, and S are (50 - 20 - 10)/50 = 0.4, 20/50 = 0.4, and 10/50 = 0.2, respectively.

M: EV = 0.4(18) + 0.4(33) + 0.2(46) = 29.6
B: EV = 0.4(25) + 0.4(27) + 0.2(31) = 27.0
E: EV = 0.4(30) + 0.4(34) + 0.2(37) = 33.0

Since 27.0 < 29.6 < 33.0, Wendy should take the back streets based on the EV criterion.

(4) We see from (2) above that the best expected payoff based on the EV criterion is 27.0 minutes. Moreover, the expected payoff with perfect information is 0.4(18) + 0.4(27) + 0.2(31) = 24.2 minutes. It follows that the EVPI = |24.2 - 27.0| = |-2.8| = 2.8 minutes. Hence, Wendy would save an average of 2.8 minutes per day by buying the radio.

5. Decision Trees

- Another important tool for decision-making under uncertainty - probabilistic case is a decision tree. The decision tree is an analytical tool for decision making particularly useful when a sequence of decisions need to be made over several time periods in the future.

- A decision tree consists of the following elements:

  (1) Decision node (□): A point where various alternatives are available. (Note: The first or leftmost decision node is termed the "root" of the decision tree.)

  (2) Event node (O): A point where various states of nature can occur according to a probability distribution.

  (3) Terminal node (|): A point which signals the completion of the decision-making process

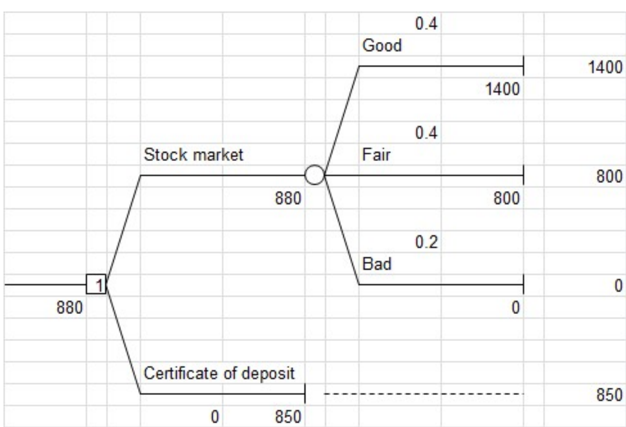
  (4) Branch (-): An alternative if emanating from a decision node or a state of nature if emanating from an event node.

- While a decision tree is constructed and read from left to right, it is analyzed from right to left. At each event node, the EP for the various states of nature is computed. At each decision node, the alternative leading to the best expected payoff is selected.

- Remark: Throughout this chapter, we will follow the so-called "settle-up-at-termination" convention in developing and evaluating a decision tree. This means that all the intermediate payoffs along the path from the root of the tree to a terminal note will be taken into account and the net payoffs will be shown to the right of the terminal node.

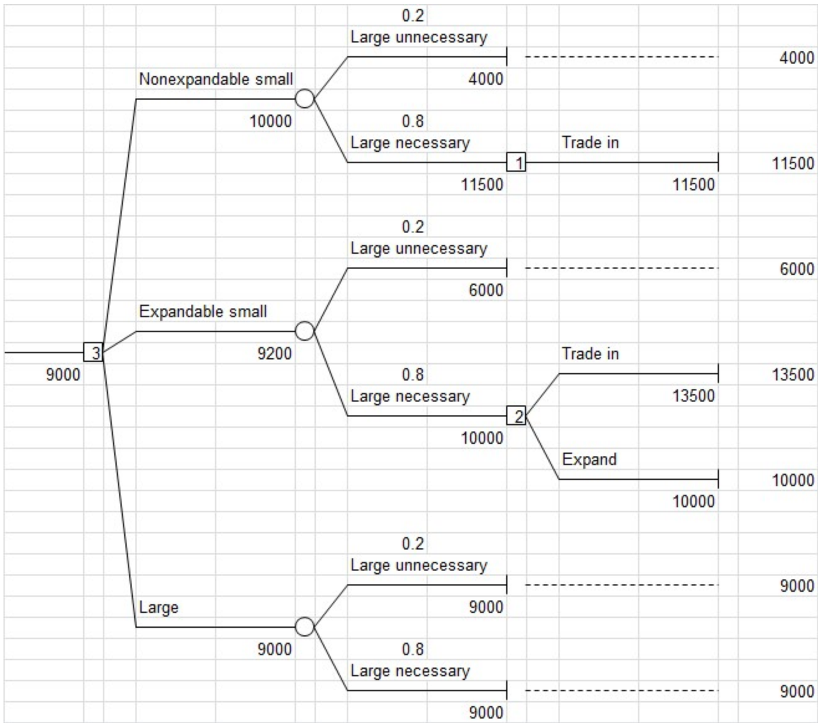
- **Example 14.6:** Allen Young must decide whether to invest \$10,000 in the stock market (SM) or in a six-month certificate of deposit (CD) at an interest rate of 8.5 percent. If he invests in the stock market and the market is good (G), he could get a 14 percent return on his money. If the market is fair (F), he expects to get an 8 percent return. If the market is bad (B), however, he will get no return at all. Suppose that the probabilities of a good market, a fair market, and a bad market are 0.4, 0.4, and 0.2, respectively. What should Allen do based on the result from a decision tree analysis?

  [Solution] We see from the following decision tree that Allen should invest the money in the stock market. The expected maximum return is \$880.



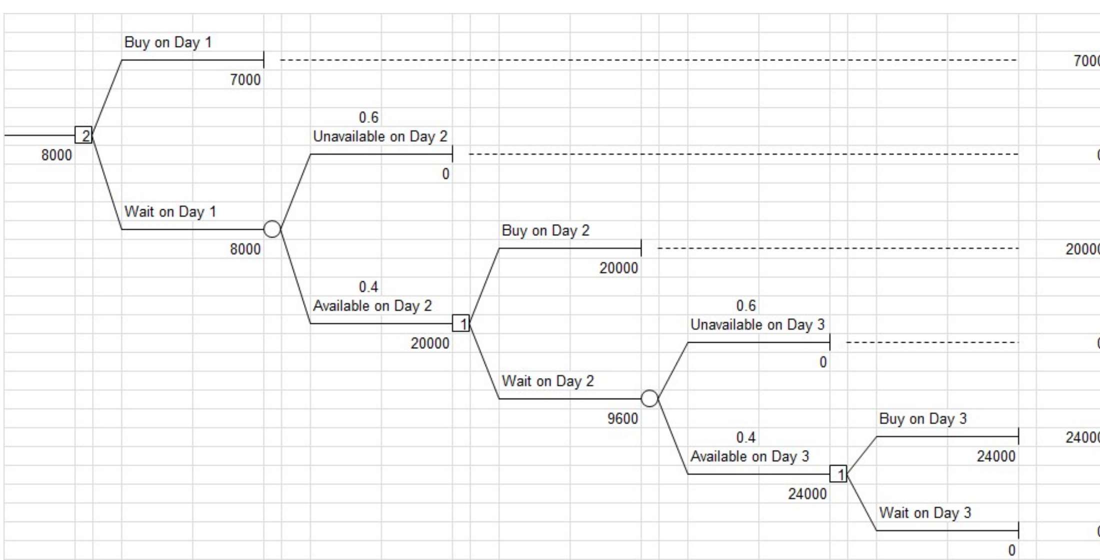
- **Example 14.7:** A small businessman decides to purchase a computer to keep track of his billing, payroll, inventory, and client records. He needs a small system now, but growth of the business could make a small system inadequate in a few years. After hearing one too many sales pitches, the businessman narrows his choices down to buying a small system, a small system that can be expanded, or a large system at the respective prices of \$4,000, \$6,000, and \$9,000. In three years, he can either trade in the small system for a large one at a cost of \$7,500 or expand the expandable small system at a cost of \$4,000. He puts the likelihood of needing the large system in three years at 80 percent. Ignoring the time value of money, employ the decision tree approach to determine which computer the businessman should purchase.

[Solution] We see from the following decision tree that a large computer system should be purchased. The expected minimum cost is $9,000.



. **Example 14.8:** A client is willing to pay $50,000 to buy the painting *Sunplant*. The art dealer can buy the painting for her today for $43,000 or can wait a day and buy the painting tomorrow (if it has not been sold) for $30,000. The dealer may also wait another day and buy the painting (if it is still available) for $26,000. At the end of the third day, the painting will no longer be available. In each day, there is a 0.6 probability that the painting will be sold. Use the decision tree approach to determine what the dealer should do to maximize his expected profit.

[Solution] It is seen from the following decision tree that the art dealer should wait on Day 1. If the painting is still available on Day 2, he should buy it then. His expected maximum profit is $8,000.

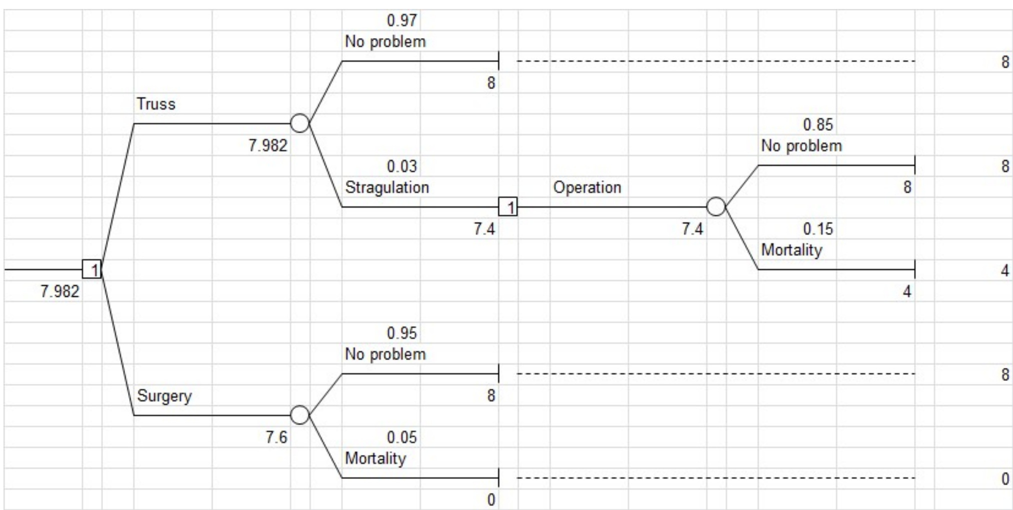


- **Example 14.9:** A 66-year-old man has an inguinal hernia. His internist says he should wear a truss, which will hold the hernia in and thereby avoid the risk of operation. The patient has been wearing the truss. It is not painful, but when he goes to his friend's swimming pool he is embarrassed by it. To avoid the embarrassment, he decides to be admitted for surgery.

  According to the anesthesiologist, if the patient wears a truss there is a 97% chance of living out the rest of his life without having the hernia strangulate. There is a 3% chance of strangulation, however. If strangulation occurs, there will be an emergency operation that has a 15% operative mortality. This patient is expected to live eight years. The emergency operative mortality would on the average be expected to shorten his life to four years. This is because strangulation is assumed to occur halfway through his expected lifetime.

  If the patient chooses the elective surgery, he faces a 5% operative mortality with a loss of eight years of life and a 95% chance of surviving the operation and living out his remaining eight years. Perform a decision tree analysis to determine what the patient should do.

  [Solution] It is seen from the following decision tree that the patient should choose to wear a truss. In case strangulation occurs, he needs an emergency operation. His expected maximum life expectancy is 7.982 more years.



- **Example 14.10:** A group of medical professionals is considering the construction of a clinic. If it is built and the medical demand is high, the doctors could realize a net profit of $100,000. If the demand is low, however, they could lose $40,000. Of course, there will be no cost if they don't proceed to build the clinic. In the absence of any market data, the best they can guess is that there is a 50 percent chance that the demand for the medical services will be high.

  A local consulting firm approaches the doctors and offers to perform a study of the medical market at a fee of $5,000. The researchers claim that their experience enables them to make the following statements:

  (1) The probability of a positive result from the study is 0.55.

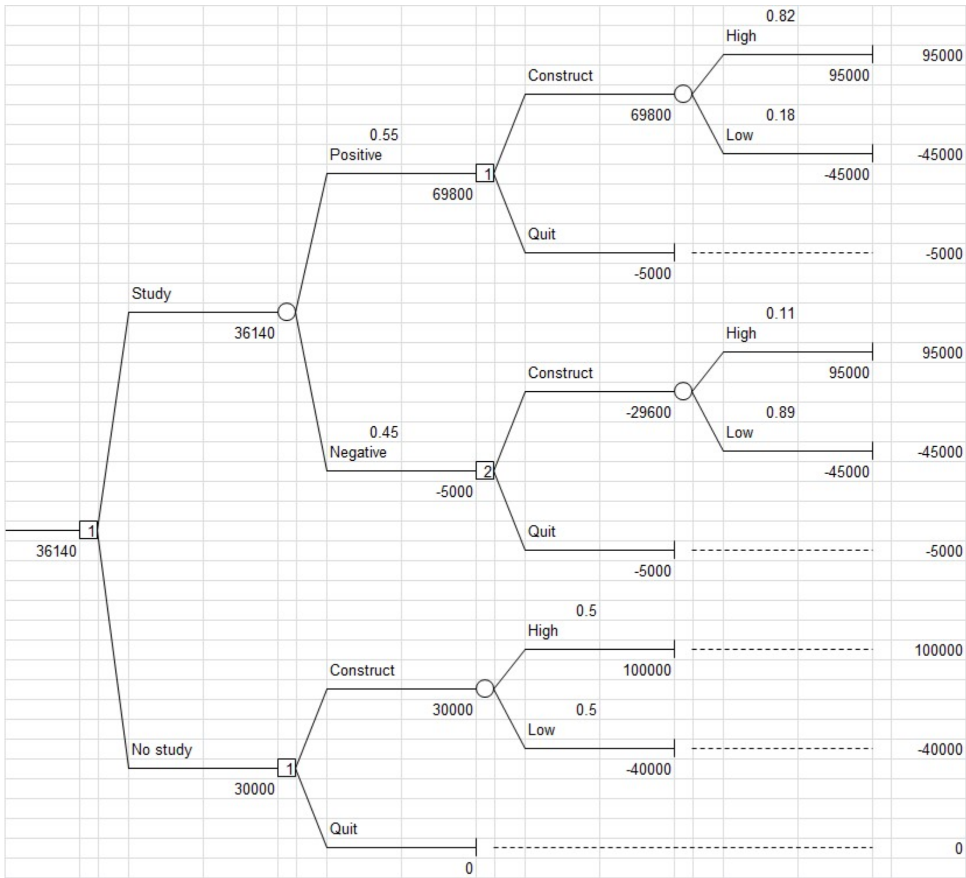
  (2) The probability of a negative result from the study is 0.45.

  (3) Given a positive result, the probability of a high medical demand is 0.82.

  (4) Given a negative result, the probability of a high medical demand is 0.11.

  Use a decision tree to analyze the problem for the medical doctors and advise them what to do.

[Solution] We see from the following decision tree that the doctors should hire the consultants to study the medical market. If the result is positive, they should construct the clinic. If the result is negative, however, the clinic should not be built. The expected maximum profit is $36,140.



. **Example 14.11:** Terri Young joined the economic analysis section of Global Oil in Denton, TX, six months ago. Prescot Oil has just offered $45,000 to buy the Burns Flat lease, which gives Global the right to explore for oil under 120 acres of land in western Oklahoma. Terri has been assigned the task of recommending to Global whether to sell the lease or to drill. If Global drills, the results are uncertain. Based on the past drilling records in the particular region of Oklahoma and the current market prices, Terri prepares the following table which shows the possible outcomes, the probability of each outcome, and the return to Global:

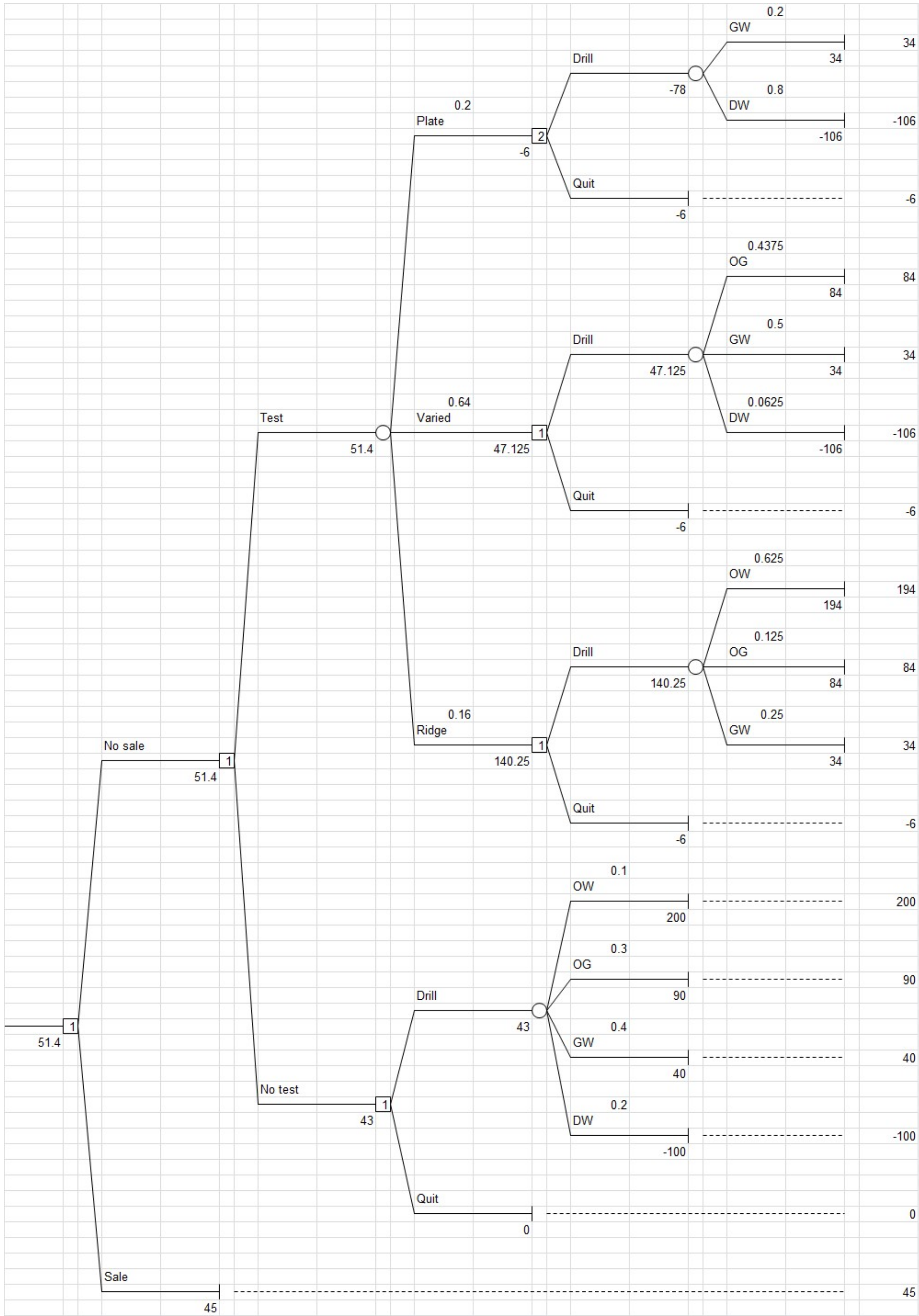
Possible outcome	Probability	Net return
Oil well	0.1	$200,000
Oil and gas	0.3	90,000
Gas well	0.4	40,000
Dry well	0.2	-100,000

Terri knows, however, that she does not have to make the recommendation simply on the basis of historical data. Petro Resource, Inc., offers to conduct a test for Global at $6,000 to evaluate the formation of the Burns Flat terrain. The result should indicate which of the three categories (plate, varied, or ridge) best describes the underground structure. The conditional probabilities of the possible outcomes for the last 50 tests follow:

Test result	Oil well	Oil and gas	Gas well	Dry well	Total	
Plate	0	0	2	8	10	
Varied	0	14	16	2	32	
Ridge	5	1	2	0	8	
	Total	5	15	20	10	50

If the study is undertaken then the opportunity to sell the lease is forfeited, because a decision to sell an oil lease after a test has been performed indicates that drilling does not appear to be profitable. Use TreePlan to carry out a decision tree analysis of the problem for Terri to determine Global's optimal strategy along with the maximum possible expected net profit.

[Solution] It is seen from the following decision tree that Global Oil should not sell the lease to Prescot Oil and should instead pay Petro Resources to perform the test. If the result is plate then it should not drill; otherwise, it should drill. The expected maximum net profit is $51,400.



51.4
1
No sale
51.4
Sale
45
Test
51.4
No test
43
0.2
Plate
-6
0.64
Varied
47.125
0.16
Ridge
140.25
Drill
-78
Quit
-6
0.2
GW
34
0.8
DW
-106
Drill
47.125
0.4375
OG
84
0.5
GW
34
0.0625
DW
-106
Quit
-6
Drill
140.25
0.625
OW
194
0.125
OG
84
0.25
GW
34
Quit
-6
Drill
43
0.1
OW
200
0.3
OG
90
0.4
GW
40
0.2
DW
-100
Quit
0
34
-106
-6
84
34
-106
-6
194
84
34
-6
200
90
40
-100
0
45