OPRE 6398.003 Prescriptive Analytics
Homework 5

Due 02/22/17
(11:30 a.m.)

****

Note: 1. Your homework submission must be typewritten.

1. Show only the solutions and do not copy the problems in the submission.

****

1. Read Readings 3 and 4.

2. A small company in Houston, TX, produces two types of noodle: MR and ZT. Each ton of MR made calls for 20 hours of labor and 4 tons of wheat, whereas each ton of ZT requires 10 hours of labor and 7 tons of wheat germ. The firm has 90 hours of labor available and 28 tons of wheat in stock for next week's production. In addition, management wants to make a total of at least one ton of noodle but the amount of ZT should be no more than three times of the amount of MR. The profit margins of MR and ZT are \$300 and \$350 per ton, respectively. It is assumed that all of the noodle to be made next week will be sold.

   The decision variables and the LP model in Excel for the product-mix problem are presented below, where the last constraint 3MR - ZT $\geq$ 0 is converted from an initial constraint ZT $\leq$ 3MR:

MR = Number of tons of macaroni to be produced next week					
ZT = Number of tons of ziti to be produced next week					
	300	350			
Z	MR	ZT			
0					
			LHS		
	20	10	0	<=	90
	4	7	0	<=	28
	1	1	0	>=	1
	3	-1	0	>=	0

   Both the Answer Report and the Sensitivity Report generated by Solver are shown below, where 1E+30 means ∞. It is seen that the optimal solution is $(MR^*, ZT^*) = (3.5, 2)$ and the optimal objective function value is $Z^* =$ 1,750. In other words, the optimal production plan is to make 3.5 tons of MR and 2 tons of ZT to maximize the total profit at \$1,750.

Microsoft Excel 12.0 Answer Report

Target Cell (Max)

Cell	Name	Original Value	Final Value
\$A\$7	Z	0	1750

Adjustable Cells

Cell	Name	Original Value	Final Value
\$B\$7	MR	0	3.5
\$C\$7	ZT	0	2

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$D\$9	LHS	90	\$D\$9<=\$F\$9	Binding	0
\$D\$10	LHS	28	\$D\$10<=\$F\$10	Binding	0
\$D\$11	LHS	5.5	\$D\$11>=\$F\$11	Not Binding	4.5
\$D\$12	LHS	8.5	\$D\$12>=\$F\$12	Not Binding	8.5

Microsoft Excel 12.0 Sensitivity Report

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B7	MR	3.5	0	300	400	100
C7	ZT	2	0	350	175	200

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
D9	LHS	90	7	90	50	34
D10	LHS	28	40	28	17	10
D11	LHS	5.5	0	1	4.5	1E+30
D12	LHS	8.5	0	0	8.5	1E+30

Answer the following questions in light of the two reports presented above without re-running Solver.

(1) At optimality, how many tons of wheat are unused? Explain your answer.

(2) At optimality, by how many tons does the total amount of noodle made exceed the minimum production level required? Explain your answer.

(3) One employee proposes to work 12 extra hours for the firm next week and wants to be paid at an hourly rate of $10. Should his offer be accepted? Why or why not?

(4) If the amount of wheat available decreases by 5 tons, what will the optimal total profit be and why?

(5) If the profit margin per ton of MR decreases by 35%, will the current optimal production plan remain optimal? Why or why not?

(6) Will the current optimal production plan change if the profit margin per ton of ZT increases to $500? What will the total profit be? Explain your answers.

3. Consider the following linear program:

$$\text{Minimize } Z = 7x + 9y - 6z$$

subject to:

$$\begin{aligned}
-8x + y + 25z &\geq 17 \\
2x - 11z &= 25 \\
14y + z &\leq 191 \\
3x + 10y &\geq 48 \\
x &\leq 0 \\
y &\geq 0 \\
z &\ \text{UIS}
\end{aligned}$$

(1) Modify the above LP to come up with an equivalent LP in standard form where all decision variables are nonnegative.

(2) Run Solver to solve the modified LP in (1). Be sure to include a copy of the Answer report and explain what the optimal solution and the optimal objective function value are.

(3) Based your findings in (2), what are the optimal solution and the optimal objective function value for the original LP?

(4) Develop the dual of the original LP (before modification).

4. The linear program for the Serendipity problem in Question 1 of Homework 1 is reproduced below:

$$
\begin{array}{lrrcr}
\text{Maximize } Z = & 140CN + & 350LS & & \\
\text{subject to:} & 5CN + & 10LS & \leq & 300 \\
 & 0.25CN + & LS & \leq & 22 \\
 & CN\ , & LS & \geq & 0
\end{array}
$$

(1) Run Solver to solve the LP. Be sure to include a copy of each of the Answer Report and the Sensitivity Report.

(2) Develop the dual of the LP.

(3) Run Solver to solve the dual LP. Be sure to include a copy of each of the Answer Report and the Sensitivity Report.

(4) Answer the following questions based on the computer outputs in (1) and (3):

(a) Is the optimal objective function value for the primal LP equal to that for the dual LP?

(b) Are the values of the optimal solution to the primal LP equal to the shadow prices of the right-hand sides of the constraints in the dual LP?

(c) Are the shadow prices of the right-hand sides of the constraints in the primal LP equal to the values of the optimal solution to the dual LP?

(5) Develop the dual of the dual LP in (2).

(6) Is the dual of the dual LP the same as the primal LP?