CHAPTER 15
PROJECT MANAGEMENT

1. Introduction

- Projects are unique, one-time operations designed to accomplish a specific set of objectives in a limited time frame. Examples of projects include launching a space shuttle, constructing a shopping mall, installing a new computer system, and so on.

- When a project is undertaken, people with diverse knowledge and skills are brought together to form a temporary organization. The organization is normally disbanded following the completion of the project. For instance, Project Gemini and Project Apollo are among the most successful projects completed by NASA.

- Both PERT (**P**rogram **E**valuation and **R**eview **T**echnique) and CPM (**C**ritical **P**ath **M**ethod) are widely used for scheduling large-scale projects. Since both PERT and CPM have a great deal in common, the remainder of this chapter will be devoted to only one of them, i.e., PERT analysis of projects.

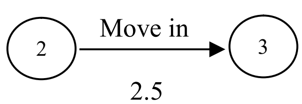
2. PERT Networks

- PERT uses a network to depict the major activities of a project and their sequential relationships. The network consists of (1) arrows, (2) nodes, and (3) numbers. An arrow represents an activity, whereas a node represents the beginning or completion of an activity. Associated with each arrow is a positive number, which is a time estimate indicating how long it would take to complete the activity.

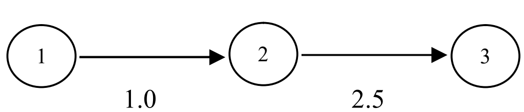
- PERT network conventions:

  (1) A PERT network has a beginning node and an ending node.

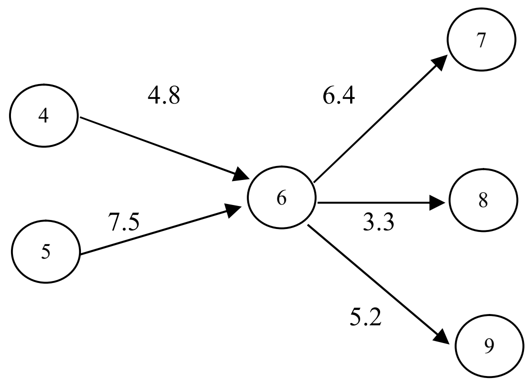
  (2) Nodes are numbered, and activities are identified by the numbers of their respective starting and ending nodes. In addition, the starting node number must be lower than the ending node number. For instance, the activity "move in" in the following diagram, which takes 2.5 hours to complete, is identified as activity 2-3 (where 2 < 3):



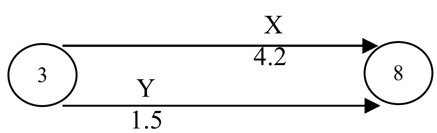
  (3) Activities must be performed in sequence. For example, in the following diagram, activity 1-2 has to be completed before 2-3 can begin:

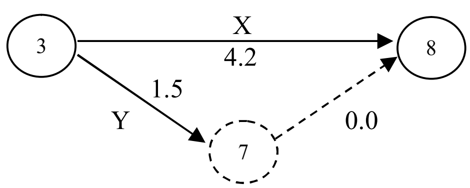


(4) At any node, all entering activities must be completed before any activity that is to begin at the node can start. For instance, in the following diagram, none of the three out going activities (6-7, 6-8, and 6-9) may be started unless both of the incoming activities (4-6 and 5-6) are completed.



(5) When two activities have the same beginning and ending nodes, a dummy node and a dummy arrow are created to form a dummy activity with zero activity time to preserve the separate identity of each activity. For example, in what follows, the fist diagram should be replaced with the second diagram in a PERT network:





(6) The use of dummy activities should be minimized.

(7) No activities may intersect.

. Glossary:

(1) Path: A sequence of activities leading from the beginning node to the ending node of a PERT network.

(2) Path length: The sum of the time estimates of the activities on a path.

(3) Critical path: The longest path in a PERT network.

(4) Critical activities: The activities on the critical path.

(5) Project duration (or completion time): The length of the critical path.

(6) Path slack: The difference between the length of a path and the project duration.

3. Deterministic PERT Analysis

- One way to analyze a PERT network with a deterministic time estimate for each activity is to calculate and compare the lengths of all paths. The critical path, critical activities, and path slacks are then determined accordingly. This approach is termed the enumeration method.

- One can obtain the following information by applying the enumeration method to analyze a PERT network:

  (1) A graphical display of all the project activities and their sequential relationships.

  (2) An estimate of how long it would take to complete the project.

  (3) An indication of which activities are critical to the timely completion of the project.

  (4) An indication of how long an activity can be delayed without affecting the project completion time.

- **Example 15.1:** Activities involved in a project along with time estimates (in weeks) are shown in the following table:

Activity	Description	Immediate predecessors	Estimated time
A	Build internal component	-	2
B	Modify roof and floor	-	3
C	Construct collection stack	A	2
D	Install frame	B	4
E	Build burner	C	4
F	Install control system	C	3
G	Install pollution device	D, E	5
H	Inspect and test	F, G	2

(1) Construct a PERT network for the project.

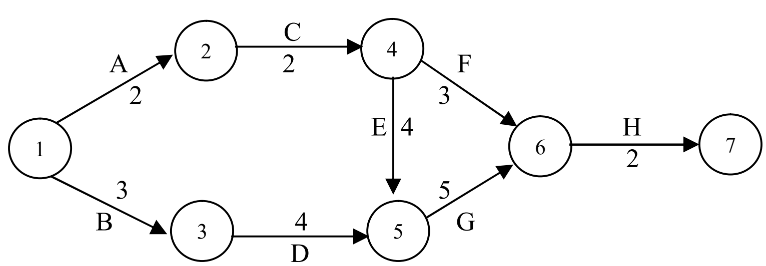
(2) Use the enumeration method to solve the problem by answering the following questions:

(a) What is the critical path? What are the critical activities?

(b) What is the duration of the project?

(c) Are there any path slacks? If so, how long is each of the slacks?

[Solution] (1) The PERT network is shown below:



(2) (a)

Path	Length	Slack
1-2-4-6-7	2 + 2 + 3 + 2 = 9	15 - 9 = 6
1-2-4-5-6-7	2 + 2 + 4 + 5 + 2 = **15**	15 - 15 = 0
1-3-5-6-7	3 + 4 + 5 + 2 = 14	15 - 14 = 1

The critical path is 1-2-4-5-6-7. The critical activities are 1-2, 2-4, 4-5, 5-6, and 6-7.

(b) The duration of the project is 15 weeks.

(c) Path 1-2-4-6-7 has a slack of 6 weeks, and path 1-3-5-6-7 has a slack of 1 week.

. **Example 15.2:** The following tasks are required to complete a software project:

Task	Immediate predecessors	Estimated time (days)
A	-	5
B	-	4
C	A	8
D	A	9
E	B	11
F	C	7
G	C, D	8
H	E	6

(1) Construct a PERT network for the project.

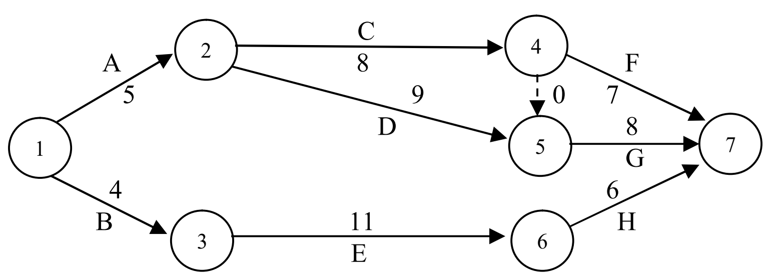
(2) Use the enumeration method to solve the problem by answering the following questions:

(a) What are the critical path and the critical activities?

(b) How long will it take to complete the entire project?

(c) Are there any path slacks? If so, how long is each of the slacks?

[Solution] (1) The PERT network is displayed below:



(2) (a)

Path	Length	Slack
1-2-4-7	5 + 8 + 7 = 20	22 - 20 = 2
1-2-4-5-7	5 + 8 + 0 + 8 = 21	22 - 21 = 1
1-2-5-7	5 + 9 + 8 = **22**	22 - 22 = 0
1-3-6-7	4 + 11 + 6 = 21	22 - 21 = 1

The critical path is 1-2-5-7, and the critical activities are 1-2, 2-5, and 5-7.

(b) It will take 22 days to complete the project.

(c) The path 1-2-4-7 has a slack of 2 days. Paths 1-2-4-5-7 and 1-3-6-7 each have a slack of 1 day.

4. Computational Algorithm

. Many real-life projects contain hundreds or thousands of activities; the necessary computational work can become exceedingly complex and time-consuming a PERT network is analyzed by using the enumeration method. Therefore, a more efficient approach called the computational algorithm is called for when the project network is large.

. The computational algorithm is used to generate the following information for each project activity with a time estimate of t:

(1) ES: The **e**arliest **s**tarting time of an activity.

(2) EF: The earliest finishing time of an activity.

(3) LS: The latest starting time of an activity.

(4) LF: The latest finishing time of an activity

. Remark: It is easily seen that EF = ES + t (or ES = EF - t) and LF = LS + t (or LS = LF - t).

. Once the ES, EF, LS, and LF are computed for each project activity, the following information can be obtained:

(1) Activity slack, which is equal to LS - ES or LF - EF.

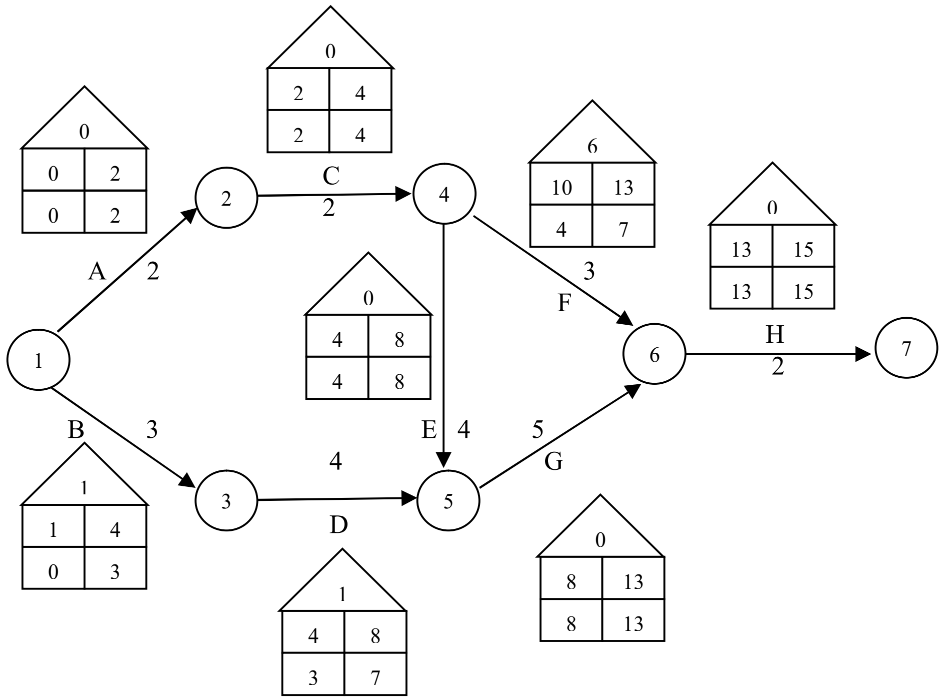
(2) Critical activity, which is an activity with zero slack.

(3) Noncritical activity, which is an activity with a positive slack.

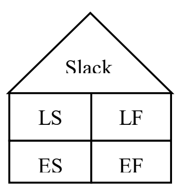
(4) Project duration, which is equal to the LF of any ending activity or the largest of the EFs of all ending activities.

. **Example 15.3:** Refer to the PERT network for Example 15.1. Apply the computational algorithm to determine (1) project completion time, (2) critical path and critical activities, and (3) activity slacks.

[Solution] The PERT network is shown below:



Legend:



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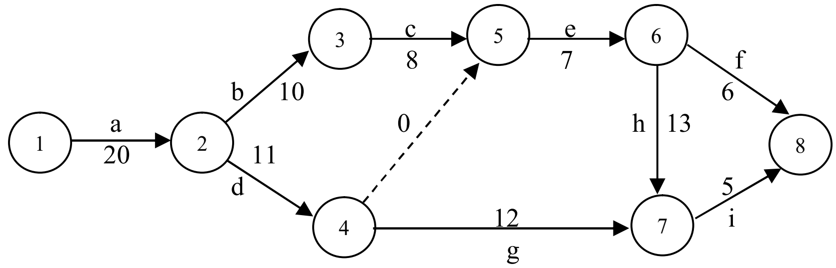
(1) The project completion time is 15 weeks from the beginning day.

(2) The critical path is 1-2-4-5-6-7. The critical activities are 1-2, 2-4, 4-5, 5-6, and 6-7.

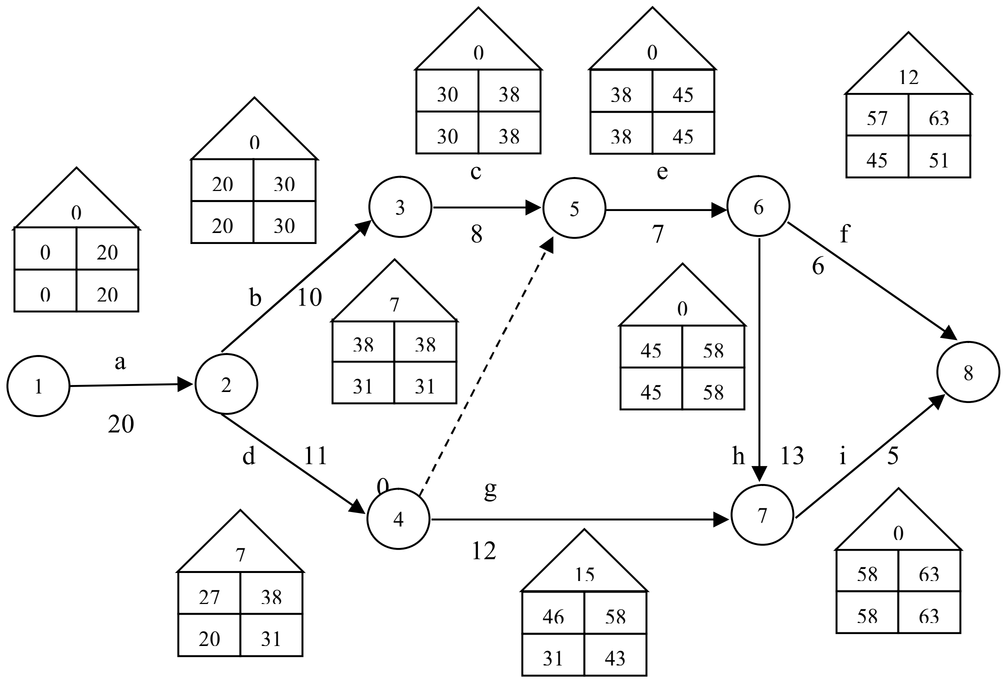
(3) The activity slacks (in weeks) are summarized below:

1-2	1-3	2-4	3-5	4-5	4-6	5-6	6-7
0	1	0	1	0	6	0	0

- **Example 15.4:** Apply the computational algorithm to analyze the following PERT network and determine the (1) project duration, (2) critical path and critical activities, and (3) activity slacks.



[Solution] The PERT network is shown below:



(1) The project completion time is 63 weeks from the beginning day.

(2) The critical path is 1-2-3-5-6-7-8, and the critical activities are 1-2, 2-3, 3-5, 5-6, 6-7, and 7-8.

(3) The activity slacks (in weeks) are summarized below:

1-2	2-3	2-4	3-5	4-7	5-6	6-7	6-8	7-8
0	0	7	0	15	0	0	12	0

. Remark: It is possible to formulate the project management problem as a linear program and then solve it to determine the project duration.

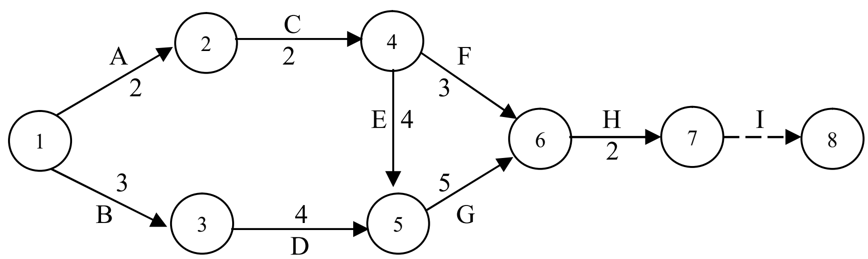
. **Example 15.5:** Reconsider the project management problem discussed in Example 15.1.

(1) Develop a linear programming model for determining the project duration.

(2) Run Solver to solve the LP in (1).

(3) Is the computer solution in (2) above consistent with the manual solution based on the enumeration method in Example 15.1 and that based on the computational algorithm in Example 15.3?

[Solution] (1) Let I be an imaginary activity starting at node 7 and ending at node 8. We then have:



Further let Xi be the earliest starting time of activity i, i = A, B, …, I. It follows that the project duration will be equal to XI. The LP formulation sought is:

$$
\begin{array}{lrcl}
\text{Minimize } Z = & XI & & \\
\text{subject to:} & XA & = & 0 \\
 & XB & = & 0 \\
 & XC & \geq & XA + 2 \\
 & XD & \geq & XB + 3 \\
 & XE & \geq & XC + 2 \\
 & XF & \geq & XC + 2 \\
 & XG & \geq & XD + 4 \\
 & XG & \geq & XE + 4 \\
 & XH & \geq & XF + 3 \\
 & XH & \geq & XG + 5 \\
 & XI & \geq & XH + 2 \\
 & XA, \ldots, XI & \geq & 0
\end{array}
$$

or

$$
\begin{array}{lrcl}
\text{Minimize } Z = & XI & & \\
\text{subject to:} & XA & = & 0 \\
 & XB & = & 0 \\
 & -XA + XC & \geq & 2 \\
 & -XB + XD & \geq & 3 \\
 & -XC + XE & \geq & 2 \\
 & -XC + XF & \geq & 2 \\
 & -XD + XG & \geq & 4 \\
 & -XE + XG & \geq & 4 \\
 & -XF + XH & \geq & 3 \\
 & -XG + XH & \geq & 5 \\
 & -XH + XI & \geq & 2 \\
 & XA, \ldots, XI & \geq & 0
\end{array}
$$

(2) Running Solver to solve the above linear program, we have (XA^*, XB^*, XC^*, XD^*, XE^*, XF^*, XG^*, XH^*, XI^*) = (0, 0, 2, 4, 4, 10, 8, 13, 15) and $Z^* = 15$. Hence, the project duration is 15 weeks.

(3) Yes, it is.

5. Probabilistic PERT Analysis

. It was assumed in the deterministic analysis of PERT networks that the project activity times can be estimated with certainty. However, the assumption may not be valid in many real-world situations where the time required to complete an activity is subject to variation. Under these circumstances, a probabilistic approach to PERT analysis is necessary for analyzing the project network.

. When performing the probabilistic PERT analysis, three time estimates are used for each activity:

(1) Optimistic time (a): The amount of time required to finish the activity under the best conditions.

(2) Pessimistic time (b): The amount of time required to finish the activity under the worst conditions.

(3) Most-likely time (m): The amount of time required to finish the activity under the average conditions.

. Remark: It should be obvious that $a \le m \le b$.

. Notation:

t = Mean (or expected) activity time

s = Standard deviation of activity time

t_{path} = Expected path length

s_{path} = Standard deviation of path length

. If the activity times follow a beta distribution, then it can be shown that

$t = (a + 4m + b)/6$

$s = (b - a)/6$ or $s^2 = (b - a)^2/36$

$t_{path} = \Sigma t$

$s_{path} = (\Sigma s^2)^{1/2}$ or $s_{path}^2 = \Sigma s^2$

. By performing the probabilistic PERT analysis of a project network, one can obtain the probability that the project will be completed within a specified period of time. The procedure involves the following steps:

(1) For each activity, compute the expected value (t) and standard deviation (s) of the completion time.

(2) For each path, compute the length (t_{path}) and the standard deviation (s_{path}) of the completion time.

(3) For each path, compute the probability that it will be completed at or before the specified time (t_0), i.e., $P(t_{path} \le t_0) = P[Z \le (t_0 - t_{path})/s_{path}]$.

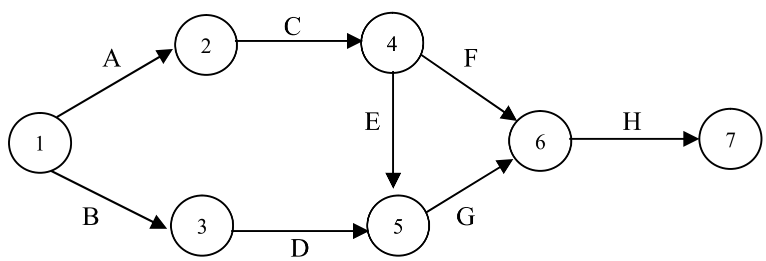
(4) Compute the probability that the entire project will be completed at or before the desired time (t_0) by multiplying the probabilities in (3) together, i.e., $P(t_{project} \le t_0) = \prod P[Z \le (t_0 - t_{path})/s_{path}]$.

. **Example 15.6:** What follows is the probabilistic version of the project management problem discussed in Example 15.1:

Activity	a	m	b	IPs
	Time (weeks)			
A	1	2	3	-
B	2	3	4	-
C	1	2	3	A
D	2	4	6	B
E	1	4	7	C
F	1	2	9	C
G	3	4	11	D, E
H	1	2	3	F, G

Perform a probabilistic analysis of the PERT network to determine the probability that the entire project will be completed within 17 weeks.

[Solution] The probabilistic PERT network without activity times is shown below:



We then prepare the following table:

Path	Activity	a	m	b	t	t_{path}	s^2	s_{path}
1-2-4-6-7	1-2 (A)	1	2	3	2	2 + 2 + 3 + 2 = 9	4/36	1.45
	2-4 (C)	1	2	3	2		4/36	
	4-6 (F)	1	2	9	3		64/36	
	6-7 (H)	1	2	3	2		4/36	
1-2-4-5-6-7	1-2 (A)	1	2	3	2	2 + 2 + 4 + 5 + 2 = 15	4/36	1.76
	2-4 (C)	1	2	3	2		4/36	
	4-5 (E)	1	4	7	4		36/36	
	5-6 (G)	3	4	11	5		64/36	
	6-7 (H)	1	2	3	2		4/36	
1-3-5-6-7	1-3 (B)	2	3	4	3	3 + 4 + 5 + 2 = 14	4/36	1.56
	3-5 (D)	2	4	6	4		16/36	
	5-6 (G)	3	4	11	5		64/36	
	6-7 (H)	1	2	3	2		4/36	

Note that $t_0 = 17$. Referring to the Z tables at the end of this chapter, we have

$Z_{1\text{-}2\text{-}4\text{-}6\text{-}7} = (17 - 9)/1.45 \approx 5.52$
$P(Z \leq 5.52) = 1.0000$

$Z_{1\text{-}2\text{-}4\text{-}5\text{-}6\text{-}7} = (17 - 15)/1.76 \approx 1.14$
$P(Z \leq 1.14) \approx 0.8729$

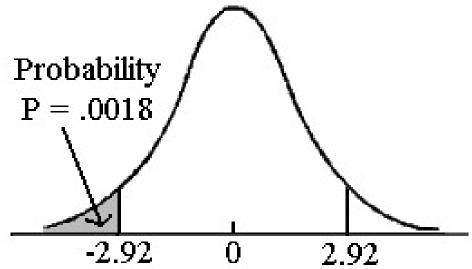
$Z_{1\text{-}3\text{-}5\text{-}6\text{-}7} = (17 - 14)/1.56 \approx 1.92$
$P(Z \leq 1.92) \approx 0.9726$

The probability that the project will be completed within 17 weeks is equal to the joint probability that each of the three paths has a length less than or equal to 17 weeks, i.e., $1.0000(0.8729)(0.9726) \approx 0.8490$.

Z Table: Negative Values

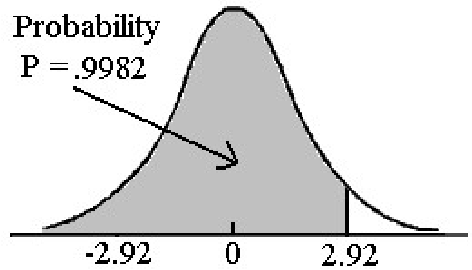
Body of table gives area under Z curve to the left of **z**.

Example: P(Z ≤ -2.92) = .0018



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.80	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
-3.70	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
-3.60	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
-3.50	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
-3.40	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.30	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.20	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.10	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.00	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.90	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.80	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.70	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.60	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.50	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.40	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.30	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.20	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.10	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.00	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.90	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.80	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.70	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.60	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.50	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.40	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.30	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.20	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.10	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.00	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.90	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.80	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.70	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.60	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.50	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.40	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.30	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.20	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.10	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.00	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Z Table: Positive Values



Body of table gives area under Z curve to the left of **z**.

Example: P(Z ≤ 2.92) = .9982

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.00	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.10	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.20	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.30	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.40	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.50	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.60	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.70	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.80	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.90	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.00	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.10	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.20	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.30	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.40	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.50	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.60	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.70	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.80	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.90	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.00	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.10	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.20	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.30	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.40	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.50	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.60	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.70	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.80	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.90	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.00	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.10	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.20	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.30	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.40	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.50	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.60	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.70	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.80	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999