CHAPTER 11
TIME SERIES FORECASTING

1. Introduction

- A forecast is a prediction of what is likely to happen in the future. While forecasts are rarely perfect, they provide an important basis for a wide range of business decisions in marketing, finance, production, etc. For example, sales forecasts help determine how many units of a product should be manufactured in the next three weeks. Also, a finance manager would be interested in the projected (or predicted) cash flows over the next two quarters.

- While useful in areas ranging from weather forecast to stock price prediction, forecasting is not an exact science. Experience, computers, and mathematical techniques all play a role in developing good forecasts.

- A classification diagram of the most widely-used forecasting methods is shown below. In the current chapter, major emphasis will be placed on some of the quantitative approaches.

		Jury of executive opinion		
		Delphi		
	Qualitative	Sales force composite		
		Consumer market survey		
				Naïve
				Simple moving average
Forecasting methods			Averaging	Weighted moving average
				Simple exponential smoothing
		Time series	Trend projection	
	Quantitative		Seasonal patterns	
		Associative		

- A time series is a time-ordered sequence of observations that are made at regular intervals over a stretch of time (e.g., 20 hourly, daily, weekly, monthly, quarterly, or yearly data points).

- **Example 11.1:** The table below shows the carpet sales in 2015 and the first half of 2016 at Hoppy Toppy in Addison, TX. Do the quarterly sales represent a time series? Why or why not?

Year	Quarter	Period number	Sales (yards)
2015	1	1	16,000
	2	2	18,500
	3	3	12,500
	4	4	15,000
2016	1	5	13,500
	2	6	17,500

[Solution] Yes, they do represent a time series since an observation was made each quarter for a period of six quarters or 1.5 years.

. Notation:

t = Index of time period

A_t = Actual demand in period t, t = 1, 2, ...

F_t = Forecast for period t, t = 1, 2, ...

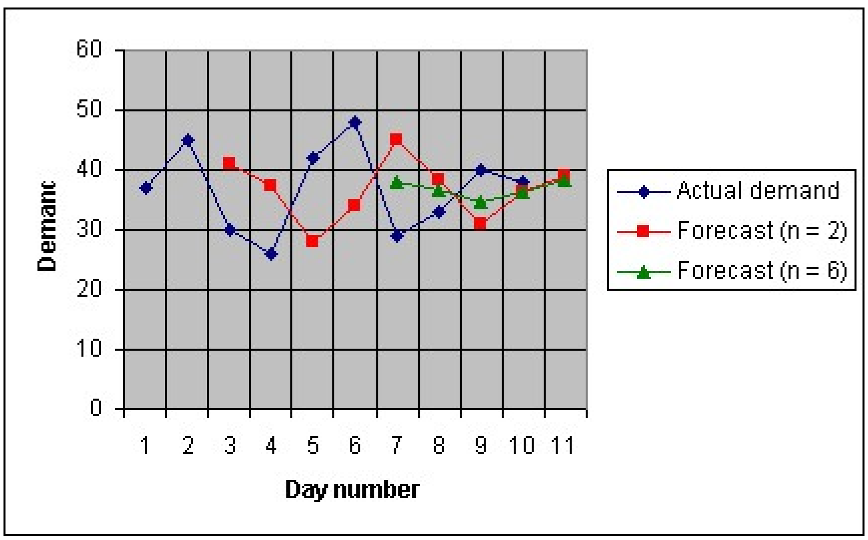
2. Naive Approach

. The naive forecast for the current period is equal to the observation in the immediately past period; namely, $F_t = A_{t-1}$.

3. Simple Moving Average (SMA)

. The SMA forecast for the current period is equal to the average value of the observations in the most recent n periods, where n is the number of time periods involved and its value is determined by the decision maker; namely, $F_t = (A_{t-1} + A_{t-2} + \ldots + A_{t-n})/n$.

. The larger the value of n is in the SMA method (i.e., more past time periods are involved), the smoother the forecasts will be. See the graph below:



4. Weighted Moving Average (WMA)

. The WMA forecast for the next period is equal to the sum of the observations in the most n periods with each being multiplied by a weight w_t, where n is defined in the same way as before; namely, $F_t = w_1A_{t-1} + w_2A_{t-2} + \ldots + w_nA_{t-n}$.

. Remarks: (1) As a general rule in applying the WMA method, heavier weights are assigned to more recent observations; namely, $w_1 \geq w_2 \geq \ldots \geq w_n$.

(2) In deriving the WMA forecasts, the weights used should sum to 1; namely, $w_1 + w_2 + \ldots + w_n = 1$.

5. Simple Exponential Smoothing (SES)

. The SES forecast for the current period is computed based on the following formula:

$F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1})$,

where $\alpha \in [0, 1]$ is called the smoothing constant.

. **Example 11.2:** Consider the table presented in Example 11.1.

(1) What is the naïve forecast of carpet sales in the third quarter of 2016 (Period 7)?

(2) Find the 5-quarter SMA forecast of carpet sales for each of Periods 6 and 7?

(3) Repeat (2) by using the 4-quarter WMA method with weights of 0.3, 0.1, 0.4, and 0.2.

(4) The owner of Hoppy Toppy estimated the sales in the first quarter of 2016 to be 16,500 yards. Apply the SES method with a smoothing constant of 0.4 to forecast the sales in the third quarter of 2016.

[Solution] (1) $F_7 = A_6 = 17{,}500$ yards

(2) $F_6 = (13{,}500 + 15{,}000 + 12{,}500 + 18{,}500 + 16{,}000)/5 = 15{,}100$ yards

$F_7 = (17{,}500 + 13{,}500 + 15{,}000 + 12{,}500 + 18{,}500)/5 = 15{,}400$ yards

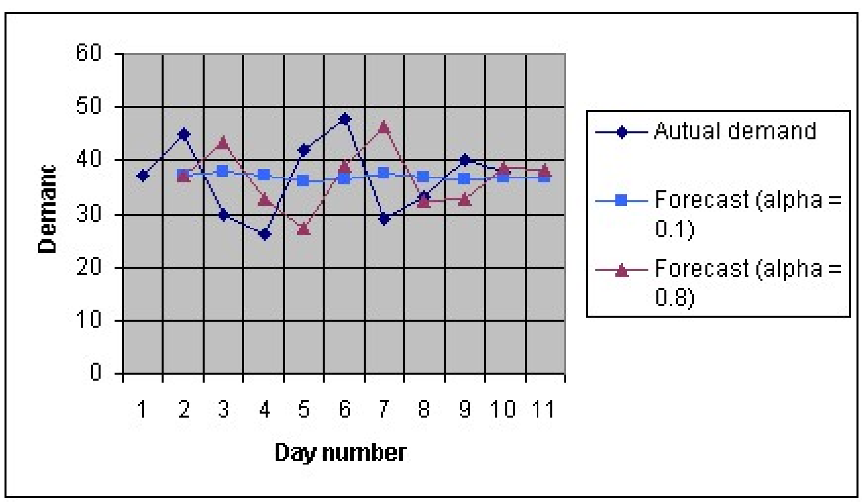
(3) $F_6 = 0.4(13{,}500) + 0.3(15{,}000) + 0.2(12{,}500) + 0.1(18{,}500) = 14{,}250$ yards

$F_7 = 0.4(17{,}500) + 0.3(13{,}500) + 0.2(15{,}000) + 0.1(12{,}500) = 15{,}300$ yards

(4) Given $F_5 = 16{,}500$ and $\alpha = 0.4$, we have $F_6 = F_5 + \alpha(A_5 - F_5) = 16{,}500 + 0.4(13{,}500 - 16{,}500) = 15{,}300$, $F_7 = F_6 + \alpha(A_6 - F_6) = 15{,}300 + 0.4(17{,}500 - 15{,}300) = 16{,}180$ yards.

. Remarks: (1) If $\alpha = 1$ then the SES forecast becomes $F_t = F_{t-1} + \alpha\,(A_{t-1} - F_{t-1}) = F_{t-1} + 1(A_{t-1} - F_{t-1}) = F_{t-1} + A_{t-1} - F_{t-1} = A_{t-1}$ or $F_t = A_{t-1}$, which is in fact the naive forecast. Hence, we see that the naive forecast is a special case of the SES forecast with $\alpha = 1$.

(2) The smaller the smoothing constant (a) is in the SES method, the smoother the forecasts will be. See the graph below:



- All of the naive, simple moving average, weighted moving average, and simple exponential smoothing methods are so-called averaging techniques in forecasting since they work best when the time series generally varies around an average. In case there is any trend present in the historical data, however, the trend projection method to be examined below should be used instead.

- Remark: A simple plotting of the historical data should indicate if there is a trend or not in the time series.

6. Trend Projection

- The trend of a time series refers to a gradual, long-term movement in the historical data. While different types of trend could exist in the real-world forecasting situations, we will limit our discussion here to the case of linear trend.

- Suppose that a linear trend is present in the time series. The main goal of trend projection analysis is searching for a linear equation that best describes the trend. Given a set of n paired data (t, A_t), $t = 1, 2, \ldots, n$, the best-fit trend line may be represented by the following equation:

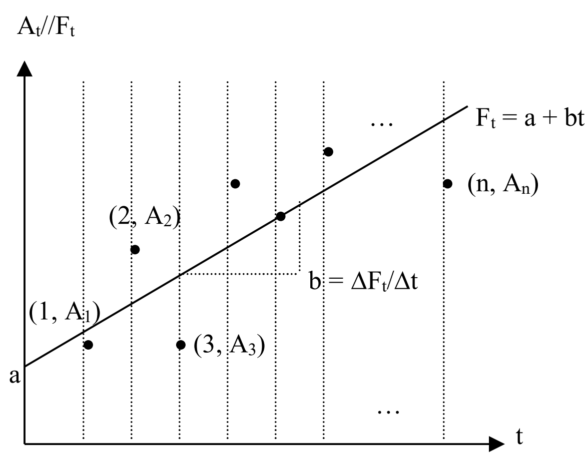
  $F_t = a + bt$,

  where t and F_t are defined in the same way as before and

  b = Slope of the trend line

  a = Intercept of the trend line at the vertical axis (note that $a = F_0$)

- The above concepts are illustrated by the following graph:



- The values of a and b for the best-fit trend line can be determined by applying the least-squares method frequently used in linear regression:

$$b = \frac{n(\sum_{t=1}^{n} tA_t) - (\sum_{t=1}^{n} t)(\sum_{t=1}^{n} A_t)}{n(\sum_{t=1}^{n} t^2) - (\sum_{t=1}^{n} t)^2}$$

$$a = \frac{(\sum_{t=1}^{n} A_t) - b(\sum_{t=1}^{n} t)}{n}$$

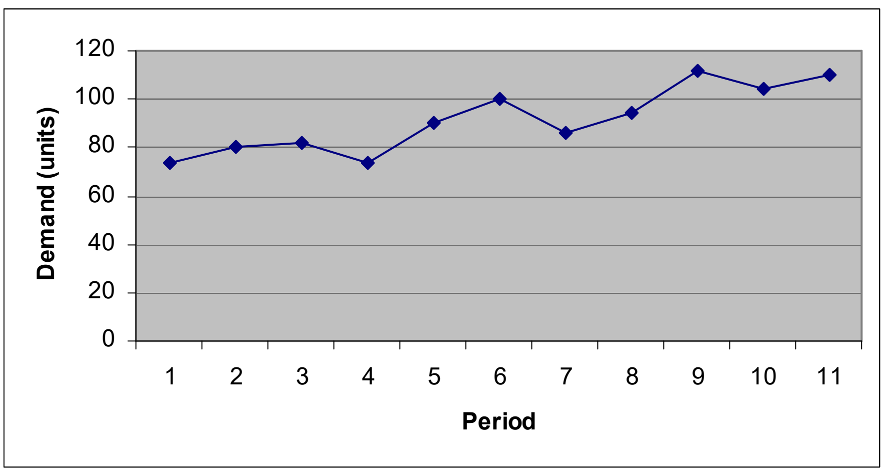
- **Example 11.3:** JM Computer Services assembles customized personal computers from generic parts. The company was formed by two part-time UTD students, John and Mary, and operates mostly at night, using other part-time students for labor. As John and Mary buy parts in volume at discount from a variety of sources whenever they see a good deal, it is important that accurate forecasts of demand be developed so that they know how many components to purchase and stock. Data on the demand for JM's computers have been accumulated for 11 months and are shown in the following table:

Month	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
Demand	74	80	82	74	90	100	86	94	112	104	110

(1) Plot the time series to determine the most appropriate forecasting approach for this problem. Be sure to justify your response.

(2) Apply the method of choice in (1) to predict how many computers will be needed in each of the coming two months.

[Solution] (1) Let February = Period 1, March = Period 2, and so on. The historical data are plotted below:



It is seen above an upward linear trend is present in the time series. As such, the trend projection approach is the most appropriate forecasting method for this problem.

(2)

t	A_t	t^2	tA_t
1	74	1	74
2	80	4	160
3	82	9	246
4	74	16	296
5	90	25	450
6	100	36	600
7	86	49	602
8	94	64	752
9	112	81	1,008
10	104	100	1,040
11	110	121	1,210
66	1,006	506	6,438

n = 11

$$b = \frac{11(6{,}438) - 66(1{,}006)}{11(506) - (66)^2} = \frac{4{,}422}{1{,}210} \approx 3.6546$$

$$a = \frac{1{,}006 - 3.6546(66)}{11} = \frac{764.7964}{11} \approx 69.5270$$

Thus, the best-fit trend line is $F_t = 69.5270 + 3.6546t$. It follows that the forecasts of demand for JM's computer in the next two months are:

January: $F_{12} = 69.5270 + 3.6546(12) = 113.3822$ or about 113 units
February: $F_{13} = 69.5270 + 3.6546(13) = 117.02368$ or about 117 units

7. Seasonal Patterns

. A seasonal pattern is a repetitive increase and decrease in a time series. There are a many products and services for which the demands exhibit seasonal behaviors. For instance, the demand for lawn mowers is usually higher in summer and lower in winter. Seasonal patterns can also occur on a monthly, weekly, or even daily basis. For example, a large number of restaurants are busier on weekends and not as busy during weekdays. In addition, traffic on an interstate highway tends to be heavier during rush hours and lighter at other times.

. There are several approaches to reflecting seasonal variations in time-series forecasts. In this section, we will only discuss a method involving the use of seasonal indexes. Specifically, a seasonal index (SI) is a numerical value determined by dividing the demand during each seasonal period by the total demand during the entire time period (e.g., quarterly demand/annual demand, monthly demand/annual demand, daily demand/weekly demand, etc.). It is then multiplied by a normal forecast of demand to obtain a seasonally adjusted forecast (SAF) of demand.

. Suppose that F_t is the normal forecast for time period t, t = 1, 2, … Let D_i, SI_i, and SAF_i be the demand, the seasonal index, and the seasonally adjusted forecast for seasonal period i in time period t, i = 1, 2, …, n. It follows that

$$SI_i = \frac{D_i}{\sum_{i=1}^{n} D_i}$$

$$SAF_i = SI_i \times F_t$$

. **Example 11.4:** Southwest County Transportation Authority needs to determine the volume of traffic that through a toll bridge by working day of the week. The numbers of vehicles using the bridge (1,000s) over the past four weeks are summarized below:

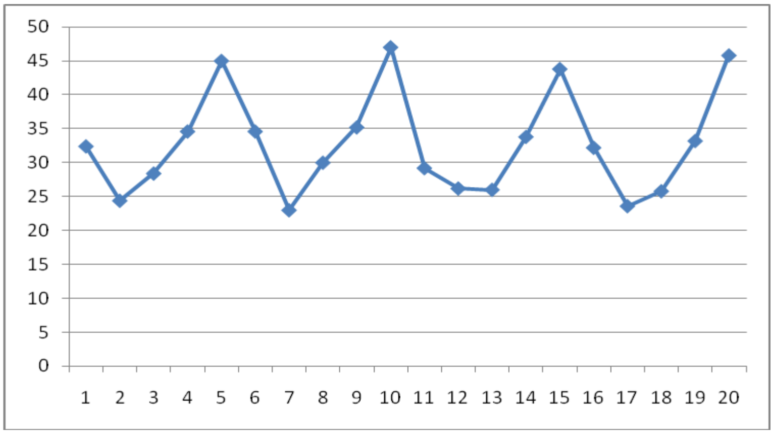
Week	Day	Volume
1	MON	32.4
	TUE	24.4
	WED	28.4
	THU	34.6
	FRI	45.0
2	MON	34.6
	TUE	23.0
	WED	30.0
	THU	35.2
	FRI	47.0
3	MON	29.2
	TUE	26.2
	WED	26.0
	THU	33.8
	FRI	43.8
4	MON	32.2
	TUE	23.6
	WED	25.8
	THU	33.2
	FRI	45.8

(1) Plot the time series and comment on the graph.

(2) Use the two-week simple moving average approach to obtain the normal forecast of traffic volume in Week 5.

(3) Based on your findings in (2), forecast the traffic volumes in each of the five working days in Week 5.

[Solution] (1) The time series is plotted below. One sees that the data varies around an average with a seasonal pattern occurring on a weekly basis.



(2) To begin, we prepare the following table based on the initial table:

Week	MON	TUE	WED	THU	FRI	Total
1	32.4	24.4	28.4	34.6	45.0	164.8
2	34.6	23.0	30.0	35.2	47.0	169.8
3	29.2	26.2	26.0	33.8	43.8	159.0
4	32.2	23.6	25.8	33.2	45.8	160.6
Total	128.4	97.2	110.2	136.8	181.6	654.2

The traffic volumes for the past four weeks are summarized below:

Week	1	2	3	4
Demand	164.8	169.8	159.0	160.6

The two-week SMA forecast of the traffic volume in Week 5 is $F_5 = (160.6 + 159.0)/2 = 159.8$ or 159,800 vehicles.

(3) To compute the seasonal indexes, we see from the table in (2) that $n = 5$, $D_1 = 128.4$, $D_2 = 97.2$, $D_3 = 110.2$, $D_4 = 136.8$, $D_5 = 181.6$, and $D_1 + D_2 + D_3 + D_4 + D_5 = 654.2$. It follows that $SI_1 = \frac{128.4}{654.2} \approx 0.1963$, $SI_2 = \frac{97.2}{654.2} \approx 0.1486$, $SI_3 = \frac{110.2}{654.2} \approx 0.1684$, $SI_4 = \frac{136.8}{654.2} \approx 0.2091$, and $SI_5 = \frac{181.6}{654.2} \approx 0.2776$. Thus, the seasonally adjusted forecasts of traffic volumes during the workdays of Week 5 are:

MON: $SAF_1 = SI_1 \times F_5 = 0.1963 \times 159{,}800 \approx 31{,}369$ vehicles
TUE: $SAF_2 = SI_2 \times F_5 = 0.1486 \times 159{,}800 \approx 23{,}746$ vehicles
WED: $SAF_3 = SI_3 \times F_5 = 0.1684 \times 159{,}800 \approx 26{,}910$ vehicles
THU: $SAF_4 = SI_4 \times F_5 = 0.2091 \times 159{,}800 \approx 33{,}414$ vehicles
FRI: $SAF_5 = SI_5 \times F_5 = 0.2776 \times 159{,}800 \approx 44{,}361$ vehicles

- **Example 11.5:** Quarterly demands for turkeys raised by Country Farms (1,000s) for the past three years are shown below:

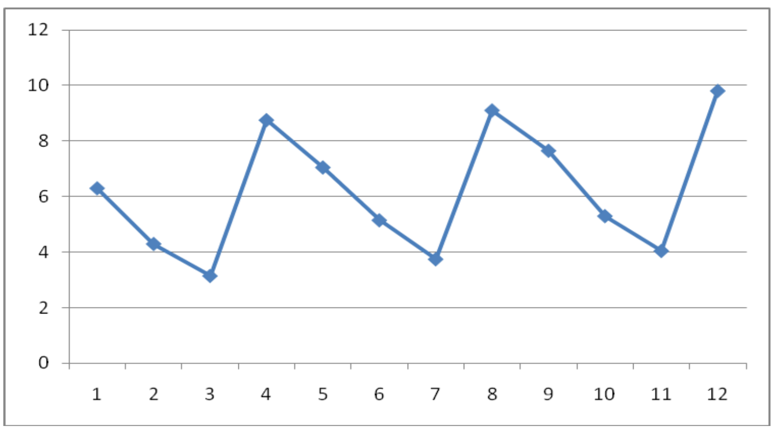
Year	Quarter	Demand
1	1	6.30
	2	4.30
	3	3.15
	4	8.75
2	1	7.05
	2	5.15
	3	3.75
	4	9.10
3	1	7.65
	2	5.30
	3	4.05
	4	9.80

(1) Plot the time series and comment on the data pattern exhibited.

(2) Which method(s) would be most appropriate for predicting demands for turkeys in the future?

(3) Based your findings in (2), forecast the demand for turkeys in each of the four quarters in Year 4.

[Solution] (1) The time series is plotted below. One sees that there is an upward trend in the data set along with a seasonal pattern occurring on a quarterly basis.



(2) The trend projection method should be applied to provide the normal forecasts. Then the season indexes should be computed and used to derive the seasonally adjusted forecasts.

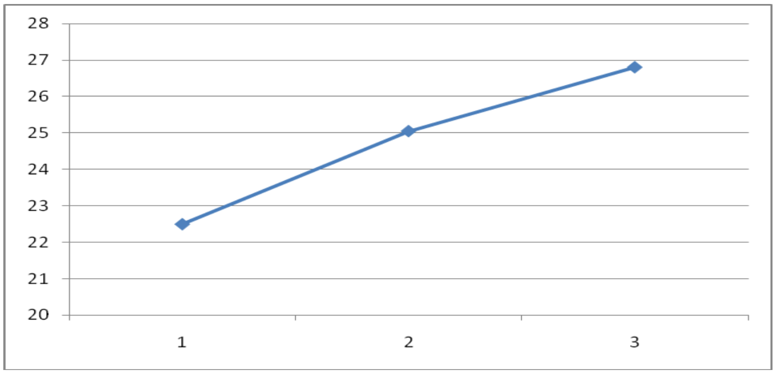
(3) To begin, the following table is prepared based on the initial table:

	Quarter				
Year	1	2	3	4	Total
1	6.30	4.30	3.15	8.75	22.50
2	7.05	5.15	3.75	9.10	25.05
3	7.65	5.30	4.05	9.80	26.80
Total	21.00	14.75	10.95	27.65	74.35

The demands for turkeys for the past three years are summarized below:

Year	1	2	3
Demand	22.50	25.05	26.80

A simple plotting of the annual data is shown below:



Since there is an upward linear trend present in the time series, the trend projection method should be used to provide normal forecasts in the future.

t	A_t	t^2	tA_t
1	22.50	1	22.50
2	25.05	4	50.10
3	26.80	9	80.40
6	74.35	14	153.00

$n = 3$

$$b = \frac{3(153.00) - 6(74.35)}{3(14) - (6)^2} = \frac{12.9}{6} = 2.15$$

$$a = \frac{74.35 - 2.15(6)}{3} = \frac{61.45}{3} \approx 20.4833$$

Thus, the best-fit trend line is $F_t = 20.4833 + 2.15t$. It follows that the normal forecast of demand for turkeys in Year 4 is $F_4 = 20.4833 + 2.15(4) = 29.0833$ or about 29,083. To compute the seasonal indexes, we see from the table at the beginning of (3) that $n = 4$, $D_1 = 21.00$, $D_2 = 14.75$, $D_3 = 10.95$, $D_4 = 27.65$, and $D_1 + D_2 + D_3 + D_4 = 74.35$. It follows that $SI_1 = \frac{21.00}{74.35} \approx 0.2824$, $SI_2 = \frac{14.75}{74.35} \approx 0.1984$, $SI_3 = \frac{10.95}{74.35} \approx 0.1473$, and $SI_4 = \frac{27.65}{74.35} \approx 0.3719$. Thus, the seasonally adjusted forecasts of demands for turkeys for the four quarters in Year 4 are:

Quarter 1: $SAF_1 = SI_1 \times F_4 = 0.2824 \times 29{,}083 \approx 8{,}213$ turkeys

Quarter 2: $SAF_2 = SI_2 \times F_4 = 0.1984 \times 29{,}083 \approx 5{,}770$ turkeys

Quarter 3: $SAF_3 = SI_3 \times F_4 = 0.1473 \times 29{,}083 \approx 4{,}284$ turkeys

Quarter 4: $SAF_4 = SI_4 \times F_4 = 0.3719 \times 29{,}083 \approx 10{,}816$ turkeys

8. Associative Models

. Associative (or causal) models in forecasting rely on the identification of related variables that can be used to predict the value of the variable of interest. For instance, crop yield is related to fertilizer application and may be predicted by the amount of fertilizer applied. Similarly, the sales volume of a product is related to and may be predicted by the advertising expenditure.

. In this section only linear associative models will be examined. The essence of a linear associative model in forecasting is the development of a linear equation that reveals the relationship between the predictor or independent variable (e.g., fertilizer, advertising expenditure) and the predicted or dependent variable (e.g., crop yield, sales).

. Given a set of n paired data (x_i, y_i), $i = 1, 2, \ldots, n$, the linear associative model sought may be represented by the following equation:

$y = a + bx$,

where

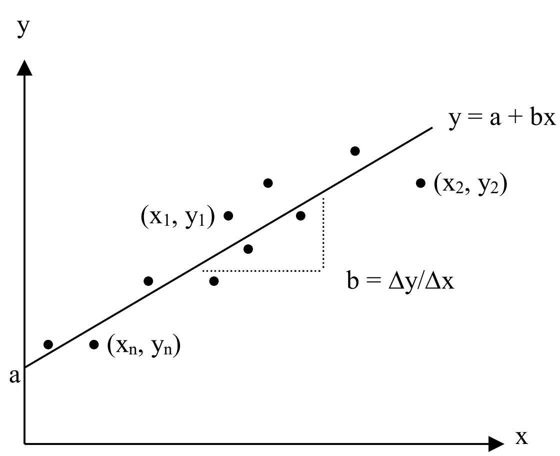
y = Predicted or dependent variable

x = Predictor or independent variable

b = Slope of the associative line

a = Intercept of the associative line at the vertical axis (note that $y = a$ when $x = 0$)

. The following diagram exhibits the relationships among the variables and parameters in the above model:



. The values of a and b in a linear associative model can be determined by applying the least-squares method in regression analysis:

$$b = \frac{n(\sum_{i=1}^{n} x_i y_i) - (\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)}{n(\sum_{i=1}^{n} x_i^2) - (\sum_{i=1}^{n} x_i)^2}$$

$$a = \frac{(\sum_{i=1}^{n} y_i) - b(\sum_{i=1}^{n} x_i)}{n}$$

. The coefficient of determination r^2 defined below represents the portion of variation in y that can be accounted for by x and is used to assess the validity of the linear associative model:

$$r^2 = \frac{[n(\sum_{i=1}^{n} x_i y_i) - (\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)]^2}{[n(\sum_{i=1}^{n} x_i^2) - (\sum_{i=1}^{n} x_i)^2][n(\sum_{i=1}^{n} y_i^2) - (\sum_{i=1}^{n} y_i)^2]}$$

. Observe that:

(1) $0 \le r^2 \le 1$.

(2) The larger the r^2 value is, the more valid the linear associative model would be.

(3) In physical and medical sciences, an association-based predictive model with $r^2 \ge 0.60$ is considered to be valid. In contrast, its counterpart in social sciences will be deemed valid so long as $r^2 \ge 0.25$.

. **Example 11.6:** Chicken Palace in Spokane, WA, periodically offers carryout five-piece chicken dinners at special prices. The following table summarizes the observations about the sales of the chicken dinner under different prices over the past six weeks:

Price ($)	No. of dinners sold
2.70	760
3.50	510
2.00	980
4.20	250
3.10	620
4.05	480

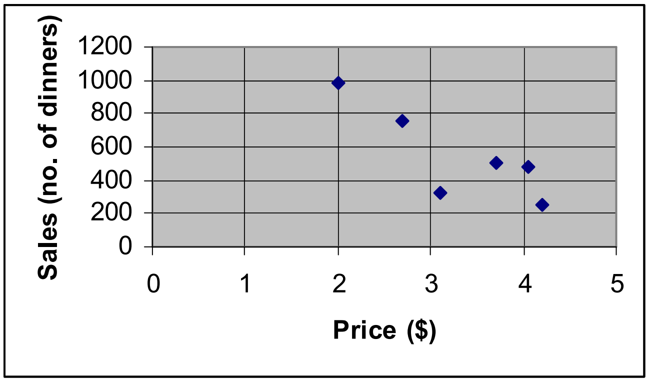
(1) Plot the price against the sales to see if a linear associative model is appropriate for the problem.

(2) Develop the linear associative model for the restaurant manager that may be used to predict future sales of the chicken dinner.

(3) Compute the coefficient of determination to assess the validity of the causal model obtained in (2).

(4) Based on your findings in (2) and (3), how many dinners can Chicken Palace expect to sell per week if the price is set at $3.90 each?

[Solution] (1) It is seen from the chart below that there is a negative linear relationship between the price and the sales of the chicken dinner. As such, a linear associative model will be appropriate for the problem.



(2) Treat the price of the chicken dinner as the predictor (or independent) variable x and the number of chicken dinners sold as the predicted (or dependent) variable y. We then have:

i	x_i	y_i	x_i^2	x_iy_i	y_i^2
1	2.70	760	7.2900	2,052	577,600
2	3.50	510	12.2500	1,785	260,100
3	2.00	980	4.0000	1,960	960,400
4	4.20	250	17.6400	1,050	62,500
5	3.10	620	9.6100	1,922	384,400
6	4.05	480	16.4025	1,944	230,400
	19.55	3,600	67.1925	10,713	2,475,400

$n = 6$

$$b = \frac{6(10{,}713) - 19.55(3{,}600)}{6(67.1925) - (19.55)^2} \approx -291.2302$$

$$a = \frac{3{,}600 - (-291.2302)(19.55)}{6} \approx 1{,}548.9251$$

Hence, the linear associative model sought is $y = 1{,}548.9251 - 291.2302x$.

(3) The coefficient of determination is

$$r^2 = \frac{[6(10{,}713) - 19.55(3{,}600)]^2}{[6(67.1925) - (19.55)^2][6(2{,}475{,}400) - (3{,}600)^2]}$$

$$= \frac{(-6{,}102)^2}{(20.9525)(1{,}892{,}400)}$$

$$\approx 0.9391$$

As 0.9391 > 0.6 (for physical and medical sciences) and 0.9391 > 0.25 (for social sciences), the causal model obtained above is valid.

(4) If the selling price is set at $3.90 each, then the number of chicken dinners to be sold per week is predicted to be y = 1,548.9251 - 291.2302(3.9) = 413.1274 or about 413.

9. Accuracy of Forecasts

. Forecasts based on time series data are rarely perfect. Predicted values always differ from actual results. The difference between them in a particular period is called the individual forecast error, i.e., $e_t = A_t - F_t$, t = 1, 2, ... The most commonly used measures of aggregate forecast errors for a forecasting method include the mean absolute deviation (MAD) and the mean squared error (MSE), whose respective definitions are given below:

$$\text{MAD} = \frac{\sum_{t=1}^{m} | e_t |}{m}$$

$$\text{MSE} = \frac{\sum_{t=1}^{m} e_t^{\,2}}{m}$$

where m is the number of forecast errors involved.

. The significance of MAD and MSE is that they may serve as the basis for comparing various forecasting methods so that the most reliable one can be selected. Obviously, the smaller the MAD or MSE is, the more accurate the forecasting approach will be.

. **Example 11.7:** Data collected on the annual promotional expenditure (in millions of dollars) of Fairface Cosmetics Company over the past seven years are shown in the following table:

Year	2009	2010	2011	2012	2013	2014	2015
Expenditure	10	8	7	9	12	14	11

(1) Use the 2-year SMA method to develop a forecast of the promotional expenditure in each year up to 2015.

(2) Repeat (1) by using the 3-year WMA method with weights of 0.25, 0.5, and 0.25.

(3) Repeat (1) by using the SES method with $\alpha = 0.5$ and the forecast for 2009 of $6 million.

(4) Which of the three methods is most reliable on the basis of MAD?

(5) Based on the findings in (4) above, provide a forecast of the promotional expenditure in 2016.

[Solution] Let Year 2009 = Period 1, Year 2010 = Period 2, and so on.

(1) 2-y-s-m-a: $F_3 = (8 + 10)/2 = 9.00$, $F_4 = (7 + 8)/2 = 7.50$, …

(2) 3-y-w-m-a: $F_4 = 0.5(7) + 0.25(8) + 0.25(10) = 8.00$, $F_5 = 0.5(9) + 0.25(7) + 0.25(8) = 8.25$, …

(3) s-e-s: With $F_1 = 6$, we have $F_2 = 6 + 0.5(10 - 6) = 8.00$, $F_3 = 8 + 0.5(8 - 8) = 8.00$, ...

(4) The results in (1), (2), and (3) are summarized in the following table:

		2-y-s-m-a		3-y-w-m-a		s-e-s	
t	A_t	F_t	e_t	F_t	e_t	F_t	e_t
1	10					6.00	4.00
2	8					8.00	0.00
3	7	9.00	-2.00			8.00	-1.00
4	9	7.50	1.50	8.00	1.00	7.50	1.50
5	12	8.00	4.00	8.25	3.75	8.25	3.75
6	14	10.50	3.50	10.00	4.00	10.13	3.87
7	11	13.00	-2.00	12.25	-1.25	12.07	-1.07

2-y-s-m-a: MAD = $(|-2.00| + |1.50| + ... + |-2.00|)/5 = 13/5 = 2.6$

3-y-w-m-a: MAD = $(|1.00| + |3.75| + ... + |-1.25|)/4 = 10/4 = 2.5$

s-e-s: MAD = $(|4.00| + |0.00| + ... + |-1.07|)/7 = 15.19/7 = 2.17$

Since $2.17 < 2.5 < 2.6$ the simple exponential smoothing method is the best and hence should be used for developing forecasts of promotional expenditures in the future.

(5) The forecast of the promotional expenditure in 2016 is $F_8 = 12.07 + 0.5(11 - 12.07) \approx 11.54$ or $11,540,000

. **Example 11.8:** Annual sales (in units) of Xerox copy machines in the Seattle area in the last five years are recorded and shown below:

Year	2011	2012	2013	2014	2015
Sales	450	495	518	563	584

(1) Use the naive approach to forecast the sales in each year up to 2015.

(2) Repeat (1) by using the linear trend projection approach.

(3) Which of the two forecasting methods provides more accurate results on the basis of MSE?

(4) Based on your findings in (3) and (4) above, provide a forecast of the sales in 2016.

[Solution] Let Year 2011 = Period 1, Year 2012 = Period 2, and so on. We then have

(1) $F_2 = 450$, $F_3 = 495$…

(2) The following table is prepared to obtain the equation for the linear trend line:

t	A_t	t^2	tA_t
1	450	1	450
2	495	4	990
3	518	9	1,554
4	563	16	2,252
5	584	25	2,920
15	2,610	55	8,166

$n = 5$

$$b = \frac{5(8{,}166) - 15(2{,}610)}{5(55) - (15)^2} \approx 33.6$$

$$a = \frac{2{,}610 - 33.6(15)}{5} \approx 421.2$$

Since the linear trend projection model is found to be $F_t = 421.2 + 33.6t$, we have $F_1 = 421.2 + 33.6(1) = 454.8$, $F_2 = 421.2 + 33.6(2) = 488.4$, etc.

(3) The results in (1) and (2) are summarized in the following table:

		Naïve		Trend	
t	A_t	F_t	e_t	F_t	e_t
1	450			454.8	-4.8
2	495	450	45	488.4	6.6
3	518	495	23	522.0	-4.0
4	563	518	45	555.6	7.4
5	584	563	21	589.2	-5.2

Naïve: MSE $= [(45)^2 + (23)^2 + (45)^2 + (21)^2]/4 = 5{,}020/4 = 1{,}255$

Trend: MSE $= [(-4.8)^2 + (6.6)^2 + ... + (-5.2)^2]/5 = 164.4/5 = 32.88$

As $32.88 < 1{,}255$, the linear trend projection method provides more accurate results based on MSE.

(4) It is seen from (3) that the linear trend projection method outperforms the naïve approach and hence should be used for developing forecasts of sales in the future. Consequently, the forecast of sales in 2016 is $F_6 = 421.2 + 33.6(6) = 622.8$ or about 623 copy machines.

. In comparing forecasting approaches, it is possible that the MAD for method A is equal to that for method B. One way to break the tie is to introduce another measure of aggregate errors such as MSE and see which of the two methods provides more accurate forecasts on the basis of the second criterion.