BUAN/OPRE 6398.003 Prescriptive Analytics
Homework 1

Due 01/23/17
(11:30 a.m.)

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Note: 1. Your homework submission must be typewritten.

2. Please refer to the sample completed homework assignment for the suggested format of your submission.

3. Show only the solutions to the problems and do not copy the problems in the submission.

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1. Serendipity

The three princes of Serendip
Went on a little trip.
They could not carry too much weight;
More than 300 pounds made them hesitate.
They planned to the ounce. When they returned to Ceylon
They discovered that their supplies were just about gone
When, what to their joy, Prince William found
A pile of coconuts on the ground.
"Each will bring 140 rupees," said Prince Richard with a grin
As he almost tripped over a lion skin.
"Look out!" cried Prince Robert with glee
As he spied some more lion skins under a tree.
"These are worth even more--350 rupees each
If we can just carry them all down to the beach."
Each skin weighed 10 pounds and each coconut, five,
But they carried them all and made it alive.
The boat back to the island was very small
22 cubic feet baggage capacity--that was all.
Each lion skin took up one cubic foot
While four coconuts the same space took.
With everything stowed they headed to sea
And on the way calculated what their new wealth might be.
"Eureka!" cried Prince Robert, "Our worth is so great
That there's no other way we could return in this state.
Any other skins or nut which we might have brought
Would now have us poorer. And now I know what--
I'll write my friend Horace in England, for surely
Only he can appreciate our serendipity."

Formulate Serendipity above as a linear program that can be used to find the three princes' maximum total new wealth. (2 pts.)

(Hint: This problem is similar to the one discussed in Example 2.1. Be sure to show your complete work including the definitions of decision variables, the objective function, and the structural as well as non-negativity constraints.)

(Note: The word *serendipity* was coined by the English writer Horace Walpole after a fairy tale entitled *The Three Princes of Serendip*. See, for example, http://en.wikipedia.org/wiki/The_Three_Princes_of_Serendip.)

2. The dietitian at a hospital in Dallas, TX, is planning the breakfast menu for the maternity ward patients. She has chosen cottage cheese and scrambled eggs for breakfast. The primary concerns are the vitamin E and iron requirements of new mothers.

   According to the American Medical Association (AMA), new mothers should get at least 32 milligrams of vitamin E and 94 milligrams of iron (fictitious figures) from breakfast. The AMA handbook reports that a scoop of cottage cheese contains 3 milligrams of vitamin E and 12 milligrams of iron. An average scoop of scrambled eggs contains 4 milligrams of vitamin E and 15 milligrams of iron. In accordance with the AMA guidelines, new mothers must eat at least 8, but no more than 14, scoops of scrambled eggs for their breakfast.

   The hospital accounting department estimates that a scoop of cottage cheese costs $0.16 and a scoop of scrambled eggs costs $0.12. The dietitian is attempting to determine the optimal breakfast menu that will satisfy all the nutritional requirements and minimize the total cost. Set up an LP for the diet problem, but there is no need to solve it. (2 pts.)

   (Hint: This is similar to the problem Example 2.2. Be sure to show your complete work including the definitions of decision variables, the objective function, and the structural as well as non-negativity constraints.)

3. Catch 22 Realty, Inc., has identified three small apartment buildings (A1, A2, and A3) in Fort Worth, TX, in which it would like to invest. Mr. Trump, an assistant manager at the firm, approaches four local banks (B1, B2, B3, and B4) for financing. Each bank has placed a credit ceiling on how much it will lend Mr. Trump in total. Moreover, each loan officer has set a different interest rate on each property. Relevant information has been summarized in the table below:

	Interest rate (%)			
Local bank	A1	A2	A3	Maximum credit line
B1	10	8	11	$130,000
B2	8	9	8	$85,000
B3	12	10	9	$120,000
B4	7	9	10	$95,000
Selling price	$150,000	$160,000	$120,000	

   Since each of the three apartment buildings is equally attractive as an investment to Mr. Trump, he decides to purchase all of them. Formulate the problem as a linear program for Mr. Trump that can be used to determine the financing plan with the minimum total interest cost. (3 pts.)

   (Hint: This problem is similar to the one discussed in Example 2.9. Be sure to show your complete work including the definitions of decision variables, the objective function, and the structural as well as non-negativity constraints.)

4. Dr. Christine Bell, the head administrator at Collin County Hospital, must determine a staffing schedule to make sure that there are enough nurses on duty throughout the day. She has broken the 24-hour period into eight three-hour segments and the demand for nurses during each of them segments is shown below:

Segment number	Time	Number of nurses required
1	6:00 A.M. - 9:00 A.M.	44
2	9:00 A.M. - 12:00 P.M.	50
3	12:00 P.M. - 3:00 P.M.	60
4	3:00 P.M. - 6:00 P.M.	86
5	6:00 P.M. - 9:00 P.M.	84
6	9:00 P.M. - 12:00 A.M.	70
7	12:00 A.M. - 3:00 A.M.	30
8	3:00 A.M. - 6:00 A.M.	20

A nurse reports for duty at the beginning of one of the eight segments and works nine consecutive hours. As an assistant to Dr. Bell, you are asked to formulate for her a linear program that may be used to determine the work schedule with the minimum number of nurses to meet the daily requirements. (3 pts.)

(Hint: This is similar to the problem in Example 2.10. Be sure to show your complete work including the definitions of decision variables, the objective function, and the structural as well as non-negativity constraints.)