MD2_Racinskis

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2. mājas darbs Mate 6029

1. uzdevums - lineāras regresijas, modeļu novērtējumi

Datu kopas ielāde:

```
df <- read.table('CMB.dat',header=TRUE)
attach(df)
```

Datu kopa - kosmiskā mikroviļņu fona novērojumi. Svarīgie parametri šajā gadījumā ir 'ell' - multipolu moments (rupji runājot, lenķiskais ekvivalents starojuma frekvencei) un starojuma spektra nobīde 'Cl' (rupji runājot, spektra temperatūras nobīde no vidējā). Pārējās trīs kolonnas ir statistiski novērojumu trokšņa u.c. raksturotāji.

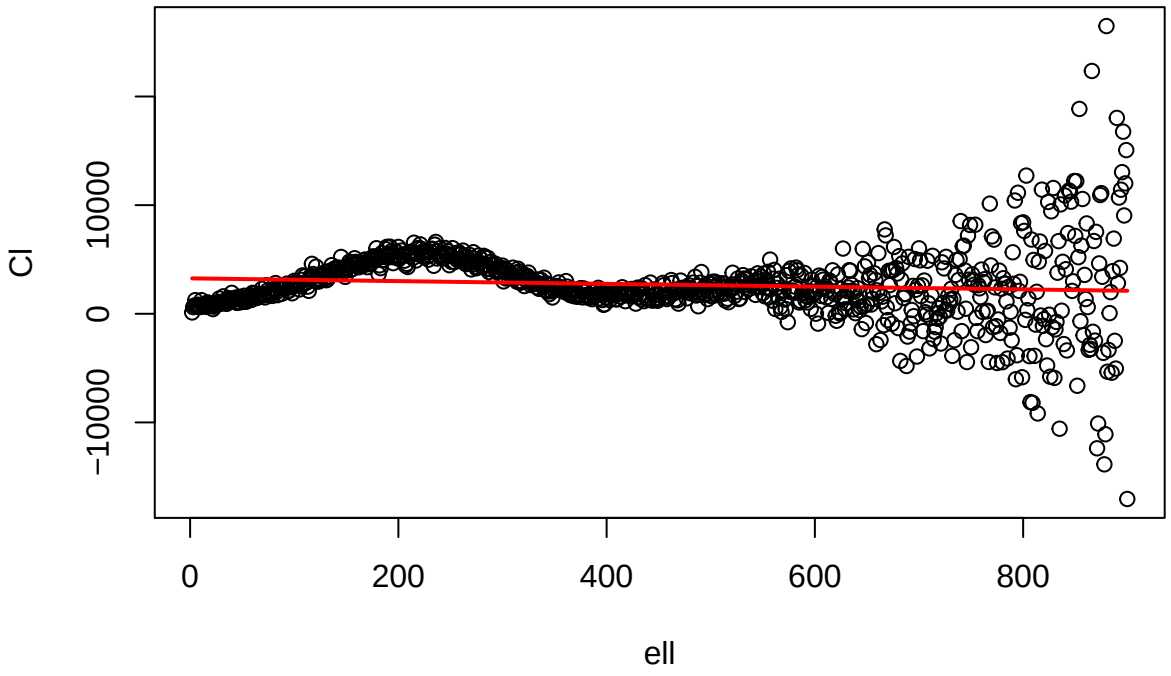
1.1. regresijas modeļu ģenerēšana

Vispārināma lineārās regresijas funkcija grafiku zīmēšanai, rezultātu izvadei uz konsoles un darbam ar augstākas pakāpes modeļiem:

```
general_lreg <- function(vec1,vec2,degree=1,plot=F,print=F,names=c("","")) {
  fit<-lm(vec2~poly(vec1,degree,raw=T))
  if(plot){
    plot(vec1,vec2,xlab=names[1],ylab=names[2])
    x <- seq(min(vec1),max(vec1),length.out = length(vec1))
    f <- predict(fit, newdata = data.frame(vec1 = x))
    lines(x,f,col="red",lwd=2)
  }
  if(print){
    print(paste("R-squared:",summary(fit)$r.squared))
  }
  fit
}
```

Vienkārša lineārā regresija parametriem ‘ell’ un ‘Cl’:

```
fit1 <- general_lreg(ell,Cl,plot=T,print=T,names=c("ell","Cl"))
```




```
## [1] "R-squared: 0.00991069470200286"
```

Funkcija labākās atbilstības polinoma meklēšanai (apstājas, kad ANOVA tests liecina, ka jaunas brīvības pakāpes pievienošana būtisku uzlabojumu modeļa atbilstībā datiem vairs nesniedz). Virzība - pa divām pakāpēm, lai ļautu modelim piekārtoties simetriskiem/asimetriskiem sadalījumiem pēc vajadzības:

```
bestfit <- function(vec1,vec2,deg=1,last_deg=1,P=0,p=0.05,max=27) {
  if((P>0.05) || (deg>max)){
    list(d=last_deg,p=P)
  } else {
    f1<-general_lreg(vec1,vec2,deg)
    next_deg<-deg+2
    f2<-general_lreg(vec1,vec2,next_deg)
    P<-anova(f1,f2)$'Pr(>F)'[2]
    bestfit(vec1,vec2,next_deg,deg,P)
  }
}
```

Labākais polinoms:

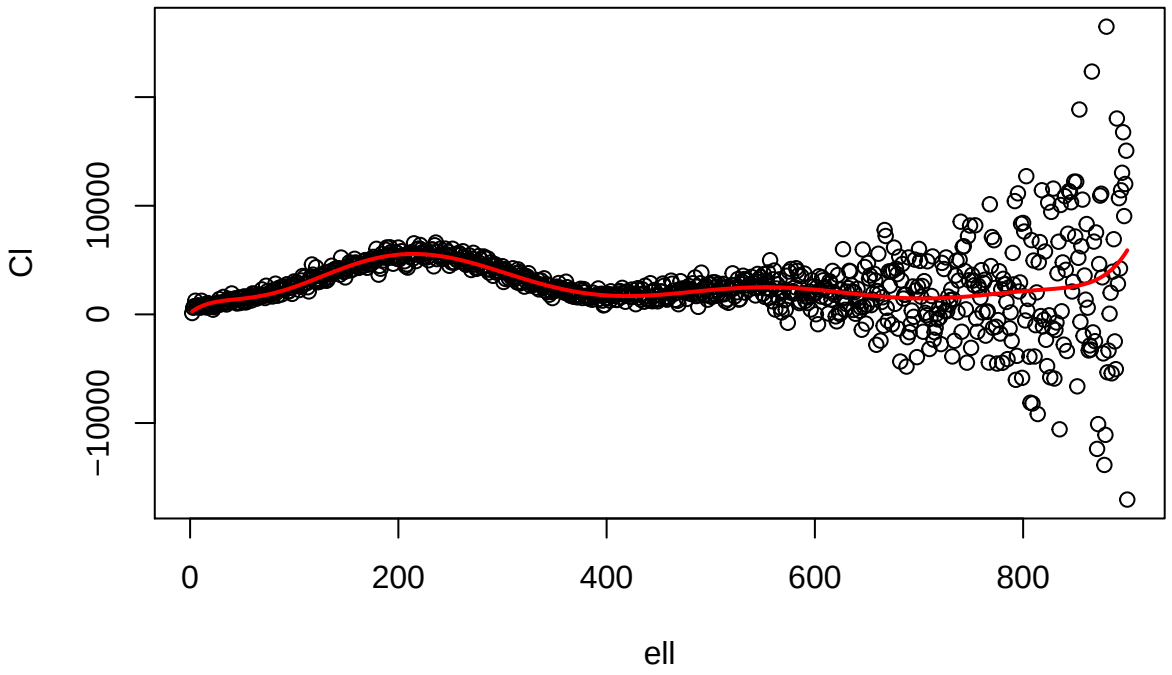
```
res <- bestfit(ell,Cl)
paste("stopped at x^", res$d,sep="")
```

```
## [1] "stopped at x^9"
```

```
paste("p value:",res$p)
```

```
## [1] "p value: 0.9633917284375"
```

```
fitmax<-general_lreg(ell,Cl,degree=res$d,plot=T,print=T,names=c("ell","Cl"))
```

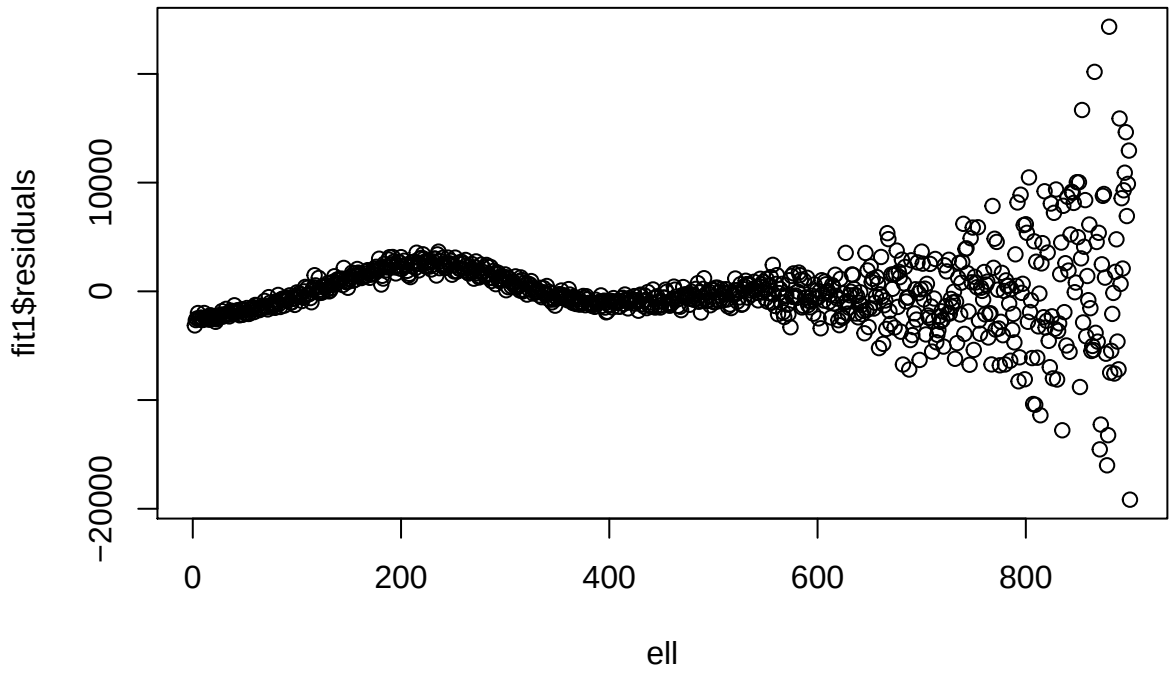



```
## [1] "R-squared: 0.14991001889"
```

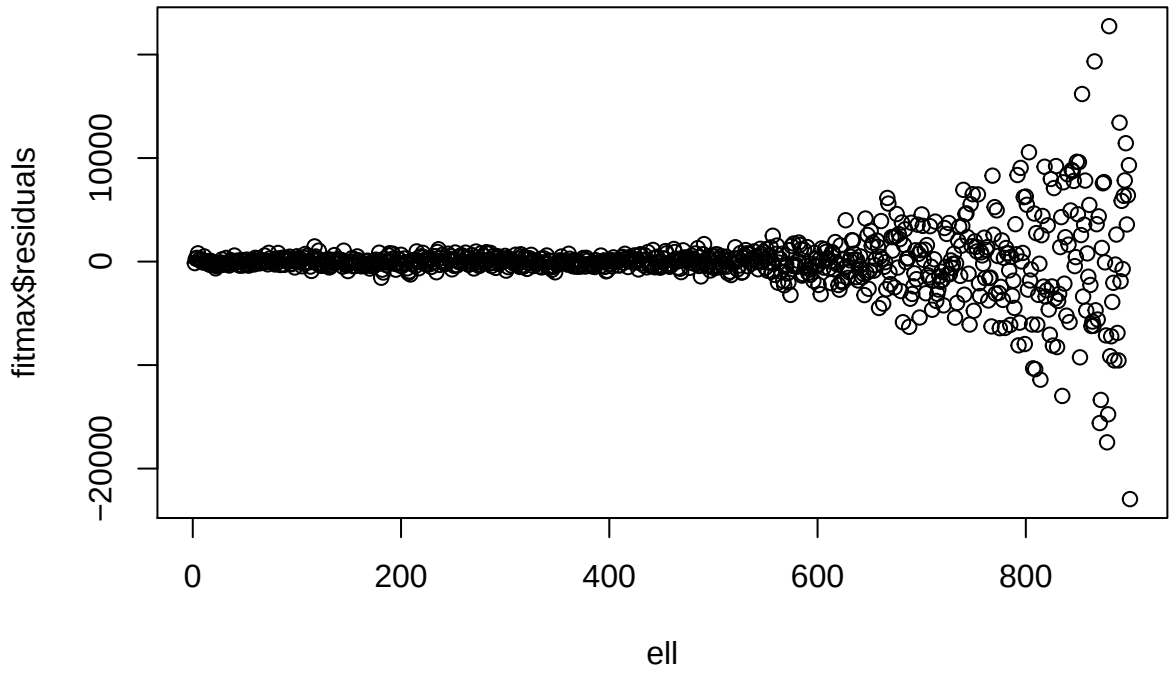
1.2. diagnostika

Atlikumu neatkarība - grafiski:

```
plot(ell,fit1$residuals)
```



```
plot(ell,fitmax$residuals)
```



Statistisko testu bibliotēkas:

```
library(car)
library(nortest)
```

Durbin-Watson tests autokorelācijai:

```
durbinWatsonTest(fit1)
```

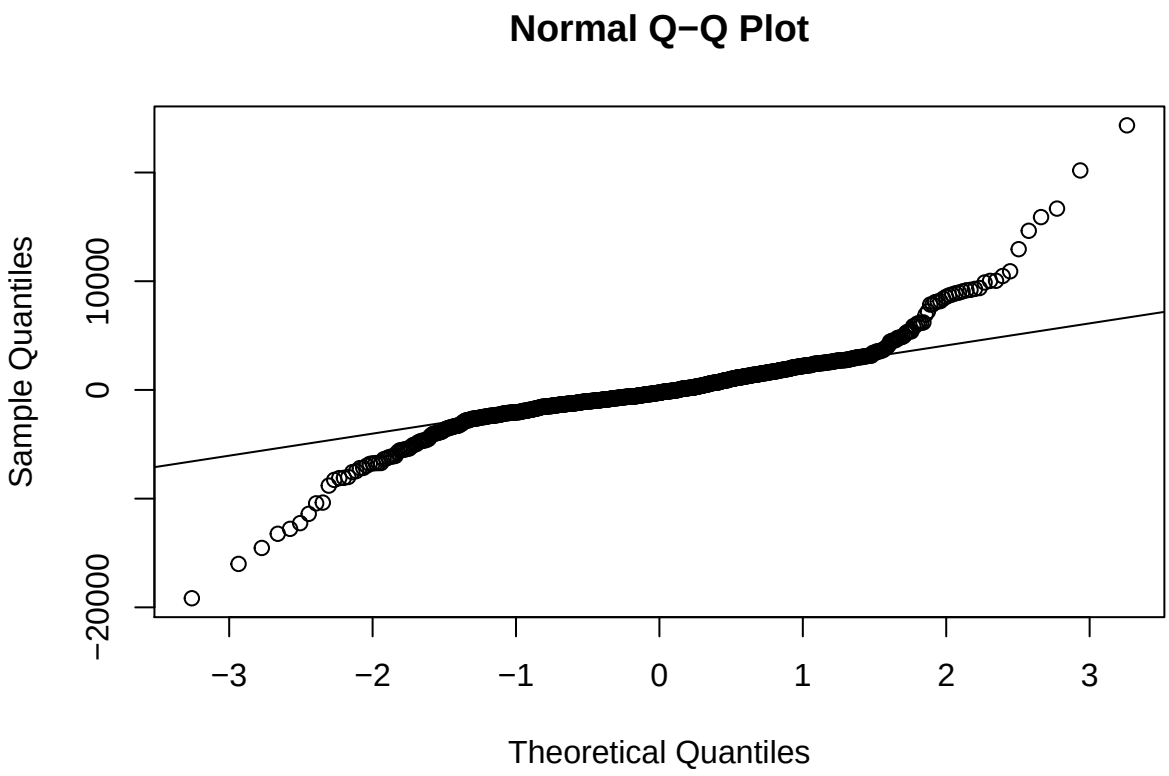
```
##  lag Autocorrelation D-W Statistic p-value
##    1      0.05972642      1.841981   0.026
##  Alternative hypothesis: rho != 0
```

```
durbinWatsonTest(fitmax)
```

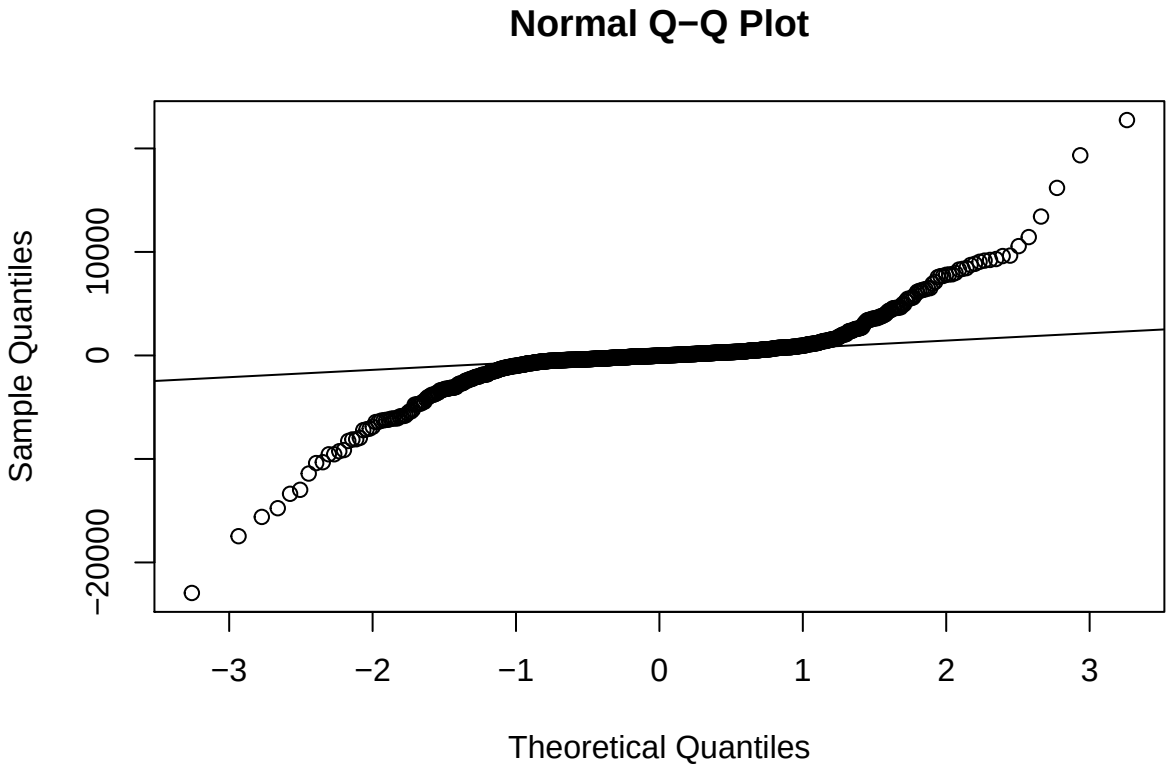
```
##  lag Autocorrelation D-W Statistic p-value
##    1      -0.1044366      2.146133   0.058
##  Alternative hypothesis: rho != 0
```

Normalitātes testi - grafiski:

```
qqnorm(fit1$residuals)
qqline(fit1$residuals)
```




```
qqnorm(fitmax$residuals)
qqline(fitmax$residuals)
```



Normalitātes testi - Kolmogorova-Smirnova tests:

```
# degree-1 approximation normality
(lillie.test(fit1$residuals)$p.value > 0.05)
```

```
## [1] FALSE
```

```
# max degree approximation normality
(lillie.test(fitmax$residuals)$p.value > 0.05)
```

```
## [1] FALSE
```

Dispersijas vienmērīguma testi:

```
# degree-1 approximation homoscedacity
(ncvTest(fit1)$p > 0.05)
```

```
## [1] FALSE
```

```
# max degree approximation homoscedacity
(ncvTest(fitmax)$p > 0.05)
```

```
## [1] FALSE
```

1.3. secinājumi

Dati acīmredzami nav lineāri sakarīgi, un to apliecina arī visas formālās metrikas. Cits jautājums ir par reģionu [0:500], kur tie diezgan cieši seko līknei, ko labi varētu aprakstīt samērā nelielas pakāpes polinoms (sk. sekciju "atlikumu neatkarība - grafiski", kur šajā reģionā atlikumi 9. pakāpes regresijas līknei ir vienmērīgi sadalīti ap 0). Taču ap 'ell' = 500 ļoti strauji pieaug novērojumu dispersija, kas pilnībā izgāž jebkādus mēģinājumus aproksimēt visu datu kopu ar vienu līkni. Šī radikālā izmaiņa dispersijā nomāc arī jebkādus dziļākus ieskatus, ko pār visu datu kopu veiktie testi varētu sniegt par sadalījuma dabu.

2. uzdevums

Datu kopas ielāde:

```
df <- LifeCycleSavings
attach(df)
```

Datu kopas kolonnas:

1. sr - uzkrājumi
2. pop15 - % iedzīvotāju zem 15
3. pop75 - % iedzīvotāju virs 75
4. dpi - ienākumi
5. ddpi - IKP pieaugums

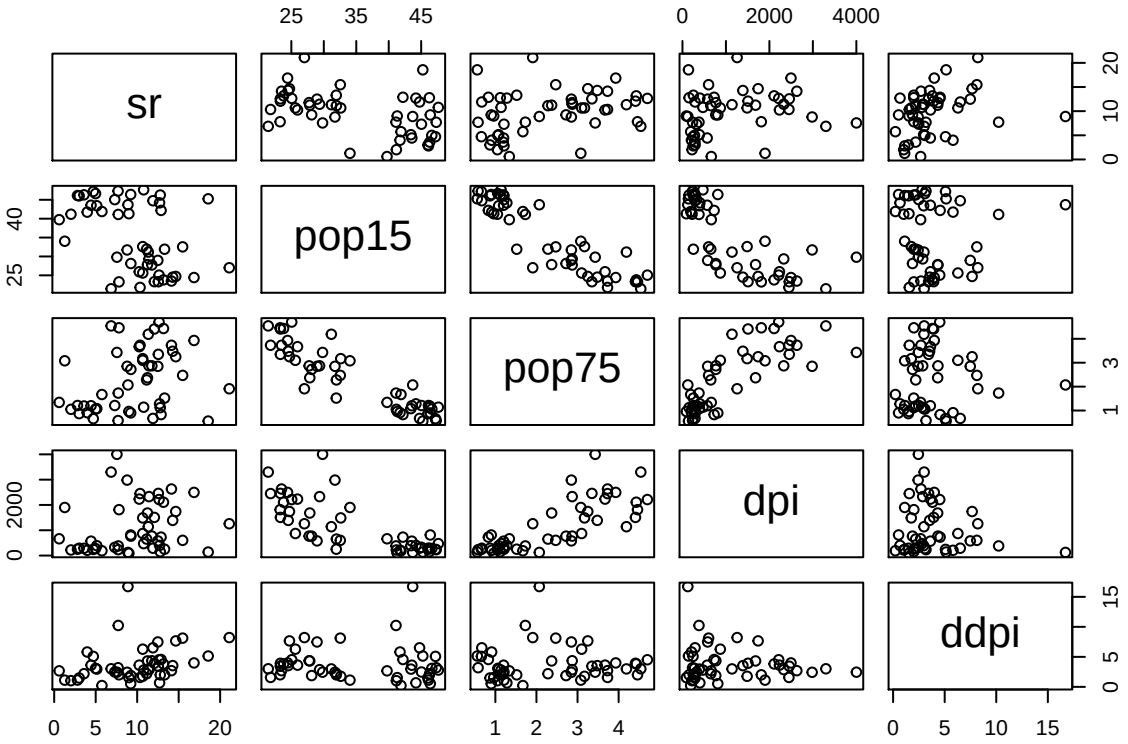
2.1. vispārīgu sakarību meklēšana

Izmantojot iebūvētās funkcijas cor() un pairs(), var ātri gūt vispārīgu priekštatu par datu kopā pastāvošajām sakarībām:

```
cor(df)
```

```
##                sr       pop15       pop75        dpi        ddpi
## sr     1.0000000 -0.45553809  0.31652112  0.2203589  0.30478716
## pop15 -0.4555381  1.00000000 -0.90847871 -0.7561881 -0.04782569
## pop75  0.3165211 -0.90847871  1.00000000  0.7869995  0.02532138
## dpi    0.2203589 -0.75618810  0.78699951  1.0000000 -0.12948552
## ddpi   0.3047872 -0.04782569  0.02532138 -0.1294855  1.00000000
```

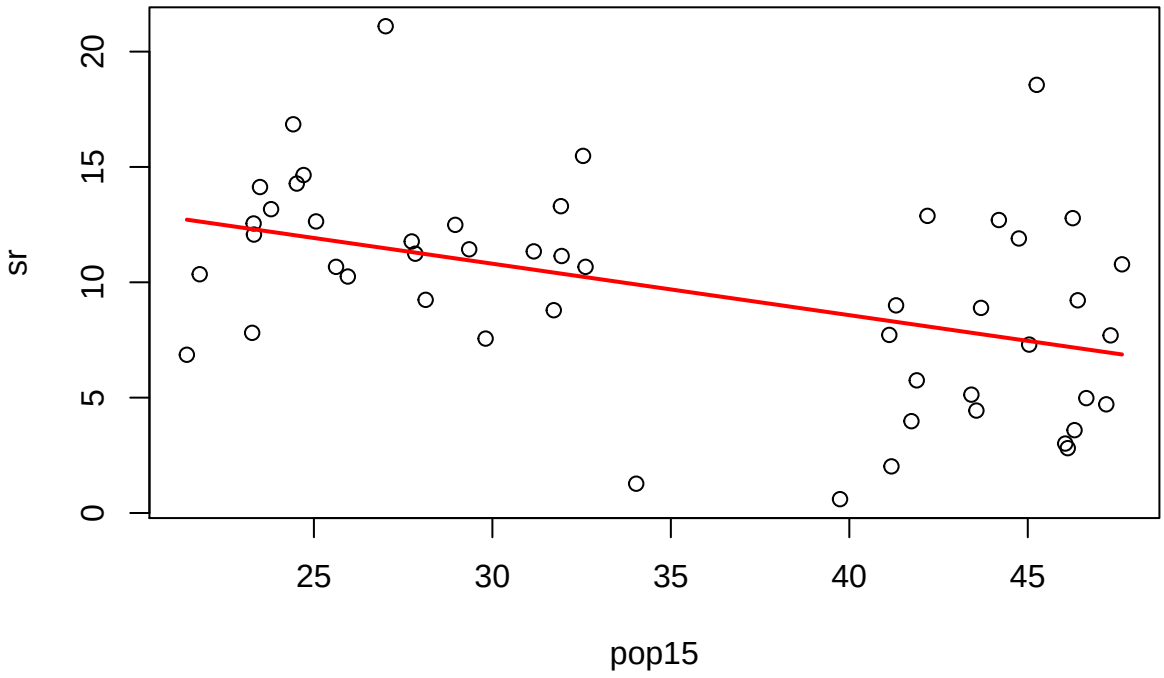
```
pairs(df)
```



Kā redzams, izteiktas sakarības nav starp nevienu parametru un uzkrājumiem, taču redzama neliela negatīva korelācija starp pop15 un sr, un nelielas pozitīvas korelācijas visos citos gadījumos.

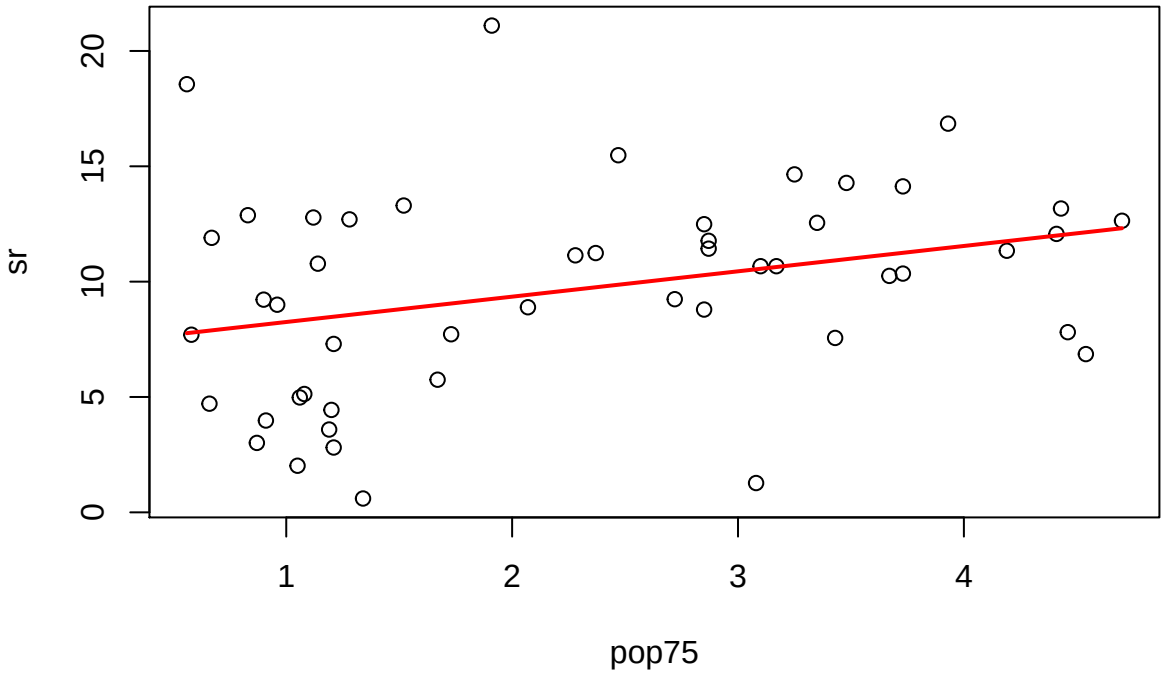
2.2. regresijas modeļu konstruēšana

```
f_pop15<-general_lreg(pop15,sr,plot=T,print=T,names=c("pop15","sr"))
```



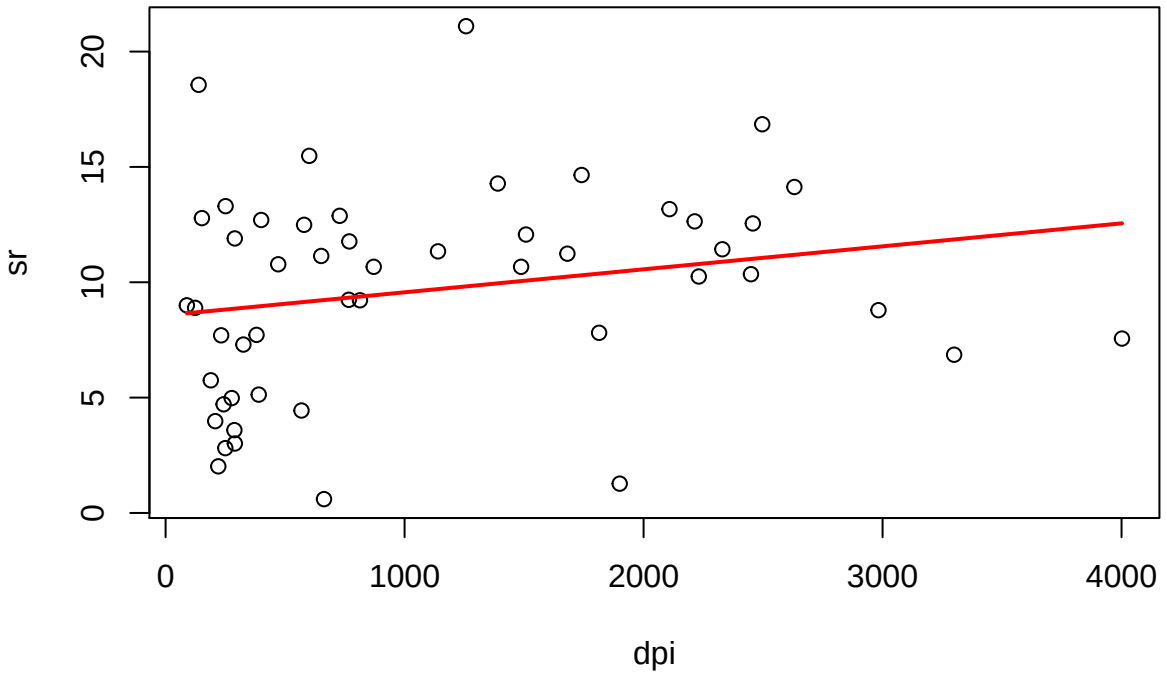

```
## [1] "R-squared: 0.20751494822826"
```

```
f_pop75<-general_lreg(pop75,sr,plot=T,print=T,names=c("pop75","sr"))
```



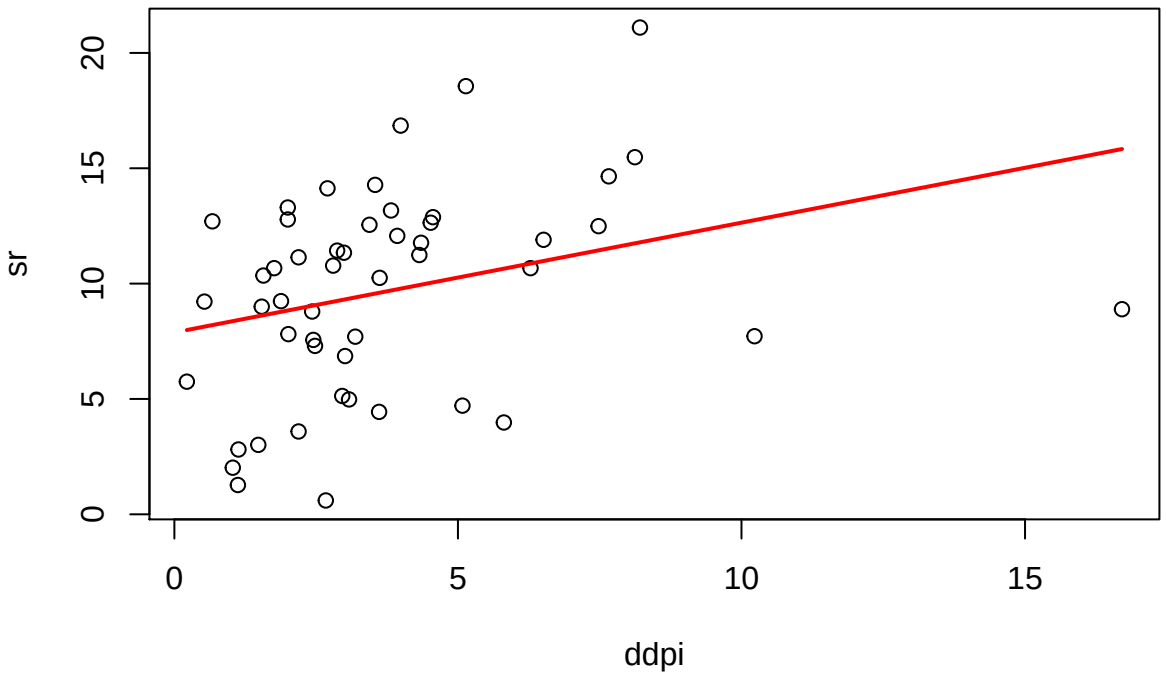

```
## [1] "R-squared: 0.100185621919712"
```

```
f_dpi<-general_lreg(dpi,sr,plot=T,print=T,names=c("dpi","sr"))
```




```
## [1] "R-squared: 0.0485580524006459"
```

```
f_ddpi<-general_lreg(ddpi,sr,plot=T,print=T,names=c("ddpi","sr"))
```



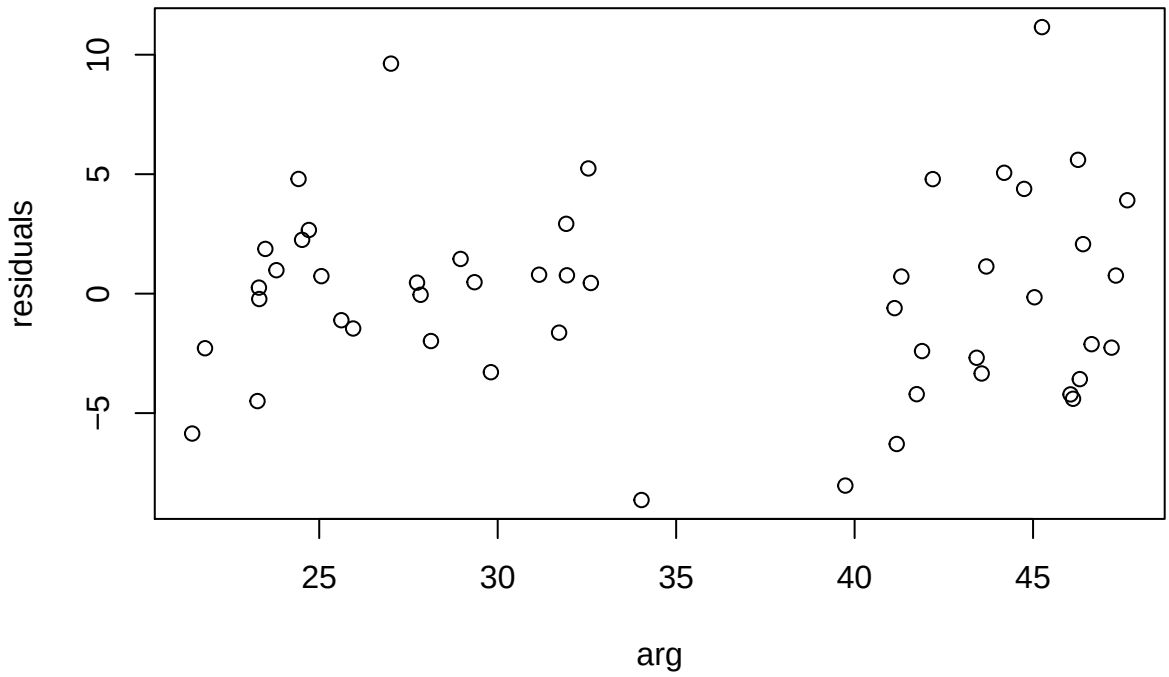

```
## [1] "R-squared: 0.092895211669384"
```

Kā jau vizuāli redzams, pop15 izskaidro lielāko frakciju (~20%) no sr dispersijas un ir negatīvi korelēts ar sr. Pārējie izskaidro ne vairāk kā 10%, bet ir pozitīvi korelēti.

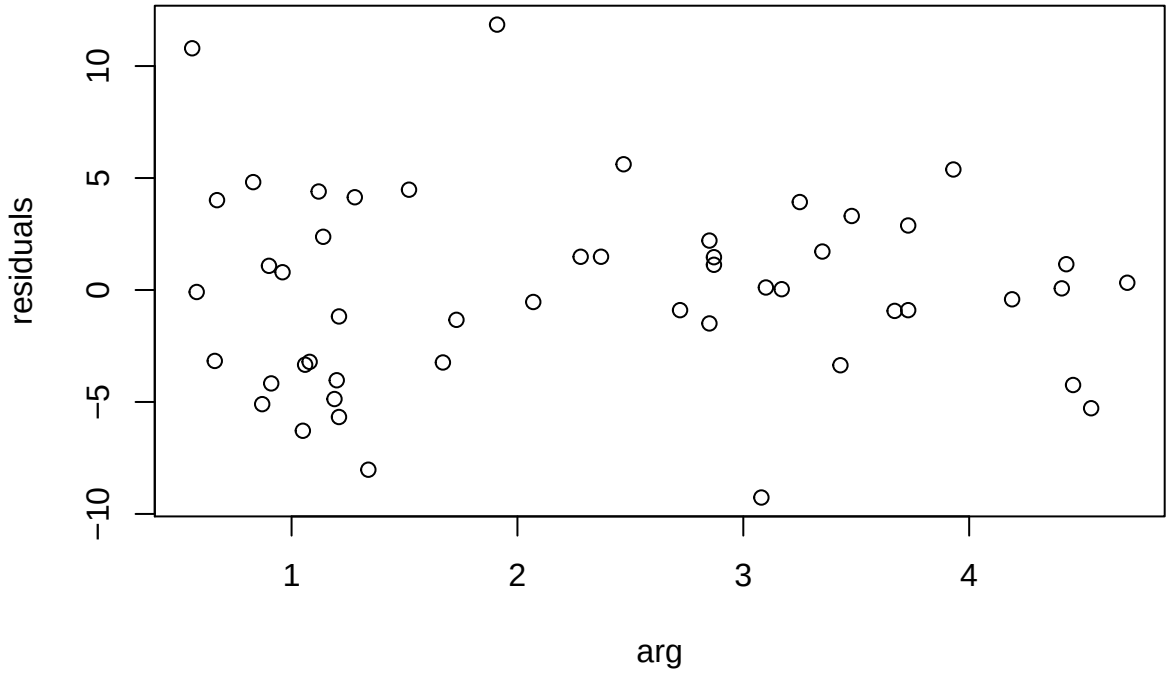
2.3. regresijas modeļu nosacījumu analīze, diagnostika

Atlikumu neatkarība (grafiski):

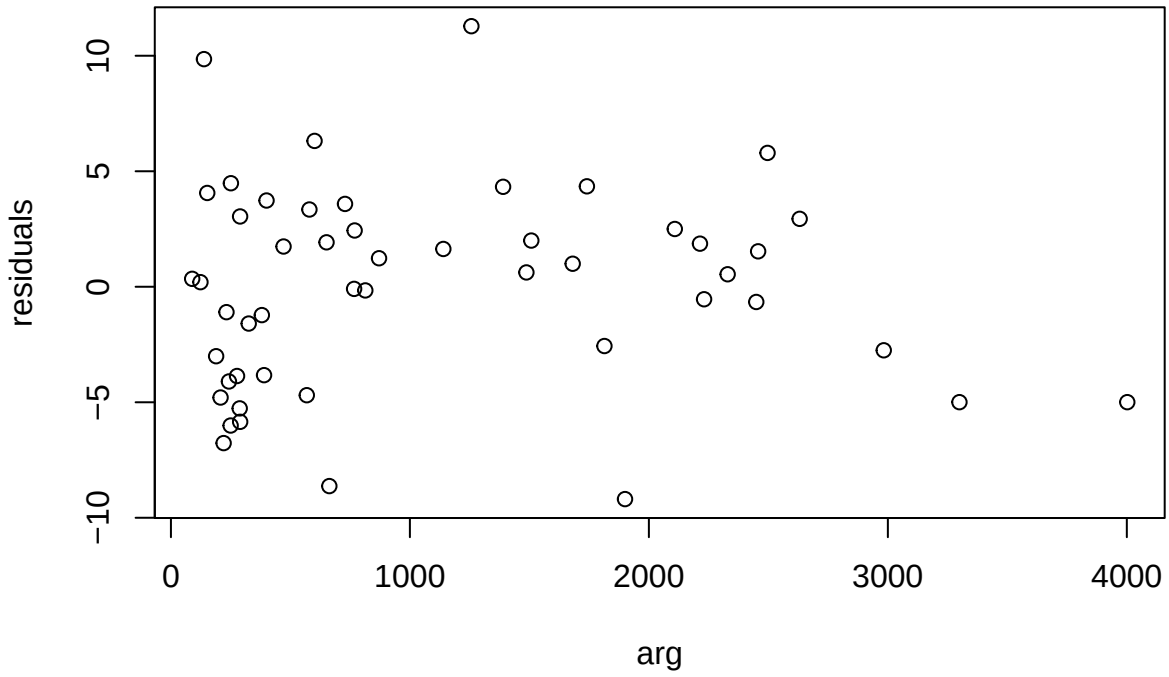
```
plot(pop15,f_pop15$residuals,xlab="arg",ylab="residuals")
```



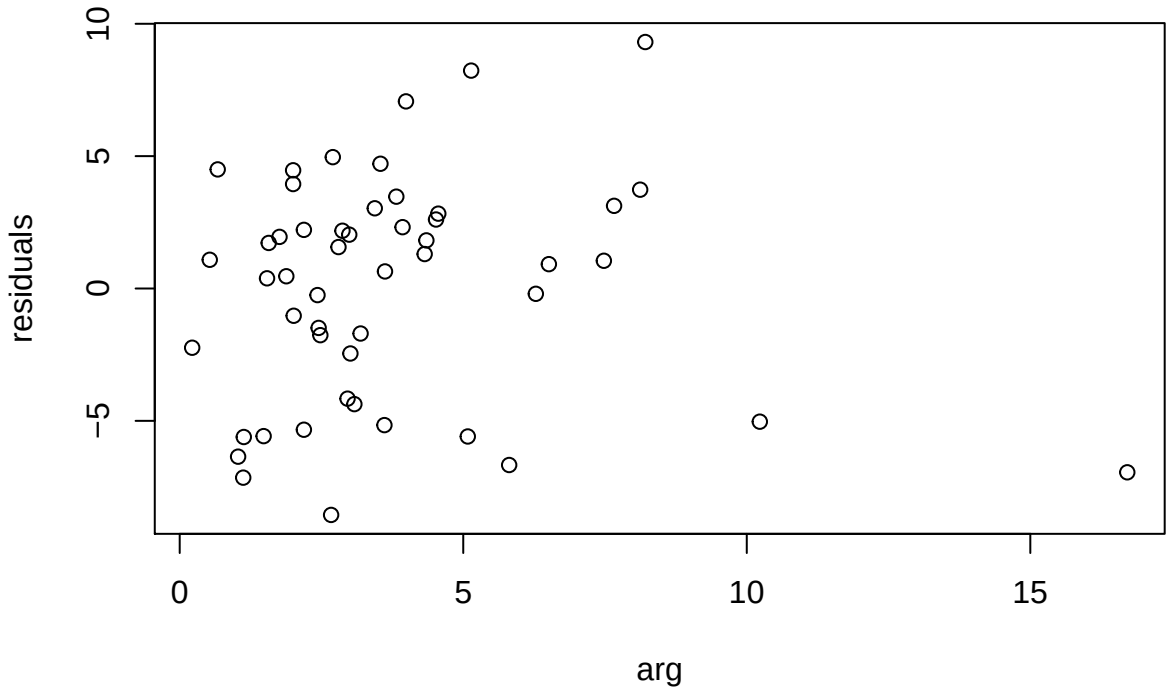

```
plot(pop75,f_pop75$residuals,xlab="arg",ylab="residuals")
```



```
plot(dpi,f_dpi$residuals,xlab="arg",ylab="residuals")
```




```
plot(ddpi,f_ddpi$residuals,xlab="arg",ylab="residuals")
```



Izteiktas sakarības nav redzamas.

Atlikumu neatkarība (Durbin-Watson autokorelācijas tests, TRUE - pastāv autokorelācija):

```
(durbinWatsonTest(f_pop15)$p < 0.05)
```

```
## [1] FALSE
```

```
(durbinWatsonTest(f_pop75)$p < 0.05)
```

```
## [1] FALSE
```

```
(durbinWatsonTest(f_dpi)$p < 0.05)
```

```
## [1] FALSE
```

```
(durbinWatsonTest(f_ddpi)$p < 0.05)
```

```
## [1] FALSE
```

Atlikumu normalitāte (Kolmogorov-Smirnov tests, TRUE - normāli sadalīti):

```
(lillie.test(f_pop15$residuals)$p.value > 0.05)
```

```
## [1] TRUE
```

```
(lillie.test(f_pop75$residuals)$p.value > 0.05)
```

```
## [1] TRUE
```

```
(lillie.test(f_dpi$residuals)$p.value > 0.05)
```

```
## [1] TRUE
```

```
(lillie.test(f_ddpi$residuals)$p.value > 0.05)
```

```
## [1] TRUE
```

Dispersijas vienmērība (TRUE - dispersija nav atkarīga no argumenta)

```
(ncvTest(f_pop15)$p > 0.05)
```

```
## [1] TRUE
```

```
(ncvTest(f_pop75)$p > 0.05)
```

```
## [1] TRUE
```

```
(ncvTest(f_dpi)$p > 0.05)
```

```
## [1] TRUE
```

```
(ncvTest(f_ddpi)$p > 0.05)
```

```
## [1] TRUE
```

2.4. daudzfaktoru regresija

Šo apakšuzdevumu gandrīz palaidu garām, jo uzdevuma nosacījumos prasīts veikt "vienkāršas lineāras regresijas", ko var pārprast kā nosacījumu veikt individuālas viena faktora regresijas. Jebkurā gadījumā, šeit veikta daudzfaktoru regresija un rezulātu analīze kā mtcars piemērā lekcijās:

```
fit_multi<-lm(sr~., data=df)
summary(fit_multi)
```

```
##
## Call:
## lm(formula = sr ~ ., data = df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
```

```
## -8.2422 -2.6857 -0.2488  2.4280  9.7509
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 28.5660865  7.3545161   3.884 0.000334 ***
## pop15       -0.4611931  0.1446422  -3.189 0.002603 **
## pop75       -1.6914977  1.0835989  -1.561 0.125530
## dpi         -0.0003369  0.0009311  -0.362 0.719173
## ddpi         0.4096949  0.1961971   2.088 0.042471 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.803 on 45 degrees of freedom
## Multiple R-squared:  0.3385, Adjusted R-squared:  0.2797
## F-statistic: 5.756 on 4 and 45 DF,  p-value: 0.0007904
```

VIF analīze:

```
vif(fit_multi)
```

```
##    pop15    pop75      dpi     ddpi
## 5.937661 6.629105 2.884369 1.074309
```

Neviena vērtība netiek izmesta.

Iteratīvā uzlabošana pēc AIC metrikas:

```
fit_multi<-step(fit_multi)
```

```
## Start:  AIC=138.3
## sr ~ pop15 + pop75 + dpi + ddpi
##
##         Df Sum of Sq    RSS    AIC
## - dpi    1     1.893 652.61 136.45
## <none>               650.71 138.30
## - pop75  1    35.236 685.95 138.94
## - ddpi   1    63.054 713.77 140.93
## - pop15  1   147.012 797.72 146.49
##
## Step:  AIC=136.45
## sr ~ pop15 + pop75 + ddpi
##
##         Df Sum of Sq    RSS    AIC
## <none>               652.61 136.45
## - pop75  1    47.946 700.55 137.99
## - ddpi   1    73.562 726.17 139.79
## - pop15  1   145.789 798.40 144.53
```

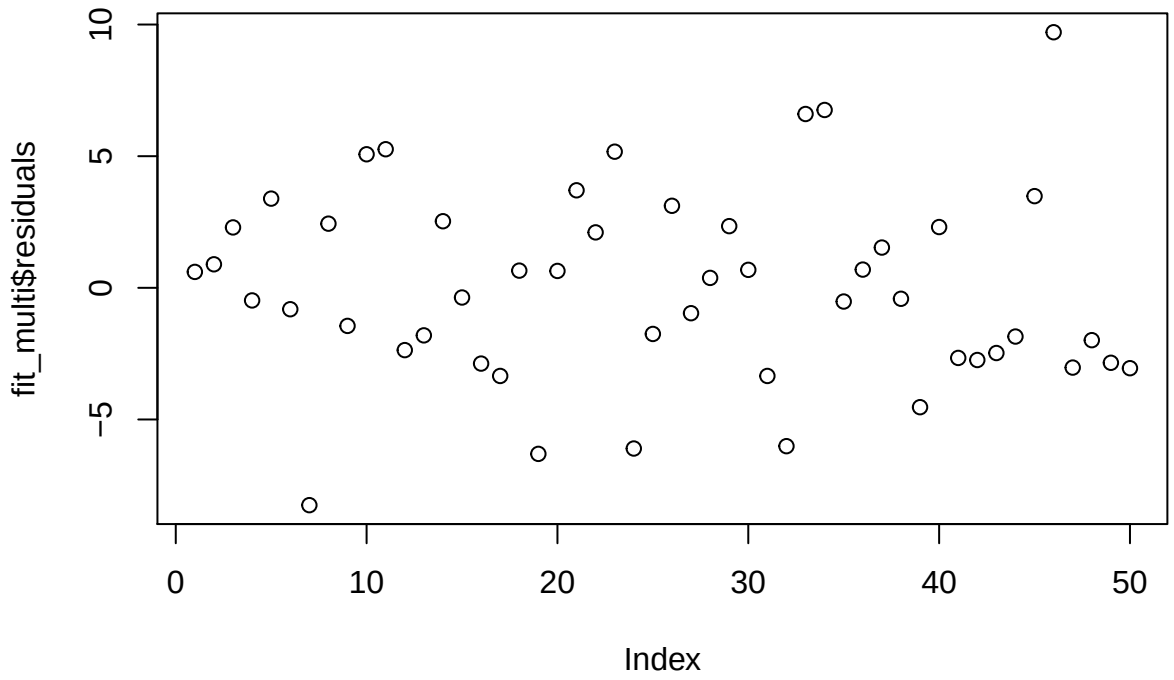
```
summary(fit_multi)
```

```
##
## Call:
## lm(formula = sr ~ pop15 + pop75 + ddpi, data = df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.2539 -2.6159 -0.3913  2.3344  9.7070
##
```

```
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  28.1247     7.1838   3.915 0.000297 ***
## pop15        -0.4518     0.1409  -3.206 0.002452 **
## pop75        -1.8354     0.9984  -1.838 0.072473 .
## ddpi          0.4278     0.1879   2.277 0.027478 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.767 on 46 degrees of freedom
## Multiple R-squared:  0.3365, Adjusted R-squared:  0.2933
## F-statistic: 7.778 on 3 and 46 DF,  p-value: 0.0002646
```

Testi tāpat kā viena faktora regresijās:

```
plot(fit_multi$residuals)
```




```
(durbinWatsonTest(fit_multi)$p < 0.05)
```

```
## [1] FALSE
```

```
(lillie.test(fit_multi$residuals)$p.value > 0.05)
```

```
## [1] TRUE
```

```
(ncvTest(fit_multi)$p > 0.05)
```

```
## [1] TRUE
```

Secinājumi

Visos gadījumos lineārās regresijas modelis ir ne īpaši tuvs datiem, taču nav novērotas nozīmīgas autokorelācijas, atlikumi ir normāli sadalīti un to dispersijas ir vienmērīgas, kas neliecina par viegli atrodamām sistemātiskām nobīdēm. Daudzfaktoru regresijas modelis sniedz mērenu uzlabojumu (det.koef 20% -> 29%).

3. uzdevums - ANOVA

Datu ielasīšana

```
df <- chickwts
attach(df)
summary(df)
```

```
##      weight             feed
##  Min.   :108.0   casein   :12
##  1st Qu.:204.5   horsebean:10
##  Median :258.0   linseed  :12
##  Mean   :261.3   meatmeal :11
##  3rd Qu.:323.5   soybean  :14
##  Max.   :423.0   sunflower:12
```

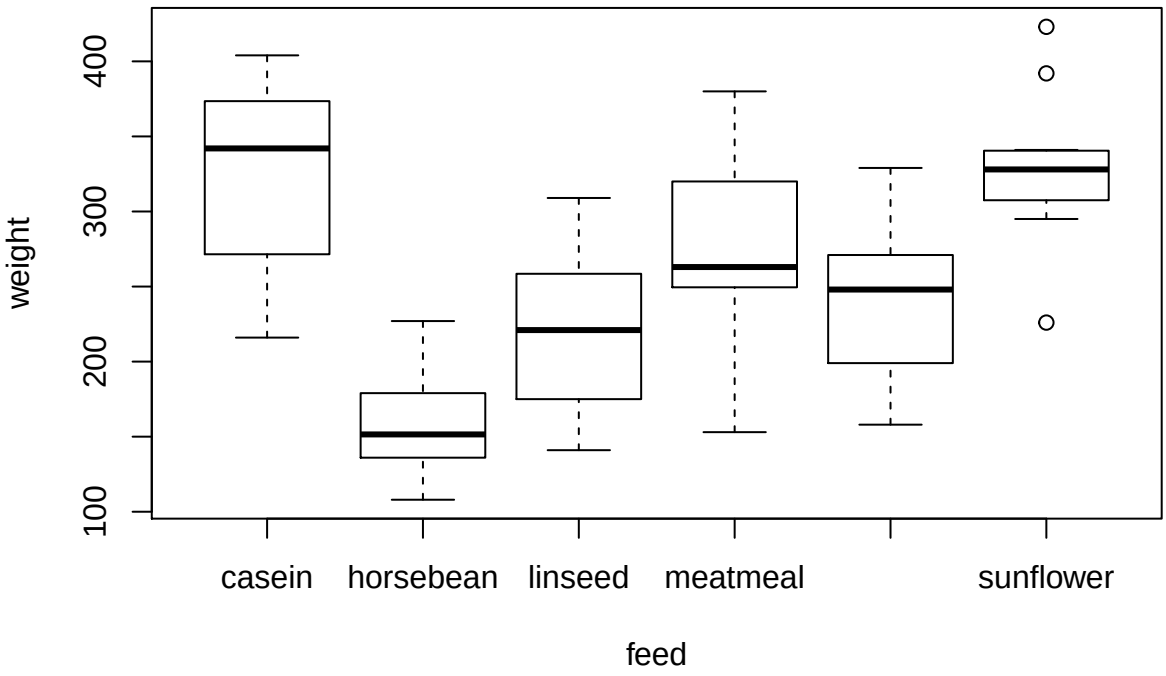
a) Kastu grafiki, aprakstošās statistikas

Bibliotēka

```
library(psych)
```

Kastu grafiki, aprakstošās statistikas:

```
boxplot(weight~feed)
```




```
describe(weight[feed=="casein"])
```

```
##    vars  n   mean    sd median trimmed   mad min max range  skew kurtosis   se
## X1    1 12 323.58 64.43    342   326.3 63.01 216 404   188 -0.46    -1.37 18.6
```

```
describe(weight[feed=="horsebean"])
```

```
##    vars  n  mean    sd median trimmed   mad min max range skew kurtosis    se
## X1    1 10 160.2 38.63  151.5  158.38 32.62 108 227   119 0.47    -1.19 12.21
```

```
describe(weight[feed=="linseed"])
```

```
##    vars  n   mean    sd median trimmed   mad min max range skew kurtosis    se
## X1    1 12 218.75 52.24    221   217.5 58.56 141 309   168 0.01    -1.33 15.08
```

```
describe(weight[feed=="meatmeal"])
```

```
##    vars  n   mean   sd median trimmed  mad min max range  skew kurtosis    se
## X1    1 11 276.91 64.9    263  279.22 77.1 153 380   227 -0.25    -0.93 19.57
```

```
describe(weight[feed=="soybean"])
```

```
##    vars  n   mean    sd median trimmed   mad min max range skew kurtosis    se
## X1    1 14 246.43 54.13    248  246.92 53.37 158 329   171 0.03    -1.17 14.47
```

```
describe(weight[feed=="sunflower"])
```

```
##    vars  n   mean    sd median trimmed   mad min max range  skew kurtosis   se
## X1    1 12 328.92 48.84    328   329.8 18.53 226 423   197 -0.05     0.06 14.1
```

b) ANOVA modelis

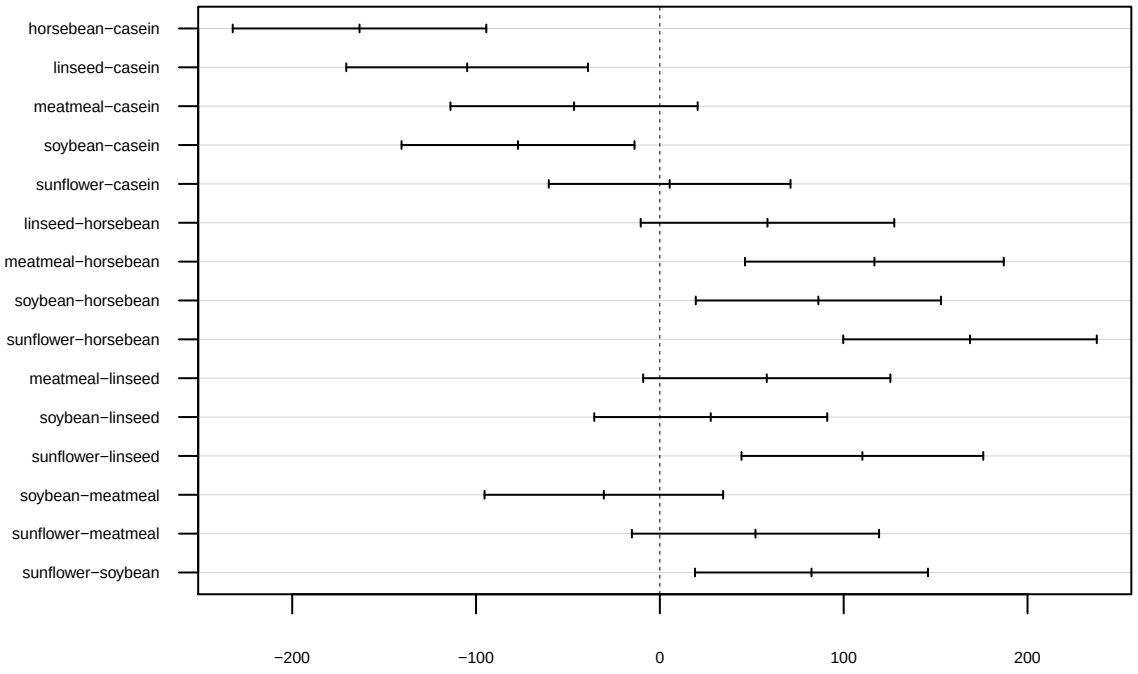
```
fit<-aov(weight~feed)
summary(fit)
```

```
##             Df Sum Sq Mean Sq F value   Pr(>F)
## feed         5 231129   46226   15.37 5.94e-10 ***
## Residuals   65 195556    3009
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Rezultāts ir gaidītais - grupas nepieder pie viena sadalījuma.

c) Post-hoc salīdzinājums pa pāriem

```
r<-TukeyHSD(fit)
op <- par(mar= c(4,5,3,3) + 0.1, cex.axis=0.5)
plot(TukeyHSD(fit),las=1)
```




```
par(op)
```

Grupas, kas varētu pārklāties:

- meatmeal-casein;
- sunflower-casein;
- linseed-horsebean;
- meatmeal-linseed;
- soybean-linseed;
- soybean-meatmeal;
- sunflower-meatmeal.

d) ANOVA pieņēmumu pārbaude

Normalitāte:

```
library(dplyr)
library(rstatix)
df %>% group_by(feed) %>% shapiro_test(weight)
```

```
## # A tibble: 6 x 4
##   feed      variable statistic     p
##   <fct>     <chr>        <dbl> <dbl>
## 1 casein    weight       0.917 0.259
## 2 horsebean weight       0.938 0.526
## 3 linseed   weight       0.969 0.903
## 4 meatmeal  weight       0.979 0.961
## 5 soybean   weight       0.946 0.506
## 6 sunflower weight       0.928 0.360
```

Nevienai grupai nevar noraidīt.

Dispersijas vienmērība:

```
leveneTest(weight,feed)
```

```
## Levene's Test for Homogeneity of Variance (center = median)
##       Df F value Pr(>F)
## group  5  0.7493 0.5896
##       65
```

```
bartlett.test(weight,feed)
```

```
##
##  Bartlett test of homogeneity of variances
##
## data:  weight and feed
## Bartlett's K-squared = 3.2597, df = 5, p-value = 0.66
```

Arī to nevar noraidīt.

e) neparametriskā ANOVA procedūra

Tā kā šajā gadījumā grupu sadalījumu normalitāti un dispersiju vienmērību noraidīt nevar, stingri runājot neparametriskas metodes nav nepieciešamas. Taču Kruskal-test procedūru var veikt jebkurā gadījumā:

```
kruskal.test(weight~feed)
```

```
##
##  Kruskal-Wallis rank sum test
##
## data:  weight by feed
## Kruskal-Wallis chi-squared = 37.343, df = 5, p-value = 5.113e-07
```

Kā iepriekš, visu sadalījumu vienādības hipotēze tiek pārliecinoši noraidīta.

Pāru salīdzināšanai var lietot pāru Vilkoksa testu:

```
pairwise.wilcox.test(weight,feed,p.adjust.method="BH")
```

```
##
##  Pairwise comparisons using Wilcoxon rank sum test
##
## data:  weight and feed
##
##           casein  horsebean linseed meatmeal soybean
## horsebean 0.00016 -         -       -        -
## linseed   0.00305 0.01191   -       -        -
## meatmeal  0.11355 0.00096   0.05451 -        -
## soybean   0.01110 0.00227   0.27306 0.28035  -
## sunflower 1.00000 9.3e-05   0.00025 0.09384  0.00334
##
## P value adjustment method: BH
```

Grupas, kas varētu pārklāties:

- meatmeal-casein;
- sunflower-casein;
- meatmeal-linseed;
- soybean-linseed;

- soybean-meatmeal;
- sunflower-meatmeal.

Salīdzinot ar iepriekšējo, parametrisko metodi atkrīt:

- linseed-horsebean.

f) neparametriskais post-hoc tests

Izmantojot Dunn testu:

```
dunn_test(df, weight~feed)
```

```
## # A tibble: 15 x 9
##    .y.    group1   group2     n1    n2 statistic         p    p.adj p.adj.signif
##  * <chr>  <chr>    <chr>   <int> <int>     <dbl>     <dbl>    <dbl> <chr>
##  1 weight casein   horseb~    12    10    -4.81    1.49e-6  2.08e-5 ****
##  2 weight casein   linseed    12    12    -3.31    9.39e-4  1.03e-2 *
##  3 weight casein   meatme~    12    11    -1.42    1.57e-1  6.27e-1 ns
##  4 weight casein   soybean    12    14    -2.50    1.24e-2  9.94e-2 ns
##  5 weight casein   sunflo~    12    12     0.183   8.55e-1  9.90e-1 ns
##  6 weight horsebe~ linseed    10    12     1.66    9.72e-2  5.83e-1 ns
##  7 weight horsebe~ meatme~    10    11     3.36    7.68e-4  9.22e-3 **
##  8 weight horsebe~ soybean    10    14     2.60    9.27e-3  8.34e-2 ns
##  9 weight horsebe~ sunflo~    10    12     4.99    6.12e-7  9.17e-6 ****
## 10 weight linseed  meatme~    12    11     1.82    6.88e-2  4.82e-1 ns
## 11 weight linseed  soybean    12    14     0.933   3.51e-1  9.90e-1 ns
## 12 weight linseed  sunflo~    12    12     3.49    4.81e-4  6.25e-3 **
## 13 weight meatmeal soybean    11    14    -0.974   3.30e-1  9.90e-1 ns
## 14 weight meatmeal sunflo~    11    12     1.59    1.11e-1  5.83e-1 ns
## 15 weight soybean  sunflo~    14    12     2.69    7.15e-3  7.15e-2 ns
```

Grupas, kuru pārklāšanos nevar izslēgt:

- meatmeal-casein;
- soybean-casein;
- sunflower-casein;
- linseed-horsebean;
- soybean-horsebean;
- meatmeal-linseed;
- soybean-linseed;
- soybean-meatmeal;
- sunflower-meatmeal;
- sunflower-soybean.

Salīdzinot ar parametrisko metodi, klāt nākušas:

- soybean-casein;
- soybean-horsebean;
- sunflower-soybean.

g) secinājumi un komentāri

- Kaut gan ANOVA normalitātes un dispersijas vienmērīguma nosacījumi it kā reti izpildās, šajā datu kopā nekādas problēmas tie nesagādā;
- Dažas grupas ļoti robusti iztur vienādības pārbaudes visās testu kategorijās, bet citas ir uz robežas - dažādi testi sniedz dažādas atbildes - kaut gan tās ir ~5% p-vērtības visos variantos.

4. uzdevums - 2-faktoru ANOVA

Ceru, ka esmu pareizi sapratis uzdevuma nosacījumus, jo šķiet, ka šajā punktā jādara daudz mazāk nekā citos.

Datu ielasīšana:

```
df <- ToothGrowth
attach(df)
summary(df)
```

```
##       len        supp         dose
##  Min.   : 4.20   OJ:30   Min.   :0.500
##  1st Qu.:13.07   VC:30   1st Qu.:0.500
##  Median :19.25           Median :1.000
##  Mean   :18.81           Mean   :1.167
##  3rd Qu.:25.27           3rd Qu.:2.000
##  Max.   :33.90           Max.   :2.000
```

Datu kolonnas:

- len - zobu garums;
- supp - uztura bagātinātājs (2 kategorijas);
- dose - doza (3 līmeņi - {0.5, 1, 2}).

2-faktoru ANOVA novērtējums:

```
summary(aov(len ~ supp*dose))
```

```
##             Df Sum Sq Mean Sq F value   Pr(>F)
## supp         1  205.4   205.4  12.317 0.000894 ***
## dose         1 2224.3  2224.3 133.415  < 2e-16 ***
## supp:dose    1   88.9    88.9   5.333 0.024631 *
## Residuals   56  933.6    16.7
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

ANOVA tests liecina, ka katra kategorija ir statistiski nozīmīga (p -> 0). Abu kategoriju mijiedarbība izdalīta kā neatkarīgs faktors nav ne tuvu tik nozīmīga kā katrs atsevišķi, taču p-vērtība tik un tā ir zem 5%, kas ļauj šo mijiedarbību arī atzīt par statistiski nozīmīgu.

Izmantojot HSD.test, var redzēt, kā kategorijas grupējas:

```
library(agricolae)
tx<-with(df, interaction(supp,dose))
amod <- aov(len ~ tx)
HSD.test(amod, "tx", group=TRUE,console=TRUE)
```

```
##
## Study: amod ~ "tx"
##
## HSD Test for len
##
## Mean Square Error:  13.18715
##
## tx,  means
##
##          len      std  r  Min  Max
## OJ.0.5 13.23 4.459709 10  8.2 21.5
## OJ.1   22.70 3.910953 10 14.5 27.3
```

```
## OJ.2   26.06 2.655058 10 22.4 30.9
## VC.0.5  7.98 2.746634 10  4.2 11.5
## VC.1   16.77 2.515309 10 13.6 22.5
## VC.2   26.14 4.797731 10 18.5 33.9
##
## Alpha: 0.05 ; DF Error: 54
## Critical Value of Studentized Range: 4.178265
##
## Minimun Significant Difference: 4.798124
##
## Treatments with the same letter are not significantly different.
##
##          len groups
## VC.2   26.14      a
## OJ.2   26.06      a
## OJ.1   22.70      a
## VC.1   16.77      b
## OJ.0.5 13.23      b
## VC.0.5  7.98      c
```

Grupās, kas izdalāmas arī ar grafisko metodi, kas apskatīta lekcijā, taču konfliktē ar Markdown kompilatoru, redzams, ka pieaugot dozai, mazinās atšķirības starp uztura bagātinātājiem.

5. uzdevums - joslas platuma meklēšana

Datu ielāde (dots speciāls 5-modāls sadalījums, kas lekcijās izmantots, lai ilustrētu pdf novērtējuma metožu trūkumus):

```
df <- as.numeric(read.delim("dati2_5.txt",header=F,sep=" "))
```

5.1. uzdevums - joslas platuma atrašana

Izmantojot R iebūvēto krosvalidācijas optimizatoru:

```
cross_validated<-bw.ucv(df)
cross_validated
```

```
## [1] 0.05095563
```

5.2. ilustrēt KDE aproksimāciju pret histogrammu

Lai uzskatāmāk parādītu arī "nepiegludinātus" un "pārgludinātus" PDF tuvinājumus, zīmētas līknes ar h = 1 un h = 0.01:

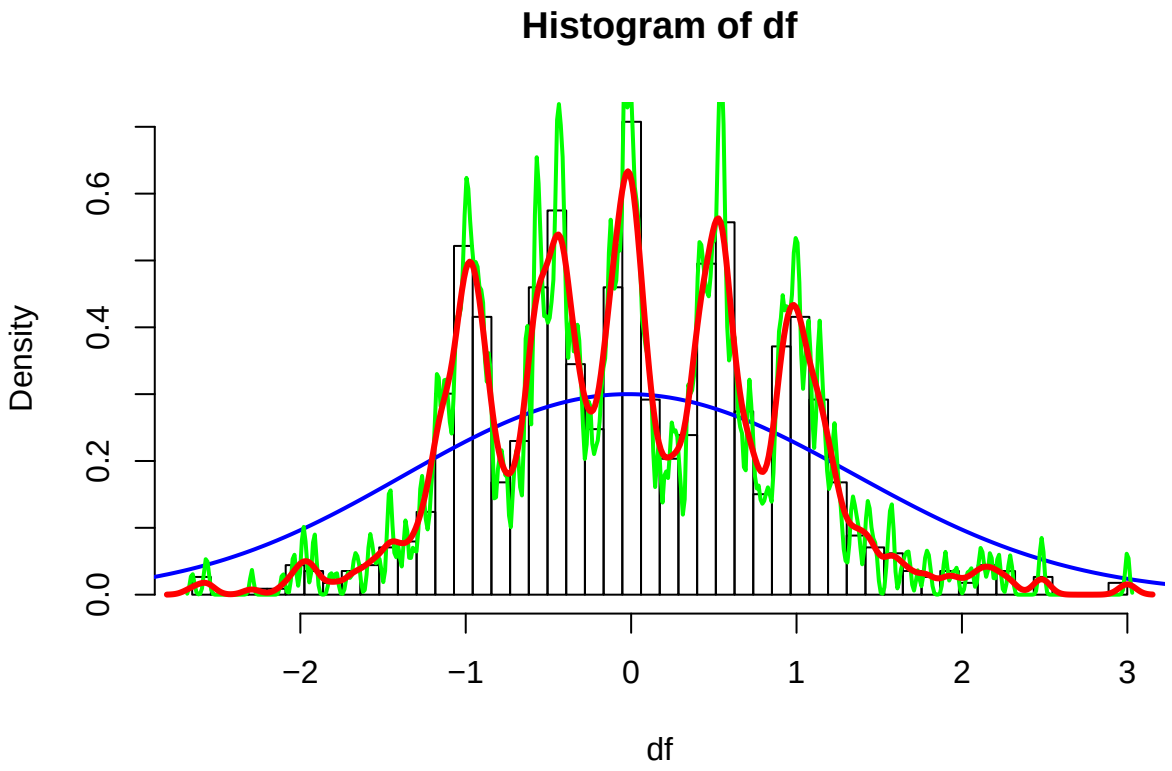
```
library(histogram)
hh<-histogram(df,type="regular",penalty="cv")
```

```
## Building regular histogram with maximum number of bins 144.
```

```
## - Choosing number of bins via leave-1-out cross validation. Using formula 1.
```

```
## - Number of bins chosen: 50.
```

```
lines(density(df,bw=1),col="blue",lwd=2)
lines(density(df,bw=0.01),col="green",lwd=2)
lines(density(df,bw="ucv"),col="red",lwd=3)
```



5.3. salīdzināt ar citām joslas platuma novērtējuma metodēm

Grāmatā "Nonparametric and Semiparametric Models" dota formula t.s. "Silverman rule of thumb" heiristikai, kas pieņem normālā sadalījuma otro atvasinājumu un līdz ar to ļauj iegūt joslas platumu no datu kopas standartnovirzes novērtējuma. Šo un nedaudz paplašinātu heiristiku arī piedāvā R iebūvētais joslas platuma noteikšanas rīks:

```
bw.nrd0(df)
```

```
## [1] 0.1952054
```

```
bw.nrd(df)
```

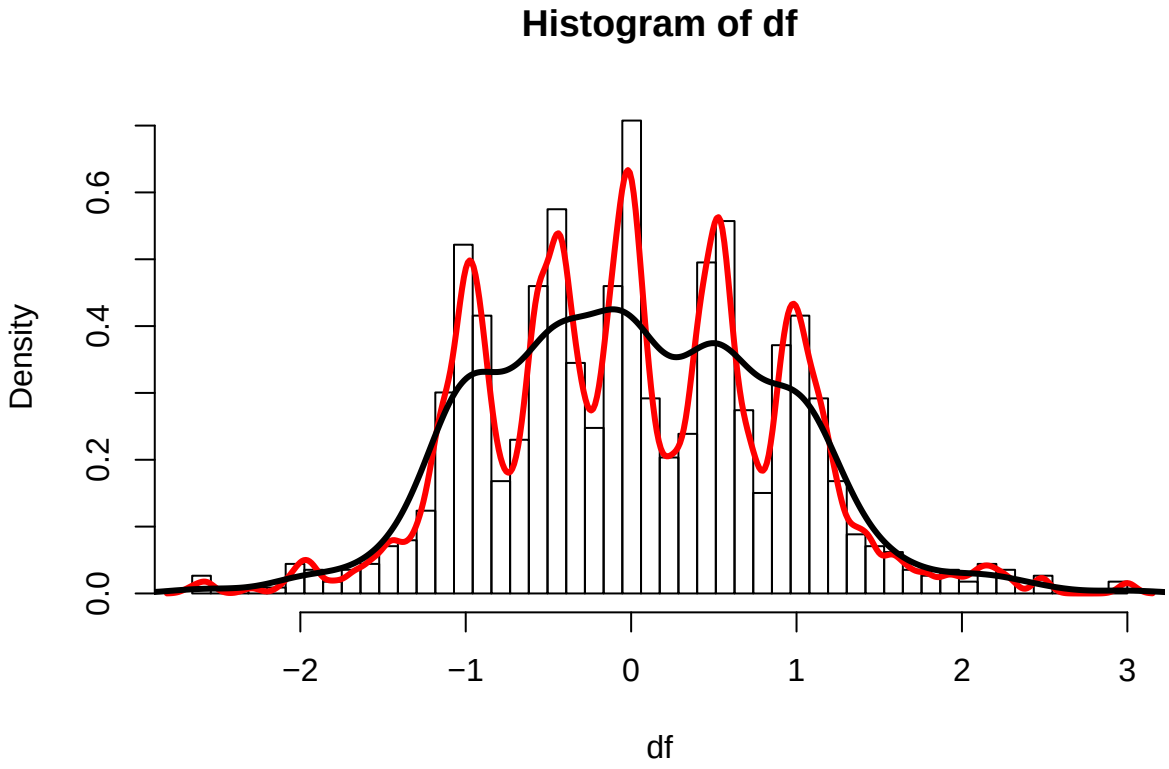
```
## [1] 0.2299086
```

Kā redzams, iegūtie rezultāti ir ļoti līdzīgi. Salīdzinot tuvāko (mazāko) ar krosvalidācijas optimizatora iegūto:

```
hh<-histogram(df,type="regular",penalty="cv")
```

```
## Building regular histogram with maximum number of bins 144.
## - Choosing number of bins via leave-1-out cross validation. Using formula 1.
## - Number of bins chosen: 50.
```

```
lines(density(df,bw="ucv"),col="red",lwd=3)
lines(density(df,bw="nrd0"),col="black",lwd=3)
```



Kā redzams, ar krosvalidācijas metodi iegūtā KDE līkne labāk seko īstajam sadalījumam.

6. uzdevums - neparametriskā regresija

Datu ielāde:

```
df <- read.table('CMB.dat',header=TRUE)
attach(df)
```

6.1. regresiju ģenerēšana

Polinomiālā regresija ar jau iepriekš noskaidroto "labāko" vērtību:

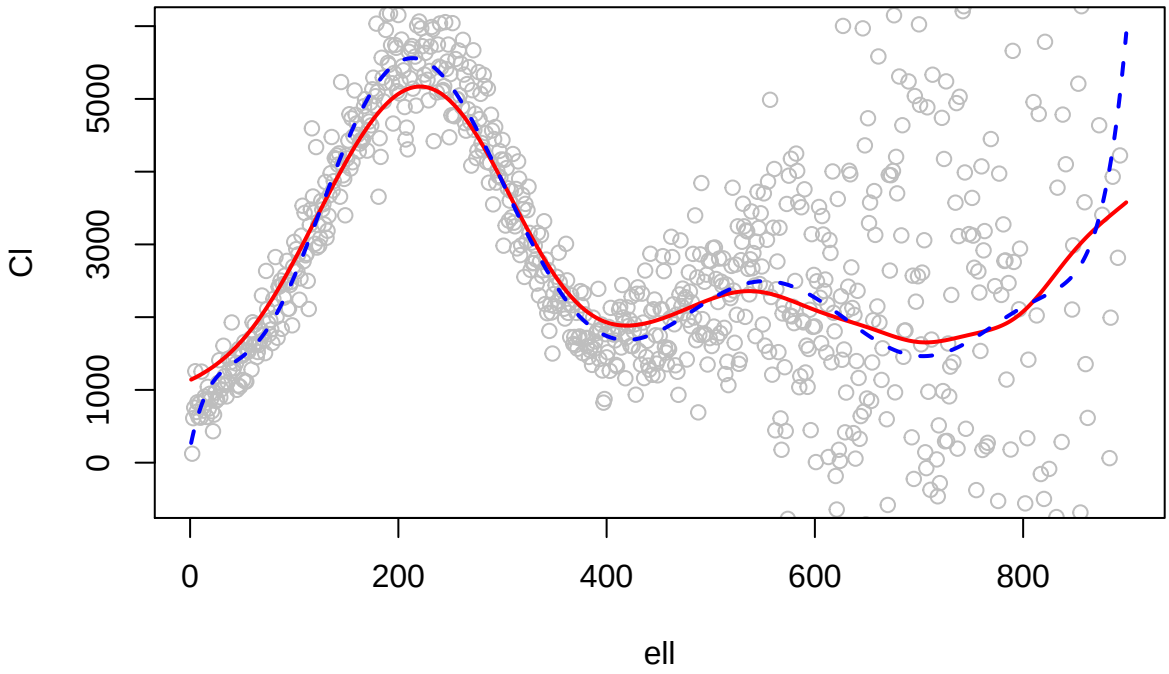
```
p <- lm(Cl~poly(ell,9,raw=T))
```

Neparametriskā regresija ar NW metodi bez papildus korekcijām (pirmā funkcija nosaka joslas platumu, otrā ģenerē pašu regresijas līkni):

```
library(np)
bw <- npregbw(Cl~ell,bwmethod="cv.ls",regtype="lc")
n <- npreg(bw,residuals=T)
```

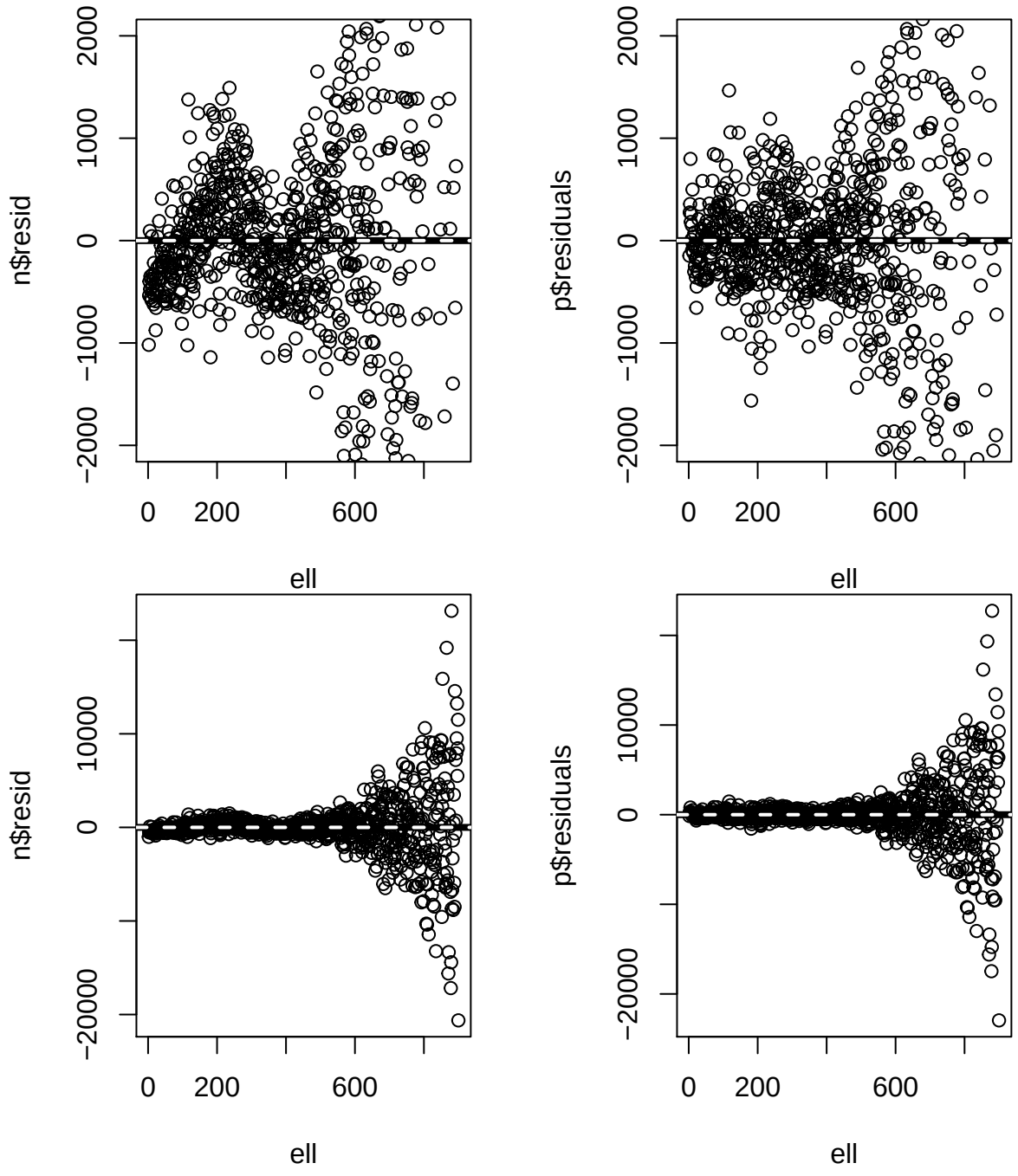
Grafisks attēlojums:

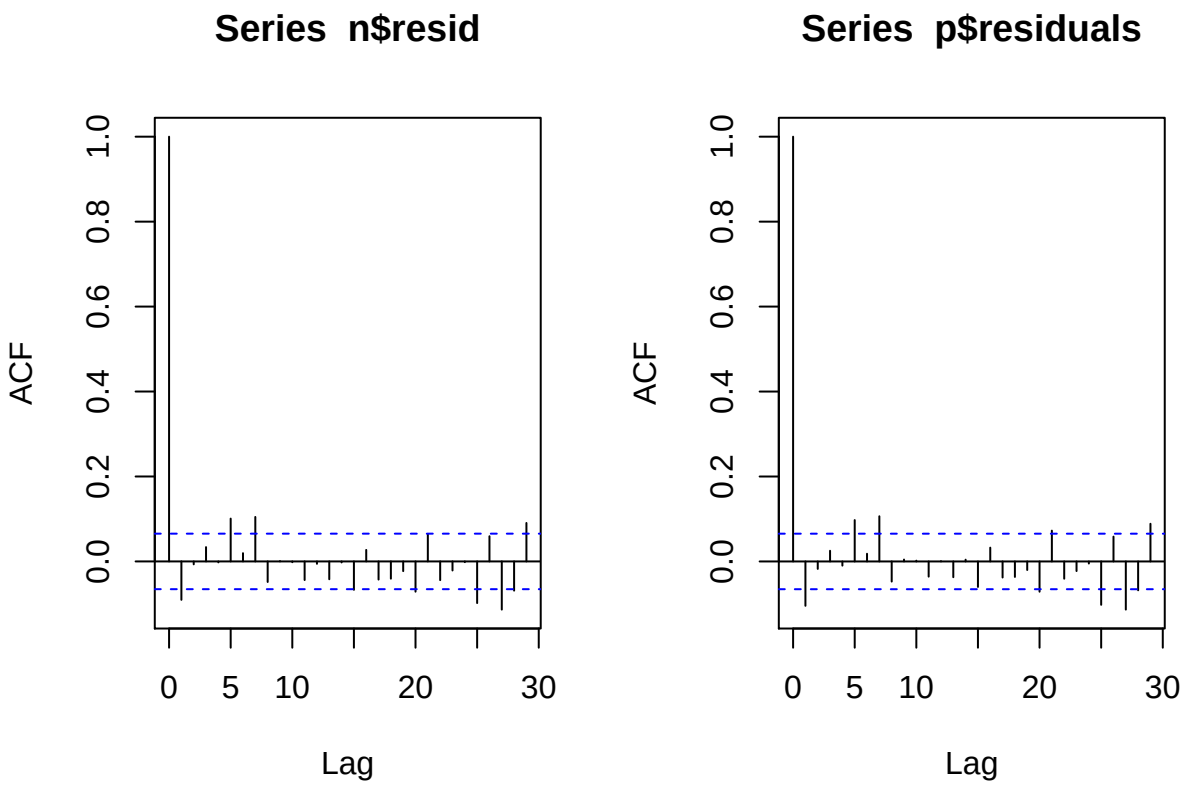
```
plot(ell,Cl,col="grey",ylim=c(-500,6000))
lines(n$mean,col="red",lwd=2)
lines(p$fitted.values,col="blue",lwd=2,lty="dashed")
```



6.2. nosacījumu pārbaude, salīdzinājums

Grafiski attēlojumi:





Grafiski redzams, ka definīcijas intervālā polinoms labāk tuvināts datiem, taču asi maina virzienu abos galos, kas to padara nederīgu paredzējumiem ārpus intervāla. Autokorelācijas abos gadījumos ir diezgan līdzīgas, taču neparametriskās regresijas gadījumā grafiski redzams, ka saglabājusies sistemātiska, periodiska nobīde starp modeli un datiem. Polinoma gadījumā atlikumi veido (vismaz vizuāli) nejauši izkliedētu punktu mākoni.

Jau pirmajā uzdevumā noskaidrots, kas neizpildās ne normalitātes, ne dispersijas vienmērīguma nosacījumi atlikumos, kas nepieciešami lineārām un polinomiālām regresijām.

6.3. joslas platuma novērtējumi

NW nav vienīgā metode, ar kuru var ģenerēt neparametriskas regresijas. Lai iegūtu joslas platumus, var izmantot t.s. "plug-in" metodes, kas izmanto vienkāršotas heiristikas. Kodola regresijas jēdzienu iespējams vispārināt, padarot izteiksmi par polinomu katrā punktā - iegūstot lokāli lineāru vai polinomiālu modeli.

```
bwll <- npregbw(Cl~ell,regtype="ll",bwmethod="cv.aic")
# would be dpill(ell,Cl), but wilcox test crashes the compiler.
bwrt <- 34.61873
bwrt <-npregbw(Cl~ell,bws=bwrt,bandwidth.compute=F)
```

```
bwll
```

```
##
## Regression Data (899 observations, 1 variable(s)):
##
##                      ell
## Bandwidth(s): 41.83265
##
## Regression Type: Local-Linear
## Bandwidth Selection Method: Expected Kullback-Leibler Cross-Validation
## Formula: Cl ~ ell
## Bandwidth Type: Fixed
```

```
## Objective Function Value: 17.07873 (achieved on multistart 1)
##
## Continuous Kernel Type: Second-Order Gaussian
## No. Continuous Explanatory Vars.: 1
```

```
bwrt
```

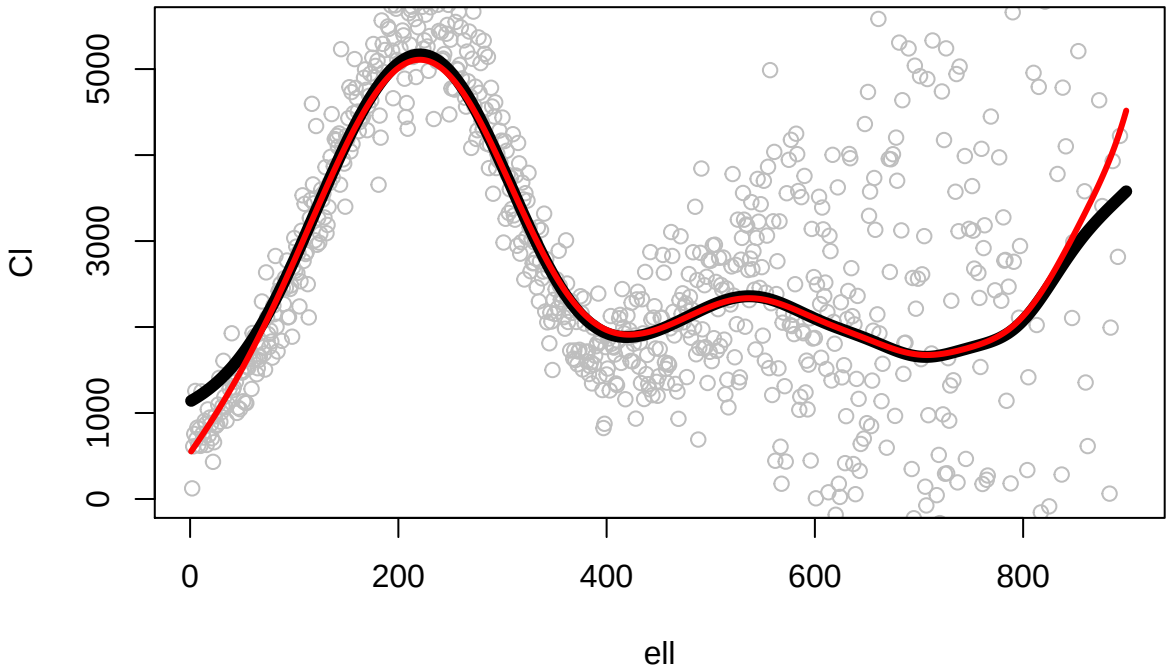
```
##
## Regression Data (899 observations, 1 variable(s)):
##
##                      ell
## Bandwidth(s): 34.61873
##
## Regression Type: Local-Constant
## Bandwidth Selection Method: Manual
## Formula: Cl ~ ell
## Bandwidth Type: Fixed
##
## Continuous Kernel Type: Second-Order Gaussian
## No. Continuous Explanatory Vars.: 1
```

Salīdzinot regresijas metodes grafiski:

```
nll <- npreg(bwll,residuals=T)
nrt <- npreg(bwrt,residuals=T)
```

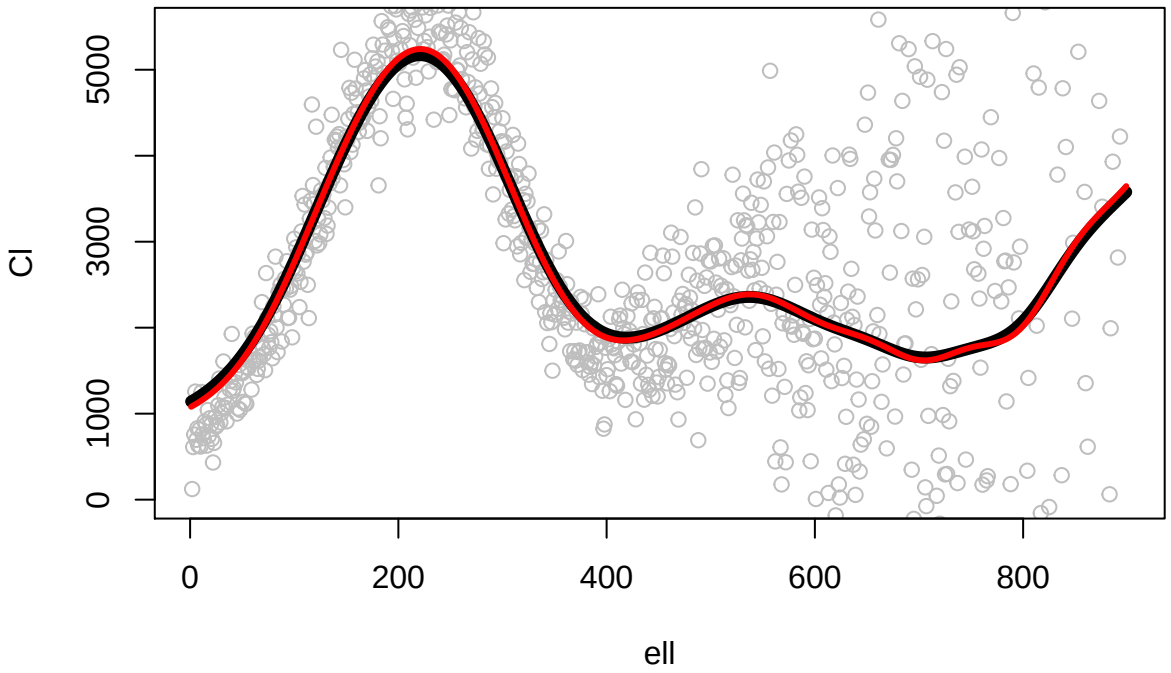
NW pret lokāli polinomiālo regresiju (metode ir ekvivalenta locpoly funkcijai, bet bez patvaļīgās argumentu kopas pārkārtošanas):

```
plot(ell,Cl,col="grey",ylim=c(0,5500))
lines(n$mean,col="black",lwd=6)
lines(nll$mean,col="red",lwd=3)
```



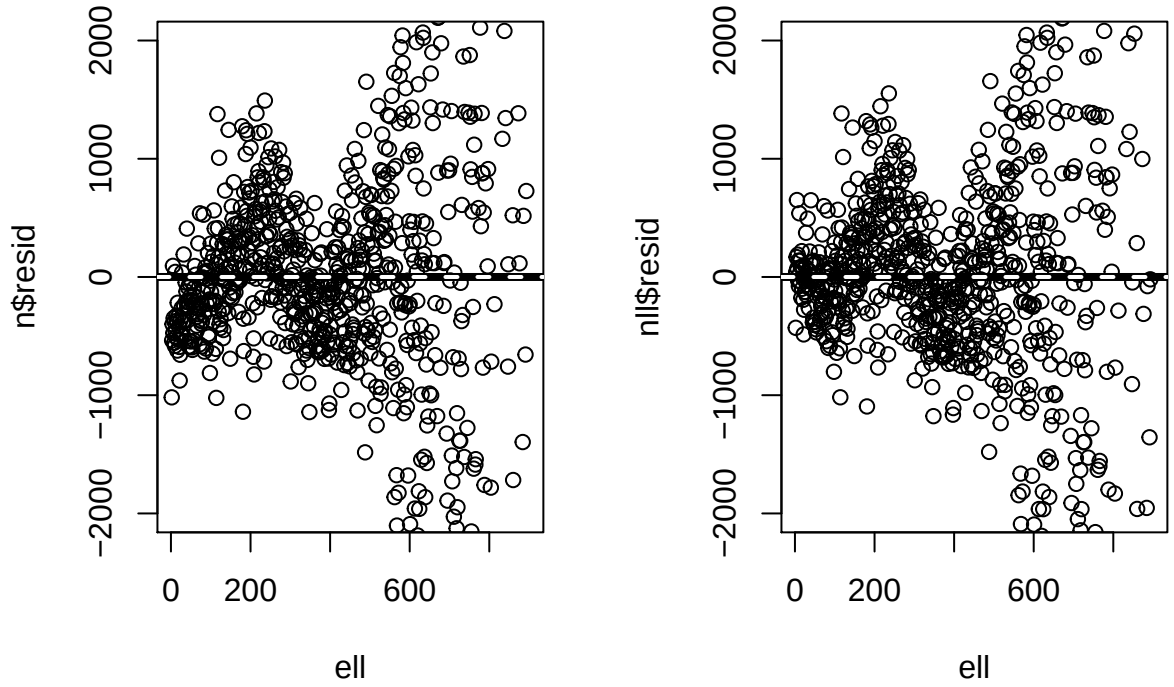
Krosvalidācija pret "plug-in" kodola aplēšanu:

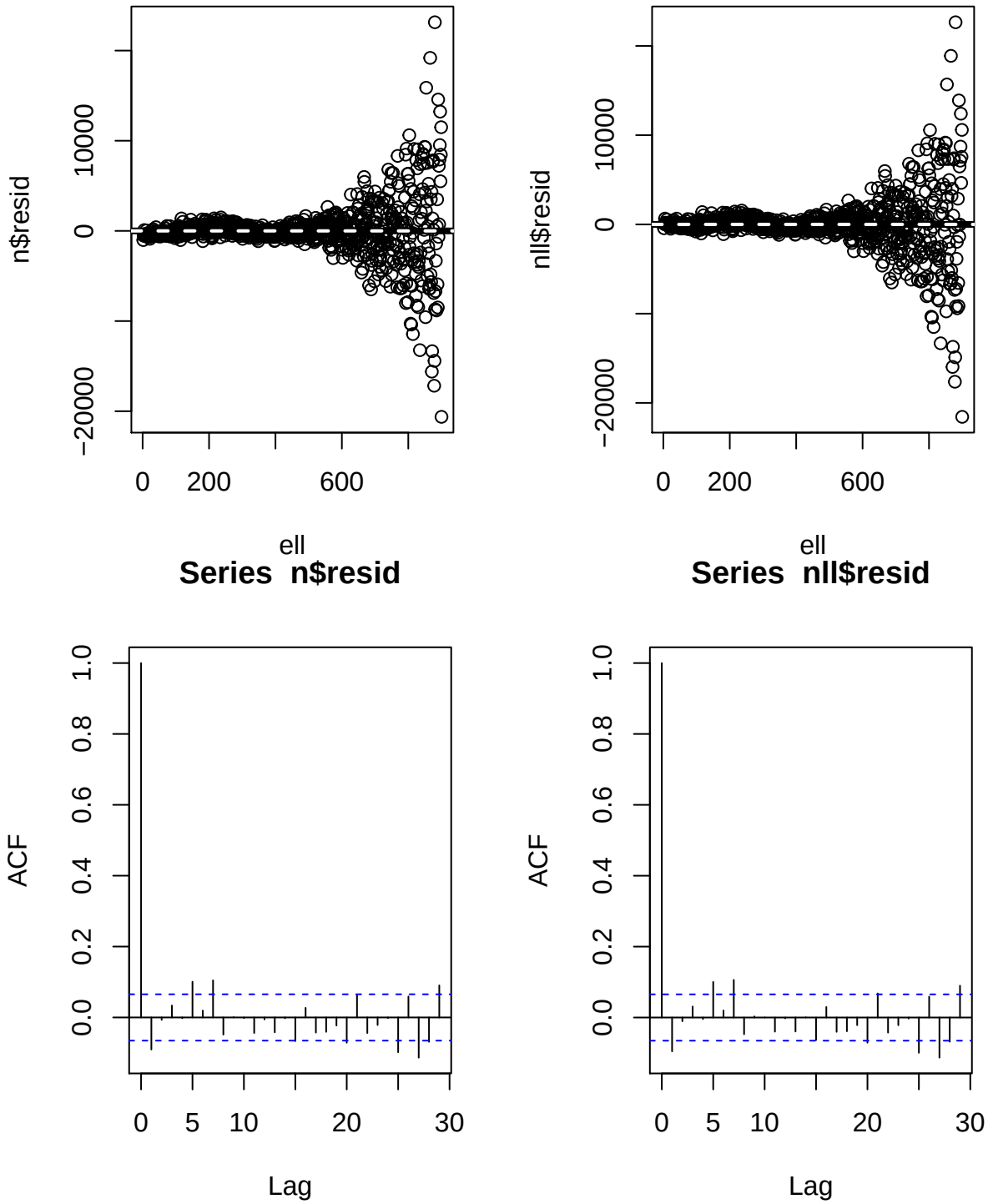
```
plot(ell,Cl,col="grey",ylim=c(0,5500))
lines(n$mean,col="black",lwd=6)
lines(nrt$mean,col="red",lwd=3)
```



Redzams, kas joslas platuma izvēles metodei ir minimāls iespaids uz rezultējošo līkni šajā gadījumā, taču lokāli polinomiālā regresija datu kopas galos uzvedas ievērojami savādāk nekā parastā NW metode.

Salīdzinot atlikumus:





Redzams, ka mazliet mazinājusies pārpalikusī struktūra atlikumos, taču tuvinājums vēl joprojām nav tik labs kā polinoma gadījumā.

7. uzdevums - bootstrap, robusti novērtētāji

Datu ielasīšana:

```
df <- InsectSprays
attach(df)
```

7.1. dažādu statistiku ticamības intervāli

Funkcijas statistiku aprēķinam:

```
library(boot)
library(e1071)
cb_mean <- function(x, i){
  mean(x[i])
}
cb_median <- function(x, i){
  median(x[i])
}
cb_skewness <- function(x, i){
  skewness(x[i])
}
```

Statistiku ticamības intervāli:

```
N<-10000
bobj_mean <- boot(count,statistic=cb_mean,R=N)
bobj_medi <- boot(count,statistic=cb_median,R=N)
bobj_skew <- boot(count,statistic=cb_skewness,R=N)
boot.ci(bobj_mean)
```

```
## BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
## Based on 10000 bootstrap replicates
##
## CALL :
## boot.ci(boot.out = bobj_mean)
##
## Intervals :
## Level      Normal              Basic
## 95%   ( 7.857, 11.159 )   ( 7.833, 11.111 )
##
## Level     Percentile            BCa
## 95%   ( 7.889, 11.167 )   ( 7.931, 11.236 )
## Calculations and Intervals on Original Scale
```

```
boot.ci(bobj_medi)
```

```
## BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
## Based on 10000 bootstrap replicates
##
## CALL :
## boot.ci(boot.out = bobj_medi)
##
## Intervals :
## Level      Normal              Basic
## 95%   ( 2.138, 10.135 )   ( 2.500,  9.000 )
##
## Level     Percentile            BCa
```

```
## 95%   ( 5.0, 11.5 )   ( 5.0, 10.5 )
## Calculations and Intervals on Original Scale
```

```
boot.ci(bobj_skew)
```

```
## BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
## Based on 10000 bootstrap replicates
##
## CALL :
## boot.ci(boot.out = bobj_skew)
##
## Intervals :
## Level      Normal              Basic
## 95%   ( 0.1901,  0.9432 )   ( 0.1764,  0.9292 )
##
## Level     Percentile            BCa
## 95%   ( 0.1889,  0.9417 )   ( 0.2074,  0.9684 )
## Calculations and Intervals on Original Scale
```

Salīdzinājums ar parametrisku un neparametrisku testu:

```
t.test(count,mu=mean(count))
```

```
##
##  One Sample t-test
##
## data:  count
## t = 0, df = 71, p-value = 1
## alternative hypothesis: true mean is not equal to 9.5
## 95 percent confidence interval:
##   7.807311 11.192689
## sample estimates:
## mean of x
##       9.5
```

```
wilcox.test(count,conf.int = T)
```

```
##
##  Wilcoxon signed rank test with continuity correction
##
## data:  count
## V = 2485, p-value = 3.535e-13
## alternative hypothesis: true location is not equal to 0
## 95 percent confidence interval:
##   7.500017 11.499944
## sample estimates:
## (pseudo)median
##       9.499973
```

7.2. robustie lokācijas novērtējumi

Vidējā vērtība pēc tradicionālā aprēķina:

```
mean(count)
```

```
## [1] 9.5
```

Robustas metodes:

```
library(robustbase)
```

```
median(count)
```

```
## [1] 7
```

```
mean(count,trim=0.2)
```

```
## [1] 8.5
```

```
huberM(count)$mu
```

```
## [1] 9.212867
```

Secinājumi: Hubera M-novērtējums un "nogrieztā" sadalījuma vidējā vērtība ir samērā tuvi vidējai, taču mediāna ir ievērojami atšķirīga.

8. uzdevums - robustā regresija

Datu ielāde:

```
library(robustbase)
df<-coleman
attach(df)
summary(df)
```

```
##     salaryP         fatherWc        sstatus           teacherSc
##  Min.   :2.070   Min.   : 9.99   Min.   :-16.0400   Min.   :21.60
##  1st Qu.:2.447   1st Qu.:19.25   1st Qu.: -0.3675   1st Qu.:24.51
##  Median :2.675   Median :30.69   Median :  5.4650   Median :25.01
##  Mean   :2.732   Mean   :40.91   Mean   :  3.1405   Mean   :25.07
##  3rd Qu.:2.955   3rd Qu.:66.17   3rd Qu.: 11.0450   3rd Qu.:25.70
##  Max.   :3.830   Max.   :86.27   Max.   : 15.0300   Max.   :28.01
##    motherLev           Y
##  Min.   :5.170   Min.   :22.70
##  1st Qu.:5.740   1st Qu.:32.77
##  Median :6.190   Median :35.85
##  Mean   :6.255   Mean   :35.08
##  3rd Qu.:6.880   3rd Qu.:39.95
##  Max.   :7.510   Max.   :43.10
```

Datu apraksts - valodas testu rezultāti skolās:

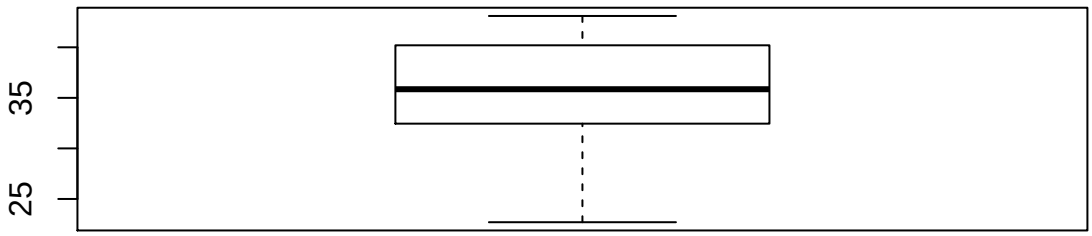
- Y - rezultāti valodas testā;
- salaryP - darbinieku algas uz vienu skolēnu;
- fatherWc - to tēvu frakcija, kas strādā "white collar" darbus;
- sstatus - socio-ekonomiskā statusa novērtējums;
- teacherSc - skolotāju testu rezultāti;
- motherLev - mātes izglītības līmenis;

8.1 aprakstošās statistikas:

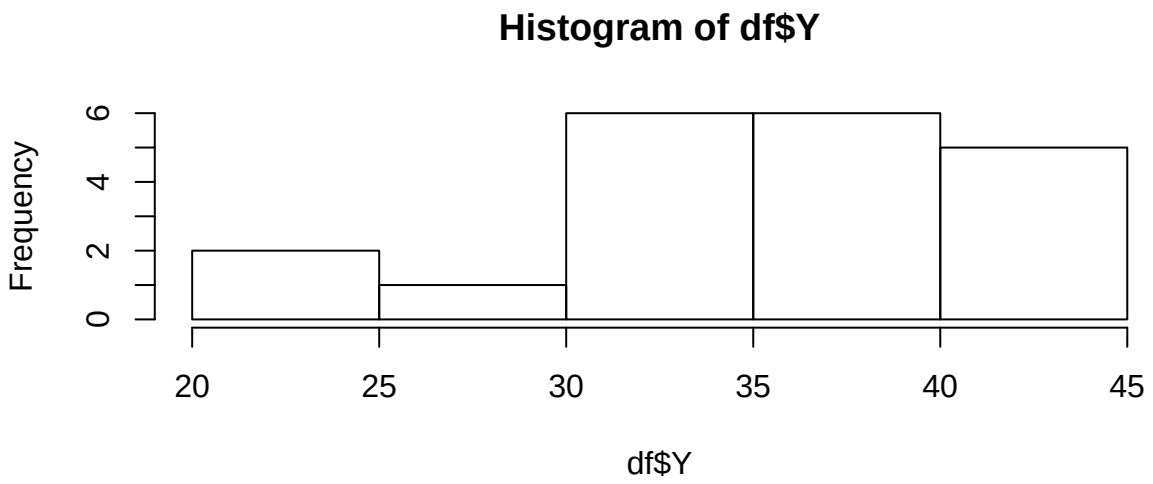
```
library(psych)
describe(df$Y)
```

```
##    vars  n  mean   sd median trimmed  mad  min  max range  skew kurtosis  se
## X1    1 20 35.08 5.82  35.86   35.67 5.86 22.7 43.1  20.4 -0.69    -0.44 1.3
```

```
boxplot(df$Y)
```



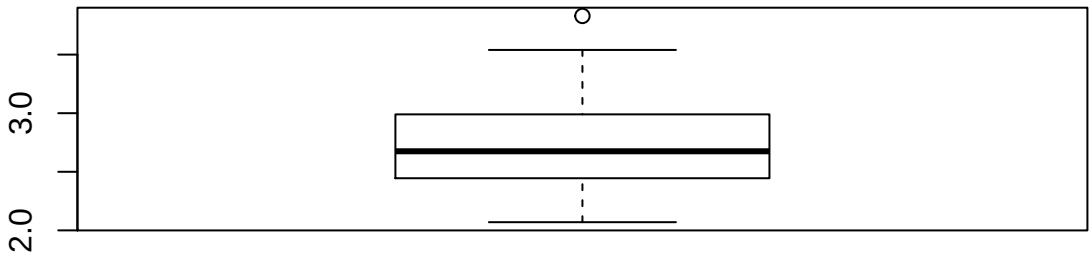
```
hist(df$Y)
```



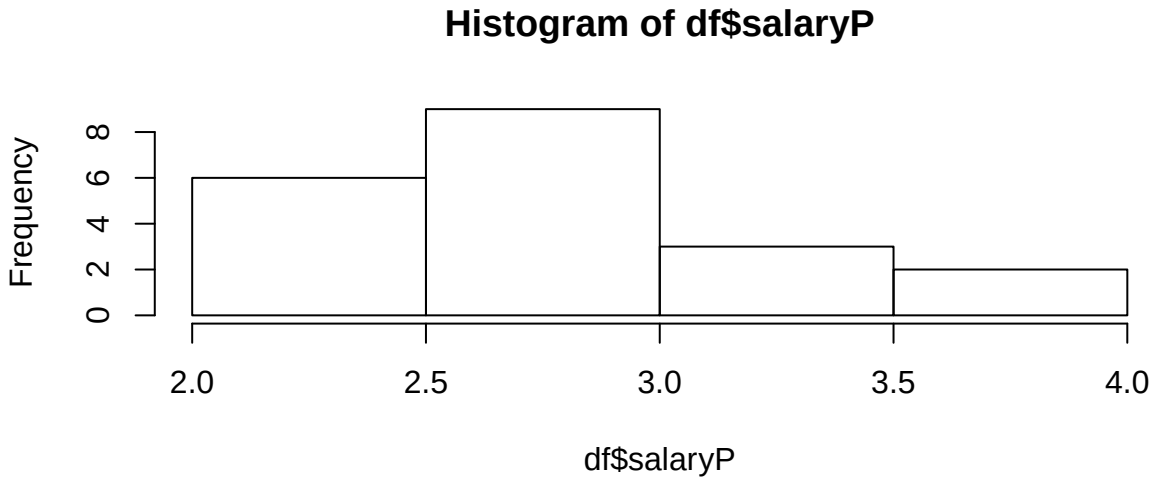

```
describe(df$salaryP)
```

```
##    vars  n mean   sd median trimmed  mad  min  max range skew kurtosis  se
## X1    1 20 2.73 0.45   2.67    2.69 0.36 2.07 3.83  1.76 0.65    -0.16 0.1
```

```
boxplot(df$salaryP)
```




```
hist(df$salaryP)
```

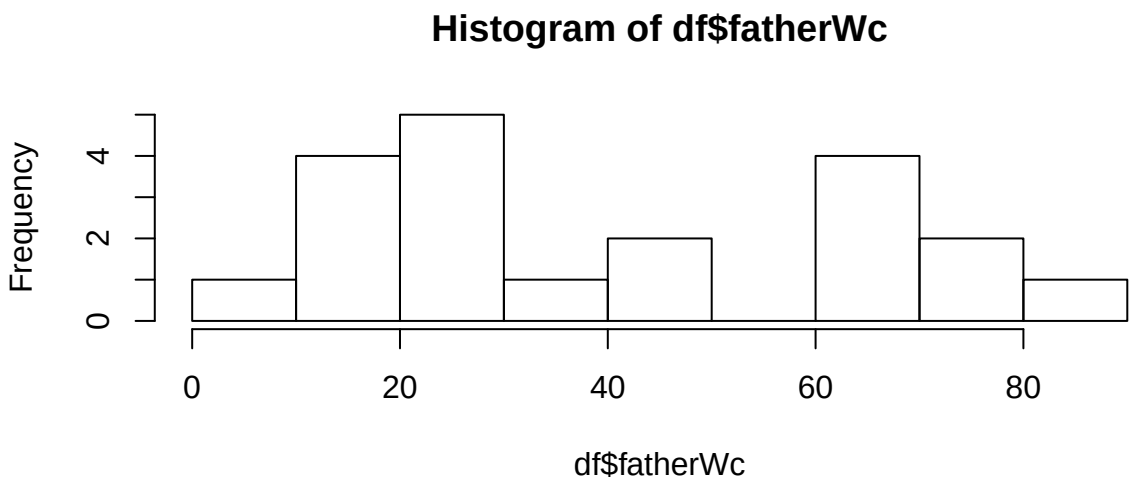



```
describe(df$fatherWc)
```

```
##    vars  n  mean   sd median trimmed   mad  min   max range skew kurtosis   se
## X1    1 20 40.91 25.9  30.69   39.56 25.86 9.99 86.27 76.28 0.36    -1.54 5.79
```

```
boxplot(df$fatherWc)
```

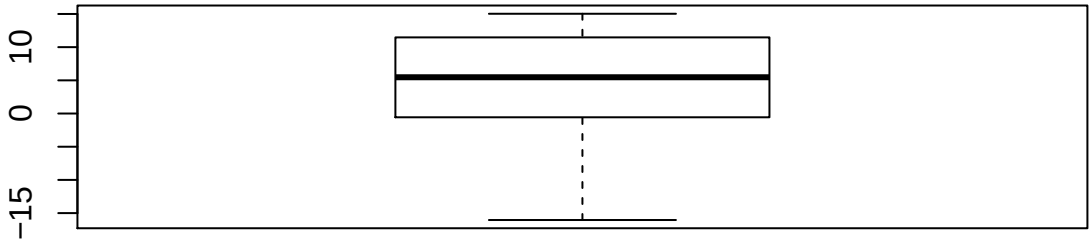

```
hist(df$fatherWc)
```



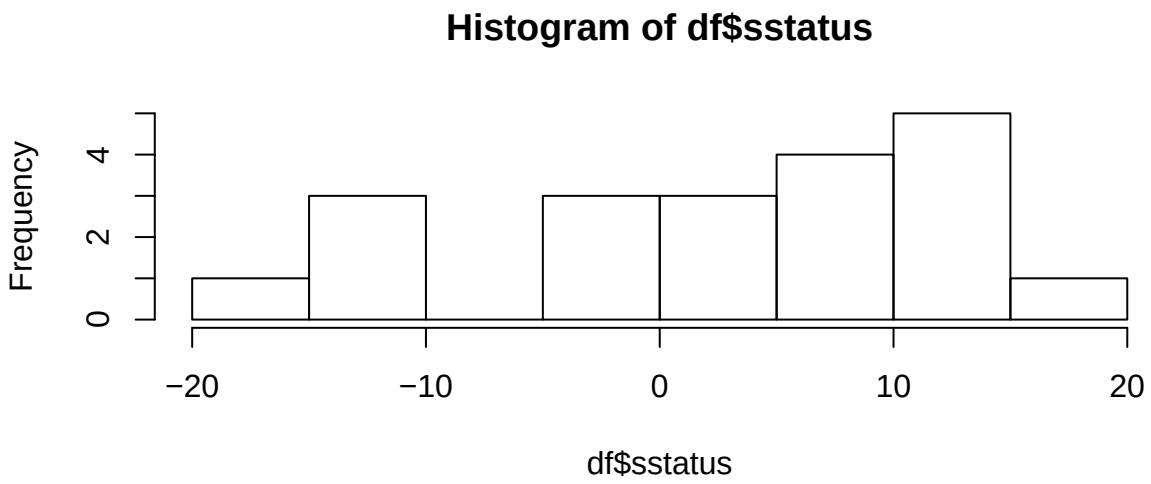

```
describe(df$sstatus)
```

```
##    vars  n mean   sd median trimmed  mad    min   max range  skew kurtosis   se
## X1    1 20 3.14 9.63   5.46     3.9 8.94 -16.04 15.03 31.07 -0.61    -0.93 2.15
```

```
boxplot(df$sstatus)
```



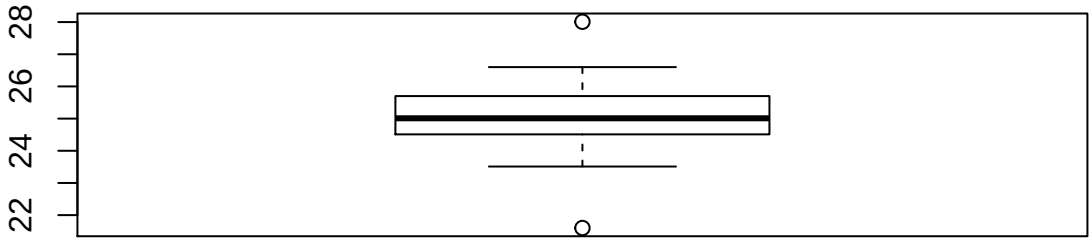

```
hist(df$sstatus)
```



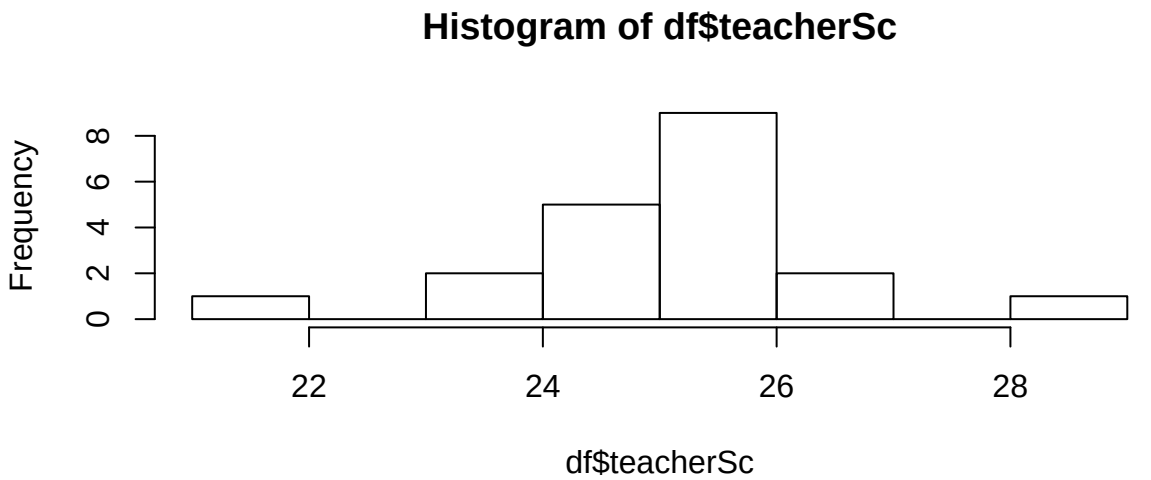

```
describe(df$teacherSc)
```

```
##    vars  n  mean   sd median trimmed  mad  min   max range  skew kurtosis   se
## X1    1 20 25.07 1.31  25.01    25.1 0.82 21.6 28.01  6.41 -0.35     1.07 0.29
```

```
boxplot(df$teacherSc)
```



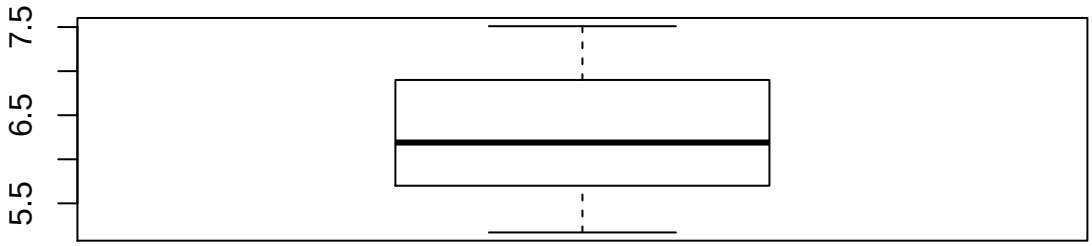

```
hist(df$teacherSc)
```



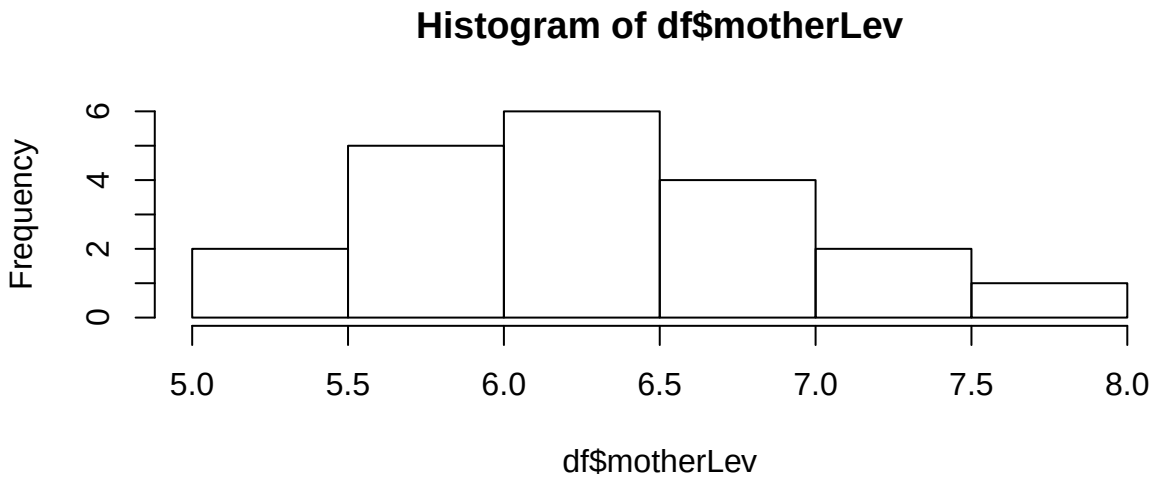

```
describe(df$motherLev)
```

```
##    vars  n mean   sd median trimmed  mad  min  max range skew kurtosis   se
## X1    1 20 6.25 0.65   6.19    6.25 0.85 5.17 7.51  2.34 0.16    -1.17 0.15
```

```
boxplot(df$motherLev)
```




```
hist(df$motherLev)
```



8.2. korelācijas

Korelācijas starp Y un argumentiem:

```
correlations<-cor(df)
correlations[6,]
```

```
##   salaryP  fatherWc   sstatus teacherSc motherLev         Y
## 0.1922916 0.7534008 0.9271611 0.3336495 0.7329859 1.0000000
```

Redzams, ka socio-ekonomiskais statuss, tēva darbavietas veids un mātes izglītības līmenis ir savstarpēji ļoti cieši korelēti:

```
correlations[3,]
```

```
##   salaryP  fatherWc   sstatus teacherSc motherLev         Y
## 0.2296278 0.8271829 1.0000000 0.1833292 0.8190633 0.9271611
```

8.3. daudzfaktoru lineārā regresija

Regresija:

```
library(car)
```

```
fit_multi<-lm(Y~., data=df)
summary(fit_multi)
```

```
##
## Call:
## lm(formula = Y ~ ., data = df)
```

```
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.9497 -0.6174  0.0623  0.7343  5.0018
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 19.94857   13.62755   1.464   0.1653
## salaryP     -1.79333    1.23340  -1.454   0.1680
## fatherWc     0.04360    0.05326   0.819   0.4267
## sstatus      0.55576    0.09296   5.979 3.38e-05 ***
## teacherSc    1.11017    0.43377   2.559   0.0227 *
## motherLev   -1.81092    2.02739  -0.893   0.3868
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.074 on 14 degrees of freedom
## Multiple R-squared:  0.9063, Adjusted R-squared:  0.8728
## F-statistic: 27.08 on 5 and 14 DF,  p-value: 9.927e-07
```

```
vif(fit_multi)
```

```
##   salaryP  fatherWc   sstatus teacherSc motherLev
##  1.385199  8.401309  3.535139  1.434130  7.770763
```

```
fit_multi<-step(fit_multi)
```

```
## Start:  AIC=34.05
## Y ~ salaryP + fatherWc + sstatus + teacherSc + motherLev
##
##             Df Sum of Sq     RSS    AIC
## - fatherWc   1     2.884  63.122 32.987
## - motherLev  1     3.433  63.671 33.160
## <none>                    60.238 34.051
## - salaryP    1     9.096  69.334 34.864
## - teacherSc  1    28.184  88.422 39.728
## - sstatus    1   153.800 214.038 57.408
##
## Step:  AIC=32.99
## Y ~ salaryP + sstatus + teacherSc + motherLev
##
##             Df Sum of Sq     RSS    AIC
## - motherLev  1     0.714  63.835 31.211
## <none>                    63.122 32.987
## - salaryP    1     8.361  71.483 33.474
## - teacherSc  1    25.495  88.616 37.772
## - sstatus    1   191.544 254.665 58.884
##
## Step:  AIC=31.21
## Y ~ salaryP + sstatus + teacherSc
##
##             Df Sum of Sq    RSS    AIC
## <none>                    63.84 31.211
## - salaryP    1      8.59  72.43 31.737
## - teacherSc  1     26.13  89.96 36.073
```

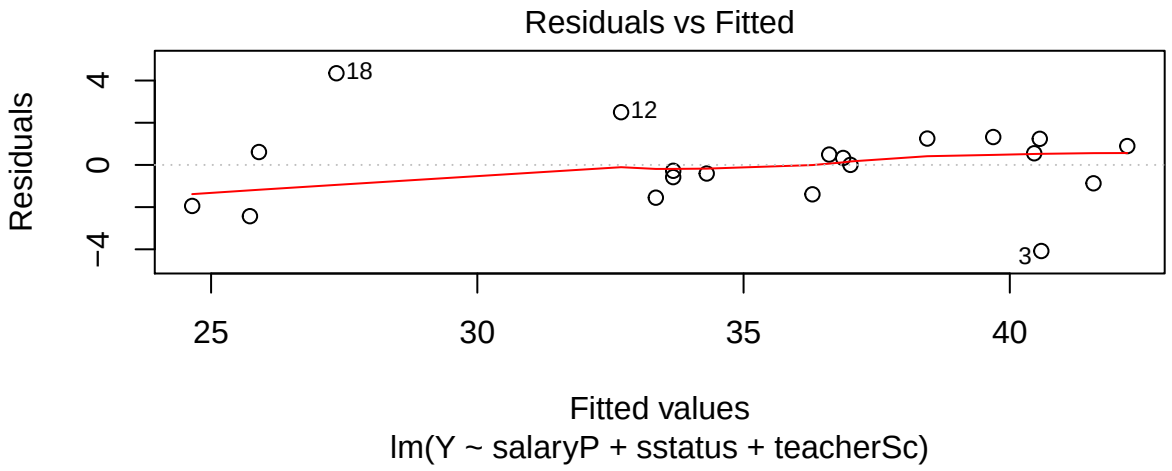
```
## - sstatus    1    507.00 570.83 73.027
```

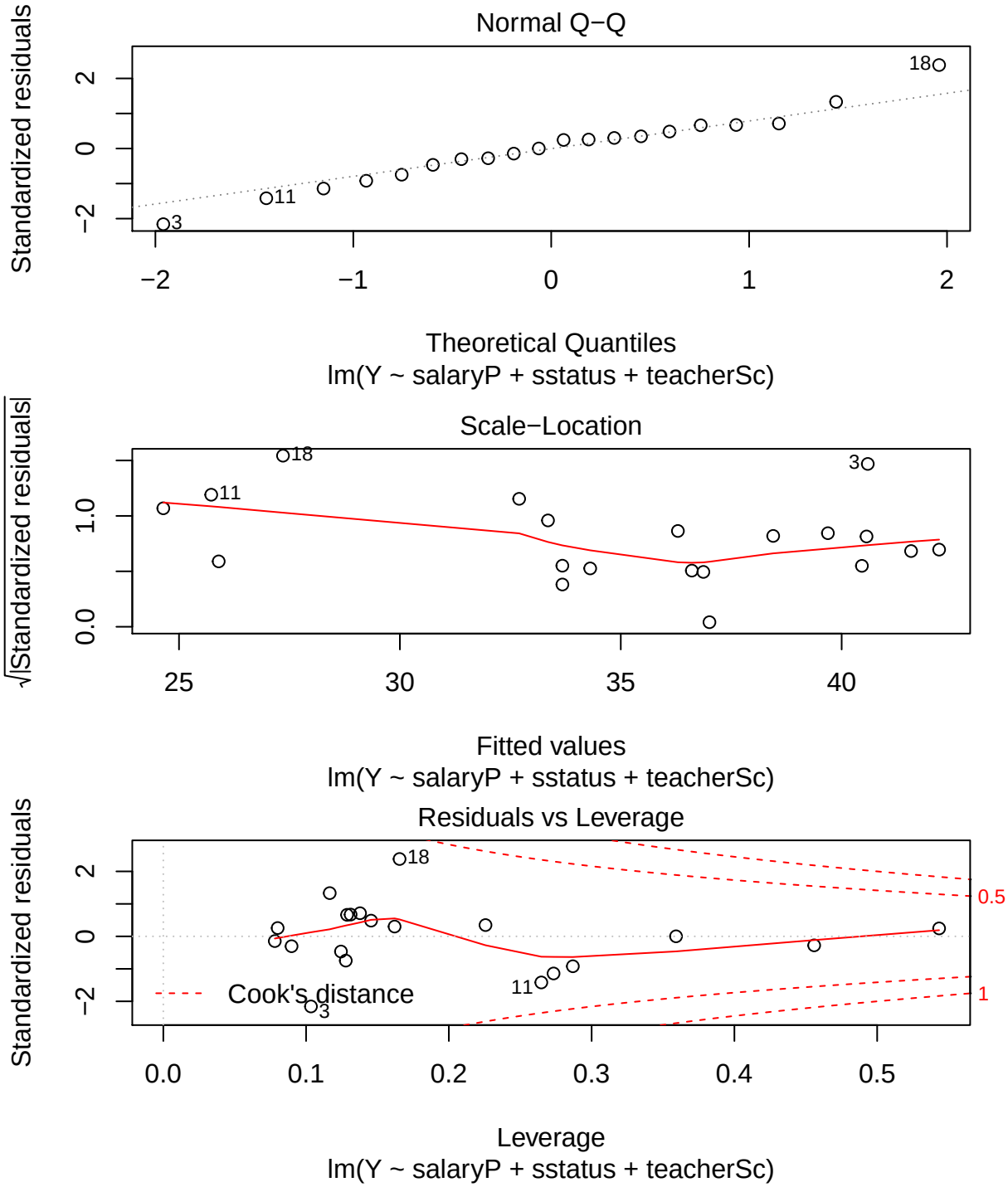
```
summary(fit_multi)
```

```
##
## Call:
## lm(formula = Y ~ salaryP + sstatus + teacherSc, data = df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.0812 -1.0017  0.1664  0.9797  4.3461
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 12.11951    9.03643   1.341    0.199
## salaryP     -1.73581    1.18290  -1.467    0.162
## sstatus      0.55321    0.04907  11.273 5.06e-09 ***
## teacherSc    1.03582    0.40479   2.559    0.021 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.997 on 16 degrees of freedom
## Multiple R-squared:  0.9007, Adjusted R-squared:  0.8821
## F-statistic: 48.38 on 3 and 16 DF,  p-value: 3.011e-08
```

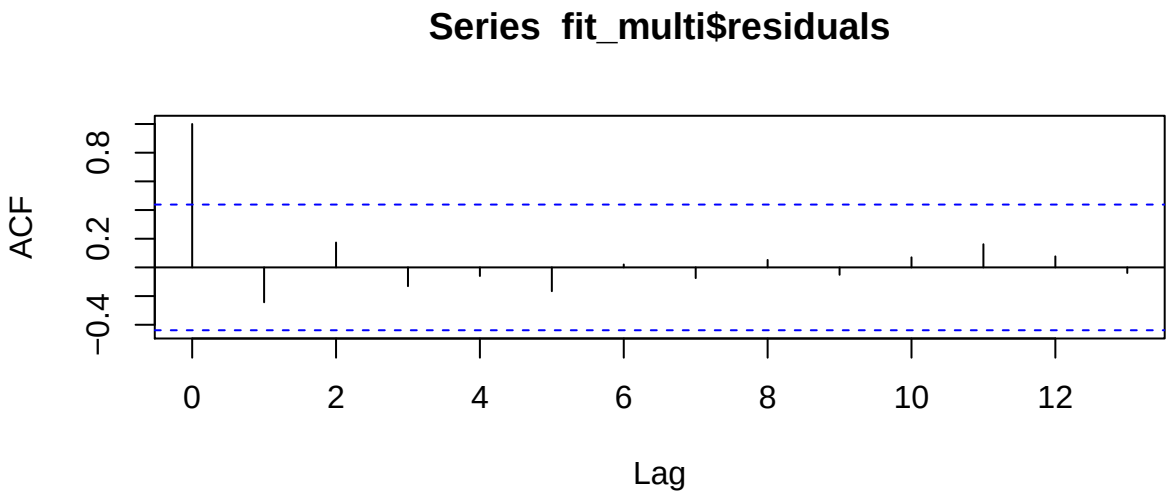
Pārbaudes - grafiski (īsi secinājumi komentāru formā):

```
plot(fit_multi)
```






```
# Residuals vs fitted: Minor visually apparent correlation
# Normal QQ: Two significant oultiers, mostly normally distributed residuals
# Scale-Location: not much apparent heteroscedacity
# Leverage: No outliers with leverage
acf(fit_multi$residuals)
```




```
# some apparent autocorrelation
```

Pārbaudes - statistiku testi:

```
library(nortest)
```

```
p<-0.05
# Are the residuals autocorrelated?
(durbinWatsonTest(fit_multi)$p < p)
```

```
## [1] FALSE
```

```
# Are the residuals normally distributed?
(lillie.test(fit_multi$residuals)$p.value > p)
```

```
## [1] TRUE
```

```
# Are the residuals homoscedastic?
(ncvTest(fit_multi)$p > p)
```

```
## [1] TRUE
```

8.4. daudzfaktoru robustā regresija

Regresija:

```
fit<-lmrob(Y~.,data=df)
summary(fit)
```

```
##
## Call:
## lmrob(formula = Y ~ ., data = df)
##  \--> method = "MM"
## Residuals:
##      Min       1Q   Median       3Q      Max
## -4.16181 -0.39226  0.01611  0.55619  7.22766
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 30.50232    6.71260   4.544 0.000459 ***
## salaryP     -1.66615    0.43129  -3.863 0.001722 **
## fatherWc     0.08425    0.01467   5.741 5.10e-05 ***
```

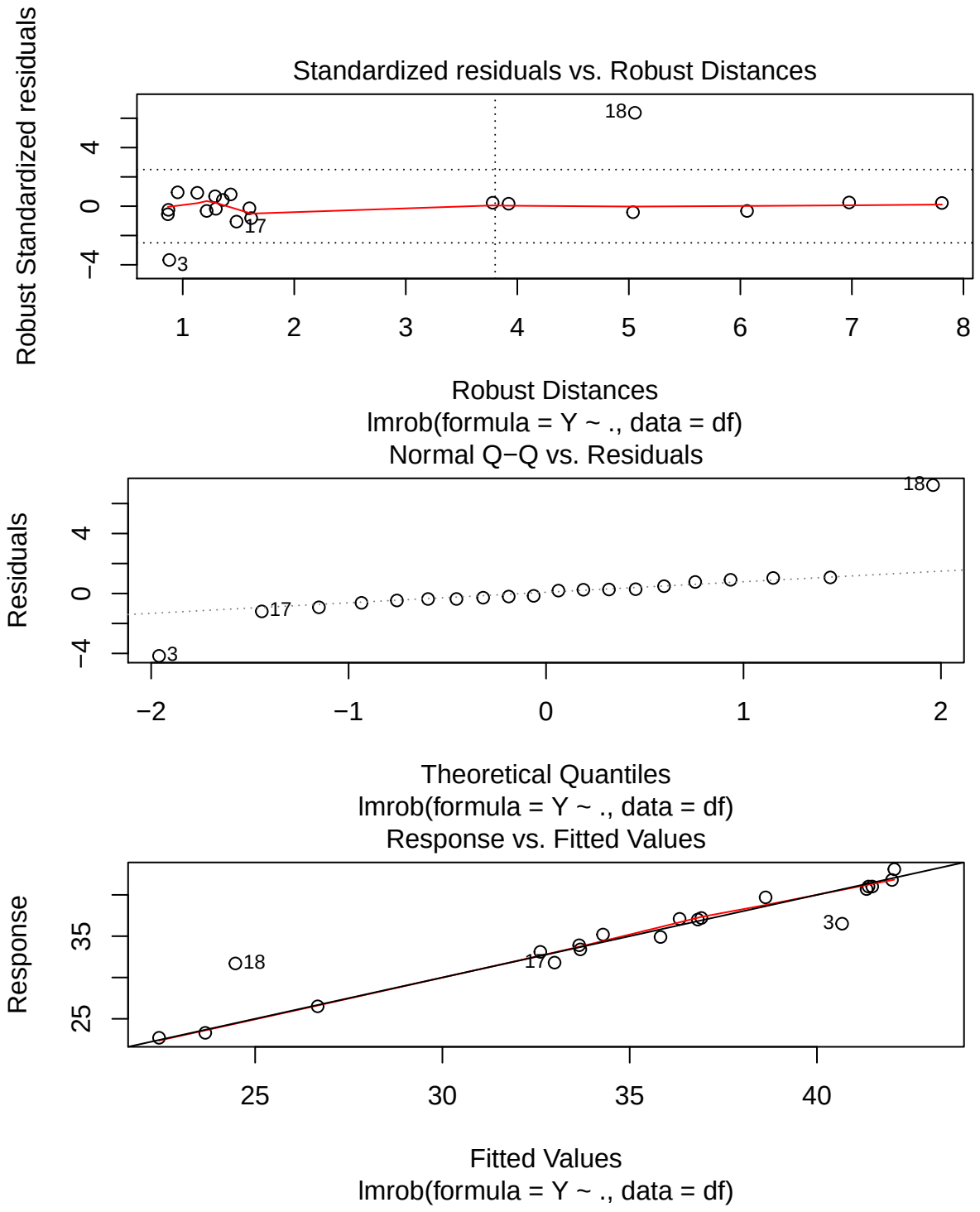
```
## sstatus      0.66774    0.03385  19.726 1.30e-11 ***
## teacherSc    1.16778    0.10983  10.632 4.35e-08 ***
## motherLev   -4.13657    0.92084  -4.492 0.000507 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Robust residual standard error: 1.134
## Multiple R-squared:  0.9814, Adjusted R-squared:  0.9747
## Convergence in 11 IRWLS iterations
##
## Robustness weights:
##  observation 18 is an outlier with |weight| = 0 ( < 0.005);
##  The remaining 19 ones are summarized as
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  0.1491  0.9412  0.9847  0.9279  0.9947  0.9982
## Algorithmic parameters:
##        tuning.chi                bb        tuning.psi        refine.tol
##         1.548e+00         5.000e-01         4.685e+00         1.000e-07
##           rel.tol         scale.tol         solve.tol       eps.outlier
##         1.000e-07         1.000e-10         1.000e-07         5.000e-03
##             eps.x warn.limit.reject warn.limit.meanrw
##         1.569e-10         5.000e-01         5.000e-01
##      nResample         max.it       best.r.s       k.fast.s          k.max
##            500             50              2              1            200
##    maxit.scale      trace.lev            mts     compute.rd fast.s.large.n
##            200              0           1000              0           2000
##                   psi           subsampling                   cov
##             "bisquare"         "nonsingular"         ".vcov.avar1"
## compute.outlier.stats
##                  "SM"
## seed : int(0)
```

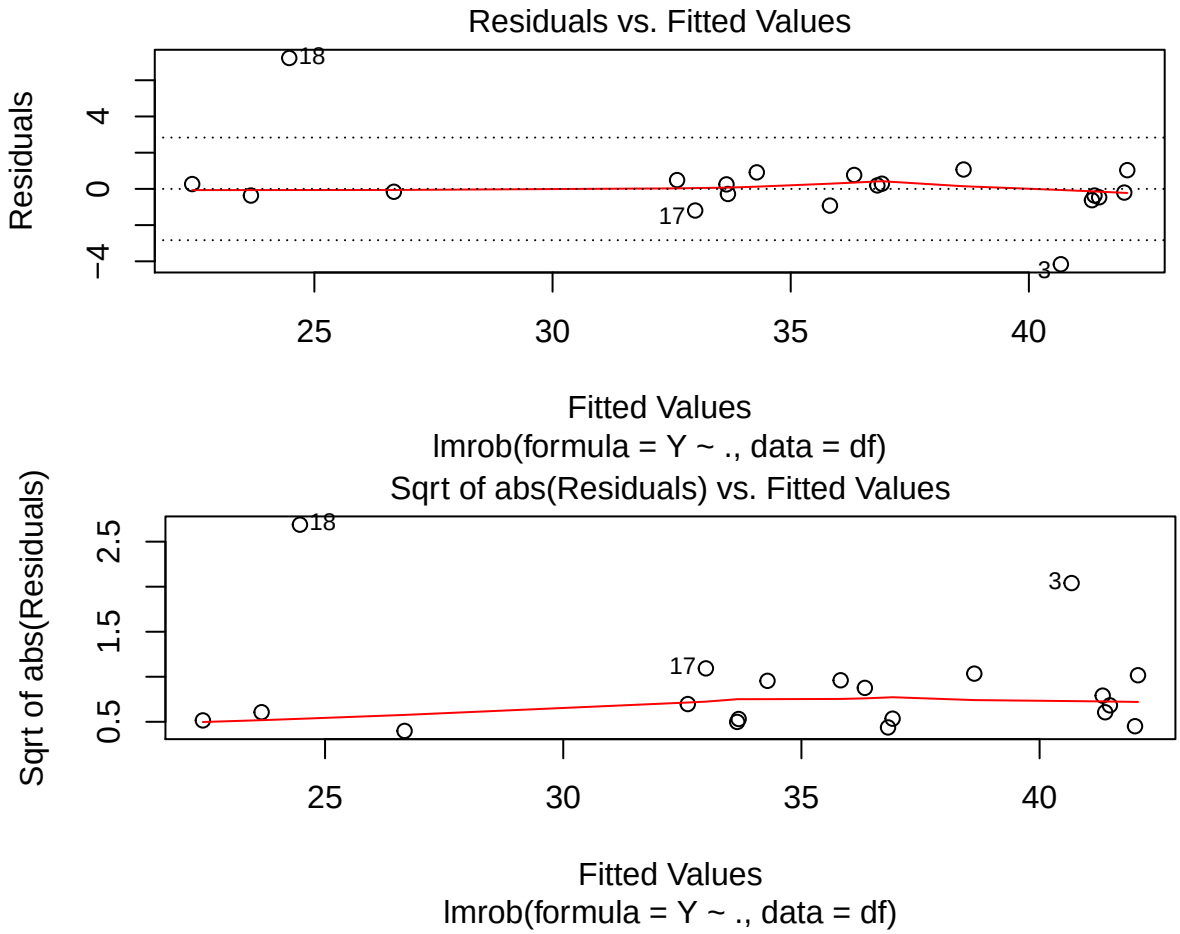
Novērtējums (tikai grafiski, jo nav atbalsta lmrob objektam; secinājumi komentāros):

```
plot(fit)
```

```
## recomputing robust Mahalanobis distances
```

```
## saving the robust distances 'MD' as part of 'fit'
```


Standardized residuals vs. Robust Distances
Robust Standardized residuals
Robust Distances
lmrob(formula = Y ~ ., data = df)
Normal Q−Q vs. Residuals
Residuals
Theoretical Quantiles
lmrob(formula = Y ~ ., data = df)
Response vs. Fitted Values
Response
Fitted Values
lmrob(formula = Y ~ ., data = df)




```
# Residuals vs leverage: One outlier with leverage (point 18)
# Normal QQ vs residuals: All error terms besides outliers normally distributed
# Response vs fitted: Two significant outliers, otherwise close fit
# Residuals vs fitted: No clear pattern, can assume residuals are IID
# Sqrt of residuals: can see some heteroscedacity (increasing with Y)
```

Salīdzinājums: robustajai regresijai augstāks determinācijas koeficients (97% vs 88%), pat neizmetot savstarpēji korelētos parametrus.