ACCQ206 exercises – Week 5

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March 28, 2022

1. Draw the circuit that implements the encoding for Shor's 9 qubit code.
2. Show how to do the error detection/correction step in the Shor 9 qubit code when the first qubit undergoes both a phase-flip error and a bit-flip error.
3. Design a quantum circuit that implements the error correction step for the 3 qubit bit-flip code but does not use measurements. (Hint: you can create gates with multiple target qubits – e.g., CNOTS with more than one target)
4. Let $\{A_k\}_k$ be linear operators $A_k \in L(H_A, H_B)$ show that $\mathcal{E} : L(H_A) \to L(H_B)$ defined as

$$\mathcal{E}(\rho) = \sum_k A_k \rho A_k^\dagger$$

   is a quantum channel.
5. Prove that the trace map is a valid quantum channel.
6. Let A and B be two quantum systems. Define the partial transpose on system A as the linear extension of the map

$$|i\rangle\langle j| \otimes |k\rangle\langle l| \mapsto |j\rangle\langle i| \otimes |k\rangle\langle l|$$

   where $\{|i\rangle\}_i$ and $\{|k\rangle\}_k$ are orthonormal bases for systems A and B respectively.
   (a) Show that any matrix M mapping from AB to AB can be written as $M = \sum_{ijkl} m_{ijkl} |i\rangle\langle j| \otimes |k\rangle\langle l|$ for some $m_{ijkl} \in \mathbb{C}$.
   (b) Write down the action of the partial transpose on an arbitrary matrix M.
   (c) Prove that the partial transpose is **not** a quantum channel. (Hint: consider its action on the maximally entangled two-qubit state $(|00\rangle + |11\rangle)/\sqrt{2}$.)