Optimal Taxation and Enforcement with Market Power

Peter Choi

October 15, 2024

Click here for latest version

Abstract

This paper explores the effect of market power on optimal tax and enforcement rates. Under a profit tax, these rates are directly tied to the relationship between avoidance capability and firm size. Under an output tax, neither the optimal tax nor enforcement rate is necessarily monotonic with respect to competition. Higher effective taxation in imperfectly competitive markets compounds pre-existing distortions, providing a downward force on policy rates, but the behavior of reported revenue has an ambiguous effect that could amplify or outweigh this first effect. Key to the tradeoff between taxation and enforcement is the degree of complementarity between size and avoidance costs for the firm. The more that firm size facilitates avoidance, the more negative the relationship between competition and avoidance rates. This incentivizes a within industry policy that more heavily favors tax rates over enforcement at higher degrees of competition.

1 Introduction

The idea that large, powerful corporations do not remit their fair share of taxes has gained considerable public traction in recent decades, both aided by and reflective of news reports of several of these top firms remitting little to no federal taxes. For example, in 2020 it was prominently reported that dozens of Fortune 500 companies remitted no federal income taxes the past year, with a subset of those remitting nothing in the previous two years as well.[1] A 2023 Government Accountability Office (GAO) study reported that about a quarter of what they considered profitable large corporations remitted on average zero taxes between 2014-2018, with average effective tax rates hovering around 15% until dropping to single digits due to the TCJA.[2]

What "fair" means is of course a subjective standpoint, further muddied by the distinction between legal avoidance and illegal evasion. Moreover, the majority, if not all, of these large firms remitting little to no taxes in a given year are reporting losses or carrying forward or back losses.[3] Yet, there is still a sense in many that total collections at the top end of the distribution should be higher. In a yearly Gallup poll taken between 2004 and 2019, the fraction of surveyed Americans who responded that corporations "pay too little" in taxes consistently hovered around two-thirds, while the fraction believing corporations provide their "fair share" remained at one fifth.[4]

This notion has potentially not been exclusive to popular discourse. Auditing rates have consistently been correlated with firm size, with top firms having rates several factors higher than those of the smallest firms. As illustrated in Table 1, while there has been an overall decrease in enforcement rates across the board, the relative relationship between size and auditing probability has largely persisted. Though, this could simply be an illustration of an administrative cost advantage of auditing one very large firm being versus auditing many smaller firms with an equal aggregate value, regardless of beliefs on underlying avoidance behavior.

Table 1: Auditing Rates by Firm Size (%)

Firm Size (Balance Sheet Assets)	2011	2013	2015	2017
\$1–\$999,999	1.00	0.81	0.55	0.32
\$1,000,000–\$99,999,999	2.48	2.03	1.94	1.45
\$100,000,000–\$999,999,999	20.50	17.31	16.15	9.41
$>$ \$1,000,000,000	48.13	37.95	32.57	24.11

Data Source: IRS Data Book 2021, Table 17. Categories used by IRS redefined into the above four broader groups. More recent years are still potentially subject to change due to the 3-year rolling auditing process and so are not included.

More recently, there have been measures with more explicit targeting of top firms remitting more taxes. Two of the provisions of the landmark Inflation Reduction Act of 2022 in the US were to increase funding for the IRS by \$80 billion over the next decade, with \$45 billion in particular earmarked for increasing and improving enforcement activities, and the introduction of a 15 percent Corporate Alternative Minimum Tax (CAMT) on corporations averaging over one billion in income over a three year span. Among the plans for improvement was a stronger focus on tax system equity

[1] https://www.forbes.com/sites/tommybeer/2021/04/02/more-than-50-major-us-corporations-including-nike-and-fedex-paid-no-federal-taxes-last-year/?sh=3a7b338521d3

[2] https://www.gao.gov/products/gao-23-105384

[3] While carryback was disallowed by the TCJA 2017, the CARES Act in 2020 allowed firms to carryback losses 5 years for tax years beginning in 2018 to 2020.

[4] https://news.gallup.com/poll/1714/taxes.aspx

through enforcement on high income individuals and large firms who appear to remit considerably less taxes than expected.[5] This increase in funding was not a unanimously agreed upon decision, illustrated by an attempt from House Republicans to block the funding in January 2023, and later by a $20 billion reduction in the funding as a part of the Fiscal Responsibility Act of 2023. From these conflicting decisions in a short time span, it is clear that there is little consensus on how enforcement should be handled and how it relates to the rest of the tax system. Should the government prioritize raising statutory rates or try to reduce evasion or avoidance at the current rates? Should these policy prescriptions differ across firms or industries based on size and/or market power?

This paper makes two primary contributions to answering these questions. First, it develops a framework, that nests alternative cases of a profit tax and an output tax, to illustrate the various forces that tax evasion/avoidance and market power affect in determining how the government should optimally set statutory tax rates and enforcement rates. In doing so, I discuss how competition and avoidance affect behavioral responses margins, profit retention, and pass-through onto consumer prices of both taxes and enforcement. Under a profit tax, the elasticity of taxable income is equivalent to the elasticity of avoidance, meaning the only determinant of the relationship between the profit tax rate and market power is the relationship between avoidance responses and market power. The same is true for the optimal enforcement rate and the elasticities with respect to enforcement, with some minor additional considerations. Under an output tax there is an excess burden factor (downward on both policy rates) that is highest for a monopoly but dissipates as competition increases. Either tool increases the effective tax rate which amplifies the pre-existing distortion related to market power. The output tax also affects the true size of the industry rather than just avoidance rates. Based on demand curvature and the region of production along the demand curve, the industry size may grow or shrink in response to an effective tax increase. The total taxable income response, a combination of both the avoidance and true revenue response, is not strictly monotonic with respect to competition, and it is entirely possible to find cases where optimal rates and competition are positively or negatively related.

Second, this paper analyzes the tradeoff between the government's two tax tools: statutory tax rates and enforcement rates. In some ways, these two tools are similar as both affect the effective tax rate faced by a firm. However, both generally incur differential responses via output and prices, avoidance, and profits. An increase in the tax rate will increase mechanical revenue but make avoidance directly more attractive as the marginal benefit (additional avoided taxes) increases. On the other hand, the direct effect of an increase in enforcement is to make avoidance less attractive as the marginal costs rise for the firm, but this incurs real resource costs to the government. We can frame these results in a manner similar to those of Keen and Slemrod (2017). Either a higher elasticity of taxable income with respect to taxation or enforcement provides a higher incentive to use the enforcement tool. Similarly, higher market distortion terms for either drives the incentive in favor of the other.

Much of these differences are affected by the complementarity between avoidance and size in the firms' cost functions. Consider a cost structure where being larger facilitates avoidance. This occurs if firm size scales marginal avoidance benefits at a faster rate than the marginal costs. Under mild assumptions, per firm size decreases as the number of firms in the market increases. Thus, increased competition drives down avoidance rates, providing a incentive for more reliance on tax rates over enforcement rates as the size of the tax base grows relative to the "enforcement base." If instead marginal benefits and costs proportionally scale with output, then competition has no

[5] See introductory quote, from U.S. Secretary of the Treasury Janet Yellen's letter to the IRS: https://home.treasury.gov/system/files/136/JLY-letter-to-Commissioner-Rettig-Signed.pdf

relative base effect, and if benefits scale more quickly there is an inverse effect.

As a starting point, I consider a fully differentiated tax system setup, similar to the Ramsey (1927) commodity tax model, that features both avoidance and noncompetitive industries. The federal government seeks to maximize consumer utility using a set of industry specific ad valorem taxes (rates) and enforcement rates. The nature of the avoidance problem follows such papers as Kopczuk et al. (2016) and Keen and Slemrod (2017) rather than the more prototypical Allingham and Sandmo (1972) approach. This means that instead of featuring a probability of detection, costs of avoidance are instead encompassed in reduced form into an avoidance portion of the cost function. Government spending on enforcement then serves to increase these costs for the firm.

While building off the Ramsey model is useful for developing intuition and to have a standard comparative baseline, a fully differentiated tax system has a weak link to the real world motivation described previously. Generally the discussion on avoidance and fairness in regards to these firms in centered around, for example, the corporate income tax rate, which is closer to a uniform profit tax. Thus, after completing the analysis of a fully differentiated system, this paper moves to an analysis on a uniform tax system, though enforcement may still be targeted. While targeting suffers to some degree due to imperfect instruments, the general analysis carries through.

This paper connects to several strands of the literature. First it extends the optimal taxation literature that integrates agent evasion decisions and/or tax agency administrative decisions (Kaplow (1990), Cremer and Gahvari (1993), Dharmapala et all. (2011), Keen and Slemrod (2017)). A particularly relevant point from Kaplow (1990) mirrors the previous discussion: despite similarities in the two types of tax tools, differences in behavioral distortions and resource costs can create a need for an optimal allocation. I primarily bridge these studies with the those that introduce market power into the optimal tax problem (Besley (1989), Myles (1989), Kaplow (2021), Eckhout et al. (2021)). This area raises the relevance of considering the impact of firm entry/exit, tax rate pass-through, profit retention, and pre-existing output distortions in response to taxation. Pass-through in particular has received recent attention in how it relates to market power (Anderson et al. (2001), Weyl and Fabinger (2013), Pless and Van Benthem (2019), Miklós-Thal and Shaffer (2021), Adashi and Fabinger (2022), Ritz (2024)) and evasion (Kopczuk et al. (2016)). Finally, this paper relates to the broad literature on the role of firms in the tax system (Kopczuk and Slemrod (2006), de Paula and Scheinkman (2010), Slemrod and Velayudhan (2018)). In much of the optimal tax literature, the direct role of firms is largely abstracted away from. However, in reality, firms remit the vast majority of taxes, and therefore being more direct in how their avoidance and enforcement may evolve is integral to understanding the tax system.

The empirical evidence for the connections between market power and avoidance rates is small, but has developed recent attention in the accounting literature. Kubick et al. (2015) find a negative association between product market power and effective tax rates, their proxy for tax avoidance, hypothesizing the the persistent profitability that market power allows firms the ability to better predict future income streams and thus increases the value of tax avoidance strategies. Martin et al. (2022) examine the reverse direction and illustrate that firms that engage in tax avoidance have higher sales, aiding in higher concentration ratios. While these may not be enough to conclusively determine the true relationship between firm size and avoidance, they provide suggestive evidence.

The rest of this paper is structured as follows. Section 2 provides the details of the model for each type of agent: consumer, firm, and government. Section 3 focuses on how both taxation and enforcement affect the firms in the markets, in their profits, and in how their output and avoidance decisions affect reported taxable incomes. Section 4 introduces and discusses optimal policy expressions, both for the levels and in the tradeoff between the two policy tools. Section 5 illustrates features of the model and examine these interactions through a simulated environment. Section 6 relaxes the fully differentiated tax rates to a uniform system and examines how the intuition from the Ramsey setup carries over. Section 7 covers a few extensions and alternations of the model. Section 8 discusses the main results of the paper and concludes. The majority of the mathematical derivations are relegated to the Appendix.

2 Model Setup

There is a set of K industries with potentially varying degrees of competition in the economy. Though a perfectly competitive or monopoly industry may not exist in this set, I will sometimes discuss the results of these two extreme cases to help build intuition. Each of these industries produces a unique good k such that the terms good and industry interchangeably when referring to the differentiated tax or enforcement rates (e.g., a "tax on good k" or a "tax on industry k"). For goods in each industry, consumers face a tax (rate) inclusive price q_k while producers receive the net of tax price of $p_k = q_k(1 - t_k)$.

2.1 Consumer Problem

In the baseline model, a representative consumer chooses an optimal consumption bundle x of the K goods in addition to leisure (or, equivalently, choosing labor L). Labor is treated as the numeraire good such that the wage is set to 1. The consumer receives all profits Π_k from each industry. Thus, the consumer solves

$$\max_{\vec{x},L} u(\vec{x}, L)$$
$$\text{s.t. } I + L + \sum_k \Pi_k \geq \vec{q} \cdot \vec{x}$$

While I denotes additional nonlabor income, this will typically be set to 0 and its purpose here is to define the marginal utility of income, $\frac{\partial v}{\partial I} = \alpha$, where v is the indirect utility. I assume that the individual does not consider their own impact on either prices or on profits. I will also assume away income effects in the demand for each good via quasilinear utility and cross-price effects across markets to simplify exposition of the key results. Finally, if not explicitly stated, I will generally assume log-concave demand forms.

2.2 Firm Problem

A firm in industry k chooses output y_k and an avoidance rate γ_k, while subject to a tax rate t_k and an enforcement rate β_k, to maximize their profits. While I will generally refer to avoidance, I make no distinction here between illegal evasion and legal avoidance, and γ_k serves to capture both of these types of activities. One major limitation with this assumption is that marginal costs for these types of activities may not ever align in practice. Thus, using a single variable to capture both activities ignores a potentially important decision margin in how the firm decides to reduce tax remittance and in how the government decides to limit these activities.

Firms have producer price $p_k(x_k) = q_k(x_k)(1-t_k)$ and are allowed to deduct a fraction $\mu \in [0, 1]$ of their direct production costs $\phi_k(y_k)$. We define firm size to be pre-avoidance taxable income: $r_k \equiv y_k q_k - \mu\phi_k(y_k)$. Avoidance costs $H_k(r_k, \gamma_k, \beta_k)$ are a function of firm size, the avoidance rate, and the enforcement rate. Total costs are simply a sum of direct production costs and avoidance costs. In other words, $C_k(y_k, r_k, \gamma_k, \beta_k) = \phi_k(y_k) + H_k(r_k, \gamma_k, \beta_k)$. Furthermore, I will frequently assume that H_k is multiplicatively separable in each of its arguments. From this point on, the k subscript in the cost function will be dropped in favor of partial derivative representation, e.g.,

$H_\gamma \equiv \frac{\partial H(r_k,\gamma_k,\beta_k)}{\partial \gamma_k}$, but it should be remembered that C may functionally differ across industries. While I will later describe how the features of the cost function determine certain results, in order to obtain feasible and interior solutions for output and avoidance, I generally assume that ϕ and H are continuous and twice differentiable in all arguments and $\phi_y > 0$ for $y \geq 0$, $H_\gamma > 0$ for $\gamma \geq 0$, and $H_{\gamma\gamma} > 0$. As increasing enforcement should naturally increase the costs of avoidance, I also assume $H_\beta > 0$ for $\gamma > 0$ and $H_{\gamma\beta} > 0$. While not explicitly needed for all aspects of the problem, an assumption of constant marginal production costs (or not too decreasing) may be imposed to leave aside discussions of situations where a less concentrated industry may be more desirable from a production standpoint. Formally, the firm solves

$$\max_{y_k,\gamma_k} y_k q_k(x_k) - C_k(y_k, r_k, \gamma_k, \beta_k) - t_k((1-\gamma_k)(y_k q_k - \mu\phi_k(y_k)))$$

When $\mu = 0$, no costs are deductible meaning the firm is facing a pure output tax, while if $\mu = 1$, all production costs are deductibile and the firm is facing a pure profit tax. This affects the relevant tax base for the firm–they hide revenues under an output tax and production profits under a profit tax. The relevant tax base is considered to be what enters as size into the firm's cost function with the implicit assumption that the exact functional form for avoidance costs may differ whether the firm is hiding revenues or profits.

Another key note here is that in this version of the model avoidance costs are not included as deductible costs for the firm or as part of the portion of income that the firm avoids. Most avoidance costs in practice are likely to be cost deductible to the firm (e.g., accountants, lawyers). However, including deductibile avoidance costs does little to change the results while significantly complicating the exposition of the expressions. Thus, the version of the model with deductible avoidance costs version is left to the appendix.

Imperfectly competitive markets have a fixed number N_k of Cournot competitors. Unless otherwise noted, these competitors are assumed to be identical. This Cournot framework has a few implications. First, it implies that within a market, number of firms, firm size, and market power are isomorphic. Between markets, however, this is not true. It possible for equivalently sized firms in two industries to face very different market structures. Thus, we must be careful when making "within market" statements versus "between market" statements.

The firm's problem leads to the following production condition:

$$(1-(1-\gamma_k)t_k - H_r)q_k\left[1 - \frac{1}{N_k\varepsilon_{x_k}}\right] = (1-\mu[(1-\gamma_k)t_k + H_r])\phi_y$$

where $\varepsilon_{x_k} \equiv \frac{\partial x_k}{\partial q_k}\frac{q_k}{x_k}$ is the elasticity of demand. Importantly, under a pure profit tax ($\mu = 1$) the solution for the optimal choice of output is independent of the tax rate and the avoidance rate. In other words, as is generally the case in models that do not feature avoidance, the production choice is undistorted by the tax and the equilibrium quantity is the same as if there were no tax.

Also to note is that this expression implies that

$$1 - \frac{1}{N_k\varepsilon_{x_k}} > 0$$

for nonzero marginal costs. One important takeaway from this formulation is that firms in the industry must be producing in the portion of the demand curve where $1 - \frac{1}{N_k\varepsilon_{x_k}} > 0$, or $\varepsilon_{x_k} > \frac{1}{N_k}$. A commonly used version of this result is that a monopoly must produce in the elastic part of the demand curve, i.e., where $\varepsilon_{x_k} > 1$. This conclusion, and the more general Cournot conclusion, still hold with avoidance. As N_k increases, this lower bound on the allowable portions of the demand curve for production falls.

Now we turn to the avoidance condition, which can be written as

$$t_k r_k = H_\gamma(r_k, \gamma_k, \beta_k)$$

which follows the same intuition of equalizing marginal revenues and costs. The marginal benefit of avoidance, the LHS of the FOC, increases as firm size increases in a linear manner. If the net marginal costs of avoidance also rise linearly with firm size, such that $H(r_k, \gamma_k, \beta_k) = r_k H(1, \gamma_k, \beta_k)$, then the avoidance rate is independent of firm size and correspondingly the concentration of firms. If instead $H(\cdot)$ is convex in firm size, then the optimal avoidance rate is inversely related to firm size. Since per-firm income of an industry decreases as the number of firm increases, meaning average firm size decreases, we can say that, for a given tax-enforcement combination, the avoidance rate increases as competition increases. The intuition is that if avoidance costs are convex in size, then growing larger hinders a firm's ability to avoid as costs outpace benefits. The reverse is true if the cost of avoidance is concave in size.

The degree of convexity matters here. As convexity (concavity) is amplified, so are the directionalities of the above statements. In order to get at this idea, we can define an elasticity of the marginal costs of avoidance with respect to firm size

$$\varepsilon^r_{H_\gamma} = H_{\gamma r} \frac{r_k}{H_\gamma}$$

An elasticity of 1 represents the situation of marginal avoidance costs that scale linearly with size, a special subcase which we will refer to as constant returns to scale avoidance technology. An elasticity greater than 1 represents the case where marginal avoidance costs are convex in size, decreasing returns to scale avoidance technology, and less than 1 represents the case where costs are concave in size, increasing returns to scale avoidance technology. We summarize the importance of this relationship in the following Lemma.

Lemma 1. *As the elasticity of marginal avoidance costs with respect to firm size, $\varepsilon^r_{H_\gamma}$, increases, the more positive is the within industry relationship between avoidance rate and competition. For values of the elasticity less than 1, the relationship is negative. For values of the elasticity greater than 1, the relationship is positive. At an elasticity of 1, there is no relationship between the number of firms and the avoidance rate.*

It should be noted that term "constant" is on a rate basis. If the rate of avoidance is the same for two differently sized firms, then the larger firm is clearly avoiding more on a level basis. Even under decreasing returns to scale technology, a larger firm could be avoiding more on a level basis.

Importantly, this elasticity does not directly imply anything about the relationship between firm size and avoidance rates across markets unless we assume that the same avoidance technology is available to all firms regardless of industry. This would be true if there is neglible industry-specific benefits or access (e.g., specialized accountants in one industry that have more expertise than those in another industry). This assumption is provided as follows and will be applied where needed.

Assumption 1. *Avoidance technology is fully general to all industries, and no industry has an advantage in its use. Therefore, avoidance related costs are functionally the same for all firms regardless of their industry.*

If this assumption holds, we can directly link between firm size and avoidance in the theory regardless of market. As stated earlier, however, we cannot link avoidance and market power unless we focus on a within industry style comparative static. A middle ground would be to do a between industry comparison, but assume that that the two markets are fundamentally identical except one has a higher number of firms.

2.3 Government

The government seeks to maximize the utility of the representative agent subject to the use of two sets of policy instruments, industry specific tax rates t and industry specific enforcement efforts β in order to satisfy the following budget constraint:

$$\sum_j t_j(1-\gamma_j)R_j - a(R_j, \beta_j) \geq G$$

where R_j is aggregate industry size (pre-avoidance taxable production profits), $a(\cdot)$ is the administrative cost function, and G is an exogenous revenue requirement that will ultimately determine the value of the government budget multiplier λ.[6] Administrative costs are a function of both the effort put in by the government into the industry, β_j, as well as the size of the industry. While sometimes left separated out into real production income and avoidance components, define total taxable income of an industry $Z_j = (1-\gamma_j)R_j$. The Lagrangian of the full problem is then

$$\mathcal{L} = v(q) + \lambda \left[\sum_j t_j Z_j - a(R_j, \beta_j) - G \right]$$

Throughout much of the paper, I assume a simple linear form for administrative costs $a(R_j, \beta_j) = R_j\beta_j$. In this form, β_j can directly be interpreted as a rate similar to the tax rate t_j, and the cost per unit of enforced income is normalized to one. Built into this assumption is the ability to identify $t_j(1-\gamma_j) - \beta_j$ as the mechanical net rate of revenue raised by the government for a given policy combination.

I begin with a model of differentiated tax rates to derive intuition. However, such a system is not reflective of the corporate tax rates commonly observed in the world, which drives the motivation of the paper. Thus, in section 6, I consider an alternative model where the tax is kept uniform across all industries, though enforcement rates can still be differentiated in line with the assumption that enforcement authorities such as the IRS have relatively more freedom in how they choose to target their resources.

3 Market Responses to Taxation and Enforcement

Important to the discussion of the optimal tax and enforcement rates are how firms and therefore the markets respond to these tools. As is typical in optimal tax problems, the two primary components to balance are the elasticity of the base of the tax tool and the welfare impacts on consumers. The welfare impact of a policy change is composed of the direct price effects, as behavioral impacts of bundle re-optimization are zeroed out by the envelope condition and income and cross-price effects are assumed zero, and profit responses, since consumers receive all profits from each industry:

$$\frac{dv}{d\theta_k} = -\frac{\partial q_k}{\partial \theta_k} x_k + \frac{\partial \Pi_k}{\partial \theta_k}$$

[6] A note on the value of λ when performing "within market" comparisons. In the hypothetical comparison we often perform, which is to alter the number of firms (and therefore market power) within an industry, we are implicitly also structurally changing the overall economy as the equilibrium aggregates absent taxation are altered. In this sense, λ is endogenous to the degree of market power we attribute to a given industry. To bypass this issue, we can either (1) assume that that the relative size of the rest of the economy is large enough such that structural changes in one industry neglibly affect the value of λ or (2) instead of assuming an exact revenue requirement, we instead assume there is a public good with a fixed marginal value of $\lambda > \alpha$.

where v is the indirect utility of the consumer and $\theta_k \in \{t_k, \beta_k\}$ represents one of the tax tools. A relevant exercise that will help determine the tradeoff between the use of taxation and enforcement is an offsetting change in one tool versus the other. One such exercise is to offset the consumer price change (or equivalently the output change) by adjusting the second tool to just offset the output distortion. In this case, the net welfare effect is

$$\frac{dv}{dt_k} - \frac{dv}{d\beta_k}\frac{\frac{\partial q_k}{\partial t_k}}{\frac{\partial q_k}{\partial \beta_k}} = \frac{\partial \Pi_k}{\partial t_k} - \frac{\partial \Pi_k}{\partial \beta_k}\frac{\frac{\partial q_k}{\partial t_k}}{\frac{\partial q_k}{\partial \beta_k}}$$

On the fiscal side, the base of the tax rates are the reported taxable income in each industry Z_k. Define the elasticities of reported income with respect to each of the tax rates and enforcement rates respectively as

$$\varepsilon_{Z_k} = \frac{\partial Z_k}{\partial(1-t_k)}\frac{1-t_k}{Z_k}, \qquad \varepsilon^{\beta}_{Z_k} = \frac{\partial Z_k}{\partial \beta_k}\frac{\beta_k}{Z_k}$$

These will be the two relevant elasticities in determining an inverse elasticity type rule, similar to the demand elasticity in the standard Ramsey commodity tax framework and the elasticity of taxable income in an indvidual income tax framework. Before the full discussion of how these elasticities and consumer welfare are affected by the presence of avoidance and market power, we will first discuss in detail the individual components: price responses, avoidance responses, and profit responses.

3.1 Price Response

The pass-through rates of each tax tool onto consumer prices are important for both welfare and reported income behavior. In a standard Ramsey framework, free entry and constant returns production technology in perfectly competitive markets ensure that the entire burden of the tax is shifted to the consumer, i.e., $\frac{\partial q_k}{\partial t_k} = 1$ if t_k is an excise tax and $\frac{\partial q_k}{\partial t_k} = \frac{q_k}{1-t_k}$ if t_k is ad valorem, and thus the price change is always the same no matter where the tax is applied (on a level basis for excise taxes and percentage basis for ad valorem). The introduction of both avoidance and market power into the problem alters this idea.

With avoidance, firms do not bear the full burden of taxation and thus they also do not pass the full burden of taxation onto consumers either. Rather, they pass on the portion of the tax burden they do experience, ie., the effective tax.[7]

Likewise, the lack of free entry in imperfectly competitive markets and the (partial) control of pricing implies that pass-through does not necessarily have to be full or can even be overshifted. As discussed in depth in Weyl and Fabinger (2013), the degree of pass-through in imperfect markets hinges on the concavity of the demand function. Defining the negative of marginal consumer surplus $ms_k = -\frac{\partial q_k}{\partial x_k}x_k$, then log-concave demand functions imply $\frac{1}{\varepsilon_{ms_k}} > 0$, which ensures that pass-through is below the competitive level for a monopoly and rises as competition increases so long as costs are not too convex (Ritz, 2024).

Combining these two considerations, we define and derive the tax pass-through rate as

$$\rho_k \equiv \frac{1}{q_k}\frac{\partial q_k}{\partial t_k} = \frac{\left(1-\gamma_k - t_k\frac{\partial \gamma_k}{\partial t_k}[1-\varepsilon^{r}_{H_\gamma}]\right)\left[1-\frac{1}{N_k\varepsilon_{x_k}}\right]}{(1-t_k+\gamma_k t_k)\left(1+\frac{1}{N_k\varepsilon_{ms_k}}+\frac{\varepsilon_{x_k}-\frac{1}{N_k}}{\varepsilon_{s_k}}\right)}$$

[7]In a problem with avoidance costs as specified in our model, the "effective tax" faced by the firm is not the same as the "effective tax" from the point of view of the government. The prior includes includes additional changes in net margins due to altering firm size.

and the enforcement pass-through rate as

$$\rho_k^\beta \equiv \frac{1}{q_k}\frac{\partial q_k}{\partial \beta_k} = \frac{(H_{r\beta} - t_k \frac{\partial \gamma_k}{\partial \beta_k}[1-\varepsilon^r_{H_\gamma}])\left[1 - \frac{1}{N_k \varepsilon_{x_k}}\right]}{(1 - t_k + \gamma_k t_k)\left(1 + \frac{1}{N_k \varepsilon_{ms_k}} + \frac{\varepsilon_{x_k} - \frac{1}{N_k}}{\varepsilon_{s_k}}\right)}$$

where ε_{x_k} is the elasticity of demand and ε_{s_k} is the elasticity of the inverse marginal cost ("pseudo-supply") curve. The derivation of these pass-through rates follows closely to Weyl and Fabinger (2013) and Adashi and Fabinger (2022), with modifications associated with the inclusion of avoidance.

One key term that appears in both expressions is the effect of endogenous change in the avoidance rate: $-\frac{\partial \gamma_k}{\partial \theta_k}[1-\varepsilon^r_{H_\gamma}]$. The term in the brackets respresents how the net margins of avoidance are related to firm size. If this is positive, additional avoidance by the firm will increase the net margins of the firm. The industry passes this net benefit onto the consumer through a lower pass-through rate. Since $\frac{\partial \gamma_k}{\partial t_k} > 0$, this is exactly what happens with the tax rate. For the enforcement rate, $\frac{\partial \gamma_k}{\partial \beta_k} < 0$, and so the firm will incur net costs from changing avoidance and pass on those costs to the individual instead. All effects are reversed if $\varepsilon^r_{H_\gamma} > 1$, and firm size is negatively related to the net margins of avoidance.

A final difference in these expressions with the avoidance-less versions is in the pseudo-supply elasticity. The marginal costs defined in this term are inclusive of how additional production affects avoidance costs. Therefore, even if there are constant marginal production costs, which would typically imply $\varepsilon_{s_k} = \infty$ and this overall term would vanish, this term would remain.

For constant scale avoidance, this endogenous avoidance effect drops out. Importantly, the qualitative relationship between these expressions and market power are the same as if there were no avoidance. One useful implication of this is that for log-concave demand functions, the pass-through rate for both tools is unambiuously increasing as competition increases. We repeat this in the following lemma.

Lemma 2. *Under constant scale avoidance technology, if a market faces a log-concave demand function and the firm costs are not too convex, the pass-through rate increases as the number of firms, and therefore competition, increases.*

The degree to which enforcement must be changed in order to net out the price and output effect of a change in the tax rate is given by the ratio of the two pass-through expressions

$$\frac{\rho_k}{\rho_k^\beta} = \frac{1-\gamma_k - t_k \frac{\partial \gamma_k}{\partial t_k}[1-\varepsilon^r_{H_\gamma}]}{H_{r\beta} - t_k \frac{\partial \gamma_k}{\partial \beta_k}[1-\varepsilon^r_{H_\gamma}]} = \frac{1-\gamma_k - t_k \frac{r_k}{H_{\gamma\gamma}}[1-\varepsilon^r_{H_\gamma}]}{H_{r\beta} + t_k \frac{H_{\beta\gamma}}{H_{\gamma\gamma}}[1-\varepsilon^r_{H_\gamma}]}$$

As before, under constant scale avoidance, the net margin term drops out of the numerator and denominator. We note that the above is a measure of the relative impacts of the two policy tools on effective tax rates (to the firm). From the firm's point of view, the effective tax it faces is

$$\tau(t,\beta) = (1-\gamma)t + H_r$$

Thus, we have that

$$\frac{\partial \tau}{\partial t} = (1-\gamma) - \frac{\partial \gamma}{\partial t}(t - H_{r\gamma}), \quad \frac{\partial \tau}{\partial \beta} = H_{r\beta} - \frac{\partial \gamma}{\partial \beta}(t - H_{r\gamma})$$

where $t - H_{r\gamma} = t(1-\varepsilon^r_{H_\gamma})$ gets us the result. Then changing the enforcement rate by this ratio of pass-throughs can be thought of as an effective tax neutral policy change, i.e., we consider an alternate tax-enforcement combination that maintains the same effective tax rate (from the firm's perspective).

3.2 Avoidance Response

We have discussed in Section 2 the implications of the elasticity of marginal avoidance costs on the levels of avoidance. Here we discuss the implications on the behavioral response of avoidance, which is an important component of the reported taxable income response. The change in the avoidance rate in response to an increase in the tax rate is

$$\frac{1}{1-\gamma_k}\frac{\partial \gamma_k}{\partial t_k} = \underbrace{\frac{r_k}{(1-\gamma_k)H_{\gamma\gamma}}}_{\textit{Direct}} + \underbrace{\frac{y_k q_k}{(1-\gamma_k)H_{\gamma\gamma}}(1-\varepsilon^r_{H_\gamma})t_k\rho_k\left[1-\left(1-\mu\frac{\phi_y}{q_k}\right)\varepsilon_{x_k}\right]}_{\text{Endogenous Size Response}}$$

which breaks down the response into a direct effect and an indirect size related effect. The direct effect is simple. The tax increase makes avoidance more attractive as the marginal benefit of avoidance increases. However, the change in the tax rate also leads to a change in optimal firm size through the production decision. This change in firm size is given by the last term in the brackets (multiplied by the pass-through rate).

The term $1-\varepsilon^r_{H_\gamma}$, as before, determines whether this change in firm size is beneficial or not. Again, this term represents the change in the net margins of avoidance as firm size increases. Therefore, if $1-\varepsilon^r_{H_\gamma} > 0$, this implies that increasing firm size increases the net margin of avoidance, making further avoidance more attractive.

Combining these terms gets us the sign of the overall term. We see that if $\varepsilon_{H^r_\gamma} < 1$, then a demand elasticity greater than 1 (under an output tax such that $\mu = 0$) indicates a negative overall response while an elasticity lower than 1 indicates a positive response. These directions are inverted if $\varepsilon^r_{H_\gamma} > 1$. Thus, the endogenous size response may either amplify or mute the direct response depending on where along the demand curve the firm produces in.

Lastly, under constant scale avoidance, we have that $1-\varepsilon^r_{H_\gamma} = 0$, indicating no endogenous change in the avoidance margin, leaving only the direct effect. Since firm size is unrelated to the avoidance decision, the endogenous change in firm size has no further impact on attractiblity of avoidance. This assumption also implies that the direct effect is unrelated to firm size. This is because

$$\frac{r_k}{(1-\gamma_k)H_{\gamma\gamma}(r_k,\gamma_k,\beta_k)} = \frac{1}{(1-\gamma_k)H_{\gamma\gamma}(1,\gamma_k,\beta_k)}$$

under constant scale avoidance. Since γ_k does not change with firm size, the direct effect is independent of firm size and market power. Under increasing scale avoidance technology, however, the direct effect would increase with market power. Since the marginal benefit of avoidance scales faster with size than do marginal costs, larger firms, conditional on current firm size, will want to engage in relatively more avoidance.

The corresponding expression for the change in the enforcement rate is given by

$$\frac{1}{1-\gamma_k}\frac{\partial \gamma_k}{\partial \beta_k} = \underbrace{-\frac{H_{\gamma\beta}}{(1-\gamma_k)H_{\gamma\gamma}}}_{\textit{Direct}} + \underbrace{\frac{y_k q_k}{(1-\gamma_k)H_{\gamma\gamma}}(1-\varepsilon^r_{H_\gamma})t_k\rho^\beta_k\left[1-\left(1-\mu\frac{\phi_y}{q_k}\right)\varepsilon_{x_k}\right]}_{\text{Endogenous Size Response}}$$

A similar discussion to the tax counterpart can be had here. The key difference is that the direct effect is to incentivize lower avoidance as enforcement directly increases the marginal cost of avoidance. Thus, while the endogenous size response may amplify or mute the direct effect here as well, it works in the opposite direction as in the tax case.

Similarly, the relationship between the direct effect of an enforcement change and market power is generally reverse to that of a tax change (both increase in magnitude, but in opposite signs).

There is no direct size related component here, but the baseline level of avoidance matters. Under increasing returns avoidance technology, larger firms engage in more avoidance for a given enforcement and tax rate. Increasing the enforcement rate directly impacts the marginal avoidance costs of the firm. Since avoidance costs are strictly convex in the avoidance rate, this implies a larger impact at higher levels of avoidance.

We summarize the relationships between tax and enforcement with market power in the following lemma.

Lemma 3. *Under constant returns to scale avoidance technology, there is no relationship between market power and the avoidance response for a given industry. Under increasing (decreasing) returns to scale avoidance technology, there is a positive (negative) relationship between market power and the direct effect of taxes on avoidance and a negative (positive) relationship between market power and the direct effect of enforcement on avoidance. Under a profit tax, $\mu = 1$, this direct effect represents the entire avoidance response.*

The net effect of these two tools in our constant effective tax rate exercise is

$$\frac{\partial \gamma_k}{\partial t_k} - \frac{\partial \gamma_k}{\partial \beta_k}\frac{\rho_k}{\rho_k^\beta} = \frac{r_k}{H_{\gamma\gamma}} + \frac{H_{\gamma\beta}}{H_{\gamma\gamma}}\frac{\rho_k}{\rho_k^\beta}$$

3.3 Profit Response

Our final building block component to discuss are the profit responses to the tax tools. As all firms are homogeneous, we simply add up the individual firm profit responses to obtain the aggregate profit response to a tax change as

$$\frac{\partial \Pi_k}{\partial t_k} = \underbrace{-Z_k}_{Mechanical} + \underbrace{(1-(1-\gamma_k)t_k - H_r)\frac{N_k - 1}{N_k}\rho_k x_k q_k}_{Competition}$$

The first term represents the mechanical change in profits for the industry, which is simply the aggregate reported income. Under a monopoly ($N_k = 1$), any behavioral impact is zeroed out by the envelope condition. However, with $N_k > 1$ competitors, as each firm in the market has some control over prices, a joint behavioral impact still arises, which we label the competition effect. As the number of firms rises, this term approaches the (negative) of the mechanical effect and the overall change tends toward zero.

Similarly, the impact of a change in the enforcement rate

$$\frac{\partial \Pi_k}{\partial \beta_k} = -N_k H_\beta + (1-(1-\gamma_k)t_k - H_r)\frac{N_k - 1}{N_k}\rho_k^\beta x_k q_k$$

where we see the same separation into the direct effect of the increase in enforcement rates on profits and the competitive adjustment due to the partial price control each firm has. The direct effect here is the marginal increase in avoidance costs for the firms with respect to enforcement, sometimes referred to as the marginal compliance cost.

Lastly, we again continue our price neutral comparison and calculate the net profit change as

$$\frac{\partial \Pi_k}{\partial t_k} - \frac{\partial \Pi_k}{\partial \beta_k}\frac{\rho_k}{\rho_k^\beta} = -Z_k + N_k H_\beta \frac{\rho_k}{\rho_k^\beta}$$

As this exercise nets out price changes, the net profit effect expression no longer contains the competitive price effect that appeared in each of the two expressions separately. Thus, the net profit effect is the sum of the direct effects, where the second is adjusted by the degree enforcement must be altered to net out the tax effect.

3.4 Welfare Effect

We now establish the first consideration of optimal problem in the welfare effect, which we have discussed as the sum of the effect on consumers and profits (which feeds back to the consumer since they obtain all profits). Using our profit response expression, we can write the welfare effect in response to a tax change as

$$\frac{1}{\alpha}\frac{dv}{dt_k} = -Z_K - \rho_k x_k q_k \left[1 - (1 - (1-\gamma_k)t_k - H_r)\frac{N_k - 1}{N_k}\right]$$

Breaking down this expression, we see that the first term is simply the reported revenue in the industry, which is exactly equals the mechanical gain in tax revenue. Thus, the second term represents the additional welfare effect beyond the mechanical transfer in tax revenue from firms/consumers to the government.

If we divide through by the reported income, this expression becomes the welfare effect per mechanical dollar raised:

$$\frac{1}{\alpha Z_k}\frac{dv}{dt_k} = -1 - \rho_k \frac{x_k q_k}{Z_k}\left[1 - (1 - (1-\gamma_k)t_k - H_r)\frac{N_k - 1}{N_k}\right]$$

We can thus describe this second term as the marginal excess burden of taxation per dollar raised. We label this as EB_k,

$$EB_k = \rho_k \frac{x_k q_k}{Z_k}\left[1 - (1 - (1-\gamma_k)t_k - H_r)\frac{N_k - 1}{N_k}\right]$$

One thing to note is that the excess burden term does not trend exactly to zero even in perfect competition. The importance of the term, however, is that it grows as market power increases and therefore is indicative of the increasing distortion in the market.

Similarly, combining our results to obtain the welfare effect per reported revenue on the enforcement side, we have

$$\frac{1}{\alpha Z_k}\frac{dv}{d\beta_k} = -\frac{N_k H_\beta}{Z_k} - \rho_k^\beta \frac{x_k q_k}{Z_k}\left[1 - (1 - (1-\gamma_k)t_k - H_r)\frac{N_k - 1}{N_k}\right]$$

while this expression does not have as natural of a breakdown into an expected transfer of a dollar from the consumers or firms to the government and then excess cost, we can still apply a similar logic. The first term is the mechanical impact on profits and is thus our "standard" burden. As before, the second term in the expression shrinks as the number of firms grows, and as such we may still define this as a type of excess burden, which we will label EB_k^β.

Finally, we attempt to describe this excess burden effect in terms of potentially observable characteristics. Under the profit tax the excess burden is zero, so we focus on the output tax case here. First, using the production FOC, we have that

$$(1 - (1-\gamma_k)t_k - H_r) = \frac{\frac{\phi_y}{q_k}}{1 - \frac{1}{N_k \varepsilon_{x_k}}}$$

Second, as is typical in the market power related studies, we denote an elasticity-adjusted Lerner Index

$$\xi_k \equiv \frac{q_k - \frac{\phi_y}{1-(1-\gamma_k)t_k - H_r}}{q_k}\varepsilon_{x_k} = \frac{1}{N_k}$$

Thus, we can think of the number of firms being inversely related to the adjusted industry price-cost margins. Regardless of fundamentals, a firm in one industry with the same number of competitors as the firm in another industry will have the same Lerner index. This measure then is in some sense market neutral. We then can then redefine the excess burden as

$$EB_k^\theta = \rho_k^\theta \frac{\frac{\phi_y}{q_k}}{1 - \frac{1}{N_k \varepsilon_{x_k}}}(1 - \xi_k)$$

Within a market, the excess burden is increasing with market power, but we can use it also as a way to compare market power across industries. For example, consider two industries under which the firm sizes are equal. Under the same tax rate, and under Assumption 1 of the same avoidance technology across industries, the center term in the expression above should be equivalent as their avoidance rates and costs should be equivalent. If these two industries also have identical pass-through rates, we can say that the market with higher market power (lower number of firms) has the higher excess burden. All else equal, this will provide a stronger downward force on both the tax and enforecement levels, as we will see in Section 4.

The most difficult portion of this term to empirically observe are marginal costs of production. These are sometimes backed out in empirical industrial organization research. A crude alternative is to assume constant marginal costs and no fixed costs. In this case, marginal costs can be calculated as total costs, which are often reported, divided by total quantity.

The last term in the welfare expressions thus far not described in terms of potential observables is the marginal complicance cost to the firm H_β. There has been recent work in this field uncovering compliance costs, both directly related to administration and enforcement (Harju et al. (2019)) or related to other regulatory compliance (Trebbi et al. (2023)).

3.5 Reported Income

Finally, we tie everything together on the fiscal side and describe the behavior of the tax base, reported industry income $Z_k = (1 - \gamma_k)R_k$. The elasticity of reported revenue to the tax rate is

$$\frac{\varepsilon_{Z_k}}{1 - t_k} = \underbrace{\frac{\partial(1 - \gamma_k)}{\partial(1 - t_k)} \frac{1}{1 - \gamma_k}}_{\text{Avoidance Response}} - \underbrace{\frac{(1 - \gamma_k)x_k q_k}{Z_k} \rho_k \left[1 - \left(1 - \mu\frac{\phi_y}{q_k}\right)\varepsilon_{x_k}\right]}_{\text{True Taxable Income Response}}$$

which can be broken down into the avoidance response and the true taxable income (or industry size) response. We have previously detailed the avoidance response. In the avoidance response, there is a size response response that can be joined to the second response here but we leave these as separate for now.

On the other side, the reported revenue is also relevant for the enforcement parameter's behavioral response. Thus, we find the corresponding change in the reported revenue to a change in the enforcement rate as

$$\frac{\varepsilon_{Z_k}^{\beta_k}}{\beta_k} = \frac{\partial(1 - \gamma_k)}{\partial \beta_k} \frac{1}{1 - \gamma_k} - \frac{(1 - \gamma_k)x_k q_k}{Z_k} \rho_k^\beta \left[1 - \left(1 - \mu\frac{\phi_y}{q_k}\right)\varepsilon_{x_k}\right]$$

To simplify exposition, we will consider the two poles, profit taxation ($\mu = 1$) and output taxation ($\mu = 0$), in turn.

3.5.1 Profit Taxation

Under profit taxation, the true taxable income response is zeroed out since the production decision is unaffected by the effective tax rate, leaving only the avoidance response. Thus, the relationship between the reported income response and market power exactly follows the relationship between avoidance respose and market power, which we have previously described in Lemma 3. Under constant scale avoidance technology, there is no relationship, and under increasing scale technology, the size of the response increases as market power increases.

3.5.2 Output Taxation

Under an output tax, the true income response does not drop out, but we can simplify the overall response to

$$\frac{\varepsilon_{Z_k}}{1-t_k} = \frac{\partial(1-\gamma_k)}{\partial(1-t_k)}\frac{1}{1-\gamma_k} - \rho_k\left[1-\varepsilon_{x_k}\right]$$

Consider the constant avoidance technology case. Then the avoidance response is independent of firm size. Then the only thing that matters is how the pass-through rate and the demand elasticity change with number of firms, i.e., how the elasticity of true revenue changes with the number of firms. As discussed previously, it can be shown that log-concave demand functions imply increasing pass-through rates with competition, while log-convex demand functions imply decreasing pass-through rates. We can then make the following assertions.

Proposition 1. *Under constant scale avoidance technology, the relationship between market power and the elasticity of reported revenue with respect to the relevant tax tool is qualitatively the same as the relationship between market power and elasticity of true revenue with respect to the tool.*

(a) For log-concave demand functions and production costs that are not too convex, an increase in competition decreases the elasticity of reported revenue to taxation if $\varepsilon_{x_k} < 1$. For log-convex demand functions and costs that are not too concave, an increase in competition increases the elasticity of reported revenue if $\varepsilon_{x_k} < 1$.

(b) For log-concave demand functions and production costs that are not too convex, an increase in competition increases the elasticity of reported revenue to enforcement if $\varepsilon_{x_k} < 1$.

Under non-constant returns avoidance techonology, the "tipping point" of the reported revenue response changes as the avoidance response shifts where the change from a positive to a negative industry size effect on the overall response occurs.

4 Optimal Rates and Tradeoff

Having discussed the various forces of the problem, particularly in the welfare effects and reported taxable income elasticities, we are now ready to dicuss the features of the optimal policy. Much of these results will simply be formalizations of moving parts decribed in the previous section.

4.1 Optimal Tax Rates and Enforcement Rates

Bringing everything together, we arrive at our expression for the optimal tax rate in the presence of market power and avoidance:

Proposition 2. *Given an enforcement rate, the optimal tax rate in an industry k is given by the following expression*

$$\frac{t_k}{1-t_k} = \frac{1 - \frac{\alpha}{\lambda}\left[1 + EB_k\right] - \frac{\beta_k}{Z_k}\frac{\partial R_k}{\partial t_k}}{\varepsilon_{Z_k}}$$

Given a tax rate, the optimal enforcement rate in an industry k is given by the following expression

$$\beta_k = \frac{t_k \varepsilon^{\beta}_{Z_k}}{\frac{R_k}{Z_k} + \frac{\beta_k}{Z_k}\frac{\partial R_k}{\partial \beta_k} + \frac{\alpha}{\lambda}\left[\frac{N_k H_\beta}{Z_k} + EB^{\beta}_k\right]}$$

Again EB^{θ}_k is representative of the additional distortion taxation (or enforcement in the corresponding enforcement expression) has due to the markets being imperfectly competitive. As the number of firms rises, excess burden decreases, providing a driving force upward on the tax rate as competition rises (alternatively stated, as market power increases this provides a downward force on the tax rate). This is true for both policy tools as either will increase the effective tax rate of the firm, which what the firm responds to and distorts the market.

However, as we have discussed in the previous section, the behavior of reported income does not necessarily follow the same direction. Since the reported taxable income elasticity is in the denominator of the tax expression, a lower response drives up the tax rate while a higher response drives down the tax rate. Since the excess burden term decreases with competition, if the reported income elasticity also decreases with competition, then we can unambiguously say that the optimal tax rate should increase with competition. As discussed in Proposition 1, one case where this is true is under constant scale avoidance technology, log-concave demand, and an elasticity of demand below unity (under an output tax).

While we cannot guarantee that for the same conditions, but an elasticity greater than 1, that increasing market power should increase the optimal tax rate, we can say that elasticity greater than 1 is a necessary additional condition for this relationship to be possible. Relaxing the constant scale avoidance technology will alter the benchmark elasticity that separates the first region from the second. We will illustrate in our simulations that this possibility for optimal tax rates (and enforcement rates) to be increasing with market power in certain regions.

The optimal enforcement expression follows much of the same intuition. The imperfect market related distortion is positively in the denominator, indicating that a higher value decreases the optimal enforcement rate, as should be expected. The enforcement elasticity is in the numerator, indicating a higher value increases the optimal enforcement rate. This too makes sense as a higher enforcement elasticity indicates that enforcement is highly effective in increasing reported revenue which allows the tax rate to be more effective. As discussed in Proposition 1, the same conditions that allow for the elasticity of taxable revenue and the market distortion factor to go in the same direction also allow the corresponding two effects to go in the same direction for the enforcement expression.

Not discussed yet are the enforcement costs in each expression. For the tax rate, there are no direct enforcement costs, but there is an indirect effect since enforcement costs are based on firm size. Since a change in the tax rate alters the (true) firm and industry size, it will change how much is spent on total enforcement. If the industry grows, then this leads to higher total enforcement costs, and so this puts a downward effect on the tax rate. This same indirect effect also occurs in the enforcement expression, and similarly an indirect increase in industry size would be a negative factor in the enforcement rate and drive it down. The enforcement rate also has a "mechanical" resource cost, but this is simply the current industry size. If we include these administrative costs into the income elasticity, we can obtain an "enforcement elasticity of tax revenue" in a vein identical to Keen and Slemrod (2017).

Under a pure profit tax, these relationship becomes even simpler. There is no excess burden related to production and market power, leaving only the reported taxable income elasticity in the tax expression. For enforcement, the excess burden also drops out out, but there is still an administrative cost consideration.

Corollary 2.1. *Under a profit tax, the optimal tax and enforcement rates are given by*

$$\frac{t_k}{1-t_k} = \frac{1-\frac{\alpha}{\lambda}}{\varepsilon_{1-\gamma_k}}, \qquad \beta_k = \frac{t_k \varepsilon^{\beta}_{\gamma_k}}{\frac{R_k}{Z_k} + \frac{\alpha}{\lambda}\frac{H_\beta}{z_k}}$$

Under constant scale avoidance technology, the levels of both the profit tax and enforcement rate are independent of market power. Under increasing (decreasing) scale avoidance technology, the tax (enforcement) is negatively (positively) related to market power within an industry, and, under Assumption 1, firm size across industries.

Though this corollary replaces the elasticity of reported taxable income with the elasticity of avoidance to highlight the lack of a production distortion under the profit tax, the elasticity of reported income is the sufficient statistic for the optimal tax rate.

Reitering the final separation between within market and between market comparisons: the avoidance response is tied to the relationship between firm size and the avoidance cost function. For within market comparative statics, this is directly linked to a market power comparison as well. Across industries, we can only make statements based on firm size. The above corollary would dictate the same policies for two industries with same individual firm sizes regardless of market power.

4.2 Policy Choice: Taxation vs. Enforcement

Above we have derived the condition for an optimal tax conditional on a not necessarily optimal enforcement rate and vice versa for the optimal enforcement rate. What's equally as important, however, is the optimal balance of these two activities, as this accounts for both tools being set optimally. The condition that governs this tradeoff is

$$\underbrace{(1-\gamma_k) - \frac{\partial \gamma_k}{\partial t_k} t_k + \frac{\alpha}{\lambda}\frac{1}{x_k}\frac{\partial \pi_k}{\partial t_k}}_{\text{Marginal Tax Effect}} = \frac{\rho_k}{\rho^{\beta}_k} \underbrace{\left[-1 - \frac{\partial \gamma_k}{\partial \beta_k} t_k + \frac{\alpha}{\lambda}\frac{1}{x_k}\frac{\partial \pi_k}{\partial \beta_k}\right]}_{\text{Marginal Enforcement Effect}}$$

or, alternatively stated,

$$\underbrace{\left[(1-\gamma_k) + \frac{\rho_k}{\rho^{\beta}_k}\right]}_{\text{Mechanical Change}} + \underbrace{\left[-\frac{\partial \gamma_k}{\partial t_k} + \frac{\partial \gamma_k}{\partial \beta_k}\frac{\rho_k}{\rho^{\beta}_k}\right] t_k}_{\text{Behavioral Change}} + \underbrace{\frac{\alpha}{\lambda}\frac{1}{x_k}\left[\frac{\partial \pi_k}{\partial t_k} - \frac{\partial \pi_k}{\partial \beta_k}\frac{\rho_k}{\rho^{\beta}_k}\right]}_{\text{Profit Change}} = 0$$

While the above expression speaks to the optimum, we can also use the LHS of the same expression to dictate whether a policy change consisting of a tax increase and an enforcement decrease is welfare beneficial (if the LHS is greater than 0). In this format, we can easily state the factors that could positively or negatively push for tax rates versus enforcement. Taxes become preferable if (1) the mechanical revenue gain from tax rates is relatively high due to low avoidance rates, (2) the fiscal loss due to avoidance rates increasing both from the increase in taxes and reduction in enforcement is relatively low, and (3) the negative profit responses of taxes are outweighed by the positive profit response from reducing enforcement. All of these factors are impacted by how responsive prices are to enforcement relative to taxes, which dictates how much change in enforcement is needed to compensate for the change in taxes.

The profit effect is the only part of the tradeoff that does not exist in the perfectly competitive version. As profits are zero and remain zero for the competitive industry, this effect is zeroed out. Here, we must consider how raising enforcement versus taxes differentially affects the profits of the industry and feed back to the individual.

We've discussed these factors along the way, but we summarize the main connections here. The more production facilitates avoidance, i.e., the lower the elasticity of marginal cost of avoidance with respect to firm size $\varepsilon^{r}_{H_\gamma}$ is, the more competition drives down avoidance rates. Since avoidance rates are negatively related to the tax base and directly related to the enforcement base, this pushes the tradeoff in favor of taxes at high levels of competition. In other words, if an industry has a low level of avoidance, there is simply more to gain via additional taxation then there is in trying to further reduce avoidance. The relationship between competition and the avoidance response follows in the same direction as discussed in Section 3. While competition may have conflicting impacts on profit margins, ultimately the magnitude of this effect likely does not overpower the previous two effects due to the lower weight on profits since $\frac{\alpha}{\lambda} < 1$. We can equivalently demonstrate this tradeoff by combining our two expressions from Proposition 2.

Proposition 3. *The tradeoff between taxes and enforcement follows*

$$\frac{\frac{t_k}{1-t_k}}{\beta_k} = \frac{\left(1 - \frac{\alpha}{\lambda}\left[1 + EB_k\right]\right)\left(\frac{\alpha}{\lambda}\left[\frac{N_k H_\beta}{Z_k} + EB^{\beta}_{k}\right]\right)}{t_k \varepsilon_{NR_k} \varepsilon^{\beta}_{NR_k}}$$

where we combined taxable income responses with adminstrative costs into singular net elasticities $\varepsilon^{\theta}_{NR_k}$.

This result is exactly comparable to the tradeoff discussed in Keen and Slemrod (2017). Like their conclusion, having a higher value of either elasticity pushes the argument in favor of enforcement, as a higher tax elasticity indicates higher efficiency costs of taxation, while a higher enforcement elasticity indicates more effective enforcement. New to these expressions as compared to their paper are the two excess burden terms. Having a high excess burden for either tool drives down the incentive to use that tool. Recalling our derivation of excess burden, the primary distinction between the two types is in the pass-through rates.

As before, under a pure profit tax, the excess burden terms drop out and only avoidance related behavioral responses remain. For this case then, we can directly state that the more firm size facilitates avoidance ($\varepsilon^{r}_{H_\gamma}$ decreases), the more the government should prioritize using enforcement rates over statutory tax rates.

Lastly, regardless of the type of tax, under constant scale avoidance technology, we get the unique result that relative desirability of tax versus enforcement is completely independent of the number of firms, and thereby the market power.

Corollary 3.1. *Under constant scale avoidance technology, the optimal ratio of taxation to enforcement for a given industry is independent of the market power in the industry. Under a profit tax, for increasing (decreasing) returns avoidance technology, this ratio decreases (increases) as the market power increases.*

As we can see by combining our profit tax and pass-through expressions from Section 3, the profit change under constant technology equals zero. This is because the relative erosion of profits via taxes versus via enforcement is exactly equal to relative price effect. Since this is true for any number of firms, this term completely drops out. What this then means is that welfare considerations have zero impact on this tradeoff, and thus the optimization condition is exactly equivalent to if the government had just been maximizing revenue. Second, again using our results from Section 3, the avoidance responses and the pass-through ratio are both independent of firm size. Therefore, the three terms are all independent of firm size and therefore the optimal relationship between taxation and enforcement is independent of market power. We combine these statements into the following corollary.

Corollary 3.2. *Let $\frac{t_k^*}{\beta_k^*}$ represent the ratio of the optimal tax rate to the optimal enforcement rate set by a representative consumer welfare maximizing government in an industry k. Let $\frac{t_k^R}{\beta_k^R}$ represent the ratio of revenue maximizing rates. If the market has firms with constant scale avoidance technology, then*

$$\frac{t_k^*}{\beta_k^*} = \frac{t_k^R}{\beta_k^R}$$

This is true for any exogenous revenue requirement between 0 and the maximum tax revenue possible.

5 Simulations

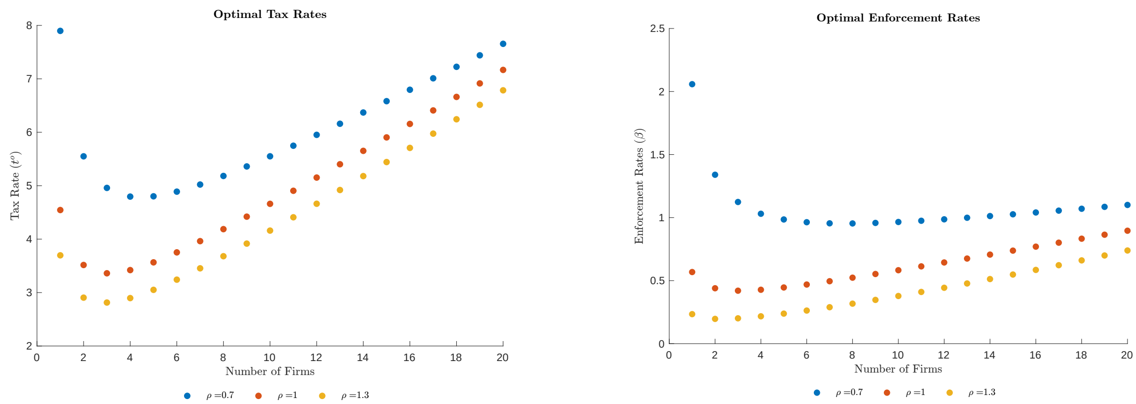
To provide better intution into how the structure of the cost function can affect the results, it is fruitful to illustrate some of these findings with a more concrete cost function. Consider the specifications of the form

$$C(y, r, \gamma, \beta) = \phi(y) + D\beta G(r)\gamma^2$$

where our used specification will be $H(r)$ of the form r^ρ as a measure of increasing (cost-reducing) complementarity between the level of evasion and the output. In this specification, $\rho = 1$ is equivalent to a elasticity of marginal avoidance costs $\varepsilon_{H_\gamma}^r = 1$, and a higher (lower) ρ increases (decreases) this elasticity. β, in conjunction with the constant D, parameterizes how impactful enforcement can be on the avoidance portion of the firm's cost function. The avoidance fraction γ is squared to allow for strictly convex evasion costs.

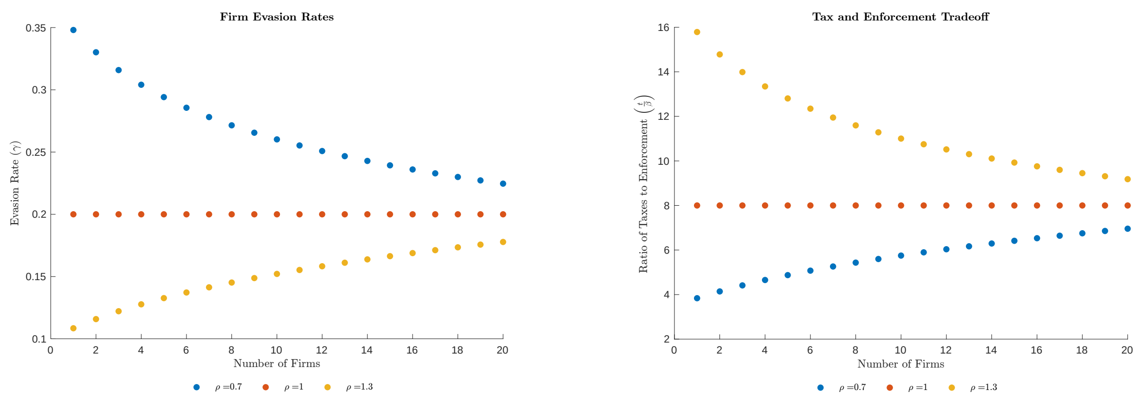
5.1 Simulation Results: Output Tax

Here we describe the results of simulations of the model presented above. For the starting simulation environment, we examine variants of a single industry ("within market" comparison) and employ a linear demand curve of the form $q(x) = A - Bx$, and the format of the cost function is described as above. The cost function is further parameterized with squared direct production costs, $F(y) = Ky^2$, and a power form for size in the avoidance cost, $G(y) = (yq)^\rho$. The strength of enforcement D is arbitrarily assigned to obtain somewhat reasonable values for these illustrations, but ultimately the scale of the y-axes in the following figure have no explicit meaning. Particularly, in the tradeoff figure, a value of "8" implies that taxes are 8 times more valuable than enforcement, but this is highly dependent on the assumed cost of enforcement (normalized to 1) and strength of enforcement parameter.



(a) (b)

Figure 1: Optimal Policy Tools



(a) (b)

Figure 2: Avoidance Rates and Policy Tradeoff

All simulation results have two layers of heterogeneity. First, each graph should be framed as a comparative static diagram of competition. Number of firms increase along the x-axis, representative of a reduction in individual firm market power, and we observe how the equilibrium values respond. Second, each figure demonstrates three sets of points to show results at different degrees of firm size-avoidance complementarity. The $\rho = 0.7$ points indicate an edge for larger firms in avoidance costs, while $\rho = 1.3$ indicates higher costs for larger firms.

Figures 1a and 1b illustrate the transition paths of the optimal tax and enforcement rates. We can see clearly here that it is possible that tax rates are higher for firms with higher market power in certain regions, while the reverse is true in other regions. The "tipping point" in both figures occurs near the point where ε_{x_k} transitions from a value above one to a value below 1, but as previously stated this does not necessarily have to be the case except under stricter conditions.

A key subplot is Figure 2b, which illustrates the tradeoff between taxes and enforcement. The implications of this figure reflects the same conclusions as the previous theory. If production facilitates evasion (ex. $\rho = 0.7$ in the simulations), more market power means the government should shift more priority to enforcement over tax rates. If production reduces the ability to evade (by raising marginal costs faster than benefits), then more market power means the government should have higher priority on tax rates over enforcement.

We can see this figure inversely mirrors Figure 2a on avoidance rates. This aligns with our discussions on the relationship between the tax and enforcement bases as a function of the avoidance rates. When ρ is high, the elasticity of marginal costs of evasion is high, and increasing competition is tied to higher avoidance rates. High avoidance rates generally reduces the tax rate base and increases the enforcement rate base, which is primarily reflected in the downward trend in the tax-enforcement ratio. The sizes of the other effects are not large enough to overturn this primary effect.

5.2 Simulation Results: Profit Tax [In Progress]

5.2.1 Elasticity of Demand [In Progress]

The simulations thus far have been under the assumption of a linear demand function. However, there are several ways the assumption on the convexity of the demand function may affect the problem. For example, it is well known in the literature that a linear demand function implies an inverse relationship between tax pass-through and competition since linear demand falls under the log-concave classification. Under other log-convex demand functions, however, it is also possible that the reverse is true.

6 Uniform Taxation

In this section, we build a bridge to real world connections by translating our results to a uniform tax system, while still allowing for differentiation on the lines of enforcement. Intuitively, we should see much of the same intuition as in the previous sections but enforcement must also capture part of the problem previously captured by the tax rate. First, the tax rate satisfies

$$\frac{t}{1-t} = \frac{\sum_j Z_j \left[1 - \frac{\alpha}{\lambda}\left[1 + EB_j\right] - \frac{\beta_j}{Z_j}\frac{\partial R_j}{\partial t}\right]}{\sum_j Z_j \varepsilon_{Z_j}}$$

For the uniform tax rate, the correlation between reported income and the elasticity of reported income becomes important. If industries that have high reported incomes are also the ones that have the highest behavioral responses of reported incomes, this leads to a larger loss in the tax base than one would expect if the relationship were random. This is also true for the excess burden term. If the industries with the highest excess burden are the ones that have the highest reported revenue, there is a greater total distortion. Thus, positive correlations between the reported income with either or both the excess burden or the tax elasticity would push the uniform rate lower than it would otherwise be. As before, under a profit tax, this relationship becomes even simpler

$$\frac{t_k}{1-t_k} = \frac{1 - \frac{\alpha}{\lambda}}{\varepsilon_Z}$$

where $\varepsilon_Z = \frac{\sum_j Z_j \varepsilon_{Z_j}}{\sum_j Z_j}$ is simply the weighted average income elasticity.

Since tax policy cannot be differentiated, all differentiation must be done through the enforcement rates. To get the ratio of enforcement rates across industries, we simply divide our optimal enforcement condition from before with the same condition for a different market, i.e.,

$$\frac{\beta_m}{\beta_n} = \frac{\frac{H_\beta(z_n, \gamma_n, \beta_n)}{z_n} + EB_n}{\frac{H_\beta(z_m, \gamma_m, \beta_m)}{z_m} + EB_m} \frac{\varepsilon^\beta_{NR_m}}{\varepsilon^\beta_{NR_n}}$$

which follows a similar comparison across industries as before. The key difference is that at the optimum, this ratio does not account for the ratio in differentiated tax rates. Under the profit tax, we have

$$\frac{\beta_m}{\beta_n} = \frac{\frac{H_\beta(z_n,\gamma_n,\beta_n)}{z_n}}{\frac{H_\beta(z_m,\gamma_m,\beta_m)}{z_m}} \frac{\varepsilon^\beta_{Z_m}}{\varepsilon^\beta_{Z_n}}$$

With differentiated taxes, this term would be adjusted by the ratio of tax rates $\frac{t_m}{t_n}$. If this ratio of tax rates exceeds one, we expect that part of the differentiation that could be done with enforcement rates is instead done via tax rates. When taxes are forced to be uniform, the government must instead increase β_m relative to β_n to compensate. If the ratio is below 1, then we have the inverse case. The government desired lower relative tax rates in industry m, but cannot do so. Thus, enforcement of industry m should decrease relative to n. Connecting back to the relationship between firm size and avoidance gives the following proposition:

Proposition 4. *Under a uniform profit tax, if firm size facilitates (hinders) avoidance,* $\varepsilon^r_{H_\gamma} < (>) 1$*, the enforcement rate is relatively lower (higher) than it would have been in a world with a differentiated profit tax.*

We can explain this imperfect instruments logic as follows. The choice of tax and enforcement for each industry can be equivalently thought of as the choice of effective tax rates and the tax and enforcement ratio. When all both tools are perfectly differentiated, the government can exactly select the differentiated effective tax rates that they want in each industry and use the most efficient ratio of tax to enforcement to reach that effective tax rate. When moving to uniform taxes, this is no longer the case. If the government sets effective tax rates targets, they can only reach them by changing the enforcement rate, leading to inefficient ratios. In reverse, if the government only used efficient ratios, they would not be targeting the correct effective tax rates.

7 Extensions and Alternative Specifications of the Model

In this section, we consider a few extensions and alternative specifications of the model.

7.1 Per-Firm Administrative Costs

In the baseline version of the model, enforcement costs are linear with respect to true industry size. One issue with this assumption is that the government may not be fully aware of true industry size as they may only observe reported revenues. Thus, this version of the model implicitly assumes that they are able to observe both true revenues and avoidance rates, but are artifically restricted on only taxing based on true firm size. This may be be a valid assumption if, for example, we believe that all or the types of avoidance firms engage in are activities that the government is aware of but may not fully deem worth enforcing (or cracking down on legal avoidance).

An alternative version of the enforcement costs that does not assume that the government is aware of avoidance levels is per-firm administrative costs, which only assumes that the government is aware of number of firms in the industry. Thus, the government's budget constraint is

$$\sum_j t_j Z_j - \eta(N_j) a(\beta_j)$$

where $\eta_j(\cdot)$ is a possibly nonlinear, but increasing function of the number of firms, indicative of the rising costs for the government as the number of firms they must potentially enforce increases. In this case, we make a simple adjustment to our optimum expressions,

$$\frac{t_k}{1-t_k} = \frac{1-\frac{\alpha}{\lambda}\left[1+EB_k\right]}{\varepsilon_{Z_k}}$$

and

$$\beta_k = \frac{t_k \varepsilon^{\beta}_{Z_k}}{\frac{\alpha}{\lambda}\left[1+EB^{\beta}_k\right] + \frac{\eta(N_k)}{Z_k} a_{\beta}}$$

In the tax expression, there is no longer an effect on the tax rate due to the true revenue changing sizes affecting total enforcement costs. In the enforcement expression, administrative cost term appears in the denominator, divided by reported revenue. Clearly total administrative costs increase with the number of firms, but in order for the overall term to increase, it must be the case that these costs increase at a rate faster than the reported revenue. Thus, as long as $\frac{d}{dN}\left(\frac{\eta}{Z}\right) > 0$, then this provides a downward force on the enforcement rate.

On a cross-industry basis, this becomes a simpler comparison. If two industries have the same aggregate reported income, then the enforcement cost factor pushes the enforcement rate down on the one that has a higher number of firms as the average enforcement cost per dollar is higher.

7.2 Heterogenous Firms [In Progress]

Suppose that the firms within a given industry are heterogeneous in their productive capability, i.e., different levels of marginal costs. Ultimately, the expressions should remain the same in formulation, though the underlying substance may change. Both elasticities (with respect to either tax and enforcement) of reported taxable income and the excess burden terms are weighted averages of individual firm responses where weights are not equal as in the homogeneous case.

The details of these changes can be found in the appendix.

7.3 Differential Welfare Weights on Profits [In Progress]

We have thus far ignored any consideration of redistribution in our problem. One manner in which we may simply incorporate some degree of redistribution is to put a different weight on welfare associated with profits and those associated with consumer surplus. Let ζ represent the weight on profits, such that the welfare effect of a tax tool change is

$$\frac{dv}{d\theta_k} = -\alpha\left[\frac{\partial q_k}{\partial \theta_k}x_k + \zeta\frac{\partial \Pi_k}{\partial \theta_k}\right]$$

where $\zeta < 1$ would indicate profits (producer surplus) are valued less than consumer surplus. Since there is no consumer surplus effect under the profit tax case, there is ultimately no change to the previous analysis as all forces work solely through profits. For non-full deductibility, however, this is no longer the case.

7.4 Market Power as a Choice Variable [In Progress]

An alternative potential instrument to the government in this problem is directly addressing market power via regulatory action. A potentially simple way to illustrate this idea in the context of the current model is to have N_k be a control variable for the government. Increasing N_k is to increase the competition within an industry (at some regulatory cost).

8 Discussion and Conclusion

Whether increasing market concentration and supernormal firm sizes is cause for additional enforcement attention or more directly higher statutory tax rates has become an increasingly relevant policy question. While this paper has several limitations in fully bridging theory to these real world concerns, including a heavily stylized enforcement parameter and lack of firm heterogeneity, it suggests a potentially more nuanced mindset in this regard. A pure profit tax provides potentially the most tenable structure for prevailing sentiments. In this setup, a positive relationship between firm size and avoidance would encourage higher enforcement on large firms (but with relatively lower statutory taxes). If costs are not deductible, neither the relationship between optimal tax rates or optimal enforcement rates and the competitivity of an industry are unambiguously monotonically negative (or positive), meaning that a prescription to tax or enforce a noncompetitive industry at a higher rate does not necessarily bear fruit. Though we are sometimes able to find conditions under which we can sign the relationship, there is no singular rule. Core to this issue is that reported revenue can behave non-monotonically, depending on the demand curve and the avoidance technology. So, even though there is a downward force on the tax and enforcement rates due to increased market distortion as market power rises, the overall direction of either of the two tools can be ambiguous. What may be more directly prescriptive, however, is in the relative ratio of tax rates versus enforcement. Depending on the relationship between avoidance costs and production costs, avoidance rates may be directly tied to firm size and competition within a market. Provided that potentially counteracting behavioral responses are not overly large, this would provide a strong incentive for the government to prioritize the use of one tool over the other.

To understand the importance of considering industry composition and behavior in optimal tax systems, this paper has developed a model in which industries can be (symmetric) oligopolistic via fixed numbers of Cournot competitors. Firms in each industry maximize private profits by choosing outputs and avoidance rates. The government can only tax based on the reported taxable incomes of each industry. Most of this paper explores a fully differentiated system of output tax rates and enforcement rates similar in nature to a Ramsey commodity tax setup. While this setup may not be particularly realistic, especially as it pertains to the motivation of large firms, particularly in the US, potentially avoiding the corporate tax rate, it simplifies the analysis and we use it to derive intuition that may otherwise be partially obsfucated by a system that more closely mirrors the real world. However, this paper later makes attempts to bridge the gap to the real world motivation by considering a version of the model where the tax is uniform (but enforcement rates are still differentiable). Ultimately, the results are frequently qualitatively similar.

Much of the results heavily depend on the complementarity of production and avoidance costs for the firm. It is theoretically possible for production to aid or harm (or do nothing) a firm's ability to avoid taxes. If we are in the special case where it is neither, such that a firm will choose the same avoidance rate regardless of what size they are, then a lot of results simplify and it is the case that under the profit tax the optimum is independent of the market structure. This is not the case with the levels of taxation or enforcement in an output tax, as the tradeoff between reported revenue elasticities and market distortion still apply. It does, however, apply to the relative tradeoff between the two tools. Despite, the actual levels of each changing with different number of firms, the ratio of taxes to enforcement remains the same.

If instead firm size aids avoidance, then an increase in competition in a market drives down avoidance rates. This increases the preferability of using tax rates over enforcement rates (higher relative base). The reverse is true if size is negatively associated with avoidance.

While measures of avoidance levels and behavior are difficult to obtain, this paper offers some guidance in connecting terms in the optimal expressions to empirics. Behavioral elasticities are in terms of reported income, and market related excess burden terms are in terms of pass-through rates, demand elasticities, Lerner indices (isomorphic to number of firms here), price-marginal cost ratios, and marginal complicance costs. The final two are the hardest to empirically found, but there have been growing efforts directly related and semi-related spheres.

9 References

1. Adachi, Takanori, and Michal Fabinger. 2022. "Pass-through, Welfare, and Incidence Under Imperfect Competition." *Journal of Public Economics*, 211: Article 104589.

2. Allingham, Michael G., and Agnar Sandmo. 1972. "Income Tax Evasion: A Theoretical Analysis." *Journal of Public Economics*, 1: 323-338.

3. Anderson, Simon P., Andrè de Palma, and Brent Kreider. 2001. "Tax Incidence in Differentiated Product Oligopoly." *Journal of Public Economics*, 81: 173-192.

4. Besley, Timothy. "Commodity Taxation and Imperfect Competition." *Journal of Public Economics*, 40: 359-367.

5. Cremer, Helmuth, and Firouz Gahvari. 1990. "Tax Evasion and Optimal Commodity Tax." *Journal of Public Economics*, 50: 261-275

6. de Paula, Áureo, and José A. Scheinkman. 2010. "Value Added Taxes, Chain Effects, and Informality. *American Economic Journal: Macroeconomics*, 2: 195-221.

7. Dharmapala, Dhammika, Joel Slemrod, and John Douglas Wilson. 2011. "Tax Policy and the Missing Middle: Optimal Tax Remittance with Firm-Level Administrative Costs." *Journal of Public Economics*, 95: 1036-1047.

8. Eckhout, Jan, Chunyang Fu, Wenjian Li, Xi Weng. 2021. "Optimal Taxation and Market Power." Working Paper.

9. Kaplow, Louis. 1990. "Optimal Taxation with Costly Enforcement and Evasion." *Journal of Public Economics*, 43: 221-236.

10. Kaplow, Louis. 2021. "Market Power and Income Taxation." *American Economic Journal: Economic Policy*, 13(4): 329-354.

11. Keen, Michael, and Joel Slemrod. 2017. "Optimal Tax Administration." *Journal of Public Economics*, 152: 133-142.

12. Kopczuk, Wojciech, and Joel Slemrod. 2006. "Putting Firms into Optimal Tax Theory." *The American Economic Review*, 96(2): 130-134.

13. Kopczuk, Wojciech, Justin Marion, Erich Muehlegger, and Joel Slemrod. 2016. "Does Tax Collection Invariance Hold? Evasion and Pass-Through of State Diesel Taxes." *American Economic Journal: Economic Policy*, 8(2): 251-286.

14. Kubick, Thomas R., Daniel P. Lynch, Michael A. Mayberry, and Thomas C. Omer. 2015. "Product Market Power and Tax Avoidance: Market Leaders, Mimicking Strategies, and Stock Returns." *The Accounting Review*, 90(2): 675-702.

15. Martin, Julien, Mathieu Parenti, and Farid Toubal. 2022. "Corporate Tax Avoidance and Sales: Micro Evidence and Aggregate Implications." Working Paper.

16. Miklós-Thal, Jeanine, and Greg Shaffer. 2021. "Pass-Through as and Economic Tool: On Exogenous Competition, Social Incidence, and Price Discriminiation." *Journal of Political Economy*, 129(1): 323-335.

17. Myles, Gareth D. 1989. "Ramsey Tax Rules for Economies with Imperfect Competition." *Journal of Public Economics*, 38:95-115.

18. Pless, Jacquelyn, and Arthur A. van Benthem. 2019. "Pass-Through as a Test for Market Power: An Application for Solar Subsidies." *American Economic Journal: Applied Economics*, 11(4): 367-401.

19. Ramsey, Frank P. 1927. "A Contribution to the Theory of Taxation." *The Economic Journal*, 37(145): 47-61.

20. Ritz, Robert A. "Does Competition Increase Pass-Through?" *The RAND Journal of Economics*, 55(1): 140-165.

21. Slemrod, Joel and Tejaswi Velayudhan. 2018. "Do Firms Remit at Least 85% of Tax Everywhere? New Evidence from India." *Journal of Tax Administration*, 4(1): 24-37.

22. Weyl, E. Glen, and Michal Fabinger. 2013. "Pass-Through as an Economic Tool: Principles of Incidence under Imperfect Competition." *Journal of Political Economy*, 121(3): 528-583.

A Mathematical Appendix

A.1 A Pure Profit Tax

Under a truly pure profit tax where all costs are deductible, neither the production or avoidance margins are distorted. To see this, cosider the maximization problem for the firm

$$\max_{y_k,\gamma_k} y_k q_k(x_k) - C_k(y_k, r_k, \gamma_k, \beta_k) - t_k(1-\gamma_k)(y_k q_k - \mu C_k)$$

where $C = F(y_k) + H(y_k q_k - \mu F - \mu H, \gamma, \beta)$. Then the two FOC are

$$y_k : (1-(1-\gamma_k)t_k - H_r)\left[q_k + \frac{\partial q_k}{\partial x_k} y_k\right] = (1-\mu[(1-\gamma_k)t_k + H_r])\phi_y$$
$$\gamma_k : t_k(y_k q_k - \mu C_k) = H_\gamma - t_k(1-\gamma_k)\mu H_\gamma$$

A.2 Firm and Market Responses

In this section, we derive firm and market response margins to each of the tax tools. The relevant response margins are profits, price, and avoidance. Recall that the firm's problem is

$$\max_{y_k,\gamma_k} y_k q_k(x_k) - C_k(y_k, r_k, \gamma_k, \beta_k) - t_k((1-\gamma_k)(y_k q_k - \mu\phi_k(y_k)))$$

To help us with these responses, we recall our first order conditions for production and for avoidance:

$$y_k : (1-(1-\gamma_k)t_k - H_r)\left[q_k + \frac{\partial q_k}{\partial x_k} y_k\right] = (1-\mu[(1-\gamma_k)t_k + H_r])\phi_y$$
$$\gamma_k : t_k(y_k q_k - \mu\phi_k) = H_\gamma$$

We can also rearrange the first production condition in terms of equating (net) marginal revenues and marginal costs, which will be helpful for later derivations

$$(1-(1-\gamma_k)t_k)\left[q_k + \frac{\partial q_k}{\partial x_k} y_k\right] = (1-\mu(1-\gamma_k)t_k)\phi_y + H_r\left[q_k + \frac{\partial q_k}{\partial x_k} y_k - \mu\phi_y\right]$$

A.2.1 Profit Responses

We differentiate the firm's after-tax profits π_k with respect to the tax rate:

$$\begin{aligned}\frac{\partial \pi_k}{\partial t_k} = &(1-H_r)\left[\frac{\partial y_k}{\partial t_k} q_k + y_k \frac{\partial q_k}{\partial t_k}\right] - \phi_y \frac{\partial y_k}{\partial t_k} + H_r \mu \phi_y \frac{\partial y_k}{\partial t_k} - H_\gamma \frac{\partial \gamma_k}{\partial t_k} - z_k \\ &- t_k\left[-\frac{\partial \gamma_k}{\partial t_k}(y_k q_k - \mu\phi_k) + (1-\gamma_k)\left[\frac{\partial y_k}{\partial t_k} q_k + y_k \frac{\partial q_k}{\partial t_k} - \mu\phi_y \frac{\partial y_k}{\partial t_k}\right]\right]\end{aligned}$$

where as noted in the main body z_k is the firm's reported taxable income. Using the envelope condition, we can simplify this down to

$$\frac{\partial \pi_k}{\partial t_k} = -z_k + (1-(1-\gamma_k)t_k - H_r)\frac{N_k - 1}{N_k}\frac{\partial q_k}{\partial t_k} y_k$$

Similarly, a firm's profit change due to an increase in enforcement is given by

$$\frac{\partial \pi_k}{\partial \beta_k} = -H_\beta + (1-(1-\gamma_k)t_k - H_r)\frac{N_k - 1}{N_k}\frac{\partial q_k}{\partial \beta_k} y_k$$

The aggregate industry profit (Π_k) responses are then given by

$$\frac{\partial \Pi_k}{\partial t_k} = -Z_k + (1-(1-\gamma_k)t_k - H_r)\frac{N_k-1}{N_k}\frac{\partial q_k}{\partial t_k}x_k$$
$$\frac{\partial \Pi_k}{\partial \beta_k} = -N_k H_\beta + (1-(1-\gamma_k)t_k - H_r)\frac{N_k-1}{N_k}\frac{\partial q_k}{\partial t_k}x_k$$

Under an output tax, we can then use our production FOC to get this in terms of price-cost margins via the Lerner Index. The production FOC can be rearranged as

$$\frac{\phi_y}{q_k} = (1-(1-\gamma_k)t_k - H_r)\left[1 - \frac{1}{N_k \varepsilon_{x_k}}\right]$$

which turns the two profit conditions to

$$\frac{\partial \pi_k}{\partial t_k} = -z_k + \frac{N_k-1}{N_k}\frac{\partial q_k}{\partial t_k} y_k \frac{\phi_y}{q_k}$$

A.2.2 Price Responses

To obtain the price responses, we must differentiate the first order condition of production with respect to each of the two tax tools. As before, we start with the tax rate:

$$\begin{aligned}
&\left(\frac{\partial \gamma_k}{\partial t_k}t_k - (1-\gamma_k) - \left[H_{rr}\left[\frac{\partial y_k}{\partial t_k}q_k + y_k\frac{\partial q_k}{\partial t_k} - \mu\phi_y\frac{\partial y_k}{\partial t_k}\right] + H_{r\gamma}\frac{\partial \gamma_k}{\partial t_k}\right]\right)\left(q_k + \frac{\partial q_k}{\partial x_k}y_k\right) \\
&\qquad + (1-(1-\gamma_k)t_k - H_r)\left(\frac{\partial q_k}{\partial t_k} + \frac{\partial y_k}{\partial t_k}\frac{\partial q_k}{\partial x_k} + y_k\frac{\partial^2 q_k}{\partial x_k^2}\frac{\partial x_k}{\partial t_k}\right) \\
&= \mu\left(\frac{\partial \gamma_k}{\partial t_k}t_k - (1-\gamma_k) - \left[H_{rr}\left[\frac{\partial y_k}{\partial t_k}q_k + y_k\frac{\partial q_k}{\partial t_k} - \mu\phi_y\frac{\partial y_k}{\partial t_k}\right] + H_{r\gamma}\frac{\partial \gamma_k}{\partial t_k}\right]\right)\phi_y \\
&\qquad + (1-\mu[(1-\gamma_k)t_k + H_r])\phi_{yy}\frac{\partial y_k}{\partial t_k}
\end{aligned}$$

which we can rearrange as

$$\begin{aligned}
&\frac{\partial q_k}{\partial t_k}\left[(1-(1-\gamma_k)t_k - H_r)\left(1 + \frac{1}{N_k}\left[1 + \frac{\partial^2 q_k}{\partial x_k^2}\frac{\partial x_k}{\partial q_k}\right]\right) - (1-\mu[(1-\gamma_k)t_k + H_r])\frac{\phi_{yy}}{N_k}\frac{\partial x_k}{\partial q_k}\right. \\
&\qquad \left. - \frac{H_{rr}}{N_k}\left(\frac{\partial x_k}{\partial q_k}q_k + x_k - \mu\phi_y\frac{\partial x_k}{\partial q_k}\right)\left(q_k + \frac{\partial q_k}{\partial x_k}y_k - \mu\phi_y\right)\right] \\
&\qquad = \left(1 - \gamma_k - \frac{\partial \gamma_k}{\partial t_k}(1-\mu\phi_y)\left[t_k - H_{r\gamma}\right]\right)\left(q_k + \frac{\partial q_k}{\partial x_k}y_k\right)
\end{aligned}$$

Before moving on, we first note that if $\mu = 1$, then the original expression simplifies to

$$\begin{aligned}
&\left(\frac{\partial \gamma_k}{\partial t_k}t_k - (1-\gamma_k) - \left[H_{rr}\left[\frac{\partial y_k}{\partial t_k}q_k + y_k\frac{\partial q_k}{\partial t_k} - \mu\phi_y\frac{\partial y_k}{\partial t_k}\right] + H_{r\gamma}\frac{\partial \gamma_k}{\partial t_k}\right]\right)\left(q_k + \frac{\partial q_k}{\partial x_k}y_k - \mu\phi_y\right) \\
&\qquad (1-(1-\gamma_k)t_k)\left(\frac{\partial q_k}{\partial t_k} + \frac{\partial y_k}{\partial t_k}\frac{\partial q_k}{\partial x_k} + y_k\frac{\partial^2 q_k}{\partial x_k^2}\frac{\partial x_k}{\partial t_k} - \phi_{yy}\frac{\partial y_k}{\partial t_k}\right) = 0
\end{aligned}$$

The first line is zeroed since the second bracketed term is zero by the production condition. Then, we are left with

$$(1-(1-\gamma_k)t_k - H_r)\frac{\partial q_k}{\partial t_k}\left(1 + \frac{1}{N_k}\left(1 + \frac{\partial^2 q_k}{\partial x_k^2}\frac{\partial x_k}{\partial t_k}x_k - \phi_{yy}\frac{\partial x_k}{\partial q_k}\right)\right) = 0$$

which implies that $\frac{\partial q_k}{\partial t_k} = 0$. This makes sense as the output decision is independent of the tax rate and avoidance decision when costs are fully deductible. We can find the same consequence for the pass-through rate of the enforcement tool.

Returning to the case where $\mu < 1$, to simplify this expression, we first define a few elasticities. First, as in Weyl and Fabinger (2013), define the (negative) marginal surplus of quantity expansion as $ms_k = \frac{\partial q_k}{\partial x_k} y_k = \frac{x_k}{N_k}\frac{\partial q_k}{\partial x_k}$. Then

$$\frac{\partial ms_k}{\partial x_k} = -\frac{1}{N_k}\frac{\partial q_k}{\partial x_k}\left[1 + \frac{\partial^2 q_k}{\partial x_k^2}\frac{\partial x_k}{\partial q_k} x_k\right]$$

Then we have that the inverse of the elasticity of marginal surplus is

$$\begin{aligned}
\frac{1}{\varepsilon_{ms_k}} &= \frac{\partial ms_k}{\partial x_k}\frac{x_k}{ms_k} \\
&= -\frac{1}{N_k}\frac{\partial q_k}{\partial x_k}\left[1 + \frac{\partial^2 q_k}{\partial x_k^2}\frac{\partial x_k}{\partial q_k} x_k\right]\frac{N_k}{\frac{\partial q_k}{\partial x_k}} \\
&= 1 + \frac{\partial^2 q_k}{\partial x_k^2}\frac{\partial x_k}{\partial q_k} x_k
\end{aligned}$$

Second, we have that the net marginal costs as

$$\begin{aligned}
MC_k &= (1 - \mu(1-\gamma_k)t_k)\phi_y + H_r\left[q_k + \frac{\partial q_k}{\partial x_k} y_k - \mu\phi_y\right] \\
&= (1 - \mu(1-\gamma_k)t_k)\phi_y + H_r q_k\left[1 - \frac{1}{N_k\varepsilon_{x_k}} - \mu\frac{\phi_y}{q_k}\right]
\end{aligned}$$

Using the production FOC, this gives

$$\frac{MC_k}{q_k} = (1 - (1-\gamma_k)t_k)\left[1 - \frac{1}{N_k\varepsilon_{x_k}}\right]$$

We then define the elasticty of the inverse marginal cost curve (a pseudo "supply") as

$$\begin{aligned}
\frac{1}{\varepsilon_{s_k}}\frac{MC_k}{x_k} &= \frac{\partial MC_k}{\partial x_k} \\
&= (1 - \mu(1-\gamma_k)t_k)\frac{1}{N_k}\phi_{yy} + \frac{1}{N_k}H_{rr}\left[q_k + x_k\frac{\partial q_k}{\partial x_k} - \mu\phi_y\right]^2 \\
&\quad + H_r\left[\frac{\partial q_k}{\partial x_k} + \frac{1}{N_k}\left(\frac{\partial q_k}{\partial x_k} + x_k\frac{\partial^2 q_k}{\partial x_k^2}\right) - \mu\frac{1}{N_k}\phi_{yy}\right] \\
&= ((1 - \mu[(1-\gamma_k)t_k + H_r)])\frac{1}{N_k}\phi_{yy} + \frac{1}{N_k}H_{rr}\left[q_k + x_k\frac{\partial q_k}{\partial x_k} - \mu\phi_y\right]^2 \\
&\quad + H_r\left[\frac{\partial q_k}{\partial x_k} + \frac{1}{N_k}\left(\frac{\partial q_k}{\partial x_k} + x_k\frac{\partial^2 q_k}{\partial x_k^2}\right)\right]
\end{aligned}$$

Multiplying through by $\frac{\partial x_k}{\partial q_k}$, then we can convert our pass-through expression as

$$\begin{aligned}
&\frac{\partial q_k}{\partial t_k}\left[(1 - (1-\gamma_k)t_k)\left(1 + \frac{1}{N_k}\left[1 + \frac{\partial^2 q_k}{\partial x_k^2}\frac{\partial x_k}{\partial q_k}\right]\right) + \frac{\varepsilon_{x_k}}{\varepsilon_{s_k}}\frac{MC_k}{q_k}\right] \\
&\qquad = \left(1 - \gamma_k - \frac{\partial \gamma_k}{\partial t_k}(1 - \mu\phi_y)\left[t_k - H_{r\gamma}\right]\right)\left(q_k + \frac{\partial q_k}{\partial x_k} y_k\right)
\end{aligned}$$

and plugging in our value of $\frac{MC_k}{q_k}$ and the marginal surplus elasticity from before,

$$\frac{\partial q_k}{\partial t_k}\left[(1-(1-\gamma_k)t_k)\left(1+\frac{1}{N_k\varepsilon_{ms_k}}+\frac{\varepsilon_{x_k}-\frac{1}{N_k}}{\varepsilon_{s_k}}\right)\right]$$
$$=\left(1-\gamma_k-\frac{\partial\gamma_k}{\partial t_k}(1-\mu\phi_y)\left[t_k-H_{r\gamma}\right]\right)\left(q_k+\frac{\partial q_k}{\partial x_k}y_k\right)$$

The process for the pass-through rate of the enforcement parameter is very similar. Thus, we can write our final pass-through rates as

$$\frac{\partial q_k}{\partial t_k}=\frac{\left(1-\gamma_k-\frac{\partial\gamma_k}{\partial t_k}(1-\mu\phi_y)\left[t_k-H_{r\gamma}\right]\right)\left(q_k+\frac{\partial q_k}{\partial x_k}y_k\right)}{(1-(1-\gamma_k)t_k)\left(1+\frac{1}{N_k\varepsilon_{ms_k}}+\frac{\varepsilon_{x_k}-\frac{1}{N_k}}{\varepsilon_{s_k}}\right)}$$

$$\frac{\partial q_k}{\partial \beta_k}=\frac{\left(H_{\gamma\beta}-\frac{\partial\gamma_k}{\partial \beta_k}(1-\mu\phi_y)\left[t_k-H_{r\gamma}\right]\right)\left(q_k+\frac{\partial q_k}{\partial x_k}y_k\right)}{(1-(1-\gamma_k)t_k)\left(1+\frac{1}{N_k\varepsilon_{ms_k}}+\frac{\varepsilon_{x_k}-\frac{1}{N_k}}{\varepsilon_{s_k}}\right)}$$

A.2.3 Proof of Lemma X

The derivation follows closely to Adachi and Fabinger (2022). Suppose that the firm faces both an excise (v_k) and an ad valorem tax t_k on output. Then the production FOC is

$$(1-(1-\gamma_k)t_k-H_r)\left[q+\frac{\partial q_k}{\partial x_k}y_k\right]=\phi_y+v_k(1-\gamma_k)$$

The effective marginal cost to the firm is

$$\tilde{MC}_k=\frac{\phi_y+v_k(1-\gamma_k)}{1-(1-\gamma_k)t_k-H_r}$$

Consider changes in each tool to keep this effective marginal cost constant

$$\frac{\partial}{\partial t_k}\left(\frac{\phi_y+v_k(1-\gamma_k)}{1-(1-\gamma_k)t_k-H_r}\right)dt_k+\frac{\partial}{\partial v_k}\left(\frac{\phi_y+v_k(1-\gamma_k)}{1-(1-\gamma_k)t_k-H_r}\right)dv_k=0$$

which becomes

$$\left(\frac{(1-(1-\gamma_k)t_k-H_r)(-v_k\frac{\partial\gamma_k}{\partial t_k})+(1-\gamma_k-\frac{\partial\gamma_k}{\partial t_k}(t_k-H_{r\gamma}))(\phi_y+v_k(1-\gamma_k))}{(1-(1-\gamma_k)t_k-H_r)^2}\right)dt_k$$
$$+\left(\frac{(1-(1-\gamma_k)t_k-H_r)(1-\gamma_k-\frac{\partial\gamma_k}{\partial v_k}v_k)+(\phi_y+v_k(1-\gamma_k))(\frac{\partial\gamma_k}{\partial v_k}(t_k-H_{r\gamma}))}{(1-(1-\gamma_k)t_k-H_r)^2}\right)dv_k=0$$

which simplifies to

$$\left(\frac{(1-(1-\gamma_k)t_k-H_r)(-v_k\frac{\partial\gamma_k}{\partial t_k})+(1-\gamma_k-\frac{\partial\gamma_k}{\partial t_k}(t_k-H_{r\gamma}))(\phi_y+v_k(1-\gamma_k))}{(1-(1-\gamma_k)t_k-H_r)^2}\right)dt_k$$
$$+\left(\frac{(1-(1-\gamma_k)t_k-H_r)(1-\gamma_k-\frac{\partial\gamma_k}{\partial v_k}v_k)+(\phi_y+v_k(1-\gamma_k))(\frac{\partial\gamma_k}{\partial v_k}(t_k-H_{r\gamma}))}{(1-(1-\gamma_k)t_k-H_r)^2}\right)dv_k=0$$

Clearly this does not work. Instead, if we did things in terms of "effective tax rates" from the firm's perspective, i.e., $\tau_k = (1-\gamma_k)t_k + H_r$ and $\nu_k = (1-\gamma_k)t_k$. We get that

$$\frac{\partial}{\partial \tau_k}\left(\frac{\phi_k + \nu_k}{1-\tau_k}\right) d\tau_k + \frac{\partial}{\partial \nu_k}\left(\frac{\phi_k + \nu_k}{1-\tau_k}\right) d\nu_k = 0$$

which will then lead to the exact same conclusion and Adachi and Fabinger (2022).

A.2.4 Avoidance Response

For the avoidance response, we differentiate the avoidance first order condition with respect to our two tax tools. As before, we start with the tax rate.

$$(y_k q_k - \mu F) + (t_k - H_{\gamma r})\left(\frac{\partial y_k}{\partial t_k} q_k + y_k \frac{\partial q_k}{\partial t_k} - \mu \phi_y \frac{\partial y_k}{\partial t_k}\right) = H_{\gamma\gamma}\frac{\partial \gamma_k}{\partial t_k}$$

which can be rearranged as

$$H_{\gamma\gamma}\frac{\partial \gamma_k}{\partial t_k} = y_k q_k - \mu F + (t_k - H_{\gamma r}) y_k \frac{\partial q_k}{\partial t_k}\left(1 - \varepsilon_{x_k} + \mu \frac{\phi_y}{q_k}\varepsilon_{x_k}\right)$$

or

$$\frac{\partial \gamma_k}{\partial t_k} = \frac{r_k}{H_{\gamma\gamma}} + \frac{(t_k - H_{\gamma r})}{H_{\gamma\gamma}} y_k \frac{\partial q_k}{\partial t_k}\left[1 - \left(1 - \mu\frac{\phi_y}{q_k}\right)\varepsilon_{x_k}\right]$$

On the enforcement side, we have that

$$(t_k - H_{\gamma r})\left(\frac{\partial y_k}{\partial \beta_k} q_k + y_k \frac{\partial q_k}{\partial t_k} - \mu \phi_y \frac{\partial y_k}{\partial t_k}\right) = H_{\gamma\gamma}\frac{\partial \gamma_k}{\partial \beta_k} + H_{\gamma\beta}$$

which can be rearranged as

$$\frac{\partial \gamma_k}{\partial \beta_k} = -\frac{H_{\gamma\beta}}{H_{\gamma\gamma}} + \frac{(t_k - H_{\gamma r})}{H_{\gamma\gamma}} y_k \frac{\partial q_k}{\partial \beta_k}\left[1 - \left(1 - \mu\frac{\phi_y}{q_k}\right)\varepsilon_{x_k}\right]$$

Lastly, taking the avoidance FOC, we have that

$$t_k r_k = H_\gamma$$

Using our definition of the elasticity of marginal evasion costs, we have that

$$t_k - H_{\gamma r} = t_k\left(1 - \frac{H_{\gamma r}}{t_k}\right) = t_k\left(1 - H_{\gamma r}\frac{H_\gamma}{r}\right) = t_k(1 - \varepsilon^r_{H_\gamma})$$

Thus,

$$t_k - H_{\gamma r} > 0 \iff \varepsilon^r_{H_\gamma} < 1$$

A.2.5 Welfare Effect

The effect of a change in one of the tax tools on welfare is the effect on the representative consumer, i.e.,

$$\frac{dW}{d\theta_k} = \alpha\left[-\frac{\partial q_k}{\partial \theta_k}x_k + \frac{\partial \Pi_k}{\partial \theta_k}\right]$$

where $\theta_k \in \{t_k, \beta_k\}$. Starting with the tax rate, we plug in our value for the profit change calculated previously to get

$$\frac{1}{\alpha \Pi_k^P}\frac{dW}{dt_k} = -\left[1 + \frac{\partial q_k}{\partial t_k}\frac{x_k}{Z_k}\left[1 - (1-(1-\gamma_k)t_k - H_r)\frac{N_k - 1}{N_k}\right]\right]$$

We can define the "1" as the mechanical dollar raised, while the rest of the expression is the excess burden. Thus, we can simplify this expression as

$$\frac{1}{\alpha Z_k}\frac{dW}{dt_k} = -\left[1 + EB_k\right]$$

Similarly, for the enforcement, we have that the welfare effect is

$$\frac{1}{\alpha Z_k}\frac{dW}{d\beta_k} = -\left[\frac{N_k H_\beta}{Z_k} + \frac{\partial q_k}{\partial \beta_k}\frac{x_k}{Z_k}\left[1 - (1-(1-\gamma_k)t_k - H_r)\frac{N_k - 1}{N_k}\right]\right]$$

where we can equivalently define an excess burden type term

$$\frac{1}{\alpha \Pi_k^P}\frac{dW}{d\beta_k} = -\left[\frac{N_k H_\beta}{Z_k} + EB_k^\beta\right]$$

As we have previously shown, under a pure profit tax where $\mu = 1$, then the price pass-through is zero for both tool, and therefore, we have that $EB_k = EB_k^\beta = 0$. Thus, the per taxable income welfare effect of taxation is independent of competition (and firm size). Meanwhile, the welfare effect of enforcement is dependent on

A.2.6 Taxable Income Effect

The relevant elasticity is the elasticity of taxable income, which is reported revenue is the output tax ($\mu = 0$) case and reported profits in the profit tax case ($\mu = 1$). Then the defined elasticity of reported taxable income in the paper can be derived as

$$\begin{aligned}
\frac{\varepsilon_{Z_k}}{1-t_k} &= \frac{1}{Z_k}\frac{\partial Z_k}{\partial(1-t_k)} \\
&= \frac{\partial(1-\gamma_k)}{\partial(1-t_k)}\frac{R_k}{Z_k} - \frac{(1-\gamma_k)x_k}{Z_k}\frac{\partial q_k}{\partial t_k}\left[1 - \varepsilon_{x_k} + \mu\frac{\phi_y}{q_k}\varepsilon_{x_k}\right] \\
&= \frac{\partial(1-\gamma_k)}{\partial(1-t_k)}\frac{1}{1-\gamma_k} - \frac{(1-\gamma_k)x_k}{Z_k}\frac{\partial q_k}{\partial t_k}\left[1 - \left(1 - \mu\frac{\phi_y}{q_k}\right)\varepsilon_{x_k}\right]
\end{aligned}$$

Under a profit tax, the second term drops out, and thus, we have that

$$\varepsilon_{Z_K} = \varepsilon_{1-\gamma_k}$$

i.e., that the elasticity of reported taxable income is exactly equal to the tax elasticity of avoidance rate. Similarly, for our elasticity of enforcement, we have

$$\begin{aligned}
\frac{\varepsilon_{Z_k}^{\beta_k}}{\beta_k} &= \frac{1}{Z_k}\frac{\partial Z_k}{\partial \beta_k} \\
&= \frac{\partial(1-\gamma_k)}{\partial \beta_k}\frac{R_k}{Z_k} - \frac{(1-\gamma_k)x_k}{Z_k}\frac{\partial q_k}{\partial \beta_k}\left[1-\varepsilon_{x_k} - \mu\frac{\phi_y}{q_k}\varepsilon_{x_k}\right] \\
&= \frac{\partial(1-\gamma_k)}{\partial \beta_k}\frac{1}{1-\gamma_k} - \frac{(1-\gamma_k)x_k}{Z_k}\frac{\partial q_k}{\partial \beta_k}\left[1-\left(1-\mu\frac{\phi_y}{q_k}\right)\varepsilon_{x_k}\right]
\end{aligned}$$

and once again, under a profit tax the enforcement elasticity of taxable income is equal to enforcement elasticity of avoidance

$$\varepsilon_{Z_K}^{\beta_k} = \varepsilon_{1-\gamma_k}^{\beta_k}$$

Returning to our tax expression, if we plug in our expression for the evasion response, we

$$\begin{aligned}
\frac{\varepsilon_{Z_k}}{1-t_k} &= \left(\frac{r_k}{H_{\gamma\gamma}} + \frac{(t_k - H_{\gamma r})}{H_{\gamma\gamma}}y_k\frac{\partial q_k}{\partial t_k}\left[1-\left(1-\mu\frac{\phi_y}{q_k}\right)\varepsilon_{x_k}\right]\right)\frac{1}{1-\gamma_k} \\
&\quad - \frac{(1-\gamma_k)x_k}{Z_k}\frac{\partial q_k}{\partial t_k}\left[1-\left(1-\mu\frac{\phi_y}{q_k}\right)\varepsilon_{x_k}\right] \\
&= \frac{r_k}{(1-\gamma_k)H_{\gamma\gamma}} + \left(\frac{(t_k - H_{\gamma r})y_k}{(1-\gamma_k)H_{\gamma\gamma}} - \frac{(1-\gamma_k)x_k}{Z_k}\right)\frac{\partial q_k}{\partial t_k}\left[1-\left(1-\mu\frac{\phi_y}{q_k}\right)\varepsilon_{x_k}\right]
\end{aligned}$$

A.3 Perfectly Competitive Benchmarks

In this section, we benchmark market responses to the competitive limit. First, the production FOC reduces to

$$(1-(1-\gamma_k)t_k - H_r)q_k = (1-\mu[(1-\gamma_k)t_k + H_r])\phi_y$$

while the avoidance condition has the same form as before.

A.3.1 Price Response

To get the price response for a competitive market, rather than differentiating the production condition, we utilize the zero profit condition. For a change in the tax tool, the change in profits must remain zero, so we have by the envelope condition that

$$y_k\frac{\partial q_k}{\partial t_k} - H_r\left[y_k\frac{\partial q_k}{\partial t_k}\right] - z_k - t_k\left[(1-\gamma_k)\left[y_k\frac{\partial q_k}{\partial t_k}\right]\right] = 0$$

which we can rearrange as

$$\frac{\partial q_k}{\partial t_k} = \frac{z_k}{y_k(1-t_k(1-\gamma_k)-H_r)}$$

The enforcement pass-through in a competitive market is

$$\frac{\partial q_k}{\partial \beta_k} = \frac{H_\beta}{y_k(1-t_k(1-\gamma_k)-H_r)}$$

A.3.2 Welfare

Since the change in profits is zero, the only effect is the change in the consumer's income due to the price effect. This means

$$\frac{1}{\alpha Z_k}\frac{dv}{dt_k} = -\frac{\partial q_k}{\partial t_k}\frac{x_k}{Z_k} = -1 - \left[\frac{\partial q_k}{\partial t_k}\frac{x_k}{Z_k} - 1\right]$$

$$\frac{1}{\alpha Z_k}\frac{dv}{d\beta_k} = -\frac{\partial q_k}{\partial \beta_k}\frac{x_k}{Z_k} = -1 - \left[\frac{\partial q_k}{\partial \beta_k}\frac{x_k}{Z_k} - 1\right]$$

Plugging in the pass-through expressions we found above, we get

$$\frac{1}{\alpha Z_k}\frac{dv}{dt_k} = -\frac{1}{1 - t_k(1-\gamma_k) - H_r} = -1 - \left[\frac{1}{1 - t_k(1-\gamma_k) - H_r} - 1\right]$$

$$\frac{1}{\alpha Z_k}\frac{dv}{d\beta_k} = -\frac{N_k H_\beta / Z_k}{1 - t_k(1-\gamma_k) - H_r} = -1 - \left[\frac{N_k H_\beta / Z_k}{1 - t_k(1-\gamma_k) - H_r} - 1\right]$$

A.3.3 Reported Income

A.4 Core Results

In this section, we derive the core results of the paper–primarily the optimal tax and enforcement level and tradeoff expressions.

A.4.1 Optimal Levels (Proposition 2)

We begin with the government's Lagrangian:

$$\mathcal{L} = v(q) + \lambda\left[\sum_j t_j Z_j - \beta_j R_j - G\right]$$

Differentiating this expression with respect to the tax rate of an inudstry k, we get

$$\alpha\left[-\frac{\partial q_k}{\partial t_k}x_k + \frac{\partial \Pi_k}{\partial t_k}\right] + \lambda\left[Z_k + t_k\frac{\partial Z_k}{\partial t_k} - \beta_k\frac{\partial R_k}{\partial t_k}\right]$$

Dividing through the previous expression by reported income, we get

$$\frac{\alpha}{\lambda}\frac{1}{Z_k}\left[-\frac{\partial q_k}{\partial t_k}x_k + \frac{\partial \Pi_k}{\partial t_k}\right] + 1 - \frac{t_k}{1-t_k}\varepsilon_{Z_k} - \beta_k\frac{1}{z_k}\frac{\partial R_k}{\partial t_k}$$

where the elasticity of reported taxable income is defined with respect to the retention rate $1 - t_k$. Using our work from previous section, we define

$$1 + EB_k = \frac{1}{z_k}\left[\frac{\partial q_k}{\partial t_k}x_k - \frac{\partial \Pi_k}{\partial t_k}\right]$$

and thus, write this tax result in the simple form

$$\frac{t_k}{1-t_k} = \frac{1 - \frac{\alpha}{\lambda}\left[1 + EB_k\right] - \frac{\beta_k}{z_k}\frac{\partial R_k}{\partial t_k}}{\varepsilon_{z_k}}$$

or, if we include enforcement costs with the taxable income elasticity to get a "net revenue elasticity", we can write this as

$$\frac{t_k}{1-t_k} = \frac{1 - \frac{\alpha}{\lambda}\left[1 + EB_k\right]}{\varepsilon_{NR_k}}$$

Similarly, on the enforcement side, we can differentiate the expression with respect to enfocement to get

$$\alpha\left[-\frac{\partial q_k}{\partial \beta_k}x_k + \frac{\partial \Pi_k}{\partial \beta_k}\right] + \lambda\left[t_k\frac{\partial Z_k}{\partial \beta_k} - R_k - \beta_k\frac{\partial R_k}{\partial \beta_k}\right]$$

which we can similarly convert to

$$-\frac{\alpha}{\lambda}\left[1 + EB_k^{\beta}\right] + \frac{t_k}{\beta_k}\varepsilon_{z_k}^{\beta_k} - \frac{R_k}{z_k} - \frac{\beta_k}{z_k}\frac{\partial R_k}{\partial \beta_k}$$

giving us

$$\beta_k = \frac{t_k\varepsilon_{z_k}^{\beta_k}}{\frac{R_k}{z_k} + \frac{\beta_k}{z_k}\frac{\partial R_k}{\partial \beta_k} + \frac{\alpha}{\lambda}\left[\frac{N_k H_\beta}{Z_k} + EB_k^{\beta}\right]}$$

or, once again using a net revenue elasticity that includes enforcement costs into the income elasticity,

$$\beta_k = \frac{t_k\varepsilon_{NR_k}^{\beta_k}}{\frac{\alpha}{\lambda}\left[\frac{H_\beta}{z_k} + EB_k^{\beta}\right]}$$

Alternatively, instead of explicity finding an enforcement rate, we simply illustrate that the enforcement elasticity is

$$\varepsilon_{z_k}^{\beta} = \frac{\beta_k\frac{\alpha}{\lambda}\left[\frac{H_\beta}{z_k} + EB_k^{\beta}\right] + \beta_k^2\frac{\partial R_k}{\partial \beta_k}}{t_k Z_k}$$

as in Keen and Slemrod (2017). In this form, the RHS is, as they describe, an adjusted cost-to-revenue ratio and thus the elasticity of enforcement fully captures the tradeoff.

A.4.2 Proof of Corollary X1

Under a profit tax ($\mu = 1$), we have that $EB_k = EB_K^{\beta} = 0$. Additionally, we have that $\varepsilon_{Z_k} = \varepsilon_{1-\gamma_k}$ and $\varepsilon_{Z_k}^{\beta} = \varepsilon_{1-\gamma_k}^{\beta}$. Thus, we can simplify our two prior expressions down to

$$\frac{t_k}{1-t_k} = \frac{1-\frac{\alpha}{\lambda}}{\varepsilon_{\gamma_k}}, \qquad \beta_k = \frac{t_k\varepsilon_{\gamma_k}^{\beta_k}}{\frac{R_k}{Z_k} + \frac{\alpha}{\lambda}\frac{H_\beta}{z_k}}$$

Note that while we rewrote the behavioral elasticity to just be the avoidance elasticity to highlight the lack of production based distortion, the sufficient statistic would still be the elasticity of taxable income.

A.4.3 Proof of Corollary X2

Under constant scale avoidance, we have that ε_{γ_k} is independent of the market power within a market. Thus, the tax rate does not depend on market concentration. Similarly, $\varepsilon^{\beta}_{\gamma_k}$ is independent of market power. Under constant scale avoidance, we have that $\frac{H_\beta}{z_k} \not\propto z_k$, and therefore the enforcement level is also independent of market power.

A.4.4 Tradeoff (Proposition X2)

We will now also get our expression for the optimal tradeoff, which we will do so in two ways. First, we will simply divide our previous two expressions and obtain a Keen and Slemrod (2017) style tradeoff between policy elasticities in addition to the excess burden costs.

$$\frac{\frac{t_k}{1-t_k}}{\beta_k} = \frac{\left(1 - \frac{\alpha}{\lambda}\left[1 + EB_k\right]\right)\left(\frac{\alpha}{\lambda}\left[\frac{N_k H_\beta}{Z_k} + EB_k^\beta\right]\right)}{t_k \varepsilon_{NR_k} \varepsilon^{\beta}_{NR_k}}$$

We alternatively manipulate our original two first order conditions to get a price neutral change in the two tax tools. For the tax, we separate out the revenue and avoidance responses

$$\alpha\left[-\frac{\partial q_k}{\partial t_k} x_k + \frac{\partial \Pi_k}{\partial t_k}\right] + \lambda\left[Z_k + t_k\left[\frac{\partial(1-\gamma_k)}{\partial t_k} R_k + (1-\gamma_k)\frac{\partial R_k}{\partial t_k}\right] - \beta_k \frac{\partial R_k}{\partial t_k}\right]$$

We've previously found that

$$\begin{aligned}\frac{\partial R_k}{\partial t_k} &= \frac{\partial x_k}{\partial t_k} q_k + x_k \frac{\partial q_k}{\partial t_k} - \mu\phi_y \frac{\partial x_k}{\partial t_k} \\ &= \frac{\partial q_k}{\partial t_k} x_k \left[1 - \left[\frac{q_k - \mu\phi_y}{q_k}\right]\varepsilon_{x_k}\right]\end{aligned}$$

which means, we can convert our expression to

$$\frac{\partial q_k}{\partial t_k}\left[-\frac{\alpha}{\lambda} x_k + x_k\left[1 - \left[\frac{q_k - \mu\phi_y}{q_k}\right]\varepsilon_{x_k}\right]\left[(1-\gamma_k)t_k - \beta_k\right]\right] = -Z_k - t_k \frac{\partial(1-\gamma_k)}{\partial t_k} R_k - \frac{\alpha}{\lambda}\frac{\partial \Pi_k}{\partial t_k}$$

For the enforcement tool, an equivalent process gets us

$$\frac{\partial q_k}{\partial \beta_k}\left[-\frac{\alpha}{\lambda} x_k + x_k\left[1 - \left[\frac{q_k - \mu\phi_y}{q_k}\right]\varepsilon_{x_k}\right]\left[(1-\gamma_k)t_k - \beta_k\right]\right] = R_k - t_k \frac{\partial(1-\gamma_k)}{\partial \beta_k} R_k - \frac{\alpha}{\lambda}\frac{\partial \Pi_k}{\partial \beta_k}$$

Dividing the two equations and rearranging brings us to the expression in the proposition

$$\left[1 - \gamma_k + \frac{\frac{\partial q_k}{\partial t_k}}{\frac{\partial q_k}{\partial \beta_k}}\right] - t_k\left[\frac{\partial \gamma_k}{\partial t_k} - \frac{\partial \gamma_k}{\partial \beta_k}\frac{\frac{\partial q_k}{\partial t_k}}{\frac{\partial q_k}{\partial \beta_k}}\right] + \frac{\alpha}{\lambda}\frac{1}{R_k}\left[\frac{\partial \Pi_k}{\partial t_k} - \frac{\partial \Pi_k}{\partial \beta_k}\frac{\frac{\partial q_k}{\partial t_k}}{\frac{\partial q_k}{\partial \beta_k}}\right] = 0$$

Note that we could have instead derived an equation for when an increase in the tax rate and a decrease in the enforcement rate is welfare positive (outside of an optimum). To do so, we differentiate the Lagagrangian with respect to the tax rate without assuming we are at an optimum (not setting the first expression equal to 0). Then we differentiate the Lagrangian with respect to the enforcement rate at an amount that cancels out the price change of the first change (i.e., the ratio of the two pass-throughs). Adding these two expressions together gets us the same above expression, and we can ask when it is greater than or less than 0 (equal to 0 at the optimum).

Since there is no output distortion under a profit tax, we instead add only the RHS of the two FOCs (zeroing out the price related terms) to get

$$\left[1 - \gamma_k + 1\right] - t_k\left[\frac{\partial \gamma_k}{\partial t_k} - \frac{\partial \gamma_k}{\partial \beta_k}\right] + \frac{\alpha}{\lambda}\frac{1}{R_k}\left[\frac{\partial \Pi_k}{\partial t_k} - \frac{\partial \Pi_k}{\partial \beta_k}\right] = 0$$

A.4.5 Proof of Corollary X2

Under a profit tax ($\mu = 1$), there is no price/output distortion. Then we take our two optimal expressions and simply divide them to get

$$\frac{\frac{t_k}{1-t_k}}{\beta_k} = \frac{\left(1-\frac{\alpha}{\lambda}\right)\left(\frac{R_k}{Z_k}+\frac{\alpha}{\lambda}\frac{1}{Z_k}\frac{H_\beta}{z_k}\right)}{t_k\varepsilon_{\gamma_k}\varepsilon^{\beta}_{\gamma_k}}$$

Again, outside of the optimum, we can simply ask when the above is greater than 0 for a tax-increase, enforcement decrease policy change.

A.5 Firm Costs, Avoidance, and Relation to Market Power

Of central importance to this paper is the relationship between firm size (or probability in the case of the profit tax) and avoidance in the firm's cost function. To examine this in more detail, again consider the firm's avoidance FOC

$$t_k r_k = (1-\mu t_k)H_\gamma(r_k, \gamma_k, \beta_k)$$

For expositional simplicity, we will consider the class of multiplicatively separable cost functions

Assumption 2. *Avoidance costs to the firm are multiplicatively separable such that* $H(r_k, \gamma_k, \beta_k) = G(r_k)I(\gamma_k)J(\beta_k)$. $I(\cdot)$ *is a strictly increasing monotonic, twice continuously differentiable, strictly convex function.*

A.5.1 Proof of Lemma X

Under Assumption 2, we have that the optimal choice in the avoidance rate satisfies

$$I(\gamma_k) = \frac{r_k}{G(r_k)}\frac{t_k}{(1-\mu t_k)J(\beta)}$$

Since $I_{\gamma\gamma} > 0$, then I_γ is strictly monotone and therefore has an inverse I_γ^{-1}. This implies that

$$\gamma_k^* = I_\gamma^{-1}\left(\frac{r_k}{G(r_k)}\frac{t_k}{(1-\mu t_k)J(\beta)}\right)$$

where I_γ^{-1} is a monotonic increasing function (since we assumed I_γ is). Clearly, this implies that if $G(\cdot)$ is strictly convex, and thus the denominator inside the parentheses increases faster than the numerator, then γ decreases as individual firm size increases. This occurs, in our Cournot framework, when the number of firms decreases, i.e., lower market power. Thus, we conclude that a strictly convex $G(\cdot)$ implies decreasing avoidance as market power increases, and a strictly concave $G(\cdot)$ implies increasing avoidance as market power decreases.

A.5.2 Proof of Lemma X2

Total avoidance costs for an optimally chosen leve of avoidance are given by

$$H(r_k, \gamma_k^*, \beta_k) = G(r_k)I\left(I_\gamma^{-1}\left(\frac{r_k}{G(r_k)}\frac{t_k}{(1-\mu t_k)J(\beta)}\right)\right)J(\beta_k)$$

And total costs per dollar of firm size ("average cost" of sorts)

$$\frac{H(r_k,\gamma_k^*,\beta_k)}{r_k} = \frac{G(r_k)}{r_k} I\left(I_\gamma^{-1}\left(\frac{r_k}{G(r_k)}\frac{t_k}{(1-\mu t_k)J(\beta)}\right)\right) J(\beta_k)$$

Let $\nu \equiv \frac{r_k}{G(r_k)}$. Then the relationship between average costs and firm size will be determined by the functional form of

$$\frac{1}{\nu} I(I_\nu^{-1}(D\nu))$$

where D is just some constant. We know that I is montonic increasing and strictly convex, and I_ν^{-1} is monotonic strictly increasing. Then $I(I_\nu^{-1}(D\nu))$ is convex in ν. Thus, we can conclude that the above expression is weakly proportional to ν (it may be constant). Thus, when firm size facilitates avoidance, average costs weakly increase. Note the same relation holds for average marginal costs of enforcement activity on the firm, i.e., $\frac{H_\beta}{r_k}$.

A.6 Comparative Statics on Market Power

We first reexamine our elasticity of reported revenue, but paricularly focus on the true income response:

$$\frac{\varepsilon_{R_k}}{1-t_k} = \frac{1}{q_k}\frac{\partial q_k}{\partial t_k}\left[1-\left(1-\mu\frac{\phi_y}{q_k}\right)\varepsilon_{x_k}\right]$$

Under constant scale avoidance, this is the only term that matters for determining how the reported income elasticity changes with respect to market power. To derive intuition, we first consider how this changes under an output tax $\mu = 0$. Differentiating this condition, we get

$$\frac{d}{dN_k}\left(\frac{\partial q_k}{\partial t_k}\frac{1}{q_k}(1-\varepsilon_{x_k})\right) = \frac{d}{dN_k}\left(\frac{\partial q_k}{\partial t_k}\frac{1}{q_k}\right)(1-\varepsilon_{x_k}) + \frac{\partial q_k}{\partial t_k}\frac{1}{q_k}\frac{d}{dN_k}(1-\varepsilon_{x_k})$$

Under log-concave demand, the pass-through rate increases with the number of firms, meaning that the first term is positive when $\varepsilon_{x_k} < 1$ and negative when $\varepsilon_{x_k} > 1$.

A.6.1 Proof of Proposition X3

Proof. We differentiate the elastity of reported revenue with respect to the number of firms. Under constaint avoidance evasion technology, the first term is independent of firm size, and therefore independent of the number of firms. Thus, the relationship hinges on how the second term changes with a rising number of firms. More directly, the relevant expression is

$$\frac{d}{dN_k}\left(\frac{\partial q_k}{\partial t_k^o}\frac{1}{q_k}(\varepsilon_{x_k}-1)\right) = \frac{d}{dN_k}\left(\frac{\partial q_k}{\partial t_k^o}\frac{1}{q_k}\right)(\varepsilon_{x_k}-1) + \frac{\partial q_k}{\partial t_k^o}\frac{1}{q_k}\frac{d}{dN_k}(\varepsilon_{x_k}-1)$$

Under log-concave demand, the pass-through rate increases with the number of firms, meaning that the first term is negative when $\varepsilon_{x_k} < 1$. Similarly, the demand elasticity is decreasing with the number of firms, which means the second term is also negative. Thus, the overall expression is negative, meaning the elasticity of reported revenue is smaller. For log-convex demand functions, the pass-through rate is decreasing with number of firms, and the demand elasticity is increasing with number of firms. As long as $\varepsilon_{x_k} < 1$, both terms are positive and thus the elasticity of reported revenue is larger. $\square$

A.6.2 Proof of Proposition X4

Proof. We differentiate the elastity of reported revenue with respect to the number of firms. Under constaint avoidance evasion technology, the first term is independent of firm size, and therefore independent of the number of firms. Thus, the relationship hinges on how the second term changes with a rising number of firms. More directly, the relevant expression is

$$\frac{d}{dN_k}\left(\frac{H_{\gamma\beta}}{H_{\gamma\gamma}}\frac{1}{(1-\gamma_k)x_k q_k}+\frac{\partial q_k}{\partial t_k^o}\frac{1}{q_k}(\varepsilon_{x_k}-1)\right)=\frac{d}{dN_k}\left(\frac{\partial q_k}{\partial t_k^o}\frac{1}{q_k}\right)(1-\varepsilon_{x_k})+\frac{\partial q_k}{\partial \beta_k}\frac{1}{q_k}\frac{d}{dN_k}(1-\varepsilon_{x_k})$$
$$+\frac{d}{dN_k}\frac{H_{\gamma\beta}}{H_{\gamma\gamma}}\frac{1}{(1-\gamma_k)x_k q_k}$$

Under log-concave demand, the pass-through rate increases with the number of firms, meaning that the first term is positive when $\varepsilon_{x_k} < 1$. Similarly, the demand elasticity is decreasing with the number of firms, which means the second term is also positive. The sign of the last term depends on the firm revenue in the denominator. Since the revenue change with respect to the number of firms is given by

$$\frac{d}{dN}x_k q_k = \frac{\partial q_k}{\partial N_k}(1-\varepsilon_{x_k})x_k$$

The price should decrease with the number of firms, and therefore if $\varepsilon_{x_k} < 1$, the total revenue should decrease. Since this term is in the denominator, the overall change of this term is positive. Thus, all three terms are positive and the overall expression is positive, meaning the elasticity of reported revenue is larger. For log-convex demand functions, the pass-through rate is decreasing with number of firms, the demand elasticity is increasing with number of firms. However, the last term is still positive and we cannot explicitly sign the overall expression. □

A.7 Uniform Taxation

Under uniform taxation, the Lagrangian is

$$\mathcal{L} = v(q) + \lambda\left[\sum_j tZ_j - \beta_j R_j - G\right]$$

Differentiating this expression with respect to the tax rate, we get

$$\sum_j \alpha\left[-\frac{\partial q_j}{\partial t}x_j + \frac{\partial \Pi_j}{\partial t}\right] + \lambda\left[Z_j + t_j\frac{\partial Z_j}{\partial t_j} - \beta_j\frac{\partial R_j}{\partial t}\right] = 0$$

which we can restate as

$$\sum_j \frac{\alpha}{\lambda}\frac{Z_j}{Z_j}\left[-\frac{\partial q_k}{\partial t_k}x_k + \frac{\partial \Pi_k}{\partial t_k}\right] + Z_j - \frac{t_k}{1-t_k}Z_j\varepsilon_{Z_j} - \beta_j\frac{Z_j}{Z_j}\frac{\partial R_k}{\partial t_k} = 0$$

using the same definition of excess burden from before, we can rearrange this expression to get

$$\frac{t}{1-t} = \frac{\sum_j Z_j\left[1-\frac{\alpha}{\lambda}[1+EB_j]-\frac{\beta_j}{Z_j}\frac{\partial R_j}{\partial t}\right]}{\sum_j Z_j\varepsilon_{Z_j}}$$

or, if we include enforcement costs with the taxable income elasticity to get a "net revenue elasticity", we can write this as

$$\frac{t}{1-t} = \frac{\sum_j Z_j \left[1 - \frac{\alpha}{\lambda}\left[1 + EB_j\right]\right]}{\sum_j Z_j \varepsilon_{NR_j}}$$

The enforcement side, however, has the same expression as before since we still allow these to be differentiated. The only change is to replace the differentiated tax rate with the uniform tax rate.

A.7.1 Corollary Y

Suppose we have two industries m and n. Then we can divide the two optimal enforcement conditions to get the relative ratio of enforcement

$$\frac{\beta_m}{\beta_n} = \frac{\frac{H_\beta(z_n,\gamma_n,\beta_n)}{z_n} + EB_n}{\frac{H_\beta(z_m,\gamma_m,\beta_m)}{z_m} + EB_m} \frac{\varepsilon^{\beta}_{NR_m}}{\varepsilon^{\beta}_{NR_n}}$$

When taxes are differentiated, this expression is multiplied by $\frac{t_m}{t_n}$

A.8 Heterogenous Firms

Now we assume that the firms in each industry may differ along their productivity margin. For simplicity, assume that we have constant marginal costs and thus firms in an industry are indexed by their marginal cost scalar. Let i index each firm within an industry k. Then the Lagrangian of the government. Expression wise, the optimal formulae will all appear nearly the same since these are written in terms of aggregates. We will have to make a few adjustments.

A.9 Alternative Enforcement Costs

In this section, we consider several alternatives to the enforcement costs specified in the baseline model which are directly related to the (true) size of the industry. Again, this specification implicitly assumes that the government knows in advance or has perfect expectations on the degree of avoidance in all industries, which is how they are able to balance their budget. We can specify alternative versions of the model that do not require this assumption.

A.9.1 Per-Firm Enforcement Cost

A.10 Assumptions on the Cost Function

Note that the Hessian of the profit function of a monopoly is as follows

$$H = \begin{bmatrix} \frac{\partial q_k}{\partial y_k} + \frac{\partial^2 q_k}{\partial y_k^2} y_k + \frac{\partial q_k}{\partial y_k} - C_{yy} & t_k - C_{y\gamma} \\ t_k - C_{y\gamma} & -C_{\gamma\gamma} \end{bmatrix}$$

Then the determinant is

$$-\left[\frac{\partial^2 q_k}{\partial t_k^2} y_k + 2\frac{\partial q_k}{\partial t_k} - C_{yy}\right] C_{\gamma\gamma} - (t - C_{x\gamma})^2$$

In order to be at an optimum, it must be true that this determinant is greater than 0, and thus it be be the case that

$$(t - C_{x\gamma})^2 < -\left[\frac{\partial^2 q_k}{\partial t_k^2} y_k + 2\frac{\partial q_k}{\partial t_k} - C_{yy}\right] C_{\gamma\gamma}$$

where the right side is positive since $C_{\gamma\gamma} > 0$ and the term in the brackets is < -0 in order to be a maximum.

A.11 Monopoly Taxation: A Pigouvian Argument?

In this section, we discuss the relationship between "correcting" the monopoly problem and correcting an externality via a typical Pigouvian tax. As a monopoly leads to underproduction of a good, the naive equivalent would be a positive extenernality such that there is a gap between the private marginal benefit and social marginal benefit. To make this comparison we consider the following exercise: For a given monopoly market we find an equivalent external marginal benefit in a competitive market that would make it such that the level of production is the same across the two markets. In other words, the demand function in the monopoly market is equal to the social marginal benefit curve in the externality market. Next, we find the subsidy in each market that would close this wedge and return each market to the social equilibrium. In the externality market, this is simply equal to the marginal benefit at the point of production. The exercise then is to see whether the same is true for the monopoly market.

Consider a monopoly market producing a quantity x_m, where x_m must satisfy the profit condition equating marginal benefit and cost

$$q_m + \frac{\partial q_m}{\partial x_m} x_m = C_y$$

Let ξ_b represents the marginal externality benefit in the competitive market. The social marginal benefit is the $q_c + \xi_b$, and therefore the private equilibrium is governed by the condition

$$q_c + \xi_b = C_y$$

Thus, it should be clear that the equivalent marginal externality benefit that makes the competitive case ex ante similar to a monopoly is

$$\xi_b = \frac{\partial q_m}{\partial x_m} x_m$$

Now suppose that the government can impose a tax in each market to restore output to the socially competitive level. As stated before, if $t_c = -\xi_b$, then the competitive condition becomes

$$p_c = C_y$$

which would mean that the competitive industry is producing exactly where price equals marginal costs (inverse supply) and so producing efficiently. Is the same true for the monopoly industry? Not quite. This is because the monopoly will endogenously react to the tax rates, and so the equivalent marginal damage changes along the pathway back to the social equilibrium point. When applying the same tax rate as in the externality problem, the monopoly's first order condition becomes

$$p_m(x'_m) + \frac{\partial q_m}{\partial x'_m} x'_m - \frac{\partial q_m}{\partial x_m} x_m = C_y(x'_m) \iff p_m(x'_m) - C_y(x'_m) = \frac{\partial q_m}{\partial x_m} x_m - \frac{\partial q_m}{\partial x'_m} x'_m$$

Notice that x_m' is *not* the output which equates price and marginal cost, meaning it is not the socially optimal output. It misses the mark by the difference between the marginal surplus between the two points. So what is actual subsidy needed? It must be the case that the marginal revenue at this output equals the net marginal cost. The output, of course, should be x_c' where x_c' is the externality corrected output in the competitive side. Thus, we need that

$$p_m(x_c') + \frac{\partial q_m}{\partial x_c'} x_c' = C_y(x_c') + s_m \iff s_m = p_m(x_c') + \frac{\partial q_m}{\partial x_c'} x_c' - C_y(x_c')$$

Well the price should equal marginal cost at the social optimum and thus, it must be the case that

$$s_m = \frac{\partial q_m}{\partial x_c'} x_c'$$

Thus, the required subsidy to push the monopoly to the socially competitive level of output is the marginal surplus at this social optimum. Thus the difference between the monopoly correction and the equivalent Pigouvian correction is

$$s_m - \xi_b = \frac{\partial q}{\partial x}(x_c') x_c' - \frac{\partial q}{\partial x}(x_m) x_m$$