

CMPT360

Assignment 1 [100 points] [Sub. Deadline: Canvas]

General Instructions

Please read all the questions carefully. The questions have been designed from scratch. If you find any mathematical error in the question, please report it by email with the subject line 'COMM360'.

There are three types of questions and the rules for collaboration are stated below.

Type X: No discussion on the Canvas discussion board, but you can ask clarification questions over email. The reason this type disallows discussion on Canvas is that they are easy and the slightest hint may reveal the answer. No discussion with any individual within or beyond the class. No online search or AI search or posting.

Type Y: Any clarification question must be asked on the Canvas discussion board. No discussion with any individual within or beyond the class. No online search or AI search or posting.

Type Z: Any clarification question must be asked in the Canvas discussion board. You can discuss this only with the members of your team. Your team can consist of at most 3 students including yourself. No online search or AI search or posting.

Assume all the variables to be integers unless something else is explicitly defined in the context. To receive timely help, it is recommended to ask clarification questions as early as possible. Please see the syllabus for late submission penalty or related policy.

1 Quiz [Type X], 16 points

Please answer the multiple choice questions in Quiz 1.

Marking criteria: Automatic on Canvas, no partial mark.

2 Palindrome [Type X], 10 points

Prove or disprove: There is a prime number that divides (with no remainder) every homogeneous digit sequence of size 3.

Further Information: A prime number is a natural number greater than 1 that is not a product of two smaller natural numbers. A homogeneous digit sequence of size k is a number with k digits where all the digits are the same.

Submission Instruction: The first line of your answer should clearly state whether you are trying to prove or disprove the statement. If you want to prove, you must provide the number and argue why the statement holds. If you want to disprove, you must provide a detailed justification.

Please upload the answers to the questions 2-6 in a single .pdf file.

Marking criteria:

1: Following the submission instructions

+

1: Correctly identifying whether the statement is true or false

+

0/2/6/8: Incorrect proof, irrelevant thoughts, hard-to-understand arguments / some idea / Almost correct answer without unnecessary arguments/ A correct answer without unnecessary arguments.

3 Prime [Type X], 10 points

Prove or disprove: There exists a prime number such that the sum of its digits is 44.

Submission Instruction: The first line of your answer should clearly state whether you are trying to prove or disprove the statement. If you want to prove, you must provide the number and argue why that number is prime. If you want to disprove, you must provide a detailed justification.

Please upload the answers to the questions 2-6 in a single .pdf file.

Marking criteria:

1: Following the submission instructions

+

1: Correctly identifying whether the statement is true or false

+

0/2/6/8: Incorrect proof, irrelevant thoughts, hard-to-understand arguments / some idea / Almost correct answer without unnecessary arguments/ A correct answer without unnecessary arguments.

4 Comparing Asymptotic Complexities [Type Y], 20 points

Prove or disprove: $n^{7/2} = O(2^{\sqrt{n}})$. If you must use the definition of $O()$ notation, i.e., find appropriate constants, for your argument.

Submission Instruction: The first line of your answer should clearly state whether you are trying to prove or disprove the statement. Then you must provide a detailed justification.

Please upload the answers to the questions 2-6 in a single .pdf file.

Marking criteria:

1: Following the submission instructions

+

1: Correctly identifying whether the statement is true or false

+

0/6/12/18: Incorrect proof, irrelevant thoughts, hard-to-understand arguments / some idea / Almost correct answer without unnecessary arguments/ A correct answer without unnecessary arguments.

5 Time Complexity Analysis [Type Y], 10 points

Give the best possible asymptotic lower bound in terms of n for the following code block. Explain your answer in detail.

```

c = 1
i = n
while(i > 0){
    i = i/2
    c = c + 1
    j = 1
    while(j < c){
        j = j*2
        k = c
        while(k > j){
            k = k/4
        }
    }
}

```

Submission Instruction: Please upload the answers to the questions 2-6 in a single .pdf file.

Marking criteria:

- 0: Incorrect answer, irrelevant thoughts, hard-to-understand arguments
- 2: Limited understanding of lower bound time complexity inappropriate use of notations
- 4: Some understanding of lower bound time complexity
- 8: Almost correct answer without unnecessary arguments
- 10: A correct answer without unnecessary arguments

6 Find a Recurrence Relation [Type Y], 24 points

Define Δ_b^n , where $1 < b < n$, is the number of ways we can watch n movies with $(b-1)$ coffee breaks in between.

For example, $\Delta_2^3 = 3$ because for a set $S = \{\text{'trap'}, \text{'twisters'}, \text{'watchers'}\}$ there are three options as follows: $\{\text{'trap'}\} \odot \{\text{'twisters'}\text{'watchers'}\}$, $\{\text{'trap'}\text{'twisters'}\} \odot \{\text{'watchers'}\}$, $\{\text{'twisters'}\} \odot \{\text{'watchers'}\text{'trap'}\}$. Similarly, $\Delta_2^4 = 7$ and $\Delta_3^4 = 6$.

Write down a recurrence relation for Δ_b^n , give a justification for the recurrence, and prove by induction that $\Delta_b^n \leq n^b b^n$.

Submission Instruction: Please upload the answers to the questions 2-6 in a single .pdf file.

Marking criteria:

- 0: Incorrect answer, irrelevant thoughts, hard-to-understand arguments
- 2: A relevant recurrence relation without any correctness argument
- 4: A partially correct recurrence relation with some relevant attempt to provide correctness and inductive proof
- 10: An almost correct recurrence relation with a preliminary idea for correctness and inductive proof
- 16: An almost correct recurrence relation with some concrete justification for the correctness and correct inductive proof
- 20: A correct recurrence relation with justification and proof, but with minor problems
- 24: A correct answer

7 Spend Money [Type Z], 10 points

The following problem is a coding problem stated in the style of Kattis. If you are not familiar with Kattis, it is recommended that you solve this problem

<https://open.kattis.com/problems/addtwonumbers>

to have an understanding of the system. We will use a similar system for the evaluation of your code.

One day Mr. Minion plans to do something different in his life. He decides to spend each day half the money he currently has to plant banana seedlings and spend 50 cents to buy bananas to eat. After repeating this for k days, when Mr. Minion does not have any money left, he wants to find the original dollar amount n that he started with. Mr. Minion can derive k because the first day when he decided to take this action was his birthday. Please write a code to determine n .

Input:

The first line is the number of test cases. Each test case consists of a number representing k , where $1 \leq k \leq 30$. For example, in the following input, we have two test cases. The first test case has $k = 1$ and the second has $k = 3$.

Sample input:

```
2
1
3
```

Output:

The output consists of the value of n for each test case. For example, if $k = 1$, the initial dollar amount must be 1 because Mr. Minion spent 50 cents to plant and 50 cents to eat. If $k = 3$, the initial dollar amount must be 7 because Mr. Minion spent 4 dollars on the first day, 2 dollars on the second day, and 1 dollar on the third day.

Sample output:

1
7

Submission Instruction:

You must edit and submit the given file Minion.java

Marking criteria: The code must run without error and pass all our test cases. No partial mark. If you add/remove/edit the code at any other place except for the suggested area, then the submission will automatically be graded with 0.