Recursion.

September 6, 2024 1:07 PM

Let's write a recursive algo to sum $A[1 \ldots n]$

```
Sum ( A[i...j] )
    if i = j
        return A[i]
    else return A[i] + Sum(A[i+1...j])
```

$$A[i \ldots j] = \begin{cases} A[i] \; ; \text{ if } i = j \\ A[i] + Sum(A[i+1 \ldots j]) \end{cases}$$

bas

$$T(n) = T(n-1) + d$$

this is a const

to add A[

and to

$$= [T(n-2) + d] + d$$

$$= T(n-2) + 2d$$

$$= T(n-3) + 3d$$

$$\vdots$$

$$T(n-(n-1)) + (n-1)d$$

$= \boxed{T(1)} + (n-1)d.$

$= k + (n-1)d$

cost for which

I want to prove $T(n) = O(n)$

Assume $k + (n-1)d \leq cn.$

$\Rightarrow c \geq \frac{k}{n} + \frac{(n-1)d}{n}$

$= \frac{k}{n} + \frac{\cancel{n}d}{\cancel{n}} -$

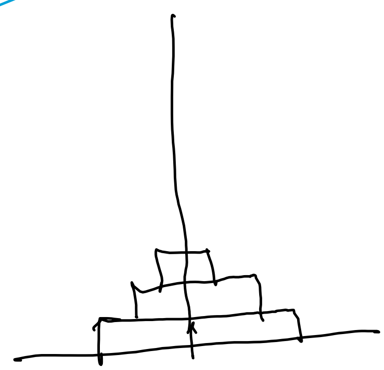
$= \frac{k}{n} - \frac{d}{n} + d$

If $n \to \infty$ then c app

I can chose a constant $c = k + d$ and

$$c = k + d \geq \frac{k}{n} - \frac{d}{n} + d \quad \text{for}$$

Tower of



A

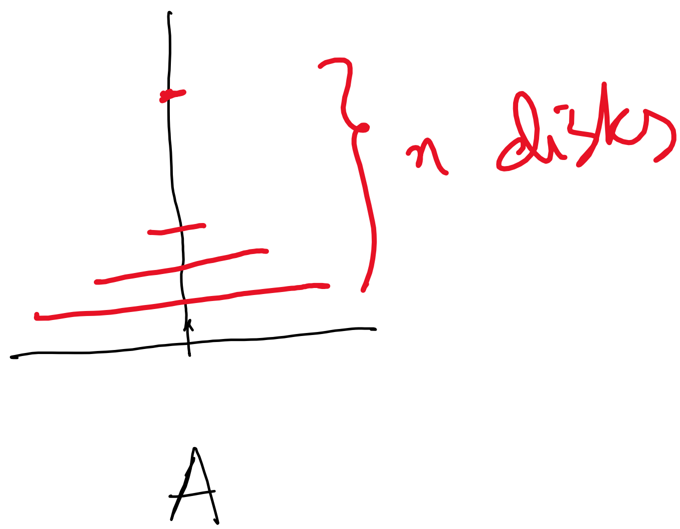
Move all the disks fr

At each step you must

At no point you co

How many steps do

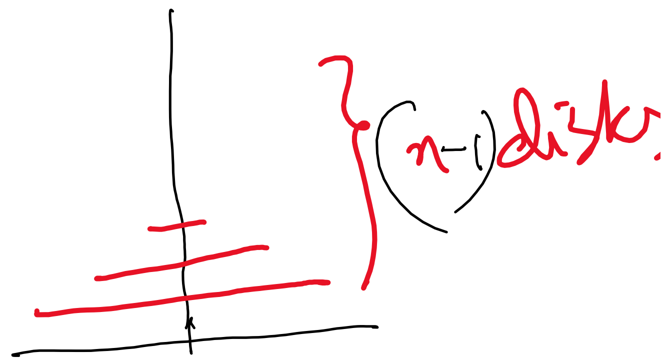
A wrong proof



I have a problem of si

I move the top disk to

problem of size (n-1)



A

By induction I will solv

$$T(n) = T(n-1)$$

$$= [T(n-2)$$

$$= T(n-2)$$

$$\vdots$$

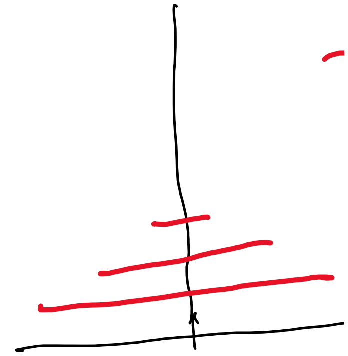
$$= T(\underbrace{n - \quad}_{T(1)}$$

$= 1 + n$

$= n$

Recursion works

for all smal



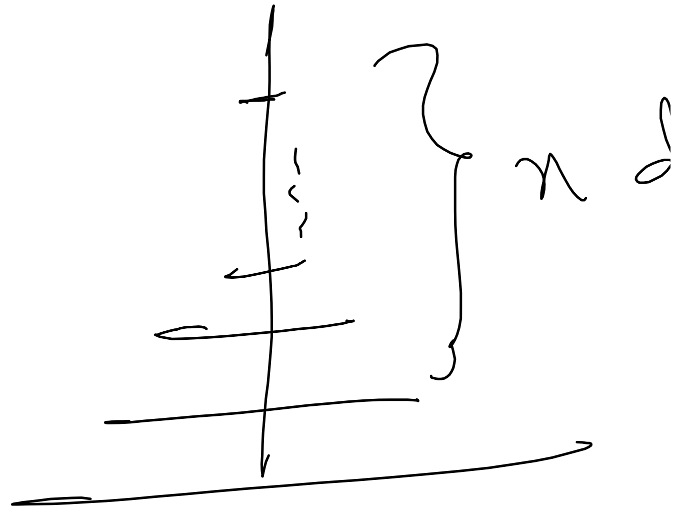
A

We cannot use

the input of

— C

Correct way to v



A

<u>Obs 1</u> If g have

using

Same

C

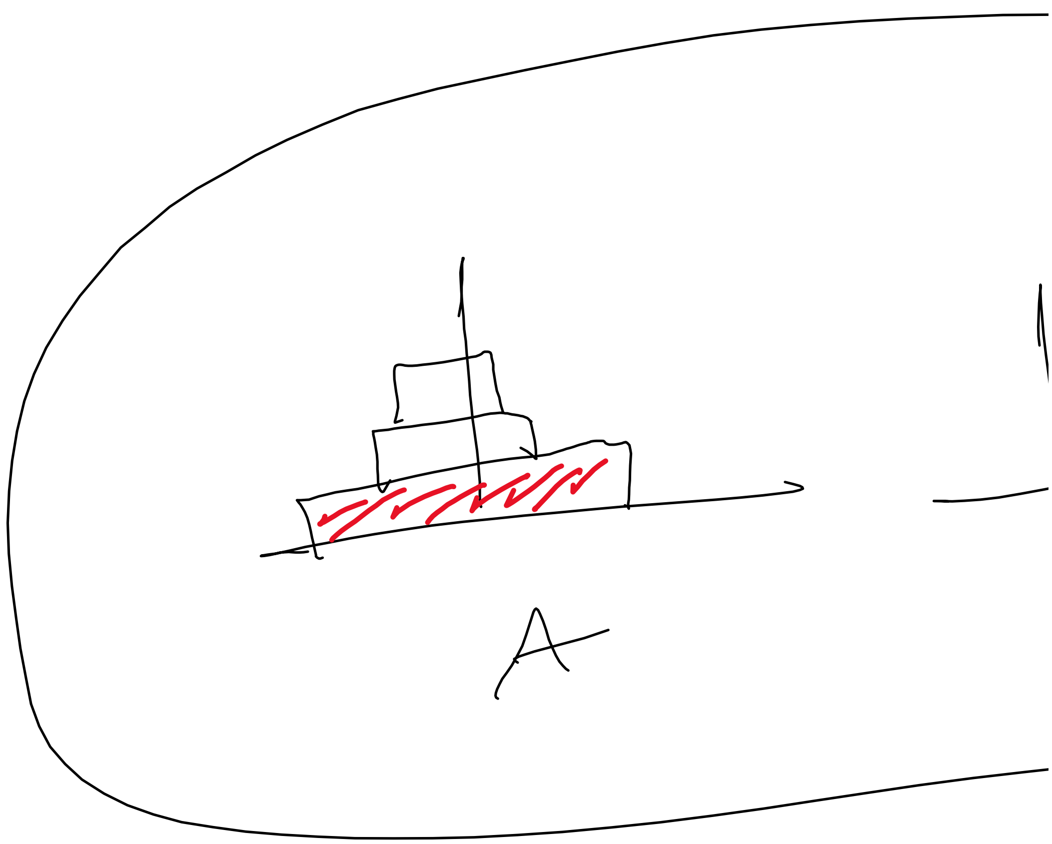
Obs 2

If f

disla

use

addi



I

II

III

Now I know (

empty.

Now my intention

I want to

— I

-9

111

Algorithm

Step 1 Move the

Step 2 Move the

Step 3 Move 𝑑

$T(n) =$

$=$

$=$

Solve Recurrence

We can try to

To solve this

Case 1 If

then

Example T

The term

is polyn

Therefore,

Case 2: If

then

Example

T (

The term

is polyno

Therefore,

Case 3

If x

then

Example

$T($

The term

is asympto

Therefore

Example of a

$T($

$n^{\log_b a}$

We can see

as follows:

If $f(n) = n$

we (

Solving Recurre

$T(n)$

The Cost T

If we look

n

$\emptyset$

$$\left(\frac{2}{3}\right)^n$$

$\Rightarrow$

The cost TCn

How many

We have

If $k = 1$

If $k = 2$

$\mathscr{H}\ k = 3$

Let's Think

In Case 2

The

In

Cure

Let's see

$n =$

Case 1

g a

y σ

Case 2

9

The (

differen

Hence

student