Law of Quadratic Reciprocity

Let p, q be distinct odd primes, and define the following symbols:

$p\boldsymbol{R}q$	$x^2 \equiv p \pmod{q}$ for some $x \in \mathbb{Z}$ p is a ***quadratic residue*** of q
$p\boldsymbol{N}q$	$x^2 \not\equiv p \pmod{q}$ for all $x \in \mathbb{Z}$ p is a ***quadratic nonresidue*** of q

Case I

If $p = 1 + 4n$, for some $n \in \mathbb{Z}$
then either pRq and qRp or pNq and qNp

Case II

If $p = 3 + 4m$ and $q = 3 + 4n$ for some $m, n \in \mathbb{Z}$
then either pRq and qNp or pNq and qRp

— *Karl Friedrich Gauss (1801)*

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							$\boldsymbol{q}$								
		3	5	7	11	13	17	19	23	29	31	37	41	43	47
	3	–	N	N	**R**	**R**	N	N	**R**	N	N	**R**	N	N	**R**
	5	N	–	N	**R**	N	N	**R**	N	**R**	**R**	N	**R**	N	N
	7	**R**	N	–	N	N	N	**R**	N	**R**	**R**	**R**	N	N	**R**
	11	N	**R**	**R**	–	N	N	**R**	N	N	N	**R**	N	**R**	N
	13	**R**	N	N	N	–	**R**	N	**R**	**R**	N	N	N	**R**	N
	17	N	N	N	N	**R**	–	**R**	N	N	N	N	N	**R**	**R**
	19	**R**	**R**	N	N	N	**R**	–	N	N	**R**	N	N	N	N
$\boldsymbol{p}$	23	N	N	**R**	**R**	**R**	N	**R**	–	**R**	N	N	**R**	**R**	N
	29	N	**R**	**R**	N	**R**	N	N	**R**	–	N	N	N	N	N
	31	**R**	**R**	N	**R**	N	N	N	**R**	N	–	N	**R**	**R**	N
	37	**R**	N	**R**	**R**	N	N	N	N	N	N	–	**R**	N	**R**
	41	N	**R**	N	N	N	N	N	**R**	N	**R**	**R**	–	**R**	N
	43	**R**	N	**R**	N	**R**	**R**	**R**	N	N	N	N	**R**	–	N
	47	N	N	N	**R**	N	**R**	**R**	**R**	N	**R**	**R**	N	**R**	–

R $\rightarrow$ p is a quadratic residue of q

N $\rightarrow$ p is a quadratic nonresidue of q