Law of Quadratic Reciprocity

Let p, q be odd primes and $p \neq q$
define **Legendre Notation** as

$$\left(\frac{p}{q}\right) = \begin{cases} 1 & \text{if } x^2 \equiv p \bmod q \text{ for some } x \in \mathbb{Z} \\ -1 & \text{otherwise} \end{cases}$$

Then

$$\left(\frac{p}{q}\right)\left(\frac{q}{p}\right) = (-1)^{\frac{p-1}{2}\frac{q-1}{2}}$$

- *Karl Friedrich Gauss (1801)*

Law of Quadratic Reciprocity

Let p, q be odd primes and $p \neq q$
Let pRq mean $x^2 \equiv p \bmod q$ for some $x \in \mathbb{Z}$
Let pNq mean $x^2 \not\equiv p \bmod q$ for all $x \in \mathbb{Z}$

Case I

If $p = 1 + 4n$, for some $n \in \mathbb{Z}$
then either pRq and qRp or pNq and qNp

Case II

If $p = 3 + 4m$, $q = 3 + 4n$ for some $m, n \in \mathbb{Z}$
then either pRq and qNp or pRq and qNp

- *Karl Friedrich Gauss (1801)*