Start with L_{PT2}:

$$L_{PT2} = \sum_{MN} \langle \tilde{M}|\hat{H}|\tilde{N}\rangle R^*_{MP} R_{NP} + \sum_{LMN} \langle \tilde{M}|\hat{T}^\dagger_{LM}\hat{H}|\tilde{N}\rangle R^*_{LP} R_{NP} - E_s \sum_{LMN} \langle \tilde{M}|\hat{T}^\dagger_{LM}\hat{T}_{LN}|\tilde{N}\rangle R^*_{LP} R_{LP}$$

$$+ \sum_{LMN} \langle \tilde{M}|\hat{\lambda}^\dagger_{LM}(\hat{f} - \epsilon^{(0)}_L + \epsilon_s)\hat{T}_{LN}|\tilde{N}\rangle + \sum_{LM} \langle \tilde{M}|\hat{\lambda}^\dagger_{LM}\hat{H}|\tilde{L}\rangle \tag{1}$$

First Term:

$$\frac{\partial}{\partial \tilde{c}_{Q,K}} \left[\sum_{MN} \langle \tilde{M}|\hat{H}|\tilde{N}\rangle R^*_{MP} R_{NP} \right]$$

$$= \sum_{M} \langle \tilde{M}|\hat{H}|\tilde{K}\rangle R^*_{MP} R_{QP} + \sum_{N} \langle \tilde{K}|\hat{H}|\tilde{N}\rangle R^*_{QP} R_{KP}$$

$$= R_{QP} \sum_M R^*_{MP} \sum_J \langle \tilde{I}|\tilde{c}^\dagger_{M,J}\hat{H}|\tilde{K}\rangle + R^*_{QP} \sum_N R_{NP} \sum_I \langle \tilde{K}|\hat{H}\tilde{c}^\dagger_{N,I}|\tilde{I}\rangle$$

$$= R_{QP} \left[\sum_{wxyz} \sum_M R^*_{MP} \sum_J \langle \tilde{J}|\tilde{c}^\dagger_{M,J}(\hat{E}_{wx}\hat{E}_{yz} - \delta_{xy}\hat{E}_{wz}) g(1,2)|\tilde{K}\rangle \right]$$

$$+ R^*_{QP} \left[\sum_{wxyz} \sum_N R_{NP} \sum_I \langle \tilde{K}|(\hat{E}_{wx}\hat{E}_{yz} - \delta_{xy}\hat{E}_{wz}) g(1,2)\tilde{c}^\dagger_{N,I}|\tilde{I}\rangle \right]$$

$$+ R_{QP} \left[\sum_{wx} \sum_M R^*_{MP} \sum_J \langle \tilde{J}|\tilde{c}^\dagger_{M,J}\hat{E}_{wx} h(1)|\tilde{K}\rangle \right] + R^*_{QP} \left[\sum_{wx} \sum_N R_{NP} \sum_I \langle \tilde{K}|\hat{E}_{wx} h(1)\tilde{c}^\dagger_{N,I}|\tilde{I}\rangle \right]$$

$$= R_{QP} \left(\sum_M R^*_{MP} \sum_{wxyz} \Theta^{M,K\dagger}_{wxyz} A^{(g)}(wxyz) + \sum_{wx} \Theta^{M,K\dagger}_{wx} A^{(h)}(wx) \right)$$

$$R^*_{QP} \left(\sum_N R_{NP} \sum_{wxyz} \Theta^{N,K}_{wxyz} A^{(g)}(wxyz) + \sum_{wx} \Theta^{N,K}_{wx} A^{(h)}(wx) \right) \tag{2}$$

Second Term:

$$\frac{\partial}{\partial \tilde{c}_{Q,K}} \left[\sum_{LMN} \langle \tilde{M}|\hat{T}^\dagger_{LM}\hat{H}|\tilde{N}\rangle R^*_{LP} R_{NP} \right] \tag{3}$$

$$= \sum_{LM} \langle \tilde{M}|\hat{T}^\dagger_{LM}\hat{H}|\tilde{K}\rangle R^*_{LP} R_{QP} + \sum_{LN} \langle \tilde{K}|\hat{T}^\dagger_{LQ}\hat{H}|\tilde{N}\rangle R^*_{LP} R_{NP}$$

$$= \sum_L R^*_{LP} \left(R_{QP} \sum_{M,J} \langle \tilde{J}|\tilde{c}^\dagger_{M,J}\hat{T}^\dagger_{LM}\hat{H}|\tilde{K}\rangle + \sum_N R_{NP} \sum_I \langle \tilde{K}|\hat{T}^\dagger_{LQ}\hat{H}\tilde{c}_{N,I}|\tilde{I}\rangle \right) \tag{4}$$

Third Term:

$$\frac{\partial}{\partial \tilde{c}_{Q,K}}\Bigg[- E_s \sum_{LMN} \langle \tilde{M}|\hat{T}^{\dagger}_{LM}\hat{T}_{LN}|\tilde{N}\rangle R^{*}_{LP}R_{LP}\Bigg]$$

$$= -E_s\Bigg(\sum_{LM}\langle \tilde{M}|\hat{T}^{\dagger}_{LM}\hat{T}_{LQ}|\tilde{K}\rangle + \sum_{LN}\langle \tilde{K}|\hat{T}^{\dagger}_{LQ}\hat{T}_{LN}|\tilde{N}\rangle\Bigg)$$

$$= -E_s\sum_{L}|R^{*}_{LP}|^2\Bigg(\sum_{M,J}\langle \tilde{J}|\tilde{c}^{\dagger}_{M,J}\hat{T}^{\dagger}_{LM}\hat{T}_{LQ}|\tilde{K}\rangle + \sum_{N,I}\langle \tilde{K}|\hat{T}^{\dagger}_{LQ}\hat{T}_{LN}\tilde{c}_{N,I}|\tilde{I}\rangle\Bigg) \tag{5}$$

Fourth Term:

$$\frac{\partial}{\partial \tilde{c}_{Q,K}}\Bigg[\sum_{LMN}\langle \tilde{M}|\hat{\lambda}^{\dagger}_{LM}(\hat{f} - \epsilon^{(0)}_{L} + \epsilon_s)\hat{T}_{LN}|\tilde{N}\rangle\Bigg]$$

$$= \sum_{LM}\langle \tilde{M}|\hat{\lambda}^{\dagger}_{LM}(\hat{f} - \epsilon^{(0)}_{L} + \epsilon_s)\hat{T}_{LQ}|\tilde{K}\rangle + \sum_{LN}\langle \tilde{K}|\hat{\lambda}^{\dagger}_{LQ}(\hat{f} - \epsilon^{(0)}_{L} + \epsilon_s)\hat{T}_{LN}|\tilde{N}\rangle$$

$$+ \sum_{LMN}\langle \tilde{M}|\hat{\lambda}^{\dagger}_{LM}\Big(\frac{\delta \hat{f}}{\delta \tilde{c}_{Q,K}}\Big)\hat{T}_{LN}|\tilde{N}\rangle$$

Fifth Term:

$$\frac{\partial}{\partial \tilde{c}_{Q,K}}\Bigg[\sum_{LM}\langle \tilde{M}|\hat{\lambda}^{\dagger}_{LM}\hat{H}|\tilde{L}\rangle\Bigg] \tag{6}$$

$$= \sum_{M}\langle \tilde{M}|\hat{\lambda}^{\dagger}_{LM}\hat{H}|\tilde{K}\rangle + \sum_{L}\langle \tilde{K}|\hat{\lambda}^{\dagger}_{LQ}\hat{H}|\tilde{L}\rangle$$

$$= \sum_{M,J}\langle \tilde{J}|\tilde{c}^{\dagger}_{M,J}\hat{\lambda}^{\dagger}_{LM}\hat{H}|\tilde{K}\rangle + \sum_{L,I}\langle \tilde{K}|\hat{\lambda}^{\dagger}_{LQ}\hat{H}\tilde{c}_{L,I}|\tilde{I}\rangle \tag{7}$$

$$\frac{\partial}{\partial \tilde{c}^{Q}_{K}}\Bigg[\sum_{LMN}\langle \tilde{M}|\hat{T}^{\dagger}_{LM}\hat{H}|\tilde{N}\rangle R^{*}_{LP}R_{NP}\Bigg]$$

$$= \sum_{L}\Bigg[\sum_{M}\sum_{J}\langle J|\tilde{c}^{M\dagger}_{J}\hat{T}^{\dagger}_{LM}\hat{H}|K\rangle R_{QP} + \sum_{N}\sum_{I}\langle K|\hat{T}^{\dagger}_{LM}\hat{H}\tilde{c}^{N}_{I}|I\rangle R_{NP}\Bigg]R^{*}_{LP}$$

$$\sum_{L}\Bigg[\sum_{M}\sum_{J}\Big(\langle J|\tilde{c}^{M\dagger}_{J}T^{\dagger}_{LM,abcd}(\hat{E}_{ab}\hat{E}_{cd} - \delta_{bc}\hat{E}_{ad})(\hat{E}_{wx}\hat{E}_{yz} - \delta_{xy}\hat{E}_{wz})g(1,2)|K\rangle + \tag{8}$$

$$\langle J|\tilde{c}^{M\dagger}_{J}T^{\dagger}_{LM,abcd}(\hat{E}_{ab}\hat{E}_{cd} - \delta_{bc}\hat{E}_{ad})\hat{E}_{wx}h(1)|K\rangle\Big)R_{QP} + \tag{9}$$

$$\sum_{N}\sum_{I}\Big(\langle K|T^{\dagger}_{LQ,abcd}(\hat{E}_{ab}\hat{E}_{cd} - \delta_{bc}\hat{E}_{ad})(\hat{E}_{wx}\hat{E}_{yz} - \delta_{xy}\hat{E}_{wz})g(1,2)\tilde{c}^{N}_{I}|I\rangle \tag{10}$$

$$\langle K|T^{\dagger}_{LQ,abcd}(\hat{E}_{ab}\hat{E}_{cd}-\delta_{bc}\hat{E}_{ad})\hat{E}_{wx}h(1)\tilde{c}^{N}_{I}|I\rangle\Big)R_{NP}\Big]R^{*}_{LP} \tag{11}$$

$$(1)=\sum_{M}\sum_{J}$$