

This is the kind of thing to calculate :

$$W(i, j, l) = X(i) + \sum_{j, k \in R} Y(i, j) Z(k, l) \delta_{jk}$$

The $W(i, j, l)$ takes three integers, i, j, l , as inputs, and returns a number, e.g.,

$$W(1, 2, 3) = 7$$

It is a better to think of, $W(i, j, l)$, as a function, not an array. It is to described by an instance of the term class, which contains this function.

$$W(i, j, l) = X(i) + \sum_{j,k}^{j,k \in R} Y(i, j)Z(k, l)\delta_{jk}$$

The \sum sums over all indices, j, k , in the range R . E.g., if $R = \{0, 1\}$

$$\sum_{j,k}^{j,k \in R} Y(i, j)Z(k, l)\delta_{jk} =$$

$$Y(i, 0)Z(0, l)\delta_{00} + Y(i, 0)Z(1, l)\delta_{01} + Y(i, 1)Z(0, l)\delta_{10} + Y(i, 1)Z(1, l)\delta_{11}$$

The δ_{jk} constrains the indices according to

$$\delta_{jk} = \begin{cases} 1 & \text{if } j == k \\ 0 & \text{otherwise} \end{cases}$$

So

$$\sum_{j,k}^{j,k \in R} Y(i, j)Z(k, l)\delta_{jk} =$$

$$Y(i, 0)Z(0, l) + Y(i, 1)Z(1, l)$$

This is how I'm splitting it up into classes :

$$W(i, j, l) = X(i) + \sum_{j \in R} Y(i, j) Z(k, l) \delta_{jk}$$

Diagram illustrating the classification of terms in the equation $W(i, j, l) = X(i) + \sum_{j \in R} Y(i, j) Z(k, l) \delta_{jk}$:

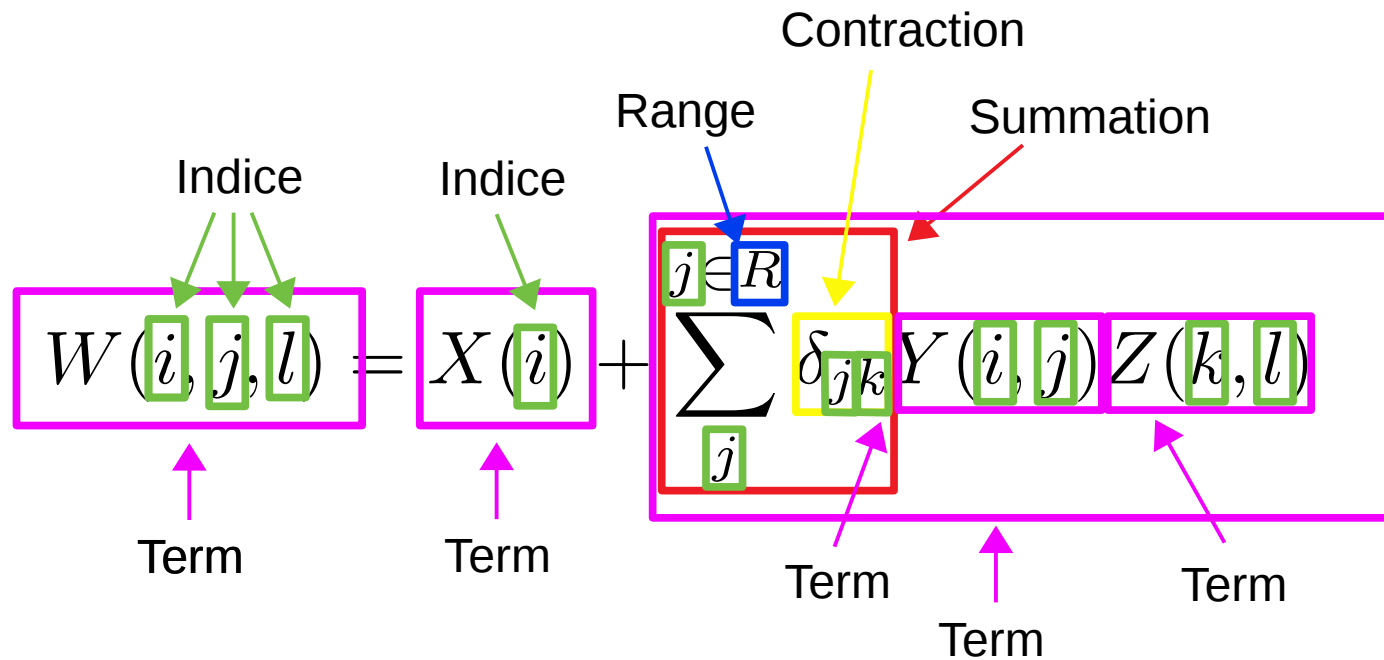
- Indice**: Points to the variables i, j, l in $W(i, j, l)$.
- Range**: Points to the summation index j in $\sum_{j \in R}$.
- Summation**: Points to the summation symbol \sum .
- Term**: Points to $W(i, j, l)$, $X(i)$, and the entire summation term $\sum_{j \in R} Y(i, j) Z(k, l) \delta_{jk}$.

It's important that terms can be defined recursively. For example, in the below $W(i, j, k)$ is being used to define $Q(j)$:

$$Q(j) = U(j) + \sum_{m, n} \sum_{i, k} V(m, n) W(i, j, k)$$

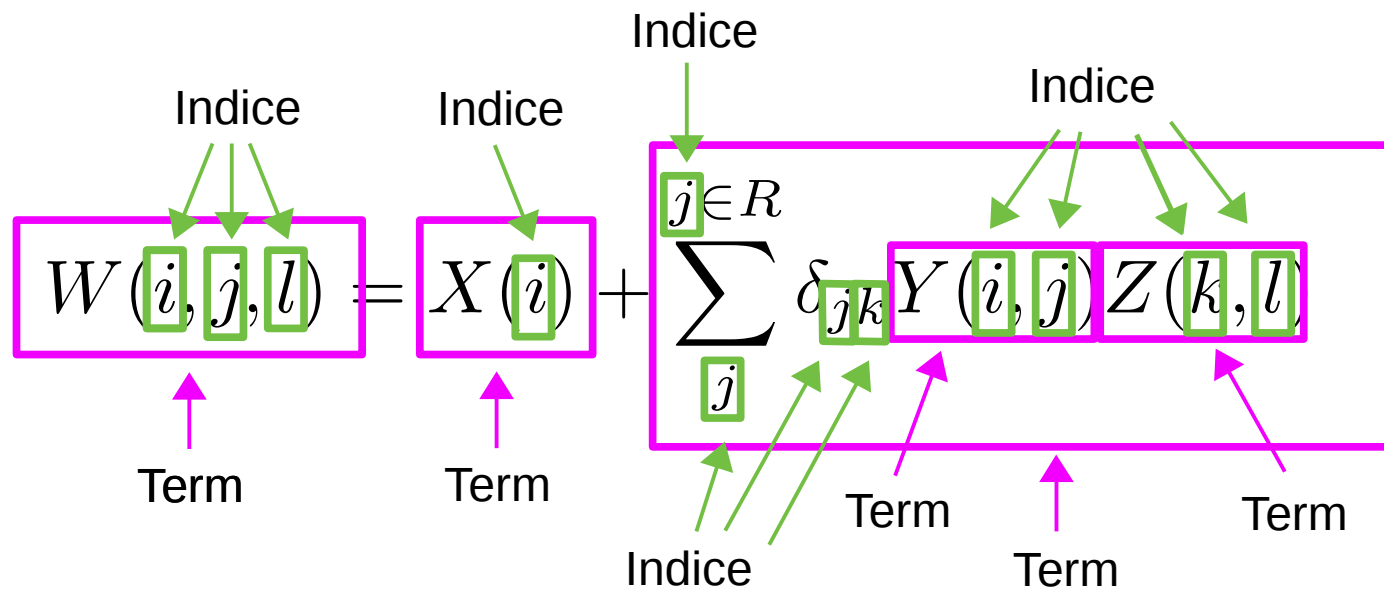
Diagram illustrating the recursive definition of $Q(j)$ using $W(i, j, k)$:

- Term**: Points to $U(j)$.
- Summation**: Points to the summation symbol $\sum_{m, n}$.
- Term**: Points to $V(m, n)$.
- Term**: Points to $W(i, j, k)$.
- Term**: Points to the entire right-hand side expression $U(j) + \sum_{m, n} \sum_{i, k} V(m, n) W(i, j, k)$.



This is hopefully a bit clearer.... If “Box A” is inside “Box A”, then the definition of the class corresponding to “Box A” contains a field which is of the class corresponding to “Box B”.

The following slides break it down into smaller chunks for clarity.



The diagram illustrates the equation $W(i, j, l) = X(i) + \sum_{j \in R} \delta_{jk} Y(i, j) Z(k, l)$ with magenta boxes and arrows highlighting specific terms. The boxes are placed around $W(i, j, l)$, $X(i)$, the summation $\sum_{j \in R}$, the product $\delta_{jk} Y(i, j) Z(k, l)$, and the individual components $Y(i, j)$ and $Z(k, l)$. Arrows point from the word "Term" to each of these boxed elements.

$$W(i, j, l) = X(i) + \sum_{j \in R} \delta_{jk} Y(i, j) Z(k, l)$$

Term

Term

Term

Term

Term

$$W(i, j, l) = X(i) + \sum_{j \in R} \delta_{jk} Y(i, j) Z(k, l)$$

Diagram illustrating the indices of the variables in the equation:

- W(i, j, l):** The indices i , j , and l are each labeled "Indice" with arrows pointing to them.
- X(i):** The index i is labeled "Indice" with an arrow pointing to it.
- Summation:** The summation is over the index j , which is labeled "Indice" with an arrow pointing to the summation symbol.
- Y(i, j):** The indices i and j are each labeled "Indice" with arrows pointing to them.
- Z(k, l):** The indices k and l are each labeled "Indice" with arrows pointing to them.
- Delta:** The indices j and k are each labeled "Indice" with arrows pointing to them.


$$W(i, j, l) = X(i) + \sum_j \delta_{jk} Y(i, j) Z(k, l)$$

Range


Summation

Contraction

Summation


$$W(i, j, l) = X(i) + \sum_{j \in R} \delta_{jk} Y(i, j) Z(k, l)$$

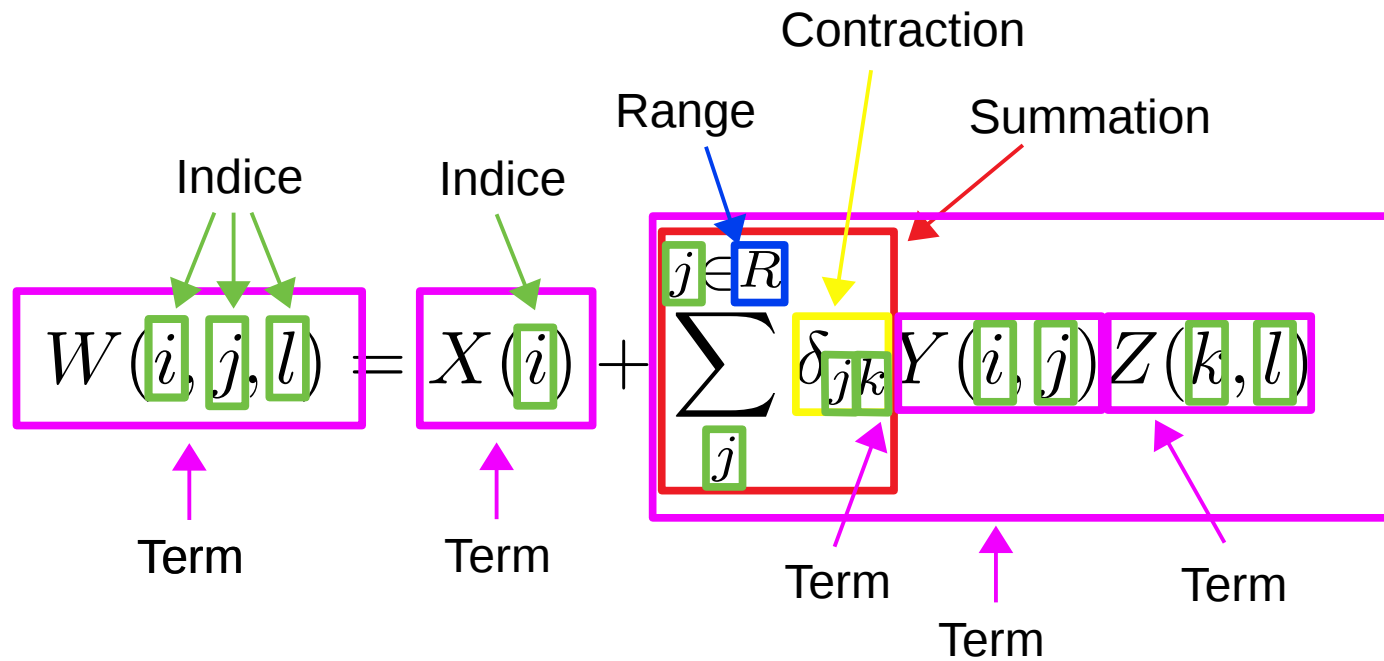
Range


$$W(i, j, l) = X(i) + \sum_j^{j \in R} \delta_{jk} Y(i, j) Z(k, l)$$

$$W(i, j, l) = X(i) + \sum_j^{j \in R} \delta_{jk} Y(i, j) Z(k, l)$$

Contraction





$$Q(j) = U(j) + \sum_{m, n} \sum_{i, j \in R^i} V(m, n) W(i, j, k)$$

Diagram illustrating the summation structure of the equation above:

- Summation:** A bracket above the double summation indicates the summation over the indices m, n and i, j .
- Term:** A bracket below the double summation indicates the term $V(m, n) W(i, j, k)$.