

This is the kind of thing to calculate :

$$W(i, j, l) = X(i) + \sum_{j, k \in R} Y(i, j) Z(k, l) \delta_{jk}$$

The $W(i, j, l)$ takes three integers, i, j, l , as inputs, and returns a number, e.g.,

$$W(1, 2, 3) = 7$$

It is a better to think of, $W(i, j, l)$, as a function, not an array. It is to described by an instance of the term class, which contains this function.

$$W(i, j, l) = X(i) + \sum_{j,k}^{j,k \in R} Y(i, j)Z(k, l)\delta_{jk}$$

The \sum sums over all indices, j, k , in the range R . E.g., if $R = \{0, 1\}$

$$\sum_{j,k}^{j,k \in R} Y(i, j)Z(k, l)\delta_{jk} =$$

$$Y(i, 0)Z(0, l)\delta_{00} + Y(i, 0)Z(1, l)\delta_{01} + Y(i, 1)Z(0, l)\delta_{10} + Y(i, 1)Z(1, l)\delta_{11}$$

The δ_{jk} constrains the indices according to

$$\delta_{jk} = \begin{cases} 1 & \text{if } j == k \\ 0 & \text{otherwise} \end{cases}$$

So

$$\sum_{j,k}^{j,k \in R} Y(i, j)Z(k, l)\delta_{jk} =$$

$$Y(i, 0)Z(0, l) + Y(i, 1)Z(1, l)$$

This is how I'm splitting it up into classes :

$$W(i, j, l) = X(i) + \sum_{j \in R} Y(i, j) Z(k, l) \delta_{jk}$$

Diagram illustrating the classification of terms in the equation $W(i, j, l) = X(i) + \sum_{j \in R} Y(i, j) Z(k, l) \delta_{jk}$:

- Indice**: Points to the indices i, j, l in $W(i, j, l)$.
- Range**: Points to the range $j \in R$ of the summation.
- Summation**: Points to the summation symbol \sum .
- Term**: Points to the three terms being added: $W(i, j, l)$, $X(i)$, and the summation term $\sum_{j \in R} Y(i, j) Z(k, l) \delta_{jk}$.

It's important that terms can be defined recursively. For example, in the below $W(i, j, k)$ is being used to define $Q(j)$:

$$Q(j) = U(j) + \sum_{m, n} \sum_{i, k} V(m, n) W(i, j, k)$$

Diagram illustrating the recursive definition of $Q(j)$ using $W(i, j, k)$:

- Term**: Points to the term $U(j)$.
- Summation**: Points to the summation $\sum_{m, n}$.
- Term**: Points to the term $V(m, n)$.
- Term**: Points to the term $W(i, j, k)$.

A large arrow at the bottom indicates that the term $W(i, j, k)$ is used to define $Q(j)$.

$$W(i, j, l) = X(i) + \sum_{j \in R} Y(i, j) Z(k, l) \delta_{jk}$$

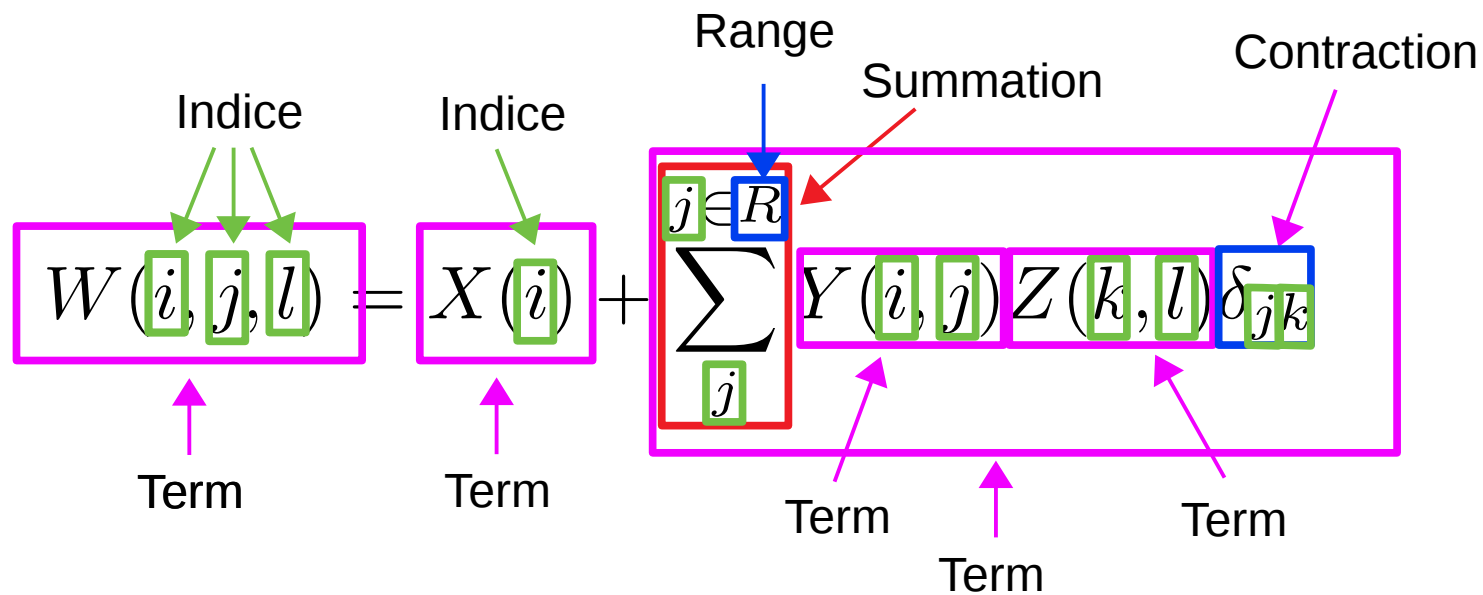
Diagram illustrating the components of the equation:

- Indice:** Points to the indices i, j, l in $W(i, j, l)$.
- Term:** Points to $W(i, j, l)$ and $X(i)$.
- Range:** Points to the summation index $j \in R$.
- Summation:** Points to the summation symbol \sum .

$$Q(j) = U(j) + \sum_{m, n} \sum_{i, k} V(m, n) W(i, j, k)$$

Diagram illustrating the components of the equation:

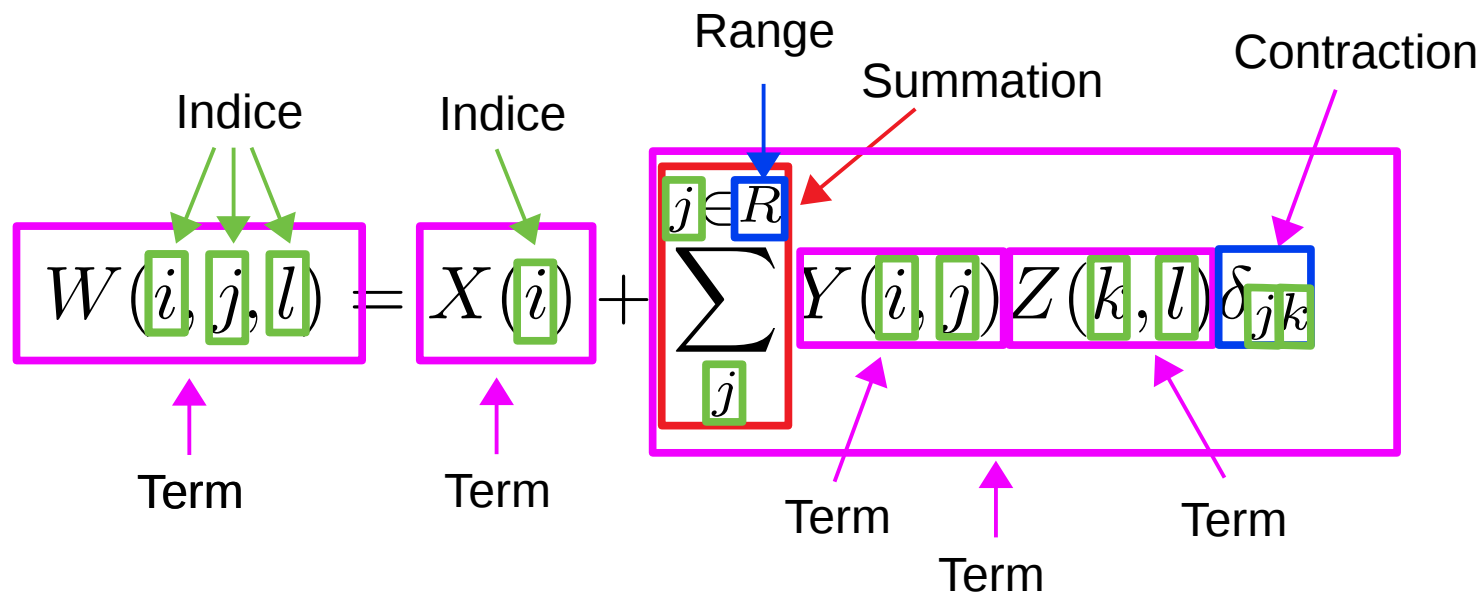
- Summation:** Points to the summation symbols $\sum_{m, n}$ and $\sum_{i, k}$.
- Term:** Points to the entire expression $\sum_{m, n} \sum_{i, k} V(m, n) W(i, j, k)$.



$$Q(j) = U(j) + \sum_{m, n} \sum_{i, k}^{m, n \in R^m \quad i, j \in R^i} V(m, n) W(i, j, k)$$

Diagram illustrating the summation structure of the equation:

- Summation:** Points to the summation indices m, n and i, k .
- Term:** Points to the entire right-hand side expression $U(j) + \sum_{m, n} \sum_{i, k} V(m, n) W(i, j, k)$.



$$Q(j) = U(j) + \sum_{m, n} \sum_{i, k}^{m, n \in R^m \quad i, j \in R^i} V(m, n) W(i, j, k)$$

Diagram illustrating the summation structure:

- Summation:** Points to the summation indices m, n and i, k .
- Term:** Points to the entire expression $Q(j) = U(j) + \sum_{m, n} \sum_{i, k} V(m, n) W(i, j, k)$.