Spring 2012: Geometry/Topology Graduate Exam

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Problem 1. *(Topology)*

Proof. □

Problem 2. *(Topology)*

Proof. $\square$

Problem 3. *(Topology)*

Proof. $\square$

Problem 4. Does there exist a smooth embedding of the projective plane $\mathbb{R}P^2$ into $\mathbb{R}^2$? Justify your answer.

Proof. $\square$

Problem 5. Let M be a manifold and let $C^\infty(M)$ be the algebra of C^∞ functions $M \to \mathbb{R}$. Explain the relationship between vector fields on M and $C^\infty(M)$. if we consider vector fields as maps $C^\infty(M) \to C^\infty(M)$ is the composition map XY also a vector field? What about $[X, Y] = XY - YX$?

Proof. $\square$

Problem 6. Let S be the unit sphere defined by $x^2 + y^2 + z^2 + w^2 = 1$ in $\mathbb{R}^4$. Compute $\int_S \omega$ where

$$\omega = (w + w^2)\, dx \wedge dy \wedge dz$$

Proof. $\square$

Problem 7. Does the equation $x^2 = y^3$ define a smooth submanifold in $\mathbb{R}^3$?

Proof. $\square$