Math 225 - Midterm 2

Fall 2019

Last Name: ______________________________

First Name: ______________________________

USC ID: ______________________________

Instructions:

- Unless otherwise indicated, please clearly show all of your work. Correct answers without justification may not receive credit.
- No calculators or other electronics are permitted for use. In particular, cell phones must be turned **off** and stored away.
- You may use one single-sided, handwritten A4 note sheet.
- No other notes or books may be used.

Question:	1	2	3	4	5	6	Total
Points:	5	9	7	8	6	5	40
Score:							

1. (5 points) Let V be an inner product space, and let u_1, u_2 be fixed (nonzero) vectors in V. Define $T\colon V \to \mathbb{R}^2$ by

$$T(v) = (\langle u_1, v\rangle, \langle u_2, v\rangle).$$

Use the properties of the inner product to show that T is a linear transformation.

2. (9 points) Consider the transformation $T\colon P_2(\mathbb{R}) \to P_1(\mathbb{R})$ defined by

$$T(ax^2 + bx + c) = (a+b)x + (a-c),$$

where a, b and c are arbitrary real numbers.

(a) (3 points) Show that T is a linear transformation.

(b) (6 points) Determine $\ker(T), \mathrm{Im}(T)$, and their dimensions.

3. (7 points) Use the Gram-Schmidt process to determine an orthogonal basis for the subspace of $C^0[-1,1]$ spanned by $f_1(x) = 1, f_2(x) = x^3, f_3(x) = x^4$, with respect to the inner product $\langle f, g\rangle = \int_{-1}^{1} f(x)g(x)dx$.

4. (8 points) Let $A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 7 & 9 \\ 1 & 5 & 21 \end{bmatrix}$, $b = \begin{bmatrix} 4 \\ 9 \\ 6 \end{bmatrix}$. Find all the solutions of the system $Ax = b$.

5. (6 points) Let $V = C^0[-1, 0]$ and for f and g in V, consider the mapping

$$\langle f, g \rangle = \int_{-1}^{0} x^3 f(x) g(x) dx.$$

Does this define a valid inner product on V? Show why or why not.

6. (5 points) Define $T\colon P_2(\mathbb{R}) \to P_1(\mathbb{R})$ by $T(ax^2 + bx + c) = c$. Determine whether T is 1-1, onto, or neither. Find T^{-1} or explain why it does not exist.