Week 3 Quiz Solutions.

1. (*5 points*)

   (a) Find a vector $\vec{v}$ of the form $\langle c, 2c, c-1\rangle$ (for some constant c) such that $\vec{v}\cdot\langle 3,2,1\rangle = 15$

   (b) Find the vector projection of $\vec{v}$ onto the vector $\langle -2,1,2\rangle$

   Solution.

   (a) We compute the dot product using the coordinates

   $$\begin{aligned} 3(c) + 2(2c) + 1(c-1) &= 15 \\ 8c - 1 &= 15 \\ c &= 2 \\ \vec{v} &= \langle 2,4,1\rangle \end{aligned}$$

   (b) We can use the projection formula,

   $$\text{proj}_a(b) = \left(\frac{a\cdot b}{|a|}\right)\frac{a}{|a|},$$

   with $a = \langle -2,1,2\rangle$ and $b = \langle 2,4,1\rangle$ giving

   $$\text{proj}_{\langle -2,1,2\rangle}(\vec{v}) = \left(\frac{-4+4+2}{\sqrt{9}}\right)\frac{\langle -2,1,2\rangle}{\sqrt{9}} = \frac{2}{9}\langle -2,1,2\rangle$$

2. (*5 points*)

   (a) Two forces $\vec{F}$ and $\vec{G}$ act on a wagon. $\vec{F}$ has magnitude 4 N and acts at an angle of $60°$ from the **negative** x-axis. $\vec{G}$ has magnitude $2\sqrt{2}$ N and acts at an angle of $45°$ from the **postive** x-axis. Find the vector components of the net force.

   (b) If this wagon is pulled 500 meters in the **positive** x direction by this net force, find the total work done by the net force on the wagon.

   Solution.

   (a) Writing the vectors as magnitude and direction gives

   $$\begin{aligned} \vec{F} &= 4\text{ N}\langle -\cos(60°), -\sin(60°)\rangle = \langle -2, -2\sqrt{3}\rangle\text{ N} \\ \vec{G} &= 2\sqrt{2}\text{ N}\langle \cos(45°), \sin(45°)\rangle \;\; = \langle 2,2\rangle\text{ N} \\ \vec{F}+\vec{G} &= \langle 0, 2-2\sqrt{3}\rangle\text{ N} \end{aligned}$$

   (b) The formula for work is $W = \vec{F}_{\text{net}}\cdot\vec{d}$. In this case, $\vec{F}_{\text{net}} = \vec{F}+\vec{G}$.

   $$W = \langle 0\text{ N}, 2-2\sqrt{3}\text{ N}\rangle\cdot\langle 500\text{ m}, 0\text{ m}\rangle = 0\text{ Nm}.$$