Matrix Analysis: Main ideas

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1 Definitions

1.1 M^λ

Define M^λ to be the complex vector space with basis the tabloids $\{T\}$ of shape λ.

Example. When $\lambda = (3, 2)$, the basis of M^λ is

$$\left\{ \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & 5 \\ \cline{1-2} \end{array}, \begin{array}{|c|c|c|}\hline 1 & 2 & 4 \\ \hline 3 & 5 \\ \cline{1-2} \end{array}, \ldots, \begin{array}{|c|c|c|}\hline 3 & 4 & 5 \\ \hline 1 & 2 \\ \cline{1-2} \end{array} \right\}$$

1.2 v_T

For each numbering T of shape λ, there is an element $v_T \in M^\lambda$ given by

$$v_T = \sum_{q \in C(T)} \operatorname{sgn}(q)\{q \cdot T\}$$

where $C(T)$ is the column group of T: the set of permutations which preserve the columns of T.

1.3 Specht module

The Specht modules, denoted S^λ is the subspace of M^λ given by $\operatorname{span}\{v_T : T \text{ is shape } \lambda\}$.