1. (5 points) Simplify the expression as much as possible.

(a) $6^{-2} \cdot 3^3$ $= 2^{-2} \cdot 3^{-2} \cdot 3^3 = \frac{3^1}{2^2} = \frac{3}{4}$

(b) $5^{3/2} \cdot \sqrt{5}$ $= 5^{3/2} \cdot 5^{1/2} = 5^2 = 25$

(c) $\sqrt{50} + \sqrt{32} + \sqrt{18}$ $= \sqrt{25 \cdot 2} + \sqrt{16 \cdot 2} + \sqrt{9 \cdot 2}$

$= 5\sqrt{2} + 4\sqrt{2} + 3\sqrt{2} = 12\sqrt{2}$

(d) 6^0 $= 1$

(e) $(2^{3/4})^2$ $= 2^{6/4} = 2^{3/2}$

2. (4 points) Evaluate the following expressions

(a) $3^a + 2^a$ when $a = -2$

$\frac{1}{3^2} + \frac{1}{2^2} = \frac{1}{9} + \frac{1}{4} = \frac{4}{36} + \frac{9}{36} = \frac{13}{36}.$

(b) $(b^2 + c)^{1/2}$ when $b = -3$ and $c = 7$

$\sqrt{(-3)^2 + 7} = \sqrt{9+7} = \sqrt{16} = 4.$

(c) $3d\left(\frac{d^2}{d^{-1}}\right)^2$ when $d = 10$

$3d\,(d^3)^2 = 3d \cdot d^6 = 3d^7 = 3 \times 10^7.$

(d) $\left(\frac{1}{x}\right)^{-3}$ when $x = 7^2$ *(Write in the form a^b.)*

$\left(\frac{1}{7^2}\right)^{-3} = (7^{-2})^{-3} = 7^6.$

3. (8 points) Right now the UN approximates that roughly 800 000 000 people in the world lack the necessary food to live a healthy lifestyle. The UN also estimates ending world hunger each year would cost about \$30 000 000 000 per year. In 2019, the US defense budget was approximately \$700 billion.

   (a) Write 800 000 000 in scientific notation.

   (b) Write \$30 000 000 000 in scientific notation.

   (c) Write \$700 billion in scientific notation.

   (d) Ending world hunger would cost what percentage of the US defense budget?

   (e) How much does the UN Estimate it would cost on a *per person* basis to end world hunger?

(a) 8×10^8

(b) 3×10^{10}

(c) 7×10^{11} (700,000,000,000)

(d) $\dfrac{3 \times 10^{10}}{7 \times 10^{11}} = \dfrac{3}{7 \times 10} = \dfrac{3}{70} \approx 0.043 = 4.3\%$

(e) $\dfrac{3 \times 10^{10}}{8 \times 10^{8}} = \dfrac{3}{8} \times 10^{2} = \dfrac{300}{8} = 37.5.$ \$37.50/person.

4. (6 points) Escape velocity is the minimum speed an object must reach to escape the pull of a planet's gravity (ignoring wind resistance). The escape velocity from planet earth is v where

$$v = \sqrt{\frac{2Gm}{r}}$$

where

$G = 6.5 \times 10^{-11}\ \mathrm{m}^3/(\mathrm{kg} \cdot \mathrm{s}^2)$ (universal gravitational constant)

$m = 6.0 \times 10^{24}$ kg (mass of earth)

$r = 6.5 \times 10^6$ m (radius of earth)

(a) What is the escape velocity from earth in meters per second?

$$v = \sqrt{\frac{2 \cdot \cancel{6.5} \times 10^{-11} \cdot 6 \times 10^{24}}{\cancel{6.5} \times 10^6}} = \sqrt{\frac{12 \cdot 10^{13}}{10^6}} = \sqrt{12 \cdot 10^7}$$

$$= \sqrt{1.2 \cdot 10^8} = 10^4\sqrt{1.2} \approx 10954\ ^{m}/_{s}$$

(b) Using the information that one mile is 1.6 kilometers, what is the escape velocity in miles per hour?

$$\frac{10954\ \cancel{m}}{\cancel{s}} \cdot \frac{60\ \cancel{s}}{\cancel{min}} \cdot \frac{60\ \cancel{min}}{\text{hour}} \cdot \frac{1\ \cancel{km}}{1000\ \cancel{m}} \cdot \frac{1\ mi}{1.6\ \cancel{km}} \approx 24{,}600\ mph$$

5. (8 points) Given a square based pyramid with equilateral triangles for faces and side length ℓ, the height, surface area, and volume is given by

$$h = \frac{\sqrt{2}}{2}\ell \qquad \text{(height)}$$
$$A = (1+\sqrt{3})\ell^2 \qquad \text{(surface area)}$$
$$V = \frac{\sqrt{2}}{6}\ell^3 \qquad \text{(volume)}$$

If the volume of the Great Pyramid of Giza is 2 600 000 m^3, what is its height?

FIRST, solve for ℓ:

$$2.6 \times 10^6 = \frac{\sqrt{2}}{6}\ell^3$$

$$1.1 \times 10^7 \, m^3 = \ell^3$$

$$\sqrt[3]{1.1 \times 10^7 \, m^3} = \ell \approx 220 \, m.$$

Now use formula for h:

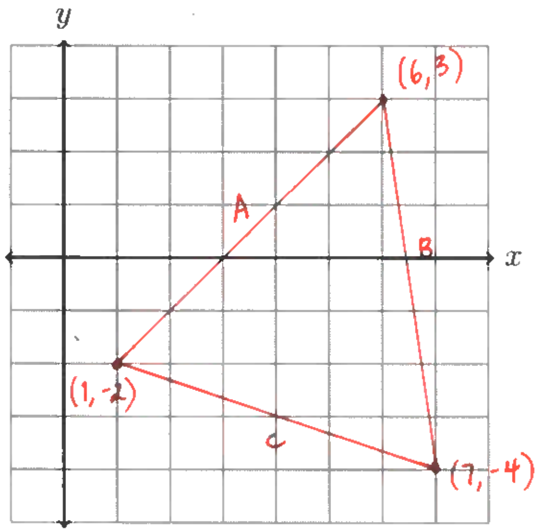
$$h = \frac{\sqrt{2}}{2}\ell \approx 157 \, m.$$

6. (10 points) The distance between two points (x_1, y_1) and (x_2, y_2) is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Consider the triangle with vertices $(1, -2)$, $(6, 3)$, and $(7, -4)$.

(a) Draw the triangle.



A: $\sqrt{(6-1)^2 + (3+2)^2} = \sqrt{50} = 5\sqrt{2}$

B: $\sqrt{(7-6)^2 + (-4-3)^2} = \sqrt{50} = 5\sqrt{2}$

C: $\sqrt{(7-1)^2 + (-4+2)^2} = \sqrt{40} = 2\sqrt{10}$

(b) What is the length of the longest side? (Simplify the radical as much as possible.)

$5\sqrt{2}$.

(c) What is the length of the shortest side?

$2\sqrt{10}$

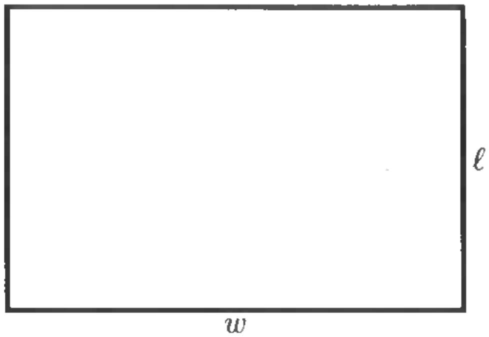
(d) What is the perimeter of the triangle?

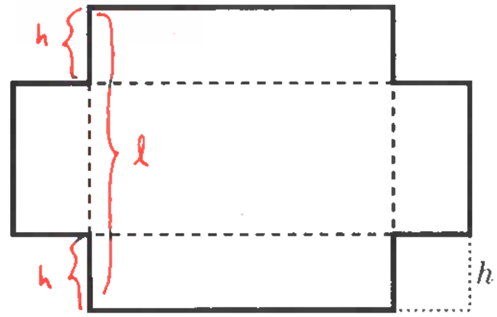
$2 \cdot 5\sqrt{2} + 2\sqrt{10} = 10\sqrt{2} + 2\sqrt{10}$.

(e) Is this an equilateral triangle, and isosceles triangle, or a scalene triangle?

A = B, so isosceles

7. (10 points) One way to create a box (with no top) is to take a rectangular piece of cardboard, cut squares from each of the corners, and fold up the sides. Suppose you start with a piece of cardboard of width w and length ℓ and cut out corners of size h, as shown below in the diagram, where the dashed region is the base of the box.





(a) What is the height of the box?

$h.$

(b) What are the dimensions of the base of the resulting box?

$(w-2h)(l-2h)$

(c) What is the surface area of the box?

$lw - 4h^2$

(d) What is the volume of the box in terms of w, h, and ℓ?

$h(w-2h)(l-2h)$

(e) What is the volume of the box specifically when $w = 9$, $h = 2$, and $\ell = 8$?

$2(9-4)(8-4) = 2 \cdot 5 \cdot 4 = 40.$

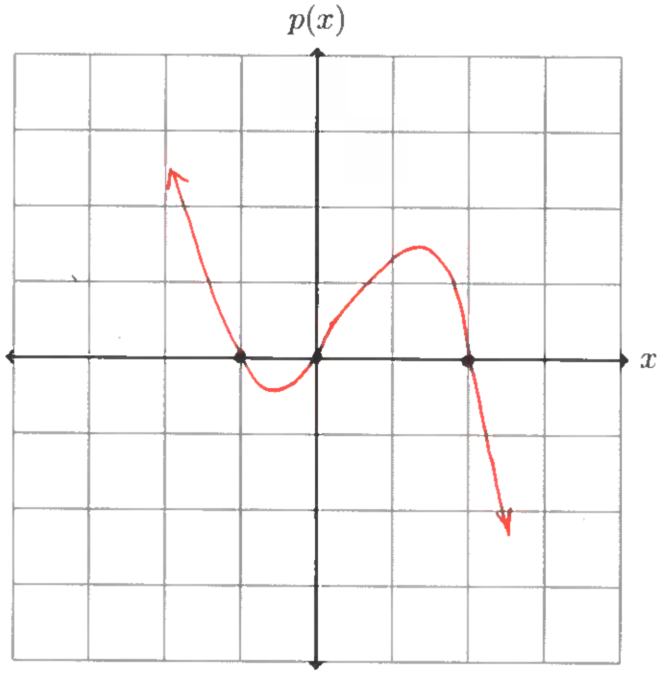
8. (8 points) Consider the polynomial $p(x) = -x^3 + x^2 + 2x$.

(a) Factor $p(x)$. What are its roots?

$-x(x^2 - x - 2) = -x(x-2)(x+1)$

Roots: $0, -1, 2$.

(b) Graph $p(x)$ including its roots and end behavior.



(c) What is the domain and range of $p(x)$?

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

9. (12 points) (Miscellaneous problem-solving questions.)

(a) At Rogelio's restaurant, *Los Muchachos Nachos*, there are three choices of chips, four choices of cheese, five choices of salsa, and eight additional toppings (e.g. jalapeños, olives, sour cream) that you can either put on or leave off.
The nachos must include exactly one kind of chip, one kind of cheese, and one kind of salsa. How many different nacho platters are possible to make?

$$3 \cdot 4 \cdot 5 \cdot \underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{2^8} = 3 \cdot 4 \cdot 5 \cdot 2^8 = 16080.$$

(b) Suppose that Olivia can paint a fence in four hours and Keenan can paint a fence in five hours. How long does it take Olivia and Keenan to paint a fence together?

Olivia: $\frac{1 \text{ fence}}{4 \text{ hour}}$ Keenan: $\frac{1 \text{ fence}}{5 \text{ hour}}$

Together $\left(\frac{1}{4} + \frac{1}{5}\right) \frac{\text{fences}}{\text{hour}} = \left(\frac{5}{20} + \frac{4}{20}\right) \frac{\text{fence}}{\text{hour}} = \frac{9 \text{ fence}}{20 \text{ hour}} = \frac{1 \text{ fence}}{\left(\frac{20}{9}\right) \text{hour}}$

$\frac{20}{9}$ hours ≈ 2.22 hours.

(c) Jessica swims at 2 miles per hour in a swimming pool (which has no current). When she swims in a river with a slight current, it takes her 50% longer to swim upstream from Dock A to Dock B than it takes for her to swim downstream from Dock B to Dock A. Using this information, what is the speed of the current in the river?

Say that the docks are c miles apart and the current of the river is v miles per hour. Then:

Time to swim upstream: $\dfrac{c \text{ miles}}{2-v \ \frac{\text{miles}}{\text{hour}}} = \dfrac{c}{2-v}$ hours.

Time to swim downstream: $\dfrac{c \text{ miles}}{2+v \ \frac{\text{miles}}{\text{hour}}} = \dfrac{c}{2+v}$ hours

$$\frac{c}{2-v} = 1.5\left(\frac{c}{2+v}\right)$$

$$\cancel{c}(2+v) = 1.5\cancel{c}(2-v)$$

$$2+v = 3 - 1.5v$$

$$2.5v = 1$$

$$v = 0.4 \text{ mph}.$$

10. (10 points) The position of a basketball thrown from a height of six feet at a speed of v feet per second upward is given by

$$h_v(t) = -16t^2 + vt + 6$$

where $h_v(t)$ is the height of the ball t seconds after the ball is thrown.

(a) Use the quadratic equation to find the roots of h_v where v is a variable.

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow \frac{-v \pm \sqrt{v^2 + 384}}{-32}$$

For the remaining questions, assume $v = 20$.

(b) What is the value of t at which the ball hits the ground?

$$\frac{-20 \pm \sqrt{400 + 384}}{-32} = \frac{-20 \pm 28}{-32} = \frac{-48}{-32} = \boxed{1.5}$$

OR $\frac{8}{-32} = -0.25$

(c) If the ball is at its highest point at the *average* of its two roots, what is the value of t at which the ball attains its maximum height? How high is it? *(Hint: one of the roots should be negative.)*

$$t = \frac{1.5 - 0.25}{2} = 0.625$$

$$h_{20}(0.625) = -16 \cdot 0.625^2 + 20 \cdot 0.625 + 6$$

$$= 12.25 \text{ ft.}$$

(d) Suppose the ball is a basketball being shot up to a 10 foot rim. At what value of t does the basketball go through the basket? *(Hint: The time should be **after** the time found in part (c).)*

$-16t^2 + 20t + 6 = 10$ so $-16t + 20t - 4 = 0.$

after 6.25.

$$\frac{-20 \pm \sqrt{400 - 256}}{-32} = \frac{-20 \pm \sqrt{144}}{-32} = \frac{-20 - 12}{-32} = \boxed{1}$$

OR $= \frac{-20 + 12}{-32} = 0.25$

$t = 1$ second.