Math 225 - Midterm

Fall 2019

Last Name: ______________________________

First Name: ______________________________

USC ID: ______________________________

Instructions:

- Unless otherwise indicated, please clearly show all of your work. Correct answers without justification may not receive credit.
- No calculators or other electronics are permitted for use. In particular, cell phones must be turned **off** and stored away.
- You may use one single-sided, handwritten A4 note sheet.
- No other notes or books may be used.

Question:	1	2	3	4	5	6	Total
Points:	5	6	6	9	6	4	36
Score:							

1. (5 points) Show that $\det(A) = (1 + 2x^2)^3$ where

$$A = \begin{bmatrix} 1 & -2x & 2x^2 \\ 2x & 1 - 2x^2 & -2x \\ 2x^2 & 2x & 1 \end{bmatrix}.$$

2. (6 points) Consider the system of linear equations $Ax = b$, where

$$A = \begin{bmatrix} 1 & -k & k^2 \\ 1 & 0 & k \\ 0 & 1 & -1 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad \text{and } b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

For what value(s) of k does this have

(a) a unique solution?

(b) infinitely many solutions?

Justify your answer.

3. (6 points) Find the (complex) eigenvalues of the matrix

$$A = \begin{bmatrix} -5 & -5 & 0 \\ -8 & 1 & 0 \\ -5 & 3 & 7 \end{bmatrix}.$$

4. (9 points) Assume that A is skew-symmetric $n \times n$ matrix, that is, $A^T = -A$.

   (a) (6 points) Prove that $\text{rank}(A) < n$ if n is odd.

   (b) (3 points) Find a 2×2 skew-symmetric matrix A such that $\text{rank}(A) = 2$.

5. (6 points) Let S be the subspace of $M_3(\mathbb{R})$ consisting of all 3×3 symmetric matrices (matrices such that $A = A^T$). Find a basis for S. What's the dimension of S?

6. (4 points) Let $p_1(x) = x-4$ and $p_2(x) = x^2-x+3$. Determine whether $p(x) = 2x^2-x+2$ lies in $\text{span}\{p_1, p_2\}$