Week 4 Quiz Solutions.

1. (*5 points*) Find a parametric equation of the line through the point $(1, 2, -1)$ and perpendicular to both $\mathbf{i} + \mathbf{j}$ and $\mathbf{j} - 2\mathbf{k}$.

   Solution.
   The parametric equation of a line through (x_0, y_0, z_0) parallel to $\langle a, b, c\rangle$ is given by the parametric equations

   $$\begin{aligned} x &= x_0 + at \\ y &= y_0 + bt \\ z &= z_0 + ct. \end{aligned}$$

   We can find a vector $\langle a, b, c\rangle$ perpendicular to both $\mathbf{i} + \mathbf{j}$ and $\mathbf{j} - 2\mathbf{k}$ by taking the cross product.

   $$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 0 \\ 0 & 1 & -2 \end{vmatrix} = \mathbf{i}\begin{vmatrix} 1 & 0 \\ 1 & -2 \end{vmatrix} - \mathbf{j}\begin{vmatrix} 1 & 0 \\ 0 & -2 \end{vmatrix} + \mathbf{k}\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = -2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$$

   Therefore (one choice for) the desired parametric equations is

   $$\begin{aligned} x &= 1 - 2t \\ y &= 2 + 2t \\ z &= -1 + t. \end{aligned}$$

2. (*5 points*) Find the equation of the plane through the point $(2, 2, 9)$ and perpendicular to the line $x = 2t$, $y = 1 + 2t$, and $z = 4 - t$.

   Solution. The paramteric equations described a line that is parallel to the vector $\langle 2, 2, -1\rangle$. Thus the desired plane is of the form

   $$2x + 2y - z + d = 0.$$

   Choosing our point on the plane, $(x, y, z) = (2, 2, 9)$, allows us to solve for d

   $$\begin{aligned} 2(2) + 2(2) - 9 + d &= 0 \\ d &= 1. \end{aligned}$$

   So the desired equation is

   $$2x + 2y - z + 1 = 0.$$