

# Complex Analysis: Homework 14

Peter Kagey

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**Problem 2.** (page 227)

Show that the functions  $z^n$ ,  $n$  a nonnegative integer, form a normal family in  $|z| < 1$ , also in  $|z| > 1$ , but not in any region that contains a point on the unit circle.

*Proof.*

□

**Problem 3.** (page 227)

If  $f(z)$  is analytic in the whole plane, show that the family formed by all functions  $f(kz)$  with constant  $k$  is normal in the annulus  $r_1 < |z| < r_2$  if and only if  $f$  is a polynomial.

*Proof.*

□

**Problem 1.** (page 232)

If  $z_0$  is real and  $\Omega$  is symmetric with respect to the real axis, prove (by the uniqueness) that  $f$  satisfies with the symmetry relation  $f(\bar{z}) = \overline{f(z)}$

*Proof.*

□

**Problem 2.** (page 232)

What is the corresponding conclusion if  $\Omega$  is symmetric with respect to the point  $z_0$ ?

*Proof.*

□