# Counting structures on the $n \times k$ grid graph

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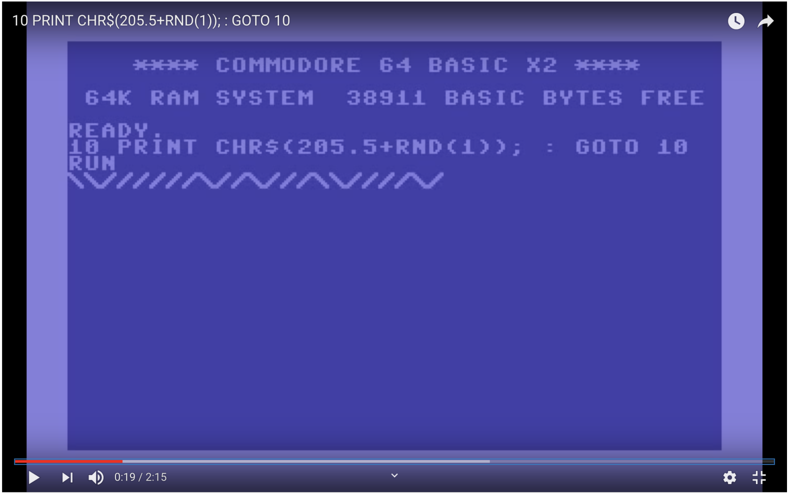
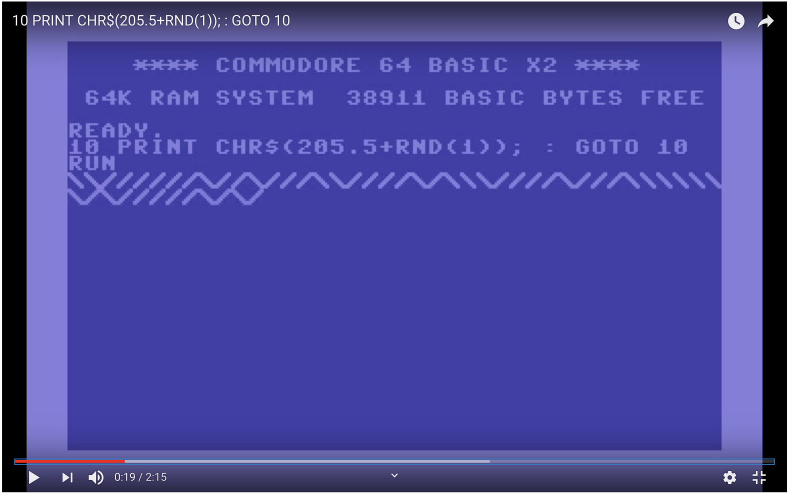
# Commodore 64

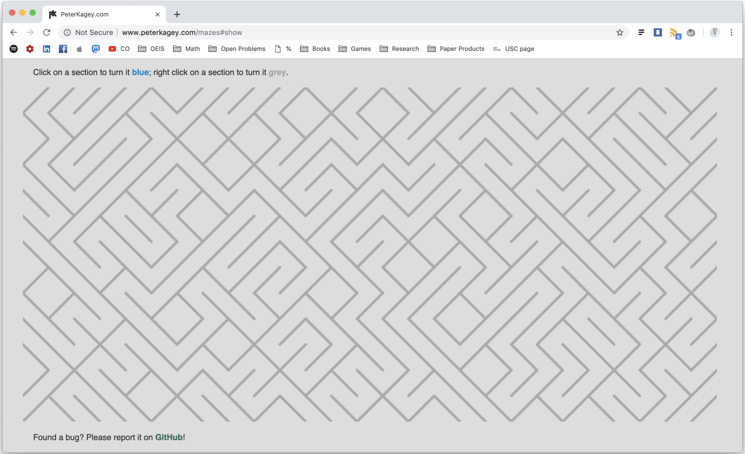


# Commodore 64



# Commodore 64

# Javascript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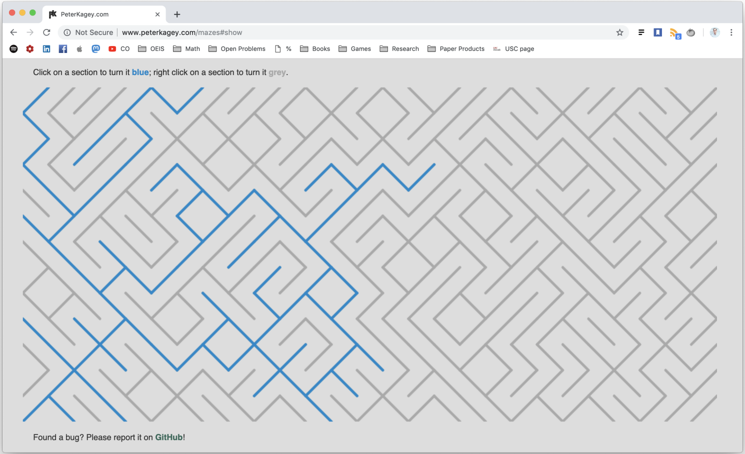
# Javascript



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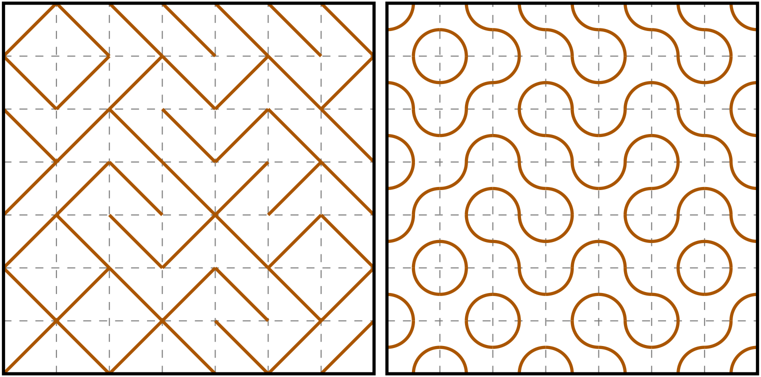
# Counting grids



# A295229: Number of tilings of the $n \times n$ grid, using diagonal lines to connect the grid points.

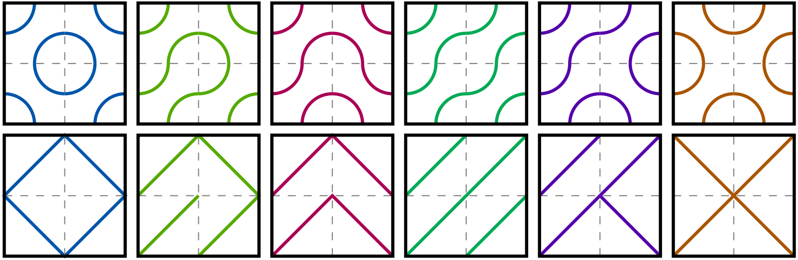
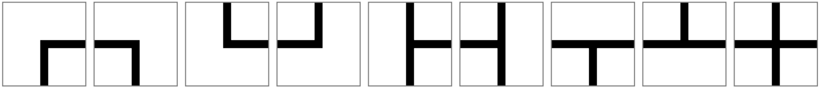
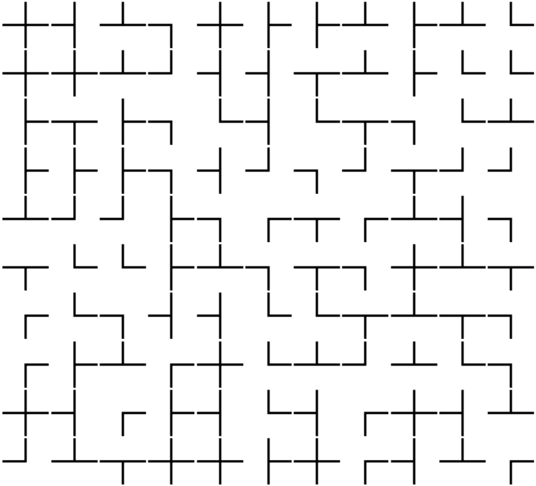


Figure 1: An example of the $a(2) = 6$ different ways to fill the $2 \times 2$ grid with diagonal tiles up to dihedral action of the square.

$$a(n) = \begin{cases} \frac{1}{8}\left(2^{n^2} + 2 \cdot 2^{n(n+1)/2} + 3 \cdot 2^{n^2/2} + 2 \cdot 2^{n^2/4}\right) & n \text{ even} \\ \frac{1}{8}\left(2^{n^2} + 2 \cdot 2^{n(n+1)/2} + 2^{(n^2+1)/2}\right) & n \text{ odd} \end{cases}$$

# Other tiles





# Baby's first corollary

## Theorem (Corollary of Burnside's Lemma)

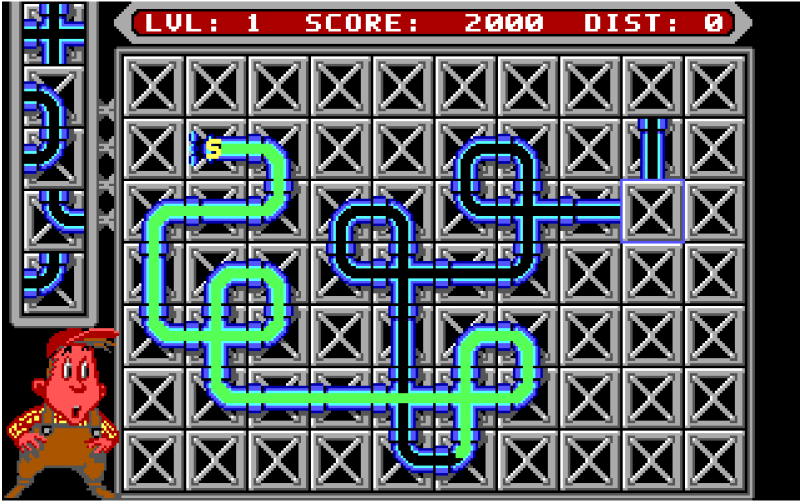
*Let*

- $t$ *be the number of tiles,*
- $q$ *be the number of tiles symmetric under a* $90^\circ$ *rotation,*
- $h$ *be the number of tiles symmetric under a* $180^\circ$ *rotation,*
- $d$ *be the number of tiles symmetric under a diagonal reflection, and*
- $v$ *be the number of tiles symmetric under a vertical reflection.*

*Then the number of tilings up to symmetries of the square is given by*

$$a(n) = \begin{cases} \frac{1}{8}\left(t^{n^2} + 2qt^{(n^2-1)/4} + ht^{(n^2-1)/2} + (v^n + d^n)t^{(n^2-n)/2}\right) & n \text{ odd} \\ \frac{1}{8}\left(t^{n^2} + 3t^{n^2/2} + 2t^{n^2/4} + 2d^n t^{(n^2-n)/2}\right) & n \text{ even} \end{cases}$$

# Pipe Mania



# Leaf Free Grids

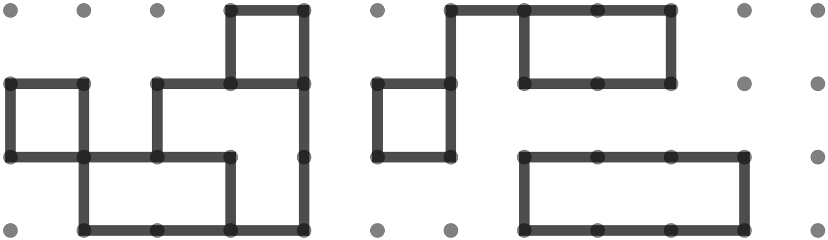
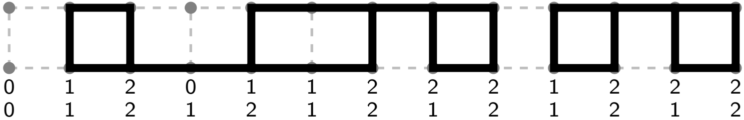


Figure 2: One of the $a_4(12) = 42650154782713601$ grids on the $12 \times 4$ grid.

$$a_2(1) = 1, a_2(2) = 2$$
$$a_2(n) = 5a(n-1) - 5a(n-2)$$

$$a_3(1) = 1, a_3(2) = 5, a_3(3) = 43, a_3(4) = 463$$
$$a_3(n) = 12a(n-1) - 6a(n-2) - 20a(n-3) - 5a(n-4)$$

# Leaf Free Grids: The System of Recurrences



# Leaf Free Grids: The System of Recurrences

$$a_{(0,0)}(n) = a_{(0,0)}(n-1) + a_{(2,2)}(n-1)$$
$$a_{(1,0)}(n) = a_{(0,0)}(n-1) + a_{(2,2)}(n-1)$$

# System of Recurrences: Getting one long recurrence

# Mazes and Spanning Trees

# Systems of linear equations

# Generalizations