

Week 2 Quiz Solutions.

1. (5 points) Determine whether or not the points $A(0, 1, -1)$, $B(1, -1, 0)$, $C(-1, 3, -2)$ are collinear.

Solution.

The points are collinear if \overrightarrow{AB} is a scalar multiple of \overrightarrow{AC} .

$$\overrightarrow{AB} = \vec{B} - \vec{A} = \langle 1, -1, 0 \rangle - \langle 0, 1, -1 \rangle = \langle 1, -2, 1 \rangle$$

$$\overrightarrow{AC} = \vec{C} - \vec{A} = \langle -1, 3, -2 \rangle - \langle 0, 1, -1 \rangle = \langle -1, 2, 1 \rangle$$

So $\overrightarrow{AB} = -\overrightarrow{AC}$, and the points are collinear.

2. (5 points)

- (a) Show that the equation $x^2 + y^2 + z^2 - 2x - 4y - 6z = -13$ represents a sphere (put it into standard form).
- (b) The sphere in part (a) and the plane $x = 1$ intersect in a circle. Find the radius and the coordinates of the center of this circle.

Solution.

- (a) Completing the square gives

$$\begin{aligned} (x^2 - 2x + 1) + (y^2 - 4y + 4) + (z^2 - 6z + 9) &= -13 + 1 + 4 + 9 \\ (x - 1)^2 + (y - 2)^2 + (z - 3)^2 &= 1. \end{aligned}$$

- (b) Let $x = 1$, then

$$(1 - 1)^2 + (y - 2)^2 + (z - 3)^2 = (y - 2)^2 + (z - 3)^2 = 1.$$

So this circle is centered at $(x, y, z) = (1, 2, 3)$, with radius 1.