Math 533: Homework 7

Peter Kagey

April 24, 2019

Problem 1. Write the character table for S_4.

Proof. First, we know that the trivial and the signed representation have to be in there. Next, we know that there is only one way to write $4! = 24$ as a sum of five squares, namely $1 + 1 + 4 + 9 + 9$.

	$(1,1,1,1)$	$(2,1,1)$	$(2,2)$	$(3,1)$	(4)
$\chi^{(1,1,1,1)}$	1	1	1	1	1
$\chi^{(2,1,1)}$	3	?	?	?	?
$\chi^{(2,2)}$	2	?	?	?	?
$\chi^{(3,1)}$	3	?	?	?	?
$\chi^{(4)}$	1	-1	1	1	-1

We know that the defining representation has

$$\begin{aligned}
\chi^{\text{def}}((1)(2)(3)(4)) &= 4 \\
\chi^{\text{def}}((12)(3)(4)) &= 2 \\
\chi^{\text{def}}((12)(34)) &= 0 \\
\chi^{\text{def}}((123)(4)) &= 1 \\
\chi^{\text{def}}((1234)) &= 0
\end{aligned}$$

And by taking the inner products with the two known characters gives

$$\begin{aligned}
\frac{1}{4!}\langle \chi^{\text{def}}, \chi^{(1,1,1,1)} \rangle &= \frac{1 \cdot 4 + 6 \cdot 2 + 3 \cdot 0 + 8 \cdot 1 + 6 \cdot 0}{4!} = 1 \\
\frac{1}{4!}\langle \chi^{\text{def}}, \chi^{(4)} \rangle &= \frac{1 \cdot 4 - 6 \cdot 2 + 3 \cdot 0 + 8 \cdot 1 - 6 \cdot 0}{4!} = 0
\end{aligned}$$

□

Problem 3.

Proof.

(a)

$\square$