Week 5 Quiz Solutions.

1. (*5 points*) Find a parametric equations of the tangent line to the curve $\vec{r}(t) = \langle \frac{\pi}{t}, \cos^2(t), e^t \rangle$ at the point $(1, 1, e^\pi)$.

   Solution.
   First, by solving for the z coordinate and checking the others, it is clear that when $t = \pi$, $\vec{r}(\pi) = (1, 1, e^\pi)$. Then, it is enough to find the direction of a tangent vector at this point

   $$\begin{aligned} \vec{r'}(t) &= \langle -\pi t^{-2}, -2\cos(t)\sin(t), e^t \rangle \\ \vec{r'}(\pi) &= \langle -1/\pi, 0, e^\pi \rangle. \end{aligned}$$

   Using a point on the line and its direction, the parametric equations of the tangent line can be written

   $$\begin{aligned} x &= 1 - t/\pi \\ y &= 1 \\ z &= e^\pi + e^\pi t. \end{aligned}$$

2. (*5 points*)

   (a) Let $\vec{r}(t) = \langle 0, t, t^2 \rangle$ and $\vec{s}(u) = \langle u\sin(\pi u), u\cos(\pi u), u \rangle$. Find the intersection points of $\vec{r}(t)$ and $\vec{s}(u)$.

   Solution.

   $$\begin{aligned} 0 &= u\sin(\pi u) && (1) \\ t &= u\cos(\pi u) && (2) \\ t^2 &= u && (3) \end{aligned}$$

   - By the first equation, $u = 0$ or $\sin(\pi u) = 0$, so u must be an integer.
   - This means that, in the second equation, $\cos(\pi u) = \pm 1$, so $t = u$ or $t = -u$.
   - Substitution into the third equation gives $u = (\pm u)^2 = u^2$ so $u = 0$ or $u = 1$.
   - By the third equation, if $u = 0$ then $t = 0$.
   - By the second equation, if $u = 1$ then $t = -1$.

   Checking these values,

   $$\begin{aligned} \vec{s}(0) &= \langle 0, 0, 0 \rangle = \vec{r}(0) \\ \vec{s}(1) &= \langle 0, -1, 1 \rangle = \vec{r}(-1). \end{aligned}$$

   (b) Which (if any) of these intersection points are collision points?

   Solution.
   Collisions happens when $\vec{r}(t) = \vec{s}(t)$. In this case, this only happens when $t = u = 0$.