

**Difficulty:** 2/4    **Interest:** 2/4

Let  $a_3(n)$  be the least  $k > n$  such that  $nk$  or  $nk^2$  is a cube, and let  $A299117$  be the image of  $a_3(n)$ .

$$a_3(1) = 8$$

$$a_3(2) = 4$$

$$a_3(3) = 9$$

$$a_3(4) = 16$$

$$a_3(5) = 25$$

$$a_3(6) = 36$$

$$a_3(7) = 49$$

$$a_3(8) = 27$$

$$a_3(9) = 24$$

**Question.** Is there another way to characterize what integers are in  $A299117$ ?

**Note.**  $A299117$  contains every cube, because  $a(n^3) = (n+1)^3$ .

$A299117$  contains the square of every prime, because  $a(p) = p^2$ .

**Related.**

1. Does  $A299117$  contain every square?
2. Does  $A299117$  contain any squarefree number?
3. What about the generalization: the image of  $a_\beta$  where  $a_\beta(n)$  is the least  $k > n$  such that  $nk, nk^2, \dots, nk^{\beta-2}$ , or  $nk^{\beta-1}$  is a  $\beta$ -th power? Prime  $\beta$  is an injection—is this well behaved?

**References.**

<https://oeis.org/A277781>

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