Difficulty: 3/4 Interest: 3/4

Consider an n-coloring of a triangular grid such that no upright sub-triangle has the same coloring as any other (up to rotation).

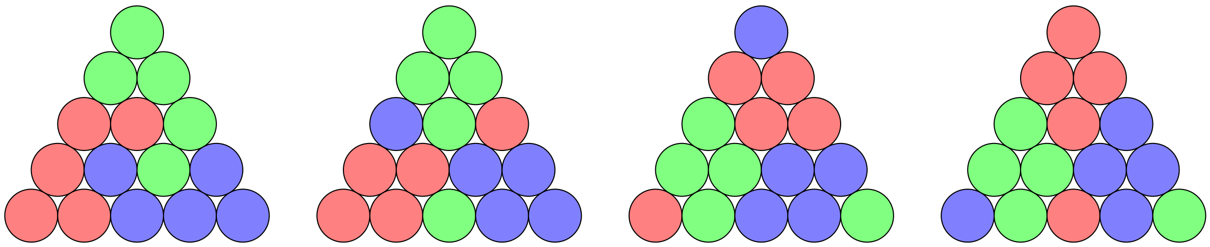

Figure 1: Four examples of 3-colorings of the length 5 triangle. In all cases, 10 different colorings appear exactly once. In the first example, starting from the top: (1) GGG, (2) RRG, (3) RGG, (4) RRB, (5) RGB, (6) GGB, (7) RRR, (8) RBB, (9) GBB, and (10) BBB. (Incidentally, this is *all* of the colorings, so $a(3) = 5$.)

Question. Given n colors, what is the biggest triangle that can be constructed? Call the side length of such a triangle $a(n)$.

Related.

1. What if inverted triangles are counted too?
2. What if two triangles with the same coloring but different rotations are counted as different?
3. How many patterns exist for a triangle of length k with the minimum number of labels?
4. What if diagonal equilateral triangles are also considered? (For example, take the second circle on every side as measured clockwise from each corner.)
5. What if this is done on a square grid?
6. What if this is done on hexagonal shapes?
7. What if this is done on tetrahedra or cuboids?
8. Consider the lexicographically earliest infinite case. Does every triangle eventually appear?

References.

`https://math.stackexchange.com/a/2416790/121988`

`https://math.stackexchange.com/a/2636168/121988`