Richard Guy beat me to this problem by a few years. (https://arxiv.org/abs/1207.5099).
John Conway described the "Subprime Fibonacci Sequence":

$$a(1) = a, a(2) = b, a(n+1) = \text{gpd}(a(n) + a(n-1)),$$

where $\text{gpd}(k)$ is the greatest proper divisor of k.
Conway then conjectured that regardless of the starting terms, the sequence ends in a handful of cycles. Richard Guy found that there are more cycles than those that Conway conjectured.

Question. What are all of the different possible end behaviors of Conway's Subprime Fibonacci Sequence?