Suppose you have two finite groups G and H, a G-set X, and a target function $t\colon X \to H$.
You want to construct some finite sequence of maps $\{f_i\colon X \to H\}_{i=1}^{N}$ such that for any function $s_0\colon X \to H$ and any sequence $\{g_i \in G\}_{i=1}^{N-1}$ the following sequence contains t

$$\{s_0(x), \underbrace{f_1(x)\cdot s_0(x)}_{s_1}, \underbrace{f_2(x)\cdot g_1 * s_1(x)}_{s_2}, \ldots, \underbrace{f_N(x)\cdot g_{N-1} * s_{N-1}(x)}_{s_N}\}$$

where $f_i(x)\cdot s(x) = f_i(x)s(x)$ under ordinary group multiplication, and $g_i * s(x) = s(g_i^{-1}x)$.

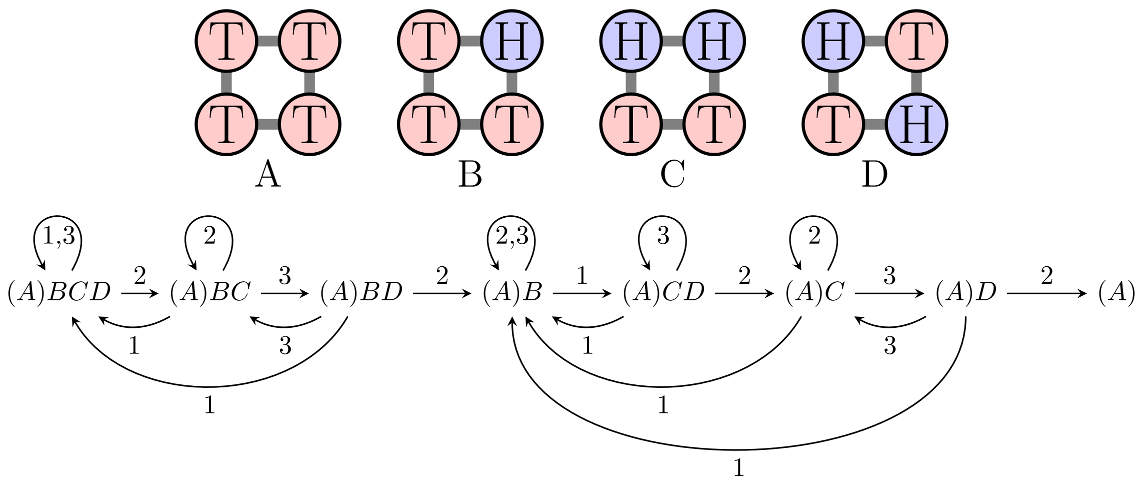

Figure 1: Let X be the vertices of the square $G = C_4$ the cyclic group which acts on the X by rotation, $H = \mathbb{Z}_2$, and $t(x) \equiv 0$. Then interspersing the above sequence of $N = 7$ moves with $f(x) = 1$ will always result in the sequence reaching the target function, where $1, 2,$ and 3 are the moves that flip one coin, two opposite coins, and two adjacent coins respectively.

Question. What conditions on G, H, X and t guarantee such a finite sequence of maps?

Related.

1. Can this be generalized to an arbitrary 2^n-gon?
2. Given G, H, X and t, what is the minimum N?
3. If the sequence $\{g_i \in G\}_{i=1}^{\infty}$ is chosen uniformly at random, what is the expected length of $\{f_i\}$ before t is reached?
4. What if there is a set of target functions, any one of which is valid. (e.g. all sets satisfying the condition that the number of heads is congruent to 2 (mod 3).)
5. Can this be generalized to multiple dimensions (e.g. a tetrahedron)?
6. What if something about s_i is told to you after the ith move for each i (and the strategy can depend on this information)?

Note. In the case of the polygon is a sequence of moves for $n = 1$, $n = 2$, $n = 4$, and $n = 8$, but no other $n < 12$. Also in the example and in the case for $n = 8$, the first and last move is to flip every other coin. Lastly, the number of flips for these cases (to get either all heads or all tails) is $0, 1, 7, 127$.

References.

http://mathriddles.williams.edu/?p=77