Difficulty: 3/4 Interest: 1/4

A country has a strange legislative procedure. For each bill, the body is split up into k_1 committees of $\lfloor n/k_1 \rfloor$ or $\lfloor n/k_1 \rfloor$ legislators each, each of which picks a representative. These k_1 representatives are split up into k_2 sub-committees with $\lfloor k_1/k_2 \rfloor$ or $\lfloor k_1/k_2 \rfloor$ legislators each, which each elect a representative, and so on until $k_T = 1$ and the final committee votes on the bill.

There are a few rules:

1. Each committee (and subcommittee and so on) much have at least ℓ members.
2. Ties are settled by a coin toss.
3. The president does not get to vote, but she does get to choose the number of committees and who goes in each one.
4. There are α supporters who will always vote in the president's interests and $n - \alpha$ who will always vote against.

Let $a_\ell(n)$ be the minimum number of supporters (α) required for the president to be able to pass every bill.

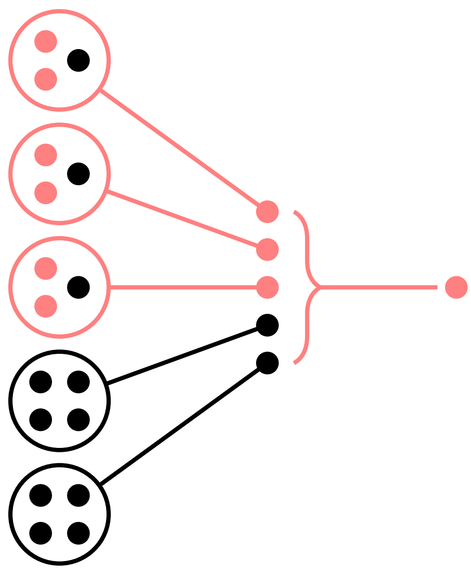

Figure 1: An example of $n = 17$ legislators with a minimum comittee size of $\ell = 3$, which demonstrates that $a_3(17) \leq 6$.

Question. What is an efficient way to compute $a_\ell(n)$ for general ℓ and n?

Related.

1. What if the president gets to choose who is on each committee but the opposition party gets to choose the committee size? Vice versa?
2. What if $k_1 \leq k_2 \leq \ldots \leq k_T$? Or $k_1 \geq k_2 \geq \ldots \geq k_T$?
3. What if ties go to the president? To the opposition?

References.

https://oeis.org/A290323

https://math.stackexchange.com/q/2395044/121988