



Suppose that we want to model Plinko/Galton board by supposing that a ball (1) is equally likely to bounce in either direction off of the first peg, and (2) will bounce in the same direction that it previously bounced with probability p . For example, when $p = 1$, it will always bounce in the same direction, when $p = 1/2$ it is equally likely to bounce in either direction at any peg, and when $p = 0$, it will alternate left-right-left-right (or vice versa).

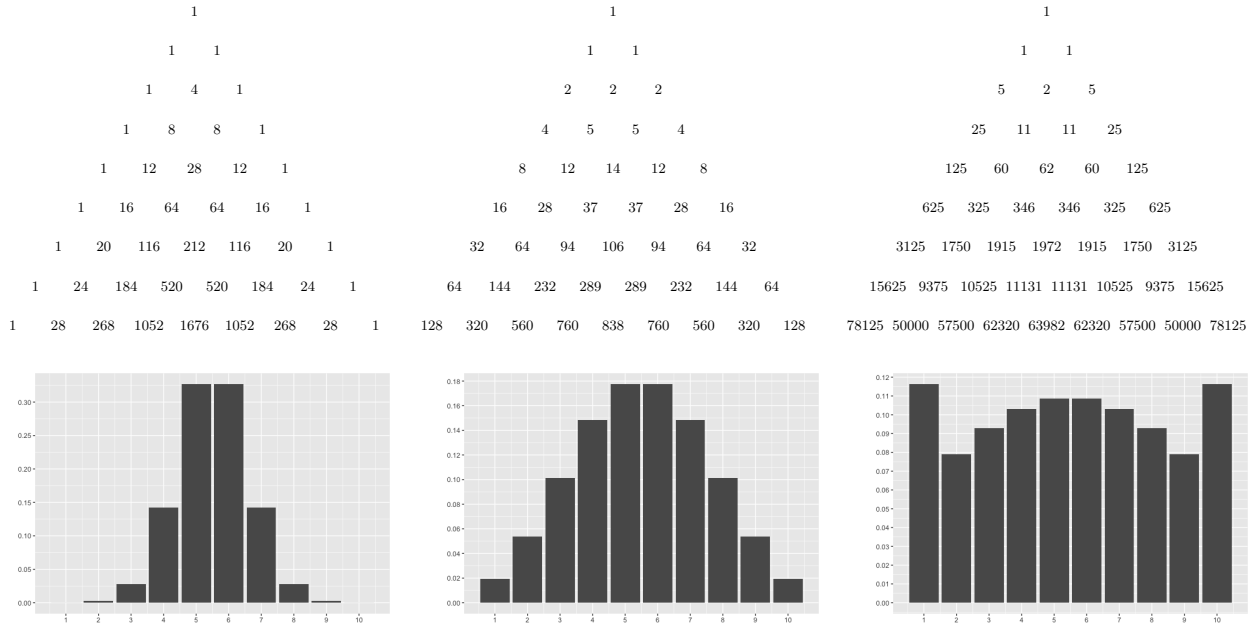


Figure 1: Illustrations for numerators of $p = 1/3$, $p = 2/3$ and $p = 5/6$, followed by probability mass functions of the respective tenth rows.

Question. For an arbitrary $p \in [0, 1]$, and a triangle with n rows, what is the distribution of balls in bin k ?

Related.

1. What is the least/greatest value of p such that at the $(2n)$ -th row, the middle is equal to the extreme? How about for the $(2n + 1)$ -st row?
2. What if this is done on a different geometry like a cylinder or a tetrahedron?
3. How does this relate to lattice walks? (E.g. see A348595.)
4. As $n \rightarrow \infty$, does this converge in distribution to a normal distribution? If so, what is the variance?

References.

A035002: Table of numerators for $p = 2/3$.

A348595: Related to table of numerators for $p = 1/3$.