Say that two sequences with distinct elements are in the same equivalence class if their first differences have the same signs. (e.g. $(1, 3, 2, 3)$ and $(7, 8, -1, 0)$ are equivalent because their first differences are both $(+, -, +)$.)

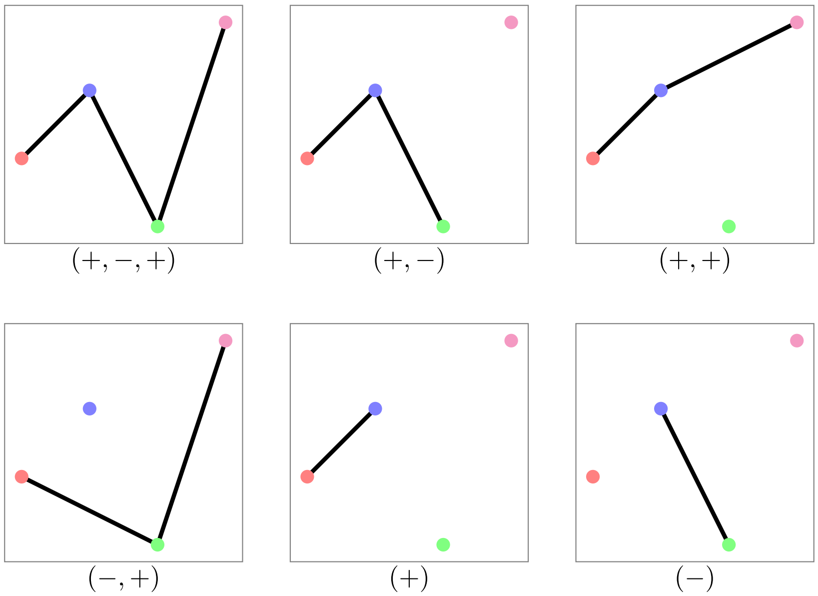

Figure 1: $(0, 1, -1, 2)$ has subsequences in the following six equivalence classes: $(+, -, +)$, $(+, +)$, $(+, -)$, $(-, +)$, $(+)$, $(-)$. No length 4 sequence has its subsequences in more equivalence classes, so $a(4) = 6$.

Question. What is the general formula for $a(n)$?

Related.

1. What if the sequences do not necessarily consist of distinct elements?
2. What if two sequences are considered to be equivalent if they are in the same "sort order"; that is, if both sequences have their biggest element in the same position, their second biggest in the same position, and so on.
3. What if $(+, +) \sim (+)$?
4. Is the number of equivalence classes for the subsequences determined by the number of local minima and maxima?

Note. A quick attempt finds that $a(2) = 1$, $a(3) = 3$, $a(4) = 6$, and $a(5) = 11$. (Fibonacci minus 2?)
For question 70.3, conjecture the answer is $a'(n) = 2n - 3$ for $n \geq 2$.
For question 70.3 without distinct elements (as in problem 70.1), the inital terms are

$$\begin{aligned}
&a(2) = 1 \text{ via } (+)\\
&a(3) = 4 \text{ via } (+,-),(+),(-),(=)\\
&a(4) = 8 \text{ via } (+,-,+),(+,-),(+,=),(=,+),(-,_)\,(+),(-),(=)\\
&a(5) = 15; a(6) = 25; a(7) = 40\\
&a(8) = 62; a(9) = 94; a(10) = 141; a(11) = 210; a(12) = 311 \text{ (conjectured*)}
\end{aligned}$$

* Assumes sequence is $(1, 2, 1, 2, 1, 2, \ldots)$.