

Difficulty: 3/4 Interest: 3/4

Suppose that there is a “cop” and a “robber” on an infinite grid, where each starts at some given position with some given orientation on the grid, and each can move according to some rule set.

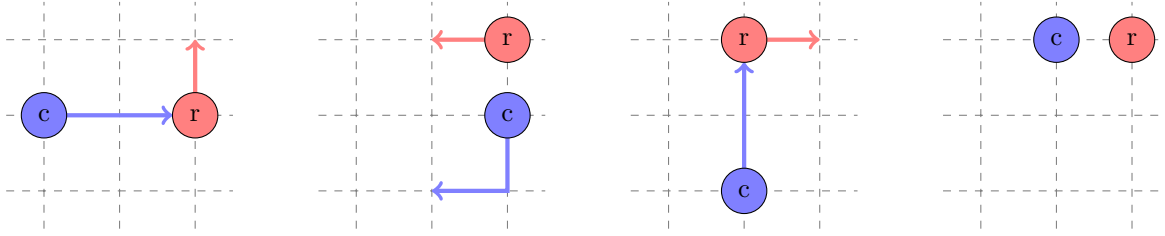


Figure 1: In this example, the cop can perform any of the following moves $C = \{ 2 \text{ units straight, } 1 \text{ unit right} + 1 \text{ unit right, } 1 \text{ unit right} + 1 \text{ unit straight} \}$ and the robber can move one unit in any direction along the grid. After three steps, the cop has not caught the robber, but if the robber moves forward, backward, or right, then she will be caught.

Question. Is there a procedure for determining in general whether the cop can catch the robber?

Related.

1. Is there a procedure that can put a bound on the number of steps it will take for the cop to catch the robber?
2. If the cop/robber perform moves in their rule set according to some distribution, what is the probability that the cop will eventually catch the robber?
3. How does this generalize to a torus, Möbius strip, cylinder, multiple dimensions or a triangular/hexagonal grid?

References.

<http://demonstrations.wolfram.com/TheHomicidalChauffeurProblem/>

https://en.wikipedia.org/wiki/Homicidal_chauffeur_problem