

**Difficulty:** 3/4    **Interest:** 2/4

Consider a 2-coloring of a triangular grid of length  $\ell$ . Then label each cell with its greatest number of neighbors of one color.

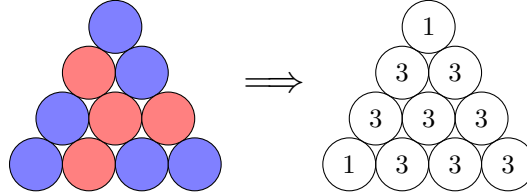


Figure 1: The second cell (reading top to bottom and left to right) is labeled with a  $\max(3, 1) = 3$  because it has 3 blue neighbors and 1 red neighbor.

**Question.** How many colorings exist of a length  $\ell$  triangle such that the maximum label is 3?

**Related.**

1. If the “number triangle” is summed for each coloring, which coloring has the smallest sum?
2. How does this generalize for a  $k$ -coloring?
3. How does this generalize to a  $n \times m$  square grid where horizontal-vertical connections are counted? Diagonal connections? Both?
4. How does this generalize to a tetrahedron, torus, Möbius strip, cylinder, or cube?
5. How many colorings exist of a length  $\ell$  triangle such that the maximum label is 4 or 5?