



According to Peter Winkler's problem "Red and Blue Dice" in *Mathematical Mind-Benders*, given two sequences of length  $n$  with letters in  $[n]$ , there must always exist a (nonempty) subsequence in each such that the sum of each of the subsequences are equal. Furthermore, there must exist a *substring* (i.e. contiguous subset) in both such that the sum of each substring is equal.

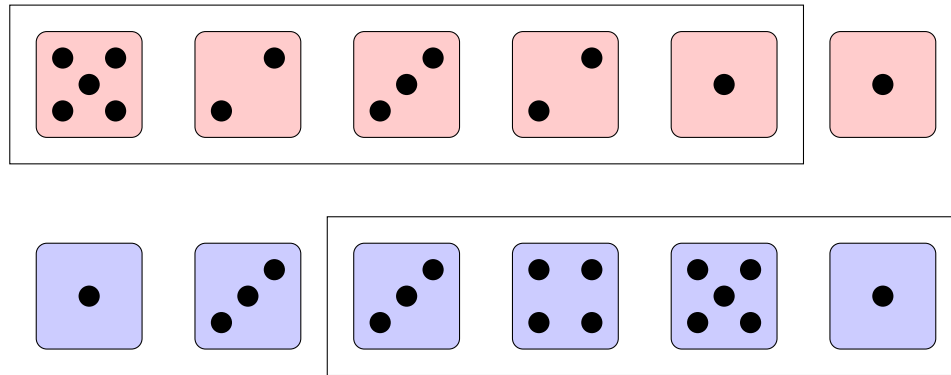


Figure 1: The first five dice in the top row have the same sum as the last four dice in the bottom row.

**Question.** How many equal-sum subsequences (respectively substrings) are guaranteed to exist?

**Related.**

1. What if the subsequence or substring must have length greater than or equal to  $k$ ?
2. What if the subsequences or substrings must be of equal length?
3. What if the substring is allowed to wrap around?
4. What if this is done over permutations  $S_n$  instead of subsets  $[n]^n$ ?
5. What if the sequences are of length  $\ell < n$ , and must be injections from  $\mathbb{N}$  to  $[n]$ ?
6. What if there are three sequences?  $m$  sequences?
7. Given two random sequences, what is the expected number of equal-sum subsequences?

**References.**

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