A snail travels along the grid in unit steps—but it hates crossing its trail, so if a step is going to cross is trail, it will only go half way.

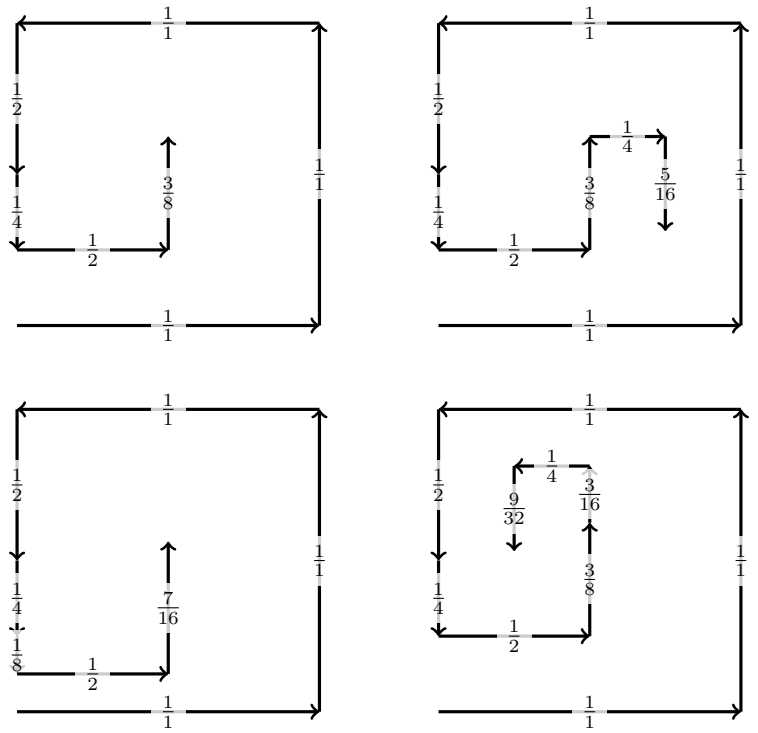

Figure 1: Left-to-right and top-to-bottom: examples of walks that end in step sizes that have numberators of 3, 5, 7, and 9.

Question. Let $a(n)$ be the minimum number of steps the snail must take before it can take a step of size $(2n-1)/2^k$. What is $a(n)$?

Related.

1. What if the snail must always turn left or right?
2. What if the snail is walking on a triangular or hexagonal grid?
3. What is the set of points the snail can step on after finitely many steps?
4. How many distinct points can the snail reach after m steps?

References.

`https://math.stackexchange.com/q/2678852/121988`

`https://oeis.org/A300444`