Consider all r-colorings of the $n \times m$ grid where no two colors are adjancent (horizontally/vertically) more than once.

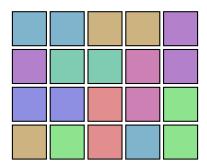


Figure 1: An 8-coloring of the 4×5 grid where no two colors are adjancent more than once. There is no 7-coloring.

Question. Let $r_{n \times m}$ be the smallest integer such that there exists an $r_{n \times m}$ -coloring of the $n \times m$ grid. What is $r_{n \times m}$?

Related.

- 1. What if colors are not allowed to be self-adjacent?
- 2. How many a(n, m)-colorings exist up to permutation of the colors?
- 3. What if this is done on a triangular or hexagonal grid?
- 4. What if orientation matters? (A horizontal adjacency is distinct from a vertical adjacency.)
- 5. What if order matters? (red-green is distinct from green-red.)
- 6. What if diagonal adjacencies are considered?

Note.

$$\begin{array}{llll} r_{1\times 1}=1 & & & \\ r_{1\times 2}=1 & r_{2\times 2}=3 & & & \\ r_{1\times 3}=2 & r_{2\times 3}=4 & r_{3\times 3}=5 & & \\ r_{1\times 4}=2 & r_{2\times 4}=5 & r_{3\times 4}=6 & r_{4\times 4}=7 & \\ r_{1\times 5}=3 & r_{2\times 5}=5 & r_{3\times 5}=7 & r_{4\times 5}=8 & r_{5\times 5}=9 \end{array}$$

References.

Problem 27.

Problem 40.

Problem 56.