Consider partitions of the $n \times m$ grid into triangles with vertices on gridpoints.

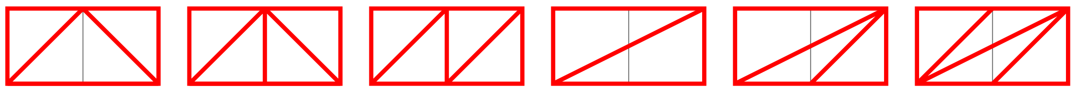

Figure 1: All six partitions of the 2×1 grid into triangles with gridpoint vertices, up to dihedral action.

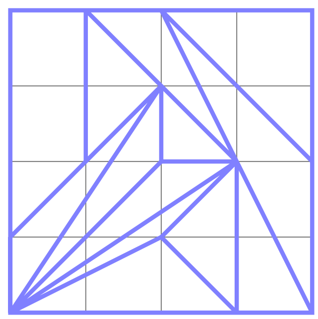

Figure 2: An example of a partitions of the 4×4 grid into triangles with no "empty" gridpoints.

Question. How many such partitions exist?

Related.

1. What if these are counted up to rotation/reflection?
2. What if this is done on a triangular/hexagonal grid?
3. How many partitions with the maximal number of triangles? With k triangles?
4. What if all triangles must be right triangles? Acute? Obtuse?
5. What if each gridpoint must touch a triangle? What is the minimum number of faces?
6. What if each gridpoint must touch as many triangles as possible? What is the minimum number of faces? What's the expected number of faces? (i.e. there's no way to draw a new edge?)
7. What if this is done on a grid in hyperbolic space?

Note. $a_1(n) = A051708(n)$

References.

`https://oeis.org/A051708`

`https://codegolf.stackexchange.com/q/176646/53884`