

Consider an $n \times n$ grid with n marked cells, where each marked cell must be contained in exactly one of n distinct continuous n-cell regions.

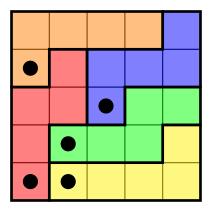


Figure 1: An example of a 5×5 grid with a unique solution.

Question. How many $n \times n$ boards exist with a unique solution?

Related.

- 1. How many $n \times n$ boards exist with no solution? Multiple solutions?
- 2. What board has the most solutions?
- 3. What if this is counted up to dihedral action?
- 4. What if this is done on an $n \times m$ board with k marked cells where k|nm and each region has nm/k cells?
- 5. What if the board is a torus? Triangular/hexagonal grid? Multiple dimensions?
- 6. What if instead of marked cells there are marked regions? (e.g. two adjacent cells are both marked "1" and must be captured by the same region)
- 7. What if cells must must be rectangular? Symmetric?

References.

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https://math.stackexchange.com/q/3072735/121988