Ron Graham's (A006255) sequence is the least integer k for which there exists a strictly increasing integer sequence

$$n = a_1 < a_2 < \cdots < a_t = k$$

such that $a_1 \cdot a_2 \cdots a_t$ is square.

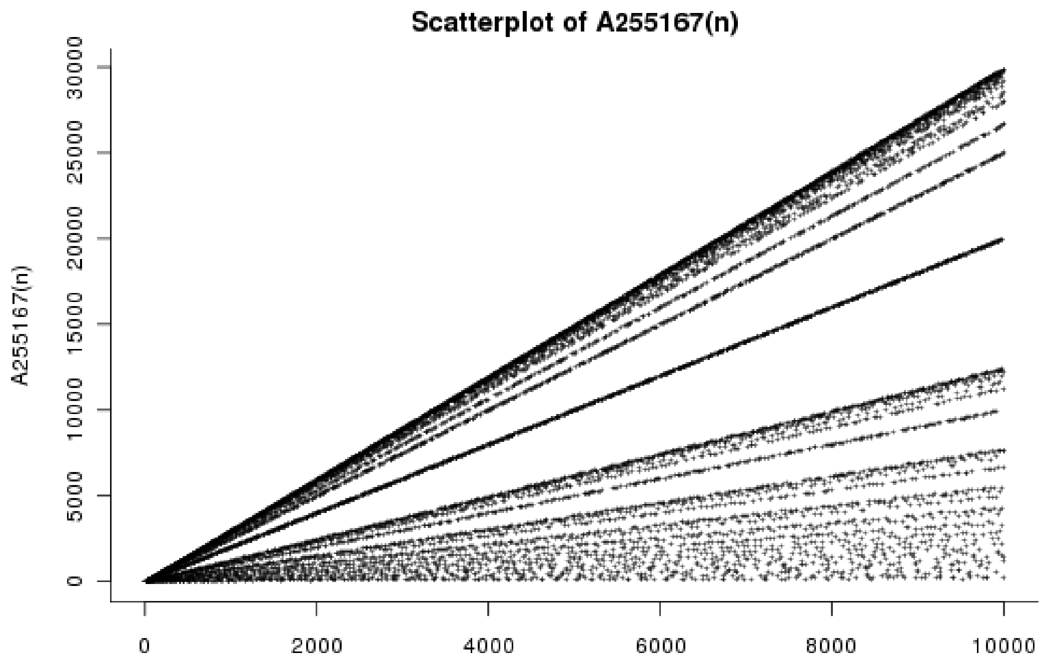

Figure 1: The scatterplot for OEIS sequence A255167 which is defined as the difference $A255167(n) = A072905(n) - A006255(n)$.

Question. Does there exist any n for which $A006255(n) = A072905(n)$. In other words, is there any non-square n for which $n \cdot A006255(n)$ is square?

Related.

1. Does the gap $A072905(n) - A006255(n)$ have a nonzero lower bound?
2. Does this idea generalize to cubes, powers of four, etc?

Note. A006255 is bounded above by A072905, the least $k > n$ such that kn is square.

This is equivalent to showing that for any $a < b$ with the same squarefree part, there is some subset of $\{a+1, a+2, \ldots, b-1\}$ such that the product of the elements of the subset has the same squarefree part as a (and b).

References.

https://oeis.org/A006255

https://oeis.org/A072905

https://oeis.org/A255167