Starting with a pair of integers (a, b), there exists an algorithm for making the two integers equal by repeated applications of the map $(x, y) \overset{\alpha}{\mapsto} (2x, y+1)$ or $(x, y) \overset{\beta}{\mapsto} (x+1, y)$.

$$\begin{array}{rlllllll}
(4,0) & \overset{\beta}{\mapsto} (5,0) & \overset{\beta}{\mapsto} (6,0) & \overset{\alpha}{\mapsto} (12,1) & \overset{\beta}{\mapsto} (13,2) & \overset{\beta}{\mapsto} (14,4) & \overset{\beta}{\mapsto} (15,8) & \overset{\beta}{\mapsto} (16,16) \\
(5,4) & \overset{\beta}{\mapsto} (6,8) & \overset{\beta}{\mapsto} (7,16) & \overset{\beta}{\mapsto} (8,32) & \overset{\alpha}{\mapsto} (16,33) & \overset{\beta}{\mapsto} (17,66) & \overset{\alpha}{\mapsto} (34,67) & \overset{\alpha}{\mapsto} (68,68) \\
(8,1) & \overset{\beta}{\mapsto} (9,2) & \overset{\beta}{\mapsto} (10,4) & \overset{\alpha}{\mapsto} (20,5) & \overset{\beta}{\mapsto} (21,10) & \overset{\alpha}{\mapsto} (42,11) & \overset{\beta}{\mapsto} (43,22) & \overset{\beta}{\mapsto} (44,44) \\
(9,6) & \overset{\beta}{\mapsto} (10,12) & \overset{\beta}{\mapsto} (11,24) & \overset{\beta}{\mapsto} (12,48) & \overset{\alpha}{\mapsto} (24,49) & \overset{\beta}{\mapsto} (25,98) & \overset{\alpha}{\mapsto} (50,99) & \overset{\alpha}{\mapsto} (100,100) \\
(11,7) & \overset{\beta}{\mapsto} (12,14) & \overset{\beta}{\mapsto} (13,28) & \overset{\beta}{\mapsto} (14,56) & \overset{\alpha}{\mapsto} (28,57) & \overset{\beta}{\mapsto} (29,114) & \overset{\alpha}{\mapsto} (58,115) & \overset{\alpha}{\mapsto} (116,116)
\end{array}$$

Figure 1: Five examples of (shortest) seven-step paths to equality, starting from $(4, 0)$, $(5, 4)$, $(8, 1)$, $(9, 6)$, and $(11, 7)$.

Question. What is an algorithm for the shortest path to equality?

Related.

1. What are some good upper bounds for the shortest path length?
2. Can this be generalized to other maps (e.g. $(x, y) \mapsto (3x, y + 2)$)?
3. What is the least k such that there is a path from (a, b) to (k, k)? Is there a way to characterize all such values for k?

References.

`https://oeis.org/A304027`

`https://codegolf.stackexchange.com/q/164085/53884`