Suppose that we have a total order of some finite set of combinatorial objects, such as permutations, derangements, labeled graphs, partitions, compositions, Dyck paths, set partitions, signed partitions, labeled trees, etc.

Moreover, let's say we index them so that the minimum in the total order is indexed 1, the next element is indexed 2, and so on, and the maximum in the total order is labeled N, the size of the set.

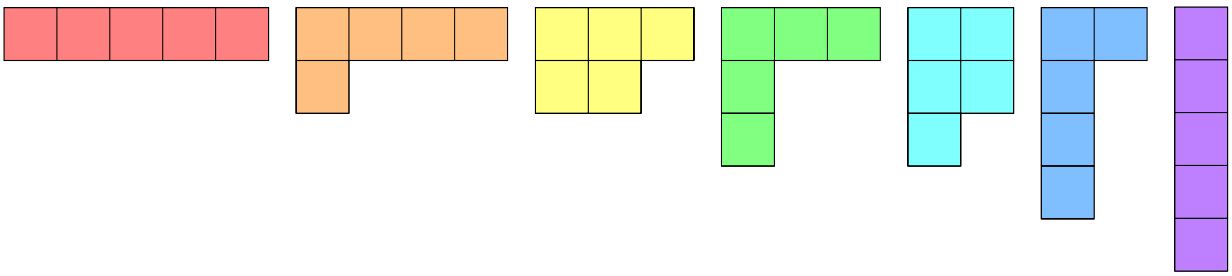

Figure 1: An illustration of the partitions of 5, corresponding to $4+1$, $3+2$, $3+1+1$, $2+2+1$, $2+1+1+1$, and $1+1+1+1+1$ respectively. Because the 4th partition is $3+1+1$, $\mathrm{unrank}(4) = 3+1+1$ and $\mathrm{rank}(3+1+1) = 4$.

Question. What combinatorial objects have efficient ranking and unranking algorithms, and what are the algorithms?

Related.

1. In particular, when do objects that are counted with the principle of inclusion–exclusion have such algorithms? With Burnside's lemma?
2. For which permutation statistics $\mathrm{st}\colon S_n \to \mathbb{Z}$ is it possible to unrank the set $\{\pi \in S_n \mid \mathrm{st}(\pi) = k\}$, for a given value of k?

Note. If we have two sets X and Y with an (efficient) bijection $X \to Y$, a total order on X which induces a total order on Y, and efficient ranking/unranking algorithms for X, then we have efficient ranking/unranking algorithms for Y. For instance, if we order labeled trees by lexicographic order of their Prüfer codes, then we have efficient ranking/unranking for labeled trees under this ordering.