Consider Ron Graham's sequence for LCM, that is, look at sequences such that

$$n = b_1 < b_2 < \ldots < b_t = k \text{ and } \mathrm{LCM}(b_1, \ldots, b_t) \text{ is square.}$$

Question. Let $A300516(n)$ be the least k (as a function of n) such that such a sequence exists?

$a(1) = 1$ via (1)	$a(11) = 121$ via $(11, 121)$	$a(21) = 49$ via $(21, 36, 49)$
$a(2) = 4$ via $(2, 4)$	$a(12) = 18$ via $(12, 18)$	$a(22) = 121$ via $(22, 64, 121)$
$a(3) = 3$ via $(3, 9)$	$a(13) = 169$ via $(13, 169)$	$a(23) = 529$ via $(23, 529)$
$a(4) = 4$ via (4)	$a(14) = 49$ via $(14, 16, 49)$	$a(24) = 48$ via $(24, 36, 48)$
$a(5) = 25$ via $(5, 25)$	$a(15) = 25$ via $(15, 16, 18, 25)$	$a(25) = 25$ via (25)
$a(6) = 12$ via $(6, 9, 12)$	$a(16) = 16$ via (16)	$a(26) = 169$ via $(26, 64, 169)$
$a(7) = 49$ via $(7, 49)$	$a(17) = 289$ via $(17, 289)$	$a(27) = 81$ via $(27, 81)$
$a(8) = 16$ via $(8, 16)$	$a(18) = 25$ via $(18, 20, 25)$	$a(28) = 49$ via $(28, 49)$
$a(9) = 9$ via (9)	$a(19) = 361$ via $(19, 361)$	$a(29) = 841$ via $(29, 841)$
$a(10) = 25$ via $(10, 16, 25)$	$a(20) = 25$ via $(20, 25)$	$a(30) = 50$ via (30)

Figure 1: Examples of $A300516(n)$ for $1 \leq n \leq 30$.

Related.

1. For what values n is $A300516(n)$ nonsquare?
2. For what values n does the corresponding sequence have three or more terms?
3. What is the analogous sequence for perfect cubes, etc?

References.

`https://oeis.org/A300516`