Figure 1: For $(n, k) = (4, 2)$, it appears that there are $C(4) = 14$ valid patterns in six equivalence classes.

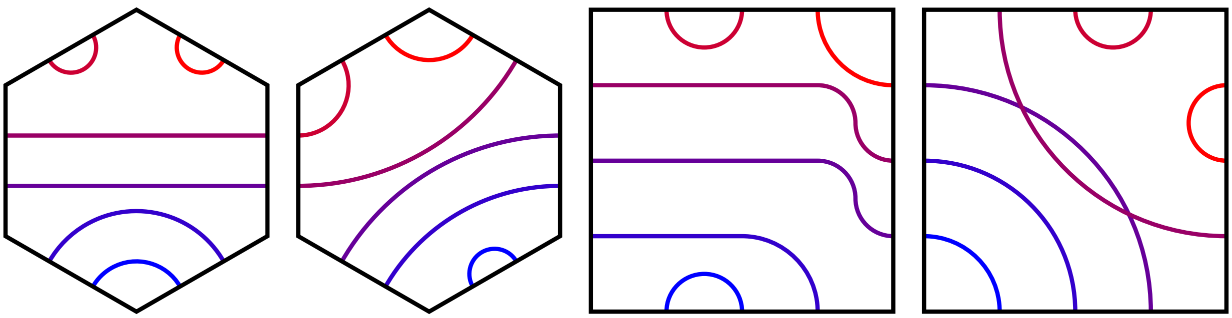

Figure 2: A valid $(6, 2)$-pattern does not always have a corresponding valid $(4, 3)$-pattern: the first square's pattern has curves that are not line segments or circular arcs, and the second square's pattern is self-overlapping.

**Question.** Given an $n$-gon with $k$ markings on each side, how much such patterns can be made using circular arcs and line segments such that each curve meets the boundary at a right angle?

**Note.** An $(n, k)$-pattern has or $C(nk/2)$ fewer realizations, where $C(m)$ is the $m$-th Catalan number.

**Related.**

1. How many $(n, k)$-patterns up to dihedral action of the $n$-gon?
2. For some fixed $k$, which values of $N$ allow for $C(Nk/2)$ $(N, k)$-patterns? If none, what are the obstructions?
3. How does this generalize to non-regular polygons or to higher dimensional polytopes?
4. What if curves other than circular arcs and line segments are allowed?

**References.**

Problems 28, 31, and 92.