Ron Graham's (A006255) sequence is the least k for which there exists a strictly increasing sequence

$$n = a_1 \leq a_2 \leq \ldots \leq a_T = k \text{ where } a_1 \cdot \ldots \cdot a_T \text{ is square.}$$

There is a known way to efficiently compute analogous sequences wherein $a_1 \cdot \ldots \cdot a_T$ is a p-th power, where p is a prime and where any term appears at most $p-1$ times.

Question. What is an efficient way to compute analogous sequences wherein $a_1 \cdot \ldots \cdot a_T$ is a c-th power, where c is composite?

$$\begin{array}{rll}
a_4(1) = 1 & \text{via } 1 & = 1^4 \\
a_4(2) = 2 & \text{via } 2^2 \cdot 4 & = 2^4 \\
a_4(3) = 6 & \text{via } 3^2 \cdot 4 \cdot 6^2 & = 6^4 \\
a_4(4) = 4 & \text{via } 4^2 & = 2^4 \\
a_4(5) = 10 & \text{via } 5^2 \cdot 8^2 \cdot 10^2 & = 20^4 \\
a_4(6) = 9 & \text{via } 6^2 \cdot 8^2 \cdot 9 & = 12^4 \\
a_4(7) = 14 & \text{via } 7^2 \cdot 8^2 \cdot 14^2 & = 28^4 \\
a_4(8) \leq 15 & \text{via } 8^2 \cdot 9 \cdot 10^2 \cdot 15^2 & = 60^4 \\
a_4(9) = 9 & \text{via } 9^2 & = 3^4 \\
a_4(10) \leq 18 & \text{via } 10^2 \cdot 12^2 \cdot 15^2 \cdot 18^2 & = 180^4
\end{array}$$

Figure 1: Examples of $a_4(n)$ for $n \in \{1, 2, \ldots, 10\}$.

Related.

1. For what values n is $a(n) < A006255(n)$?
2. How many c-th power sequences have $a_T = a_c(n)$?
3. Do any such c-th power sequences exactly two distinct terms?