A polyform counting problem from Alec Jones: let $a_k(n)$ count the number of polyabolos with n faces and k exposed edges.

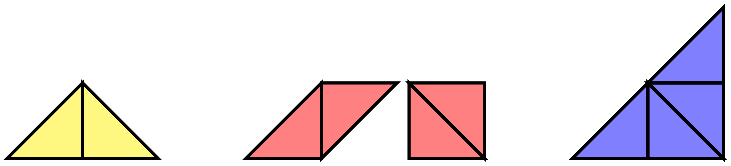

Figure 1: An example in yellow showing that $a_3(2) \geq 1$, two examples in red showing that $a_4(2) \geq 2$, and an example in blue showing that $a_3(4) \geq 1$.

Question. What is the smallest k such that for some fixed n, $a_k(n) > 0$?

Related.

1. What is the largest k such that for some fixed n, $a_k(n) > 0$?
2. What if $\hat{a}_k(n)$ counts polyiamonds instead?
3. What if concave polygons are excluded?
4. Is the following function well-defined?

$$b(k) = \max\{\, a_k(n) : n \in \mathbb{N}\,\}$$

5. Is the following function interesting?

$$c(n) = \max\{\, a_k(n) : k \in \mathbb{N}\,\}$$

References.

https://en.wikipedia.org/wiki/Polyiamond

https://en.wikipedia.org/wiki/Polyabolo