Say that an n-robot takes steps that are $1/n$ of a circle ($2\pi/n$ radians). Call a (k, j)-step pattern a walk that starts with k right turns, followed by j left turns, followed by k right turns, and so on until the robot reaches its original position in the original orientation.

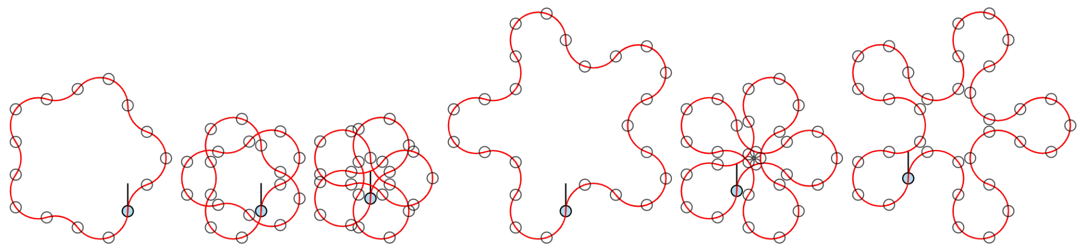

Figure 1: A 5-robot walks in $(1, 2)$, $(1, 3)$, $(1, 4)$, $(2, 3)$, $(2, 4)$, and $(3, 4)$-step patterns, respectively.

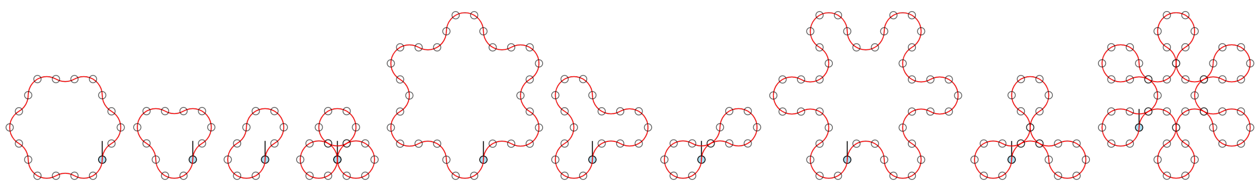

Figure 2: A 6-robot walks in $(1, 2)$, $(1, 3)$, $(1, 4)$, $(1, 5)$, $(2, 3)$, $(2, 4)$, $(2, 5)$, $(3, 4)$, $(3, 5)$, and $(4, 5)$-step patterns, respectively.

Question. For an n-robot, which of these paths encloses the most area? The least area?

Related.

1. Which of these figures has the largest convex hull? Smallest convex hull?
2. Is there a way to tell at a glance whether or not these walks will self-intersect?
3. Is there a way to tell at a glance if a (k, j)-step pattern will "go off to infinity"?
4. Are the areas enclosed by these figures "nice" numbers?
5. How does this generalize to $(a_1, a_2, \ldots, a_k)$-step patterns?
6. How many steps are taken before the figure "reconnects"?
7. For what step patterns are the "footprints" (the small grey circles in the figure) closest together (the $(2, 4)$-step pattern for the 5-robot)? How many steps are required to get two footprints within ε?
8. What if the robot turns $1/n$ of a circle when it turns right, but $1/m$ of a circle when it turns left?
9. What if the robot turns with some other rational number a/b of a circle?
10. What if the robot only needs to reach the original position, but not original orientation?

Note. It is likely that 3, 4, and 6-robots are special cases because the footprints appear at lattice points.

References.

Problem 41.

https://cemulate.github.io/project-euler-208/