Difficulty: 3/4 **Interest: 3/4**

A problem inspired by a Project Euler problem: suppose an n-robot takes steps that are $1/n$ of a circle, and turns right or left after every step.

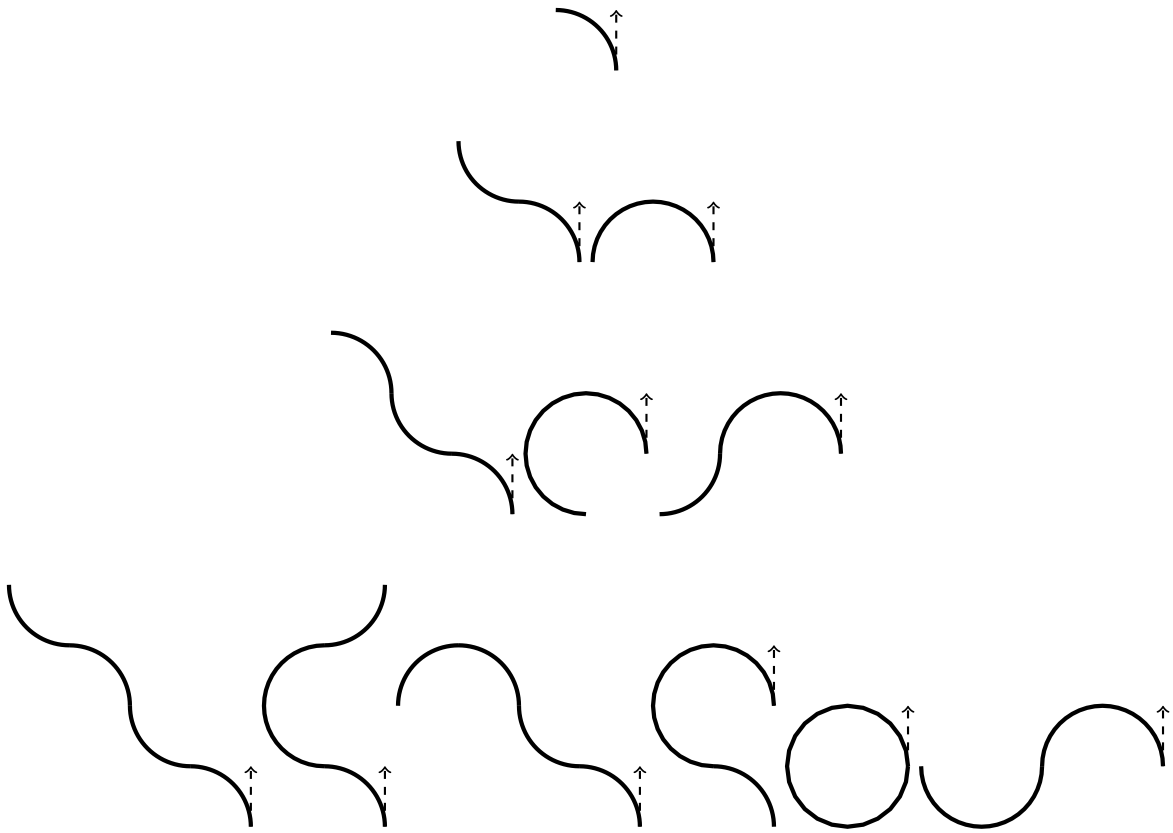

Figure 1: An example of distinct paths of k steps (up to dihedral action) for a 4-robot. $a(1) = 1$, $a(2) = 2$, $a(3) = 3$, and $a(4) = 6$.

Question. How many distinct paths exist for an n-robot, where the robot never retraces its steps?

Related.

1. What if the robot is allowed to retrace its steps?
2. What is the smallest radius that can contain a k-step walk if the robot cannot retrace its steps?
3. What if only smooth loop paths are counted? (The robot returns to where it started in the same direction that it started.)
4. Can smooth loop paths occur when the number of steps is not a multiple of n?
5. What if the orientation of the path matters (i.e. *not* counted up to dihedral action)?
6. What if this is done on a torus, cylinder, or Möbius strip?
7. What if the robot cannot cross its own path?

References.

`https://projecteuler.net/index.php?section=problems&id=208`