Start with an $n \times n$ grid of boxes and place lines through gridpoints at the border. A box is considered "on" if a line travels through its interior.

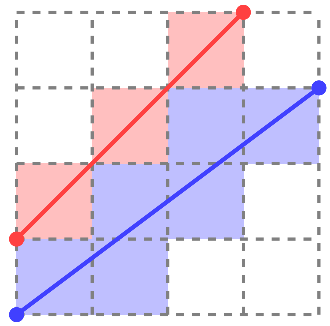

Figure 1: An example of two lines drawn on a grid. The seven white squares still require a line to be drawn through them.

Question. What is the minimum number of lines required to turn on all of the squares in and $n \times n$ grid?

Related.

1. What if touching the corner of a square also turns it on?
2. What if two triangles with the same coloring but different rotations are counted as different?
3. What if no two lines can be parallel? If no two line segments can be congruent?
4. What if no two lines can intersect?
5. How many fully "on" grids exist? How many such minimal grids? (A grid is minimal if removing any line results in a square turning off.)
6. Suppose a grid is on if an even number of lines pass through it and off if an odd number of lines pass through it. How many such grids?
7. How about on an $n \times m$ grid?
8. What if this is done on a triangular grid?
9. What if this is done on a cuboid? On a cuboid with planes passing through the cubes?

Note. In the case where "grid is on if an even number of lines pass through it and off if an odd number of lines pass through it", there exist 2^k configurations. If we further restrict to dihedral-symmetric grids, there are 2^j configurations.
It appears that $k = 5n^2 - 14n + 9$, the 12-gonal numbers. Is there a bijection between the basis elements and the 12-gonal numbers?