Problem 17.

Peter Kagey

Start with n piles with a single stone in each pile. If two piles have the same number of stones, then any number of stones can be moved between them.

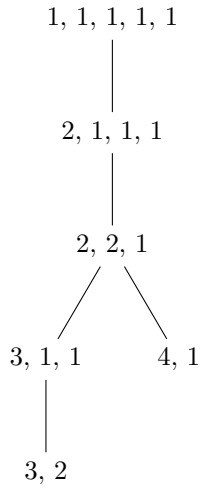

Figure 1: An illustration of all possible moves for $n = 5$.

Question. What is the greatest number of steps that can occur? Alternatively how many "levels" are in the tree of possible moves?

Related.

1. Let s be the total number of distinct states. (The example shows that $s(5) = 6$.)
2. Let c be the total number of states that *cannot* be acheived. (In the example, $c(5) = 1$ via the state (5).)
3. Is $c(p) = 1$ for all primes p?
4. Is $c(n) = 0$ if and only if n is a power of 2?
5. Let ℓ be the least number of steps to a terminal state. (In the example, $\ell(5) = 3$ ending in the state $(4, 1)$.)
6. Let g be the greatest number of steps to a terminal state. (In the example, $g(5) = 4$ ending in the state $(3, 2)$.)
7. Let p be the total number of paths. (In the example, $p(5) = 2$.)
8. Let t be the number of distinct *terminal* states. (In the example, $t(5) = 2$ with states $(4, 1)$ and $(3, 2)$.)