

Consider a 2-coloring of a triangular grid of length ℓ . Then label each cell with its greatest number of neighbors of one color.

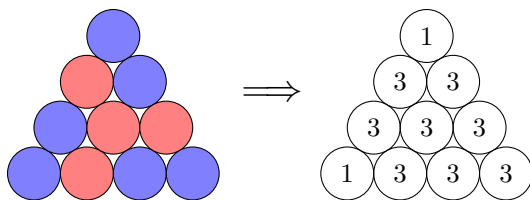


Figure 1: The second cell (reading top to bottom and left to right) is labeled with a $\max(3, 1) = 3$ because it has 3 blue neighbors and 1 red neighbor.

Question. How many colorings exist of a length ℓ triangle such that the maximum label is 3?

Related.

1. If the “number triangle” is summed for each coloring, which coloring has the smallest sum?
2. How does this generalize for a k -coloring?
3. How does this generalize to a $n \times m$ square grid where horizontal-vertical connections are counted? Diagonal connections? Both?
4. How does this generalize to a tetrahedron, torus, Möbius strip, cylinder, or cube?
5. How many colorings exist of a length ℓ triangle such that the maximum label is 4 or 5?