This one is based on correspondence from Alec Jones: Consider all of the ways of partitioning the complete graph on n vertices into smaller complete graphs.

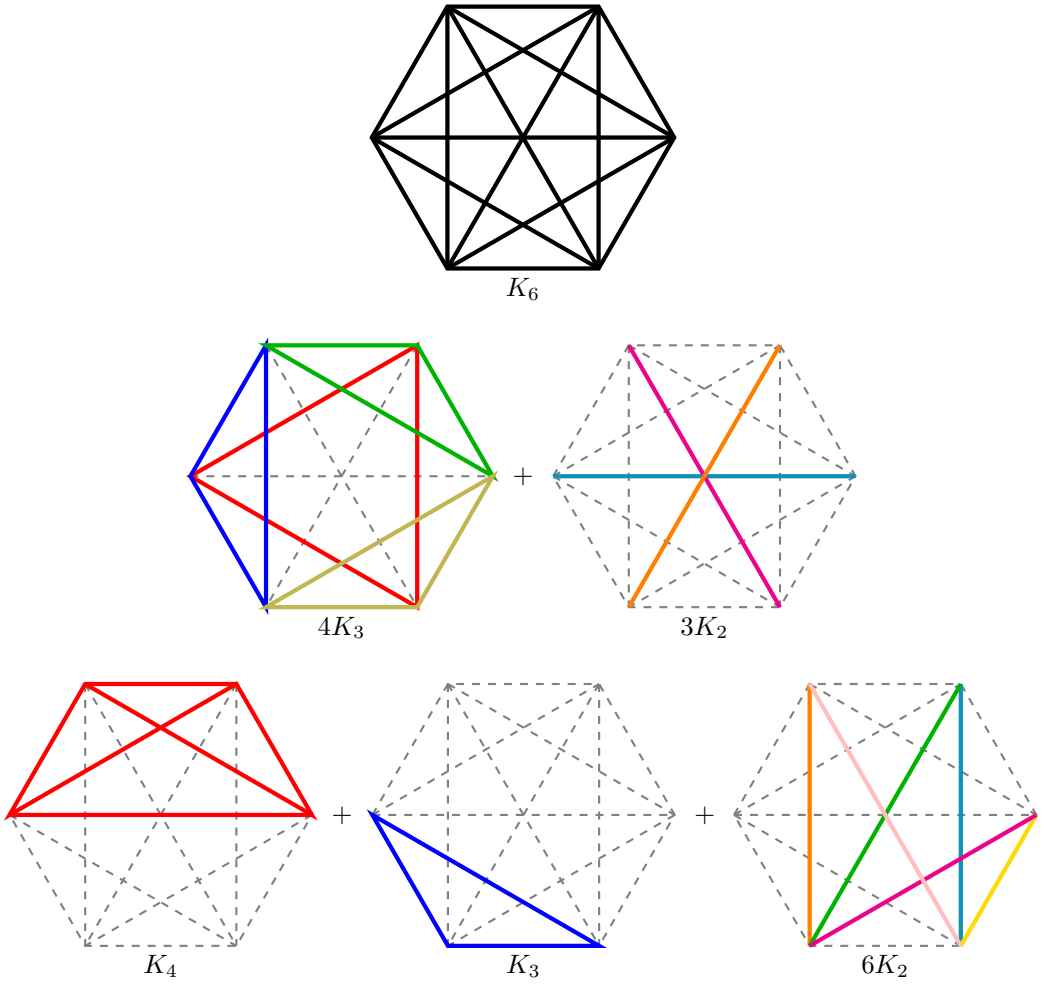

Figure 1: An example three ways to partition K_6 into complete graphs: the trivial partition, a partition into 4 copies of K_3 and 3 copies of K_2, and a partition into 1 copy of K_4, 1 copy of K_3, and 6 copies of K_2.

Question. How many such partitions exist, up to graph isomorphism?

Related.

1. What if the union of K_j graphs cannot contain a K_{j-1} subgraph?
2. What if the partition can only consist of two "sizes" of complete graphs, as in the second example?
3. How many such partitions exist up to dihedral action?