Consider square, triangular, and hexagonal grids that are filled in with with tiles of different patterns.

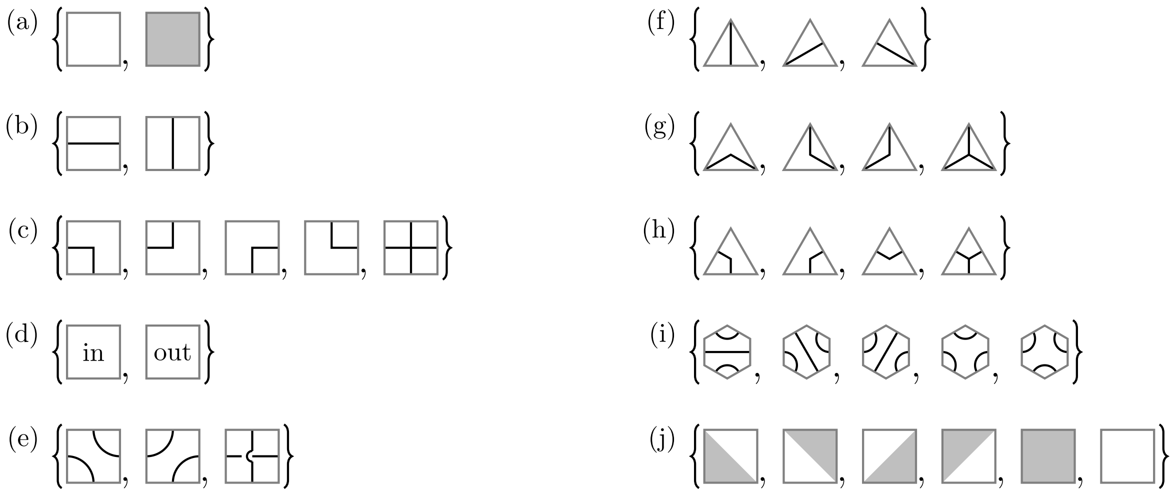

Figure 1: Ten examples of different tiles.

Question. How many essentially different grids of size n exist with these tiles? (Up to dihedral action? Up to cyclic action?)

Related.

1. The square grid can be $n \times n$ or $n \times m$.
2. The hexagonal grid can have triangles with side length n or hexagons with side length n.
3. The triangular grid can have triangles with side length n or hexagons with side length n.
4. The square grid can be quotiented to be a cylinder, torus, or Möbius strip.
5. What if shapes have to "match-up" (e.g. the lines in the third example or colors in the last example have to be "smooth".)
6. Furthermore, what if
7. How many distinct regions, as in Problem 2?

References.

Problem 2.

Problem 28.

`https://en.wikipedia.org/wiki/Burnside%27s_lemma`