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There are two popular, essentially identical, iPhone games in the app store: *AMAZE!!!* and *Roller Splat!*. The goal of the puzzle is to reach every (white) square in the board—the catch is that you can only move in as-long-as-possible rook moves.
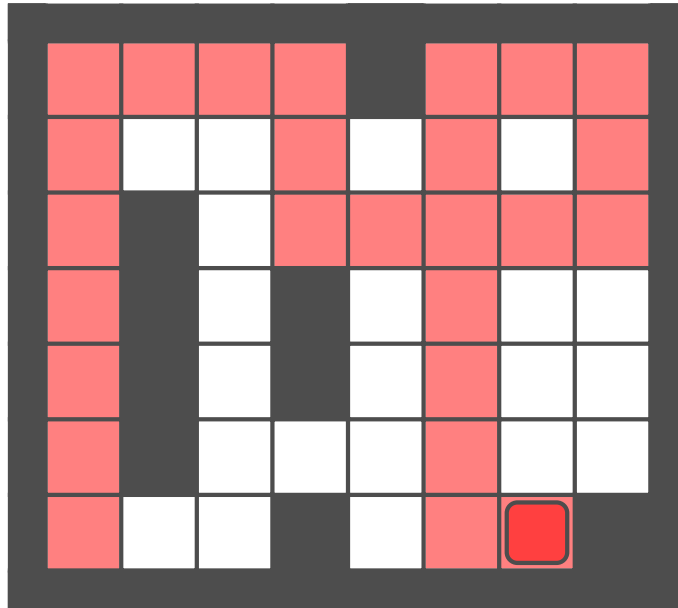

Figure 1: Starting from the lower left corner, the board can be filled using the following 25 moves: $\underbrace{\uparrow\rightarrow\downarrow\rightarrow\uparrow\leftarrow\downarrow\rightarrow}_{\text{illustrated above}}\uparrow\downarrow\leftarrow\uparrow\leftarrow\rightarrow\downarrow\leftarrow\uparrow\downarrow\leftarrow$

Question. How many solvable puzzles exist on an $n \times m$ board?

Related.

1. What if we only want to count "primitive" puzzles—those that cannot exist on a smaller board?

2. What if we count up to symmetries of the rectangle?

3. Which puzzle requires the greatest number of moves?

4. What if we do this on a torus? Möbius strip? More dimensions?

5. Given some configuration, what is an algorithm to figure out how to solve it?