

Difficulty: 3/4 Interest: 3/4

Say that a *minimally interpolable permutation* f is a permutation of $\{1, 2, \dots, n\}$ such that no $k + 2$ of the points $\{(1, f(1)), \dots, (n, f(n))\}$ fall on a degree k polynomial.

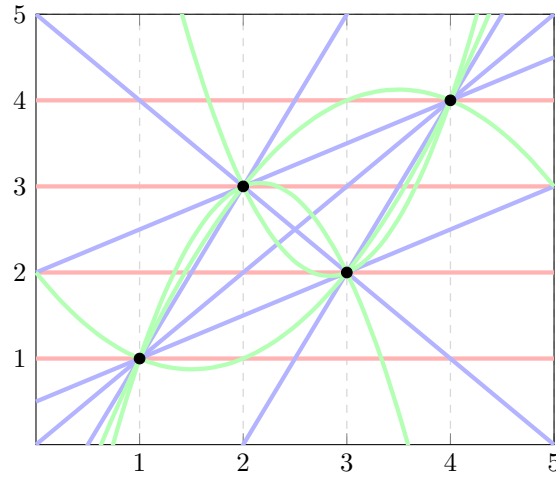


Figure 1: A minimally interpolable permutation of $\{1, 2, 3, 4\}$. (Degree 0 polynomials are plotted in red, degree 1 in blue, and degree 2 in green.)

Question. Do such permutations always exist? If not, what is the least N such that there is a minimally interpolable function from $[n]$ into $[N]$?

Related.

1. How many minimally interpolable permutations exist?
2. Does the number of minimally interpolable permutations increase as a function of n ?
3. Is there a method to explicitly construct a minimally interpolable permutation?
4. If such permutations do not always exist, what is the least M such that there exists a subset $S \subset [M]$ and a surjection $g: S \rightarrow [N]$ with the aforementioned property?

References.

Problem 23.

<https://oeis.org/A301802>

<https://codegolf.stackexchange.com/q/160382/53884>