Problem 4.

Peter Winkler's Coins-in-a-Row game works as following:

> On a table is a row of fifty coins, of various denominations. Alice picks a coin from one of the ends and puts it in her pocket; then Bob chooses a coin from one of the (remaining) ends, and the alternation continues until Bob pockets the last coin.

Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed according to some probability distribution.

Question. For some fixed ω, what is the expected first player's score of Peter Winkler's Coins-in-a-Row game when played with $X_1(\omega), X_2(\omega), \ldots, X_3(\omega)$ where both players are using a min-max strategy?

Note. Let

$$e = E[X_2 + X_4 + \ldots + X_{2n}] \text{ and } o = E[X_1 + X_2 + \ldots + X_{2n-1}]$$

When played with $2n$ coins, the first player's score is bounded below by $\max(e, o) - \min(e, o)$ by the strategy outlined by Peter Winkler.

Trivially the first player's score is bounded above by the expected value of the n largest coins minus the expected value of the n smallest coins.

Related.

1. If all possible n-coin games are played with coins marked 0 and 1, how many games exist where both players have a strategy to tie.
2. How does this change when played according to the (fair) Thue-Morse sequence?
3. What if the players are cooperating to help the first player make as much as possible (with perfect logic)?
4. What is both players are using the greedy algorithm?
5. What if one player uses the greedy algorithm and the other uses min-max? (i.e. What is the expected value of the score improvement when using the min-max strategy?)
6. What if one player selects a coin uniformly at random, and the other player uses one of the above strategies?