John Conway described the "Subprime Fibonacci Sequence":

$$a(1) = \alpha, a(2) = \beta, a(n+1) = \text{gpd}(a(n) + a(n-1)),$$

where $\text{gpd}(k)$ is the greatest proper divisor of k.
Conway then conjectured that regardless of the starting terms, the sequence ends in a handful of cycles.

Question. What are all of the different possible end behaviors of Conway's Subprime Fibonacci Sequence?

Note. Richard Guy beat me to this problem by a few years.
Richard Guy found that there are more cycles than those that Conway conjectured.

References.

https://arxiv.org/abs/1207.5099