It turns out that the rational numbers $\mathbb{Q}$ can be generated starting from 0 by iterating the two maps $f(x) = x+1$ and $g(x) = -1/x$. This is because f and g generate the modular group Γ, and $g \circ f \circ g \circ f \circ g = f^{-1}$ and $g = g^{-1}$.

We can use this fact to create the tree in OEIS sequence A226247, which is shown below.

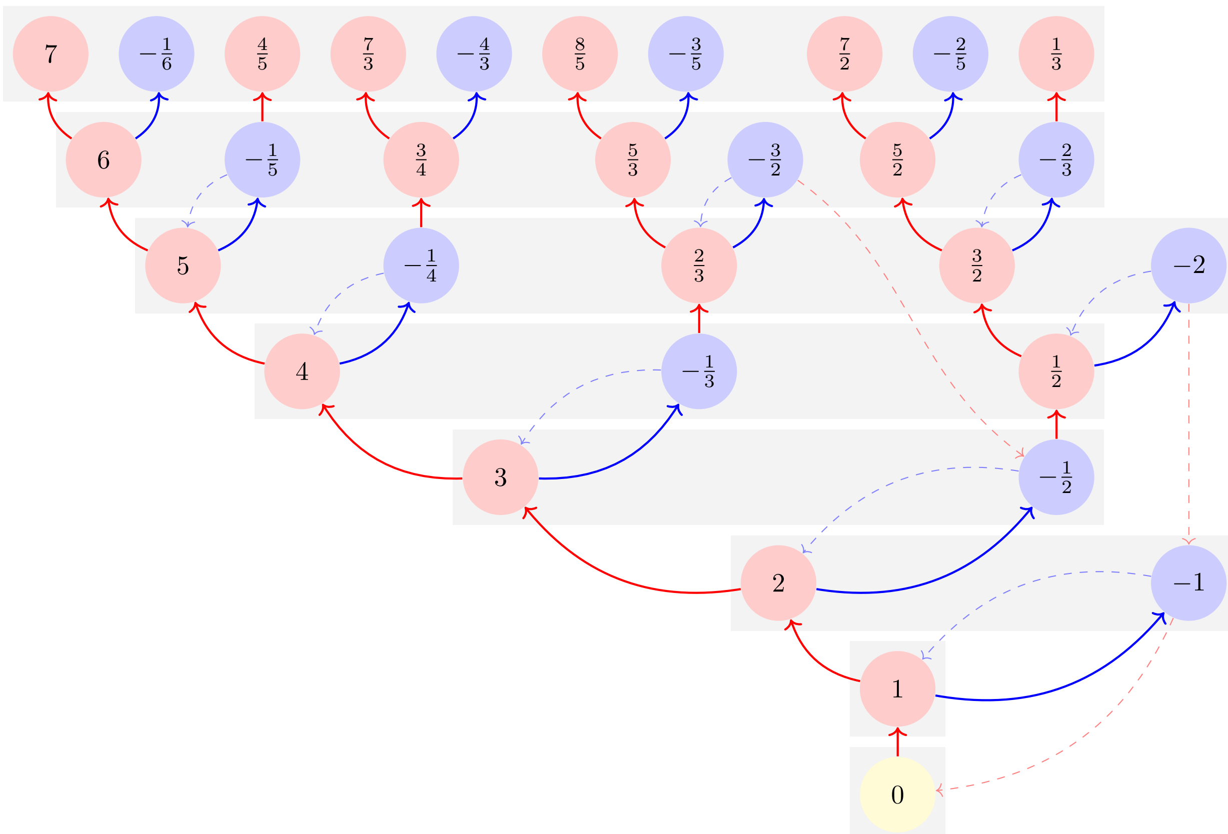

Figure 1: Caption.

Question. Let $a(n)$ be the number of elements in the n-th rank. Does $a(n) = a(n-1) + a(n-3)$ for all $n \geq 4$?

Related.

1. In the figure above, the vertices numbers whose last application is $g(x) = -1/x$ are colored blue. Is a vertex blue if and only if its value is negative?

2. Is there a way to characterize all rank n rational numbers?

Note. Math Stack Exchange, "Enumerating all fractions by $x \mapsto x + 1$ and $x \mapsto -1/x$."