The chromatic polynomial of a graph G, $\chi_G(n)$ gives the number of ways to color the vertices of the graph such that no two vertices of the same color are connected by an edge.

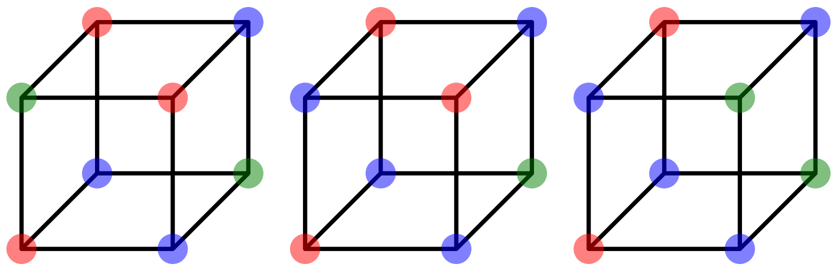

Figure 1: Three examples of 3-colorings of the cube. The chromatic polynomial of the cubic graph is $\chi_{Q_3}(n) = a(n) = n^8 - 12n^7 + 66n^6 - 214n^5 + 441n^4 - 572n^3 + 423n^2 - 133n$.

Question. Is there a way to generate the chromatic polynomial of an n-cube in polynomial time with respect to n?

Related.

1. What about up to permutations of the colors and/or isometries of the cube?
2. What about simplices, cross-polytopes, and demicubes?
3. What about other polytopes such as associahedra, permutahedra, and the 24-, 120-, and 600-cells?

References.

Problem 61.

`https://math.stackexchange.com/q/3632156/121988`

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