

A country has a strange legislative procedure. For each bill, the body is split up into k_1 committees of $\lfloor n/k_1 \rfloor$ or $\lceil n/k_1 \rceil$ legislators each, each of which picks a representative. These k_1 representatives are split up into k_2 sub-committees with $\lfloor k_1/k_2 \rfloor$ or $\lceil k_1/k_2 \rceil$ legislators each, which each elect a representative, and so on until $k_T = 1$ and the final committee votes on the bill.

There are a few rules:

1. Each committee (and subcommittee and so on) must have at least ℓ members.
2. Ties are settled by a coin toss.
3. The president does not get to vote, but she does get to choose the number of committees and who goes in each one.
4. There are α supporters who will always vote in the president's interests and $n - \alpha$ who will always vote against.

Let $a_\ell(n)$ be the minimum number of supporters (α) required for the president to be able to pass every bill.

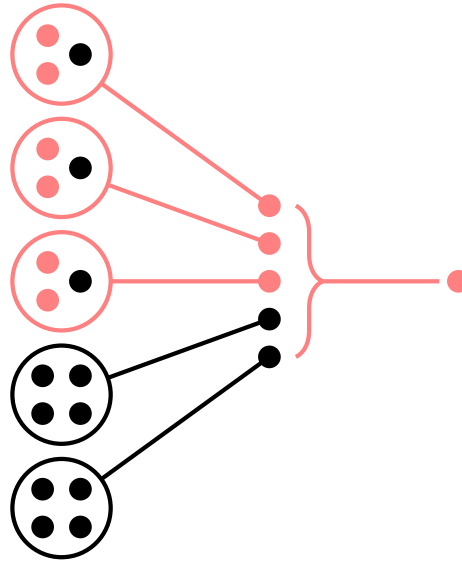


Figure 1: An example of $n = 17$ legislators with a minimum committee size of $\ell = 3$, which demonstrates that $a_3(17) \leq 6$.

Question. What is an efficient way to compute $a_\ell(n)$ for general ℓ and n ?

Related.

1. What if the president gets to choose who is on each committee but the opposition party gets to choose the committee size? Vice versa?
2. What if $k_1 \leq k_2 \leq \dots \leq k_T$? Or $k_1 \geq k_2 \geq \dots \geq k_T$?
3. What if ties go to the president? To the opposition?

References.

<https://oeis.org/A290323>

<https://math.stackexchange.com/q/2395044/121988>