



Given a regular polygon with vertices v_1, v_2, \dots, v_n , consider the following process. Begin by plotting a random point p_1 inside of the triangle. Then given p_i , let p_{i+1} be uniformly chosen from the centers of $\triangle v_1 v_2 p_i, \triangle v_2 v_3 p_i, \dots, \triangle v_n v_1 p_i$.

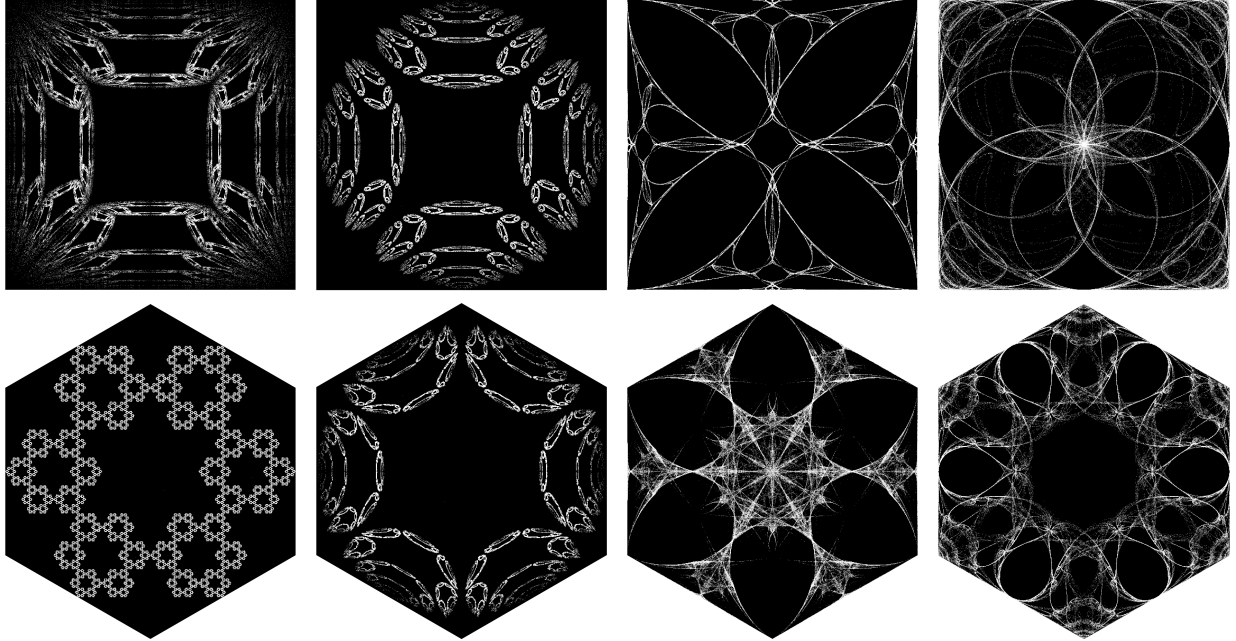


Figure 1: The four squares have triangle centers given by $X(1)$, $X(202)$, $X(220)$, and $X(244)$. The four hexagons have triangle centers given by $X(2)$, $X(202)$, $X(220)$, and $X(244)$.

Question. What are the fractals and their dimensions that occur when iterating this process for various triangle centers?

Related.

1. What if this is done where the triangles are given by the current point p and not necessarily adjacent vertices on the n -gon?
2. What is the probability density of where a point will be after an “infinite” number of moves?
3. What if this is done in higher dimensions?

References.

<https://twitter.com/matthen2/status/1268808515574886407>

Clark Kimberling’s Encyclopedia of Triangle Centers

Chaos Game, YouTube video

Wikipedia: Iterated function system