Consider ways to lay matchsticks (of unit length) on the $n \times m$ grid in such a way as to form a maze.

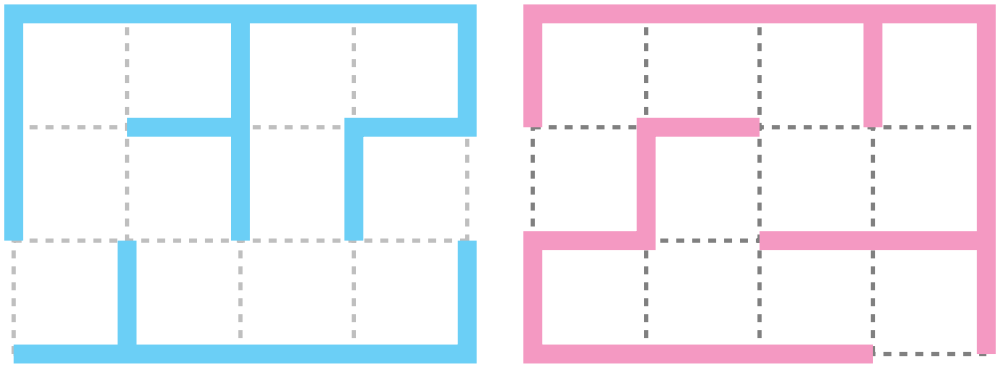

Figure 1: Two mazes on a (5×4)-cell grid.

Question. How many distinct mazes can be drawn on the grid?

Related.

1. What if every 1×1 cell must be reachable?
2. What if there are no dead ends?
3. What if there are to be identically k dead ends?
4. What if paths that loop are not allowed?
5. What if the entrance and exit have prescribed positions?
6. What if this is done on a hexagonal or triangular grid? On a torus?
7. Is there a meaningful way to assign "difficulty" to a maze?

Note. This appears to be the number of spanning trees on the $n \times m$ grid graph such that the start and end are leaves.

References.

Problem 56.

`https://oeis.org/A116469`