Consider the graded poset on $\mathbb{N}_{>0}$ given by covering relations $n - \dfrac{n}{p} \lessdot n$ for all primes $p \mid n$.

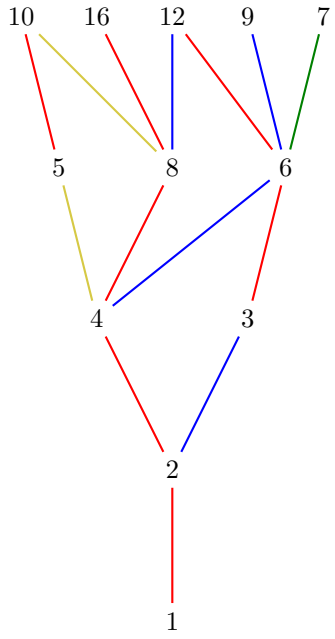

Figure 1: Ranks 0 through 4 of the poset. Prime divisors $2, 3, 5$, and 7 are displayed as red, blue, gold, and green respectively. For example, $12 \gtrdot 12 - \dfrac{12}{3} = 8$.

Question. Is this poset a lattice?

Related.

1. If not, is this poset a join- or meet-semilattice?
2. If so, is this poset a distributive lattice? A modular lattice?
3. It appears that (at least for small values of n and k) $(n \vee k) \mid \operatorname{lcm}(n, k)$. What is $\dfrac{\operatorname{lcm}(n,k)}{n \vee k}$?
4. Is there a good way to construct the least integer (in the usual sense) for a given rank number?

References.

`https://math.stackexchange.com/q/3632156/121988`

`https://oeis.org/A334230`