Consider all rectangles composed of n squares such that the greatest common divisor of all the sidelengths is 1.

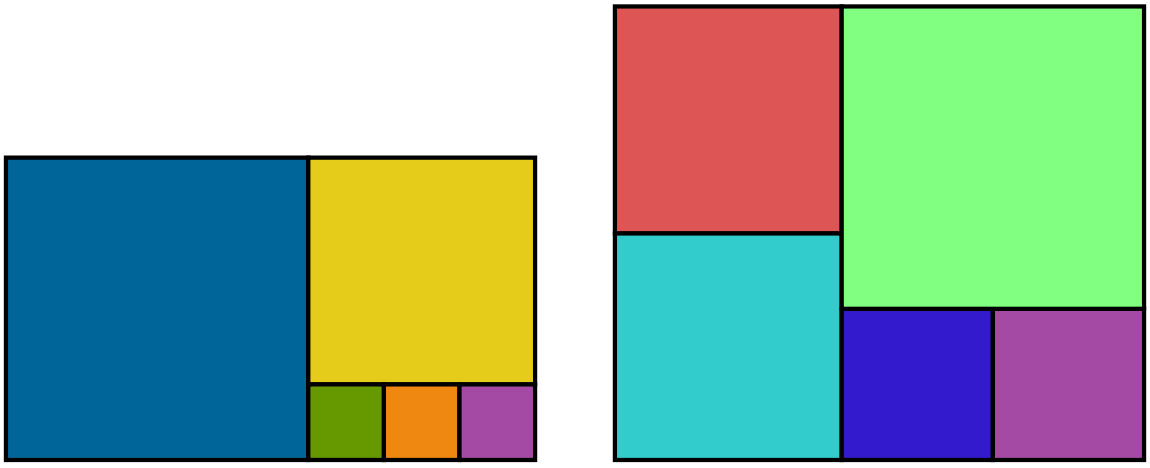

Figure 1: Two examples of rectangles made from $n = 5$ squares. In the first $\gcd(1, 1, 1, 3, 4) = 1$ and in the second $\gcd(2, 2, 3, 3, 4) = 1$.

Question. Given n squares, how many such rectangles exist?

Related.

1. How many ways are there to make convex polygons out of n equilateral triangles?
2. How many ways are there to make cuboids out of n cubes?