



Suppose you choose two points on a line segment uniformly at random, defining three smaller line segments. The probability that these three line segments satisfy the triangle inequality is  $\frac{1}{4}$ . Similarly, if you choose five points on a line segment, it is conjectured that they can form a tetrahedron with probability  $\frac{1}{79}$ .

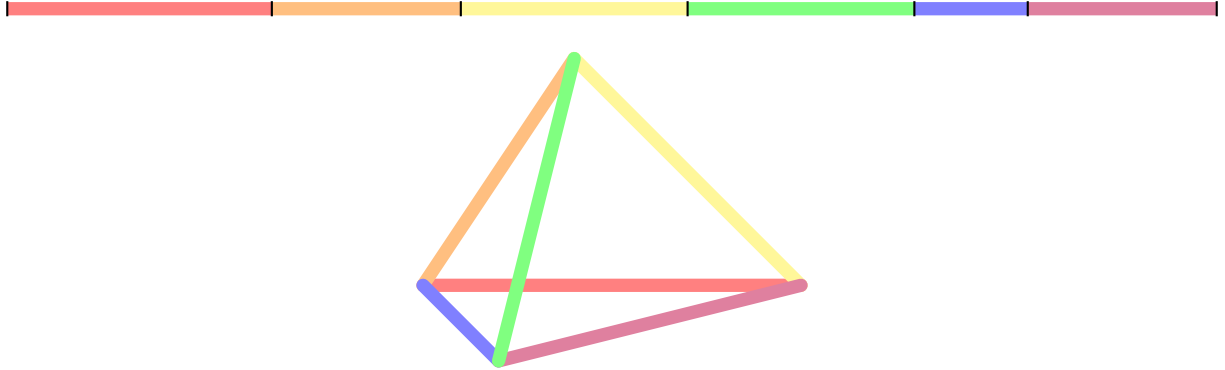


Figure 1: An example of a partition of a line segment and the resulting tetrahedron.

**Question.** If the line segment is split into  $\frac{k(k+1)}{2}$  pieces using this prescription, do the resulting segments form a  $k$ -simplex with rational probability?

**Related.**

1. What if the segments form prescribed sides of the  $k$ -simplex? What if any configuration is valid?
2. What if there are more than  $\frac{k(k+1)}{2}$  pieces, what is the probability that there exists some subset of  $\frac{k(k+1)}{2}$  of them that forms a  $k$ -simplex?
3. What is the expected hyper-volume of such a tetrahedron?

**References.**

Problem 73.

Problem 78 also deals with a generalization of a Stack Exchange question.

<https://mathoverflow.net/q/142983/104733>