



Consider attempting to fill in a  $n \times n$  Latin square with the numbers 1 through  $n$ , one number at a time. A clumsy filling is an incomplete filling in which no more valid moves are possible.

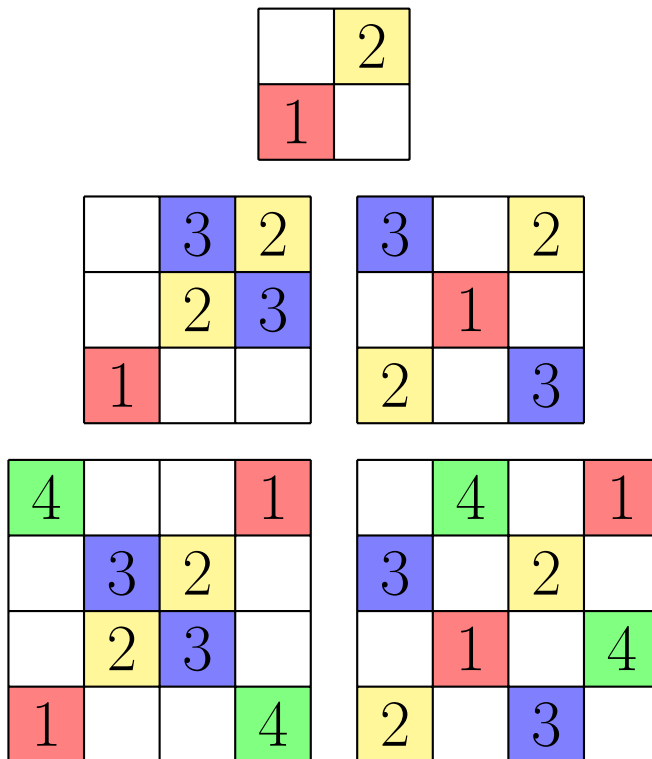


Figure 1: Clumsy filling for  $n = 2, 3, 4$  can be achieved with 2, 5, and 8 or fewer entries respectively; thus  $a(2) = 2, a(3) \leq 5$ , and  $a(4) \leq 8$ .

**Question.** Let  $a(n)$  be the fewest number of entries required for a clumsy filling. What is  $a(n)$ ?

**Related.**

1. How many “essentially different” fillings are there?  
(Two fillings are the same if related by permuting the symbols or dihedral action of the board.)
2. Can minimal clumsy fillings be built iteratively, as suggested by the leftmost diagrams in the example?
3. What if this is done on a group table instead of a Latin square (quasigroup table)?

**References.**

<https://www.youtube.com/watch?v=U5NLgivoKDQ>