Euler's well is a labeling of the $n \times k$ grid with a permutation in $S_{n\times k}$ such that the upper left corner is labeled with 1.

> Water is poured into the well from a point above the section marked 1, at the rate of 1 cubic foot per minute. Assume that water entering a region of constant depth immediately disperses to all orthogonally adjacent lower-depth regions evenly along that regions exposed perimeter (an assumption that Euler insisted on).
>
> After how many minutes will the water begin to accumulate in [the lower right corner]?

1	14	9	20	3
5	13	24	17	18
25	16	4	21	6
10	2	15	19	23
7	22	8	12	11

Figure 1: A labeling of the 5×5 grid where the labels are a permutation of the integers from 1 to 25.jj

Question. For a random permtation in $S_{n\times k}$, what is the expected amount of time it takes for water to reach the lower right hand corner of the grid?

Related.

1. What if water can flow diagonally?
2. What if the source or sink are in different places? What if there are multiple sources/sinks?
3. What if this is done on a torus? Triangular/hexagonal grid? Three dimensions?
4. What if the numbers are not neccesarily a permutation?
5. What if the well is a Latin square?
6. What is an efficient algorithm for computing this for an arbitrary permutation?
7. What is the expected value of number of "wet" squares at the end?
8. How many wells have minimal filling times?
9. How many wells up to "fill level"? (e.g. two wells are equivalent if each square has the same height after the water flows all the way)

References.

`http://chalkdustmagazine.com/blog/well-well-well/`

`https://oeis.org/A321853`