We want to understand k-dimensional degree-d Bézier curves with $d+1$ control points $p_0, p_1, \ldots, p_d$ in the set $\{0, 1, 2, \ldots, n\}^k$:

$$\vec{c}(t) = \sum_{i=0}^{d} \binom{d}{i} t^i (1-t)^{d-i} p_i$$

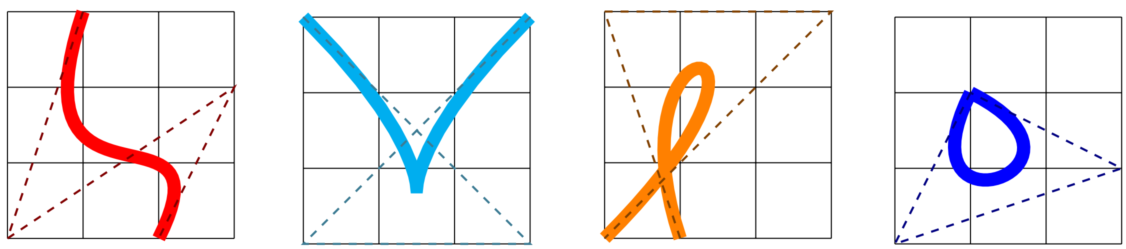

Figure 1: Four examples of cubic Bézier curves with all four control points in $\{0,1,2,3\}^2$. The first has control points $(1,3)$, $(0,0)$, $(3,2)$, $(2,0)$. The second: $(0,3),(3,0),(0,0),(3,3)$. The third: $(1,0),(0,3),(3,3),(0,0)$. The fourth: $(1,2),(0,0),(3,1),(1,2)$.

Question. For fixed d the limit as $n \to \infty$, what is the probability of self-intersection?

Related.

1. How many distinct curves are there up to symmetry of the square.
2. How many curves have a cusp?
3. What can we say about the space of these curves when control points are instead in $[0,1]^k$? Is the set of non-self-intersecting curves connected in this setting?
4. What are the extremal curves with respect to length, number of intersections, enclosed area, etc?

References.

Wikipedia, "Bézier curve."