Consider an n-coloring of a triangular grid such that no sub-triangle has corners all with the same color.

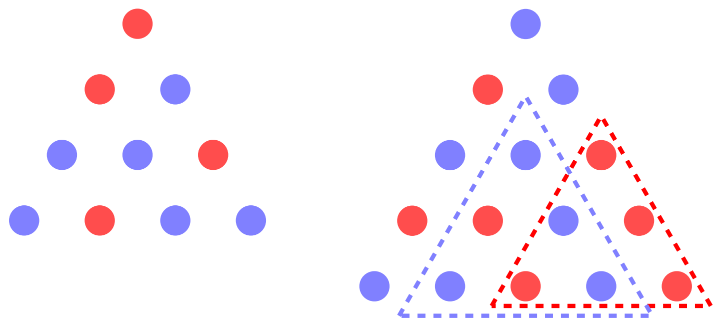

Figure 1: On the left is an example of a triangle on two labels that has no sub-triangles with equal corners. On the right is a non-example of such a triangle on two labels—it has two sub-triangles with equal corners.

Question. Given n labels, what is the biggest triangle that can be constructed? Call the side length of such a triangle $a(n)$.

Related.

1. Given an n-coloring of a triangle of side length k, what number of sub-triangles with equal corners must exist?
2. How many such triangles exist?
3. What if diagonal equilateral triangles also are not allowed to have equal corners?
4. What if this is done with hexagons instead of triangles?
5. What if this is done on a square grid?
6. What if for $n \geq 3$ no *two* corners are allowed to be equal? (This is a bit like a peaceable queens problem on a hexagonal chessboard.)

References.

`https://math.stackexchange.com/a/2416790/121988`

`https://math.stackexchange.com/a/2636168/121988`