Consider a $n \times m$ grid of ones and zeroes, which represent the heights of the cells. It rains, and the grid fills up with the rain moving horizontally and vertically.

1	0	0	1	1	1
0	1	1	0	0	1
1	0	0	1	1	0
1	0	0	0	1	1
0	1	1	1	1	1

Figure 1: An example of five non-parallel centered squares in the size 6 square, and an example of three non-parallel non-centered squares that do not share any lattice points.

Question. What is the expected area of a lake? Of the sum of all lakes?

Related.

1. What is the expected number of lakes? Of islands? Of lakes on islands?
2. What if water can flow diagonally too?
3. What if the heights can take on arbitrary values?
4. What if there is a border around the grid of height k?
5. What if the cell is height 0 with probability p and height 1 with probability $(1 - p)$?
6. How does this generalize to triangular/hexagonal grids? More dimensions?
7. How does this generalize to a cylinder?

References.

Problem 86.

`https://codegolf.stackexchange.com/q/2638/53884`