Consider a puzzle that consists of an $n \times n$ grid with n marked cells. The goal of the puzzle is to partition the grid into n-cell regions of size n, each containing exactly one marked cell.

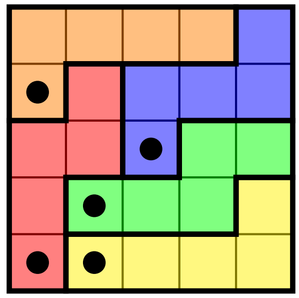

Figure 1: An example of a 5×5 grid with a unique solution.

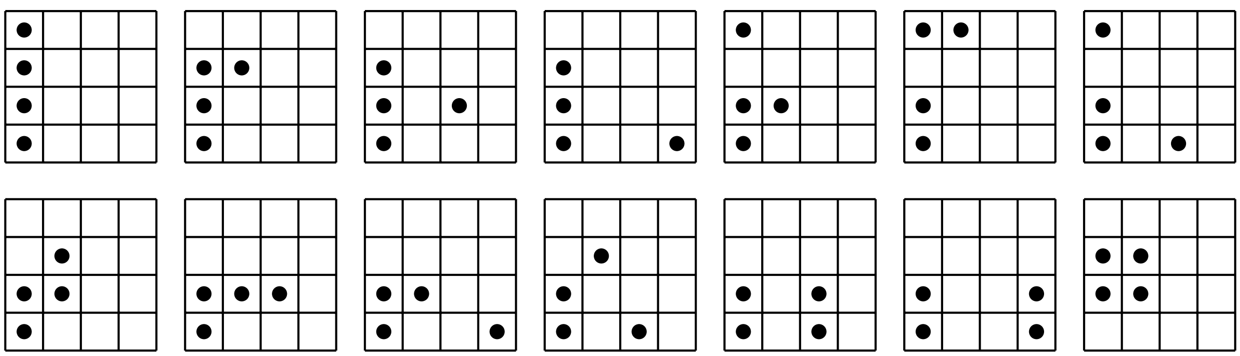

Figure 2: Fourteen (all, up to dihedral action?) markings with exactly one solution.

Question. How many $n \times n$ boards exist with a unique solution?

Related.

1. How many $n \times n$ boards exist with no solution? Multiple solutions?
2. What board has the most solutions?
3. What if this is counted up to dihedral action?
4. What if this is done on an $n \times m$ board with k marked cells where $k|nm$ and each region has nm/k cells?
5. What if the board is a torus? Triangular/hexagonal grid? Multiple dimensions?
6. What if instead of marked cells there are marked regions?
7. What if cells must must be rectangular? Symmetric?
8. What if every region must be a walk starting at a marked cell? (As in the example.)

References.

Problem 24

`https://math.stackexchange.com/q/3072735/121988`

`https://en.wikipedia.org/wiki/Flow_Free`