Suppose that you're on an $n \times m$ grid, and you'd like to place rectangles to fill up as many gridlines as possible—the catch is that if there are an even number of boxes on a gridline, then they cancel out.

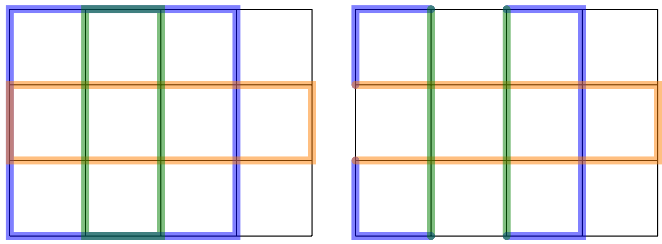

Figure 1: This arrangment with three boxes on the 4×3 grid misses only seven edges.

Question. What is the greatest number of edges that can be covered?

Related.

1. How many minimal arrangments of rectangles are there? (Right example)
2. How many maximal edge covers are there? (Left example)
3. What if the rectangles must be square?
4. What if a maximum of k rectangles is allowed?
5. What if it takes 3 (or c) rectangles to cancel out?
6. Is this equivalent to finding disjoint collections of "almost" Eulerian cycles on a grid graph?
7. How does this generalize to higher dimensions, the triangular grid, etc?
8. What if we want to cover vertices instead of edges? Facets?
9. How many edges are covered if we use every possible rectangle?
10. Placement of rectangles generates an abelian group. What is the group's structure? Is the size of the group the number of possible edge covers?

References.

`https://math.stackexchange.com/q/1579862/121988`

Problems 37 and 74.