Difficulty: 1/4 Interest: 3/4
Start with n piles with a single stone in each pile. If two piles have the same number of stones, then any number of stones can be moved between them.

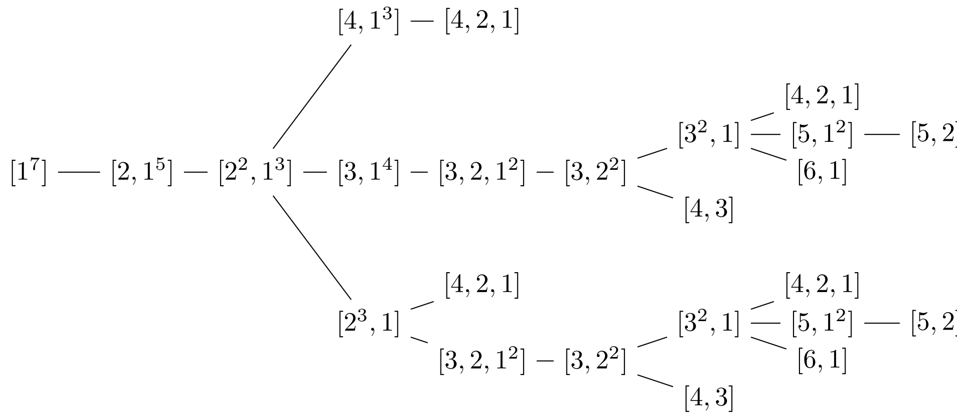

Figure 1: An illustration of all possible moves for $n = 7$.

Question. What is the greatest number of steps that can occur? Alternatively how many “levels” are in the tree of possible moves?

Related.

1. Let $A292726$ be the total number of distinct states. What is $A292726$? (e.g. $A292726(7) = 14$.)
2. Let $c = A000041 - A292726$ be the total number of states that *cannot* be acheived. (e.g. $c(5) = 1$ via the state $[5]$.)
3. Is $c(p) = 1$ for all primes p? Is $c(n) = 0$ if and only if n is a power of 2?
4. Let $\ell = A292836$ be the least number of steps to a terminal state. (e.g. $\ell(7) = 4$ ending in $[4, 2, 1]$.)
5. Let $g = A292729$ be the greatest number of steps to a terminal state. (e.g. $g(7) = 8$ ending in $[5, 2]$.)
6. Let p be the total number of paths, i.e. the number of leaves in the tree. (e.g. $p(7) = 10$.)
7. Let t be the number of distinct *terminal* states. (e.g, $t(7) = 4$ via $[4, 2, 1]$, $[4, 2]$, $[6, 1]$, and $[4, 3]$.)
8. What if you can move stones between any sets of piles that share the same number of stones? (e.g. $[2^3] \to [6]$ or $[2^3] \to [4, 1, 1]$)

References.

`https://oeis.org/A292836`