

Difficulty: 2/4 Interest: 4/4

There is a well known magic trick called “Communicating the Card” in which a spectator draws k cards from an n -card deck and shows them to the magician’s assistant. He then shows $k - 1$ of them to the magician in a particular order, after which she (the magician) can deduce the remaining card. In this variation, the largest possible deck is $k! + k - 1$ cards.

$f(1, 2) = \{1, 2, 3\}$	$f(4, 8) = \{1, 4, 8\}$	$f(7, 2) = \{2, 4, 7\}$	$f(8, 3) = \{3, 5, 8\}$
$f(2, 1) = \{1, 2, 4\}$	$f(5, 1) = \{1, 5, 6\}$	$f(8, 2) = \{2, 4, 8\}$	$f(3, 6) = \{3, 6, 7\}$
$f(1, 5) = \{1, 2, 5\}$	$f(5, 7) = \{1, 5, 7\}$	$f(2, 5) = \{2, 5, 6\}$	$f(6, 3) = \{3, 6, 8\}$
$f(1, 6) = \{1, 2, 6\}$	$f(5, 8) = \{1, 5, 8\}$	$f(5, 2) = \{2, 5, 7\}$	$f(7, 3) = \{3, 7, 8\}$
$f(1, 7) = \{1, 2, 7\}$	$f(6, 7) = \{1, 6, 7\}$	$f(8, 5) = \{2, 5, 8\}$	$f(4, 5) = \{4, 5, 6\}$
$f(1, 8) = \{1, 2, 8\}$	$f(6, 8) = \{1, 6, 8\}$	$f(6, 2) = \{2, 6, 7\}$	$f(5, 4) = \{4, 5, 7\}$
$f(1, 3) = \{1, 3, 4\}$	$f(7, 8) = \{1, 7, 8\}$	$f(8, 6) = \{2, 6, 8\}$	$f(8, 4) = \{4, 5, 8\}$
$f(3, 1) = \{1, 3, 5\}$	$f(2, 3) = \{2, 3, 4\}$	$f(8, 7) = \{2, 7, 8\}$	$f(4, 6) = \{4, 6, 7\}$
$f(6, 1) = \{1, 3, 6\}$	$f(3, 2) = \{2, 3, 5\}$	$f(3, 4) = \{3, 4, 5\}$	$f(6, 4) = \{4, 6, 8\}$
$f(7, 1) = \{1, 3, 7\}$	$f(2, 6) = \{2, 3, 6\}$	$f(4, 3) = \{3, 4, 6\}$	$f(7, 4) = \{4, 7, 8\}$
$f(8, 1) = \{1, 3, 8\}$	$f(2, 7) = \{2, 3, 7\}$	$f(3, 7) = \{3, 4, 7\}$	$f(5, 6) = \{5, 6, 7\}$
$f(1, 4) = \{1, 4, 5\}$	$f(2, 8) = \{2, 3, 8\}$	$f(3, 8) = \{3, 4, 8\}$	$f(6, 5) = \{5, 6, 8\}$
$f(4, 1) = \{1, 4, 6\}$	$f(2, 4) = \{2, 4, 5\}$	$f(3, 5) = \{3, 5, 6\}$	$f(7, 5) = \{5, 7, 8\}$
$f(4, 7) = \{1, 4, 7\}$	$f(4, 2) = \{2, 4, 6\}$	$f(5, 3) = \{3, 5, 7\}$	$f(7, 6) = \{6, 7, 8\}$

Figure 1: An example of an encoding where $k = 3$ and $n = k! + k - 1 = 8$.

Question. What if the assistant can show any number of cards less than k , and the magician must guess all of the remaining cards?

Related.

1. How many different encodings exist (up to relabeling)?
2. What if the magician just needs to guess one of the remaining cards?
3. What if there are ℓ identical copies of a deck, how many cards can the original trick support?
4. If the assistant shows $k - 2$ cards to the magician, what is the biggest deck that this trick can be done with? $k - j$?

References.

<http://oeis.org/A030495>

https://www.reddit.com/r/math/comments/71lt84/a_combinatorists_card_trick/

<https://web.northeastern.edu/seigen/11Magic/Articles/Best%20Card%20Trick.pdf>