



Say that an n -robot takes steps that are $1/n$ of a circle ($2\pi/n$ radians). Call a (k, j) -step pattern a walk that starts with k right turns, followed by j left turns, followed by k right turns, and so on.

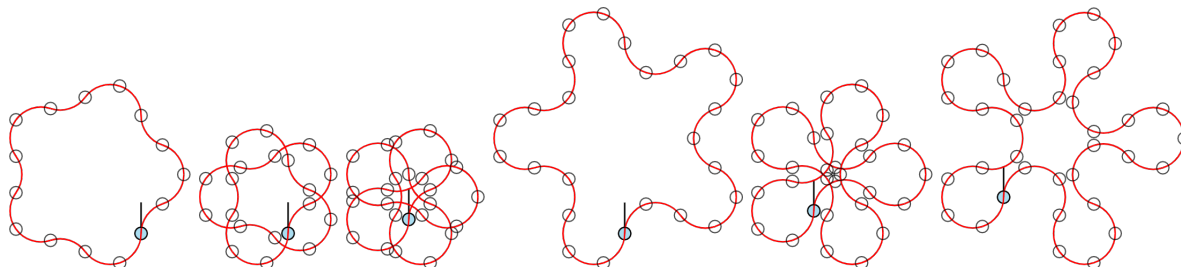


Figure 1: A 5-robot walks in $(1, 2)$, $(1, 3)$, $(1, 4)$, $(2, 3)$, $(2, 4)$, and $(3, 4)$ -step patterns, respectively.

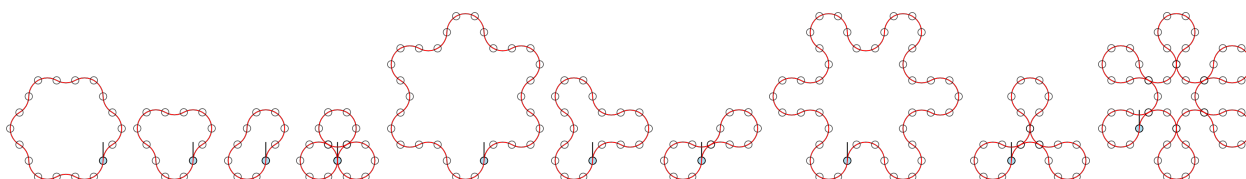


Figure 2: A 6-robot walks in $(1, 2)$, $(1, 3)$, $(1, 4)$, $(1, 5)$, $(2, 3)$, $(2, 4)$, $(2, 5)$, $(3, 4)$, $(3, 5)$, and $(4, 5)$ -step patterns, respectively.

Question. For an n -robot, which of these has the largest path by area?

Related.

1. Which of these figures has the smallest area?
2. Is there a way to tell at a glance whether or not these walks will self-intersect?
3. Is there a way to tell at a glance if a (k, j) -step pattern will “go off to infinity”?
4. Are the areas enclosed by these figures “nice” numbers?
5. Which of these figures has the largest convex hull?
6. How does this generalize to (a_1, a_2, \dots, a_k) -step patterns?
7. How many steps are taken before the figure “reconnects”?
8. For what step patterns are the “footprints” (the small grey circles in the figure) closest together? For the 5-robot, it appears to be the $(2, 4)$ -step pattern.
9. What if the robot turns $1/n$ of a circle when it turns right, but $1/m$ of a circle when it turns left?
10. What if the robot turns with some other rational number a/b of a circle?

References.

Problem 46.

<https://cemulate.github.io/project-euler-208/>