Difficulty: 1/4 Interest: 3/4

Consider Ron Graham's sequence for LCM, that is, look at sequences such that

$$n = b_1 < b_2 < \ldots < b_t = k \text{ and } \mathrm{LCM}(b_1, \ldots, b_t) \text{ is square.}$$

Question. Let $A300516(n)$ be the least k (as a function of n) such that such a sequence exists?

$$\begin{aligned}
a(1) &= 1 \quad \text{via } (1) \\
a(2) &= 4 \quad \text{via } (2,4) \\
a(3) &= 3 \quad \text{via } (3,9) \\
a(4) &= 4 \quad \text{via } (4) \\
a(5) &= 25 \text{ via } (5,25) \\
a(6) &= 12 \text{ via } (6,9,12) \\
a(7) &= 49 \text{ via } (7,49) \\
a(8) &= 16 \text{ via } (8,16)
\end{aligned}$$

Figure 1: Examples of $A300516(n)$ for $1 \leq n \leq 8$.

Related.

1. For what values n is $A300516(n)$ nonsquare?
2. For what values n does the corresponding sequence have three or more terms?
3. What is the analogous sequence for perfect cubes, etc?

References.

`https://oeis.org/A300516`