Consider walks on an $n \times m$ grid, where the walk can only self-intersect at a perpendicular step.

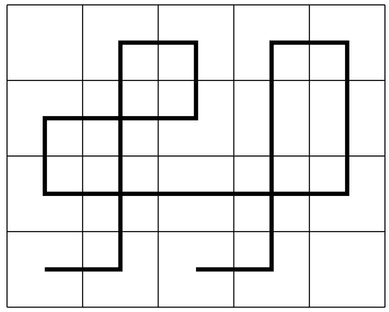

Figure 1: An example of a walk on a $5 \times 5$ grid.

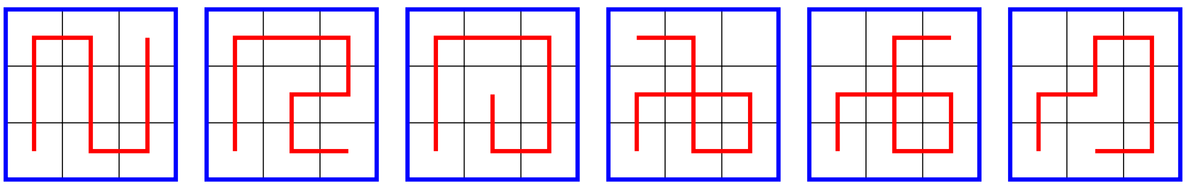

Figure 2: Six (all?) "as-long-as-possible" paths starting in the lower left corner, up to dihedral action on the $3 \times 3$ grid.

**Question.** How many $n \times n$ boards exist with a unique solution?

**Related.**

1. What if paths must be "as long as possible", in the sense that they can't be extended at either end?
2. What if this is done on a torus, triangular grid, cube, etc?
3. What if paths much touch every square at least once?
4. How many up to dihedral action? How many with dihedral symmetry?
5. What if paths must start at, say, the upper right corner?
6. What if the path must have at least one self-intersection?
7. What if paths are allowed to be loops (i.e., end on same square as they began on?) What if they must be loops?
8. What is the greatest number of king steps? Fewest on an "as-long-as-possible" path? Rook steps?
9. What if diagonal moves are allowed? Only diagonal moves?
10. What if multiple paths can be drawn on the same grid, only intersecting perpendicularly?

**References.**

Problem 31.

Problem 42.

Problem 56.