

Consider all r -colorings of the $n \times m$ grid where no two colors are adjacent (horizontally/vertically) more than once.

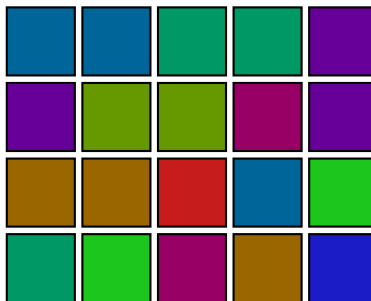


Figure 1: A 9-coloring of the 4×5 grid where no two colors are adjacent more than once. There is no 8-coloring.

Question. Let $a(n, m)$ be the minimal r such that there exists an r -coloring of the $n \times m$ grid. What is $a(n, m)$?

Related.

1. What if colors are not allowed to be self-adjacent?
2. How many $a(n, m)$ -colorings exist up to permutation of the colors?
3. What if this is done on a triangular or hexagonal grid?
4. What if orientation matters? (A horizontal adjacency is distinct from a vertical adjacency.)
5. What if order matters? (red-green is distinct from green-red.)
6. What if diagonal adjacencies are considered?

References.

Problem 27.

Problem 40.

Problem 56.