Consider Ron Graham's sequence for lcm, that is, look at sequences such that

$$n = a_1 < a_2 < \ldots < a_T = k \text{ and } \operatorname{lcm}(a_1, \ldots, a_T) \text{ is square.}$$

Question. What is the least k (as a function of n) such that such a sequence exists?

$$\begin{aligned}
a(1) &= 1 \quad \text{via } (1) \\
a(2) &= 4 \quad \text{via } (2,4) \\
a(3) &= 3 \quad \text{via } (3,9) \\
a(4) &= 4 \quad \text{via } (4) \\
a(5) &= 25 \text{ via } (5,25) \\
a(6) &= 12 \text{ via } (6,9,12) \\
a(7) &= 49 \text{ via } (7,49) \\
a(8) &= 16 \text{ via } (8,16)
\end{aligned}$$

Figure 1: Examples of $a(n)$ for $n \in \{1, 2, \ldots, 8\}$.

Related.

1. For what values n is $a(n)$ nonsquare?
2. For what values n does the corresponding sequence have three or more terms?
3. What is the analogous sequence for perfect cubes, etc?