In the grid $\mathbb{Z}^2$, the only regular polygons that you can draw are squares. In $\mathbb{Z}^3$, you can draw equilateral triangles, squares, and regular hexagons, but no other regular polygons.

In $\mathbb{Z}^3$, you can also draw regular tetrahedra, cubes, and octahedra(?), but not dodecahedra or icosahedra. (Otherwise you could draw pentagons too!)

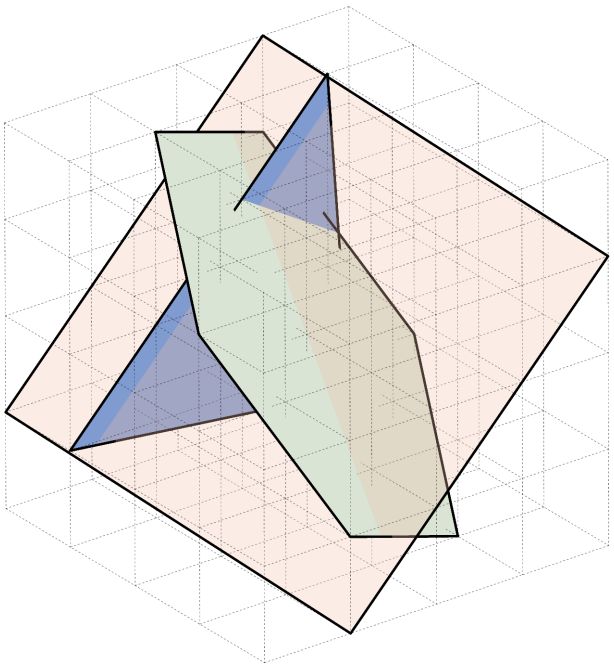

Figure 1: An example of an equilateral triangle, a square, and a regular hexagon drawn with integer coordinates in $[5]^3$.

Question. How many regular k-dimensional polytopes can be drawn with vertices in $[n]^\ell$?

Related.

1. What is the asymptotic growth of the number of k-dimensional polytopes?
2. What if other sorts of polytopes are considered? (E.g. Archimedean solids.)

References.

Problems 21, 54, 66, 94, 104.

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