Difficulty: 2/4 Interest: 4/4

Suppose you are given an $n \times m$ grid, and I then think of a rectangle with its corners on grid points. I then ask you to "black out" as many of the gridpoints as possible, in such a way that you can still guess my rectangle after I tell you all of the non-blacked out vertices that its corners lie on.

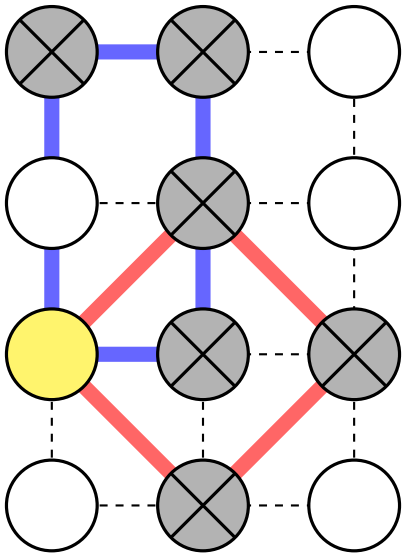

Figure 1: An example of an invalid "black out" for an 4×3 grid. The blue rectangle and the red rectangle have the same presentation, namely the gridpoint inside the yellow circle.

Question. How many vertices may be crossed out such that every rectangle can still be uniquely identified?

Related.

1. What if the interior of the rectangle is lit up instead?
2. What if all gridpoints that instersect the perimeter are lit up?
3. What if the rectangles must be square?
4. What if parallelograms are used instead of rectangles?
5. What if the rectangles must be horizontal, vertical, or 45° diagonal?
6. What if this is done on a triangular grid with equilateral triangles?
7. What if this is done in more dimensions (e.g. with a rectangular prism or tetrahedron?)

References.

`https://math.stackexchange.com/q/2465571/121988`