Say that two sequences with distinct elements are in the same equivalence class if their first differences have the same signs. (e.g. (1,3,2,3) and (7,8,-1,0) are equivalent because their first differences are both (+,-,+).)

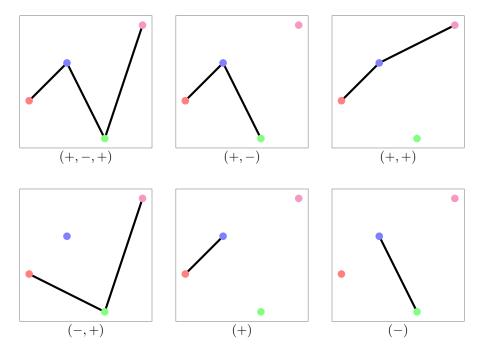


Figure 1: (0, 1, -1, 2) has subsequences in the following six equivalence classes: (+, -, +), (+, +), (+, -), (-, +), (+), (-). No length 4 sequence has its subsequences in more equivalence classes, so a(4) = 6.

Question. What is the general formula for a(n)?

Note. A quick attempt finds that a(2) = 1, a(3) = 3, a(4) = 6, and a(5) = 11.

Related.

- 1. What if the sequences do not necessarily consist of distinct elements?
- 2. What if two sequences are considered to be equivalent if they are in the same "sort order"; that is, if both sequences have their biggest element in the same position, their second biggest in the same position, and so on.
- 3. What if $(+,+) \sim (+)$?
- 4. Is the number of equivalence classes for the subsequences determined by the number of local minima and maxima?