Consider maximal non-self-intersecting polygonal chains on $[n] \times [m]$ stable under 180° rotation.

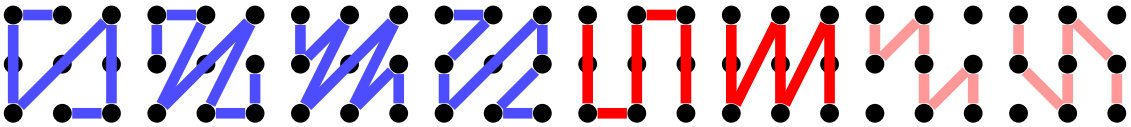

Figure 1: The 6 (or 8) maximal polygonal chains with vertices in $[3] \times [3]$.

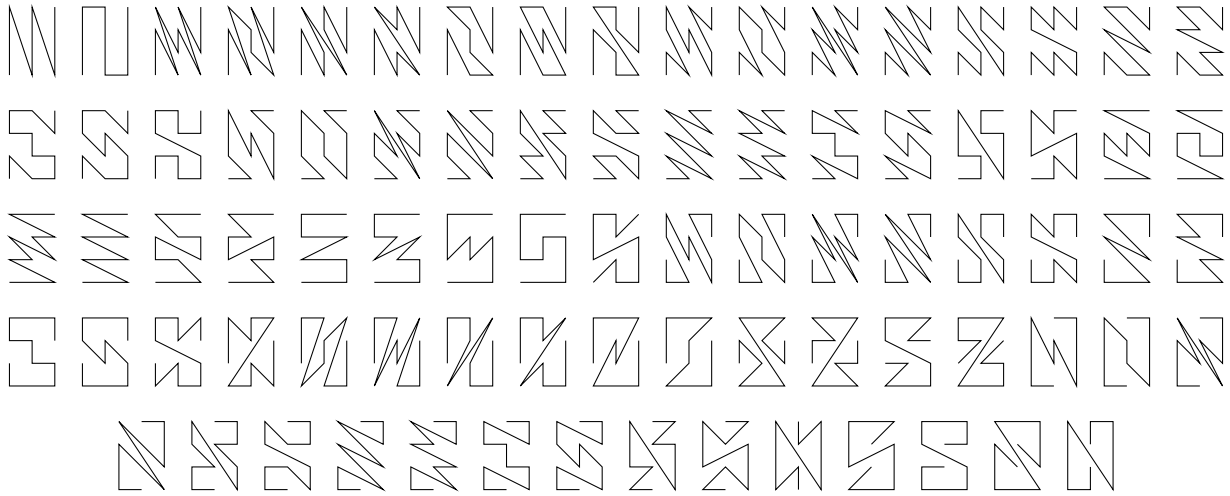

Figure 2: $f(4 \times 3) = 82$

Question. How many non-self-intersecting polygonal chains with vertex set equal to $[n] \times [m]$ are stable under 180° rotation?

Related.

1. What if this is done with other kinds of symmetry? (e.g. horizontal or vertical reflection)
2. What if this is done for polygons instead of polygonal chains?
3. What if maximal means that the polygonal chain cannot be extended, a weaker condition than that the vertex set is $[n] \times [m]$. (This includes the last two chains in the example.)
4. What is the maximal length of such a chain with respect to ℓ_1, ℓ_2, and ℓ_∞? What if the symmetry restriction is dropped?
5. What if the only allowed moves are king moves? Rook moves?
6. What if this is done with vertex set $[n_1] \times [n_2] \times \cdots \times [n_k]$?

References.

Problems 5, 44, 46, 55, 68, 74, 87, and 104.