A Heronian 2-simplex (triangle) is a triangle with both integer sides and integer area. A Heronian n-simplex is an n-simplex with integer volume and where all sides are Heronian $(n-1)$-simplices.

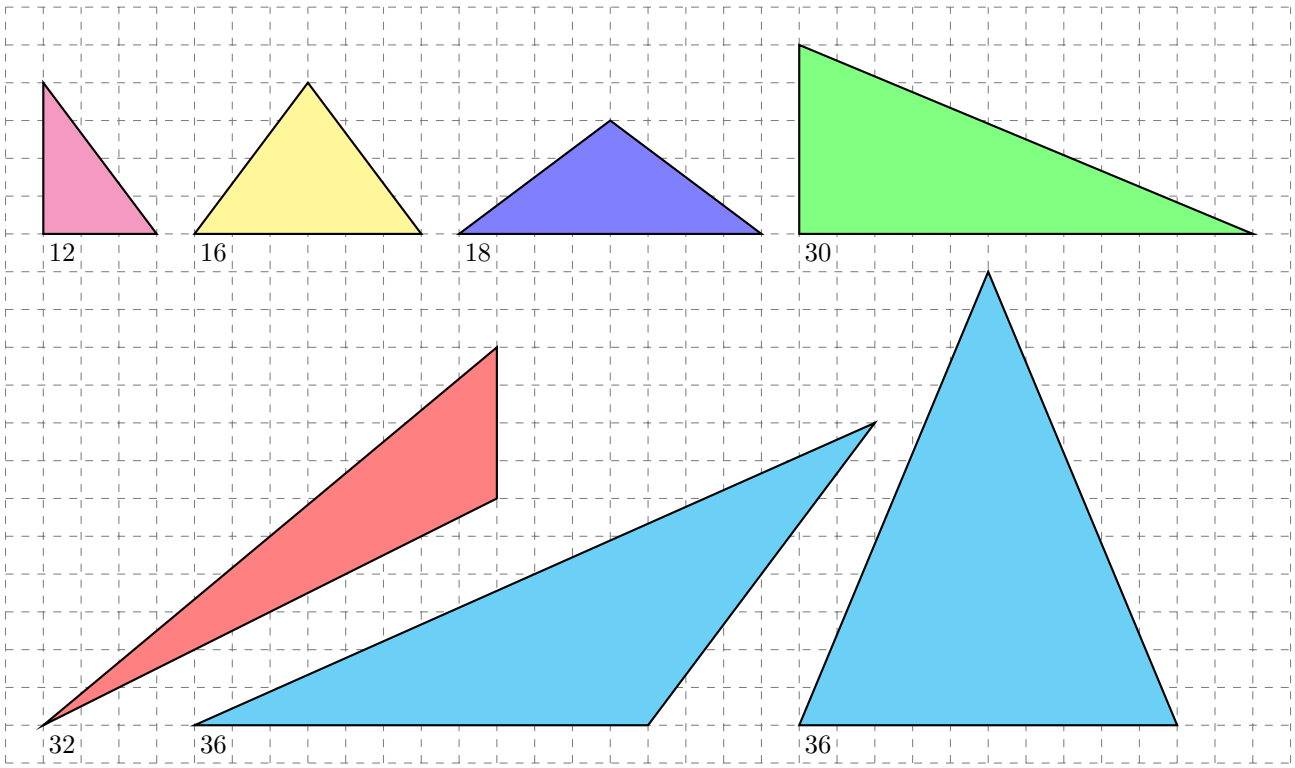

Figure 1: The seven smallest primitive Heronian triangles as measured by perimeter.

Question. Do Heronian n-simplices exist for all integers n?

Related.

1. Do infinitely many primitive Heronian n-simplices exist for each n?
2. What is the smallest Heronian n-simplex for each n as measured by volume? As measured by largest side? As measured by sum of sides? As measured by "surface area" (sum of volume of $(n-1)$-degree facets)?
3. Are all Heronian n-simplices lattice simplices?
4. What if the definition is relaxed so that only the volume and the edges must be integers?

References.

https://www.jstor.org/stable/2695390

https://oeis.org/A272388

https://en.wikipedia.org/wiki/Heronian_tetrahedron

https://en.wikipedia.org/wiki/Simplex