

Difficulty: 1/4 **Interest:** 4/4

John Conway described the “Subprime Fibonacci Sequence”:

$$a(1) = \alpha, a(2) = \beta, a(n+1) = \text{gpd}(a(n) + a(n-1)),$$

where $\text{gpd}(k)$ is the greatest proper divisor of k .

Conway then conjectured that regardless of the starting terms, the sequence ends in a handful of cycles.

Question. What are all of the different possible end behaviors of Conway’s Subprime Fibonacci Sequence?

Note. Richard Guy beat me to this problem by a few years.

Richard Guy found that there are more cycles than those that Conway conjectured.

References.

<https://arxiv.org/abs/1207.5099>