Consider integer functions f from an n-element subset of $\mathbb{N}$ such that no $k+2$ of the points $\{(j_1, f(j_1)), \ldots, (j_n, f(j_n))\}$ fall on a degree k polynomial.

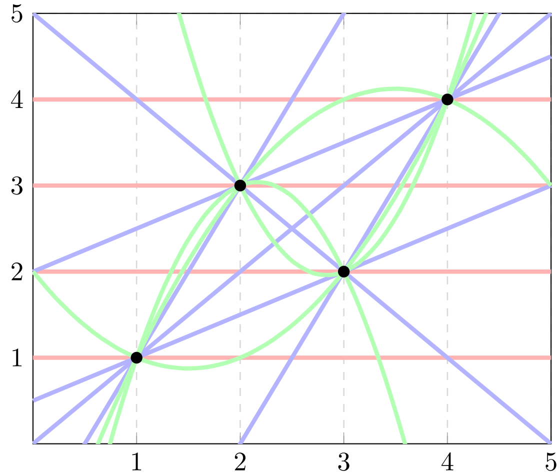

Figure 1: An example that shows that $a(4) = 4$. (Degree 0 polynomials are plotted in red, degree 1 in blue, and degree 2 in green.)

Question. What is $a(n)$, the least N such that there exists a function $f\colon \{1, 2, \ldots, n\} \to \{1, 2, \ldots, N\}$ with the above property?

Note. Trivially, $a(n)$ is bounded above by the function described in problem 23.

Related.

1. What is the least M such that there exists a subset $S \subset \{1, 2, \ldots, M\}$ and a surjection $g\colon S \to \{1, 2, \ldots, n\}$ with the aforementioned property?
2. How many such functions exist when N and M are minimized respectively?

Note. It appears that $N = M = n$.

References.

Problem 23.

https://oeis.org/A301802