

**Difficulty:** 1/4    **Interest:** 2/4

Let  $U_n$  be the set of sequences of positive integers of length  $n$  such that no substring occurs twice.

$$\begin{aligned}(1, 1, 2, 2, 1, 3, 1) &\in U_7 \\ (1, 2, 1, 2, 3) &\notin U_5 \text{ because } (1, 2) \text{ occurs twice.} \\ (1, 1, 1) &\notin U_3 \text{ because } (1, 1) \text{ occurs twice.}\end{aligned}$$

Figure 1: An example and two non-examples of sequences with no repeated substrings.

**Question.** What is the number of sequences in  $U_n$  where the sum of terms is minimized?

**Related.**

1. What is the minimum least common multiple of a sequence in  $U_n$ ? How many such minimal sequences?
2. What is the minimum product of a sequence in  $U_n$ ? How many such minimal sequences?
3. What if substrings are considered forward and backward?
4. What if only substrings of length greater  $k$  are considered?
5. What if any term can appear at most  $\ell$  times?

**References.**

<https://oeis.org/A259280>