

Difficulty: 3/4 Interest: 3/4

Let

$C_n = \{f : [n] \rightarrow \mathbb{N} \mid \text{the convex hull around } \{(1, f(1)), \dots, (n, f(n))\} \text{ forms an } n\text{-gon}\}$

and then let $a(n)$ denote the least upper bound over all functions in C_n

$$a(n) = \min\{\max\{f(k) \mid k \in [n]\} \mid f \in C_n\}$$

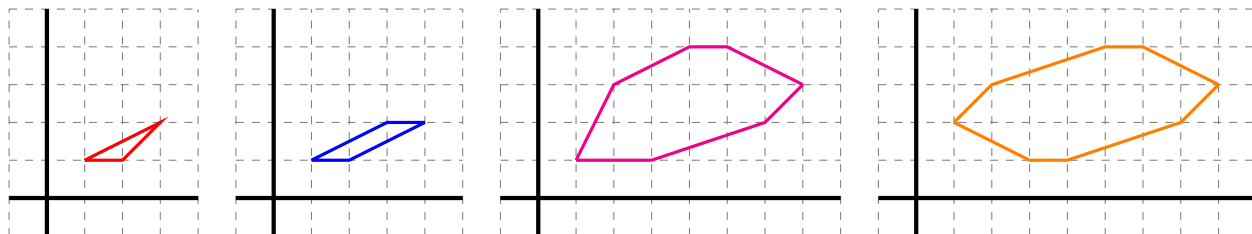


Figure 1: Examples of $a(3) = 2$, $a(4) = 2$, $a(7) = 4$, and $a(8) = 4$, where the polygons with an even number of vertices have rotational symmetry.

Question. Do these polygons converge to something asymptotically?

Related.

1. Does $a(2n) = a(2n - 1)$ for all n ?
2. Do the minimal $2n$ -gons always have a representative with rotational symmetry?
3. Are minimal $2n$ -gons unique (up to vertical symmetry) with finitely many counterexamples?
4. What is the asymptotic growth of $a(n)$?

References.

A285521: "Table read by rows: the n -th row gives the lexicographically earliest sequence of length n such that the convex hull of $(1, a(1)), \dots, (n, a(n))$ is an n -gon with minimum height." (<https://oeis.org/A285521>)