Open problem collection

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December 19, 2022

This is a catalog of open problems that I began in late 2017 to keep tabs on different problems and ideas I had been thinking about.

Each problem consists of an introduction, a figure which illustrates an example, a question, and a list related questions. Some problems also have references which refer to other problems, to the OEIS, or to other web references.

1 Rating

Each problem is rated both in terms of how difficult and how interesting I think the problem is.

1.1 Difficulty

The difficulty score follows the convention of ski trail difficulty ratings.

| | Easiest | The problem should be solvable with a modest amount of effort. |
|----------|----------------|---|
| | Moderate | Significant progress should be possible with moderate effort. |
| ♦ | Difficult | Significant progress will be difficult or take substantial insight. |
| * | Most difficult | The problem may be intractable, but special cases may be solvable. |

1.2 Interest

The interest rating follows a four-point scale. Each roughly describes what quartile I think it belongs in with respect to my interest in it.

| | Least interesting | Problems have an interesting idea, but may feel contrived. |
|--|-------------------|--|
| | More interesting | Either a somewhat complicated or superficial question. |
| | Very interesting | Problems that are particularly natural or simple or cute. |
| | Most interesting | These are the problems that I care the most about. |

Be patient towards all that is unsolved in your heart and try to love the questions themselves as if they were locked rooms or books written in a very foreign language. Don't search for the answers, which could not be given to you now because you would not be able to live them. And the point is to live everything. Live the questions now. Perhaps then, someday far in the future, you will gradually, without even noticing it, live your way into the answer.

Rainer Maria Rilke, Letters to a Young Poet

My love for this teaching and the seriousness with which I take it rests in part on my deep reverence for the gravity and the power of questions in human life. I think that this is undervalued in a culture that is in love with the form of words that is an answer — and the way with words that is an argument.

But I also find a question to be a mighty form of words, and I have learned a few things about questions. I have learned that questions elicit answers in their likeness — that answers rise or fall to the questions they meet. We've all seen this. We've all experienced it. It's very hard to respond to a combative question with anything but a combative answer. It's almost impossible to transcend a simplistic question with anything but a simplistic answer. But the opposite is also true: it's hard to resist a generous question. This is a skill that needs relearning, but I believe that we all have it in us to ask questions that invite, that draw forth searching in dignity and revelation. There is something redemptive and lifegiving about asking a better question.

And it is a deep truth in science, and also in each of our lives, that we are shaped as much by the quality of the questions we're asking at any given point as by the answers we have it in us to give. Those moments in our lives when a better quality of question rises up in us, stops us in our tracks — those are pivot points. Those are moments where discovery and new possibility break in.

Krista Tippett, On Being, "Living the Questions".