Consider partitions of the $n \times m$ grid in which every piece has $180°$ rotational symmetry.
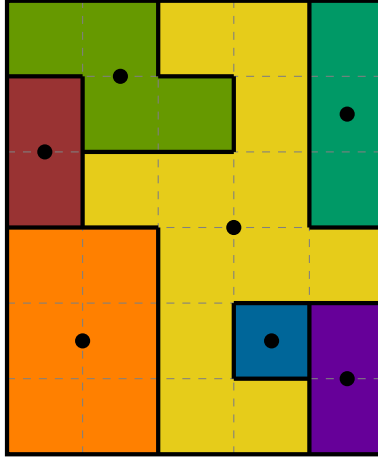

Figure 1: A partition of the 5×6 grid into 7 parts with rotational symmetry.

Question. How many such partitions of the $n \times n$ grid exist? Up to dihedral action?

Related.

1. How many partitions into exactly k parts?

2. How many partitions with other types of symmetry?

3. How many partitions of a torus? Cylinder? Möbius strip?

4. How many partitions of a triangular or hexagonal lattice?

5. How many partitions of an $n \times m \times p$ cuboid?