Difficulty: 1/4 Interest: 2/4

Consider all k-colorings of an $n \times n$ grid, where each row and column has $\lfloor n/k \rfloor$ or $\lceil n/k \rceil$ cells with each color.

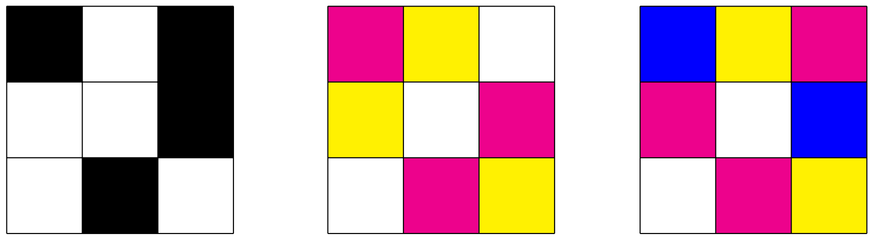

Figure 1: A valid 2-coloring, 3-coloring, and 4-coloring of an 3×3 grid.

Question. How many such k-colorings of the $n \times n$ grid?

Related.

1. What if there also must be a total of $\lfloor n^2/k \rfloor$ or $\lceil n^2/k \rceil$ cells of each color?
2. What if these are counted up to the dihedral action on the square D_4?
3. What if these are counted up to torus action?
4. What if these are counted up to permutation of the coloring?
5. Can this generalize to the cube? To a triangular tiling?