Consider polyominoes where each cell has one of n colors, and each distinct pair of colors is adjacent (horizontally or vertically) to each other somewhere in the polyomino. Let an n-minimum polyomino be one that has the minimum number of cells.

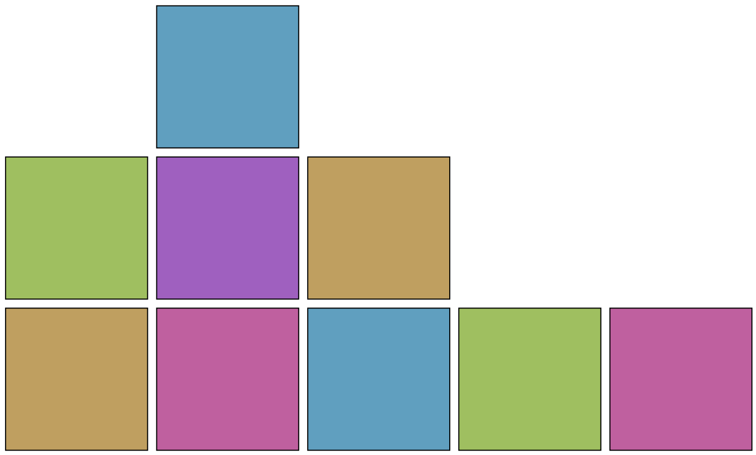

Figure 1: An example of a minimum polyomino for $n = 5$; $a(5) = 9$

Question. How many such n-minimum polyominoes exist?

Related.

1. What if the "distinct" restriction is lifted? (e.g. a blue label must somewhere be adjacent to another blue label.)
2. What is a way to determine the size of an n-minimum polyomino for large n?
3. What if this is done on a triangular or hexagonal grid?
4. What if this is done on a three dimensional cube lattice?

Note. This is closely related to the following problem: given a finite graph G, let $f(G) = n$ where n is the largest integer such that there exists a quotient $G/\sim$ that is isomorphic to the complete graph K_n. Is there an efficient way to compute f?

Is the symmetric polynomial for G which enumerates the labelings that quotient G into a complete graph known or interesting?

References.

https://oeis.org/A278299

http://www.peterkagey.com/square_games