Consider all rectangles with all corners on gridpoints on an $n \times m$ grid.

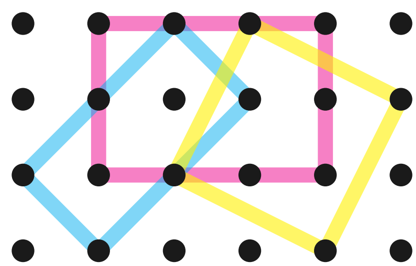

Figure 1: An example of three rectangles with all corners on gridpoints of a 4×6 grid.

Question. Given some shape, how many of these shapes can be constrained to the $n \times m$ grid?

Related.

1. What if we want to count only "primitive" squares, in the sense that the sides of the square only intersect grid points at the corners?
2. Number of rectangles on the cylinder? Torus? Möbius strip?
3. Number of "rotation classes", where two squares are equivalent if one can be transformed into the other by shifting and stretching?
4. Number of "orientation classes" where two squares are equivalent if one can be transformed into the other by shifting?
5. What if this is done on an $n \times m \times k$ grid?
6. What if the rectangles must be diagonal?
7. What if this is done on a triangular lattice with primitive equilateral triangles?

Note. Equilateral triangles in triangle is A000332; tetrahedra in a tetrahedron is A269747; triangles in a tetrahedron is A334581; tetrahedra in cube is 2*A103158; equilateral triangles in cube is A102698; rectangles in square is A085582; rectangles in rectangle is A289832; isosceles triangles in rectangle is A271910; right isosceles triangles in square is A187452; right triangles in square is A077435; convex quadrilaterals in square A189413; quadrilaterals in square A189414; trapezoids in square A189415; parallelograms in square is A189416; kites in square A189417; rhombi in square A189418.

References.

Problem 1.

`https://arxiv.org/pdf/1605.00180.pdf`

`http://people.missouristate.edu/lesreid/POW03_01.html`