Suppose that Arthur chooses an arbitrary subset $A \subseteq [n]$, and Bri attempts to discover it by repeatedly asking questions of the form, “How many elements does A have in common with $B_i \subseteq [n]$?”

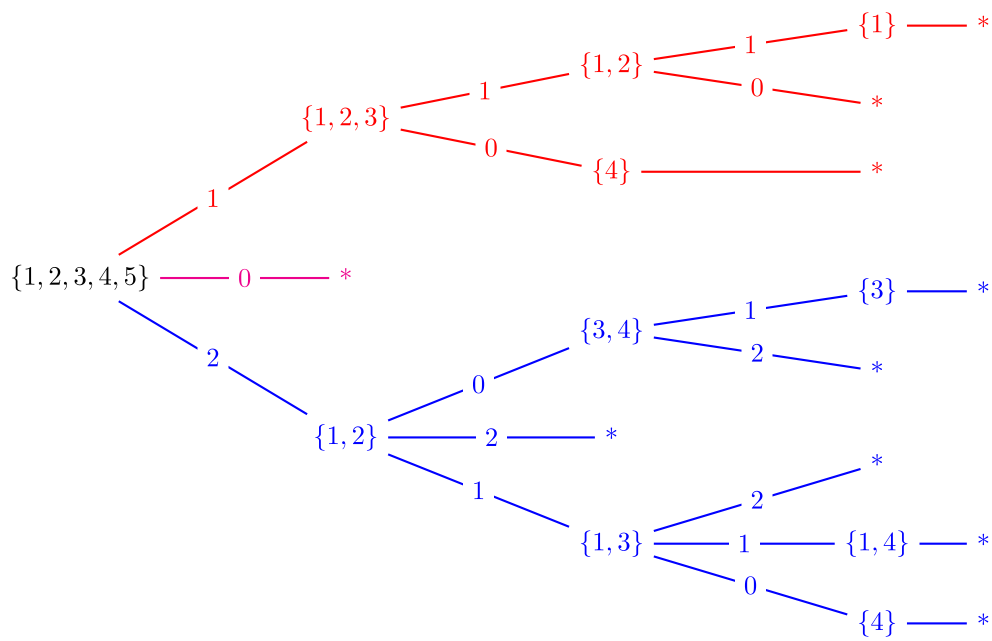

Figure 1: A strategy that Bri can use to discover A in four guesses or fewer, note that the cases where A is size $3, 4$, or 5 follow by symmetry.

Question. Let $a(n) = k$ be the least integer k such that there exists a strategy where Bri can always determine A in k guesses or fewer. What is $a(n)$?

Related.

1. What if instead of giving the size of the intersection, Arthur gives the size of the symmetric difference?
2. What if A is a multiset? Where i can occur with multiplicity at most a_i?
3. What are some upper and lower bounds?
4. How many (essentially different) optimal strategies exist? (e.g., do you always have to start by guessing the entire set?)
5. What is the best *average case* strategy?
6. What if there are restrictions on Bri’s subsets? For example, if the size of Bri’s subsets must be weakly decreasing, or if Bri’s subsets cannot simultaneously contain both i and $i + 1$?
7. What if Arthur instead picks a column from a given matrix, how many questions of the form “what is the ith entry” does Bri have to ask in order to determine the column?

References.

`https://math.stackexchange.com/a/25297/121988`