Consider ways of partitioning nonattacking rooks in such a way that no rook lies in the convex hull of its partition. Let $a(\sigma)$ be the minimum number of parts of such a partition.

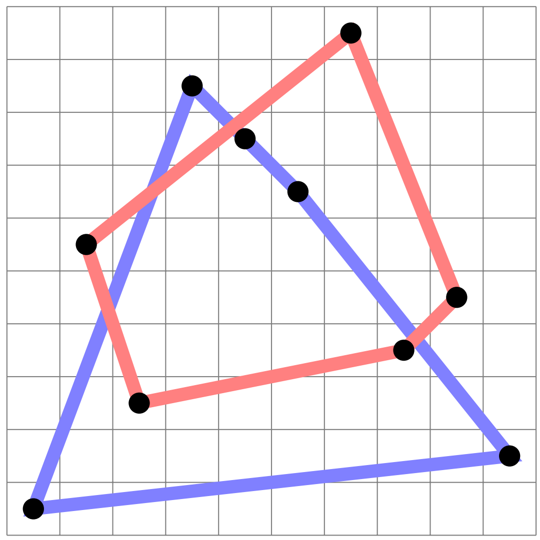

Figure 1: An illustration showing that $a(\sigma) = 2$ for $\sigma = 16398710452 \in S_{10}$.

Question. What is the expected value of $a(\sigma)$ for a uniformly random $\sigma \in S_n$?

Related.

1. What if each point must be on the corner of the convex hull?
2. What is the maximum number of convex hulls required?
3. What is the expected number of convex hulls? (i.e. how many different ways can a σ be partitioned into $a(\sigma)$ convex hulls?
4. What if the convex hulls are not allowed to overlap?
5. What is the expected value of the largest subset of $((1, \sigma(1)), \ldots, (n, \sigma(n)))$ such that no points are in the interior of the convex hull?
6. What if this is done for non-attacking queens?
7. What if this is done for an arbitrary configuration of k pieces on an $n \times m$ board?
8. What if the convex hull of the permutation is taken, and then the convex hull of the interior, and the convex hull of that interior and so on?
9. What if a no three-on-a-line rule is used instead? No $k + 2$ on a degree k polynomial?

Note.

$A156831(n) = \{\sigma \in S_n : a(\sigma) = 1\}$.

References.

`https://oeis.org/A156831`

Problem 5, 6, 7.