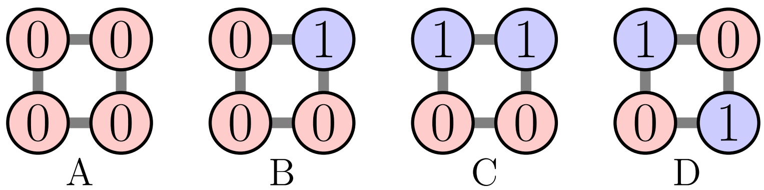
Suppose you have two finite groups G and H, and a H-set Ω. Denote the base of the wreath prooduct of G by H as

$$K = \prod_{\omega \in \Omega} g_\omega.$$

We're interested in determining when it is possible to construct a finite sequence, $\{k_i \in K\}_{i=1}^N$, such that for any choice of $\alpha \in G \wr_\Omega H$ and any sequence $\{h_i \in H\}_{i=1}^N$, there exists $n \leq N$ satisfying

$$\alpha \cdot (k_1, h_1) \cdot (k_2, h_2) \cdots (k_n, h_n) = (e_K, h)$$

for some $h \in H$.



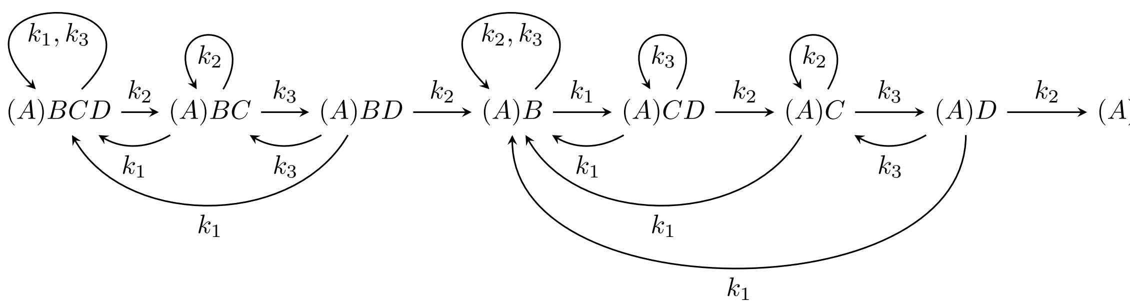


Figure 1: Suppose $G = \mathbb{Z}_2$, $H = C_4$, and Ω is the rotations of the square, with H acting in the ordinary way. Moreover, suppose that $k_0 = (1,1,1,1)$, $k_1 = (1,0,0,0)$, $k_2 = (1,0,1,0)$, and $k_3 = (1,1,0,0)$. Then the sequence $(k_0, k_2, k_0, k_3, k_0, k_2, k_0, k_1, k_0, k_2, k_0, k_3, k_0, k_2, k_0)$ satisfies the above condition.

Question. What conditions on $G \wr_\Omega H$ guarantee such a finite sequence of elements in K?

Related.

1. Given $G \wr_\Omega H$, what is N, the minimum length of the sequence?
2. If both α and the sequence $\{h_i \in H\}_{i=1}^N$ are chosen uniformly at random, what is the expected value of the minimal n such that $\alpha \cdot (k_1, h_1) \cdot (k_2, h_2) \cdot \cdots \cdot (k_n, h_n) = (e, k)$?
3. What if there is a set of elements in the base, any one of which is valid. (e.g. all sets satisfying the condition that the number of heads is congruent to 2 (mod 3).)
4. What if something about s_i is told to you after the ith move for each i (and the strategy can depend on this information)?

Note. This can be generalized to $H = C_{2^n}$ and Ω a 2^n-gon, but no other m-gon. The idea is that (1) you can't solve any m-gon with m odd, and (2) you can solve a m-gon if and only if you can solve a d-gon for every proper divisor $d \mid m$.

References.

http://mathriddles.williams.edu/?p=77