In a letter, Alec Jones asks notes that there are eleven distinct nets of the cube, considered as free polyominoes.

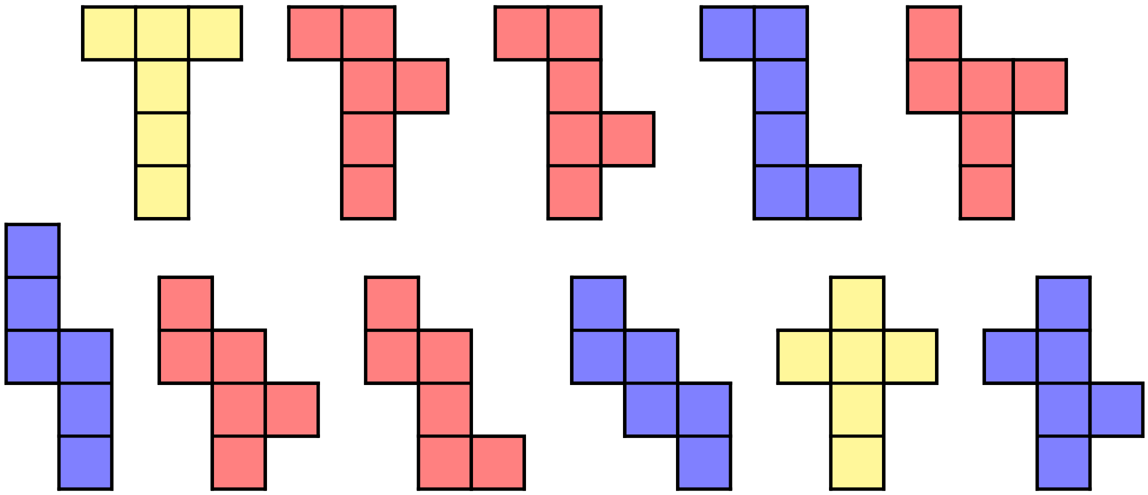

Figure 1: Eleven distinct nets of the (3-hyper)cube. Two, marked in yellow, have reflection symmetry, that is they are achiral; four, marked in blue, have rotational symmetry.

Question. Alec asks, how many nets are there of the n-hypercube?

Related.

1. How many of the nets exhibit some sort of symmetry, as shown in the example.
2. How many nets are there of the n-simplex? Other polyhedra?
3. How many nets for a rectangular analog? That is for some $a_1, a_2, \ldots, a_n$,

$$R = \{(x_1, x_2, \ldots, x_n) : x_i = 0 \text{ or } x_i = a_i\}.$$

   Note that in the case of the n-hypercube, $a_1 = a_2 = \ldots = a_n = 1$.

4. Must all of these nets "use up" n dimensions. (In the case of $n = 3$ in the example, yes, because the "straight line" 6-omino cannot be folded into a cube.)
5. What is the net with the smallest convex hull by hypervolume? Largest?
6. How many of the nets tile $(n-1)$-space?

Note. There is 1 net for the 2-hypercube, 11 nets for the 3-hypercube, and 261 nets for the 4-hypercube. The answer is unknown for the 5-hypercube. The corresponding sequence is tagged "hard" on OEIS.

References.

https://oeis.org/A091159