



Ron Graham's (A006255) sequence is the least  $k$  for which there exists a strictly increasing sequence

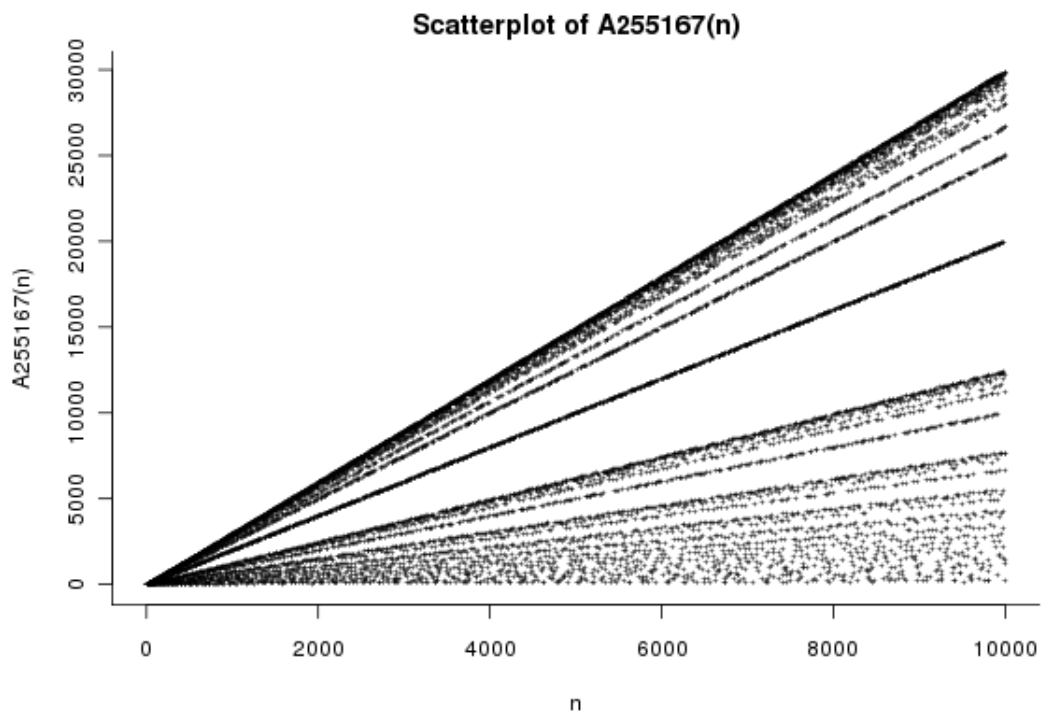
$$n = a_1 \leq a_2 \leq \dots \leq a_T = k \text{ where } a_1 \cdot \dots \cdot a_T \text{ is square.}$$

A006255 is bounded above by A072905, the least  $k > n$  such that  $k \cdot n$  is square.

**Question.** Does there exist any  $n$  for which  $A006255(n) = A072905(n)$ . In other words, is there any non-square  $n$  for which  $n \cdot A006255(n)$  is square?

**Related.**

1. Does the gap  $A072905(n) - A006255(n)$  have a nonzero lower bound?



**Note.** This is equivalent to showing that for any  $a < b$  with the same squarefree part, there is some subset of  $\{a+1, a+2, \dots, b-1\}$  such that the product of the elements of the subset has the same squarefree part as  $a$  (and  $b$ ).

**References.**

<https://oeis.org/A006255>

<https://oeis.org/A072905>

<https://oeis.org/A255167>