

**Difficulty:** 4/4    **Interest:** 1/4

Suppose a snail chooses a direction uniformly at random and walks a random distance  $d_{\text{free}}$  according to some distribution. The snail then repeats the process. However, if the snail turns in a direction that faces its trail, it walks a distance  $d_{\text{trail}}$  chosen uniformly at random between 0 and the distance to the trail.

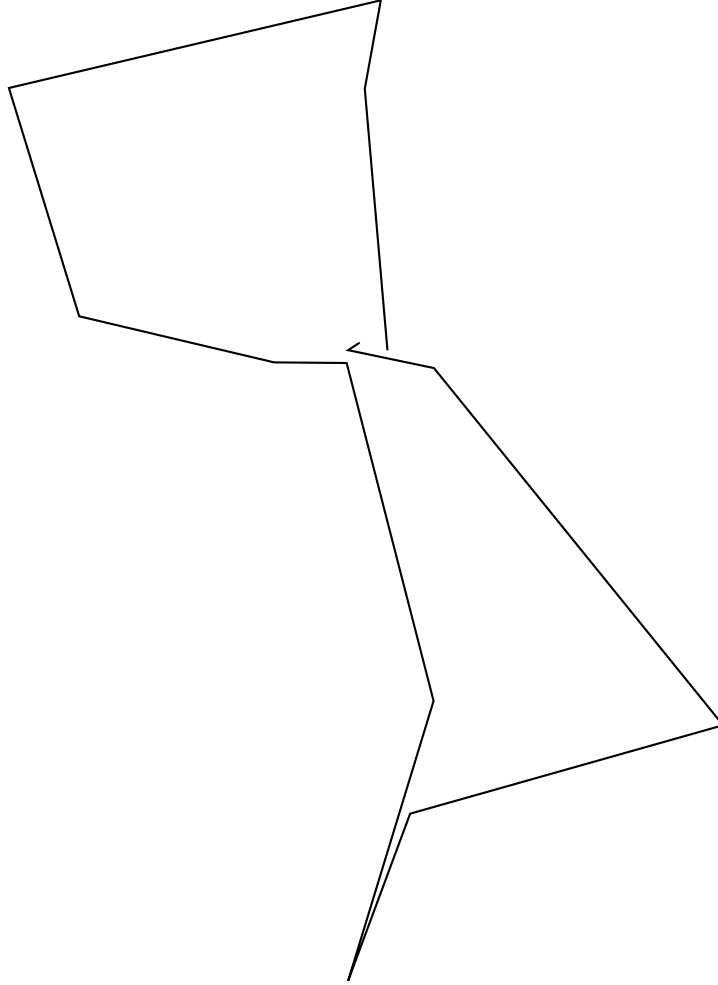


Figure 1: An (actually) random walk where the change in  $x$  and  $y$  were independently chosen from a Gaussian distribution. Is the snail likely to escape from its confine for good?

**Question.** What is the probability that the walk is unbounded?

**Related.**

1. If the walk is bounded with probability 1, what is the expected distance of the walk?
2. What if the angle is chosen uniformly at random from  $\{2\pi/n, 4\pi/n, \dots, 2(n-1)\pi/n\}$ ? And  $d_{\text{free}}$  is uniform on  $[0, 1]$ .
3. What is the expected number of (avoided) collisions after  $k$  steps?