Let $a_3(n)$ be the least $k > n$ such that nk or nk^2 is a cube, and let $A299117$ be the image of $a_3(n)$.

$a_3(1) = 8$ via $1 \cdot 8 = 2^3$	$a_3(11) = 88$ via $11 \cdot 88 = 22^3$	$a_3(21) = 168$ via $21 \cdot 168^2 = 84^3$
$a_3(2) = 4$ via $2 \cdot 4 = 2^3$	$a_3(12) = 18$ via $12 \cdot 18 = 6^3$	$a_3(22) = 176$ via $22 \cdot 176^2 = 88^3$
$a_3(3) = 9$ via $3 \cdot 9 = 3^3$	$a_3(13) = 104$ via $13 \cdot 104^2 = 52^3$	$a_3(23) = 184$ via $23 \cdot 184^2 = 92^3$
$a_3(4) = 16$ via $4 \cdot 16 = 4^3$	$a_3(14) = 112$ via $14 \cdot 112^2 = 56^3$	$a_3(24) = 72$ via $24 \cdot 72 = 12^3$
$a_3(5) = 25$ via $5 \cdot 25 = 5^3$	$a_3(15) = 120$ via $15 \cdot 120^2 = 60^3$	$a_3(25) = 40$ via $25 \cdot 40 = 10^3$
$a_3(6) = 36$ via $6 \cdot 36 = 6^3$	$a_3(16) = 32$ via $16 \cdot 32 = 8^3$	$a_3(26) = 208$ via $26 \cdot 208^2 = 104^3$
$a_3(7) = 49$ via $7 \cdot 49 = 7^3$	$a_3(17) = 136$ via $17 \cdot 136^2 = 68^3$	$a_3(27) = 64$ via $27 \cdot 64 = 12^3$
$a_3(8) = 27$ via $8 \cdot 27 = 6^3$	$a_3(18) = 96$ via $18 \cdot 96 = 12^3$	$a_3(28) = 98$ via $28 \cdot 98 = 14^3$
$a_3(9) = 24$ via $9 \cdot 24 = 6^3$	$a_3(19) = 152$ via $19 \cdot 152^2 = 76^3$	$a_3(29) = 232$ via $29 \cdot 232^2 = 116^3$
$a_3(10) = 80$ via $10 \cdot 80 = 20^3$	$a_3(20) = 50$ via $20 \cdot 50 = 10^3$	$a_3(30) = 240$ via $30 \cdot 240^2 = 120^3$

Question. Is there another way to characterize what integers are in $A299117$?

Note. The function a_3 is an injection.
$A299117$ contains every cube, because $a(n^3) = (n+1)^3$.

Related.

1. Does $A299117$ contain every square?
2. Does $A299117$ contain any squarefree number?
3. What about the generalization: the image of a_m where $a_m(n)$ is the least $k > n$ such that $n^\alpha k^\beta$ is a m-th power, with $\alpha, \beta \in \{1, 2, \ldots, m-1\}$.

References.

OEIS sequences A277781, A299117, and A343881.