A Heronian 2-simplex (triangle) is a triangle with both integer sides and integer area. A Heronian n-simplex is an n-simplex with integer volume and where all sides are Heronian $(n-1)$-simplices.
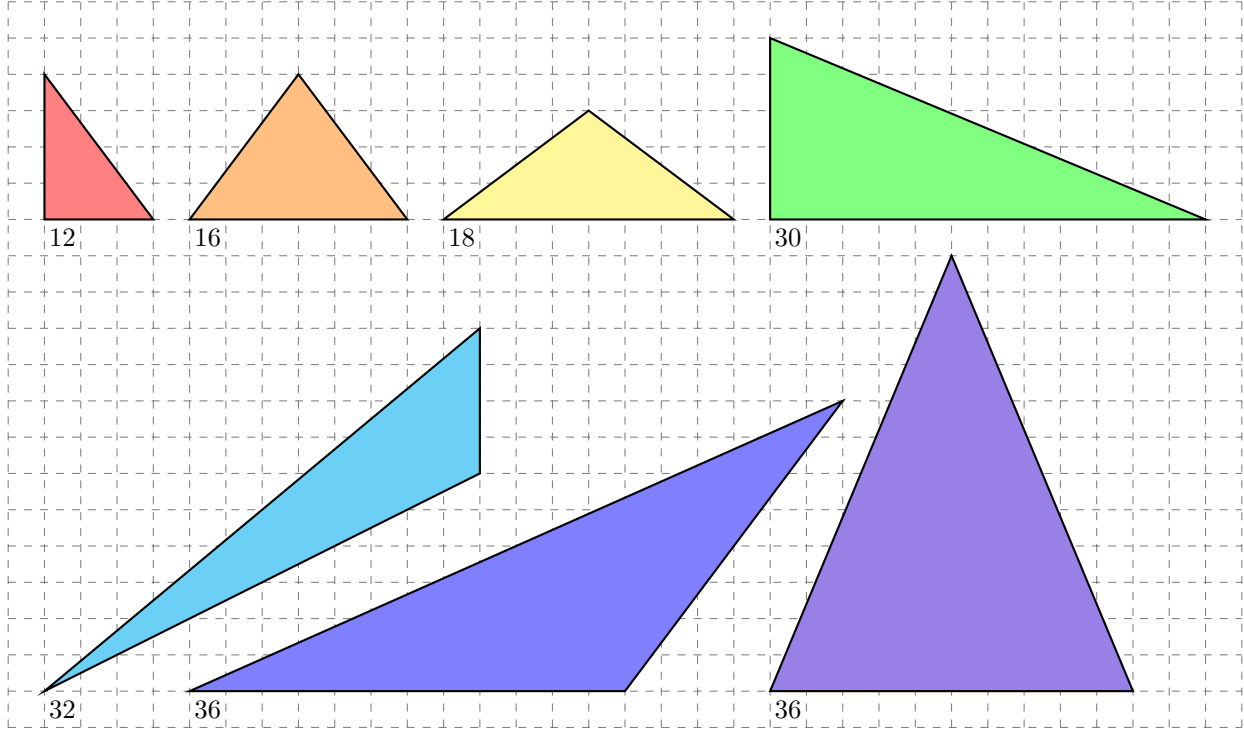

Figure 1: The seven smallest primitive Heronian triangles as measured by perimeter. The areas are 6, 12, 12, 30, 24, 60, and 36, respectively.

Question. Do Heronian n-simplices exist for all integers n?

Related.

1. Do infinitely many primitive Heronian n-simplices exist for each n?

2. What is the smallest Heronian n-simplex as measured k-dimensional volume of the largest k-face? as measured by sum volume of k-faces? (These agree when $k = n$.)

3. Are all Heronian n-simplices lattice simplices, as is the case for $n \leq 3$?

4. What if the definition is relaxed so that only, say, the volume and the edge-lengths must be integers?

5. Are other "Heronian polytopes" lattice polytopes, where a Heronian polytope is polytope where the k-dimensional volume of every k-face is an integer.

References.

https://www.jstor.org/stable/2695390

https://oeis.org/A272388

https://en.wikipedia.org/wiki/Heronian_tetrahedron

https://en.wikipedia.org/wiki/Simplex