

Say that two sequences with distinct elements are in the same equivalence class if their first differences have the same signs. (e.g. $(1, 3, 2, 3)$ and $(7, 8, -1, 0)$ are equivalent because their first differences are both $(+, -, +)$.)

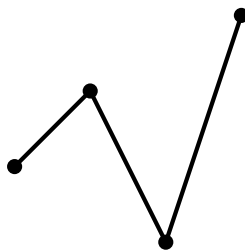


Figure 1: $(0, 1, -1, 2)$ has subsequences in the following six equivalence classes: $(+, -, +)$, $(+, +)$, $(+, -)$, $(-, +)$, $(+)$, $(-)$. No length 4 sequence has its subsequences in more equivalence classes, so $a(4) = 6$.

Question. What is the general formula for $a(n)$?

Note. A quick attempt finds that $a(2) = 1$, $a(3) = 3$, $a(4) = 6$, and $a(5) = 11$.

Related.

1. What if the sequences do not necessarily consist of distinct elements?
2. What if two sequences are considered to be equivalent if they are in the same “sort order”; that is, if both sequences have their biggest element in the same position, their second biggest in the same position, and so on.
3. What if $(+, +) \sim (+)$?
4. Is the number of equivalence classes for the subsequences determined by the number of local minima and maxima?