Consider all rectangles with all corners on gridpoints on an $n \times m$ grid.

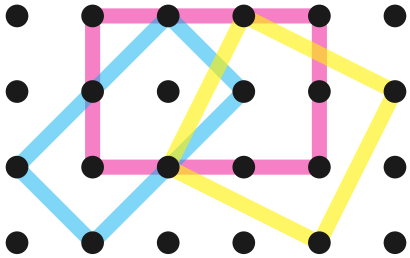

Figure 1: An example of three rectangles with all corners on gridpoints of a 4×6 grid.

Question. How many such rectangles exist?

Related.

1. How many squares exist? Rhombuses? Parallelograms? Kites? Quadrilaterals?
2. What if we want to count only "primitive" squares, in the sense that the sides of the square only intersect gridpoints at the corners?
3. Number of rectangles on the cylinder? Torus? Möbius strip?
4. Number of "rotation classes", where two squares are equivalent if one can be transformed into the other by shifting and stretching?
5. Number of "orientation classes" where two squares are equivalent if one can be transformed into the other by shifting?
6. How many right triangles?
7. What if this is done on an $n \times m \times k$ grid?
8. What if the rectangles must be diagonal?
9. What if this is done on a triangular lattice with equilateral triangles? Primitive equilateral triangles?
10. How many tetrahedra are in an n-sided tetrahedra?

References.

Problem 1.

`https://oeis.org/A000332`

`http://people.missouristate.edu/lesreid/POW03_01.html`