Let $U_n$ be the set of sequences of positive integers of length $n$ such that no substring occurs twice.

$$(1,1,2,2,1,3,1) \in U_7 \tag{1}$$
$$(1,2,1,2,3) \notin U_5 \text{ because } (1,2) \text{ occurs twice.} \tag{2}$$
$$(1,1,1) \notin U_3 \text{ because } (1,1) \text{ occurs twice.} \tag{3}$$

Figure 1: An example and two non-examples of sequences with no repeated substrings.

**Question.** What is th enumber of sequences in $U_n$ where the sum of terms is minimized?

**Related.**

1. What is the minimum least common multiple of a sequence in $U_n$? How many such minimal sequences?
2. What is the minimum product of a sequence in $U_n$? How many such minimal sequences?
3. What if substrings are considered forward and backward?
4. What if only substrings of length greater $k$ are considered?
5. What is any term can appear at most $\ell$ times?