Difficulty: 3/4 Interest: 3/4

Let G be some $n \times m$ grid as in Figure 1, where each cell has two opposite diagonals connected (uniformly at random). A cell is chosen (also uniformly at random), and the segment given by the path of diagonals that goes through the selected cell is is inspected.

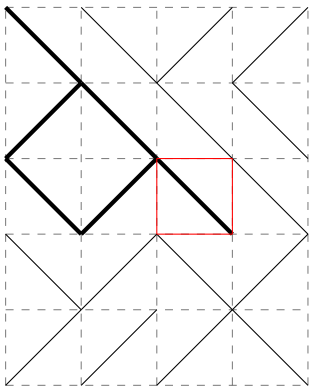

Figure 1: An example of a 4×5 grid, where a segment of size 6 has been selected.

Question. What is the expected length of the selected segment?

Related.

1. What is the expected number of segments in an $n \times m$ grid?
2. How long is the longest segment expected to be?
3. How does this change if the grid is toroidal, on a cylinder, on a Möbius strip, etc?