

Let U_n be the set of sequences of positive integers of length n such that no substring occurs twice.

$$\begin{aligned}(1, 1, 2, 2, 1, 3, 1) &\in U_7 \\ (1, 2, 1, 2, 3) &\notin U_5 \text{ because } (1, 2) \text{ occurs twice.} \\ (1, 1, 1) &\notin U_3 \text{ because } (1, 1) \text{ occurs twice.}\end{aligned}$$

Figure 1: An example and two non-examples of sequences with no repeated substrings.

Question. What is the number of sequences in U_n where the sum of terms is minimized?

Related.

1. What is the minimum least common multiple of a sequence in U_n ? How many such minimal sequences?
2. What is the minimum product of a sequence in U_n ? How many such minimal sequences?
3. What if substrings are considered forward and backward?
4. What if only substrings of length greater k are considered?
5. What if any term can appear at most ℓ times?

References.

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