Let $a_3(n)$ be the least $k > n$ such that nk or nk^2 is a cube, and let $A299117$ be the image of $a_3(n)$.

$$
\begin{aligned}
a_3(1) &= 8 \\
a_3(2) &= 4 \\
a_3(3) &= 9 \\
a_3(4) &= 16 \\
a_3(5) &= 25 \\
a_3(6) &= 36 \\
a_3(7) &= 49 \\
a_3(8) &= 27 \\
a_3(9) &= 24
\end{aligned}
$$

Question. Is there another way to characterize what integers are in $A299117$?

Note. $A299117$ contains every cube, because $a(n^3) = (n+1)^3$.
$A299117$ contains the square of every prime, because $a(p) = p^2$.

Related.

1. Does $A299117$ contain every square?
2. Does $A299117$ contain any squarefree number?
3. What about the generalization: the image of a_β where $a_\beta(n)$ is the least $k > n$ such that $nk, nk^2, \ldots, nk^{\beta-2}$, or $nk^{\beta-1}$ is a β-th power? Prime β is an injection—is this well behaved?