Consider all of the “essentially distinct” ways of starting with two points on the plane, and then with a straightedge/compass drawing n lines/circles. This forms a graded poset, where the chains the poset correspond to an algorithm for constructing that diagram.

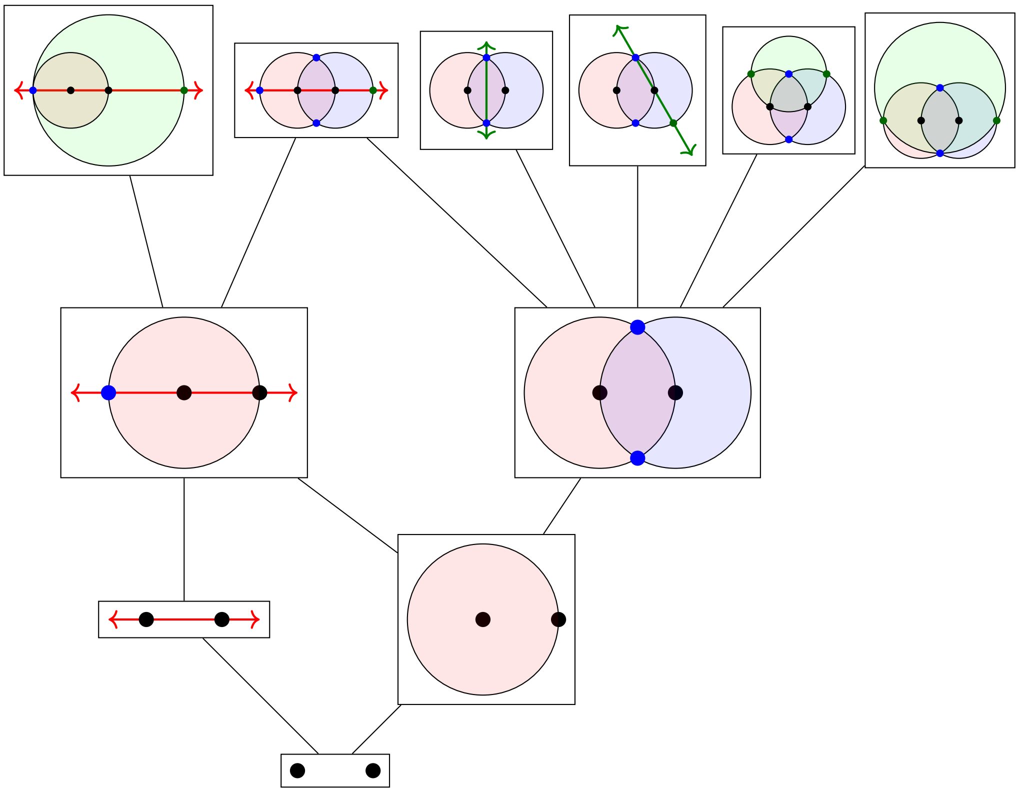

Figure 1: A ruler/straightedge poset.

Question. Consider the ranked poset of diagrams. How many diagrams are at rank n?

Related.

1. What is the greatest/least number of regions for an element at rank n?
2. What is the greatest/least number of points for an element at rank n?
3. What is the number of distinct distances over all of rank n? (e.g. rank 2 has distances of 1, 2, and $\sqrt{3}$. Rank 3 has distances of 1, 2, 3, 4, and $\sqrt{3}$.)
4. Is this poset Sperner?

Note. I suspect that it's easy to prove by induction that the least number of points for an element at rank n is $n + 2$, by continually making the biggest possible circle centered at the rightmost point.

References.

OEIS: Yuda Chen's A352903 and my A383744.

MSE: Joel David Hamkins, “What is the next number on the constructibility sequence?”