

There are two popular, essentially identical, iPhone games in the app store: AMAZE!!! and Roller Splat!. The goal of the puzzle is to reach every (white) square in the board—the catch is that you can only move in as-long-as-possible rook moves.

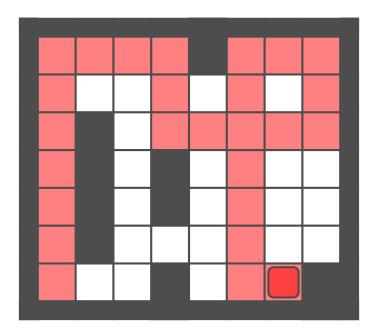


Figure 1: Starting from the lower right corner, the board can be filled using the following 25 moves: $\underbrace{\uparrow \rightarrow \downarrow \rightarrow \uparrow \leftarrow \downarrow \rightarrow}_{\downarrow \downarrow \downarrow \downarrow} \uparrow \downarrow \leftarrow \uparrow \leftarrow \rightarrow \downarrow \leftarrow \uparrow \downarrow \leftarrow$

Question. How many solvable puzzles exist on an $n \times m$ board?

Related.

- 1. What if we only want to count "primitive" puzzles—those that cannot exist on a smaller board?
- 2. What if we count up to symmetries of the rectangle?
- 3. Which puzzle requires the greatest number of moves?
- 4. What if we do this on a torus? Möbius strip? More dimensions?
- 5. Given some configuration, what is an algorithm to figure out how to solve it?