

**Difficulty:** 1/4    **Interest:** 2/4

The number of ways to draw a triangle on a triangular grid is given by

$$\sum_{k=1}^{n-1} k \cdot t(n-k) = \binom{n+2}{4}$$

where  $t(m)$  is the  $m$ -th triangular number.

The number of ways to draw a square on a square grid is given by

$$\sum_{k=1}^{n-1} k \cdot (n-k)^2 = n^2 \left( \frac{n^2-1}{12} \right)$$

the 4-dimensional pyramidal number.

The number of ways to draw a hexagon on a hexagonal grid is given by

$$\sum_{k=1}^{n-1} k \cdot h(n-k) = \sum_{k=1}^{n-1} k^3 = t(n-1)^2$$

where  $h(m)$  is the  $m$ -th hexagonal number.

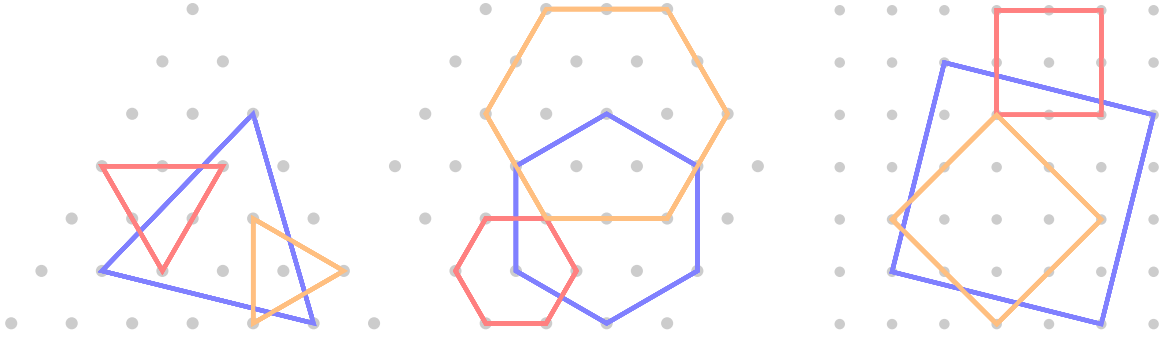


Figure 1: All connected configurations of 4 coins. Six out of the seven possible polyhexes are present.

**Question.** Is there a combinatorial explanation for why these numbers have nice closed forms?

**Related.**

1. Can this be generalized to arbitrary regular  $n$ -gons in hyperbolic space?
2. How many triangles are on the “centered triangular number” grid?

**References.**

Problem 25.

[https://en.wikipedia.org/wiki/Order-4\\_pentagonal\\_tiling](https://en.wikipedia.org/wiki/Order-4_pentagonal_tiling)