A relation on a group X is a subset $S \subseteq X \times X$. For a given relation if $(x, y) \in S$, then we say that x is related to y and denote it by xRy.
A relation is called "antitransitive" if $(x, y), (y, z) \in S$ implies $(x, z) \notin S$.

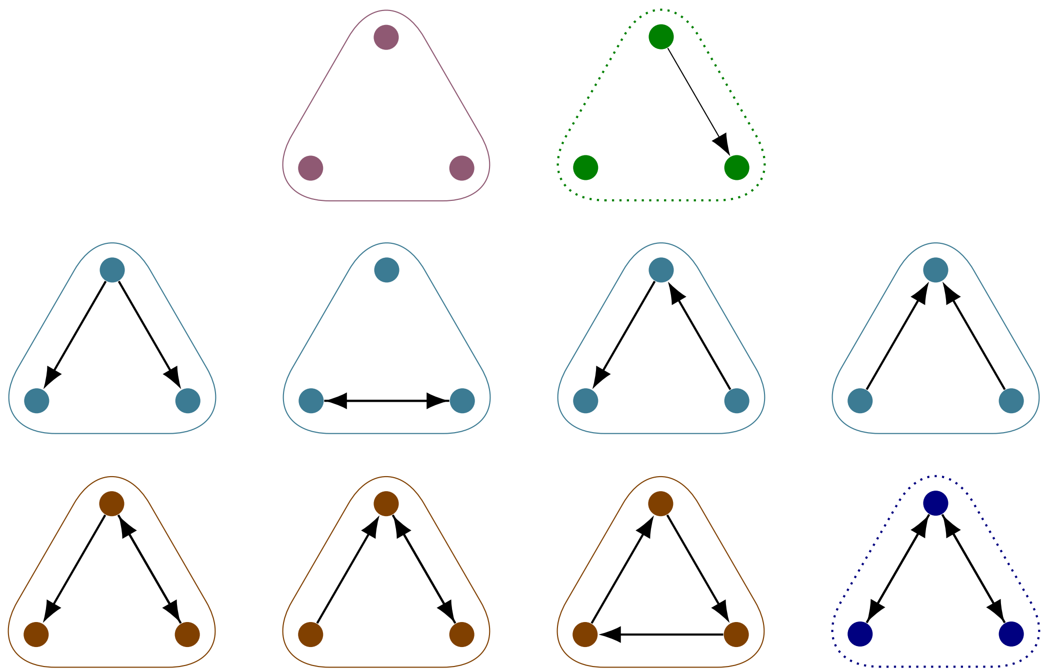

Figure 1: The ten antitransitive relations on 3 unlabeled nodes. There are 1, 1, 4, 3, and 1 relations with 0, 1, 2, 3, and 4 pairs respectively.

Question. What is the asymptotic growth of the number of antitransitive relations as a function of the number of (unlabeled) nodes?

Related.

1. On n labeled nodes?
2. Given some subset of conditions (e.g. reflexive, asymmetric, antitransitive, connex, etc.), what is the asymptotic growth?
3. What's the ratio of the number of, say, transitive relations to antitransitive relations as $n \to \infty$.
4. How many relations with exactly k pairs?
5. What's the greatest number of pairs?
6. With ℓ (strongly) connected components?

References.

Problem 39.

OEIS sequences A341471 and A341473.