Difficulty: 2/4   Interest: 2/4

Starting with an $n \times m$ grid, remove one corner at a time (uniformly at random) until the grid is gone.
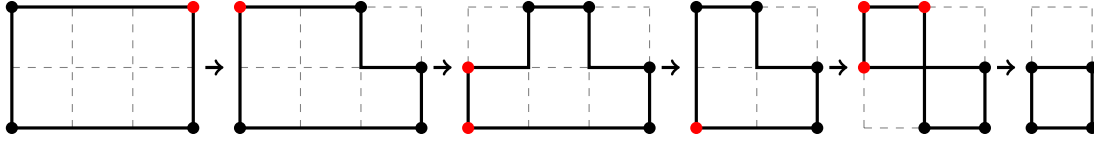

Figure 1:   An example of a process starting with a 2×3 grid.

Question. If a stopping point is chosen randomly, how many corners are expected?

Related.

1. What if the deletion is uniform with respect to faces instead of vertices?

2. How many sides are expected?

3. If all polygons in the process are considered, what is the expected number of corners on the polygon with the greatest number of corners?

4. What figure produces the greatest number of corners?

5. How many possible processes exist (up to, say, dihedral action)?

6. What if each figure must stay path connected?

7. What if paths cannot travel through corners? (e.g. the second-to-last figure is illegal.)