

Difficulty: 4/4 **Interest:** 2/4

Consider partitions of the $n \times m$ grid in which every piece has 180° rotational symmetry.

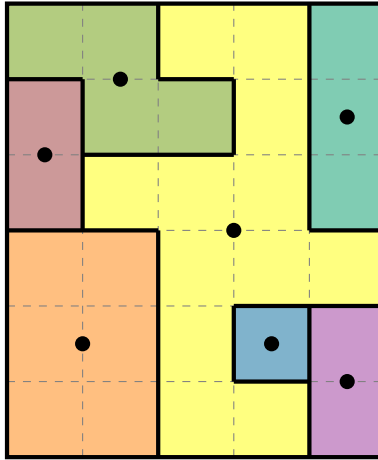


Figure 1: A partition of the 5×6 grid into 7 parts with rotational symmetry.

Question. How many such partitions of the $n \times n$ grid exist? Up to dihedral action?

Related.

1. How many partitions into exactly k parts?
2. How many partitions with other types of symmetry?
3. How many partitions of a torus? Cylinder? Möbius strip?
4. How many partitions of a triangular or hexagonal lattice?
5. How many partitions of an $n \times m \times p$ cuboid?

References.

<https://www.chiark.greenend.org.uk/~sgtatham/puzzles/js/galaxies.html>