

Consider the two-player game on a n-letter string on the alphabet $\{a,b\}$ where players take turns removing palindromic substrings of their choosing.

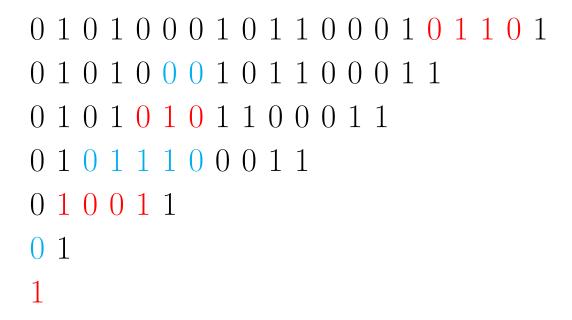


Figure 1: A seven move game. Player 1 erases red strings and Player 2 erases cyan strings. In this game, Player 1 won.

Question. How many n-letter games does Player 1 have a winning strategy?

Related.

- 1. What is a winning strategy?
- 2. If the game is chosen uniformly at random, what is the probability that the first player has
- 3. What if players take turns according to the Thue-Morse sequence?
- 4. What if players collect points based on the number of 1s they erase?
- 5. What if this is played on a larger alphabet?
- 6. What if instead of palindromes, players remove AA subwords, ABA subwords, or other patterns?
- 7. In a single-player version of the game, where the goal is to finish in as few moves a possible, which n-letter games require the most moves?

References.

Problem 3.

https://oeis.org/A298475 https://oeis.org/A298481