Given an $n \times n$ grid, consider all convex polygons with grid points as vertices. Let m(n) be the greatest integer k such that there exists a convex k-gon on the $n \times n$ grid.

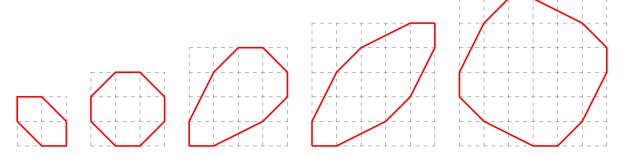


Figure 1: Examples that prove m(3) = 6, m(4) = 8, $m(5) \ge 9$, $m(6) \ge 10$, and $m(7) \ge (12)$

Question. What is m(n)?

Related.

- 1. What is a proof (or counterexample) that the examples shown are the best possible?
- 2. How does m(n) grow asymptotically?
- 3. Do the shapes do anything interesting in the limit?
- 4. Are there finitely many maximal polygons without rotational symmetry (e.g. m(5))?
- 5. How does this generalize to $m \times n$ grids?

References.

Problem 6.

Problem 7.