Given an $n \times n$ grid, consider all convex polygons with grid points as vertices. Let $m(n)$ be the greatest integer k such that there exists a convex k-gon on the $n \times n$ grid.

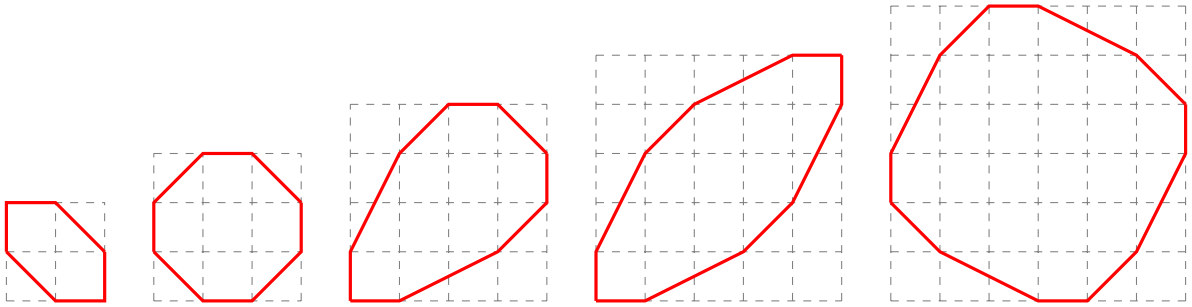

Figure 1: Examples that prove $m(3) = 6, m(4) = 8, m(5) \geq 9, m(6) \geq 10$, and $m(7) \geq (12)$

Question. What is $m(n)$?

Related.

1. What is a proof (or counterexample) that the examples shown are the best possible?
2. How does $m(n)$ grow asymptotically?
3. Do the shapes do anything interesting in the limit?
4. Are there finitely many maximal polygons without rotational symmetry (e.g. $m(5)$)?
5. How does this generalize to $m \times n$ grids?
6. See Problems 6 and 7.