Consider ways to partition the $n \times m$ grid so that no three tiles of the same partition fall on a line.

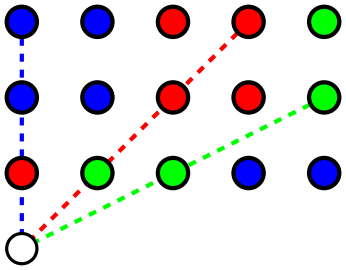

Figure 1: A 3 partition of the 5×3 grid. The white circle cannot be in any of the existing partitions, otherwise three circles of the same color would fall on the same line.

Question. How many colors are required to satisfy the "no three in a row" criterion?

Related.

1. What if this is generalized to k in a row?
2. What if this is generalized to a triangular or hexagonal grid?
3. What if this is generalized to a torus or cylinder or Möbius strip?
4. What if this only queen moves or rook moves are considered?

References.