There is a well known magic trick called "Communicating the Card" in which a spectator draws $k$ cards from an $n$-card deck and shows them to the magician's assistant. He then shows $k-1$ of them to the magician in a particular order, after which she (the magician) can deduce the remaining card. In this variation, the largest possible deck is $k! + k - 1$ cards.

| | | | |
|---|---|---|---|
| $f(1,2) = \{1,2,3\}$ | $f(4,8) = \{1,4,8\}$ | $f(7,2) = \{2,4,7\}$ | $f(8,3) = \{3,5,8\}$ |
| $f(2,1) = \{1,2,4\}$ | $f(5,1) = \{1,5,6\}$ | $f(8,2) = \{2,4,8\}$ | $f(3,6) = \{3,6,7\}$ |
| $f(1,5) = \{1,2,5\}$ | $f(5,7) = \{1,5,7\}$ | $f(2,5) = \{2,5,6\}$ | $f(6,3) = \{3,6,8\}$ |
| $f(1,6) = \{1,2,6\}$ | $f(5,8) = \{1,5,8\}$ | $f(5,2) = \{2,5,7\}$ | $f(7,3) = \{3,7,8\}$ |
| $f(1,7) = \{1,2,7\}$ | $f(6,7) = \{1,6,7\}$ | $f(8,5) = \{2,5,8\}$ | $f(4,5) = \{4,5,6\}$ |
| $f(1,8) = \{1,2,8\}$ | $f(6,8) = \{1,6,8\}$ | $f(6,2) = \{2,6,7\}$ | $f(5,4) = \{4,5,7\}$ |
| $f(1,3) = \{1,3,4\}$ | $f(7,8) = \{1,7,8\}$ | $f(8,6) = \{2,6,8\}$ | $f(8,4) = \{4,5,8\}$ |
| $f(3,1) = \{1,3,5\}$ | $f(2,3) = \{2,3,4\}$ | $f(8,7) = \{2,7,8\}$ | $f(4,6) = \{4,6,7\}$ |
| $f(6,1) = \{1,3,6\}$ | $f(3,2) = \{2,3,5\}$ | $f(3,4) = \{3,4,5\}$ | $f(6,4) = \{4,6,8\}$ |
| $f(7,1) = \{1,3,7\}$ | $f(2,6) = \{2,3,6\}$ | $f(4,3) = \{3,4,6\}$ | $f(7,4) = \{4,7,8\}$ |
| $f(8,1) = \{1,3,8\}$ | $f(2,7) = \{2,3,7\}$ | $f(3,7) = \{3,4,7\}$ | $f(5,6) = \{5,6,7\}$ |
| $f(1,4) = \{1,4,5\}$ | $f(2,8) = \{2,3,8\}$ | $f(3,8) = \{3,4,8\}$ | $f(6,5) = \{5,6,8\}$ |
| $f(4,1) = \{1,4,6\}$ | $f(2,4) = \{2,4,5\}$ | $f(3,5) = \{3,5,6\}$ | $f(7,5) = \{5,7,8\}$ |
| $f(4,7) = \{1,4,7\}$ | $f(4,2) = \{2,4,6\}$ | $f(5,3) = \{3,5,7\}$ | $f(7,6) = \{6,7,8\}$ |

Figure 1: An example of an encoding where $k = 3$ and $n = k! + k - 1 = 8$.

**Question.** What if the assistant can show any number of cards less than $k$, and the magician must guess all of the remaining cards?

**Related.**

1. How many different encodings exist (up to relabeling)?
2. What if the magician just needs to guess one of the remaining cards?
3. What if there are $\ell$ identical copies of a deck, how many cards can the original trick support?
4. If the assistant shows $k-2$ cards to the magician, what is the biggest deck that this trick can be done with? $k - j$?

**References.**


http://oeis.org/A030495

https://www.reddit.com/r/math/comments/7l1t84/a_combinatorists_card_trick/

https://web.northeastern.edu/seigen/11Magic/Articles/Best%20Card%20Trick.pdf