

Let $f_{n,m}:[n]\to[m]$ be a uniformly random function. Consider the convex hull around $\{(1,f(1)),\ldots(n,f(n))\}$

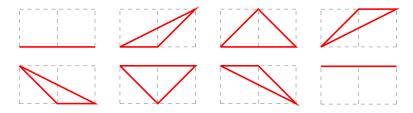


Figure 1: Examples of $f_{3,2}$. Here the expected number of sides on a convex hull is 2.75

Question. What is the probability of seeing a k-gon (for some fixed k), when given a uniformly random function $f_{n,m}$?

Related.

- 1. What value of k has the highest probability?
- 2. What is the expected value of the number of sides?
- 3. What if $f_{n,n}$ is restricted to be a permutation?
- 4. What if $f_{n,m}$ is injective?