In the "nine dots puzzle" or "thinking outside the box puzzle", a player is asked to connect dots arranged in a 3×3 grid using four lines. This can be generalized to connecting the dots of a $n \times n$ grid with $2n-2$ lines.

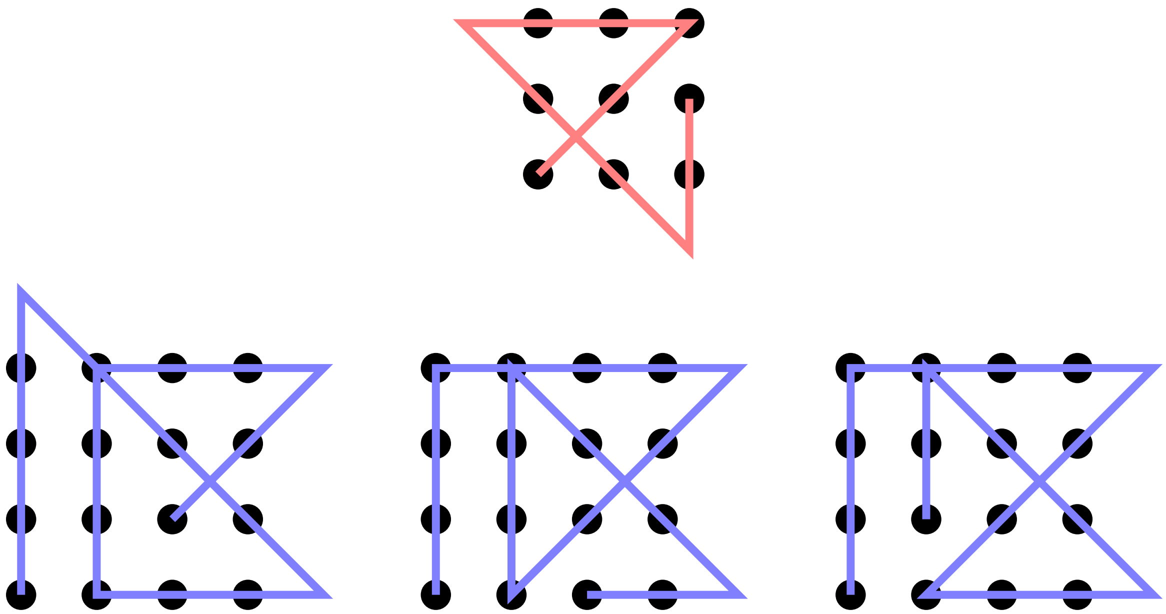

Figure 1: The unique (?) way of completing the 3×3 grid, and three distinct ways of completing the 4×4 grid.

Question. How many distinct solutions exist on the $n \times n$ grid?

Related.

1. What if you want to minimize the area "outside" of the grid?
2. What if you must start and end from the same point?
3. What if you want to minimize the path length?
4. Do any of these have lines that aren't horizontal, vertical, or 45° diagonal?
5. What if this is done on other figures? (Triangles, Diamonds, Octagons, Stars, etc.)
6. Can this be generalized into higher dimensions with lines? Planes?
7. What if the "pencil" can be lifted $k \geq 1$ times?
8. What if this is done on a torus or cylinder?

References.

`https://math.stackexchange.com/q/21851/121988`

`https://en.wikipedia.org/wiki/Thinking_outside_the_box#Nine_dots_puzzle`

`https://en.wikipedia.org/wiki/Figurate_number`