Consider ways of nesting centered squares in the $n \times n$ square lattice.

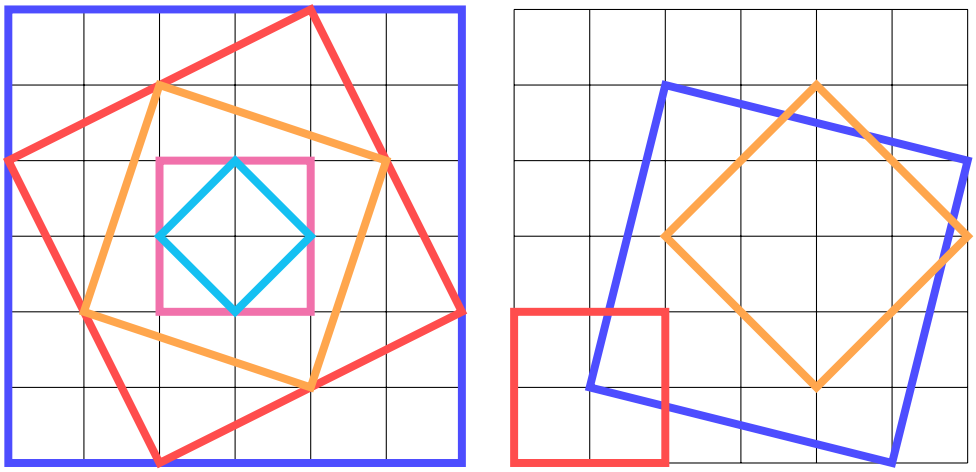

Figure 1: An example of five non-parallel centered squares in the size 6 square, and an example of three non-parallel non-centered squares that do not share any lattice points.

Question. What is the largest number of squares that can be nested?

Related.

1. What is the maximum sum of the areas? Perimeters?
2. How many maximal configurations exist?
3. What if the nested squares must be "snug", that is, all four of their corners must be on their outer neighbor.
4. What if the nested squares cannot be parallel (i.e every square must be a scaling *and* a rotation of another square)?
5. What if they don't need to be centered, but instead no two squares can share a lattice point? If no two squares can be parallel?
6. What if this is done on a triangular or hexagonal lattice?

References.

Problems 21 and 66.