According to Peter Winkler's problem "Red and Blue Dice" in *Mathematical Mind-Benders*, given two sequences of length n with letters in $[n]$, there must always exist a (nonempty) subsequence in each such that the sum of each of the subsequences are equal. Furthermore, there must exist a *substring* (i.e. contiguous subset) in both such that the sum of each substring is equal.

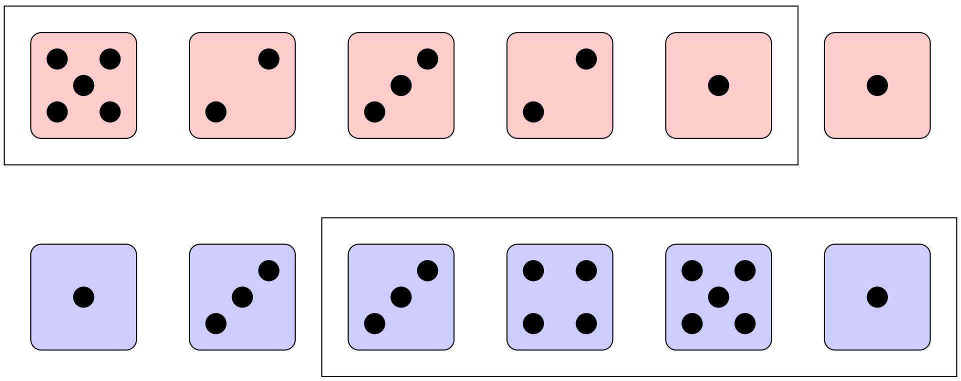

Figure 1: The first five dice in the top row have the same sum as the last four dice in the bottom row.

Question. How many equal-sum subsequences (respectively substrings) are guaranteed to exist?

Related.

1. What if the subsequence or substring must have length greater than or equal to k?
2. What if the subsequences or substrings must be of equal length?
3. What if the substring is allowed to wrap around?
4. What if this is done over permutations S_n instead of subsets $[n]^n$?
5. What if the sequences are of length $\ell < n$, and must be injections from $\mathbb{N}$ to $[n]$?
6. What if there are three sequences? m sequences?
7. Given two random sequences, what is the expected number of equal-sum subsequences?

References.

https://math.stackexchange.com/q/3035452/121988