From Alec Jones. Let $a_k(n)$ count the number of $k$-gons with vertices on the $n \times n$ grid.
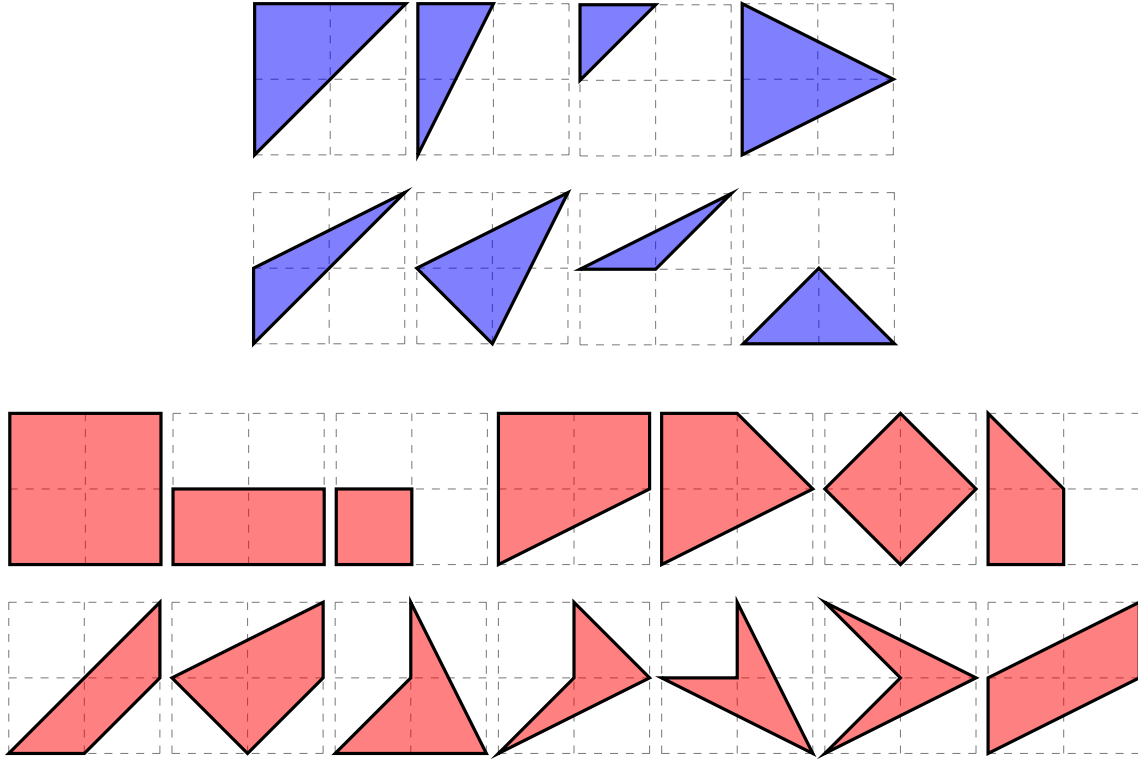


Figure 1: An example showing that $a_3(2) \geq 8$ and $a_4(2) \geq 14$.

**Question.** What is $a_k(n)$?

**Related.**

1. For a fixed $n$, what is the value of $k$ such that $a_k(n)$ is maximized?

2. Here two polygons are considered equivalent if they are congruent. What if two polygons are considered equivalent if they are similar? If they are the same under dihedral action? If they are the same over linear transformation? (e.g. stretching/skewing)

3. What if concave polygons are excluded?

4. What if this is done on an $n \times m$ grid?

5. What if we don't deduplicate based on congruence?

6. What if this is done on a hypercube or a triangular grid?