The Erdős distinct distance problem asks for the number of distinct distances determined by $n$ points in the Euclidean plane.

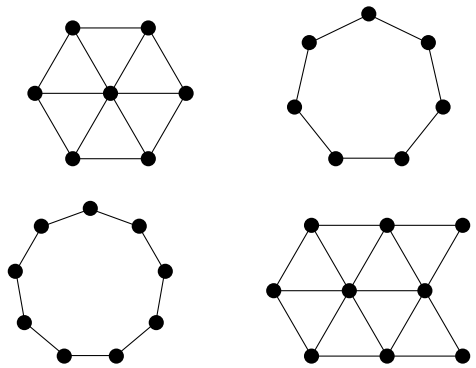

Figure 1: Sets with 3 and 4 distinct distances on 7 and 9 vertices respectively. There are no larger sets with an equal or smaller number of vertices.

**Question.** If $n$ points are constrained to the grid $\mathbb{Z}^2$, what is the minimal number of distinct distances?

**Related.**

1. How many such figures?
2. What if the figures are constrained to some subset of $\mathbb{Z}^2$, e.g. $[n] \times [n]$?
3. What about on other grids (triangular, hexagonal, etc)?
4. Can this be meaningfully done on more exotic topologies, e.g. $Z_n^2$?
5. What about $\mathbb{Z}^n$ for $n > 2$? What is the asymptotic behavior?
6. What if distance is measured via $d_1$, $d_3$, or $d_\infty$?

**References.**

Problem 30.

`https://oeis.org/A186704`: Erdős distinct distance problem.