



Consider the two-player game on a n -letter string on the alphabet $\{a, b\}$ where players take turns removing palindromic substrings of their choosing.

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0  1  0  1  0  0  0  1  0  1  1  0  0  0  1  0  1  1  0  1
0  1  0  1  0  0  0  1  0  1  1  0  0  0  1  1
0  1  0  1  0  1  0  1  1  0  0  0  1  1
0  1  0  1  1  1  0  0  0  1  1
0  1  0  0  1  1
0  1
1

```

Figure 1: A seven move game: Player 1 erases red strings and Player 2 erases cyan strings. In this game, Player 1 won.

Question. How many n -letter games does Player 1 have a winning strategy?

Related.

1. What is a winning strategy?
2. If the game is chosen uniformly at random, what is the probability that the first player has
3. What if players take turns according to the Thue-Morse sequence?
4. What if players collect points based on the number of 1s they erase?
5. What if this is played on a larger alphabet?
6. What if instead of palindromes, players remove AA subwords, ABA subwords, or other patterns?
7. In a single-player version of the game, where the goal is to finish in as few moves as possible, which n -letter games require the most moves?

References.

Problem 3.

<https://oeis.org/A298475>

<https://oeis.org/A298481>