



According to Wikipedia

A Johnson solid is a strictly convex polyhedron for which each face is a regular polygon.

Let $\text{Hull}(P)$ denote the convex hull of a polyhedron P . Say that a 1-concave Johnson solid is a polyhedron P with regular polygonal faces (such that no two faces lay the same plane) and with the property that the “concave” part $\text{Hull}(P) - P$ is a connected convex polyhedron.

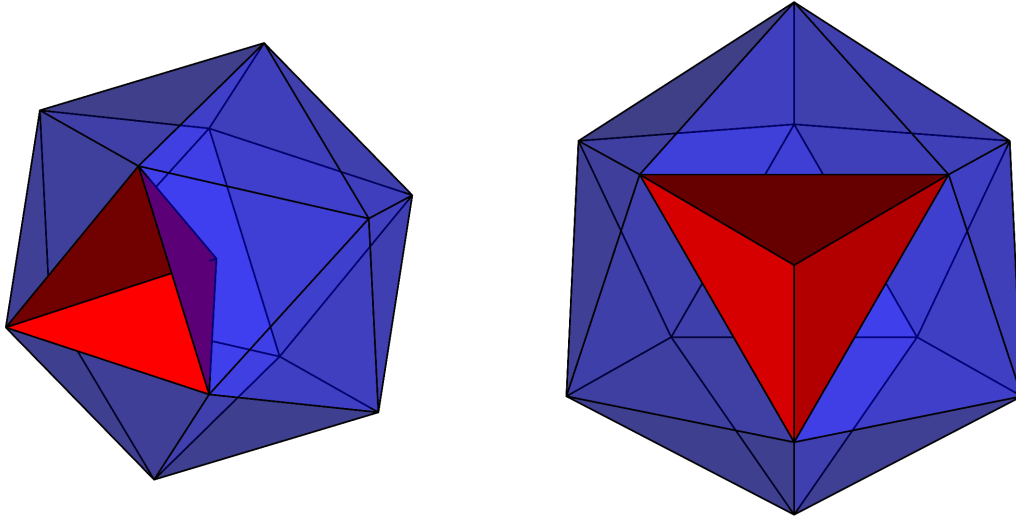


Figure 1: An example of a “1-concave” Johnson solid: an icosahedron with a tetrahedron removed.

Question. Excluding infinite families of 1-concave Johnson solids (e.g. prisms with a square pyramid removed), are there a finite number of 1-concave Johnson solids?

Related.

1. What if the “co-planar” restriction is relaxed so that no two *adjacent* faces may lie in the same plane?
2. How many 1-concave Johnson solids have a convex hull that is a Johnson solid?
3. Say that a 2-concave Johnson solid P is a polyhedron with regular polygonal faces such that $P' = \text{Hull}(P) - P$ is a connected polyhedron, and $\text{Hull}(P') - P'$ is a connected convex polyhedron. Outside of infinite families, are there a finite number of 2-concave Johnson solids? n -concave Johnson solids?
4. Are all n -concave Johnson solids homeomorphic to a sphere?
5. Do there exist k -concave Johnson solids for all k ?

References.

Problem 53.

https://en.wikipedia.org/wiki/Johnson_solid