



Consider partitions of the $n \times m$ grid in which every piece has 180° rotational symmetry.

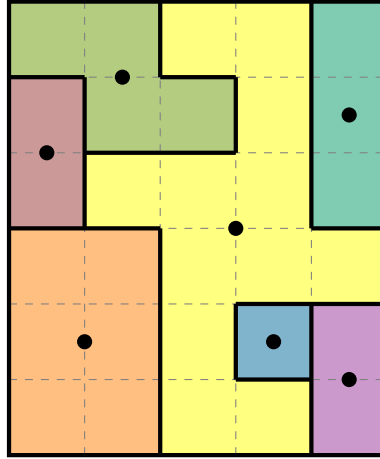


Figure 1: A partition of the 5×6 grid into 7 parts with rotational symmetry.

Question. How many such partitions of the $n \times n$ grid exist? Up to dihedral action?

Related.

1. How many partitions into exactly k parts?
2. How many partitions with other types of symmetry?
3. How many partitions of a torus? Cylinder? Möbius strip?
4. How many partitions of a triangular or hexagonal lattice?
5. How many partitions of an $n \times m \times p$ cuboid?
6. How many placements of centers results in a unique solution? Multiple solutions? No solutions?
7. What if there is the additional restriction that putting together any proper subset of adjacent parts must not exhibit symmetry? (e.g. two adjacent unit squares cannot be colored differently.)
8. What partitions have parts with the greatest average number of sides? (e.g. in the example the average part has $(5(4) + 8 + 16)/7 = 44/7 \approx 6.29$ sides.)
9. What partitions have the smallest ratio of rectangular parts? (e.g. in the example, 2 out of 7 parts are non-rectangular.)
10. What partitions have the greatest number of non-rectangular parts, total? (e.g. in the example, two of the parts are non-rectangular.)

References.

<https://www.chiark.greenend.org.uk/~sgtatham/puzzles/js/galaxies.html>