



Consider a frog hopping on a circular collection of n lily pads. The frog hops to any lily pad, and then hops with increasing steps. At the k -th step, the frog looks k steps in the clockwise direction and k steps in the counterclockwise direction and hops to whatever lily pad she has visited less. If there is a tie, she hops in the clockwise direction.

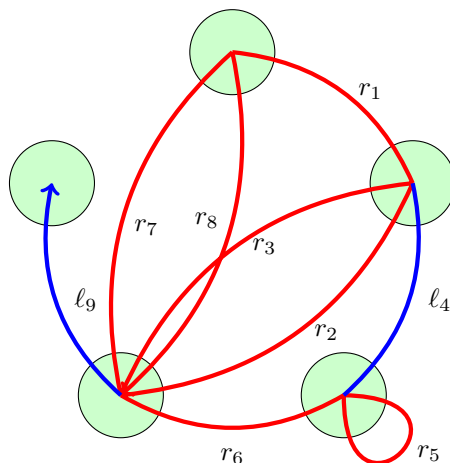


Figure 1: For $n = 5$, all lily pads will have been reached after nine hops.

Question. How many hops does it take to reach all lily pads?

Related.

1. What if ties are broken by hopping in the same direction instead of hopping clockwise?
2. What if instead of hopping with steps $1, 2, 3, \dots$, a different sequence is used?
3. How many positions are reached exactly once?
4. If the hops are in a random direction, what's the expected time to reach every lily pad? What's the expected value of the most-reached lily pad?
5. If you get to choose clockwise or counterclockwise each hop, how many ways are there to reach every lily pad in exactly n hops?

References.

<https://math.stackexchange.com/q/3418970/121988>

<https://oeis.org/A282442>

<https://oeis.org/A329230>