Consider convex polygons with integer coordinates. The notion of a best Diophantine approximation can be generalized to equilateral triangles by saying that a triangle is a better diophantine approximation if the ratio of the largest side to the smallest side is less than the ratio of any other triangle with smaller perimeter.

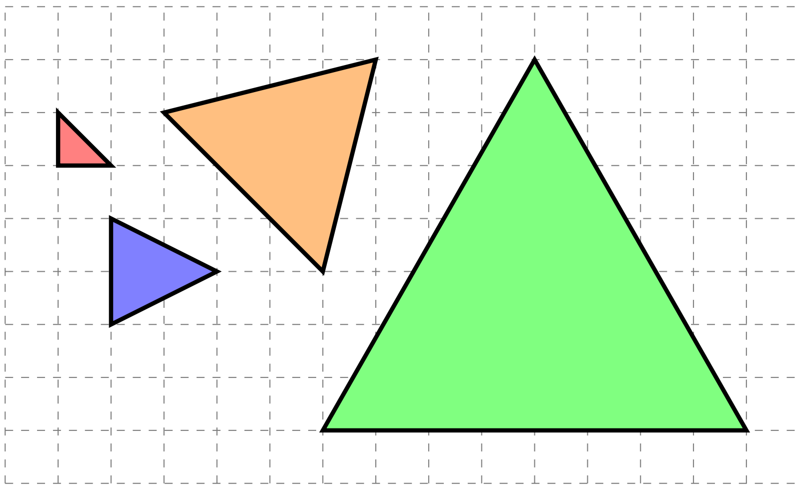

Figure 1: Four best (?) Diophantine approximations of an equilateral triangle. The red triangle has a ratio of $\sqrt{2/1} \approx 1.41$, the blue has a ratio of $\sqrt{5/4} \approx 1.118$, the orange has a ratio of $\sqrt{18/17} \approx 1.029$, and the green has a ratio of $\sqrt{64/63} \approx 1.008$.

**Question.** What is the growth of the perimeter of the $k$-th best Diophantine approximation of an equilateral triangle as a function of $k$?

**Related.**

1. How can this be generalized in a reasonable way to regular $n$-gons? (Just looking at side lengths isn't enough—angles can behave badly.)
2. What if this is done on tetrahedra?

**References.**


`https://math.stackexchange.com/q/2251555/121988`

`https://en.wikipedia.org/wiki/Near-miss_Johnson_solid`

`https://blog.plover.com/math/60-degree-angles.html`

`https://news.ycombinator.com/item?id=18536484`