Say that a *minimally interpolable permutation* f is a permutation of $\{1, 2, \ldots, n\}$ such that no $k+2$ of the points $\{(1, f(1)), \ldots, (n, f(n))\}$ fall on a degree k polynomial.

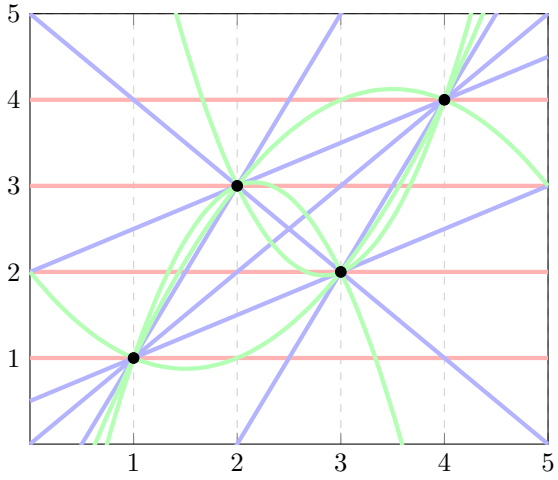

Figure 1: A minimally interpolable permutation of $\{1, 2, 3, 4\}$. (Degree 0 polynomials are plotted in red, degree 1 in blue, and degree 2 in green.)

Question. Do such permutations always exist? If not, what is the least N such that there is a minimally interpolable function from $[n]$ into $[N]$?

Related.

1. How many minimally interpolable permutations exist?
2. Does the number of minimally interpolable permutations increase as a function of n?
3. Is there a method to explicitly construct a minimally interpolable permuation?
4. If such permutations do not always exist, what is the least M such that there exists a subset $S \subset [M]$ and a surjection $g\colon S \to [N]$ with the aforementioned property?

References.

Problem 19.

`https://oeis.org/A301802`

`https://codegolf.stackexchange.com/q/160382/53884`