A "mod-n XOR-triangle" is a Pascal-like inverted triangle, where the first row consists of numbers in $\{0, 1, \ldots, n\}$, and the values in the cells of the subsequent rows are computed as the sum (mod n) of the numbers in the cells above them.

In the case of mod-3 XOR-triangles, if we additionally impose a constraint that the boundary must be rotationally symmetric, some of the resulting triangles appears to have emergent "central circles" reminiscent of the "Arctic circles" that appear in lozenge and domino tilings.

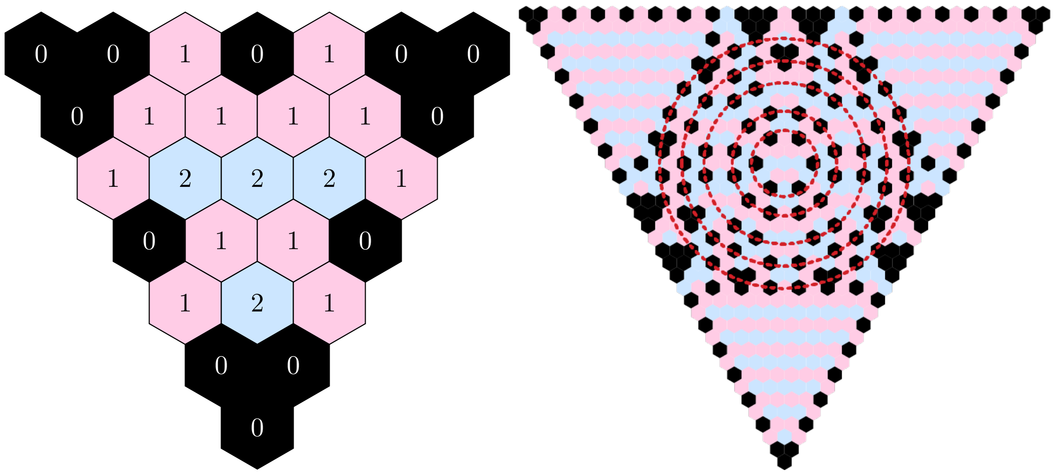

Figure 1: An illustration of the construction of a mod 3 sum triangle, and some candidates for central circles for an example of a triangle with 37 cells per side.

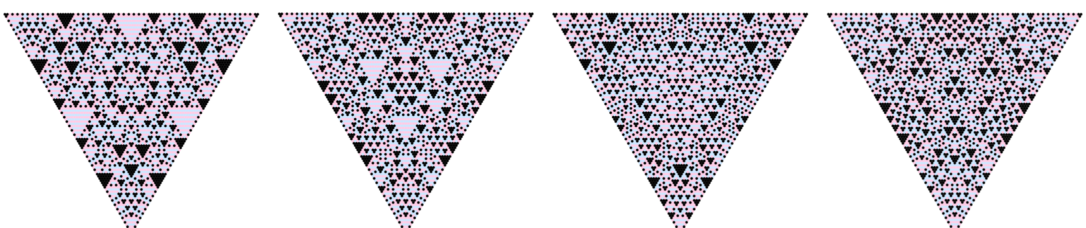

Figure 2: Four more examples of triangles that appear to have central circles

Question. Are these central circles in mod-3 XOR-triangles optical illusions or coincidences? If not, what's a mathematical explanation for why they exist?

Related.

1. What about other moduli?
2. Why does the boundary condition cause the black (0) cells to be symmetric with respect to the dihedral group of the triangle?
3. Can we create meaningful analogs where the shape is a square, tetrahedron, or another shape instead of a triangle?

References.

https://math.stackexchange.com/q/4088671/121988