The Erdős distinct distance problem asks for the number of distinct distances determined by n points in the Euclidean plane.
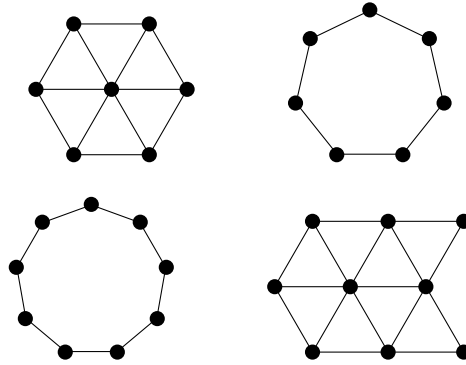

Figure 1: Sets with 3 and 4 distinct distances on 7 and 9 vertices respectively. There are no larger sets with an equal or smaller number of vertices.

Question. If n points are constrained to the grid $\mathbb{Z}^2$, what is the minimal number of distinct distances?

Related.

1. How many such figures?

2. What if the figures are constrained to some subset of $\mathbb{Z}^2$, e.g. $[n] \times [n]$?

3. What about on other grids (triangular, hexagonal, etc)?

4. Can this be meaningfully done on more exotic topologies, e.g. Z_n^2?

5. What about $\mathbb{Z}^n$ for $n > 2$? What is the asymptotic behavior?

6. What if distance is measured via d_1, d_3, or d_∞?

References.

Problem 34.

`https://oeis.org/A186704`: Erdős distinct distance problem.