

OEIS sequence A261865 describes “ $a(n)$  is the least  $k \in \mathbb{N}$  such that some multiple of  $\sqrt{k} \in (n, n+1)$ .” Clearly the asymptotic density of 2 in the sequence is  $1/\sqrt{2}$ .

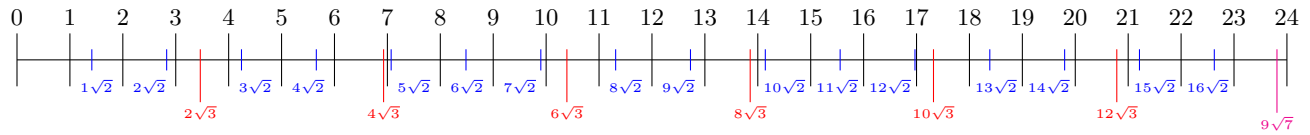


Figure 1: An illustration of  $a(n)$  for  $n \in \{1, 2, \dots, 23\}$ .

**Question.** Let  $S_\alpha \subset \mathbb{N}$  denote the squarefree integers strictly less than  $\alpha$ . Is the asymptotic density of squarefree  $j$  given by

$$\frac{1}{\sqrt{j}} \prod_{s \in S_j} \left(1 - \frac{1}{\sqrt{s}}\right)?$$

**Related.**

1. Is there an algorithm to construct a value of  $n$  such that  $a(n) > K$  for any specified  $K$ ? (Perhaps using best Diophantine approximations or something?)
2. What is the asymptotic growth of the records?
3. Given some  $\alpha$  what is the expected value of the smallest  $n$  such that  $S_\alpha \subset \{a(1), \dots, a(n)\}$ ?
4. This sequence uses the “base sequence” of  $\{\sqrt{1}, \sqrt{2}, \sqrt{3}, \dots\}$ . On what other base sequences is this construction interesting?