



A Johnson solid is a strictly convex polyhedron for which each face is a regular polygon.

Let $\operatorname{Hull}(P)$ denote the convex hull of a polygon P. Say that a 1-concave Johnson solid is a polyhedra P with regular polygonal faces (such that no two faces lay the same plane) and with the property that the "concave" part $\operatorname{Hull}(P) - P$ is a connected convex polyhedron.

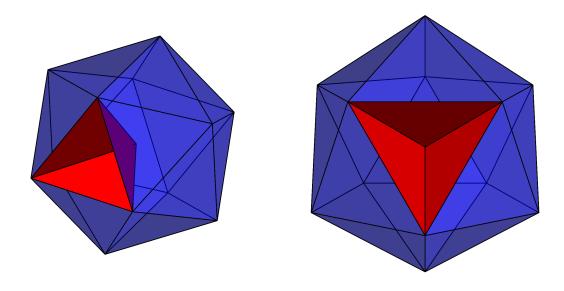


Figure 1: An example of a "1-concave" Johnson solid: an icosahedron with a tetrahedron removed.

Question. Excluding infinite families of 1-concave Johnson solids (e.g. prisms with a square pyramid removed), are there a finite number of 1-concave Johnson solids?

Related.

- 1. What if the "co-planar" restriction is relaxes so that no two adjacent faces may lie in the same plane?
- 2. How many 1-concave Johnson solids have a convex hull that is a Johnson solid?
- 3. Say that a 2-concave Johnson solid P is a polyhedra with regular polygonal faces such that $P' = \operatorname{Hull}(P) P$ is a connected polyhedron, and $\operatorname{Hull}(P') P'$ is a connected convex polyhedron. Outside of infinite families, are there a finite number of 2-concave Johnson solids? n-concave Johnson solids?
- 4. Are all *n*-concave Johnson solids homeomorphic to a sphere?

References.

Problem 53.

https://en.wikipedia.org/wiki/Johnson_solid