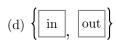


Consider square, triangular, and hexagonal grids that are filled in with tiles of different patterns.











 $(f) \left\{ \bigwedge, \bigwedge, \bigwedge \right\}$

 $(g) \left\{ \bigwedge, \bigwedge, \bigwedge, \bigwedge \right\}$

 $(h) \left\{ \bigwedge, \ \bigwedge, \ \bigwedge, \ \bigwedge \right\}$

 $(i) \left\{ \bigcirc, \bigcirc, \bigcirc, \bigcirc, \bigcirc \right\}$

Figure 1: Ten examples of different tiles.

Question. How many essentially different grids of size n exist with these tiles? (Up to dihedral action? Up to cyclic action?)

Related.

- 1. The square grid can be $n \times n$ or $n \times m$.
- 2. The hexagonal grid can have triangles with side length n or hexagons with side length n.
- 3. The triangular grid can have triangles with side length n or hexagons with side length n.
- 4. The square grid can be quotiented to be a cylinder, torus, or Möbius strip.
- 5. What if shapes have to "match-up" (e.g. the lines in the third example or colors in the last example have to be "smooth".)
- 6. How many distinct regions, as in Problem 2?

References.

Problem 2.

Problem 28.

https://en.wikipedia.org/wiki/Burnside%27s_lemma

https://en.wikipedia.org/wiki/Palago