Consider square, triangular, and hexagonal grids that are filled in with with tiles of different patterns.

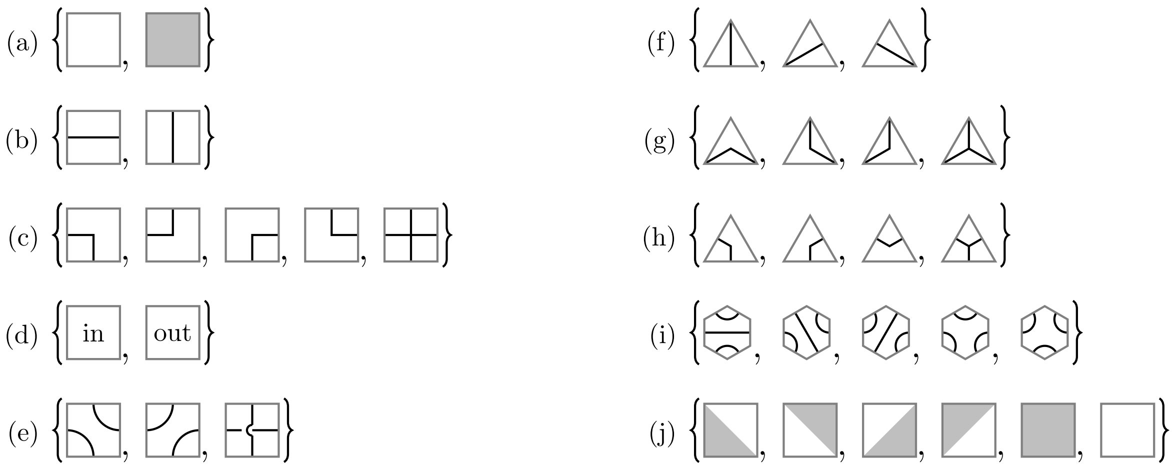

Figure 1: Ten examples of different tiles.

Question. How many essentially different grids of size n exist with these tiles? (Up to dihedral action? Up to cyclic action?)

Related.

1. The square grid can be $n \times n$ or $n \times m$.
2. The hexagonal grid can have triangles with side length n or hexagons with side length n.
3. The triangular grid can have triangles with side length n or hexagons with side length n.
4. The square grid can be quotiented to be a cylinder, torus, or Möbius strip.
5. What if shapes have to “match-up” (e.g. the lines in the third example or colors in the last example have to be “smooth”.)
6. How many distinct regions, as in Problem 2?

References.

Problem 2.

Problem 28.

`https://en.wikipedia.org/wiki/Burnside%27s_lemma`