

In the grid  $\mathbb{Z}^2$ , the only regular polygons that you can draw are squares. In  $\mathbb{Z}^3$ , you can draw equilateral triangles, squares, and regular hexagons, but no other regular polygons.

In  $\mathbb{Z}^3$ , you can also draw regular tetrahedra, cubes, and octahedra(?), but not dodecahedra or icosahedra. (Otherwise you could draw pentagons too!)

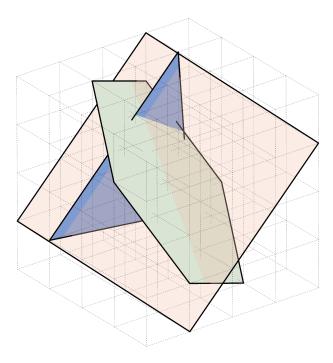


Figure 1: An example of an equilateral triangle, a square, and a regular hexagon drawn with integer coordinates in  $[5]^3$ .

Question. How many regular k-dimensional polytopes can be drawn with vertices in  $[n]^{\ell}$ ?

## Related.

- 1. What is the asymptotic growth of the number of k-dimensional polytopes?
- 2. What if other sorts of polytopes are considered? (E.g. Archimedean solids.)

## References.

Problems 21, 54, 66, 94, 104.

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