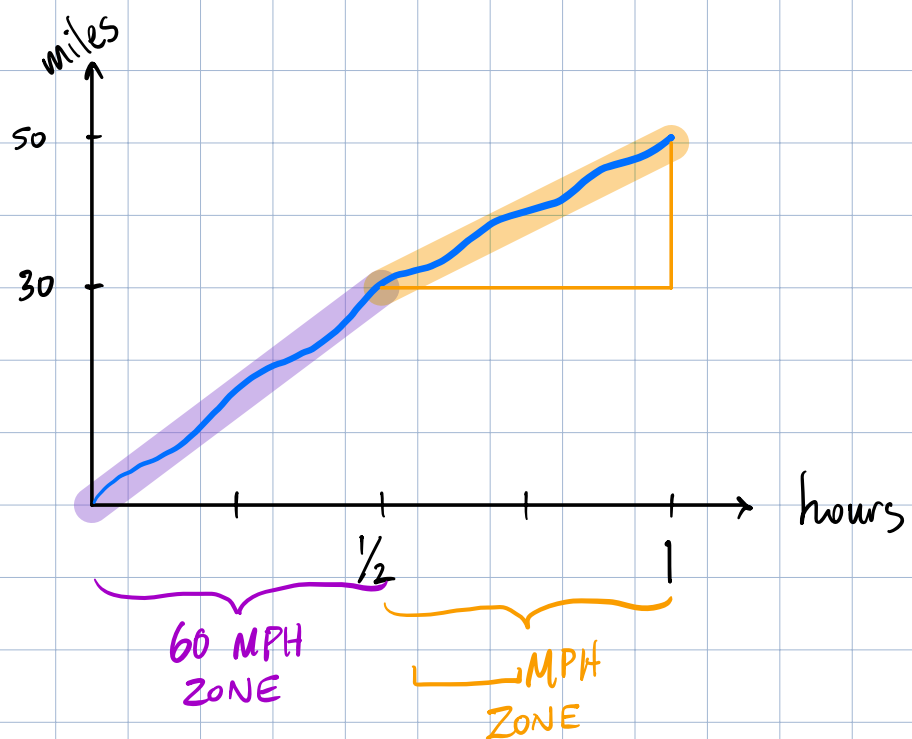
MAT 1140

CALCULUS I
MONDAY, FEB 12, 2024

Position, Velocity, Acceleration
Higher derivatives and
measuring the change of change

WARMUP



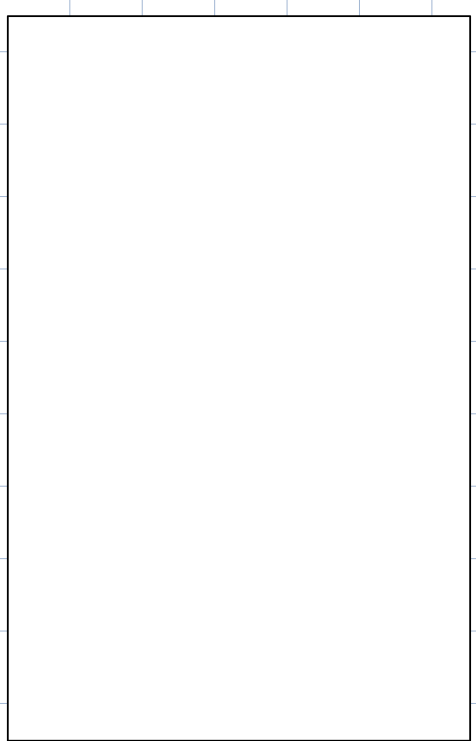
We can take derivatives of derivatives (of derivatives of derivatives...)

If $p(t)$ measures position as a function of time then

- $p'(t)$ measures velocity, sometimes $v(t)$
- $p''(t) = v'(t)$ measures acceleration $a(t)$.
- Trivia:

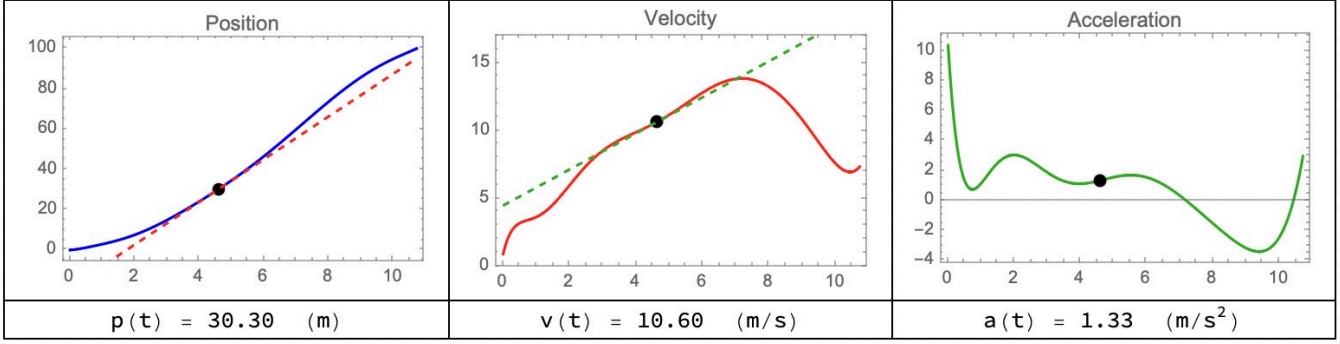
$p'''(t)$ is called "jerk"
$p^{(4)}(t)$ is called
$p^{(5)}(t)$ is called
$p^{(6)}(t)$ is called

units?



A demo! (Sha'Carri Richardson 100m sprint)

t = 4.61 (seconds)



p(t) = 30.30 (m)	v(t) = 10.60 (m/s)	a(t) = 1.33 (m/s^2)

At the 4.61 second mark, Sha'Carri is at

$p(4.61) =$

Running at a pace of

$p'(4.61) = v(4.61) =$

And is she speeding up or slowing down?

$p''(4.61) = v'(4.61) = a(4.61) =$

Example: Group work.

We often model the height of a ball at a given time by

$$p(t) = -5t^2 + 4t + 1$$

where t is given in seconds and $p(t)$ in meters.

1. What is the position (height) of the ball (with units) at times

$$t = 0, \quad t = \frac{1}{2}, \quad t = 1, \quad \text{and} \quad t = 2?$$

2. What is the velocity $p'(t) = v(t)$ of the ball at time t?

3. Specifically what is the velocity (with units) when

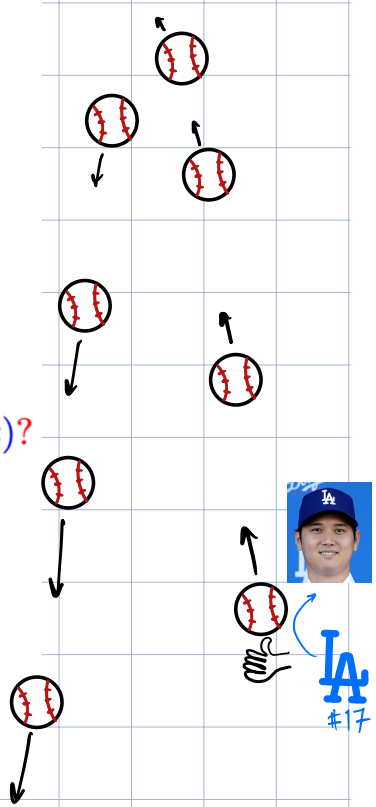
$$t = 0, \quad t = \frac{1}{2}, \quad \text{and} \quad t = 1?$$

4. At what time t does $v(t) = 0$? What does this say about $p(t)$?

5. What is the acceleration $p''(t) = v'(t) = a(t)$ at time t?

6. What is the acceleration (with units) when

$$t = 0, \quad t = \frac{1}{2}, \quad \text{and} \quad t = 1?$$



$$\boxed{p(t) = -5t^2 + 4t + 1}$$

1. What is the position (height) of the ball (with units) at times

$$t = 0, \quad t = \frac{1}{2}, \quad t = 1, \quad \text{and} \quad t = 2?$$

2. What is the velocity $p'(t) = v(t)$ of the ball at time t?

3. Specifically what is the velocity (with units) when

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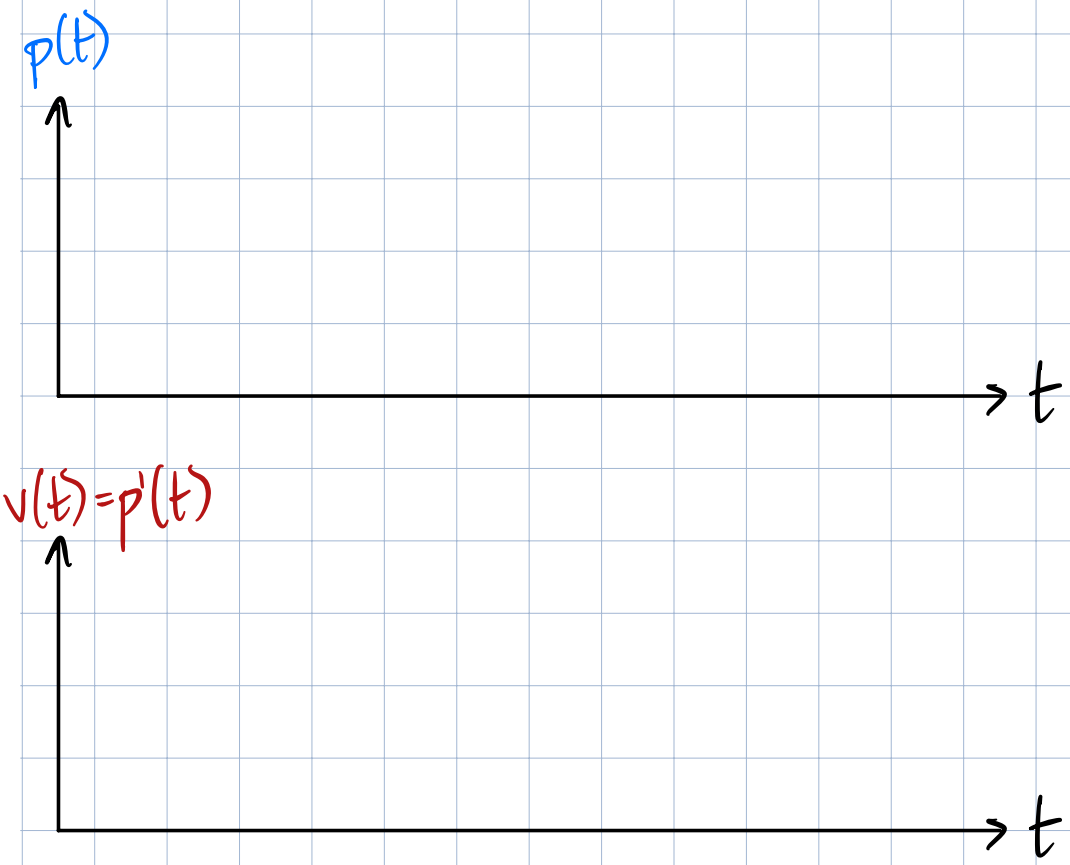
Another demo!



Overview

1. We can take derivatives of derivatives.

2. When $p(t)$ gives the position at a given time,
   - $p'(t) = v(t)$ gives velocity
   - $p''(t) = v'(t) = a(t)$ gives acceleration.

3. When $p'(t) = v(t)$ is positive, the object is moving in the positive direction. (usually up or rightward)

4. When $p''(t) = a(t)$ is positive, $v(t)$ is increasing. (⚠ but may be negative!)

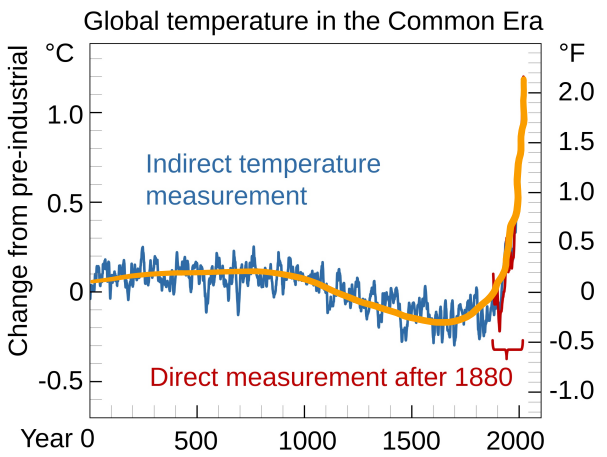
   When $p''(t) = a(t)$ is negative $v(t)$ is decreasing.

Questions to ponder

1. Suppose you go on the Tower of Terror at California Adventure, which ends with a 130 ft drop! If we say that the positive direction is up, is your acceleration at the bottom of the drop positive or negative?

2. The following chart from Wikipedia shows the change in average surface temperatures on earth, $f(t)$.

What does $f'(t)$ measure, and what are the units?
For what values of t is $f'(t) > 0$?



What does $f''(t)$ measure?

Is $f''(2024) > 0$?

Why does $f''(t)$ have to drop down below 0 for the temperatures to stop rising?