Conductors

The electric field in a conductor is 0 since otherwise things would move around. $\Rightarrow \Delta V = 0$ in the conductor since one cannot pick up energy any where from E. The distribution of charge in a conductor is all on the surface since $\nabla \cdot E = \rho/\varepsilon_0$, $\nabla \cdot 0 = 0 = \rho/\varepsilon_0 \Rightarrow \rho = 0$ so in the conductor charge density is 0.

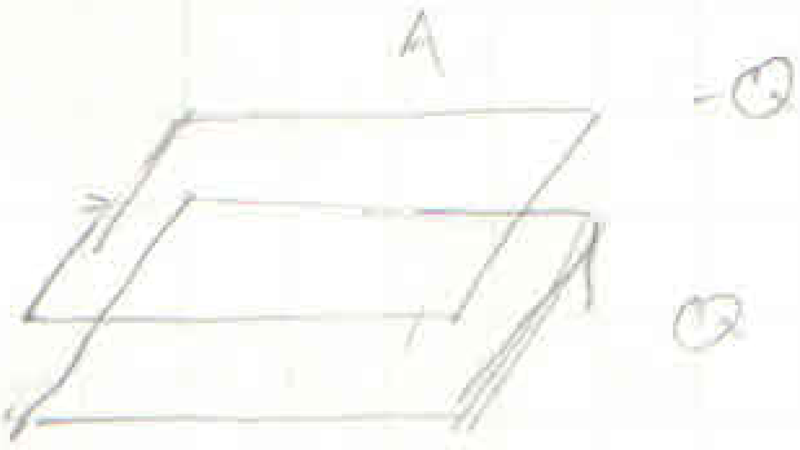
* Insert Van der Graph demo here

How much energy is in each discharge of the generator?

$\Delta x \approx 0.1m \Rightarrow \Delta V = 3e^5\ V$ since $3kV/mm$
$\Rightarrow V = Q/4\pi\varepsilon_0 R$ for spherical shell $\Rightarrow Q \approx 3\mu C$ so

$E = Q\Delta V$ so $E \approx 1J$. This is not dangerous. Now what is the capacitance of a sphere. We know $Q = CV$ so $C = Q/V = 4\pi\varepsilon_0 R$ and $dU = V\,dq = du = Q\,dQ/C$

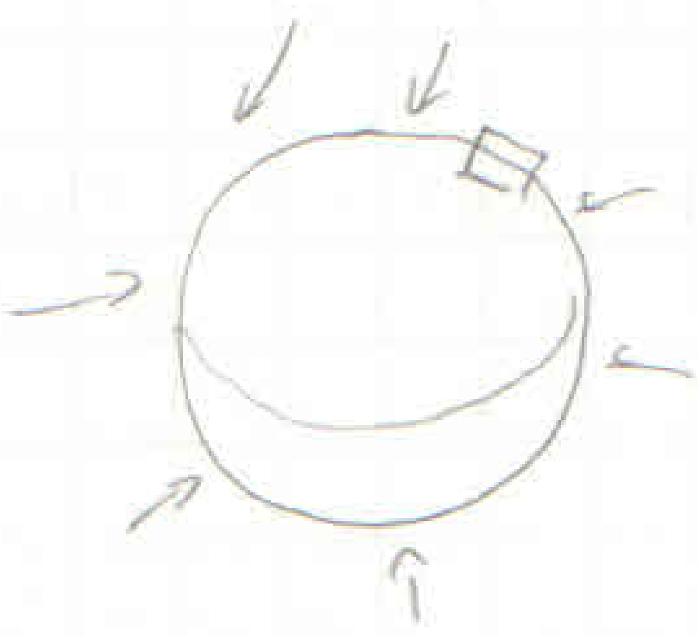
so $U = \int_0^Q \frac{Q\,dQ}{C} = \frac{1}{2}\frac{Q^2}{2C} = \frac{CV^2}{2}$. What about a parallel plate?



$EA = Q/2\varepsilon_0 = \frac{\sigma A}{2\varepsilon_0}$, $\Rightarrow E = \frac{\varepsilon_0 A}{d}$, $E_{inf} = \frac{\sigma}{\varepsilon_0}$

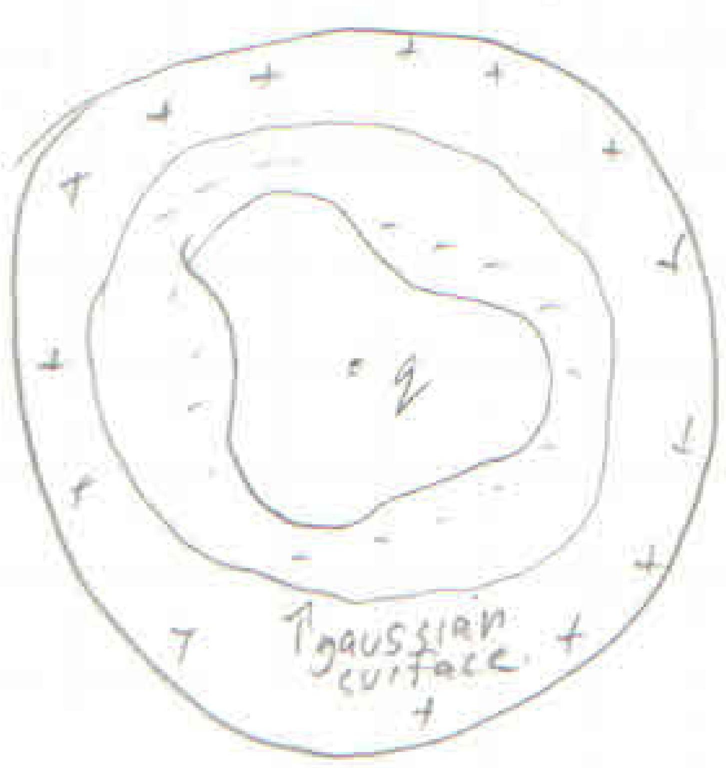
$V_{inf} = \frac{\sigma d}{\varepsilon_0}$ $\quad Q = \sigma A$, so $C = Q/V$ and

$C = \frac{\sigma A}{\sigma d} \cdot \varepsilon_0 = \frac{A\varepsilon_0}{d}$



Since the charge desity of a conductor is 0 by $\nabla \cdot E = \nabla \cdot 0 = 0 = \rho/\varepsilon_0$, we know all charges must be on the surface. But is there a way to always balance the charge on the surface in the presence of an electric field? Yes! One can even get this distribution from the flux and use a Pillbox to get $\varepsilon_0\, \hat{E} \cdot \vec{da} = \sigma$.

What if we make a cavity in a conductor and fill it with a charge $+q$. For the conductor to maintain $\vec{E} = 0$ it must move negative charges to the boundary of the cavity of $-q$. Then by charge conservation the outermost shell of the conductor obtains a charge of $+q$. All of this is implied by taking gaussian surfaces of different size and conditioning E on if it is a conductor. for example a surface in the conductor



$\int \vec{E} \cdot \vec{dA} = \int 0 \cdot \vec{dA} = 0 = q/\varepsilon_0 \Rightarrow q = 0$ so charge on surface.

Say we have a sphere and we charge it, the charges on it repel and create a pressure. From thermal we know $dU = -P\,dV$ and the work required to assemble the sphere of radius r is $U = \frac{1}{2}\frac{Q^2}{4\pi\epsilon_0 r}$ so then

$$dU = \frac{1}{2}\frac{Q^2}{4\pi\epsilon_0}\left(\frac{1}{r+dr} + \frac{1}{r}\right) \quad \text{and} \quad dV = 4\pi r^2 dr$$

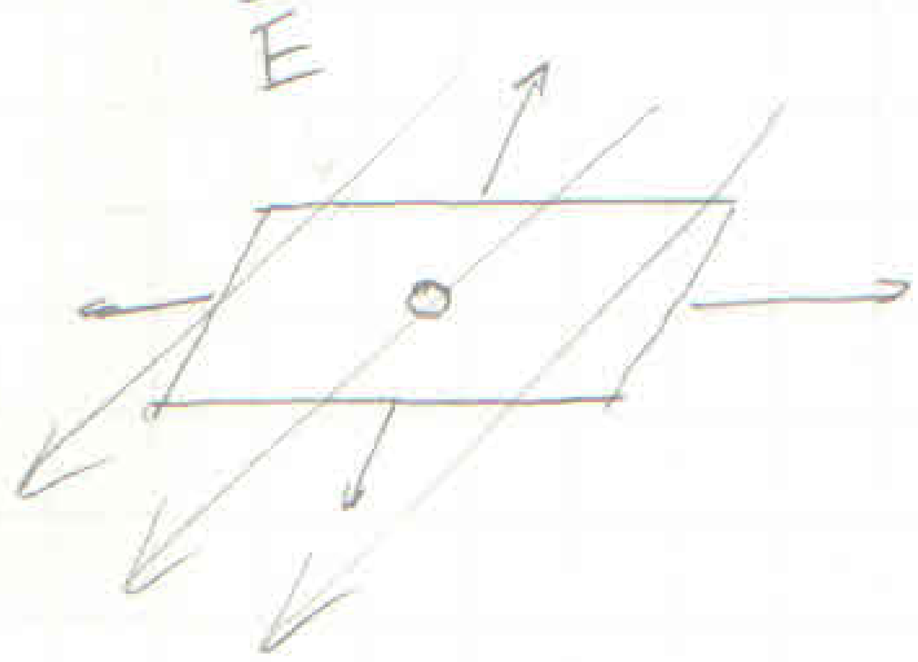
so

$$\frac{1}{2} Q^2 \frac{1}{4\pi\epsilon_0}\frac{dr}{r^2} \approx 4\pi r^2\,dr\,P$$

$$P \approx \frac{1}{2} Q^2/\epsilon_0 (4\pi r^2)^2 = \frac{1}{2}\epsilon_0 E^2$$ which is the energy density of the electric field

thus $$U = \int \frac{1}{2} E^2 \frac{1}{\epsilon_0}\, d^3x$$

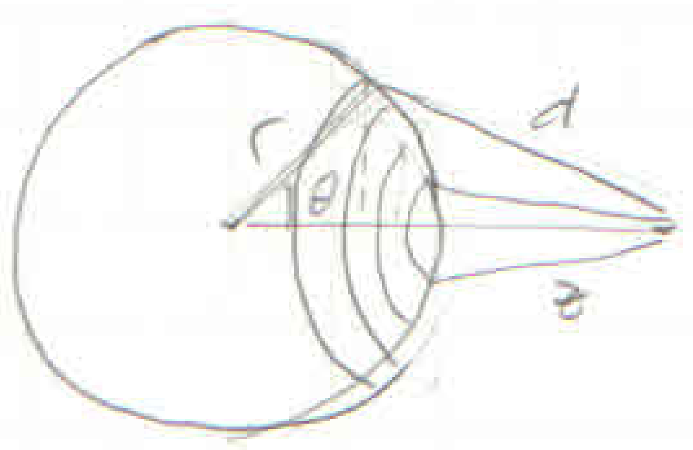
Say we have a conductor in an electric field. What is the force on a small section.



Well the conductor cannot influence itself or it will break conservation of momentum. If we remove our little chunk and freeze the charges there will be $\vec{E}$ above the plane and 0 above it? I'm not really sure what this was! Help.

Let's observe the potential $V(\vec{r}) = \int \frac{\rho(r')}{4\pi\epsilon_0 (r-r')} d^3r'$ is a scalar formula well the potential from a ring is

$$V = \frac{q}{4\pi\epsilon (r^2+z^2)^{1/2}}$$ then a sphere gives



So the surface density of a sphere is σ and $dQ = \sigma 2\pi(r\sin\theta)\cdot r\,d\theta = 2\pi r^2 \sin\theta\,\sigma\,d\theta$, by law of cosines $d^2 = r^2 + z^2 - 2rz\cos\theta$ so

$$V(z) = \int_0^{\pi} \frac{2\pi r^2 \sin\theta\,\sigma\,d\theta}{4\pi\epsilon_0 \sqrt{r^2+z^2-2rz\cos\theta}} \qquad \begin{aligned} u &= r^2+z^2-2rz\cos\theta \\ du &= -2rz\sin\theta \end{aligned}$$

$$= \int_{(r-z)^2}^{(r+z)^2} \frac{Q}{4\pi\epsilon_0}\frac{1}{2rz}\frac{du}{\sqrt{u}} = \frac{Q}{4\pi\epsilon_0}\frac{1}{2rz}\left(|r+z| - |r-z|\right)$$