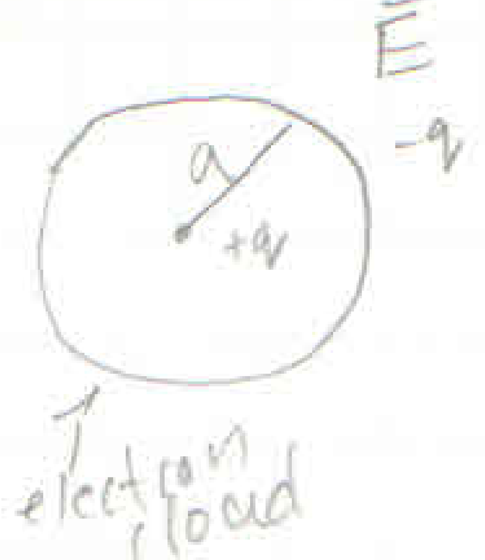


This is an atom in an electric field. We will assume that $\vec{E}$ is constant in the electric field. The electrons will move with the field and the nucleus against.

The force between the nucleus and the electrons is

$\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot z\overset{q}{e} \left(\frac{r}{a}\right)^3 \frac{1}{r^2}$ ← from shell geometry $= \frac{qr}{4\pi\varepsilon_0 a^3}$ so since $V = 4/3\pi r^3$

dipole moment ⇒ $\vec{P} = \vec{E} \cdot 4\pi\varepsilon_0 a^3 = 3V\varepsilon_0$ where V is volume!

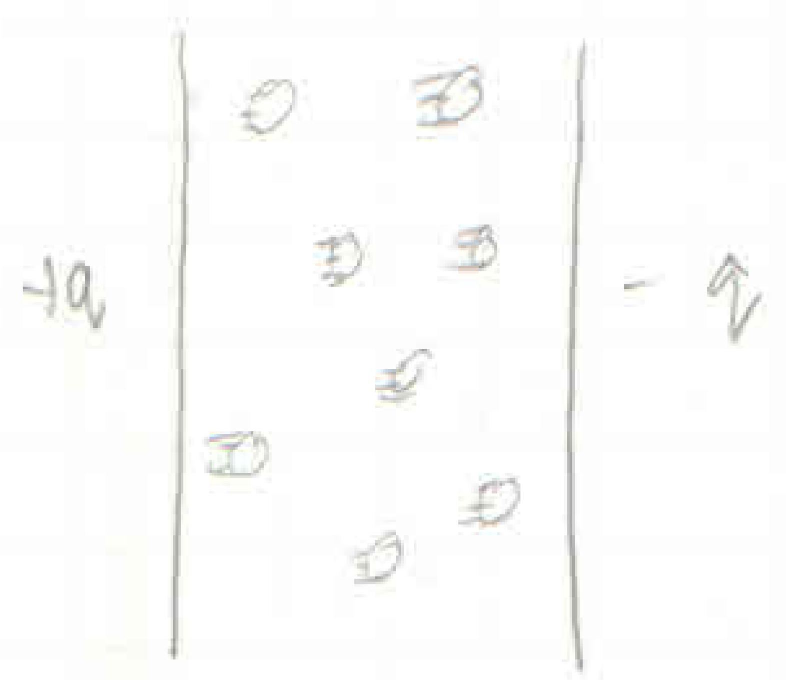
Then can we rip an atom apart? $E_{max} = 3kV/mm = 3e^6\ V/m$

⇒ $r = E_{max}\, 4\pi\varepsilon_0 a^3/q = 3 \cdot 10^{-16}\,m$ but an atom is $\sim 10^{-11}\,m$

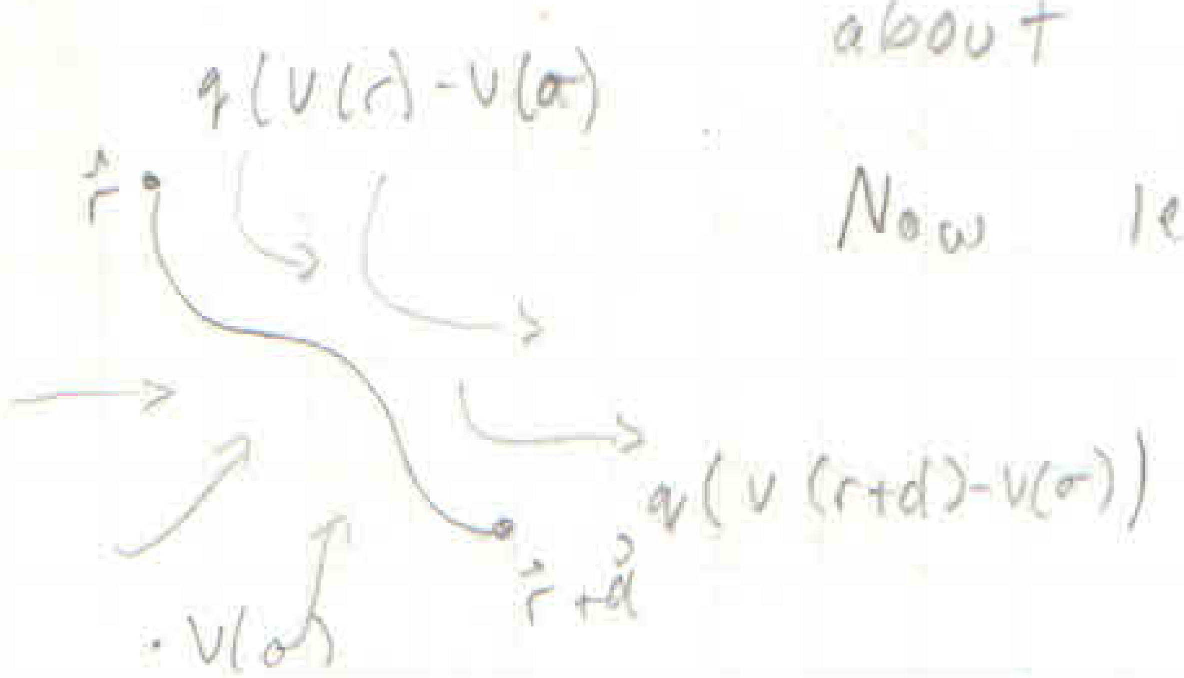
so there is no field strong enough. Instead the nucleus shifts.

Let's thing about a field applied on air?

What actually happens? Well what is air compared to water? It has $\sim 1/1000$ the density ⇒ the mean free path in air is roughly $\sim 1000x$ more. If in water the mean free path is 1 atom diameter then for air it is $1000 \cdot 1\text{atom} \approx 2 \cdot 10^{-7}m$ Or $\sim 1/1000$ of space is filled with air.



here we have a uniform field passing through air and the atoms move to cancel out their internal electric field, This cancelation lowers the ΔU of the gap by an amount proportional to the density of it's dialectric since the atoms will aim for $\Delta U = 0$. Thus we expect for air the change in ΔU is about $1/1000$.



Now let's think of a point charge moving in space. The work to move it is $\approx q\,\vec{\nabla}V \cdot d$ as seen in the diagrams below $= \vec{P} \cdot \vec{\nabla}V = -\vec{P} \cdot \vec{E}$

$\Rightarrow \Delta U = \frac{P \cdot \hat{r}\, q}{4\pi\varepsilon_0 r^2} \Rightarrow V = \frac{P \cdot \hat{r}}{4\pi\varepsilon_0 r^2}$ by division of q

Thus let $\underline{P} = P/Vol$ then we can get

$$V = \frac{1}{4\pi\varepsilon_0} \int_{Vol} \vec{P}_{Vol} \cdot \frac{\hat{r}}{r^2}\, dr^3 = \frac{1}{4\pi\varepsilon_0} \int \vec{P}_{Vol} \cdot \nabla\left(\frac{1}{|r|}\right) dr^3$$ reverse product rule!

$$= \frac{1}{4\pi\varepsilon_0} \int_{Vol} \vec{\nabla} \cdot \left(\frac{\vec{P}}{|r|}\right) - \frac{1}{|r|} \vec{\nabla} \cdot \vec{P}\, d^3r = \frac{1}{4\pi\varepsilon_0} \left[\int_{Surface} \frac{\vec{P}}{|r|} \cdot \vec{da} - \int_{Vol} \frac{1}{|r|} \vec{\nabla} \cdot \vec{P}\, d\vec{v} \right]$$

and such we recover the product rule. Now let's call $\vec{P} \cdot \vec{\hat{n}} = \sigma_{Boundary}$ and also $\vec{\nabla} \cdot \underline{P} = \rho_{Boundary}$ and this become

$$= \frac{1}{4\pi\varepsilon_0} \left[\int_{Surf.} \frac{\sigma_B\, da}{|r|} + \int_{Vol} \frac{\rho_B\, dr^3}{r} \right]$$ now what happens when we add free charges?

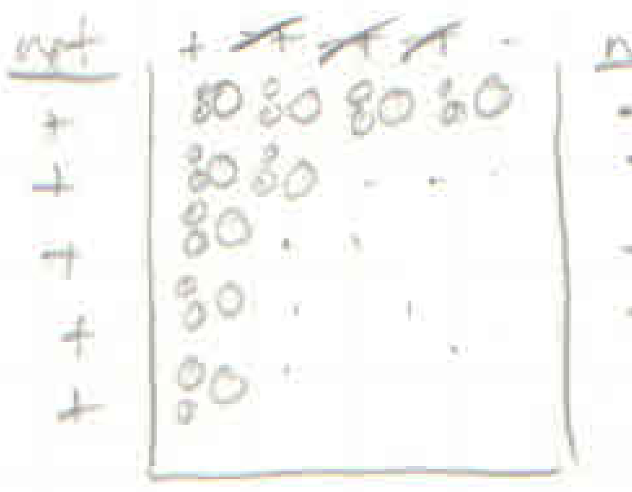
well $\vec{\nabla} \cdot \vec{E} = \rho/\varepsilon_0 \Rightarrow \varepsilon_0 \vec{\nabla} \cdot \vec{E} = \rho = \rho_{free} + \rho_{Boundary} = \rho_f - \vec{\nabla} \cdot \vec{P}$

$= \nabla \cdot (\varepsilon_0 \vec{E} + \vec{P}) = \rho_f$, we call $\vec{D} \equiv \varepsilon_0 \vec{E} + \vec{P}$ "displacement" so $\nabla \cdot (D) = \rho_f$

Now we know the divergence, what about the curl of D? It is not always zero. Sometimes, in materials called dialectrics $\vec{P} = \chi \varepsilon_0 E$ and then $D = (1-\chi)\varepsilon_0 E$ and so χ decreases the effect of ef.
we call χ the dialectric constant. We call $\vec{P}$ polarization

Let's take a look at water, because of how the bonds of the water pull electrons closer to the water, so in reality it looks like a dipole!

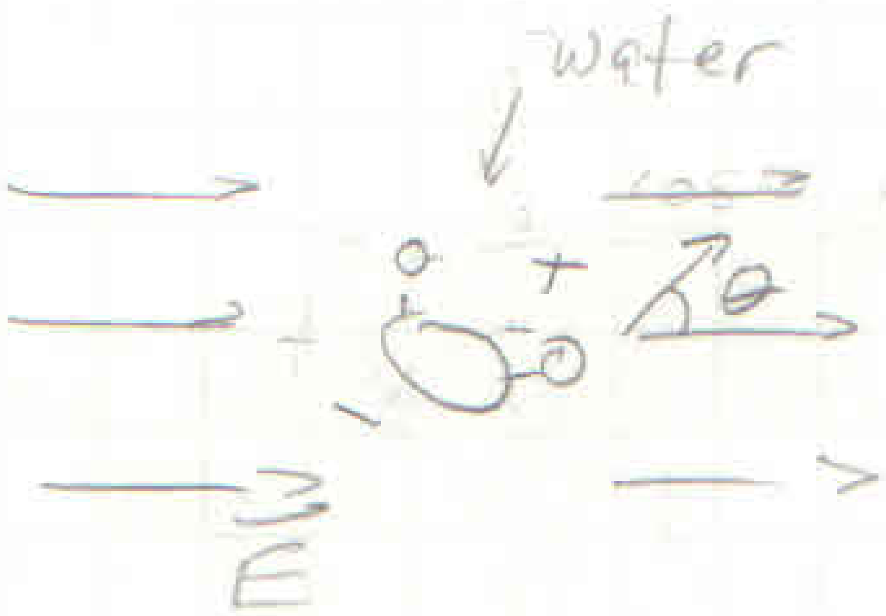
The dipole created has charge of $1.8D \approx 0.4\ e\text{Å}$. This is pretty large compared to what the maximum the electric field could do before breakdown $\sim 10^{-6}$ D. what happens if we stack water with all the dipoles alligned, what is the effect?



All the internal plusses and minuses cancel and the net effect is like that of a plate capacitor. Since we know the density of water we can actually make an estimate and we get huge fields! way bigger than anything air can support.
Why then does this not happen? well the water is at a temperature so then it has random motion? Well we can use the Boltzmann factor to get a probability of an energy state.

$$\frac{P_1}{P_2} = e^{-\Delta U/kT}$$

, so now $U = -\vec{P}\cdot\vec{E}$ in an electric field $U_1 = -PE\cos\theta$ and say we have no dipole moment in P_1 then $U_2 = 0$ so then



$$\frac{P_1}{P_2} = e^{PE\cos\theta/kT}$$

so then $\langle x \rangle = \dfrac{\int p(x)\, x\, dx}{\int p(x)\, dx}$ ← normalisation

So make $x = \cos\theta$, then
$$\langle\cos\theta\rangle = \frac{\int_0^{\pi}\int_0^{2\pi} \cos\theta\, e^{\frac{PE\cos\theta}{kT}}\, d\theta\, \cancel{d\phi}\, \sin\theta}{\int_0^{\pi}\int_0^{2\pi} e^{PE\cos\theta/kT}\, d\theta\, \cancel{d\phi}\, \sin\theta}$$
ϕ cancels

now let $\alpha \equiv PE/kT$ let $u = \cos\theta$, $du = -\sin\theta$

$$\text{so } \frac{P_1}{P_2} = +\int_1^{-1} u\, e^{\alpha u}\, du \Big/ +\int_1^{-1} e^{\alpha u}\, du = \int_{-1}^{1} u e^{\alpha u}\, du \Big/ \int_{-1}^{1} e^{\alpha u}\, du$$

$= \coth(\alpha) - 1/\alpha$ which is the Lengevin function

$\approx \frac{2\alpha}{3}/2 = \alpha/3$ for small alpha taylor expanded in the integral

But what is α gonna be? For water $\alpha = P E_{Breakdown}/kT$ with $P = 1.8D$. Then $\alpha = \dfrac{6\cdot 10^{-30}\cdot 3\cdot 10^{6}}{1.38\cdot 10^{-23}\cdot 298} \approx \dfrac{1}{200}$

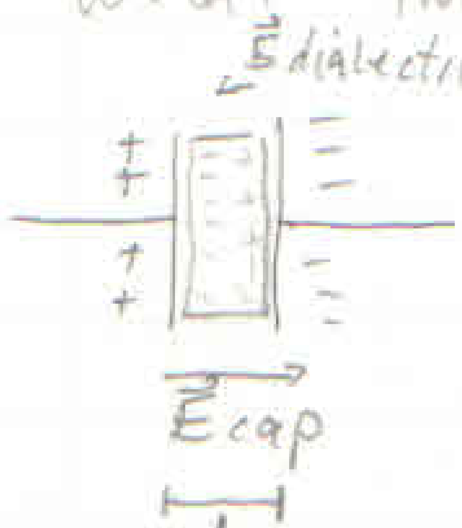
so α is quite small and the approximation should be very accurate for E being large. notice that α represents the ratio of energies.

so since $\langle \cos\theta \rangle = \alpha/3$ and so $\langle P \rangle = P\langle \cos\theta \rangle \approx \frac{P\alpha}{3} = \frac{P^2 E}{kT}$

cool so then let's try this for gasses: Steam!

$\bar{P} = n\bar{p} = \frac{\text{Pressure}}{kT} \frac{p^2 E}{kT} \approx 0.0051$ which is pairly accurate
for liquid water this fails! The micro physics are complicated.

what impact does this have on capacitors with dielectrics?

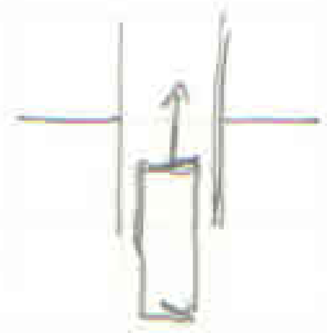


The dialectric creates a field opposing the field of the capacitor. Let's say they have charge density σ then $E_{cap} = \sigma/\varepsilon_0$, without the dialectric
$D = \varepsilon_0 E + 0 = \sigma \Rightarrow \nabla \cdot D = \rho_f$ but since for a large enough capacitor $P // -E$ we have that $\nabla \times D = 0$, which means knowing the divergence uniquely defines D. $\Rightarrow$ $D = \sigma$ even with the the dialectric in. This allows us to solve for the new $\vec{E}$ with the dialectric. Say this is a linear dialectric (i.e. $\vec{P} = \varepsilon_0 \chi \vec{E}$) then

$$D = \varepsilon_0(1+\chi)\vec{E} = \sigma \Rightarrow E = \frac{\sigma}{\varepsilon_0(1+\chi)} < \frac{\sigma}{\varepsilon_0} \;!$$

Then ΔV of the capacitor went down since $\Delta V = d \cdot E$!
Then the capacitance which is $C = Q/V$ goes up.
Spacifically $C_{cap+dia} = (1+\chi) C_{cap}$. So capacitors can store more charge for a given voltage. That means that the total work we can store.

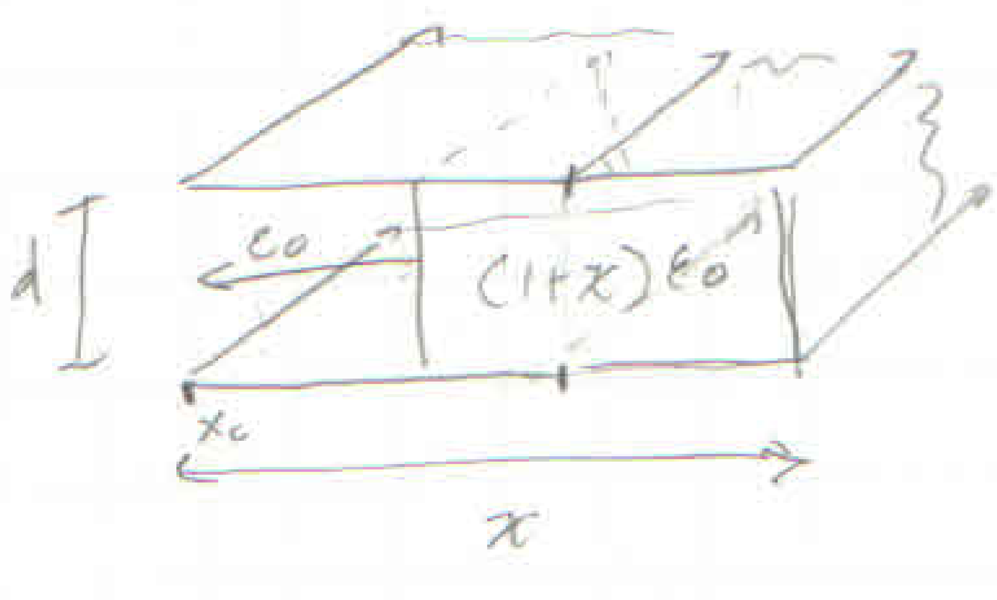
$dW = V dQ = \frac{Q}{C} dQ \Rightarrow W = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2} C V^2$ and since $C\uparrow$ each capacitor can increase its stored energy increases $W' = (1+\chi) W_{capacitor}$. Let's step it up a notch. Say we have a capacitor and we slide in a dialectric. what is the work required to place the dialectric.



① Constant voltage: here $\Delta U = \frac{1}{2} \Delta C \cdot V^2$ it is fairly straightforward but the battery does work

② Constant Q $\quad U = \frac{Q^2}{2C} \Rightarrow dU = -\frac{Q^2}{2C^2} dC = -\frac{V^2}{2} dQ$

so what happens when the dialectric is half in? Does it shove out or in? well $W = F \cdot d \Rightarrow dW = F \cdot \partial d \Rightarrow F = \frac{dW}{\partial d}$



let $a = z \cdot x_0$ then

$$\frac{a}{d} = \frac{\varepsilon_0 z(x_0 - x)}{d} + \frac{\varepsilon_0(1+\chi)(x) z}{d}$$

$$\frac{dC}{dx} = -\frac{\varepsilon z}{d} + \frac{\varepsilon_0(1+\chi) z}{d}$$