So then $\nabla \times \vec{E} = 0 \Rightarrow \oint \vec{E} \cdot d\vec{l} = 0$ So let's examine a general
and $\nabla \cdot \vec{E} = \rho/\varepsilon_0 \Rightarrow \oiint \vec{E} \cdot d\vec{n} = Q/\varepsilon_0$ vector field.

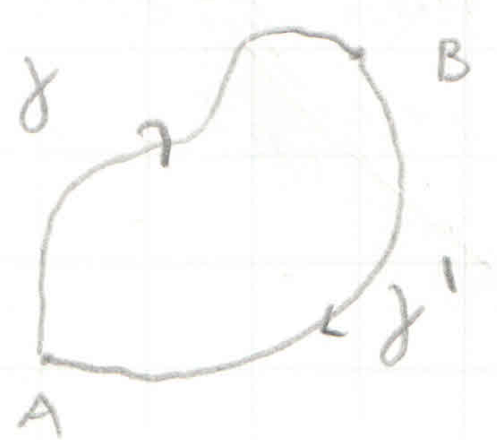
$\vec{F} = F_x(r)\hat{x} + F_y(r)\hat{y} + F_z(r)\hat{z}$ and mathematically nothing is stopping us from choosing anything for these functions. However, we need to follow the constraints above.

$$\nabla \cdot F = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}, \text{ if } = 0$$ we get 2 free parameters

$$\nabla \times F = (F_y^{(z)} - F_z^{(y)})\hat{x} + (F_x^{(z)} - F_z^{(x)})\hat{y} + (F_x^{(y)} - F_y^{(x)})\hat{z} = 0$$
$$\Rightarrow F_y^{(z)} = F_z^{(y)},\quad F_x^{(z)} = F_z^{(x)},\quad F_x^{(y)} = F_y^{(x)}$$
$$\Rightarrow F_y^{(zx)} = F_z^{(yx)},\quad F_x^{(zy)} = F_z^{(xy)},\quad F_x^{(yz)} = F_y^{(xz)}$$
$$\Rightarrow F_y^{(zx)} = F_z^{(xx)} = F_x^{(yz)}$$ which gives us only one free parameter

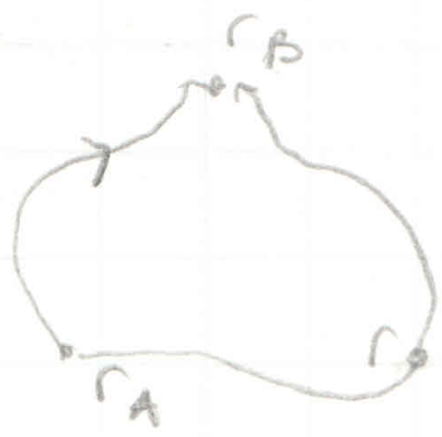
<u>Helmholtz theorem</u>:
If the curl of a vector field is 0 then given only one component of the field, the field can be recovered.

Let's think about this, since $\nabla \times E = 0 \Rightarrow \oint \vec{E} \cdot d\vec{l} = 0$ so then



$\oint_{\gamma+\gamma'} E \cdot dl = 0$ then $\int_\gamma E \cdot dl + \int_{\gamma'} E \cdot dl = 0$ so then
$\int_\gamma \vec{E} \cdot d\vec{s} = \int_{-\gamma'} \vec{E} \cdot d\vec{s}$ so the integral is path independent

That means that stopping at a different point r_0 between r_A and r_B well just add.



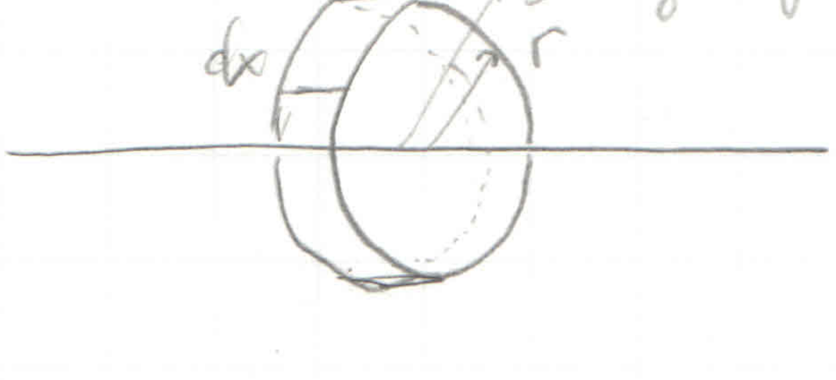
let $\partial v(r) = V(r) - V(r_0)$
$V(r_A) = V(r_0) + \partial v(r_A)$
$V(r_B) = V(r_0) + \partial v(r_B)$ and
$\Delta V_{r_A r_B} = V(r_B) - V(r_A)$

Recall that we must multiply the potential field V by the charge of the particle. V has units of Volts or J/C.

Let's get this potential of a charge with work.
For a point charge, we go along the radial vector and get

$\infty \leftarrow \infty$ is the reference point.
$$V = -\int_{r'}^{\infty} \frac{q \hat{r} k}{|r|^2} dr = -kq/r \Big|_{r'}^{\infty} = \frac{kq}{r'}$$ now what about a line of charge.

$\vec{E}$ by symmetry

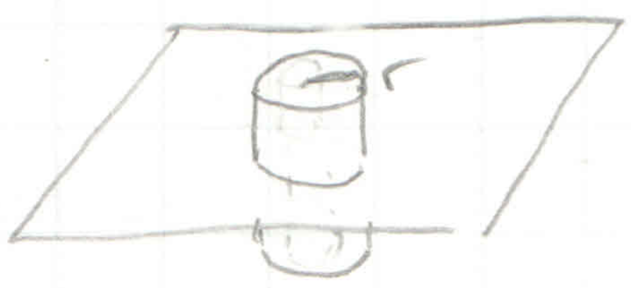


By Gauss's Law, the field is $\oiint \vec{E} \cdot dA = Q/\varepsilon_0$

so $\vec{E} \cdot 2\pi r \cancel{dx} = \lambda \cancel{dx}/\varepsilon_0$ so $\vec{E}(r) = \lambda/2\pi r \varepsilon_0 = \frac{2k\lambda}{r}$

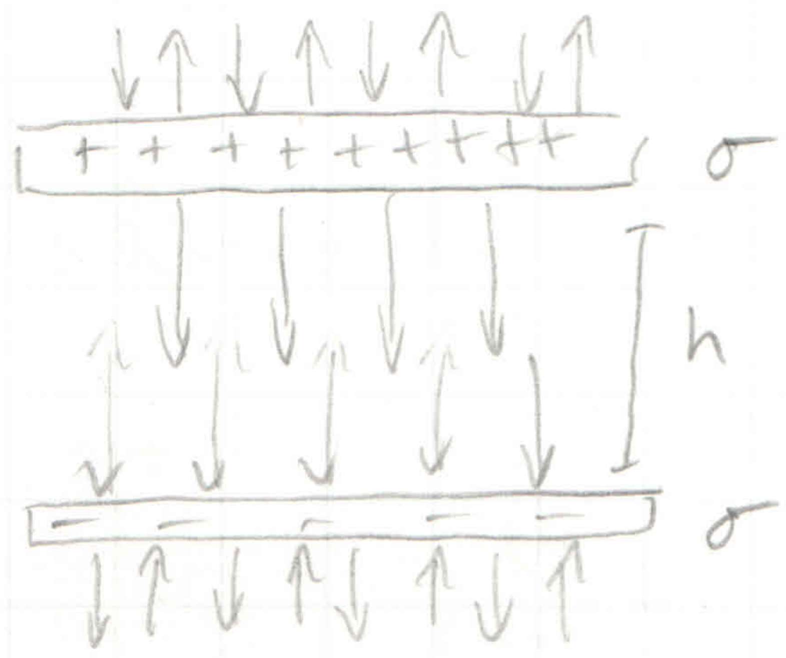
so then $V = \int_{r_1}^{r_2} \frac{2k\lambda}{r} dr = 2k\lambda \ln(r) \Big|_{r_1}^{r_2}$ which handle dimensional analysis.

What about a flat plane?



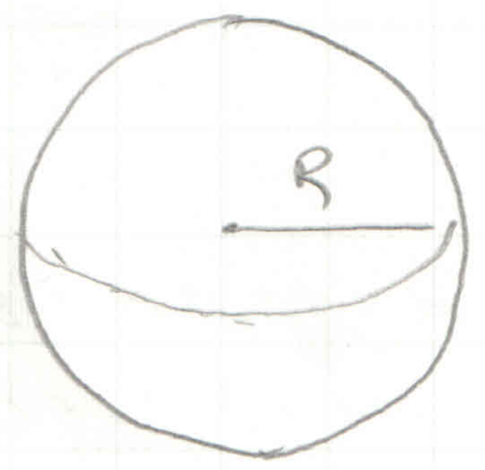
$2A\vec{E} = \frac{\sigma A}{\varepsilon_0}\hat{z}$ by Gauss's law so $\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{z}$

so $V = \frac{\sigma z}{2\varepsilon_0}$ now let's try two plates



The electric field is $E = \sigma/\varepsilon_0$ and 0 outside. So $V = \sigma/\varepsilon_0 \cdot h$.

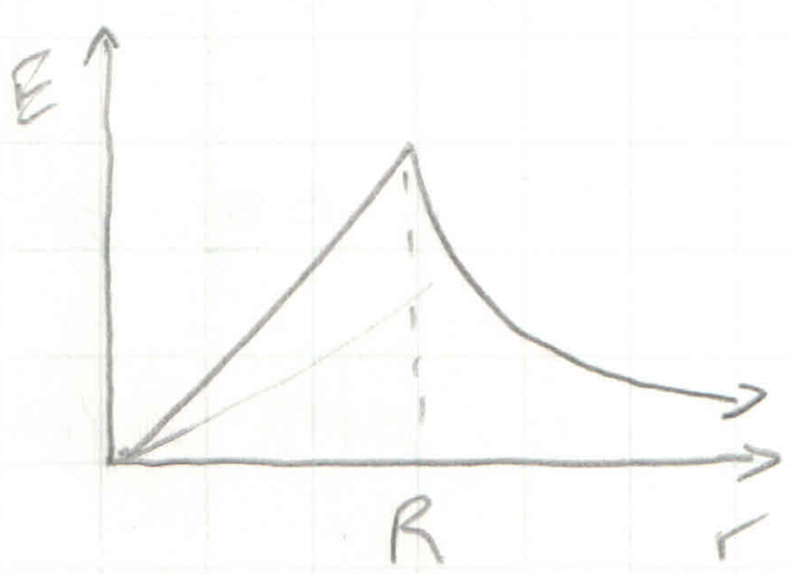
What about a spherical uniform ball of charge the



charge density ρ then $4\pi r^2 \vec{E} = \frac{4/3\pi R^3 \rho}{\varepsilon_0}$

if $r > R$ we have $\vec{E}(r) = \frac{Q_{in}}{4\pi r^2}\hat{r}$ since Q_{in} goes only to R

if $r \le R$ we have $\vec{E}(r) = \frac{r\rho}{3\varepsilon_0}$



So it is linear until $r = R$ and then drops off like a point charge at $r=0$ with value at Q_{in}