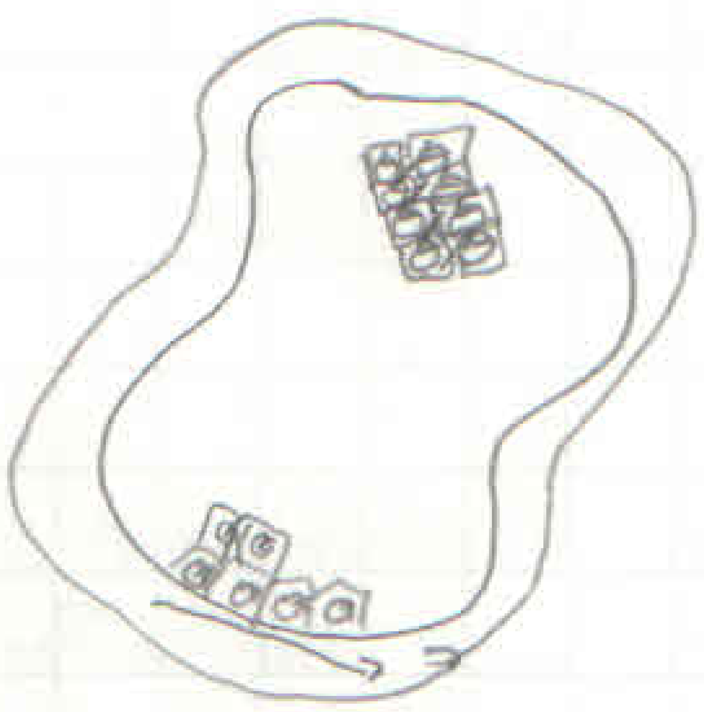
We define the magnetization to be $\vec{M} = \vec{m}/V$ then we may write A using a previous formula as:

$$\vec{A} = \frac{\mu_0}{4\pi}\int M(r') \times \frac{(r-r')}{|r-r'|^3} d^3r' \quad \text{and since} \quad B = \nabla \times A$$

$$= \frac{\mu_0}{4\pi}\int M(r')\times\nabla'\left(\frac{1}{|r-r'|}\right) d^3r' = \frac{\mu_0}{4\pi}\int \nabla' \times\left(\frac{M}{|r-r'|}\right) + \frac{1}{|r-r'|}\nabla'\times M \, d^3r'$$

$$= \frac{\mu_0}{4\pi}\int_S \frac{M \times d\vec{A}'}{r-r'} + \frac{\mu_0}{4\pi}\int_V \frac{1}{|r-r'|}\nabla' \times M \, d^3r'$$

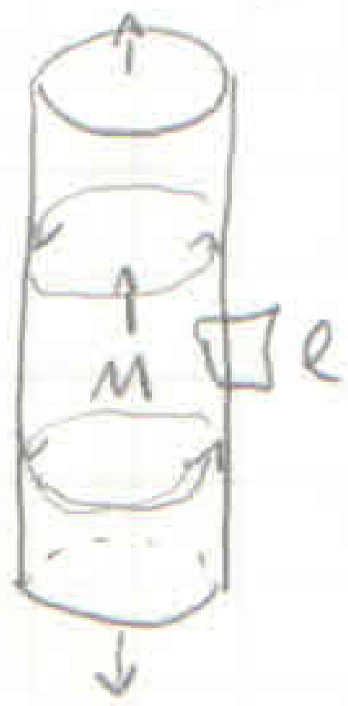
This is now familiar, we can devine $\vec{J}_B = \vec{\nabla}\times\vec{M}$, $K_B = \vec{M}\times d\vec{A}$
What do these mean? $\vec{J}_B$ is a bound current density and K_B is a surface current.



If we imagine the dipoles all being the same then all internal parts will be 0 by drawing small loops and seing that each dipole cancels on the loop.

On the outside, the dipoles on the edge will not cancel and so we will get an effective surface current!

If the dipoles are not all the same then we get $\vec{\nabla}\times\vec{M}$ as the difference!



Imagine now an infinite cylinder with constant $\underline{M}$ inside and 0 outside. Then we see that $\vec{J}_B$ is 0 inside and outside but on the surface $|K_B| = |\vec{M}\times d\vec{A}| = M\,dA$
so $J_{surf} = K_B/dA = M$ thus we get loops like an infinite solenoid. Thus the field outside is 0 and drawing a small loop around the edge and using Ampere's gives $Bl = \mu_0 M l \rightarrow B = \mu_0 M$

So now let's create an analog to displacement!

$$\vec{\nabla}\times\vec{B} = \mu_0 \vec{J} = \mu_0 (J_{free} + J_{Bound}) = \mu_0 (J_{free} + \nabla \times M)$$

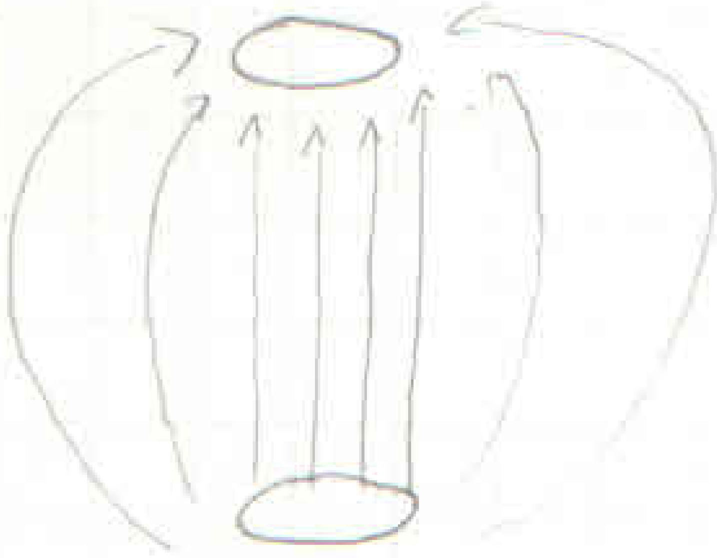
$\Rightarrow \nabla \times (B/\mu_0 - M) = J_{free}$ so define $H = B/\mu_0 - M$ which we leave nameless

so $\vec{\nabla}\times\vec{H} = \vec{J}_F$ and if we return to the problem above we have that $\vec{\nabla}\times\vec{H} = 0$ so $B/\mu_0 - M = 0 \Rightarrow \vec{B} = \mu_0\vec{M}$. But Wait! What about the divergence? Well $\vec{\nabla}\cdot\vec{H} = -\vec{\nabla}\cdot\vec{M}$ and in the case above $\vec{\nabla}\cdot\vec{M} = 0$ so $\vec{\nabla}\cdot\vec{H} = 0$ and we uniquely define $\vec{H} = 0$!

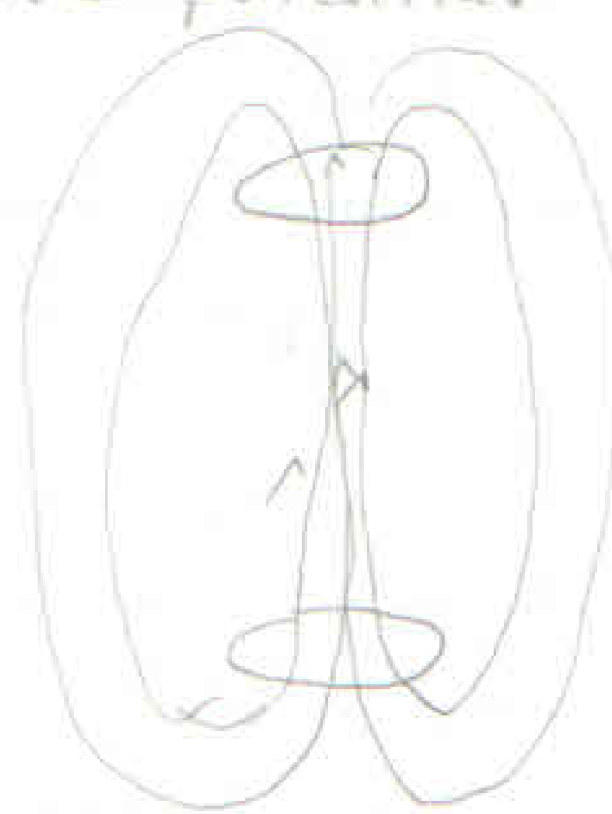
Let's now do a finite tube, now $\vec{\nabla}\cdot\vec{H} \neq 0$ at the edges so we must adjust. $\vec{\nabla}\times\vec{H} = 0$ still so now we can treat this exactly like an electric field! We may define a scalar field W s.t. $\nabla W = H \Rightarrow \Delta W = -\vec{\nabla}\cdot\vec{M}$

and we can then say $B = \mu_0 \nabla W + \mu_0 M$. Nice!

Making the analog to V, W would look like the potential from two disks of charge

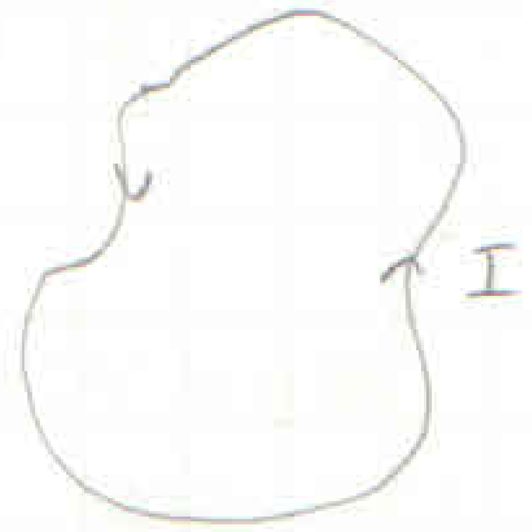


But magnetic field lines may not stop and start because $\vec{\nabla} \cdot \vec{B} = 0$ so we guess the following



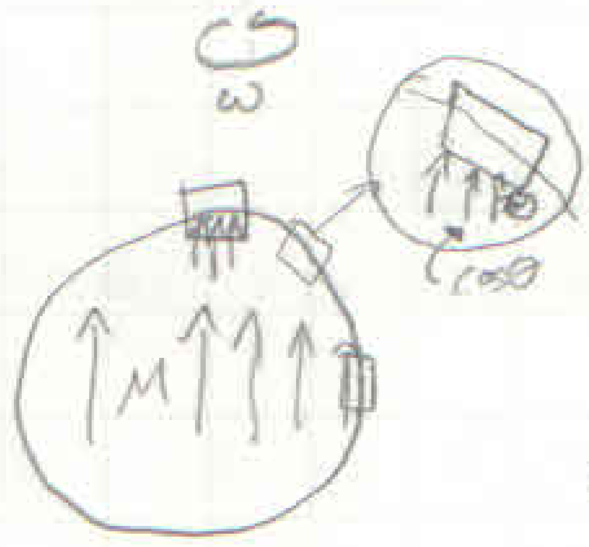
as being the sol.

Alternatively, this problem can be though of as a bound current problem.



If we have some current loop, we can think of it as a solid with no internal magnetization but with an external surface current. Thus if one can build M so that it reproduces I, then by uniqueness we can solve it again via potentials.

Say we have a sphere with surface charge density σ then $v = \omega R \sin\theta$ and $J = \sigma \omega R \sin\theta \, \hat{\phi}$ and we can integrate that. gross. Instead let's think of loops. and try and find M.



Well if we have uniform magnetization we see that $K_B = \vec{M} \times d\vec{A} = M R \sin\theta \, \hat{\phi}$ and so we recover the surface current in a similar form and so on the surface of the sphere we get net flux of M out the top and 0 on the side so $\vec{\nabla} \cdot H = M \cos\theta$ which is the induced charge from a sphere in uniform electric field. Thus we can reuse the voltage result and take the curl to get the following qualitative solution! It looks fairly like a dipole still.