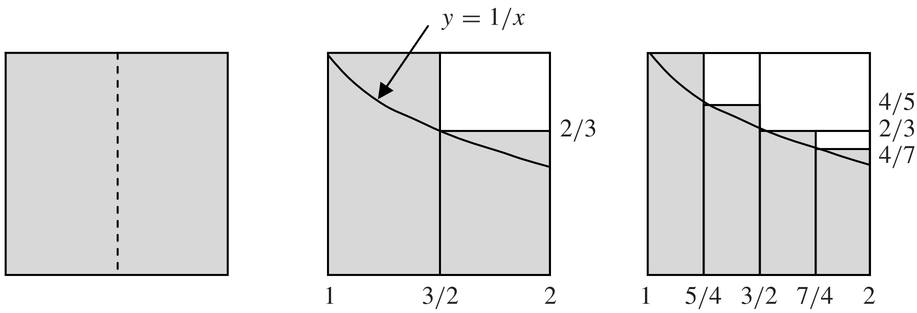
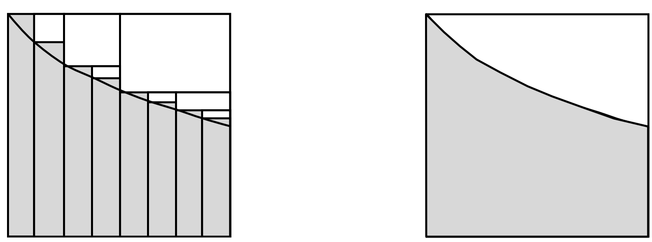
Proof Without Words: The Alternating Harmonic Series Sums to ln 2

CLAIM. $\sum_{n=0}^{\infty}(-1)^n \frac{1}{n+1} = \ln 2.$



$1 \cdots$ $\quad$ $-\frac{1}{2} + \frac{1}{3} - \cdots$ $\quad$ $-\frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \cdots$



$-\frac{1}{8} + \frac{1}{9} - \frac{1}{10} + \frac{1}{11} - \frac{1}{12} + \frac{1}{13} - \frac{1}{14} + \frac{1}{15} - \cdots$ $\quad$ $= \int_1^2 \frac{1}{x}\,dx = \ln 2$

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Summary We demonstrate graphically the result that the alternating harmonic series sums to the natural logarithm of two. This is accomplished through a sequence of strategic replacements of rectangles with others of lesser area. In the limit, we obtain the region beneath the curve $y = 1/x$ and above the x-axis between the values of one and two.

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