We thank the both referees for reading our manuscript in details and providing with valuable feedback. We think that contrition significantly improved the manuscript. Below, we give a detailed list of changes, following the referees’ suggestions:

Ref A

1) … (Turker)

2) … (Turker)

3) We added a section to the supplementary (now section IX in SI), where we evaluate <1/Delta\_{12}^2> for the case when the messenger atom is placed close to the border of the cloud. Furthermore we added a paragraph to the end of section I in SI, explaining the result.

4) We added a paragraph about the considerations of the phase matching condition in the presence of an optical cavity to the end of section IV.C in SI.

5) We added a figure (now Fig 1.) to section IV.C of SI, which illustrates the orientation of the coherent driving fields with respect to the optical cavity field.

6) Comparing our results to the standard quantum limit, as a benchmark, has the advantage of being easily comparable to other results that also compare themselves to SQL. In case of the results reported in Ref [10], they report a 70-fold increase in accuracy of phase measurement, which ideally would translate to the same enhancement in clock stability. We found a 12-fold enhancement in our analysis. To make the comparison easier, we changed the sentence about Ref [10] in the introduction to “Significant noise reduction has recently been demonstrated with spin-squeezed states in a single ensemble of atoms in [10], which reported a 70-fold enhancement of phase measurement accuracy beyond the standard quantum limit.”

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9) Yes, local oscillators of the clocks have to be phase locked prior to entangling the atoms. We added a clarification to the introduction: “… network of atomic clocks can result in substantial boost of the overall precision if multiple clocks are phase locked and connected by quantum entanglement.”

minor suggestions:

1) We added labels “M ensembles” to figure 1.

2) To improve the explanation of step 4 we changed the following:

- We modified the following sentence  
“, which promotes any population in s to r\_2, which then blocks the path via r\_1.” to   
“This promotes any population in s to r\_2, which then blocks the path g ↔ r\_1 ↔ f.”

- We moved the lower indices inside the kets in Eq. 4, so that the description in following text is easier to follow.

- We changed   
“measurement of n\_{s\_k} → m in {0,1}” to  
“measurement of n\_{s\_k}, yielding m in {0,1}”

- We replaced the arrow in “n\_{s\_k} → 0” and “n\_{s\_k} → 1” with equal signs.

- We moved the “k” index inside the ket in the expression of the GHZ state, to match with the convention used in Eq. (4).

3) We added the sentence “The kets, |n\_f>, |n\_s> for n in {0,1} stand for collective spin waves being excited by n quanta.” to the end of the paragraph of Eq 1.

Furthermore, to make the distinction between single-atom and collective states, we changed the symbol for the ground state from “|0>” to “|0\_f 0\_s>”. This way, it is clear that if a letter appears alone inside the ket, say |f>, it refers to a single atom state, while if it appears as subscript to a number, say |0\_f>, then the ket stands for a collective state. We remind the reader of this convention right after Eq. 5.

4) We added the sentence “This particular sequence results in emitting a single photon (from e → g transition) provided that the level s is empty, i.e. |0\_s>|vacuum> → |0\_s>|1 photon>.” to illustrate the immediate effect of applying the pulse sequence once.

5) Corrected typo. The sentence now reads “In such a case, the messenger atom can be used…”.

6) Added “(See Supplementary for details.)” after the sentence in question.

7) Corrected typo. Now it reads “[\pi]\_{f,r1}”, in the Supplementary.

8) Corrected typo. Now it reads "|e> -->|g> transitions", in the Supplementary.

9) We changed the symbol for cavity finesse from “f” to “I”, in and after Eq 22, in the Supplementary.

Ref B

1) We changed to title to the more specific “Quantum network of atom clocks: a possible implementation with neutral atoms”.

2) We added “Overall fidelity turns out to depend on the lattice geometry; it is the highest for 3D optical lattice.” to the end of paragraph 2 on page 4.

3) … (Jun)