Higher-categorical Strucures and Type Theories: DRAFT!

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Contents

CHAPTER 1

Introduction and background

1. Homotopy Type Theory: an introduction

1.1. Similar to the introduction to my previous paper: a quick, accessible intro to the higher-categorical view of identity types.

1.2. Refer to appendices for full background on globular higher cats & on DTT. However: include a *rough* introduction to the higher cats, & a *full* introduction to (& discussion of) identity types.

2. Survey of the field

2.1. Goals! What we're working towards in the short-term (eg sound and complete semantics, good analysis of categorical properties of Π, Σ-types, etc.

2.2. What's actually been done! Some models, a few structures, syntactic analysis in dimension 2, applications as independence results...

Lots of references should go in this, of course!

3. Outline of the present work

3.1. Overall structure (composition of 2-cells along bounding 0-cell), reason therefor (aim of analysing type theories in the well-understood quasi-categorical setting).

3.2. Overview of the "universal-algebraic aspects" setup: technical, dry, but necessary!

3.3. Results of the "syntactic structures" section.

3.4. Results of the "homotopical constructions" section.

4. Outlook: visions of a higher-categorical foundation

4.1. Write up some of what's currently just in folklore, the n-lab, the categories list, boozy nights out with the gang, etc. :

Voevodsky's model(s) + axiom; the type theorists' OTT etc.; notions of "the same"; "category theory without equality", etc.

5. Acknowledgements

(Should go before this chapter, or here at end of it?)

— Steve! Krzys/Chris. Other HTT'ers: Michael, Richard, Benno, Chris. Pittsburgh PL crowd: Bob H, Dan L, Noam Z. Also in Pittsburgh: Kohei, Henrik, James C, Rick S, Peter A, Dana. Chicago group: Mike, Emily, Claire, Daniel.

Nottingham: Thorsten and his merry men. Elsewhere on-topic: Martin H, Andrej, Pierre-Louis C., Paul-Andr M?, Thomas F?. Off-topic: Yimu, orchestras, parents!

(To do: ask people's permission for this??)

CHAPTER 2

Universal-algebraic aspects

Possibly fold this chapter into the next??

1. A category of Type Theories

1.1. If poss, use elegant Fiore et al "second-order..." as framework for this! Explain: this is *much more literal to the syntax* than Cartmell, categories with attributes etc; but on the other hand, much more categorically tractable than more ad hoc presentations of the syntax.

Mention possible tie-in with formalisation in Agda?? (*)

2. Internal algebras for operads

2.1. Give fuller account of what I rush through in my previous paper: show correspondence btn different notions of algebras for an operad! (a) models of ess. alg. (Lawvere) theory (poss with extra structure: "P-maps"); (b) Batanin: monoidal globular categories (as used + nicely expounded in [GvdB]); (c) Leinster: (weak) T-structured categories.

Lovely rarely-cited WEBER paper gives source for most of this! Possibly even *everything* I need is there, in which case possibly move this section to appendix, and fold the first part of this chapter into the next chapter???

CHAPTER 3

Globular structures from type theory

1. The fundamental weak omega-groupoid of a type

Update of my prev paper (+ more detailed comparison with Richard + Benno): type gives internal weak omega-groupoid in the classifying category.

DEFINITION 1.1. algebraic Id-type categories, $\mathbf{Cat}_{\mathrm{Id}}$.

1.2. functor $\mathbf{Th}_{\mathrm{Id}} \to \mathbf{Cat}_{\mathrm{Id}}$ over $\mathbf{Cat}$.

1.3. functor $\mathbf{FibSpans} : \mathbf{Th} \to \mathbf{MonGlobCat}$: imitate the original Batanin "higher spans" construction, but using doubly-dependent types instead of spans. Note that it's even strict since fibration was split! (is it? maybe not because of reordering!)

1.4. now for $\mathcal{C} \in \mathbf{Th}_{\mathrm{Id}}$, get for each context $X \in \mathcal{C}$ a globular element of $fibSpans(\mathcal{C})$.

2. The classifying weak omega-category of a type theory

CHAPTER 4

Homotopical constructions from globular higher categories

(NON)SECTION AIM: Motivate constructing the simplicial nerve, and hence the model structure. TODO: cut down, too discursive?? Maybe move this to end of previous section, or even to introduction, and here give an overview of simplicial structures / methods?

0.1. The great power of classifying categories in 1-categorical logic come from ['depends on'?] an analysis of the logical constructors and rules in categorical terms: substitution as pullbacks, Π- and Σ-types as adjoints, and so on. So, a natural first impulse is to try to analyse the universal properties of the type constructors within $\mathbb{C}{\ll}_\omega(\mathcal{T})$, which we would expect to be weak-higher-categorical analogues of the usual logical structure.

Unfortunately, the theory of logical structure on globular higher-categories is not yet well-understood. Of course, we can hope that the developing dictionary with type theory will help understand how such structure should behave! However, there is an alternative model of higher categories for which the relevant theory is already much further advanced: Joyal's quasi-categories. Quasi-categories are not a fully general theory of higher categories: they only model so-called (∞,1)-categories, in which all cells above dimension 1 are (weakly) invertible. However, as we have seen, the classifying categories of type theories are of this form; so quasi-categories seem potentially excellently-suited for our desired analysis, if only we can give quasi-category models for $Clw(\mathcal{T})$!

0.2. In other words, we would like to construct a functor

$$\mathbb{C}{\ll}_\omega^{qcat} \colon \mathbf{Th} \to \mathbf{QCat}.$$

TODO: Hmm, this doesn't work if the reader doesn't know yet that quas-cateogries are simplicial things! Work out how to re-organise to fit that in nicely.

There are two obvious options. Firstly, we could construct $\mathbb{C}{\ll}_\omega^{qcat}(\mathcal{T})$ directly from the theory $\mathcal{T}$. [TODO: Ask Michael whether/how much to mention simplicial type theory.] However, Id-types as they stand are inescapably globular; there seems no obvious way to extract simplicial sets from the theory as cleanly and directly as one can extract globular sets. (The intriguing approach of re-axiomatising Id-types to be "naturally simplicial in shape" has, however, been considered by Warren and Gambino [**?**].)

It thus seems natural to take a different approach: to construct $\mathbb{C}{\ll}_\omega^{qcat}$ in two steps, composing $\mathbb{C}{\ll}_\omega$ from the previous section with a functor

$$\mathcal{N}^{qcat} \colon P\text{-}\mathbf{Alg} \to \mathbf{QCat}$$

giving the "quasi-category nerve" of a globular weak n-category. This has the added payoff that such a functor would be of independent interest, since the comparison between globular and simplicial higher categories is as yet little-understood in the fully weak case.

0.3. In Section 3, we will thus construct several candidate nerve functors. Constructing simplicial objects is straightforward; the hard part is proving the requisite horn filling conditions to show that they are quasi-categories.

It is for this that we construct, in Sections 1 and 2, a Quillen model structure on categories of globular higher categories (under certain extra hypotheses). The computational tools provided by such a structure provide precisely what we need to show that the horns arising in our nerve constructions can be filled, and hence that the nerves are indeed quasi-categories.

In fact, we construct an *algebraic* model structure in the sense of [**?**]. This is a Quillen model structure in which both weak factorisation systems are NWS's and there is moreover a comparison map connecting the two. While we will not need any of the extra power of an algebraic model structure, the algebraicity comes almost for free given the form of our proof.

1. Cellular algebraic model structures

SECTION AIM: The general construction of an algebraic model structure from a collection of generating cells. (TODO: read/ask around in case I've missed where something closer to this has already been done; work out terminology for this general construction.)

1.1. Recall what an AWFSand AMSare. (Or put this in appendix?)

1.2. We start by recalling from [**Gar09**], [**Gar08**] the construction of an AWFS on **str**-n-**Cat** whose right maps are precisely the contractible maps.

...do it by the Garner small-object argument! Algebraic freeness shows the right maps are what we think. And Garner shows that this is also the adjunction with computads, fwiw (maybe leave this out if not needed).

Point out how this comes from the map $D(\mathrm{ob}\,\mathbb{G}) \to \mathbb{G} \to \widehat{\mathbb{G}} \to$ **wk**-n-**Cat** of cells/boundaries.

1.3. In fact, the remainder of the construction of the AMS(though not the proof that it really is one) can be given entirely in terms of this set of generating cofibrations; and, indeed, L' categories will give another example. Thus for the remainder of this section, we will fix a category $\mathcal{E}$ that admits the small object argument [TODO: define this here or elsewhere!], a set $\mathcal{I}$ (considered as a discrete category), and a functor $\mathcal{I} \to \mathcal{E}^{\mathbf{2}}$. The functor will remain nameless, but we will write maps in its image as $d_i \hookrightarrow c_i$, for $i \in \mathcal{I}$. (They are to be thought of as inclusions of boundaries into cells.)

1.4. Construct $(\mathbb{C}, \mathbb{F}_t)$; construct "canonical squares". Describe how to see them as equivs.

1.5. Construct $\mathbb{W}$, $\mathcal{W}$, $(\mathbb{C}_t, \mathbb{F})$ from this. Infer ξ, by [**?**, Rmk 3.6] (and hence get $\mathcal{C}_t \Rightarrow \mathcal{C}$, $\mathcal{F}_t \Rightarrow \mathcal{F}$).

1.6. Give $TF \Leftrightarrow F \cap W$, i.e. $\mathbb{F}_t$-**Alg** $\Leftrightarrow$ $\mathbb{F}$-**Alg** $\times_{\mathcal{E}^{\mathbf{2}}}$ $\mathbb{W}$-**Alg**. Give: $\mathbb{W} \to \mathcal{E}^{\mathbf{2}}$ "creates retracts", so $\mathcal{W}$ closed under retracts.

1.7. Now the hard stuff!

2. An algebraic model structure on globular higher categories?

2.1. Discuss instantiating the theorem of the previous section to (a) L'-categories, (b) P-algebras; prove as many of the lemmas as possible!

3. Simplicial nerves of globular higher categories

3.1. Give aim; different NWFS for different nerves; proof with model structure that these give nerves!

3.2. Prove from model strux that these really do give quasi-categories.
TODO: read up Dugger references properly.

APPENDIX A

Background: Martin-Löf Type Theory

Possibly just make this a presentation of the type theory?

1. Presentation overview

1.1. The type theories we consider are all essentially variants on the basic type theory originally presented in [**?**]. See also (TODO: pick out good references!)

The core of the type theory, its basic judgements and structural rules, is given in Section 1.4; this will remain constant in all the theories we consider.

The type-constructors and various other rules that will be used are given in Section 1.5; we will consider theories including various different subsets of these rules.

Section **??** contains some fundamental lemmas concerning the resulting type theories: basic proof-theoretic properties, admissible rules, and so on. TODO: this has changed! Memo to self: don't write overviews of chapters before starting the chapters themselves...

For further background, see [**Pit00**] (a notably thorough and precise presentation), [**Jac99**, §6] (for a thoughtful discussion of this system within the wider type-theoretic context), [**Hof97**], [**NPS90**, Ch.3] (discussion of the use of simply-typed calculus as the "raw language'), and [**ML75**] (the originial presentation of the theory). The presentation uses essentially the formal presentation of [**Pit00**], in the notation of [**Jac99**], slightly modified to include dependent contexts and their morphisms.

(TODO: perhaps fully expound the presentation rather than just recapping?)

Notes (re-organise and/or remove later):

— point of using λ-calculus as raw language: to separate the subtleties of binding and capture-avoiding substitution from those of dependency and well-formedness.

— point of having the raw language already simply-typed, not completely un(i)typed: to maintain decidability of $\equiv$.

— abuse of notation: making variables explicit

— dependent contexts and maps: to formulate Frobenius rule

— dependent maps: really only need $\Gamma \vdash \vec{f} : \Delta \Rightarrow \Delta'$ in case either $\Gamma = \diamond$ (for substitution rules) or $Delta = \diamond$ (for Frobenius rules). However, it's simpler to give this than those two separately, plus natural for syntactic cwa.

— syntactic sugar: $\vec{x} : \Gamma \vdash a(\vec{x}) : A(\vec{x})$ really means $\Gamma \vdash a : A$, where all of these are *closed* metaexpressions, but a, A have arities $\mathsf{term}^n \to \mathsf{term}$, $\mathsf{term}^n \to \mathsf{type}$, where n is the length of Γ; and so on.

1.2. Raw syntax

Define: "raw language". (NB: sometimes called *metalanguage*, but not meaning the language in which we reason about the type theory; rather, plays a similar rôle to that of strings (or trees) of symbols in simpler systems.)

Define: signature, pre-terms, pre-types, pre-terms, judgements.

1.3. Judgement forms

Basic judgement forms	
$\Gamma \vdash A\ \mathsf{type}$	$\Gamma \vdash a : A$
$\Gamma \vdash A = A'\ \mathsf{type}$	$\Gamma \vdash a = a' : A$

Derived judgement forms	
$\Gamma \vdash \Delta\ \mathsf{cxt}$	$\Gamma \vdash \vec{f} : \Delta \Rightarrow \Delta'$
$\Gamma \vdash \Delta = \Delta'\ \mathsf{cxt}$	$\Gamma \vdash \vec{f} = \vec{f'} : \Delta \Rightarrow \Delta'$

Explain in what sense these are derived judgements!
(Discuss in parens why we use dependent cxts and morphisms.)
Perhaps (if feeling pedantic, or can find way to say it non-pedantically) discuss arities of things in judgements, etc.?

1.4. Rules: Structural core

Rules for contexts:

$$\frac{}{\diamond \vdash \diamond\ \mathsf{cxt}}\ \text{cxt-}\diamond\diamond \qquad \frac{\diamond \vdash \Gamma\ \mathsf{cxt}}{\Gamma \vdash \diamond\ \mathsf{cxt}}\ \text{cxt-}\diamond \qquad \frac{\diamond \vdash \Gamma\ \mathsf{cxt}}{\diamond \vdash \diamond = \diamond\ \mathsf{cxt}}\ \text{cxt=-}\diamond$$

$$\frac{\Gamma \vdash \Delta\ \mathsf{cxt} \quad \Gamma, \Delta \vdash A\ \mathsf{type}}{\Gamma \vdash \Delta, A\ \mathsf{cxt}}\ \text{cxt-cons} \qquad \frac{\Gamma \vdash \Delta = \Delta'\ \mathsf{cxt} \quad \Gamma, \Delta \vdash A = A'\ \mathsf{type}}{\Gamma \vdash \Delta, A = \Delta', A'\ \mathsf{cxt}}\ \text{cxt=-cons}$$

Rules for types:

$$\frac{\Gamma \vdash A\ \mathsf{type}}{\Gamma \vdash A = A\ \mathsf{type}}\ \text{type=-refl} \qquad \frac{\Gamma \vdash A = B\ \mathsf{type}}{\Gamma \vdash B = A\ \mathsf{type}}\ \text{type=-sym}$$

$$\frac{\Gamma \vdash A = B\ \mathsf{type} \quad \Gamma \vdash B = C\ \mathsf{type}}{\Gamma \vdash A = C\ \mathsf{type}}\ \text{type=-trans} \qquad \frac{\vec{x} : \Gamma \vdash A(\vec{x})\ \mathsf{type} \quad \diamond \vdash \vec{f} : \Gamma' \Rightarrow \Gamma}{\vec{y} : \Gamma' \vdash A(\vec{f}(\vec{y}))\ \mathsf{type}}\ \text{type-subst}$$

$$\frac{\vec{x} : \Gamma \vdash A = A'\ \mathsf{type} \quad \diamond \vdash \vec{f} = \vec{f'} : \Gamma' \Rightarrow \Gamma}{\vec{y} : \Gamma' \vdash A(\vec{f}(\vec{y})) = A'(\vec{f'}(\vec{y}))\ \mathsf{type}}\ \text{type=-subst}$$

Rules for terms:

$$\frac{\Gamma \vdash A \ \mathsf{type} \qquad \Gamma, A \vdash \Delta \ \mathsf{cxt}}{\Gamma, x : A, \Delta \vdash x : A} \ \mathsf{var}$$

$$\frac{\Gamma \vdash a : A \qquad \Gamma \vdash A = A' \ \mathsf{type}}{\Gamma \vdash a : A'} \ \mathsf{term\text{-}coerce} \qquad \frac{\Gamma \vdash a = a' : A \qquad \Gamma \vdash A = A' \ \mathsf{type}}{\Gamma \vdash a = a' : A'} \ \mathsf{term{=}\text{-}coerce}$$

$$\frac{\Gamma \vdash a : A}{\Gamma \vdash a = a : A} \ \mathsf{term{=}\text{-}refl} \qquad \frac{\Gamma \vdash a = b : A}{\Gamma \vdash b = a : A} \ \mathsf{term{=}\text{-}sym}$$

$$\frac{\Gamma \vdash a = b : A \qquad \Gamma \vdash b = c : A}{\Gamma \vdash a = c : A} \ \mathsf{term{=}\text{-}trans} \qquad \frac{\vec{x} : \Gamma \vdash a(\vec{x}) : A(\vec{x}) \qquad \diamond \vdash \vec{f} : \Gamma' \Rightarrow \Gamma}{\vec{y} : \Gamma' \vdash a(\vec{f}(\vec{y})) : A(\vec{f}(\vec{y}))} \ \mathsf{term\text{-}subst}$$

$$\frac{\vec{x} : \Gamma \vdash a(\vec{x}) = a'(\vec{x}) : A(\vec{x}) \qquad \diamond \vdash \vec{f} = \vec{f}' : \Gamma' \Rightarrow \Gamma}{\vec{y} : \Gamma' \vdash a(\vec{f}(\vec{y})) = a'(\vec{f}'(\vec{y})) : A(\vec{f}(\vec{y}))} \ \mathsf{term{=}\text{-}subst}$$

Rules for context maps:

$$\frac{\diamond \vdash \Gamma \ \mathsf{cxt}}{\diamond \vdash \diamond : \Gamma \Rightarrow \diamond} \ \mathsf{cxt\text{-}\diamond} \qquad \frac{\diamond \vdash \Gamma \ \mathsf{cxt}}{\diamond \vdash \diamond = \diamond : \Gamma \Rightarrow \diamond} \ \mathsf{cxt{=}\text{-}\diamond}$$

$$\frac{\Gamma \vdash \vec{f} : \Delta' \Rightarrow \Delta \qquad \vec{x} : \Gamma, \vec{y} : \Delta(\vec{x}) \vdash A(\vec{x}, \vec{y}) \ \mathsf{type} \qquad \vec{x} : \Gamma, \vec{z} : \Delta'(\vec{x}) \vdash a(\vec{x}, \vec{z}) : A(\vec{x}, \vec{f}(\vec{x}, \vec{z}))}{\Gamma \vdash (\vec{f}, a) : \Delta' \Rightarrow (\Delta, A)} \ \mathsf{cxt\text{-}cons}$$

$$\frac{\Gamma \vdash \vec{f} : \Delta' \Rightarrow \Delta \qquad \vec{x} : \Gamma, \vec{y} : \Delta(\vec{x}) \vdash A(\vec{x}, \vec{y}) \ \mathsf{type} \qquad \vec{x} : \Gamma, \vec{z} : \Delta'(\vec{x}) \vdash a(\vec{x}, \vec{z}) : A(\vec{x}, \vec{f}(\vec{x}, \vec{z}))}{\Gamma \vdash (\vec{f}, a) : \Delta' \Rightarrow (\Delta, A)} \ \mathsf{cxt\text{-}cons}$$

1.5. Type constructors and miscellaneous rules

Rules for type-constructors go here: Id-, Π-, Σ-, with just a tip of the hat to $\mathbb{N}$-, Bool, ...

Rules for Id-types:

$$\frac{\Gamma \vdash A\ \mathsf{type}}{\Gamma, x, y : A \vdash \mathrm{Id}_A(x,y)\ \mathsf{type}}\ \text{Id-FORM} \qquad \frac{\Gamma \vdash A\ \mathsf{type}}{\Gamma, x : A \vdash r(x) : \mathrm{Id}_A(x,x)}\ \text{Id-INTRO}$$

$$\frac{\begin{array}{c}\Gamma, x, y : A, p : \mathrm{Id}_A(x,y), \vec{w} : \Delta(x,y,p) \vdash C(x,y,p,\vec{w})\ \mathsf{type} \\ \Gamma, z : A, \vec{v} : \Delta(z,z,r(z)) \vdash d(z,\vec{v}) : C(z,z,r(z),\vec{v})\end{array}}{\Gamma, x, y : A, p : \mathrm{Id}_A(x,y), \vec{w} : \Delta(x,y,p) \vdash J_{A;\ x,y,p.\Delta;\ x,y,p,\vec{w}.C}(z,\vec{v}.\ d(z,\vec{v});\ x,y,p,\vec{w}) : C(x,y,p,\vec{w})}\ \text{Id-ELIM}$$

$$\frac{\begin{array}{c}\Gamma, x, y : A, p : \mathrm{Id}_A(x,y), \vec{w} : \Delta(x,y,p) \vdash C(x,y,p,\vec{w})\ \mathsf{type} \\ \Gamma, z : A, \vec{v} : \Delta(z,z,r(z)) \vdash d(z,\vec{v}) : C(z,z,r(z),\vec{v})\end{array}}{\Gamma, x : A, \vec{w} : \Delta(x,y,p) \vdash J_{A;\ x,y,p.\Delta}(z,\vec{v}.\ d(z,\vec{v});\ x,x,r(x),\vec{w}) = d(x,\vec{w}) : C(x,y,p,\vec{w})}\ \text{Id-COMP}$$

$$\frac{\Gamma \vdash e : \mathrm{Id}_A(a,b)}{\Gamma \vdash a = b : A}\ \text{REFLECTION}$$

$$\frac{\begin{array}{c}\Gamma, x : A, p : \mathrm{Id}_A(x,x), \vec{w} : \Delta(x,p) \vdash C(x,p,\vec{w})\ \mathsf{type} \\ \Gamma, z : A, \vec{v} : \Delta(z,r(z)) \vdash d(z,\vec{v}) : C(z,r(z),\vec{v})\end{array}}{\Gamma, x : A, p : \mathrm{Id}_A(x,x), \vec{w} : \Delta(x,p) \vdash K_{A;\ x,p.\Delta;\ x,p,\vec{w}.C}(z,\vec{v}.\ d(z,\vec{v});\ x,p,\vec{w}) : C(x,p,\vec{w})}\ \text{K}$$

Include optional things: η- and extensionality rules for function-types; K and the truncation rules for Id-types.

Discuss implications between these?

1.6. Translations

Define translation, category of type theories.

2. Categories with attributes

TODO: chase up references on these! In particular: names — type-categories, categories with attributes, categories with families, split comprehension categories, . . . ?

TODO: ask around about reachable vs. "stratified" cwa's.

2.1. Equivalence with type theories

Define: full split comprehension categories [**?**, 4.10] / categories with attributes. [**Pit00**, §6.4]

Recall (from Pitts): *equivalence* (honest 1-equivalence!) between type theories with no constructors and cwa's. TO DO: read Jacobs in full (& chase further. . .) re theories with constructors!

Define: Id-structure, Π-structure, etc. on cwa's. (Reference: [**AW09**].)

Extend equivalence above; define $\mathbf{Th}_{\mathrm{Id}}$, $\mathbf{Th}_{\mathrm{Id},\Pi}$, etc.

Fact: since all essentially algebraic (or otherwise), these categories are all cocomplete!

2.2. The "strong Σ's" monad Explicitly give the monad on $\mathbf{Th}_{\mathrm{Id}}$ throwing in all dependent contexts as new types, and interpreting equality as described in loc. cit. TODO: look up that interpretation of equality! Refer to Thorsten's OTT also.

Point out how this corresponds closely (2-equivalence?) but not precisely to adding strong Σ-types in the type theory.

Ask: is this induced by the adjunction with Id-type categories? or categories with appropriate nwfs's, or something? or Hyland-Pitts style "categories with ...?"

Discuss the Kleisli of this monad: "modelling types as contexts".

APPENDIX B

Background: globular higher category theory

1. Strict higher categories

1.1. Define **str**-n-**Cat** and **str**-ω-**Cat** by enrichment.

1.2. Analyse T: pasting diagrams, Batanin trees, familial representability.

2. Weak higher categories

2.1. Contractibility.

APPENDIX C

Background: "Homotopical" higher categories

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