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Abstract

Some examples of Mathematical Equations rendered in LaTex

1 Algebra

The well known Pythagorean theorem $x^2 + y^2 = z^2$ was proved to be invalid for exponents greater than 2.

Thus the more general equation has no integer solutions:

$$x^n + y^n = z^n$$

$$\begin{aligned} A &= \frac{\pi r^2}{2} \\ &= \frac{1}{2}\pi r^2 \end{aligned} \tag{1}$$

1.1 Aligning Equations

$$\begin{aligned} a + b \times a - b &= (a + b)(a - b) & &= a^2 - b^2 \\ x + y \times x - y &= (x + y)(x - y) & &= x^2 - y^2 \\ p + q \times p - q &= (p + q)(p - q) & &= p^2 - q^2 \end{aligned}$$

$$\begin{aligned} (a + b)^2 &= a^2 + 2ab + b^2 \\ (a - b)^2 &= a^2 - 2ab + b^2 \end{aligned}$$

$$\begin{aligned} f(x) &= x^2 \\ g(x) &= \frac{1}{x} \\ F(x) &= \int_b^a \frac{1}{3}x^3 \end{aligned}$$

1.2 Various Examples

$$F = G\left(\frac{m_1 m_2}{r^2}\right)$$

$$\left[\frac{N}{\left(\frac{L}{p}\right) - (m+n)}\right]$$

$$y = 1 + \left(\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \ldots + \frac{1}{x^{n-1}} + \frac{1}{x^n}\right)$$

$$\left\{\begin{array}{ccc} 1 & 5 & 8 \\ 0 & 2 & 4 \\ 3 & 3 & \text{-8} \end{array}\right\}$$

2 Trigonometry

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

3 Operators

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

This operator changes when used alongside text $\lim_{x \to h}(x - h)$.

$$f(x) = \sum_{i=0}^{n} \frac{a_i}{1+x}$$
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3.1 Fractions and Binomials

$$\frac{1 + \frac{x}{y}}{1 + \frac{1}{1 + \frac{1}{z}}}$$

$$\frac{1}{\sqrt{x}}$$

$$a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \cfrac{1}{a_4}}}}$$

Mixing text and equations:

$$f(x) = \frac{P(x)}{Q(x)} \quad \text{and} \quad f(x) = \tfrac{P(x)}{Q(x)}$$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

4 Matrices

The identity matrix:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

4.1 Greek Letters

$$\alpha \quad \beta \quad \gamma \quad \delta \, \epsilon \, \zeta \, \eta \, \theta \, \iota \, \kappa \, \lambda \, \mu \, \nu \, \xi \, o \, \pi \, \rho \, \sigma \, \tau \, \upsilon \, \phi \, \chi \, \psi \, \omega$$

4.2 Binary Operators

$$\times \quad \otimes \quad \oplus \quad \cup \quad \cap$$

4.3 Relation Operators

$$< \quad > \quad \subset \quad \supset \quad \subseteq \quad \supseteq$$

4.4 Other Symbols

$$\int \quad \oint \quad \sum \quad \prod$$

- $\mathbb{P}$ Prime numbers
- $\mathbb{W}$ Whole numbers
- $\mathbb{N}$ Natural numbers
- $\mathbb{Z}$ Integers
- $\mathbb{Q}$ Rational numbers
- $\mathbb{R}$ Real numbers
- $\mathbb{C}$ Complex numbers
- $\mathbb{H}$ Quaternions
- $\mathbb{O}$ Octonions
- $\mathbb{S}$ Sedenions

$$\langle \big\langle \Big\langle \bigg\langle \bigg\rangle \Big\rangle \big\rangle \rangle$$

5 Complex Numbers

$$i^2 - 1$$

$$z = x + iy$$

$$\mathbb{C}$$

$$\{z, \overline{z}\} \in \mathbb{C}$$

$$\Re(z) = a$$

$$\Im(z) = b$$

$$\Re(z), \Im(z)$$

$$\mathrm{Re}(z), \mathrm{Im}(z)$$

$$\mathfrak{Re}(z), \mathfrak{Im}(z)$$

$$z = a + ib$$

$$\bar{z} = a - ib$$

$$\Re z = a$$

$$\Im z = b$$

$$z_1\bar{z}_2$$

$$\overline{z_1 z_2}$$

6 Navier-Stokes Equations

6.1 Einstein Summation Convention

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_i)}{\partial x_i} = 0 \tag{2}$$

$$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial[\rho u_i u_j]}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} + \rho f_i \tag{3}$$

$$\frac{\partial(\rho e)}{\partial t} + (\rho e + p)\frac{\partial u_i}{\partial x_i} = \frac{\partial(\tau_{ij} u_j)}{\partial x_i} + \rho f_i u_i + \frac{\partial(\dot{q}_i)}{\partial x_i} + r \tag{4}$$

The Einstein summation convention dictates that: When a sub-indice (here i or j) is twice or more repeated in the same equation, one sums across the n-dimensions.

So, in the context of Navier-Stokes in three spatial dimensions, one repeats the term three times, each time changing the indice for one representing the corresponding dimension (ie $1, 2, 3$ or x, y, z).

The first equation is therefore a shorthand representation of: $\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_1)}{\partial x_1} + \frac{\partial(\rho u_2)}{\partial x_2} + \frac{\partial(\rho u_3)}{\partial x_3} = 0$.

The second equation is actually a superposition of three separable equations which could be written in a three-line form: one line equation for each i in each of which one sums the three terms for the j sub-indice.

6.2 Classic Notation with $\longrightarrow, \otimes, \nabla$

$$\frac{\partial \rho}{\partial t} + \overrightarrow{\nabla} \cdot (\rho \overrightarrow{u}) = 0 \tag{5}$$

$$\frac{\partial(\rho \overrightarrow{u})}{\partial t} + \overrightarrow{\nabla} \cdot [\rho \overline{\overline{u \otimes u}}] = -\overrightarrow{\nabla p} + \overrightarrow{\nabla} \cdot \overline{\overline{\tau}} + \rho \overrightarrow{f} \tag{6}$$

$$\frac{\partial(\rho e)}{\partial t} + \overrightarrow{\nabla} \cdot ((\rho e + p)\overrightarrow{u}) = \overrightarrow{\nabla} \cdot (\overline{\overline{\tau}} \cdot \overrightarrow{u}) + \rho \overrightarrow{f} \overrightarrow{u} + \overrightarrow{\nabla} \cdot (\overrightarrow{\dot{q}}) + r \tag{7}$$

Here $\otimes$ denotes the tensor product which forms a tensor from the constituent vectors. A double bar denotes a tensor. These equations using the Classic Notation are equivalent to the earlier three using the Einstein Summation Convention.

6.3 Navier Stokes Equation 3D Cartesia Coordinates

$$x : \rho \left(\partial_t u_x + u_x \, \partial_x u_x + u_y \partial_y u_x + u_z \, \partial_z u_x\right) =$$
$$-\partial_x p \; + \mu \left(\partial_x^2 u_x \; + \partial_y^2 u_x \; + \partial_z^2 u_x\right) +$$
$$\tfrac{1}{3}\mu \partial_x \left(\partial_x^2 u_x \; + \partial_y^2 u_y \; + \partial_z^2 u_z\right) + p g_x$$

$$
\begin{gathered}
y : \rho \left(\partial_t u_y + u_x \, \partial_x u_y + u_y \partial_y u_y + u_z \, \partial_z u_y\right) = \\
-\partial_y p \ + \mu \left(\partial_x^2 u_y \ + \partial_y^2 u_y \ + \partial_z^2 u_y\right) + \\
\tfrac{1}{3} \mu \partial_y \left(\partial_x^2 u_x \ + \partial_y^2 u_y \ + \partial_z^2 u_z\right) + p g_y
\end{gathered}
$$

$$
\begin{gathered}
z : \rho \left(\partial_t u_z + u_x \, \partial_x u_z + u_y \partial_y u_z + u_z \, \partial_z u_z\right) = \\
-\partial_z p \ + \mu \left(\partial_x^2 u_z \ + \partial_y^2 u_z \ + \partial_z^2 u_z\right) + \\
\tfrac{1}{3} \mu \partial_z \left(\partial_x^2 u_x \ + \partial_y^2 u_y \ + \partial_z^2 u_z\right) + p g_z
\end{gathered}
$$

7 Logarithhms

$$\log xy = \log x + \log y$$

8 Calculus

$$\frac{df}{dt} = \lim_{h \to 0} \frac{f(t+h) - f(t)}{h}$$

9 Newton's Law of Gravity

$$F = G\frac{m_1 m_2}{d^2}$$

10 Complex Numbers

$$i^2 = -1$$

11 Euler's Formula for Polyhedra

$$F - E + V = 2$$

12 Maxwell's Equations

13 The Normal Distribution

$$\Phi(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

14 The Wave Equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

15 The Fourier Transform

$$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x) e^{2\pi i x \xi} \quad dx$$

16 The Second Law of Thermodynamics

$$dS \geq 0$$

17 Relativity

$$E = mc^2$$

This equation states Einstein's mass energy equivalence relationship.

18 Relativity

$$i\hbar\frac{\partial}{\partial t}\Psi = \hat{H}\Psi$$

19 Information Thoery

$$H = -\sum_{x} p(x) log\ p(x)$$

20 Chaos Thoery

$$x_{t+1} = kx_t(1 - x_t)$$

21 Black-Scholes Equation

$$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} + \frac{\partial V}{\partial t} - rV = 0$$

22 References

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