UNIVERSITY OF COPENHAGEN
FACULTY OF SOCIAL SCIENCES

Seminar: Advanced Empirical Macroeconomic Analysis

Sticky Wages or Sticky Prices?

Estimating wage and price stickiness in Denmark through impulse response matching

Author: Peter Ravn

Advisor: Michael Bergman

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Abstract

This paper examines wage and price stickiness in Denmark using a New Keynesian model with real wage imperfections. Using Danish quarterly data from 1996 to 2024, I fit a VAR(4) model and construct sign restricted impulse responses. I propose a new way to estimate the signs using probable impulse responses from the theoretical model. Matching the theoretical and empirical impulse responses, I find that prices adjust every 1.5 quarters on average, while wages adjust approximately every 3.5 years. However, the empirical and theoretical matches poorly and one should be cautious interpreting these estimates. It does however show that sticky wages are important when modeling the Danish economy.

Contents

1 Introduction

One of the main goals of central banks is to provide their country with stable prices. Stable prices are achieved by controlling inflation. Inflation can be lowered by increasing the interest rate, however the inflation rarely decreases immediately. This is because prices and wages can be rigid. But are they equally rigid or is one more rigid than the other?
Stabilizing inflation takes time due to nominal rigidities and depends on price and wage stickiness. Price stickiness appears due to imperfections in the market such as menu costs, contract agreements and information costs. Wage stickiness appears due to imperfections such as labour unions, long-term contracts and asymmetric information. Central banks have to consider both price stickiness and wage stickiness when implementing economic policies.

Many macroeconomic models in the 80s and 90s only included price rigidities, as it was commonly believed that prices were the main driver of nominal rigidity. However, Blanchard and Galí (2007) show that without real wage rigidities in the New Keynesian model, central banks can stabilize inflation without changing the output gap. They will not face a trade-off between stabilizing inflation and stabilizing the output gap. The authors call this ”the divine coincidence”. Once real wage rigidities are introduced, the divine coincidence fall apart. Modeling wage rigidities are therefore just as important as modeling price rigidities and have become more common in recent literature.

In this paper, I measure the rigidities in the Danish economy by matching empirical impulse responses to theoretical impulse responses from a New Keynesian model with real wage imperfections. The theoretical model is heavily inspired from Galí (2015), while the methodology of matching impulse responses is inspired from Christiano et al. (2005). A common approach to estimate structural impulse responses is to use the Cholesky decomposition. The Cholesky decomposition requires a causal ordering and does not model contemporaneous effects. This is not in line with the macroeconomic model. I instead use sign restrictions for the identification as proposed by Uhlig (2005) and propose a new method to determine the sign restrictions. I first construct impulse responses from a theoretical model, determine their possible signs and apply them to the empirical model. This approach assumes that the theoretical model is the true model.

To construct the empirical VAR model for impulse response matching, I use Danish quarterly data from 1996 to 2024 and include variables for the interest rate, productivity, price inflation, wage inflation, GDP and employment rate. I construct an *unanticipated* interest rate shock and a technology shock using the suggested signs of the theoretical model. I then match the theoretical model to both shocks individually and simultaneously for robustness checks. For the interest rate shock, I get that both prices and wages update on average every second quarter. However, I find that prices update immediately while wages update every three and a half year to a technology shock. Matching to both shocks, I find prices to update every one and a half quarter while wages update every three and a half years. As a robustness check, I also fit a model using the Cholesky decomposition. The model did not fit any better and therefore might be misspecified.

The difference between the results can be due to either misspecification in the empirical model or the theoretical model not fitting well in the Danish economy. An obvious extension is to model the economy from a closed economy to a small open economy like Denmark. However, this complicates the model significantly and is beyond the scope of this paper. Because of the possible misspecification, one should be cautious interpreting the exact estimate of price and wage rigidities. However, the model does show two things: that wage rigidity is important in the Danish economy, and that it is likely much more rigid than the price rigidity.

2 A brief history of sticky prices and wages

The idea of sticky prices and wages stems all the way back from Keynes (1936). Keynes proposed that wages are resistant to cuts even in tough economic times. This is because workers are resistant to pay reductions and firms therefore reduce costs elsewhere, often through layoffs. He furthermore argues that prices are dependent on wages, and hence price stickiness comes from wage stickiness. Throughout the 80s and 90s, the idea that prices are sticky regardless of wages became more prominent as noted by Blinder (1994). He argues that real wages do not show the expected counter-cyclical pattern under rigid nominal wages and flexible prices. Therefore, prices must also be rigid. One of the earliest attempts of modeling price rigidities on a microeconomic foundation is from Calvo (1983). He modeled prices in such a way, that each firm in each period has a chance of keeping their old prices measured by θ_p in the following equation:

$$p_t = \theta_p p_{t-1} + (1 - \theta_p) p_t^*$$

where p_t^* is the optimal price . It has since been known just as ”Calvo pricing”. Modeling price rigidity this way is equivalent to saying that a percentage of firms, θ_p, will keep their old prices. The macroeconomic literature through the 80s and 90s primarily believed that price stickiness was enough to model nominal rigidities.

The importance of sticky wages have regained popularity in the 2000s. This is especially due to the before mentioned paper by Blanchard and Galí (2007), who demonstrates that sticky prices without sticky wages leads to ”the divine coincidence”. They model wage stickiness on a microeconomic foundation similar to the way Calvo did it. Here, a worker have the probability of keeping their old wage from the previous period, measured as θ_w. If there is no wage stickiness, θ_w is zero and the central banks will not face a trade off. The fact that central banks can theoretically stabilize inflation without affecting output is not observed empirically. Central banks do in fact consider the trade-off between stabilizing inflation and affecting output. Measuring the wage stickiness is therefore important. This is what Barattieri et al. (2010) does. They measure wages to be very sticky, with a probability for a worker to experience a nominal wage change between 5% and 18% every quarter. This is equivalent to θ_w being between 0.82 and 0.95. While the above authors only use American data, Le Bihan et al. (2012) study the wage stickiness in France. They find that workers on average have a 38% probability of a wage change each quarter. This is higher than the other study, but sticky prices still appear.

The price and wage stickiness have been thoroughly analyzed for the US economy. However, there are only a few papers focusing on analysing the price and wage stickiness in the Danish economy in recent years. Hansen and Hansen (2006) analyzes the price stickiness using Danish microdata. They estimate that 43% of prices change every quarter which equals θ_p being 57%. Unfortunately, the authors does not analyze wage stickiness. The same is true for Hansen et al. (2013), who also use microdata to estimate price stickiness. They find roughly the same estimates.

Because of the above, there seem to be a gap in the literature about estimating the wage stickiness in Denmark. Therefore, this paper contributes to the existing literature by estimating the wage stickiness alongside the price stickiness and other relevant parameters. I implement a simple macroeconomic model that can potentially be changed to fit different aspects of the Danish economy, such as being a small open economy, a Taylor rule taking account of the fixed exchange rate or maybe a different form of wage bargaining.

3 Macroeconomic model

One way to estimate price and wage stickiness is to fit theoretical impulse response functions to empirical ones. Therefore, I set up a theoretical microfounded macroeconomic model which I construct impulse responses from. I set up a model for firms, households and find the equilibrium. The equations I end up using in the final model is marked by a number in the following section. The model is heavily inspired from different chapters of Galí (2015). I refer to his book for a full discussion of the model and its implications.

3.1 Firms

I assume there is an infinite continuum of identical firms indexed by $i \in [0, 1]$ who each produces a good i and who all have the following production function:

$$Y_t(i) = A_t N_t(i)^{1-\alpha}$$

where $Y(i)$ is the output of a firm, A_t is a technology parameter identical for all firms and $N_t(i)$ is the index of labour input used by firm i. Taking the log of each side of the equation, I get:

$$y_t = a_t + (1-\alpha)n_t \tag{3.1}$$

I let lower case letters denote log variables. The exogenous technology shock is defined as an AR(1) process:

$$a_t = \rho^a a_{t-1} + \varepsilon_t^a \tag{3.2}$$

where $|\rho^a| < 1$ in order for the process to be stationary and $\varepsilon_t^a \sim N(0,1)$.
For the prices of the model, I use the same idea as Calvo (1983). In each period, each firm has a probability of $1-\theta_p$ of adjusting prices. This means that a fraction $1-\theta_p$ of firms reset prices to their optimal level while θ_p keeps the prices from the last period. Formally, the price in each period is then:

$$P_t = \left[\theta_p (P_{t-1})^{1-\epsilon_p} + (1-\theta_p)(P_t^*)^{1-\epsilon_p}\right]^{\frac{1}{1-\epsilon_p}}$$

Dividing by P_t on both sides, I get:

$$\frac{P_t}{P_{t-1}}^{1-\epsilon_p} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon_p}$$

Log-linearizing around steady state, I get the following price inflation as[1]:

$$\pi_t^p = (1-\theta_p)(p_t^* - p_{t-1}) \Leftrightarrow p_t = \theta_p p_{t-1} + (1-\theta_p)p_t^*$$

where $\pi_t^p = p_t - p_{t-1}$. To find the price inflation, I first need to find the optimal price level, p^*, that the firms set. The idea is that the firms maximize the future discounted stream of expected profits, where they consider the chance that the prices remain the same in the future:

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left\{ \frac{\Lambda_{t,t+k}}{P_{t+k}} \left(P_t^* Y_{t+k|t} - \mathcal{C}_{t+k}(Y_{t+k|t}) \right) \right\} \quad \text{s.t.} \quad Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon_p} C_{t+k}$$

where $\Lambda_{t,t+k} = \beta^k U_{c,t+k}/U_{c,t}$ is the stochastic discount factor, $\mathcal{C}_{t+k}$ is the cost of production and Y_{t+k} is the output in period $t+k$ for a firm that has reset their prices. Inserting this into the optimal price and rewriting, one can obtain[2]:

$$\pi_t^p = \beta E(\pi_{t+1}^p) - \lambda_p \hat{\mu}_t^p$$

where $\lambda_p = \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}$, $\hat{\mu}_t^p = \mu_t^p - \mu^p$ and where:

$$\mu_t^p = -(w_t - p_t) + \frac{1}{1-\alpha}(a_t - \alpha y_t) - \log(1-\alpha)$$

[1] See Appendix A, 9.1
[2] See Appendix A, 9.2

3.2 Households

I assume there is an infinite number of identical households in the economy. In a household, each member can specialize in a different labour service indexed by $j \in [0,1]$. Each household maximizes the following with a budget constraint:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N(j)_t^{1+\varphi}}{1+\varphi} \right], \quad \sigma, \varphi > 0, \quad \sigma \neq 1$$
$$\text{s.t.} \quad \int_0^1 P_t(i) C_t(i) di + Q_t B_t \leq B_{t-1} + \int_0^1 W_t(j) N_t(j) dj + D_t$$

where $P_t(i)$ is price of a good, Q_t is the bond price, B_t is a one-period discount bond, W_t is the nominal wage, $N_t(j)$ is employment and D_t is dividends from owning a firm. Furthermore, the household consumes a number of different goods given as $C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$. Using the definitions that $\rho \equiv -\ln \beta$, $i_t \equiv -\ln Q_t$ and $\pi_{t+1} \equiv p_{t+1} - p_t$ one can find the Consumption-Euler equation[3]:

$$c_t = E_t(c_{t+1}) - \frac{1}{\sigma}(i_t^{ex} - E_t(\pi_{t+1}^p) - \rho)$$

The households are gathered in unions with bargaining power. The unions will seek to maximize the following:

$$E_t \sum_{k=0}^{\infty} (\beta \theta_w)^k \left(C_{t+k}^{-\sigma} \frac{W_t^*}{P_{t+k}} N_{t+k|t} - \frac{N_{t+k|t}^{1+\varphi}}{1+\varphi} \right) \quad \text{s.t.} \quad N_{t+k|t} = \left(\frac{W_t^*}{W_{t+k}} \right)^{-\epsilon_w} \left(\int_0^1 N_t(i) di \right)$$

where W_t^* is the optimal price that the unions want to set. Constructing the wage similarly to the Calvo parameters, the wage in each period can be written as:

$$W_t \equiv \left(\theta_w W_{t-1}^{1-\epsilon_w} + (1-\theta_w)(W_t^*)^{1-\epsilon_w} \right)^{\frac{1}{1-\epsilon_w}}$$

which when log linearized becomes:

$$w_t = \theta_w w_{t-1} + (1-\theta_w) w_t^*$$

Taking the FOC of the unions maximization problem, log-linearizing and using the above equation, one can find that the wage inflation becomes[4]:

$$\pi_t^w = \beta E_t(\pi_{t+1}^w) - \lambda_w \hat{\mu}_t^w$$

where $\lambda_w = \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w \varphi)}$, $\hat{\mu}_t^w \equiv \mu_t^w - \mu^w$ and:

$$\mu_t^w \equiv (w_t - p_t) - \sigma c_t + \varphi n_t$$

[3] See Appendix A, 9.3

[4] See Appendix A, 9.4

3.3 Equilibrium

I define the output gap as the actual output minus the natural output:

$$\tilde{y}_t \equiv y_t - y_t^n \tag{3.3}$$

Using that $w_t - p_t = \sigma y_t + \varphi n_t$[5], equation (3.1) and rewriting, I get:

$$\mu_t^p = -(\sigma y_t + \varphi n_t) + \frac{1}{1-\alpha}(a_t - \alpha y_t) - \log(1-\alpha)$$
$$\mu_t^p = -\sigma y_t - \frac{\varphi}{1-\alpha}y_t + \frac{\varphi}{1-\alpha}a_t + \frac{1}{1-\alpha}a_t - \frac{\alpha}{1-\alpha}y_t - \log(1-\alpha))$$
$$\mu_t^p = -\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)y_t + \frac{1+\varphi}{1-\alpha}a_t - \log(1-\alpha))$$

In the steady state, the price markup is constant, $\mu_t^p = \mu^p$, and y_t^n becomes:

$$y_t^n = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}a_t - \log(1-\alpha) - \mu^p = \psi_{ya}a_t - \log(1-\alpha) - \mu^p \tag{3.4}$$

where $\psi_{ya} \equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$. In the steady state, one can ignore the constants. I define the wage gap as the nominal wage minus the natural wage:

$$\tilde{\omega}_t \equiv \omega_t - \omega_t^n \tag{3.5}$$

where $\omega_t \equiv w_t - p_t$ is the real wage. The natural real wage can be found as:

$$\mu^p = -\omega_t^n + \frac{1}{1-\alpha}\left(a_t - \alpha(a_t + (1-\alpha)n_t^n)\right) - \log(1-\alpha)$$
$$\Leftrightarrow \omega_t^n = (a_t + \alpha n_t^n) - \log(1-\alpha) - \mu_t^p$$

where I use $n_t^n = \frac{1}{(1-\alpha)}\left(y_t^n - a_t^n\right)$, that $y_t^n = \psi_{ya}a_t$ in steady state and disregarding the constant, I get:

$$\omega_t^n = \psi_{wa}a_t - \log(1-\alpha) - \mu^p \tag{3.6}$$

with $\psi_{wa} \equiv \frac{1-\alpha\psi_{ya}}{1-\alpha}$. At last, I can now find an expression for the price inflation:

$$\hat{\mu}_t^p = -w_t + p_t + \frac{1}{1-\alpha}\left(a_t - \alpha y_t\right) + \log(1-\alpha) - \mu^p$$
$$\hat{\mu}_t^p = -\tilde{\omega}_t - \log(1-\alpha) - \psi_{wa}a_t + \mu^p + \frac{1}{1-\alpha}\left(a_t - \alpha y_t\right) + \log(1-\alpha) - \mu^p$$
$$\hat{\mu}_t^p = -\tilde{\omega}_t - \frac{1-\alpha\psi_{ya}}{1-\alpha}a_t + \frac{1}{1-\alpha}a_t - \frac{\alpha}{1-\alpha}y_t$$
$$\hat{\mu}_t^p = \frac{\alpha}{1-\alpha}\tilde{y}_t - \tilde{\omega}_t$$

[5] Defined in Appendix A, 9.3

Inserting this back into the price inflation equation, I get:

$$\pi_t^p = \beta E(\pi_{t+1}^p) + \frac{\alpha}{1-\alpha}\lambda_p \tilde{y}_t - \lambda_p \tilde{\omega}_t \tag{3.7}$$

A similar approach can be done for the wage equation, which becomes:

$$\pi_t^w = \beta E_t(\pi_{t+1}^w) + \left(\sigma + \frac{\varphi}{1-\alpha}\right)\lambda_w \tilde{y}_t - \lambda_w \tilde{\omega}_t \tag{3.8}$$

Furthermore, there is an equation which connects real wage gap, inflation and the natural real wage gap derived as:

$$\omega_t - \omega_t = \omega_{t-1} - \omega_{t-1} + \omega_t^n - \omega_t^n + \omega_{t-1}^n - \omega_{t-1}^n$$

$$\Leftrightarrow \tilde{\omega}_t = \tilde{\omega}_{t-1} + \pi_t^w - \pi_t^p - \Delta\omega_t^n \tag{3.9}$$

In equilibrium, output and consumption are equal, $y_t = c_t$, and the Consumption-Euler equation from earlier can be written as:

$$y_t - y_t^n = -\frac{1}{\sigma}(i_t - E_t(\pi_{t+1}^p) - \rho) + E_t(y_{t+1}) - y_t^n$$

$$\tilde{y}_t = -\frac{1}{\sigma}(i_t - E_t(\pi_{t+1}^p) - \rho) + E_t(\tilde{y}_{t+1}) - \psi_{ya} a_t + E_t(\psi_{ya}\rho_a a_t)$$

$$\tilde{y}_t = -\frac{1}{\sigma}(i_t - E_t(\pi_{t+1}^p) - r_t^n) + E_t(\tilde{y}_{t+1}) \tag{3.10}$$

with:

$$r_t^n = \rho - \sigma\psi_{ya}(1-\rho_a)a_t \tag{3.11}$$

Finally, I postulate a simple Taylor rule where the interest depends on the price inflation, output gap and a stochastic term:

$$i_t = \phi_\pi \pi_t^p + \phi_y \tilde{y}_t + i_t^{\text{ex}} \tag{3.12}$$

with:

$$i_t^{\text{ex}} = \rho_i i_{t-1}^{\text{ex}} + \varepsilon_t^i \tag{3.13}$$

In total, I have 13 equations and the following 13 endogenous variables:

$$a_t, \pi_t^p, \pi_t^w, y_t, y_t^n, \tilde{y}_t, n_t, \tilde{\omega}_t, \omega_t, \omega_t^n, r_t^n, i_t \text{ and } i_t^{\text{ex}}$$

with $\varepsilon_t^i, \varepsilon_t^a \sim N(0,1)$ as exogenous variables. Furthermore, I have the following 12 parameters:

$$\alpha, \beta, \sigma, \varphi, \theta_p, \theta_w, \rho_a, \rho_i, \phi_\pi, \phi_y, \epsilon_p, \epsilon_w$$

3.4 Solving the model in steady state

Since I want to measure how fast prices and wages return to steady state, I need to find the steady state of the model. Luckily, the model is already written in deviations from steady state and I can solve it by setting each variable equal to zero. From here, I can construct the impulse responses by shocking the exogenous variables and solving the system of 13 equations and 13 variables. For a given horizon, I can repeatedly calculate the impulse responses. In practice, I use 'Dynare' to solve the model[6].

4 Impulse response matching

4.1 Structural impulse functions

In the theoretical model, the unanticipated inflation shock and technology shock are modeled as exogenous and the impulse responses can therefore be interpreted as a causality. This is not necessarily true for the empirical impulse responses. In a VAR(p) model with p lags, the error terms are correlated. Since an impulse response function from a variable 'x' to a variable 'y' is constructed by shocking the errors of 'x', one needs to have uncorrelated error terms. Otherwise, it is not possible to determine a causal effect. The most common way to implement this is to use the Cholesky decomposition and transform the VAR model to a Structural VAR model. This makes the error terms uncorrelated, and a shock from one variable to another can therefore be interpreted as a causal effect. However, this structural form brings two new problems:

- First, one needs to estimate a causal ordering for the variables. Since the Cholesky decomposition is lower triangular, the impulse responses will be estimated in a causal chain. In the theoretical model, each impulse response is calculated simultaneously and hence no causal chain is imposed.
- Second, the Cholesky decomposition implies that there are no contemporaneous effects from shocks ordered lower in the triangular matrix on shocks ordered higher. Again, this is not in line with this theoretical model in which we assume that shocks can affect other variables in the same period. Using the Cholesky decomposition will therefore ignore any contemporaneous effects which is not desired.

One solution to these problems would be to adjust the theoretical model to include no contemporaneous effects and a causal chain. However, one would need to make new assumptions on how the microeconomic foundation is constructed. Changing the economic model to fit the empirical observations risks compromising the theoretical framework[7].

[6]Code can be found here: https://github.com/peterlravn/sticky_wages_denmark
[7]See Appendix B for a full derivation of the impulse responses and sign restrictions

Instead of changing the microfounded model, I propose to estimate the model using sign restrictions suggested by Uhlig (2005). This is also done in Jørgensen and Ravn (2022) but only as a robustness check. The idea behind sign restrictions is to constructs random orthogonal matrices and multiply them onto the Cholesky decomposition. From this, I only accept an impulse response if their sign is in accordance with economic theory. Using sign restrictions solve the two problems above. First, the structure is no longer lower triangular which results in the causal chain to disappear. Second, there will be contemporaneous effects between all variables since the shocks are no longer ordered. However, it does introduce new problems discussed in section 7.

4.2 Matching theoretical to empirical impulse responses

To match the empirical impulse responses to the theoretical impulse responses, I follow the approach proposed by Christiano et al. (2005). I subset the 12 parameters into two groups: one that I calibrate and one that I estimate. I calibrate the first group $\omega_1 = \{\beta, \alpha\}$ and estimate the second group $\omega_2 = \{\sigma, \varphi, \theta_p, \theta_w, \rho_a, \rho_i, \phi_\pi, \phi_y, \epsilon_p, \epsilon_w\}$. I set $\alpha = 0.25$ which means a capital income of 25% and I set $\beta = 0.99$ which gives an annualized interest rate in steady state of 4%[8]. These calibrations are widely used in many macroeconomic models and makes it easier to compare results. For the parameters in ω_2, I estimate them by minimizing the following function:

$$\hat{\omega}_2 = \arg\min_{\omega_2} \left(\Lambda(\omega_2) - \hat{\Lambda}\right)' W \left(\Lambda(\omega_2) - \hat{\Lambda}\right)$$

where $\Lambda(\omega_2)$ is a $hs \times 1$ vector containing s theoretical impulse responses over a horizon of h periods. $\hat{\Lambda}$ is a vector of the same size that contains the same empirical impulse responses. The idea is to find values of ω_2 that minimizes the distance between the theoretical and empirical impulse responses.

W is a weighting matrix that can be set depending on what feature of the model is most important. Since contemporaneous effects are important in this model, I propose a weighting matrix that puts more weight on horizons that are low. Therefore, I construct a weight such that each horizon, h, is weighted with $w_h = 1/h$. This way, I weight the impulse of the first horizon at 1, the second at 0.5, the third at 0.33... etc. I use 20 horizons for the estimation, however the exact number of horizons is not important because I put most weight on low horizons.

4.3 Determining signs

To match empirical and theoretical impulse responses, one would ideally use an empirical variable for each theoretical variable in the system of equations. However, some theoretical variables are hard to obatain empirically and some might not even be estimable. For example, there is no common consensus on how to estimate the output gap, and there only exist proxies for the expected inflation.

[8] Since $(1 + \frac{1-0.99}{0.99})^4 - 1 \approx 0.04$

Therefore, I choose the following six variables in the sign restricted VAR framework as they are easy to obtain: Interest rate i_t, productivity (a shock to productivity is interpreted as a technology shock) a_t, price inflation π_t^p, wage inflation π_t^w, output y_t and employment n_t. From these, I construct an interest rate shock and a technology shock.

Table 1: Percentage share of positive and negative response to an interest rate and technology shock

Interest rate shock: i_t

Response variable	π_t^p	π_t^w	y_t	n_t
Positive response	0	0	0	0
Negative response	512	512	512	512

Technology shock: a_t

Response variable	π_t^p	π_t^w	y_t	n_t
Positive response	0	86	512	120
Negative response	512	426	0	392

Note: There are 9 parameters that can affect the responses to each shock, hence the total number of simulations are $2^9 = 512$

To determine the signs of the empirical impulse responses, I use the signs suggested by the theoretical model. However, it is immediately not easy to see if a variable increases, decreases or is undetermined when a positive interest rate or technology shock appears. To determine this, I give each parameter a reasonable upper and lower bound (see Table 4) and simulate impulse responses at each bound for an interest and technology shock[9]. The responses to each shock can be seen in Table 1.

Table 2: Restrictions on variables with signs restrictions

Variable	Interest rate shock	Technology shock
Interest rate	$+$	$*$
Productivity	$*$	+
Price inflation	$-$	$-$
Wage inflation	$-$	$*$
GDP	$-$	+
Employment level	$-$	$*$

Note: "+" indicates a positive shock, "-" indicates a negative shock and "$*$" indicates a non-binding shock

For the positive interest rate shock, the responses of price inflation, wage inflation, GDP and employment are all negative. A positive interest rate shock should be positively correlated with the interest rate and have no correlation with productivity. For the technology shock, it decreases price inflation and GDP for all bounds. However, the effect on wage inflation and employment are not clear. Therefore, these are modeled as unrestricted shocks. The sign restriction from the two shocks

[9]One can discuss what a "reasonable" bound is. This could be a study by itself, so I set it in line with other papers mentioned in this paper. Furthermore, the bounds all satisfy the Blanchard-Kahn condition

can be seen in Table 2.

This methodology assumes that the theoretical model is the true model. This might not be the case. Therefore, I also use a Cholesky decomposition as a robustness check but I did not find that the model fit any better[10].

5 Data

To fit the VAR model, I use quarterly Danish data from 1996Q1 to 2023Q3. This results in 111 quarterly observations. The variables are the following:

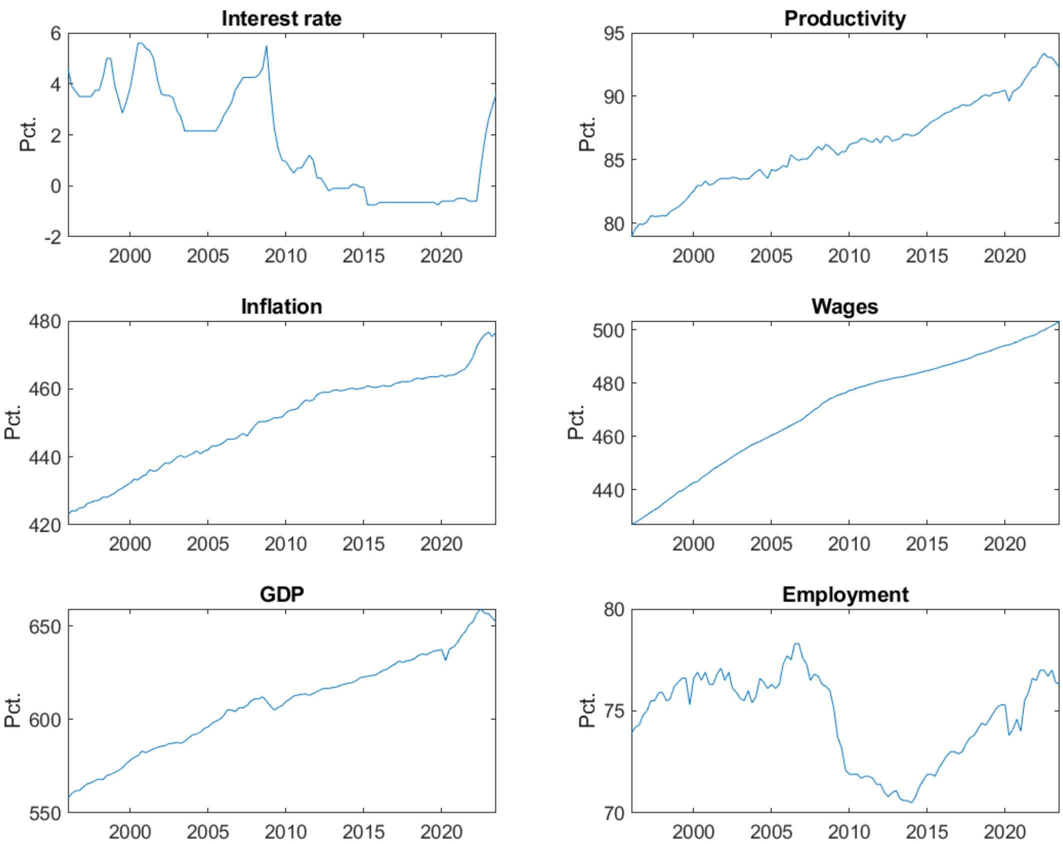
- Interest rate: Calculated as the quarterly mean of the daily deposit rates by the Central Bank of Denmark.
- Productivity: Computed as the log of seasonally adjusted gross value added divided by log of hours worked.
- Price inflation: Derived from the monthly consumer price index (CPI). February, May, August, and November represent the four quarters.
- Wage inflation: Based on the seasonally adjusted implicit wage index for the private sector.
- GDP: Seasonally adjusted gross domestic product.
- Employment: Measured by the labor force participation rate (percentage of full-time workers in the population).

I take logs of each variable not already in percentages and take the first difference to get deviations from steady state. The variables in levels are shown in Figure 1. Each variable appears to either be random walks, $I(1)$, or trending, hence taking first differences will make them all $I(0)$[11].

[10] See Appendix C

[11] Using the Augmented Dicky-Fueller test, I find each variable to be non-stationary

Figure 1: Empirical variables in log-levels or percent



Note: Productivity, inflation, wages and GDP are all in log-levels. The interest rate and employment are in percent. I use the following time series from Statistics Denmark to construct the variables: AKU101K, AKU123, ILON9, ILON12, NKN1, PRIS113, NKHO2.
Source: Statistics Denmark, Danmarks Nationalbank and own calculations.

6 Results

6.1 Fitting a VAR model

To fit a VAR model using the data above, I need to determine the number of lags. To do so, I use a bottom-up testing procedure suggested in Kilian and Lütkepohl (2017), where I increase the lag length until the residuals no longer suffer from autocorrelation. The test for autocorrelation along misspecification test for heteroskedasticity and normality can be found in Table 3.
Using the Edgerton-Shukur test, I reject the null that the residuals are autocorrelated for a VAR(4). I use the Doornik-Hendry test for multivariate heteroskedasticity but again reject the null hypothesis that the residuals are heteroskedastic. At last, I test if the residuals are normally distributed, using the tests suggested in Doornik and Hansen (2008) or Lütkepohl (2005). I reject that the errors are

Table 3: Statistical Tests Summary for Lag Length 4

Multivariate LM test for autocorrelation

Measure	Breusch-Godfrey	Edgerton-Shukur
Test statistic	196.5	1.073
P-value	0.00	0.30
Lag order	4	4

Multivariate test for heteroskedasticity

Test	Doornik-Hendry
Test statistic:	1772.2
P-value	0.44
Degrees of freedom	1764

Multivariate normality test

Test	Doornik-Hansen	Lutkepohl
Joint test statistic:	430.803	216.7
P-value	0	0
Degrees of freedom	12	12
Skewness only	42.30	27.26
P-value	0.00	0.00
Kurtosis only	388.5	189.4
P-value	0	0

normally distributed.
At last, I need to check that the model is stable. I find the norm of the largest eigenvalue of the companion matrix to be 0.91, which indicates that the model is stable. All in all, the VAR(4) model only appears to be misspecified when it comes to the normality of the residuals. However, this is not necessarily a problem since it will only affect the confidence levels and the point estimate will still be correct[12].

6.2 Shocking the interest rate

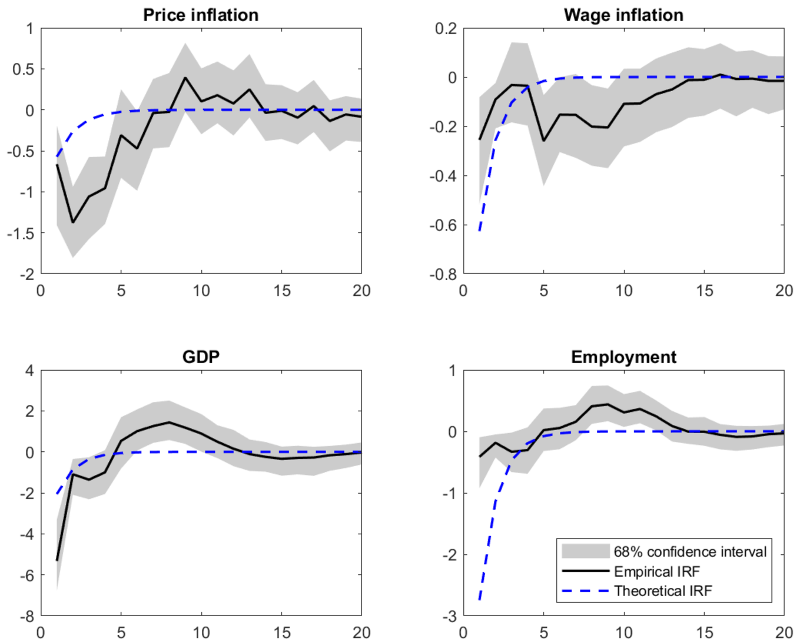
After finding a sufficient VAR model, I construct impulse responses using the sign restrictions. Using the minimizer stated earlier and matching price inflation, wage inflation, GDP and employment to an interest rate shock, I find the empirical and theoretical impulse responses seen in Figure 2.

For price inflation, the fit is almost identical in the first period. However, the decrease that follow from period 2 to 5 in the empirical response is not captured in the theoretical one. The quick return to zero for the wage inflation is captured well by the theoretical model. However, the decrease from period 5 to 15 is not entirely captured by the model. The theoretical responses are just slightly outside the 68% confidence interval. The impulse of GDP is well captured somewhat within the model, where the periods from 7 to 10 are outside the confidence interval. At last, the theoretical model

[12] Kilian and Lütkepohl (2017) argues that the asymptotic gains of modeling the errors correctly are insignificant in small samples

overshoots the negative response of employment in the first 2 periods. However, the remaining periods fit well within the 68% confidence interval

Figure 2: Empirical and theoretical impulse response functions to an interest rate shock



Note: All shocks are normalized such that the interest rate is increased by 1 unit.

The matched parameters of the theoretical model can be seen in Table 4. The Calvo parameter for prices, θ_p, is estimated to be 0.37. In this model, it means that there is a probability of 63% that a firm changes prices in each period. Furthermore, it means that prices update on average every $1/(1-0.37) = 1.58$ quarter. This is a bit lower than other literature such as Gali et al. (2001), who find that prices change between every 3 to 4.3 quarter for the EU area. For the US, Jørgensen and Ravn (2022) find that prices change every 2.5 quarters, which is also slightly higher than I find. The wages are even more sticky with a Calvo parameters for wages, θ_w, which is estimated to be 0.48. This means that wages adjust on average every $1/(1-0.48) = 1.92$ quarter. However, this is much lower than authors such as Barattieri et al. (2010), who find that wages reset from every 5 quarter to every 20 quarter. Jørgensen and Ravn (2022) find that wages change every 6.5 quarter. However, both these authors are only using data up to 1999 or 2008 and they use data for the US, so the estimates are not comparable one to one.
Of further interest, we see that both the price and wage elasticity, respectively ϵ_p and ϵ_w are 4.44 and 4.42. This means that the price and wage markup is about 28%. Galí (2015) notes that a price markup of 12.5% and a wage markup of 28% is "broadly similar to those found in the business cycle literature". The price markup appears to be too high while the wage markup matches the

those found in the business cycle literature. The inverse of the Frisch labor supply, φ, is hitting the upper bound of the model at 3. This gives a Frisch labor supply of 0.33, which means that if wages increase by 10%, the labor supply would increase by 3.3%. In macroeconometric literature, the Frisch elasticity is often too high compared to microeconomtric frameworks as noted by Chetty et al. (2011). At last, it should be noted that there is almost no interest rate response to a change in the output gap, measured by ϕ_y. It is 0.04 and almost hitting the lower bound. Furthermore, there are almost no to a change to inflation, measured by ϕ_π. It is 1.01 and hits the lower bound. This means that the central bank does not react much to changes in inflation and to changes in output.

Table 4: Parameter estimates based on an interest rate shock, technology shock and both shocks

Parameter	Lower bound	Upper bound	Interest rate shock	Technology shock	Both shocks
σ	0.50	5	0.5	0.5	0.50
φ	0.50	3	3	0.5	0.50
θ_p	0	0.99	0.37	0	0.36
θ_w	0	0.99	0.48	0.92	0.93
ρ_a	0.01	0.99	*	0.91	0.92
ρ_i	0.01	0.99	0.43	*	0.04
ϕ_π	1.01	3	1.01	3	3
ϕ_y	0.01	0.50	0.04	0.01	0.01
ϵ_p	0.10	9	4.44	1.76	5.260
ϵ_w	0.10	9	4.42	4.02	6.190

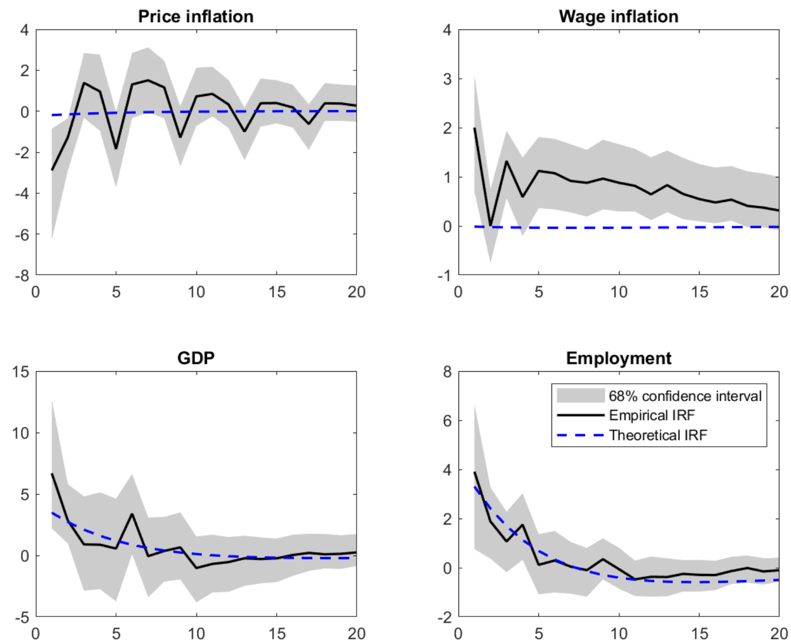
Note: A "$*$" indicates that the variable is not affected by the shock and therefore cannot be estimated. The columns named "Interest rate shock", "Technology shock" and "Both shocks" include the parameter estimates for each different fit to the different shocks.

6.3 Shocking technology

When matching impulse responses to a theoretical model, it is common to only shock one variable and estimate the parameters from that single shock. This is the approach of Christiano et al. (2005) and Jørgensen and Ravn (2022). The approach is understandable since two different shocks (or more) should in theory produce the same parameter values. However, in practice this might not be true. If the estimated parameters are not the same for the two shocks, it could mean the following: either the theoretical model could be wrong in such a way that different reactions to shocks are not modeled properly, or the empirical shock could be misrepresented. Therefore, I suggest matching the model on a technology shock too in order to check the robustness of the results. In Figure 3, I show the empirical and theoretical impulse responses to a technology shock.

The dynamics of the impulse responses for price inflation, GDP and employment are captured rather well by the theoretical model. Price inflation starts negative, by construction, but quickly returns to zero in quarter 3. From here, it oscillates around zero with confidence bands including zero. For GDP, the theoretical impulse response is slightly lower in the first period, but it lies within the confidence bands for the whole period. The same is true for employment. However, for wage inflation the theoretical impulse responses do not fit to the empirical. The wage inflation immediately jumps to zero in quarter two but then increases again to one. From here, it slowly returns to zero over the next 20 periods, only becoming insignificant at quarter 20. The theoretical model is not able to capture the slow decline towards zero very well, but instead it lies at zero for the whole horizon.

Figure 3: Empirical and theoretical impulse response functions to a technology shock



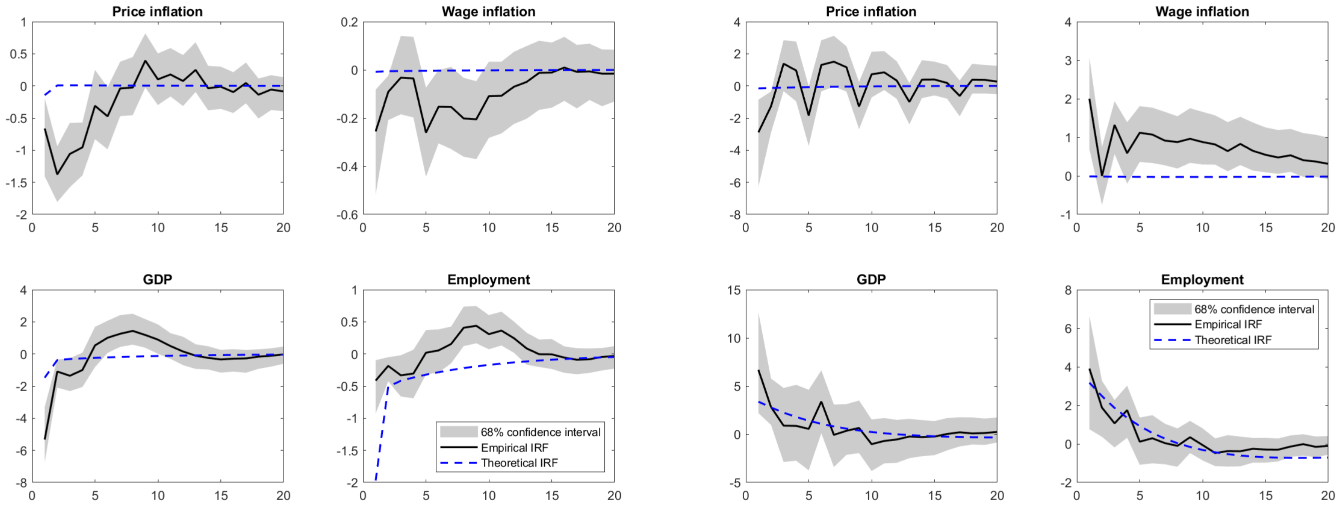
Note: All shocks are normalized such that productivity is increased by 1 unit.

Even though the theoretical model does not fit wage inflation that well, I still report the parameter estimates in Table 4. These estimates differ from the interest rate shock. The Calvo parameter for price inflation has become zero, while the Calvo parameter for wages have increased to 0.92. This means that prices reset every quarter while wages reset on average every 12.5 quarters or every 3.5 years. This change in price and wage stickiness is large and shows how important it is to include more than one shock for robustness checks. Furthermore, it should be noted that the inverse Frisch elasticity changes from 2.99 to 0.5, which results in a Frisch elasticity of 2. Whalen and Reichling (2015) performs a literature review in which they find that the Frisch elasticity is between 0.27 to 0.53. A Frisch elasticity of 2 therefore seems to be too large. The central now responds highly to inflation with ϕ_π being 3 and hitting the upper bound while not reacting at all to the output gap.

6.4 Shocking interest rate and technology

At last, I suggest fitting the theoretical model to both the interest rate shock and the technology shock. I get the responses shown in Figure 4. The theoretical interest rate shock now fit rather poorly on the empirical interest rate shock, while the impulse to technology shock have not changed.

Figure 4: Impulse responses to an interest rate shock (left side) and a technology shock (right side)



Note: All shocks are normalized such that the interest rate is increased by 1 unit.

Note: All shocks are normalized such that productivity is increased by 1 unit.

Fitting on both shocks, I now get that only the Calvo parameter of price changed from zero to 0.36. This now implies that prices change on average every 1.5 quarter. There is no other major parameter difference between fitting on the technology shock versus fitting on both shocks.

7 Discussion

The basis of the analysis relies on the assumption that the theoretical model is the true model. This is contrary to many other macroeconomic models which assumes that the true model is the empirical model. This identification has its strength and weaknesses. On one hand, it becomes possible to construct very tight sign restrictions based on a microeconomic foundation. This is in line with the reasoning of Uhlig (2017), who says the following when it comes to imposing sign restrictions: ”if you know it, impose it!”. On the other hand, any puzzling behavior is not captured by the model. For example, if employment always increase to an unanticipated interest rate shock in the true data generating process, this would not be captured by the model. The question then becomes: do we believe more in the theoretical model or in the empirical model?

To unfold this question, I also construct the empirical impulse responses from the Cholesky decomposition where the technology shock is ordered first, and the interest rate shock is ordered last. This approach is common in macroeconomic literature because it is easy to implement and easy to interpret. Using the same estimation method as above, I find that this structural identification does not fit the model any better. Since it is difficult to tell whether the theoretical model is wrong, the empirical one is or if both are, I propose possible options for further research that might increase the accuracy of the estimation.

Since Denmark is a small open economy, an obvious extension of the theoretical model is to extend it from being a closed economy to a small open economy. In the open economy, monetary policy and stabilization policies are affected by the rest of the world. Here, one would need to model how the central bank construct their Taylor rule with a fixed exchange rate in mind. Furthermore, one would need to estimate how shocks from the rest of the world would affect the economy, such as the financial crisis in 2008-09, Brexit or the war in Ukraine in 2022. In practice, the small open economy is much more complex than the closed economy and the model would then need to be greatly altered. A smaller change to the model could include changing the wage formation to include the large public sector in Denmark, which could possibly better capture the slow return to steady state of the wage inflation.

On the empirical side, an often used solution to misspecification is to include more variables in the model. This could be everything from government spending, taxes, central bank assets, private consumption etc. Including missing variables can often solve puzzling observations in the empirical models. Another option could be to change the identification to include both short and long run restrictions, or one could use high frequency data to construct exogenous shocks. This could help by identifying the exogenous shock more precisely.

8 Final remarks

Even though the theoretical and empirical model does not match particularly well, I still argue that it shows how important sticky wages are in the Danish economy. However, one should be cautious when interpreting the exact value of the price and wage stickiness in this model. This paper should be seen as a baseline model that can be improved rather than an exact identification of the Danish economy.

9 Appendix A

9.1 Log-linearizing prices

I have the following equation I want to log-linearize around the steady state denoted with bars. In the steady state, it is assumed that $P_{t-1} = P_t^* = P_t$. Initially, I have:

$$\frac{P_t}{P_{t-1}}^{1-\epsilon_p} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon_p}$$

Taking logs, I get:

$$(1-\epsilon_p)\log\left(\frac{P_t}{P_{t-1}}\right) = \log\left(\theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon_p}\right)$$

First, I log-linearize the left hand side as:

$$(1-\epsilon_p)\log\left(\frac{\bar{P}_t}{\bar{P}_{t-1}}\right) + \frac{1-\epsilon_p}{\frac{\bar{P}_t}{\bar{P}_{t-1}}}\left(\frac{P_t}{P_{t-1}} - \frac{\bar{P}_t}{\bar{P}_{t-1}}\right) = (1-\epsilon_p)\left(\frac{\bar{P}_t}{\bar{P}_{t-1}} - 1\right) = (1-\epsilon_p)\frac{P_t}{P_{t-1}} - (1-\epsilon_p)$$

where I use that $\frac{\bar{P}_t}{\bar{P}_{t-1}} = 1$. Now, the right hand side becomes:

$$\log\left(\theta + (1-\theta)\left(\frac{\bar{P}_t^*}{\bar{P}_{t-1}}\right)^{1-\epsilon_p}\right) + \frac{(1-\theta)(1-\epsilon_p)\left(\frac{\bar{P}_t^*}{\bar{P}_{t-1}}\right)^{-\epsilon_p}}{\theta + (1-\theta)\left(\frac{\bar{P}_t^*}{\bar{P}_{t-1}}\right)^{1-\epsilon_p}}\left(\frac{P_t^*}{P_{t-1}} - \frac{\bar{P}_t^*}{\bar{P}_{t-1}}\right) =$$

$$\log(\theta + (1-\theta)) + \frac{(1-\theta)(1-\epsilon_p)}{\theta + (1-\theta)}\left(\frac{P_t^*}{P_{t-1}} - 1\right) = \frac{(1-\theta)(1-\epsilon_p)}{\theta + (1-\theta)}\left(\frac{P_t^*}{P_{t-1}} - 1\right)$$

Setting the left and right side equal to each other, I get:

$$(1-\epsilon_p)\frac{P_t}{P_{t-1}} - (1-\epsilon_p) = \frac{(1-\theta)(1-\epsilon_p)}{\theta + (1-\theta)}\left(\frac{P_t^*}{P_{t-1}} - 1\right)$$

$$\Leftrightarrow \frac{P_t}{P_{t-1}} - 1 = (1-\theta)\frac{P_t^*}{P_{t-1}} - (1-\theta) \Leftrightarrow \frac{P_t}{P_{t-1}} = (1-\theta)\frac{P_t^*}{P_{t-1}} + \theta$$

Taking logs and ignoring the constant θ as it is not relevant in the steady state, I get inflation:

$$\pi_t = (1-\theta)(p_t^* - p_{t-1})$$

where $\pi_t = \log(P_t) - \log(P_{t-1})$, $\log(P_t^*) = p_t^*$ and $\log(P_{t-1}) = p_{t-1}$.

9.2 Finding an expression of price inflation

Inserting the budget constraint into the profit maximization function, the FOC becomes:

$$0 = \sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \left\{ \Lambda_{t,t+k} \left[\left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} C_{t+k} + P_t^*(-\epsilon_p) \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon_p - 1} \frac{C_{t+k}}{P_{t+k}} - \Psi_{t+k|t}(-\epsilon_p) \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon_p - 1} \frac{C_{t+k}}{P_{t+k}} \right] \right\}$$

where $P_t^* = M\Psi_{t+k|t}$, $M \equiv \frac{\epsilon_p}{\epsilon - 1}$ and $\Psi_{t+k|t} = \mathcal{C}'_{t+k} \left(\left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon_p} C_{t+k} \right)$.
Using the expression for $Y_{t+k|t}$, the above can be rewritten as:

$$0 = \sum_{k=0}^{\infty} \theta^k E_t \left\{ \frac{\Lambda_{t,t+k}}{P_{t+k}} \left[Y_{t+k|t} - \epsilon_p Y_{t+k|t} + \Psi_{t+k|t} \epsilon_p \frac{Y_{t+k|t}}{P_t^*} \right] \right\} \Leftrightarrow$$

$$0 = \sum_{k=0}^{\infty} \theta^k E_t \left\{ \frac{\Lambda_{t,t+k}}{P_{t+k}} \left[1 - \epsilon_p + \epsilon_p \frac{1}{P_t^*} \Psi_{t+k|t} \right] \right\} \Leftrightarrow$$

$$0 = \sum_{k=0}^{\infty} \theta^k E_t \left\{ \frac{\Lambda_{t,t+k}}{P_{t+k}} \left[P_t^* - \mathcal{M}\Psi_{t+k|t} \right] \right\}$$

where I have used that $1 - \epsilon_p + \epsilon_p \frac{1}{P^*} \Psi_{t+k|t} = 0$ and $\mathcal{M} = \frac{1}{P_t^*}$. Rewriting this to:

$$0 = \sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,t+k} \frac{Y_{t+k|t}}{P_{t+k}} \left[\frac{P_t^*}{P_{t-1}} - \mathcal{M} MC_{t+k|t} \frac{P_{t+k}}{P_{t-1}} \right] \right\}$$

and using that in steady state the following holds:

$$\frac{P_t^*}{P_{t-1}} = 1, \quad \Pi_{t-1,t+k} = 1, \quad Y_{t+k|t} = Y, \quad MC = \mathcal{M}^{-1}, \quad \Lambda_{t,t+k} = \beta^k$$

One can then log-linearize the model and find:

$$p_t^* = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \left(\hat{mc}_{t+k|t} + p_{t+k} \right)$$

where $p_t^* \equiv \frac{P_t^* - P}{P_t}$ and $\hat{mc}_{t+k|t} \equiv \frac{MC_{t+k|t} - MC}{MC} = \left(MC_{t+k|t} - MC \right) \mathcal{M} = mc_{t+k|t} + \log \mathcal{M} = mc_{t+k|t} + \mu$. To find the marginal cost, I first define the total cost of a firm as $TC_t = W_t(P_t)^{-1} N_t(i)$. The marginal cost is therefore the first difference of this expression:

$$MC_t^p = \frac{\partial TC_t}{\partial Y_t} = \frac{\partial TC_t}{\partial N_t} = \frac{\partial TC_t}{\partial N_t} \left(\frac{\partial Y_t}{\partial N_t} \right)^{-1} = \frac{W_t}{P_t} \left(A_t (1 - \alpha) N_t(i)^{-\alpha} \right)^{-1}$$

Taking logs and using (3.1), one gets:

$$\hat{mc}_{t+k} = w_t - p_t - \frac{1}{1 - \alpha} (a_t - \alpha y_t) - \log(1 - \alpha) + \mu^p$$

which can be rewritten as:

$$\hat{mc}_{t+k|t} = \hat{mc}_{t+k} + \frac{\alpha\epsilon_p}{1-\alpha}(p_t^* - p_{t+k})$$

Inserting this into the log-linearized function for the optimal price, one can rewrite this to the following:

$$\pi_t^p = \beta E(\pi_{t+1}^p) + \lambda \hat{mc}_t^p = \beta E(\pi_{t+1}^p) - \lambda \hat{\mu}_t^p$$

I skip a large part of the derivation in this section as it is long and tedious, so I instead refer to the following note for a clear explanation of how to derive the price inflation:
https://github.com/peterlravn/sticky_wages_denmark/blob/main/Price%20inflation%20derivation.pdf

9.3 Deriving the consumption rule

The Lagrangian of the households maximazation problem can be solved as:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_t} = 0, &\quad C_t^{-\sigma} - \lambda_t P_t = 0 \Leftrightarrow C_t^{-\sigma} = \lambda_t P_t \\
\frac{\partial \mathcal{L}}{\partial N_t} = 0, &\quad N_t^{\varphi} - \lambda_t W_t = 0 \Leftrightarrow N_t^{\varphi} = \lambda_t W_t \\
\frac{\partial \mathcal{L}}{\partial B_t} = 0, &\quad \lambda_t Q_t - \beta E_t(\lambda_{t+1}) = 0 \Leftrightarrow \lambda_t Q_t = \beta E_t(\lambda_{t+1})
\end{aligned}$$

Where the following relationship can be log-linearized:

$$C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t}$$

$$\bar{C}^{\sigma}\bar{N}^{\varphi}(1 + \sigma c_t + \varphi n_t) = \frac{\bar{W}}{\bar{P}}(1 + w_t + p_t) \Leftrightarrow \sigma c_t + \varphi n_t = w_t - p_t$$

An expression for Q_t can be found in logs to be:

$$\begin{aligned}
Q_t = \beta E_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right) = \beta E_t\left(\frac{P_{t+1}^{-1}C_{t+1}^{-\sigma}}{P_t^{-1}C_t^{-\sigma}}\right) &= \beta E_t\left(\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{P_t^{-1}}{P_{t+1}}\right) \\
1 = E_t\left\{\exp\left[\ln\left(\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{P_t^{-1}}{Q_t P_{t+1}}\right)\right]\right\} \\
1 = E_t\left\{\exp\left[\ln(\beta) - \ln\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} - \ln Q_t - \frac{P_t^{-1}}{P_{t+1}}\right]\right\} \\
1 = E_t\left\{\exp\left[-\rho - \ln\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} + i_t - \pi_{t+1}\right]\right\}
\end{aligned}$$

where $\rho \equiv -\ln\beta$, $i \equiv -\ln Q_t$, $\Delta c_{t+1} = c_{t+1} - c_t \equiv \ln C_{t+1} - \ln C_t$, $\pi_{t+1} \equiv p_{t+1} - p_t \equiv \ln P_{t+1} - \ln P_t$.

In steady state, consumption is constant, $\Delta c = 0$, and the above equation becomes:

$$1 = E_t \left\{ \exp\left[-\rho + i - \pi\right] \right\}$$
$$\ln(1) = \ln(\exp\left[-\rho + i - \pi\right])$$
$$\rho = i - \pi$$

Taking a first-order Taylor approximation around the above equation gives us:

$$1 \simeq E_t \left\{ \exp(0) + \exp(0)\left[-\sigma(\Delta c_{t+1} - \Delta c) + (i_t - i) - (\pi_{t+1} - \pi)\right] \right\} \Leftrightarrow$$
$$1 = E_t \left\{ 1 - \sigma \Delta c_{t+1} + (i_t - i) - (\pi_{t+1} - \pi) \right\} \Leftrightarrow$$
$$1 = 1 - \sigma E_t(\Delta c_{t+1}) + i_t - E_t(\pi_{t+1}) - (i - \pi) \Leftrightarrow$$
$$\sigma E_t(\Delta c_{t+1}) = i_t - E_t(\pi_{t+1}) - \rho \Leftrightarrow$$
$$\sigma c_t = \sigma E_t(c_{t+1}) - i_t + E_t(\pi_{t+1}) + \rho \Leftrightarrow$$
$$c_t = E_t(c_{t+1}) - \frac{1}{\sigma}\left(i_t - E_t(\pi_{t+1}) - \rho\right)$$

9.4 Finding an expression of price inflation

The first order condition of the unions' maximizing problem becomes:

$$\sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left\{ N_{t+k|t} C_{t+k}^{-\sigma} \left(\frac{W_t^*}{P_{t+k}} - \frac{\epsilon_w}{\epsilon_w - 1} C_{t+k}^{\sigma} N_{t+k|t}^{\varphi} \right) \right\} = 0$$

This can be log-linearized to become:

$$w_t^* = \mu^w + (1 - \beta\theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left\{ mrs_{t+k|t} + p_{t+k} \right\}$$

where $\mu^w = \frac{\epsilon_w}{\epsilon_w - 1}$ and $mrs_{t+k|t} = \sigma c_t + \varphi n_{t+k|t}$. I use the following relationship:

$$mrs_{t+k|t} = mrs_{t+k} + \varphi(n_{t+k|t} - n_{t+k}) = mrs_{t+k} - \epsilon_w \varphi (w_t^* - w_{t+k})$$

Then, the optimal wage becomes:

$$w_t^* = \frac{1 - \beta\theta_w}{1 + \epsilon_w \varphi} \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left\{ \mu_w + mrs_{t+k} + \epsilon_w \varphi w_{t+k} + p_{t+k} \right\}$$

$$= \frac{1 - \beta\theta_w}{1 + \epsilon_w \varphi} \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left\{ (1 + \epsilon_w \varphi) w_{t+k} - \hat{\mu}_{t+k}^w \right\}$$

$$= \beta\theta_w E_t(w_{t+1}^*) + (1 - \beta\theta_w)\left(w_t - (1 + \epsilon_w \varphi)^{-1} \hat{\mu}_t^w \right)$$

where $\hat{\mu}_t^w \equiv \mu_t^w - \mu^w$ and $\mu_t^w \equiv (w_t - p_t) - \sigma c_t + \varphi n_t$. Isolating w_t^* in the wage equation for the steady state, I get:

$$\begin{aligned}
w_t - w_{t-1} &= \theta_w w_{t-1} - w_{t-1} + (1-\theta_w) w_t^* \\
\pi_t^w &= -(1-\theta_w) w_{t-1} + (1-\theta_w) w_t^* \\
\frac{\pi_t^w + (1-\theta_w) w_{t-1}}{(1-\theta_w)} &= w_t^* \\
\frac{\pi_t^w}{(1-\theta_w)} + w_{t-1} &= w_t^*
\end{aligned}$$

Inserting this into the optimal wage equation, one can get equation (3.3):

$$\begin{aligned}
\frac{\pi_t^w}{(1-\theta_w)} + w_{t-1} &= \beta\theta_w E_t\left(\frac{\pi_{t+1}^w}{(1-\theta_w)} + w_t\right) + (1-\beta\theta_w) w_t - \frac{1-\beta\theta_w}{1+\epsilon_w\varphi}\hat{\mu}_t^w \\
\frac{\pi_t^w}{(1-\theta_w)} + w_{t-1} &= \beta\theta_w E_t\left(\frac{\pi_{t+1}^w}{(1-\theta_w)}\right) + \beta\theta_w w_t + (1-\beta\theta_w) w_t - \frac{1-\beta\theta_w}{1+\epsilon_w\varphi}\hat{\mu}_t^w \\
\frac{\pi_t^w}{(1-\theta_w)} &= \beta\theta_w E_t\left(\frac{\pi_{t+1}^w}{(1-\theta_w)}\right) + w_t - w_{t-1} - \frac{1-\beta\theta_w}{1+\epsilon_w\varphi}\hat{\mu}_t^w \\
\frac{\pi_t^w - (1-\theta_w)\pi_t^w}{(1-\theta_w)} &= \beta\theta_w E_t\left(\frac{\pi_{t+1}^w}{(1-\theta_w)}\right) - \frac{1-\beta\theta_w}{1+\epsilon_w\varphi}\hat{\mu}_t^w \\
\frac{\theta_w\pi_t^w}{(1-\theta_w)} &= \beta\theta_w E_t\left(\frac{\pi_{t+1}^w}{(1-\theta_w)}\right) - \frac{1-\beta\theta_w}{1+\epsilon_w\varphi}\hat{\mu}_t^w \\
\pi_t^w &= \beta E_t\left(\pi_{t+1}^w\right) - \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w\varphi)}\hat{\mu}_t^w
\end{aligned}$$

10 Appendix B

Consider a VAR(p) model given as:

$$x_t = A_1 x_{t-1} + A_2 x_{t-2} + \ldots + A_p x_{t-p} + u_t$$

where:

$$A_i = \begin{bmatrix} a_{11,i} & \ldots & a_{1k,i} \\ \vdots & & \vdots \\ a_{k1,i} & \ldots & a_{kk,i} \end{bmatrix}, \quad x_t = \begin{bmatrix} x_{1,t} & \ldots & x_{k,t} \end{bmatrix}, \quad u_t = \begin{bmatrix} u_{1,t} & \ldots & u_{k,t} \end{bmatrix}$$

for $i \in (1, 2, ..., p)$ and where k is the number of variables. This VAR(p) model can be written on companion form as:

$$X_t = AX_{t-1} + U_t$$

with the usual companion form representation:

$$X_t = \begin{pmatrix} x_t \\ \vdots \\ x_{t-p} \end{pmatrix}, A = \begin{pmatrix} A_1 & A_2 & \cdots & A_{p-1} & A_p \\ I_k & 0 & & 0 & 0 \\ 0 & I_k & & 0 & 0 \\ \vdots & & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I_k & 0 \end{pmatrix}, U_t = \begin{pmatrix} u_t \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

The error terms are written in a ways such that $u_t|\mathcal{I}_{t-1} \xrightarrow{d} N(0, \Omega)$ with:

$$\Omega = E_{t-1}\left[\begin{pmatrix} u_{1,t} \\ \vdots \\ u_{k,t} \end{pmatrix}\begin{pmatrix} u_{1,t} & \cdots & u_{k,t} \end{pmatrix}\right] = \begin{pmatrix} \Omega_{11} & \cdots & \Omega_{k1} \\ \vdots & \ddots & \vdots \\ \Omega_{1k} & \cdots & \Omega_{kk} \end{pmatrix}$$

The contemporaneous effects in this model are not directly parametrized but are instead captured in the covariances of the error terms. This is important when one wants to construct impulse responses from one variable to another. The impulse responses can be found by representing the VAR(p) in a moving average representation. The moving average representation can be found by repeatedly substituting in X_t on X_{t-1} and left multiplying $J = [I_k, 0_{k\times k(p-1)}]$ which results in the following:

$$X_t = A^T X_0 + \sum_{i=0}^{T} A^i U_{t-i} \Leftrightarrow JX_t = JA^T X_0 + \sum_{i=0}^{T} JA^i JJ' U_{t-i}$$
$$\Leftrightarrow x_t = JA^T x_0 + \sum_{i=0}^{T} \Phi_i u_{t-i} \rightarrow x_t = \sum_{i=0}^{T} \Phi_i u_{t-i}$$

where $\Phi = JA^i J$. This holds for $T \to \infty$ if the absolute value of the largest eigenvalue of A are less than one. The impulse responses can then be calculated as:

$$\frac{\partial x_t}{\partial u_t'} = I_p, \quad \frac{\partial x_{t+1}}{\partial u_t'} = \Phi_1 \quad \frac{\partial x_{t+2}}{\partial u_t'} = \Phi_2, \ldots$$

Since u_t is non-diagonal, the error term includes effects from other shocks, and it is therefore not possible to interpret the impulse responses in a causal way. A solution to this is to construct the impulse responses as structural impulse responses by multiplying them by $I_k = B_0^{-1}B_0$:

$$x_t = \sum_{i=0}^{T} \Phi_i B_0^{-1} B_0 u_{t-i} = \sum_{i=0}^{T} \Theta_i v_{t-i}$$

where $\Theta_i = \Phi_i B_0^{-1}$ and $v_{t-i} = B_0 u_{t-i}$. Setting B_0^{-1} equal to a lower triangular matrix such that $\Omega = B_0^{-1}(B_0^{-1})'$ will make the new error terms orthogonal and variance-covariance matrix will be diagonal:

$$Var(v_t) = B_0 Var(u_t) B_0' = B_0 \Omega B_0' = B_0 B_0^{-1}(B_0^{-1})' B_0' = I_k$$

To construct sign restrictions, one multiply $\Phi_i B_0^{-1}$ by a random matrix Q' such that $Q'Q = I$. The new matrix is also orthogonal since:

$$Q'var(v_t) = Q'B_0 B_0^{-1}(B_0^{-1})'B_0'Q = I$$

The common way to construct the Q matrix is by using the Householder Transformation as proposed by Rubio-Ramírez et al. (2010). If the sign of the impulse response is as imposed, the impulse response is kept. Otherwise, it is discarded, and a new matrix is drawn.

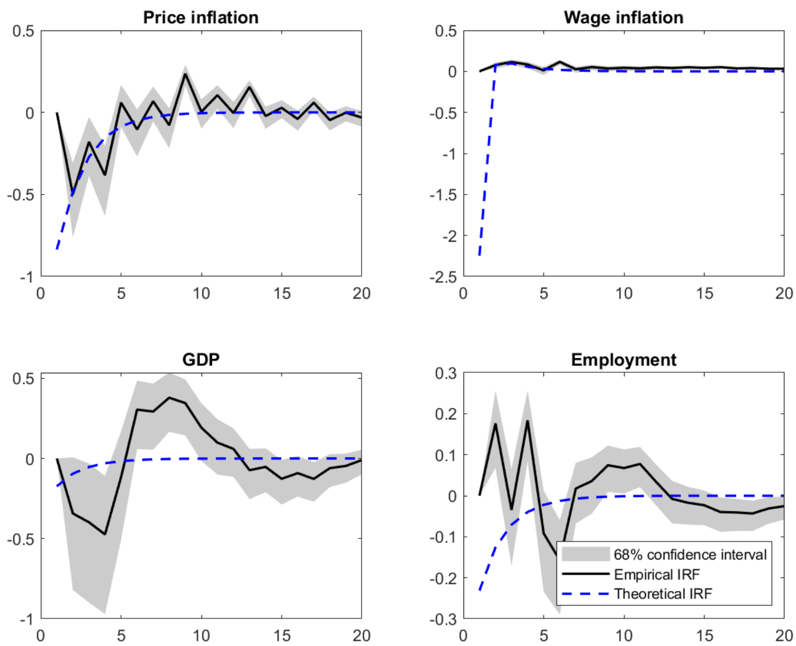
11 Appendix C

For the Cholesky decomposition, I order the variables the following way:

$$\text{Productivity} \rightarrow \text{Price inflation} \rightarrow \text{Wage inflation} \rightarrow \text{GDP} \rightarrow \text{Employment} \rightarrow \text{Interest rate}$$

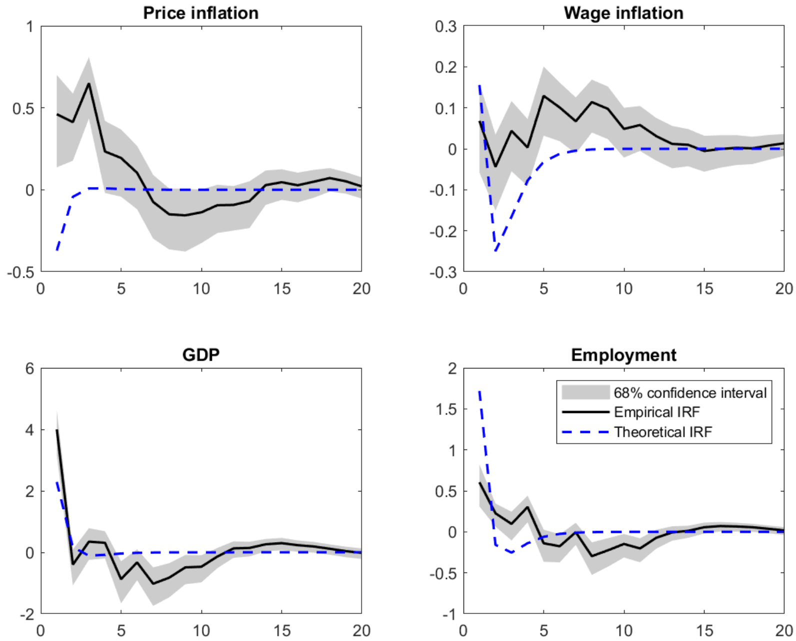
Fitting the theoretical model to the empirical model using the Cholesky decomposition, I get the following results:

Figure 5: Empirical and theoretical impulse response functions to an interest rate shock



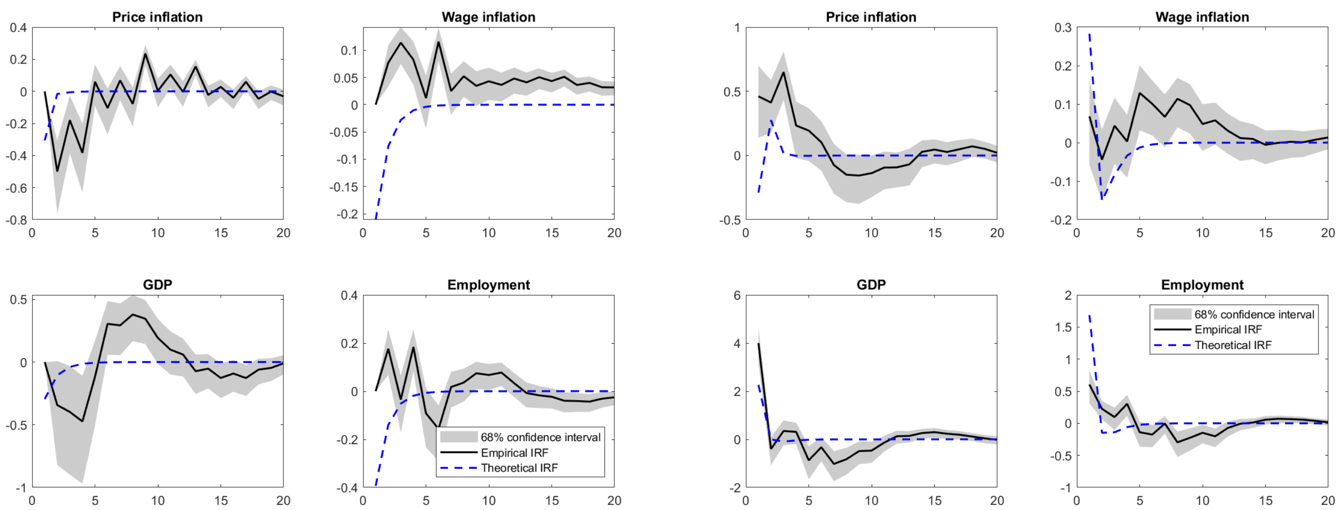
Note: All shocks are normalized such that the interest rate is increased by 1 unit.

Figure 6: Empirical and theoretical impulse response functions to an technology shock



Note: All shocks are normalized such that productivity is increased by 1 unit.

Figure 7: Impulse responses to an interest rate shock (left side) and a technology shock (right side)



Note: All shocks are normalized such that the interest rate is increased by 1 unit.

Note: All shocks are normalized such that productivity is increased by 1 unit.

Table 5: Parameter estimates based on an interest rate shock, technology shock and both shocks

Parameter	Lower bound	Upper bound	Interest rate shock	Technology shock	Both shock
σ	0.50	5	5	0.5	0.50
φ	0.50	3	3	0.5	0.50
θ_p	0.01	0.99	0.44	0.06	0.01
θ_w	0.01	0.99	0.02	0.41	0.37
ρ_a	0.01	0.99	*	0.29	0.11
ρ_i	0.01	0.99	0.56	*	0.99
ϕ_π	1.010	3	1.2	3	3
ϕ_y	0.01	0.50	0.48	0.01	0.01
ϵ_p	0.10	9	6.13	3.19	0.10
ϵ_w	0.10	9	0.59	4.17	4.860

Note: A ”$*$” indicates that the variable is not affected by the shock and therefore cannot be estimated. The columns named ”Interest rate shock”, ”Technology shock” and ”Both shocks” include the parameter estimates for each different fit to the different shocks.

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