Newton Day 2023:
Bernoulli

by Peter H. Mao and Chris Mindas
for Margaret and Owen

December 25, 2023

Abstract

For Sir Isaac Newton's 381st birthday we consider the contributions of one of his younger European contemporaries, Daniel Bernoulli, who experimentally *and* theoretically determined the relationship between flow speed and pressure in moving fluids. In Part I, Peter discusses Bernoulli's principle in the context of his original experiments, the "Bernoulli" vs "Newton" pedagogical controversy, and some easy but elucidating Pitot-static tube experiments you can try at home. In Part II, Chris presents an overview of lift and compressible flow to satisfy those who need equations to feel that Newton is properly celebrated.

Part I
Bernoulli, Newton and leaky pressure vessels

1 Introduction

For several years now, Deirdre and the kids have been requesting a Newton Day focused on airfoil lift and I have been telling them that I'm totally unqualified to do that. At the introductory level, I felt that it has already been endlessly hashed out, with excellent material on Wikipedia, NASA Glen Research Center's "Beginners Guide to Aeronautics" (BGA) [1], and anywhere else one would care to look. At the detailed level, airflow around solid objects is so complicated that I would not dare to compactify it into a December missive. And yet, because I have carried (for decades now) some *denkfehlers* regarding Bernoulli, I figured that this year's celebration of Newton would be an opportunity for me to sand down the rough edges of my understanding of Bernoulli's principle.

This year, as indicated in the byline, I enlisted the help of one of my classmates from MIT, Chris Mindas. On RDO Friday mornings, starting in October, Chris would lend me a few hours of his time to help me get unstuck from mental impasses as we worked our way through the physics teaching literature on Bernoulli. As mid-December approached, I lamented to Chris over email that I had to put "pen to paper" soon. He replied:

C'mon man, this is 2023 - that's a job for ChatGPT!

[Chris]: Write a description of Bernoulli's theorem.

ChatGPT: Bernoulli's theorem, named after the Swiss mathematician and physicist Daniel Bernoulli, is a fundamental principle in fluid dynamics that describes the behavior of an ideal fluid as it flows through a system. The theorem is derived from the principle of conservation of energy, stating that in a streamline flow of an inviscid, incompressible fluid, the total energy per unit mass remains constant along any streamlines.

The key components of Bernoulli's theorem include kinetic energy, potential energy, and pressure energy. These components contribute to the total energy of the fluid particle as it moves through a flow field. The theorem is often expressed mathematically as:

$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$

where:

P is the pressure of the fluid,
ρ is the density of the fluid,
v is the velocity of the fluid,
g is the acceleration due to gravity, and
h is the height of the fluid above a reference plane.

The equation reveals that as the fluid moves through different sections of a flow, changes in velocity are accompanied by corresponding changes in pressure and elevation. When the fluid speeds up (increasing velocity), its pressure decreases, and vice versa. This relationship helps explain phenomena such as lift in aerodynamics and the functioning of various fluid-based systems, like water pipes and blood vessels.

It is important to note that Bernoulli's theorem assumes idealized conditions, such as steady, incompressible, and frictionless flow, which may not be fully realized in real-world scenarios. Nonetheless, the theorem provides valuable insights into fluid behavior and has widespread applications in engineering and physics. ChatGPT can make mistakes. Consider checking important information.

[Chris]: Looks good to me - Done!

Thank you, AI, for lifting the burden of a coherent introduction. What's left is a bit of historical background and untangling the confusions of my squishy grey-matter.

2 Bernoulli's *Hydrodynamicae*

We associate Bernoulli's principle with fluids and gases in both constrained (e.g., pipe) and open flows, but in most of *Hydrodynamicae*, Bernoulli was only concerned with the dynamics of fluids into, out of, and through vessels and conduits.

What we now call "Bernoulli's principle" is theoretically derived in Book 12, Section 5 [2, 3]. Bernoulli asks the reader to determine the pressure exerted on the walls of a pipe which is connected at one end to a continuously filled reservoir and at the other end an exit aperture

to regulate the flow out of the reservoir. Bernoulli's derivation, which is a mix of algebraic manipulation and physical argument, is a very difficult read (original discovery rarely comes with exemplary exposition). Furthermore, conservation of energy (*vis viva*), which Bernoulli relies upon heavily, was not the settled dogma that it is today (see George Smith's *Physics Today* article on this topic [5]) and his publication came "before Euler had made it customary to express general physical principles as mathematical equations"[6]. Despite lacking the generality that Euler brought to the exposition, Bernoulli clearly established the relationship between pressure and velocity for water flowing in a pipe.

Bernoulli backs up his theoretical derivation with experimental results at the end of Book 12. The apparatus is shown in Figures 1 and 2. The reservoir has an area of 3 square feet and the (presumably internal) diameter of the pipe is $\frac{7}{12}$ inch, so when the pipe is fully open, the rate at which the reservoir level drops is less than $1/1500$ of the the water's exit velocity. The height of the water in vertical glass tube indicates the pressure in the pipe. If we ignore capillary effects, when the pipe is plugged, the height of the water in the tube will match the level in the reservoir. In Figure 2, the height under static conditions, labeled n, sits above the reservoir level because Bernoulli used a tube with inner diameter small enough to induce $\frac{5}{12}''$ of capillary action ($\sim$ 3 mm ID).

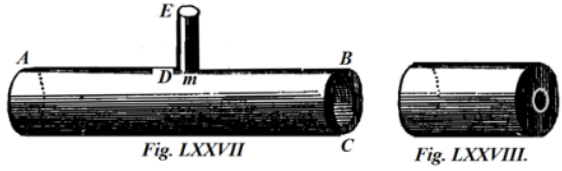

Figure 1: Bernoulli's schematics of the iron pipe (Fig. LXXVII) and cap (Fig. LXXVIII) he used to conduct is flow-rate/pressure experiments described in Book 12 of *Hydrodynamicae*. The pipe has an inner diameter of $\frac{7}{12}''$ and caps were made with apertures of $\frac{2.2}{12}''$, $\frac{3.4}{12}''$, $and \frac{5}{12}''$ in diameter.

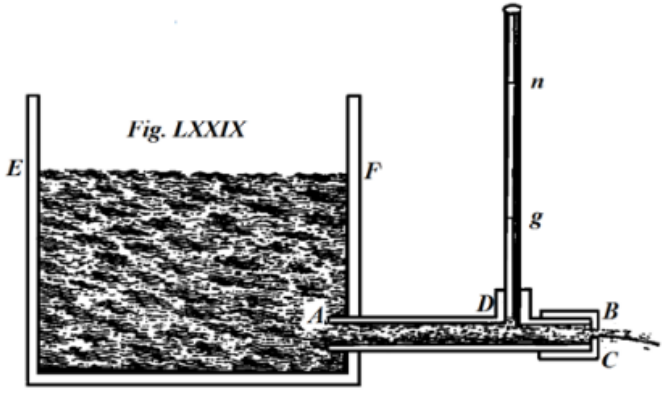

Figure 2: Bernoulli's experimental setup. The reservoir is $3' \times 1'$ and filled to $9\frac{7}{12}''$ above point A. Capillary action brings water to point n in the vertical tube when the pipe is blocked at BC. With an open aperture the level in the tube drops to a lower point, marked g.

If we had a valve on the end of Bernoulli's pipe and slowly cracked it open, you would not be surprised to see the water level in the glass tube fall below its static height, and that it would even fall to zero height for a fully open pipe. Aside from the fact that the reservoir is continuously pressurizing the pipe, this is similar to what happens when we open any pressurized vessel, be it a bottle of Champagne, a can of soda or any aerosol can – when the vessel is opened, the pressure decreases. In the case of pressurized vessels, the size of the aperture governs the speed at which the pressure drops; in the case of Bernoulli's experiment, the size of the aperture governs the steady-state pressure drop in the pipe from hydrostatic. Because the size of the aperture also governs the outflow speed from the pipe, there is a well-defined relationship between the pressure drop and the steady-state flow velocity in the pipe – Bernoulli's principle.

Rather than using a valve, Bernoulli conducted his experiments with a collection of caps with apertures ranging from $\frac{2.2}{12}''$ to $\frac{5}{12}''$ in diameter and also with the open pipe. This allowed him to run the experiments according to his theoretical derivation, which predicted pressure drops as a function of pipe-to-aperture cross sectional area ratios. The pressure differences he reported in *Hydrodynamicae* fell within 4% of his theoretical predictions, with the largest errors corresponding to the highest flow rates. Bernoulli closes out Book 12 by describing experiments in which he achieves negative pressure in a vertically-oriented conical flow pipe, roughly 60 years before Giovanni Battista Venturi published his works on essentially the same thing.

Bernoulli's seminal contribution here is the relation between steady state the velocity of the flow in a pipe with the decrease in pressure: $\Delta P = -\rho v^2/2$. With this relation, under the right conditions, which we recognize to mean an incompressible, inviscid fluid in a steady isentropic (ie, thermodynamically reversible) flow, we can infer velocity from pressure (Pitot tube, Venturi meter) or pressure from velocity (any science-class demo of Bernoulli [4]), which ever is easier to measure.

3 Airfoils, baseballs, Bernoulli, and Newton

Neither a wing nor a baseball is a pipe, nor is air the same as (liquid) water, yet the Bernoulli principle is often called upon to explain lift on wings and the curving trajectory of a baseball with significant spin. Is this justified? In these cases of open flow around an obstruction, there is no pipe to uncork, but the relationship between velocity and pressure still holds. Some educators [7, 8] have argued that the Bernoulli principle is misleading in this context, and that lift (which includes curve-balls) is better explained by momentum conservation – a wing provides lift and a spinning ball's path curves because each of these objects redirects the airflow to alter their trajectories. Smith points out that helicopters and fixed-wing aircraft both use airfoils to generate lift, and that the physics is identical. With a helicopter, the rotor blades generate significant downwash when they are spinning, and it is easy to see that lift is generated by pushing air downwards. Although this is much harder to visualize with an airplane, the same thing is happening – the jet or the propeller(s) pushes air backwards to propel the airplane forwards, and the wings push air down (making downwash) to generate lift (see Figure 3). The BGA calls this the *"Newton" position* in opposition to the traditional *"Bernoulli" position* [1]. Note that the BGA puts their names in quote because neither of them ever weighed in on aerodynamic lift.

I found Smith's and Weltner's papers compelling when I first read them. Momentum

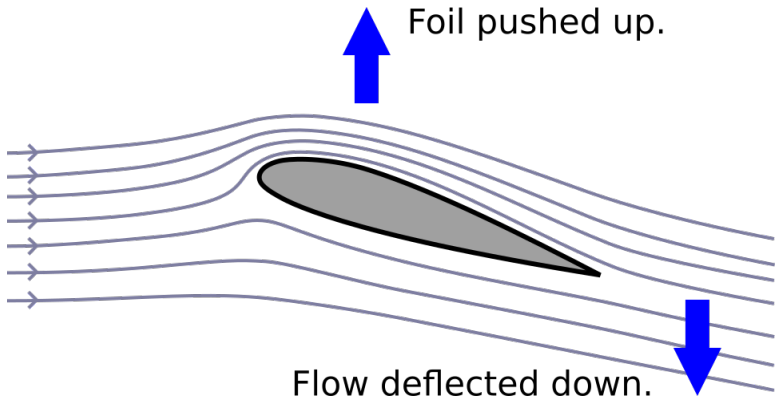

Figure 3: Airflow around an airfoil. In the Bernoullian view, the the flow is disrupted so the flow speed above is greater than the flow speed below the airfoil, resulting in a pressure difference between the top and bottom surfaces. In the Newtonian view, the airfoil deflects airflow downwards, resulting in lift by Newton's principle of action and reaction. Adapted from Michael Paetzold, License: https://creativecommons.org/licenses/by-sa/4.0/legalcode.de

transfer is easy to visualize, and there is some reductionist satisfaction in lumping rockets, jets, and wings all into the same mental bucket. I was all ready to declare myself a Newtonian, but the case of the baseball held me back. To be clear, I am not doubting that a spinning ball directs the airflow around it in the $\vec{v} \times \vec{\omega}$ direction. It's just that it's *easier* for me to think about what deflection will be imparted on the ball by considering the airflow velocities and the implicated pressures on either side of the ball, and that is very much a Bernoullian way of thinking.

The BGA declares that the Newtonian and Bernoullian approaches to lift are equivalent, but a better resource for the resolution of this pedagogical controversy is "An Aerodynamicist's View of Lift, Bernoulli and Newton" by Charles Eastlake [9]. With only a single equation in the paper (the one you already know about) and a handfull of diagrams, Eastlake covers conservation of momentum, energy and mass in the airflow around a wing and concludes:

> Both pictures can be expressed as mathematical models that correctly calculate the forces being generated. Which one is preferable depends only on which one is simpler to use with the data available. Neither is inherently more accurate or more correct.

4 The Pitot-Static Tube

The Pitot-static tube[1] is a simple instrument used on all aircraft to measure the air speed of the vehicle (see Figure 4). The part of the tube facing the direction of flow measures the stagnation pressure of the fluid, while the part(s) of the tube perpendicular to the flow

[1]Invented by the French engineer Henri Pitot in 1832 (5 years before Bernoulli published *Hydrodynamicae*) in order to measure the flow of the Seine River.

measure the "static" pressure, which is the pressure that Bernoulli measured in his pipe flow experiments. Personally, I find the terminology "static" confusing here because it is the pressure of the (nominally) undisturbed moving fluid. It is called the static pressure because, when properly installed, the reading on the static tube is the *same* as the local atmospheric pressure.

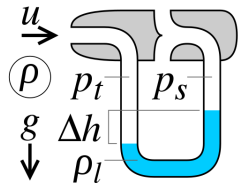

Figure 4: Typical Pitot-static tube arrangement. The stagnation pressure is labeled p_t and the static pressure, which is the same as the local atmospheric pressure, is labeled p_s. Air speed u is determined by measuring Δh and applying the relation $\frac{1}{2}\rho u^2 = \rho_l g \Delta h$. Credit: Cmglee, CC BY-SA 4.0 <https://creativecommons.org/licenses/by-sa/4.0>, via Wikimedia Commons

The fact that the static tube measures atmospheric pressure seems to contradict Bernoulli's principle. Weltner cites this phenomenon as one of his primary arguments in favor of teaching the Newtonian explanation of lift. Experimental confirmation is called for here, but if you don't want to do it yourself, take a look at the first figure in "Thinking about Bernoulli" by Martin Kamela [10], which clearly shows that the "static" pressure of the flow is the same as atmospheric pressure. I didn't have time to obtain permission to reproduce that figure, but the experiment is fairly easy to reproduce. The materials needed are:

- Several feet of clear tubing from a hardware store
- A hair dryer or other blower
- A flange for the measurement-end of the tube (I used a plastic lid with a hole and some putty)
- Two children

We find that, indeed, with a flat flange parallel to the direction of flow (Figure 5, left), the the pressure reading in the flow is atmospheric. Does this contradict Bernoulli's principle? No – Bernoulli tells us that in this situation, the pressure of the moving air is lower than that of the stationary air by the kinetic energy, but that comparison has to be made in the same air stream. The reference pressure in this situation is the pressure generated by the blower – the stagnation pressure that the Pitot tube measures (Figure 5, right).

We can relate this measurement to Bernoulli's original experiments. Just as we are doing, Bernoulli's measurements of pressure, height of water, are all taken relative to atmospheric pressure. Where he uses a reservoir of water as both his fluid and nearly-constant pressure supply, we use the air in the room as our fluid and the hairdryer as the constant pressure source. Bernoulli knows his stagnation pressure is the height of the reservoir surface above the pipe inlet; we measure our stagnation pressure by arranging our tube into the Pitot configuration, pointing into the flow. Bernoulli measured of the pressure in the pipe at varying flow rates by observing the height of the water in the vertical glass tube; we make the same

Figure 5: Static (left) and Pitot (right) pressure measurements taken with respect to atmospheric pressure. As Bernoulli predicts, the free-streaming "static" pressure is significantly lower than the reservoir, or stagnation, pressure generated by the hair dryer. The pressure in the free stream is equal to atmospheric pressure because the free streaming condition is equivalent to the open-pipe condition in Bernoulli's original experiments.

measurement with the tube in the "static" pressure arrangement. What about the plug restricting the flow? There is no plug – as Bernoulli found in his 4th experiment of Book 12, with no restriction to the flow, the "static" pressure drops to atmospheric.[2]

5 Loose ends

I've left you with a lot of loose ends this year, but as I mentioned from the start, airfoils are complicated, I am no expert on this topic, and the BGA is a fantastic existing resource for the curious.

Here are a few comments on things I intentionally skipped over:

5.1 Cause and effect

In Bernoulli's original experiment, when the end of the pipe is opened, the pressure in the pipe drops and the flow velocity changes from zero to some steady state value. Does the velocity increase because the pressure dropped, or does the pressure drop because the velocity increased? The latter statement is a bit facetious, but it is the typical mode of thought when we use Bernoulli's principle to analyze systems in which it applies. Perhaps it is better to say that when we see a velocity change, we infer a pressure gradient.

[2]Bernoulli observed 1/4 inch above capillary action, which he attributed to "impediments which the water encounters in the flow from D [the point of measurement] to B [the end of the pipe]."

5.2 Curving flows

In the same way we can infer a pressure gradient along a pipe flow from a change in fluid velocity, we can also infer a transverse pressure gradient from a flow with a curved trajectory. This can be shown by giving the flange in the static tube measurement slight hump instead of leaving it flat. The effect is fairly subtle – experimentally, we could only get a ~ 1 cm of water in vacuum with the hair dryer blowing air across the flange (Figure 6).

Figure 6: With the flange curved a bit, the free streaming air from the hairdryer can generate a modest vacuum. This took a while to set up, and you can see that Mag and Owen are getting impatient with me.

5.3 Compressibility of gases

Gases are, of course, compressible, violating the "incompressibility" requirement for Bernoulli's principle to hold true. One can still write down a Bernoulli-like energy conservation equation for a general gas flow, but it's much messier, and in the low velocity limit, the equation reduces to Bernoulli's equation. In this context, the reference speed is the speed of sound. Now *why* that's the case is a whole 'nother can of worms. Sound *is* compression; furthermore, it is evidence of energy dissipation (so now we can't claim constant entropy anymore either). This year, take my word on it. In the meantime, for a peek at the other side of the sonic barrier, I refer you to the November 2017 cover article of Physics Today, "The Relentless Pursuit of Hypersonic Flight" by Ivett A. Leyva [11].

5.4 Bernoulli and Kinetic theory

When I started my reading this year, I thought that I wanted to present Bernoulli from a purely "kinetic theory of gasses" point of view with a Venturi as the physical example, but (for me) it remains an unsimplifiable relationship. For the uninitiated, the kinetic theory of

gases derives the ideal gas equation ($PV = nkT$) from the Gaussian velocity distribution of a gas and considering the momentum transfer of that gas on the walls of a test volume. This topic is mostly covered in "Interpretation of Bernoulli's Equation" by Bauman and Schwaneberg [12] with the omitted details covered in MIT Aero/Astro's "Unified" course notes [13].

6 Conclusion

It is undeniably useful to have multiple ways to understand a problem. With aerodynamic lift, the difference between an energy and mass conservation approach (Bernoulli) and a momentum conservation approach (Newton) led to some heated debate on how lift should be taught. The current consensus is that both ways of looking at the problem are fruitful. Going back to the original source, we find an experimental setup that is essentially the same as the "pipe with varying cross sectional areas" picture in all modern texts, but presents a slightly different way to understand the phenomenon – as the uncorking of a pressurized vessel. So now you have one more tool in your mental toolbox.

References

[1] NASA/Glen Research Center, "Beginner's Guide to Aeronautics," website, accessed December 2023. Their discussion of Bernoulli and Newton is under Guide to Aerodynamics $\rightarrow$ Lift $\rightarrow$ Bernoulli and Newton. Hats off to GRC. This is an amazing resource.

[2] Daniel Bernoulli, "Hydrodynamicae," Translated by Ian Bruce, (2014). Website accessed January 2023. Thank you, Ian Bruce, for all your translations. In "Loeb Library" fashion, the original Latin is supplied with the translation. In addition the translator provides helpful annotations to the work. Regarding the title, most online sources call it "Hydrodynamica," as the 1738 cover shows. On the first page of text, though, Bernoulli uses "Hydrodynamicae." I follow Bruce's example of using the plural because in English, we call the field "hydrodynamics," not "hydrodynamic."

[3] Daniel Bernoulli, "Hydrodrodynamics," Translated by Thomas Carmody and Helmut Kobus, Dover (1968). The preface in this volume quotes a letter from Bernoulli to Euler in which Bernoulli complains about his father Johann trying to steal credit from him. It's interesting to note that Daniel is especially proud of his proof of his eponymous principle.

[4] NASA/Aeronautics Research Mission Directorate, Bernoulli's Principle website, accessed November 2023. K-8 classroom demo kit.

[5] George E. Smith, "The *vis viva* dispute: A controversy at the dawn of dynamics," *Physics Today* **59** (10), 31–36 (2006). Imagine living without the assurance that both energy and momentum are conserved quantities.

[6] C. Truesdell, "Notes on the History of the General Equations of Hydrodynamics," *American Mathematical Monthly* **60** (7) 445-458 (1953).

[7] Norman F. Smith, "Bernoulli and Newton in Fluid Mechanics," *Phys. Teach.* **10**, 451-455 (1972). This is the earliest paper I could find drawing a line in the sand between Bernoulli and Newton.

[8] Klaus Weltner, "A comparison of explanations of the aerodynamic lifting force," *Am. J. Phys.* **55**, 55-54 (1987). Another hard-core Newtonian still flying the flag as of 2015. A worthwhile read nonetheless.

[9] Charles N. Eastlake, "An aerodynamicist's view of Lift, Bernoulli, and Newton," *Phys. Teach.* **40**, 166-173 (2002). The consensus view is laid out in this paper.

[10] Martin Kamela, "Thinking about Bernoulli," *Phys. Teach.* **45**, 379-381 (2007). This was the inspiration for this year's experiment.

[11] Ivett A. Leyva, "The relentless pursuit of hypersonic flight," *Physics Today* **70** (11), 30–36 (2017). One of these days, I'll get Ivett to write Newton Day with me. The only problem is that hypersonic experiments generally aren't safe to do at home.

[12] Robert P. Bauman and Rolf Schwaneberg, "Interpretation of Bernoulli's equation," *Phys. Teach.* **32**, 478–488 (1994). This paper pays a lot of heed to thermodynamics and kinetic theory. I didn't have time to fully digest it this year.

[13] OpenCourseWare 16.01 aka "Unified.". Very few classes at MIT are referred to by name. Unified is one of them.

Part II
Airplane Physics

7 Lift

Many people use Bernoulli's theorem to explain how an aircraft wing is able to generate lift – the air moves faster along the top of the wing and as a result the pressure is lower. While is the certainly correct, it isn't much of an explanation. After all, *why* does the air flowing over the wing pick up speed? Here, we often get hand waving arguments about how the air moving on top of the wing has to move faster because it has a longer distance to travel in order to rejoin the flow below the wing. The problem is that there is simply no reason that the flow streams have to have equal transit times, and in actuality they do not.

Let's try to make a simpler explanation. Imagine that we in sitting in the aircraft watching the air move backwards. As air moves over the top of the wing, it has to move downwards in order to prevent a vacuum from forming. There are two ways that this can happen, the air can flow smoothly along the wing or the flow can become turbulent. Let's begin by considering the first case, laminar flow.

In the case of laminar flow, the airflow is forced to curve over the wing. We know from basic physics that circular motion requires a centripetal force. In this case, the only thing that can provide the force is a pressure gradient – i.e. the pressure must be increasing as we move from the top surface of the wing upwards. Since we know that high above the wing

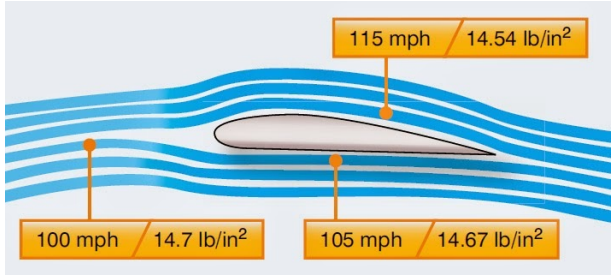

Figure 7: Typical laminar airflow around a wing, taken from https://aircraftsystemstech.com.

the pressure is simply atmospheric, we are forced to conclude that the pressure on the top surface of the wing is lower than atmospheric.

The same analysis can be performed considering flow underneath the wing. In this case, the flow curves downward away from the wing. As a result, the pressure on the bottom of the wing is higher than atmospheric.

We can make this analysis slightly more formal by writing the second Navier-Stokes equation (momentum conservation) for steady state flow,

$$\rho(\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla P, \tag{7.1}$$

in cylindrical coordinates and considering circular flow with $\mathbf{v}_\theta = r\omega$. We obtain

$$\rho r\omega^2 = \frac{\partial P}{\partial r}. \tag{7.2}$$

In words, as we move away from the center of curvature, the pressure increases.

As we increase the angle of the wing with respect to the incoming airflow (the angle of attack), we reach a point where the flow becomes turbulent. We no longer have nicely curved flow, we have swirling vortices of air on the top surface of the wing. As we might expect, the lift decreases dramatically. This is called an aerodynamic stall.

8 Compressible Flow

While air is compressible, at low velocities we can treat it as approximately incompressible. In order to justify this, we assume that the flow is isentropic and combine the second Navier-Stokes equation (momentum conservation) with the equation for isentropic flow

$$P = k\rho^\gamma, \tag{8.1}$$

where k is some constant. For steady state conditions, we obtain

$$-M^2\frac{dv}{v} = \frac{d\rho}{\rho}, \tag{8.2}$$

where M is the Mach number, i.e. the ratio of the flow velocity to the speed of sound. Most authors define the "boundary" between compressible and incompressible flow to lie at $M =$

0.3, at which point changing the fractional velocity dv/v results in a change in $d\rho/\rho$ that is 9% as large.

Compressible flow can behave considerably differently than incompressible flow. Let's consider a practical example. In a jet engine, the trust simply given by

$$T = \dot{m}(v_{out} - v_{in}) \tag{8.3}$$

where $\dot{m}$ is the mass flow rate. In order to maximize the trust, we want to increase v_{out} as much as possible[3]. Well, we know how to do that for incompressible flow – we simply neck down the flow. If you take a close look at a jet engine, you will notice that the engine exhaust does, in fact, neck down.

What happens when the flow velocity becomes large? In order to figure this out, we make the usual assumptions about isentropic, laminar flow in a pipe of area A. We combine the first two Navier-Stokes equations (mass and momentum conservation) with the equation for isentropic flow, and find

$$(1 - M^2)\frac{dv}{v} = -\frac{dA}{A}. \tag{8.4}$$

For velocities below Mach 1, decreasing the area of the pipe increases the flow velocity. For velocities above Mach 1, the opposite occurs!

Jet engines on supersonic aircraft, such as military fighter jets, need to have an exhaust that first converges in order to increase the flow velocity to Mach 1, and then diverges in order to continue increasing it even further. These are called de Laval nozzles (or simply converging-diverging nozzles). Many military fighters actually have adjustable exhaust nozzles that automatically optimize thrust for different airflow velocities[4].

References

[1] `https://www.grc.nasa.gov/WWW/k-12/airplane/nozzled.html`.

[3] Basically, what we are doing is converting as much of the internal energy into mean kinetic energy.

[4] If you watch the opening scene of the movie *Top Gun*, you can see the adjustable jet nozzles on the F-14 Tomcat.

Figure 8: Owen and Mag Dec 24, 2023.