Newton Day 2014:
There *is* gravity in space!

by Peter H. Mao for Margaret and Owen

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Abstract

For Sir Isaac Newton's 372nd birthday, we will experimentally study weightlessness (also known, in common parlance, as "zero gravity") by launching an accelerometer into sub-orbital flights.

1 Introduction

This past spring, on our daily commute to Owen's childcare center (Child Educational Center (CEC) in La Cañada, CA) Owen told me that, "Dad, there's no gravity in space." I thought he was joking, but I answered, "Sure there is – there's gravity everywhere." He dug in, "No, you're wrong, Dad. My teachers told me – there's no gravity in space." I countered, "Then why is the Moon still anywhere near us? How can we be in orbit around the Sun?" We argued back and forth like that for the rest of the ride in.

When we got to CEC, I took a look at several of the books on astronauts and spaceflight in his classroom, and sure enough, there were references to "zero gravity," which easily turns into "no gravity" for 3 year-olds. So now, at least I understood the source of the misunderstanding. The short answer, of course, is that the astronauts in orbit do not experience gravity as we do on the surface of the Earth because they are falling the entire time they are in orbit. Newton, who founded what we now call classical physics, gives the classical pedagogical example of a cannon on a mountain top.

This year, for the first time in several years, experiments will take center stage. First, we must establish that the acceleration due to gravity is our measure of the "strength" of the force and then I will run through a few simple calculations on the strength of gravity so that you have some sense of scale. Finally, I'll outline the experiments on weightlessness that we'll perform on Newton's birthday.

2 Gravity and acceleration

Newton's seminal insight was that the force that makes the apple drop from the tree is the same one that keeps the Moon in orbit around the Earth, and all the planets in orbit around the Sun. In the physics oral-history version of events[1], Newton used Johannes Kepler's

[1] Meaning, that I totally neglected to check any sources this time.

analysis of Tycho Brahe's data to derive the inverse square law for the gravitational force:

$$\mathbf{F} = -\frac{G_N M m}{r^2}\hat{\mathbf{r}}. \tag{2.1}$$

The force is attractive (the negative sign), proportional to the masses and inversely proportional to the square of the separation of the objects in question. G_N is the gravitational constant which establishes the proportionality between force, masses and distance.

Newton also gave us the philosophically fundamental equation of physics relating force, mass and acceleration:

$$\mathbf{F} = m\mathbf{a}. \tag{2.2}$$

From these equations, we extract the gravitational acceleration due to mass M:

$$\mathbf{a_g} = -\frac{GM}{r^2}\hat{\mathbf{r}}. \tag{2.3}$$

When the mass is the mass of the Earth $M_\oplus$ and the radial distance is the radius of the Earth $r_\oplus$, the magnitude of acceleration is $g_N = 9.8\mathrm{m/s}^2$.

3 Gravity here and there

Returning to the issue of "no gravity in space": What do we mean by "space" and "no gravity"? Outer space is is generally thought of as anywhere outside of the Earth's atmosphere. The scale-height of our atmosphere (where the pressure or density drops by $1/e$) is about 10 km, roughly the height of Mt. Everest. Scientific balloons fly to about 30 km, where the remaining atmosphere is thin enough to detect x-rays from astronomical sources. The International Space Station sits a bit above 400 km, losing about 3 km/month due to atmospheric drag. NuSTAR is at 600 km, and loses altitude much more slowly than the ISS.

"No gravity" should mean that the acceleration due to gravity is zero. This can indeed happen at special locations in space, but they depend on the locations of everything else in the entire Universe! In space, the gravitational acceleration due to the Earth can only be zero at $r = \infty$. What happens if you burrow into the Earth? Does the magnitude of the gravitational acceleration, a_g, go up or down?

How much gravity is there in space? On the surface of the Earth, for simplicity, let's use $g_N \approx 10$ m/s^2 and $r_\oplus \approx 6000$ km.[2] We can rewrite Equation 2.3 for Earth explicitly in terms of the height (h) above the surface (dropping vector notation and only considering the magnitude of the acceleration):

$$\begin{aligned} a_\oplus(h) &= \frac{GM_\oplus}{(r_\oplus + h)^2} && (3.1)\\ &= \frac{GM_\oplus}{r_\oplus^2}\frac{1}{(1 + h/r_\oplus)^2} && (3.2)\\ &= g_N(1 + h/r_\oplus)^{-2}. && (3.3) \end{aligned}$$

[2]The metric system was meant to be a recoverable system of units, so the meter was originally defined such that the distance from the North pole to the equator is 10 million meters (10^4 km). A more exact estimate of $r_\oplus$ is $4 \times 10^4/(2\pi)$, or 6366 km. The mean equatorial radius of Earth, 6378 km, is slightly greater due to the spin of the Earth.

All of the altitudes that I suggested for outer space are small compared to the radius of Earth. For these "low Earth orbits," we can use one of my favorite approximations

$$(1+\epsilon)^n \approx 1 + n\epsilon. \tag{3.4}$$

For $h << r_\oplus$,

$$a_\oplus(h) \approx g_N(1 - 2h/r_\oplus). \tag{3.5}$$

NuSTAR is the highest one that I mentioned, and the calculation is particularly easy: $h/r_\oplus \approx 0.1$, so the gravitational acceleration due to the Earth at NuSTAR's altitude is about 8 m/s^2. Not so different from the gravity on the surface!

How about the Moon? The Earth-Moon distance is about 60 times the radius of the Earth. In this case because h is much bigger than $r_\oplus$, we can use the approximation $1+h/r_\oplus \approx h/r_\oplus$. You can easily verify that the Earth changes the Moons velocity by less than 3 mm/s every second, but that's all it takes to keep the Moon in orbit around the Earth!

For your future entertainment, compare the gravitational acceleration due to the Sun and the Moon here on Earth. Which is greater? How can you reconcile that with the observational evidence that ocean tides track the Moon and not the Sun?

4 Weightlessness

Weightlessness happens, not necessarily when gravity is weak, but rather when gravity is the main force on a body. Typically, on Earth, the two major forces that act on us are gravity and the electrostatic repulsion between our bodies and the ground. When we are at rest on Earth, as Newton pointed out, the two forces are equal and opposite in direction.[3] If you jump, either off something or just straight up, as soon as you take flight, gravity becomes the major force on you, so you momentarily experience weightlessness. Electrostatic repulsion won't come into effect until you land.

When you fly in an airplane, you generally do not feel weightless. At constant altitude, aerodynamic forces on the wings of the airplane balance gravity. Along well planned flight paths, one can experience weightlessness in an airplane. From the late 1950's until earlier this year, NASA ran a so-called Reduced Gravity Program, which took astronauts on parabolic trajectories in an airplane (also known as the "Vomit Comet"). The typical flight pattern would give the test subjects 25 seconds of weightlessness every 65 seconds. In this case, the pilot throws the passengers and then flies the plane along the ballistic trajectory of the passengers.

A sky diver feels weightlessness for the first few seconds of his or her jump. Eventually, the diver reaches terminal velocity, where the force of gravity is balanced by the force of aerodynamic drag. A parachute reduces the terminal velocity of the sky diver so that he or she can land safely.

Astronauts in orbit are weightless because they have only gravity acting on their bodies. The atmosphere is too thin to produce appreciable drag.

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[3] Of course, Newton did not know that the source of the normal force opposing gravity was electrostatic.

5 The experiment

An accelerometer is a device that measures its own acceleration. We are fortunate to live in a time when such devices are small, inexpensive, and do not require much power to run them. Our Newton Day experiment will consist of packaging my accelerometer so that it can survive hard landings and then performing experiments on it to demonstrate weightlessness in sub-orbital flights.

5.1 Hardware

Our instrument will consist of several Adafruit-sourced parts that I have collected over the last few years:

- accelerometer: Analog Devices ADXL335 ($\pm 3 g_N$) on an Adafruit evaluation board (163)
- data logger: Arduino UNO R3 (50) with Adafruit's SD-card Data Logger (1141)
- power: 6 AA batteries in a battery holder (248)

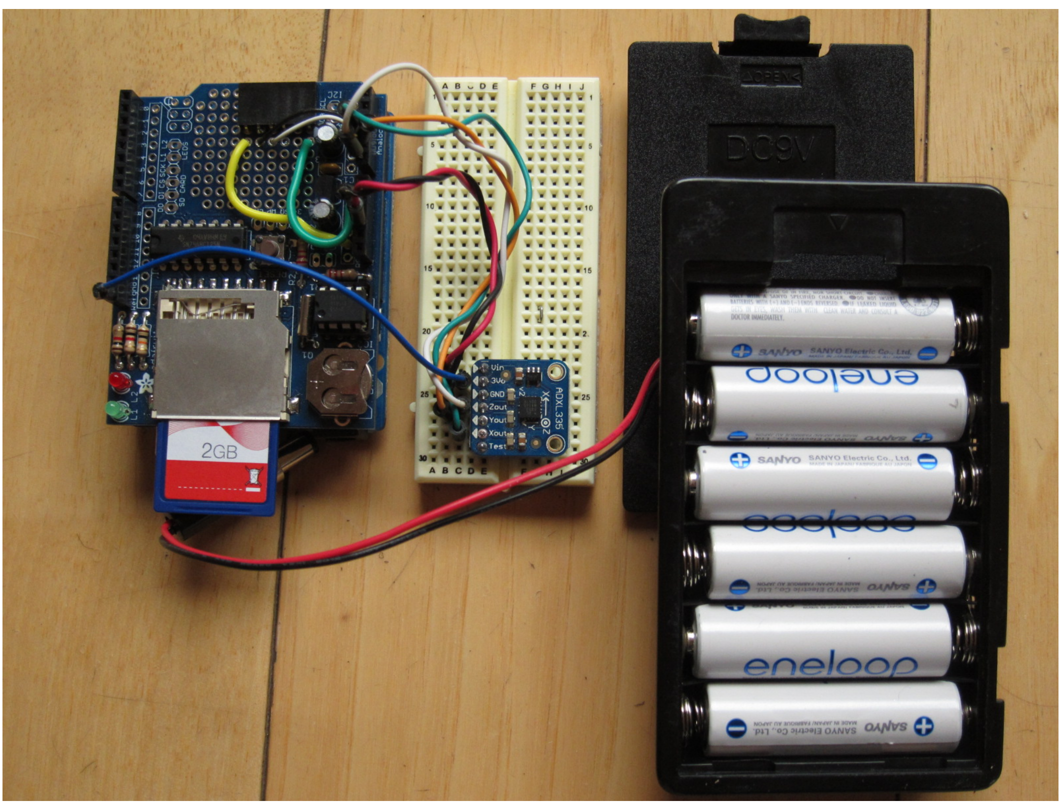

Figure 1: Accelerometer (center), data logger (left) and power source (right).

The accelerometer is mounted on a solderless breadboard and wired to the the data logger. The experiment will be limited by the life of the batteries, which should power us for several hours. The SD card can hold 20 days of continuous data.

When the accelerometer is at rest on the surface of Earth, it senses only gravitational acceleration reports it as an upward acceleration. When the accelerometer is weightless, it will sense no acceleration in any direction – this is what we will attempt to observe in our experiments.

5.2 Measurement limitations

Any measurement in any experiment is subject to measurement errors and limits of the device. I was hoping to thoroughly calibrate and characterized our instrument before Newton Day, but this year, I could not find the time to make that happen.

The best I could calibrate the zero point was to $0.03g_N$, meaning that when I flip over the accelerometer, the X and Y zero readings shift by that amount. This is fairly consistent with the spirit-level applications I've seen on phones (which use a similar sensor) but I was hoping to do better! If there are no other issues (and there will be!) then an acceleration reading less than $0.05g_N$ is consistent with zero.

Rotation of the instrument in flight will register an acceleration. This will depend on both the rotation rate and distance between the center of mass of the instrument and the sensor. We will try to place the sensor at the center of mass of the full assembly, and we will try to launch it without spin. I also have a gyroscope that we could fly, but I decided that I didn't want to deal with the analysis. Maggie and Owen can do that in the future if they are motivated.

Aerodynamic drag will be an issue at high velocities. We will need low density materials to protect the electronics on impact, but we may also need to look for some denser materials to keep terminal velocity at bay.

5.3 Procedures

After packing up the instrument, the reset button will probably not be accessible, so we will be taking continuous data until the batteries run out. Each person will leave a signature in the data stream by placing the instrument in a sequence of orientations. Then we will try jumping with the instrument and throwing it. If we can identify a safe location from which to drop it, we will do so. If time permits, we may take the instrument to the Santa Monica Pier to measure accelerations on the rides there.

6 Conclusion

There is gravity in space. Weightlessness can be achieved for long periods of time in low Earth orbit, and for short periods of time on the surface of Earth. Results will be included post-holiday.

A Additional materials

Figure 2: Owen & Mag. August, 2014. My cousin Cathy complained that I didn't have any pictures of them in last year's paper. The lack of punctuation around *Cathy* indicates that this is a restrictive apposative, since I have >> 1 cousins.