Quiz 1 MATH 175, COMBINATORICS, 18 JAN, 2018

STUDENT ID:

NAME:

Please show all of your work, as partial credit may be given. Please give your explanations in complete sentences. You may use the back of the pages if you need more space.

Problem 1. For a positive integer n, let (a) denote the property that the digits of n are distinct, and let (b) denote the property that n is even. How many four digit numbers whose digits are either 1,2,3,4, or 5 satisfy

(a) Neither property? Odd w/ Repeated digit

#odd − #odd w/ repeated digit.

$3 \cdot 5^3 - 3 \cdot 4! = \boxed{303}$

Idea: (b) and (d) are easy to answer. use your answers from those to answer (a) and (c).

(b) Property (a) only? odd w/ distinct digits:

$\underline{\quad} \ \underline{\quad} \ \underline{\quad} \ \underline{\quad}$
  2   3   4   3 ← # of choices

$\boxed{3 \cdot 4!}$

(c) Property (b) only? Even w/ repeated digit.

#Even − #Even w/ distinct digits.

$= 2 \cdot 5^3 - 2 \cdot 4! = \boxed{202}$

(d) Both properties? Even with distinct digits:

$\underline{\quad} \ \underline{\quad} \ \underline{\quad} \ \underline{\quad}$
  2   3   4   2

$\boxed{2 \cdot 4!}$

Explain your answers.

Problem 2. How many distinct positive divisors of $2^4 \cdot 3^3 \cdot 11^2$ are there? Explain your answer.

The number of positive divisors of $2^4 \cdot 3^3 \cdot 11^2$ is equal to the number of sub-multisets of $\{2,2,2,2,3,3,3,11,11\}$, which is $5 \cdot 4 \cdot 3 = 60$.