Solution

Quiz 3 (44371)

Please write your name and student ID number at the back of the paper. No calculators or phones allowed.

Problem 1.(5 points.) Evaluate

$$\int x \ln(1+x)\, dx$$

Let $y = 1+x$. then $dy = dx$

$$= \int (y-1)\ln y \, dy = \underbrace{\int y \ln y \, dy}_{I} - \underbrace{\int \ln y \, dy}_{II} = I - II.$$

~~For I. let $u = \ln y$, $dv = y \, dy \Rightarrow v = \frac{1}{2}y^2$.~~

$$\therefore I = \frac{1}{2}y^2 \ln y - \int \frac{1}{2}y^2 \cdot \frac{1}{y} \, dy = \frac{1}{2}y^2 \ln y - \frac{1}{2}\int y \, dy = \frac{1}{2}y^2 \ln y - \frac{1}{4}y^2 + C.$$

For II. let $u = \ln y$, $dv = dy \Rightarrow v = y$.

$$\therefore II = y \ln y - \int y \cdot \frac{1}{y} \, dy = y \ln y - \int dy = y \ln y - y + C.$$

$$\therefore I - II = \frac{1}{2}y^2 \ln y - \frac{1}{4}y^2 - y \ln y + y + C$$

$$= \frac{1}{2}(x+1)^2 \ln(x+1) - \frac{1}{4}(x+1)^2 - (x+1)\ln(x+1) + (x+1) + C.$$

Problem 2.(5 points.) Evaluate

$$\int \tan^4 x \sec^6 x \, dx$$

$$= \int \tan^4 x \sec^4 x \cdot (\sec^2 x \, dx)$$

$$= \int \tan^4 x (1 + \tan^2 x)^2 (\sec^2 x \, dx). \quad \text{Let } u = \tan x. \text{ then } du = \sec^2 x \, dx$$

$$= \int u^4 (1 + u^2)^2 \, du$$

$$= \int u^4 (1 + 2u^2 + u^4) \, du = \int (u^4 + 2u^6 + u^8) \, du$$

$$= \frac{1}{5}u^5 + \frac{2}{7}u^7 + \frac{1}{9}u^9 + C$$

$$= \frac{1}{5}\tan^5 x + \frac{2}{7}\tan^7 x + \frac{1}{9}\tan^9 x + C.$$