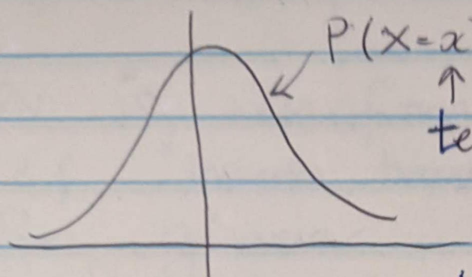
# QQ plots and why p-values are uniform under null hypothesis



$P(X=x)$ : null hypothesis.

↑ test statistic.

Perform multiple ~~tests~~ experiments, measuring the test statistic in each case.

By definition:

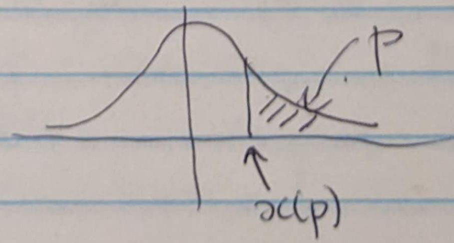
$$p\text{-value} =: PV$$
$$= P(X > x)$$

Now regard PV as a R.V.

Then

$$P(PV < p) = P(X > x(p))$$

where




But:

$$P(X > x(p)) = p$$

by definition.

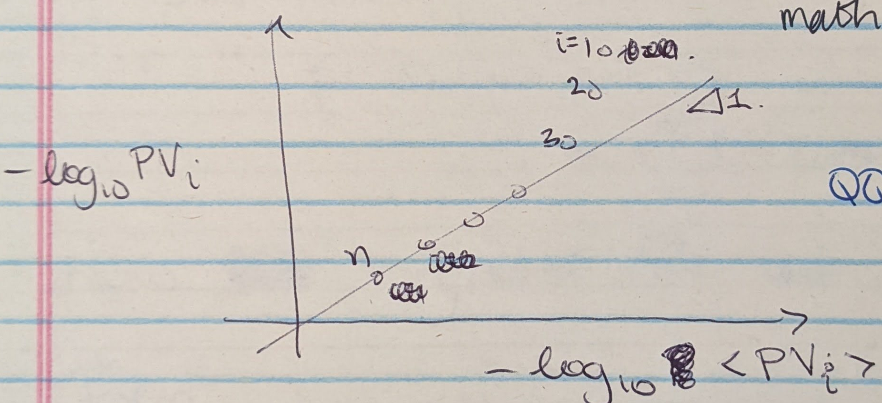
$\Rightarrow \quad P(PV<p) = p$

$\Rightarrow$ PV is uniformly distributed
[c.f. "inverse transform sampling"]

So, given an ordered set of p-values ~~[illegible]~~ $PV_1, \ldots, PV_n$, we may now plot them against their expected values:

$$\langle PV_i \rangle = \frac{i}{n+1}$$

[see wiki article on "order statistic" for math. pf.]




QQ-plot

Deviations from the line are tests where the observed p-value is ~~much~~ smaller than expected, given the ~~num~~ number of tests performed.

Note: since the distribution of p-values is independent of shape of null distribution everything holds even if each expt. has different null dist'ns