Homework 1

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1) a) $\boxed{L_v}$

<u>Base Case</u>: Every propositional atom $p_1, p_2 \ldots p_i$ is a well-formed formula

<u>Inductive Clause</u>: Given that P + Q are well-formed formula, then $(P \wedge Q)$ is a well formed formula

<u>Closure Clause</u>: Nothing else is a well-formed formula unless it can be obtained by finitely many applications of the rules above

$\boxed{\text{pmincon}}$

b) <u>Base Case</u>: For any propositional atom p_i, pmincon is given as $\text{pmincon}(p_i) = 1$

<u>Inductive Clause</u>: Given that P + Q are well-formed formula under L_v, then $\text{pmincon}(P \wedge Q) = \text{pmincon}(P) + \text{pmincon}(Q) - 1$

2) c) $\boxed{\text{Proof that for all wff } P \text{ of } L_{\neg\vee\rightarrow}\text{, translate}(P) \text{ doesn't contain } \rightarrow}$

<u>Base Case</u>: $P = p_i$, p_i is a wff of $L_{\neg\vee\rightarrow}$ and a propositional atom

$\text{translate}(p_i) = p_i$ $\because$ By rule 1

p_i does not contain $\rightarrow$

<u>Inductive Hypothesis</u>: $P = Q$, Q is an arbitrary wff of $L_{\neg\vee\rightarrow}$

For every wff Q of $L_{\neg\vee\rightarrow}$, translate contains no implications $(\rightarrow)$

<u>Inductive Step</u>:

<u>Case 1</u>: $P = \neg Q$, Q is a wff of $L_{\neg\vee\rightarrow}$

$\text{translate}(\neg Q) = \neg\,\text{translate}(Q)$ $\because$ By rule 2

$\text{translate}(Q)$ has no $\rightarrow$ $\because$ By IH

$\therefore$ $\text{translate}(\neg Q)$ has no $\rightarrow$

<u>Case 2</u>: $P = Q \vee R$, Q and R are wff of $L_{\neg\vee\rightarrow}$

$\text{translate}(Q \vee R) = \text{translate}(Q) \vee \text{translate}(R)$ $\because$ By rule 3

$\text{translate}(Q)$ and $\text{translate}(R)$ have no $\rightarrow$ $\because$ By IH

$\therefore$ $\text{translate}(Q \vee R)$ has no $\rightarrow$

<u>Case 3</u>: $P = Q \rightarrow R$, Q and R are wff of $L_{\neg\vee\rightarrow}$

$\text{translate}(Q \rightarrow R) = \neg\,\text{translate}(Q) \vee \text{translate}(R)$ $\because$ By rule 4

$\text{translate}(Q)$ + $\text{translate}(R)$ have no $\rightarrow$ $\because$ By IH

$\therefore$ $\text{translate}(Q \rightarrow R)$ has no $\rightarrow$

<u>Conclusion</u>:

$\therefore$ we have shown by induction on P that for all P that are wff of $L_{\neg\vee\rightarrow}$, translate(P) doesnt contain implications $(\rightarrow)$