



# How Itten's color diagram fails to illustrate color mixing of paints

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**Abstract:** Itten's color diagram, published in 1961, is still considered by many to be the cornerstone of color education. We show experimentally and theoretically that by mixing oil paints it is hardly possible to reproduce Itten's primary colors red, yellow and blue such that their mixtures produce Itten's secondary colors orange, green and purple. Optical models show why it is highly unlikely that paints can be created that follow the color mixing rules from Itten's color diagram. Our results confirm and explain earlier anecdotal evidence. We conclude that Itten's color diagram does not show how paint colors mix, and disagrees with optical theory and experimental evidence.

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## 1. Introduction

In modern textbooks for students of design and art, color theory is based mainly on a circular representation of the relations between colors. This color circle is then used to explain the relations between colors. In the most common graphical representation, the circle is subdivided into twelve sectors, each with a different "color" (mainly differing in hue). First, three "perceptually pure" colors (red, yellow and blue) are placed along an equilateral triangle inside the circle. Between each pair of these colors, three intermediate hues are shown. For example, between yellow and blue we find yellow-green, green and blue-green. This color circle is shown in Fig. 1. It is hard to find a textbook on "color theory" for designers or artists that does not include this diagram [1,2] (rare counter-examples are Refs. [3–6]). As a result, its authority as being the cornerstone of color theory is almost unchallenged among artists and designers.

This twelve-hue color diagram may have been created by Johannes Itten in the 1920s when teaching color theory at the Bauhaus in Weimar, Germany [10]. But its first publication dates from after Itten's retirement with *Kunst der Farbe* (1961) [8] and its English translation *The Art of Color* (1961) [11] (with an abridged version as *The Elements of Color* [12]). With this color diagram Itten aimed at offering a simple representation that combines various elements of color perception (different types of color contrast and color harmony) as well as color mixing. In the text where the color diagram is discussed, Itten describes that the "primary colors" (in German he uses the word *Grundfarben*, i.e., basic colors) red, blue and yellow need to be carefully selected, but purely on a perceptual basis: the red should not look blueish or yellowish, the yellow should look neither greenish nor reddish, and the blue should not look reddish or greenish. Between each combination of these "primary colors", Itten placed secondary colors (in German: *Farben zweiter Ordnung*, colors of the second grade) orange, green and purple, each of which, according to Itten, is "formed from two primary colors". Itten adds that the three secondary colors can only be produced by very careful mixing, and he warns that "experience shows that it is no easy task to obtain the secondary colors by mixture" [7–9,11].

Although virtually unchallenged among artists and designers up to the present day, Itten's color diagram has received a few critical sidenotes from a few academic scholars. They showed that Itten consistently confuses between color as visual sensations and the paints that have these colors [13,14]. Also, it has been noted that in Itten's color diagram the so-called "primary colors"



**Fig. 1.** Twelve-hue color circle by Johannes Itten, as reproduced in three different editions from his main work *Kunst der Farbe* [7–9].

are not related to paint properties but to three of the four elementary hues from colour perception (p. 88 in Ref [5]). Itten mentions that color mixing by lights involves additive mixing whereas paint mixing follows subtractive mixing, but his choice of color primaries for the painter's color diagram (red, yellow and blue) was apparently based on the traditional primary colors of painters, developed since d'Aguilon in 1613. It was not based on the insights from von Helmholtz in 1852 that paints largely follow subtractive mixing (with its cyan, magenta and yellow primaries) whereas light follows additive mixing (with its red, green and blue primaries).

In a scientific biography of Itten, Shamey and Kuehni remark that Itten "largely excluded scientific developments from the mid-nineteenth century onwards" [15]. Indeed, Itten's color circle ignores for example the fact that for color perception both blue-yellow and red-green form opponent pairs, known since 1878 (Hering) [15]. Also, already at the time of publication Itten's color circle represented outdated views on color harmony [16]; recent research confirms that Itten's color circle does not produce reliable predictions for color harmony [1,17–20]. Shamey and Kuehni [15] point to Itten's view that all reflectance colors can be viewed as mixtures of red, yellow and blue, although scientific developments since the 1850s showed this view is untenable.

These criticisms are valid, but they may be too academic and abstract for, e.g., art students wishing to understand color mixing. Teachers may have the impression that although Itten's color circle is inaccurate in describing color perception (opponent colors, color harmony) and in detailed predictions of mixing reflectance colors, it may be useful for teaching the laws of color mixture.

However, in a few independent publications several teachers of color theory present case studies in which art students are not able to reproduce Itten's secondary colors when mixing the primary colors he prescribed [2,3,6,21] (p. 123) [22] (p. 56). These isolated cases raise doubts if Itten's color diagram is useful as a tool to clarify the rules of color mixing: the question arises if it is not simply fundamentally wrong. In this article we will systematically investigate if Itten's color diagram correctly describes color mixing. This will be investigated experimentally by using artist oil paints as described in section 2, and theoretically by calculations introduced in section 3 and in *Supplement 1*. The experiments with paint mixing are described in section 4, and theoretically analyzed in the next section. After a discussion of the results in section 6 we finish this article with a summary of the main results.

## 2. Method

### 2.1. Paint samples

We use twelve pigmented oil paints from product line Gamblin. These paints contain two white paints (commercial names: Titanium white and Zinc white), one black (Ivory black), two red (Quinacridone red and Cadmium red medium), two green (Viridian green and Emerald green),

three blue (Cobalt blue, Ultramarine blue and Prussian blue) as well as two yellow (Cadmium lemon and Cadmium yellow medium) paints. Not surprisingly, none of the twelve oil paints closely matched the “primary colors” from Itten’s color diagram.

Following a procedure that we described in detail before [23], we determined the optical properties of these twelve paints by preparing standardized mixtures with Titanium white and Ivory black, and we applied these mixtures on both white canvas and Ivory black substrate. We measured the reflectance of these paints over black and over white using a Spectro1 spectrophotometer (Variable inc, Chattanooga, TN USA), and also determined all paint layer thicknesses. By fitting the resulting reflectance data with the non-hiding Kubelka-Munk model summarized below, we determined the values for the absorption parameter  $K$  and scattering parameter  $S$ , both as a function of wavelength.

Table 1 shows the resulting values, taking median values for wavelength ranges 400-500, 500-600 and 600-700 nm as an indication of how the parameters vary with wavelength. This table makes clear that most blue paints are blue because of having large absorption  $K$  values for longer wavelengths (500-700 nm), and not because of large scattering  $S$  values for short wavelengths (400-500 nm). Similarly, most red paints are red because of having large absorption  $K$  values for shorter wavelengths (400-600 nm), and not because of large scattering  $S$  values for long wavelengths (600-700 nm). Also for green and yellow paints we find that their colors are primarily driven by high absorption rather than by high scattering. The corresponding reflectance curves for all oil paints in Table 1 are presented in the Supplement.

## 2.2. Mathematical constraints

We combine reflectance measurement data with calculations on optimum concentrations when mixing the oil paints to reproduce Itten’s color diagram with oil paints. For this we use the Kubelka-Munk theory [24] (for more details see Refs. [25] and p. 102 in [24]). In Kubelka-Munk theory the optical properties of a paint are determined by two parameters: absorption  $K$  and scattering  $S$ , both a function of wavelength. The theoretical reflectance  $R_t$  of the paint is calculated by [26]:

$$R_t = \frac{(a+b)(a-b-R_g) \exp(-2bSD) - (a+b-R_g)(a-b)}{(a-b-R_g) \exp(-2bSD) - (a+b-R_g)} \quad (1a)$$

$$a = 1 + \frac{K}{S} \quad (1b)$$

$$b = \sqrt{a^2 - 1} \quad (1c)$$

Here,  $R_g$  is the reflectance of the underlying substrate and  $D$  is the paint film thickness. For hiding paints, Eq. (1) can be replaced by Eq. (2):

$$\frac{K}{S} = \frac{(1-R_t)^2}{2R_t} \quad (2)$$

Since instruments measure paint reflectance  $R_m$  outside the paint layer, we need to account for light reflection at the paint-air interface by using the Saunderson correction [24]:

$$R_m = \alpha k_1 + \frac{(1-k_1)(1-k_2)R_t}{1-k_2R_t} \quad (3)$$

For the measurement geometry of the Spectro1 instrument we use parameter values  $k_1 = 0.04$  and  $k_2 = 0.49$ ,  $\alpha = 0$  that we also used before [23,24]. For mixtures of  $N$  different paints, the

**Table 1.** Optimized values for a range of oil paints in different colors, available under the brand names Gamblin (indicated as G), Rembrandt, and Van Gogh (indicated as VG).<sup>a</sup>

	K parameter			S parameter		
	400-500nm	500-600nm	600-700nm	400-500nm	500-600nm	600-700nm
Titanium white (G)	0.0031	0.00092	0.00065	0.45	0.39	0.32
Zinc white (G)	0.0010	0.00026	0.00015	0.066	0.054	0.043
Ivory black (G)	1.3	1.6	1.6	0.19	0.20	0.16
Quinacridone red (G)	0.17	1.0	0.011	0.000045	0.00015	0.012
Cadmium red medium (G)	0.84	0.90	0.0019	0.0011	0.032	0.20
Cadmium yellow lemon (G)	1.1	0.0012	0.00041	0.0099	0.12	0.099
Cadmium yellow medium (G)	1.3	0.0089	0.00070	0.016	0.16	0.13
Viridian green (G)	0.0090	0.072	0.22	0.0072	0.00015	0.00011
Emerald green (G)	0.85	0.097	2.3	0.13	0.16	0.058
Cobalt blue (G)	0.021	0.40	0.40	0.022	$1 \times 10^{-7}$	0.0040
Ultramarine blue (G)	0.023	0.62	0.68	$1 \times 10^{-7}$	$1 \times 10^{-7}$	$1 \times 10^{-7}$
Prussian blue (G)	0.21	1.2	5.2	$1 \times 10^{-7}$	0.0022	0.0047
Königsblau (Rembrandt)	0.0086	0.052	0.064	0.068	0.052	0.055
Sèvres blue (VG)	0.0050	0.046	0.24	0.078	0.075	0.059
Sèvres blue (Rembrandt)	0.0083	0.076	0.46	0.062	0.053	0.045
Cerulæum blue hue (G)	0.20	1.7	12	0.49	0.47	2.0
Cerulæum blue Phthalo (VG)	0.0060	0.056	0.37	0.021	0.017	0.018

<sup>a</sup>Numbers indicate median values for the indicated wavelength range. The last 5 blue pigments are discussed in section 3.4.

reflectance can be calculated by using the relationship proposed originally by Duncan:

$$\left(\frac{K}{S}\right)_{mixture} = \frac{c_1 K_1 + c_2 K_2 + \dots + c_N K_N}{c_1 S_1 + c_2 S_2 + \dots + c_N S_N} \quad (4)$$

where  $c_i$  is the weight concentration of paint number  $i$ , and  $K_i$  and  $S_i$  are the Kubelka-Munk parameters for paint number  $i$  [27]. In the analysis we follow the commonly held assumption that in Itten's work, the secondary mixtures orange, green and purple are based on 50%-50% mixtures of the primary colors [28,29]. Similarly, we assume that tertiary mixtures are based on a mixture of 50% primary color with 50% secondary color. In a separate section below, we will investigate the consequences when lifting this assumption.

For reproducing Itten's color diagram, we need to reproduce the red, yellow and blue so-called primary colors by mixing the twelve oil paints introduced in the previous section. We will assume here that Itten's color diagram is relevant to painters: it should be possible by mixing these three so-called primary colors to reproduce the other 9 colors in Itten's diagram. From all the paints

listed in Table 1 there is no selection of red, yellow and blue that gives a close visual match of Itten's red, yellow and blue.

Based on Kubelka-Munk theory it is possible to define relations between the optical parameters of the red, yellow and blue paints. Table 2 summarizes these relations, which are derived in the Supplement. The second column of Table 2 lists the relations for the so-called primary colors red, yellow and blue. The third column shows that in order to also reproduce the so-called secondary colors in Itten's color diagram, six more relations need to be satisfied.

**Table 2. Relations between Kubelka-Munk optical parameters  $K, S$  for different wavelength ranges.<sup>a</sup>**

Wavelengths	Blue, Yellow, Red	Green, Orange, Purple	Consequence if latter criterium is not satisfied
400–500 nm	$S_{Blue} >> K_{Blue}$ (S3a)	$K_{Yellow} > S_{Blue}$ (S6a)	Blue-Yellow mixture becomes blue
	$S_{Yellow} << K_{Yellow}$ (S4a)		
	$S_{Red} << K_{Red}$ (S7a)	$K_{Red} < S_{Blue}$ (S9a)	Blue-Red mixture becomes dark and/or red
500–600 nm	$S_{Blue} << K_{Blue}$ (S3b)	$K_{Blue} < S_{Yellow}$ (S6b)	Blue-Yellow mixture becomes dark green
	$S_{Yellow} >> K_{Yellow}$ (S4b)	$K_{Red} \approx S_{Yellow}$ (S8)	Red-Yellow mixture becomes dark brown (if $K_{Red} >> S_{Yellow}$ ) or yellow (if $K_{Red} << S_{Yellow}$ )
	$S_{Red} << K_{Red}$ (S7b)		
600–700 nm	$S_{Blue} << K_{Blue}$ (S3c)	$K_{Blue} > S_{Yellow}$ (S6c)	Blue-Yellow mixture becomes yellow
	$S_{Yellow} >> K_{Yellow}$ (S4c)		
	$S_{Red} >> K_{Red}$ (S7c)	$K_{Blue} < S_{Red}$ (S9b)	Blue-Red mixture becomes dark and/or blue

<sup>a</sup>The second column lists relations needed to produce so-called primary colors Blue, Yellow, and Red. The third column lists relations needed to be satisfied in order to produce so-called secondary colors Green, Orange, and Purple. Consequences of failing these criteria are listed in the fourth column.

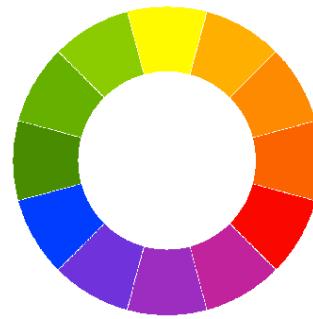
Using Table 2 it is mathematically possible to find combinations of values for the Kubelka-Munk optical parameters for blue, yellow and red such that their mixtures produce the secondary colors green, orange and purple as in Itten's color diagram. Table 3 shows such a set of values with  $K/S$  ratio's that correspond to reflectance values  $R = 0.99$  and  $R = 0.01$  (we assume these reflectance values are valid throughout the respective wavelength ranges of 400-500, 500-600 or 600-700 nm). We calculated the resulting CIE-Lab color coordinates of all resulting paints and their corresponding representation in a digital image [30], Fig. 2 shows that these values create a combination of colors that nicely matches Itten's color categories.

**Table 3. Numerical values for Kubelka-Munk  $K$  and  $S$  values.<sup>a</sup>**

	$K_{Blue}$	$K_{Red}$	$K_{Yellow}$	$S_{Blue}$	$S_{Red}$	$S_{Yellow}$
400–500 nm	$2.0 \times 10^{-7}$	0.001	300	0.004	$2.0 \times 10^{-5}$	6.1
500–600 nm	0.0001	0.0003	$1.5 \times 10^{-8}$	$2.0 \times 10^{-6}$	$6.1 \times 10^{-6}$	0.0003
600–700 nm	0.1	$1.0 \times 10^{-5}$	$1.0 \times 10^{-6}$	0.002	0.2	0.02

<sup>a</sup>If blue, red, and yellow paints would be found with these values of optical parameters, their mixtures would agree with Itten's color diagram (Fig. 2).

We have thus derived that Itten's secondary colors can only be expected to be produced by mixing when the primary colors satisfy the mathematical/optical constraints from Table 2. In the remainder of the article, we will show that when using actual pigments, it is probably



**Fig. 2.** Itten color diagram, with theoretical colors corresponding to well-chosen set of numerical values for Kubelka-Munk optical parameters.

impossible to satisfy these constraints. In this way we will demonstrate that Itten's color diagram is misleading as a method to illustrate color mixing of paints.

### 3. Results

#### 3.1. Results from paint experiments

The only instruction that Itten gave on the red, blue and yellow that he took as primary colors is that they need to be carefully selected as pure colors: the red should look neither blueish nor yellowish, etc. [7–9,11]. This leaves considerable room for color variations. No spectral reflectance data is available on the twelve color hues that Itten referred to in his diagram. Itten's color diagram in three editions of his main work show considerable color variations visually (Fig. 1), confirmed by color measurements (Table 4).

**Table 4. Color coordinates of the so-called primary colors Red, Yellow, and Blue from three different editions of Itten's *Kunst der Farbe* [7–9].**

		Itten 1975 [7]	Itten 1983 [8]	Itten 2021 [9]
Red	$L^*$	52.0	44.8	53.7
	$a^*$	48.4	57.4	50.3
	$b^*$	35.8	22.3	26.5
Yellow	$L^*$	82.1	82.6	81.0
	$a^*$	8.3	7.3	6.5
	$b^*$	82.0	89.6	79.8
Blue	$L^*$	47.7	44.6	49.1
	$a^*$	3.6	-10.1	1.1
	$b^*$	-44.6	-41.9	-35.0

We chose here to use the colors of the red, yellow and blue sectors in Itten's color diagram from the oldest edition [7] of 1975. We combined reflectance measurements for these three sectors with the  $K$  and  $S$  values of the Kubelka-Munk model that we determined for the oil paints. This enabled us to match Itten's red, blue and yellow colors by optimizing the concentrations of all twelve oil paints. We used a procedure explained in an earlier publication [23]. The paint recipes are shown in Table 5 with label A. The resulting colors can be seen in the painted color diagram shown in Fig. 3(a).



**Fig. 3.** Imitating Itten's color diagram, best matches by using (a) mixtures labeled as A: combinations of 12 oil paints, (b) mixtures B: combinations with only one red pigment for red and similarly for yellow and blue, (c) mixtures C: combinations with other red pigment for red, and similarly for yellow and blue.

**Table 5. Recipes in Gamblin artist oil paints for the three so-called primary colors in Itten's color diagram.<sup>a</sup>**

	Best matching recipe	$L^*$	$a^*$	$b^*$
Red_A	0.19 {Quinacridone red} + 0.81 {Cadmium red medium}	41.2	49.1	31.4
Yellow_A	0.007 {Titanium white} + 0.006 {Ivory black} + 0.76 {Cadmium yellow lemon} + 0.23 {Cadmium yellow medium}	82.0	7.7	90.5
Blue_A	0.06 {Titanium white} + 0.005 {Quinacridone red} + 0.003 {Cadmium red medium} + 0.93 {Cobalt blue}	35.5	4.8	-44.3
Red_B	0.001 {Titanium white} + 0.50 {Quinacridone red} + 0.50 {Cadmium yellow medium}	38.9	33.2	22.9
Yellow_B	0.0009 {Ivory black} + 0.0017 {Emerald green} + 0.98 {Cadmium yellow lemon}	71.4	3.3	66.5
Blue_B	0.06 {Titanium white} + 0.69 {Zinc white} + 0.25 {Prussian blue}	38.6	-2.7	-29.8
Red_C	0.11 {Zinc white} + 0.57 {Cadmium red medium} + 0.08 {Cobalt blue} + 0.25 {Cadmium yellow medium}	39.4	34.0	25.0
Yellow_C	0.29 {Titanium white} + 0.71 {Cadmium yellow medium}	78.9	8.5	70.2
Blue_C	0.47 {Zinc white} + 0.02 {Quinacridone red} + 0.14 {Viridian green} + 0.36 {Cobalt blue}	38.3	-2.1	-30.9

<sup>a</sup>The last three columns show measurement results for the dried paints. Mixtures labeled as A show best matching candidates, while mixtures labeled as B and C show candidates containing only one red paint in the red mixture, only one yellow paint in the yellow mixture and only one blue paint in the blue mixture (see text).

Comparing Fig. 3(a) with Fig. 1(a) we conclude that the visual match between the painted and Itten's primary colors from printed edition [7] is very reasonable: the measured color differences between the paint imitations and the colors in Itten's printed edition [7] are smaller than those between different printed editions from Itten's book. This visual match should be sufficiently good to use these primary color paints for testing the validity of Itten's color diagram to clarify color mixing of paints.

Using these red, yellow and blue oil paints we created the corresponding binary mixtures in 25:75, 50:50 and 75:25 ratios (Fig. 3(a)). Although the primary colors in Fig. 3(a) closely resemble

those in Itten's own representation of his color circle (Fig. 1(a)), we find large deviations for the binary mixtures, especially those between red and blue. Instead of Itten's bright purple-violets, we find dark browns. The same result was found in earlier studies [2,21] (pp. 123, 124 in Ref. [6] (p. 56 in Ref. [22]).

Obviously these dark brown mixtures could be made lighter by adding white to the mixture. However, this would go against the purpose of Itten's color diagram, which is to illustrate the mixing behavior of red, yellow and blue without further adding other paints.

We investigated if Itten's purple-violets would have been found if we base the red and blue "primary colors" on different pigment combinations. For example, in Table 5 the red paint is a mixture of Quinacridone red and Cadmium red medium. Would the mixing behavior of this red paint change if we would replace it by a mixture based on only Quinacridone red (mixture Red\_B), or only Cadmium red medium (Red\_C), excluding any other red paint in the mixture?

Table 5 shows two alternative formulations that match Itten's red "primary", and similarly also two alternatives for Itten's blue and yellow. The formulations labeled as B in Table 5 were mixed and applied on canvas. The color diagram is shown in Fig. 3(b). Similarly, the mixtures labeled as C resulted in the color diagram of Fig. 3(c).

Comparing the colors of paint mixtures (Fig. 3) with those from Itten's publication (Fig. 1(a)) we conclude that even when visually the same so-called primary colors red, yellow and blue are chosen, after mixing them as paints the resulting colors are often very different from Itten's diagram. Blue and yellow result in olive greens in Fig. 3(a) and 3(c), but in dark grass greens in Fig. 3(b). In contrast with Itten's color diagram, none of the 50:50 mixtures produced a bright grass green, and none of the 75:25 mixtures produced a bright blue. This proves that one of the key assumptions in Itten's color diagram is incorrect: the resulting color after mixing two paints cannot be predicted based on only the color of each of those two paints (cf. p. 124 of Ref. [6]). The color (or reflectance) of a particular paint contains insufficient information to predict what color will result if we mix it with other paints. For accurate predictions more detailed information is required, such as the spectral information captured in the Kubelka-Munk parameters  $K$  and  $S$ . Many scientists and artists before and after Itten incorrectly expected that the color of a paint fully determines what the color will be after mixing it with other paints.

The mixtures of red and blue provide another clear example of this. The bright purple that Itten shows in his diagram (Fig. 1(a)) was not found in any of our paint mixtures. Instead, when mixing red and blue paints dark red and brown are produced (Fig. 3(a) and 3(c)), up to almost black (Fig. 3(b)).

### 3.2. Theoretical analysis

The color deviations found in the paint experiments are easy to understand by calculating the Kubelka-Munk  $K$  and  $S$  parameters for formulations A, B and C. The results, shown in Table 6, indicate that for mixtures B and C for wavelengths 600–700 nm the value of  $S_{Yellow} > S_{Red}$ , making it mathematically impossible to satisfy both relation S9b ( $K_{Blue} < S_{Red}$ ) and S6c ( $K_{Blue} > S_{Yellow}$ ) from Table 2. As a consequence, paint mixtures B and (to less extend also) C both fail relation S9b. This explains why in Fig. 3(b) and 3(c) the red-blue mixture does not show Itten's bright purple, and also why for mixtures B the resulting mixture is too dark and too red.

Table 6 shows that for mixtures A the values for  $K$  and  $S$  parameters fail to satisfy equations S9a and S9b (explaining why in Fig. 3(a) blue-red mixtures become dark red rather than purple) as well as equation S6b (explaining why in Fig. 3(a) blue-yellow mixtures become dark green rather than light green). Mixtures A fail on all three conditions, by not satisfying (i)  $K_{Red} < S_{Blue}$  for 400–500 nm, (ii)  $K_{Blue} < S_{Yellow}$  for 500–600 nm, and (iii)  $K_{Blue} < S_{Red}$  for 600–700 nm. Identifying these failures makes clear that Itten's diagram would be reproduced better if we would be able to modify mixture Blue\_A as follows: (i) for 400–500 nm, the value of  $S_{Blue}$  needs to increase from 0.047 to values between 0.71 and 1.1; (ii) for 500–600 nm, the value of  $K_{Blue}$  needs to decrease

**Table 6. Numerical values for Kubelka-Munk  $K$  and  $S$  values for the color formulations of mixtures A-G.<sup>a</sup>**

		$K_{Blue}$	$K_{Red}$	$K_{Yellow}$	$S_{Blue}$	$S_{Red}$	$S_{Yellow}$
Mixtures A	400–500 nm	0.024	0.71	1.1	0.047	0.00091	0.016
	500–600 nm	0.38	0.90	0.012	0.023	0.026	0.13
	600–700 nm	0.37	0.0037	0.010	0.023	0.16	0.11
Mixtures B	400–500 nm	0.057	0.8	1.1	0.072	0.0087	0.010
	500–600 nm	0.31	0.59	0.018	0.061	0.081	0.12
	600–700 nm	1.3	0.0059	0.0060	0.050	0.071	0.10
Mixtures C	400–500 nm	0.026	0.81	0.95	0.040	0.014	0.15
	500–600 nm	0.19	0.57	0.0073	0.026	0.079	0.22
	600–700 nm	0.18	0.032	0.0036	0.024	0.15	0.18
Mixtures D	400–500 nm	0.010	0.328	1.117	0.334	0.163	0.016
	500–600 nm	0.112	0.619	0.012	0.283	0.155	0.135
	600–700 nm	0.109	0.004	0.010	0.233	0.180	0.107
Mixtures E	400–500 nm	0.010	0.304	1.117	0.334	0.062	0.016
	500–600 nm	0.112	0.430	0.012	0.283	0.065	0.135
	600–700 nm	0.109	0.002	0.010	0.233	0.110	0.107
Mixtures F	400–500 nm	0.010	0.304	1.117	0.334	0.062	0.016
	500–600 nm	0.112	0.430	0.012	0.283	0.065	0.135
	600–700 nm	0.109	0.002	0.010	0.233	0.110	0.107
Mixtures G	400–500 nm	0.041	0.507	1.117	0.146	0.104	0.016
	500–600 nm	0.414	0.717	0.012	0.116	0.109	0.135
	600–700 nm	0.223	0.004	0.010	0.101	0.183	0.107

<sup>a</sup>The corresponding paint mixtures are given in Tables 5 and 7. Note that for mixtures E and G, these values have not been scaled for the presence of (virtual) paint medium (see text).

from 0.38 to values between 0 and 0.13; (iii) for 600–700 nm, the value of  $K_{Blue}$  needs to decrease from 0.37 to values between 0.11 and 0.16.

For this we used three main approaches: (i) adapting the blue primary paint, (ii) adapting the red and yellow primary paints, (iii) allowing mixtures other than 50%–50%.

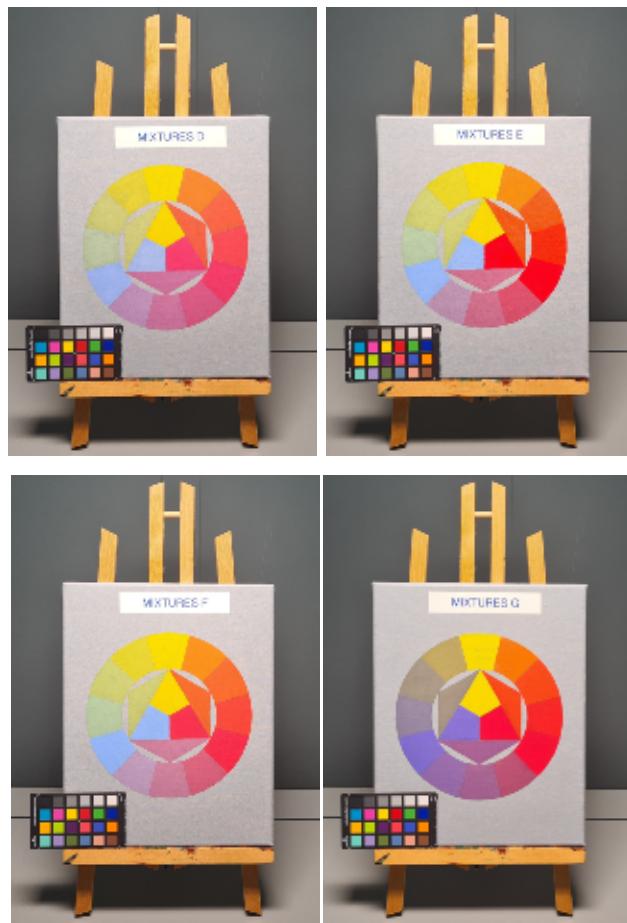
### 3.3. Adapting the blue primary paint

If we add Titanium white to the blue mixture the values of  $S_{Blue}$  increase whereas  $K_{Blue}$  decreases, but the calculations show that there is no quantity of Titanium white possible that brings all parameters to the required range.

If we only focus on creating purple and temporarily ignore the other conditions from Table 2, we find that only if large amounts of Titanium white are added to both the blue and the red mixture, the combination of blue and red will produce purple. However, the color of the blue mixture will be a much lighter shade of blue than what Itten showed in his book, and in addition to that the red mixture will look like a very bleak red. Also, relations S9a as well as S6b will still be violated. By adding Titanium white to the yellow mixture, the value for  $S_{Yellow}$  is increased making it easier to satisfy relation S6b, without changing the appearance of the yellow mixture too much. In that way we can satisfy all relations except S9a ( $S_{Blue} > K_{Red}$  for 400–500 nm).

Since we should not try to increase the value of  $S_{Blue}$  by adding even more Titanium white to the blue mixture, the only alternative is to change the red mixture such that its value for  $K$  strongly

decreases for 400-500 nm. The best way to accomplish this was found to be by reducing the amount of Cadmium red (which has a relatively large value for  $K$  for 400-500 nm, as evidenced in Table 1) with respect to the other two components, Quinacridone red and Titanium white. A good compromise that either satisfies or nearly satisfies the relations from Table 2 was found with mixtures D. Here, the blue mixture is calculated to be still lighter than in Itten's printed edition, and the red, while still purplish red rather than full red, is not too bleak or too light. Figure 4(a) gives an indication of the resulting colors, based on a photograph of the painted color circle when following the paint formulations of mixtures D.



**Fig. 4.** More attempts to reproduce Itten's color diagram, (a) mixtures D: formulations with Kubelka-Munk K and S values that all satisfy the relations from Table 2, (b) mixtures E: further optimization of formulations, by introducing paint medium in the calculations and (c) making that paint medium only virtual in mixtures F, (d) mixtures G: alternative formulation by using more virtual paint medium.

Starting from mixtures D, the red mixture can be made less purple and more saturated by reducing the amounts of the purplish Quinacridone red and Titanium white in the red mixture but not increasing the concentration of Cadmium red medium. Since this mixture has no other ingredients, we need to add optically neutral paint medium to the mixture. In the calculations for mixtures E, we chose to add a maximum of 40% paint medium to the red mixture since at this

amount the paint becomes too thin to apply. According to Table 6, the calculated values for  $K$  and  $S$  parameters for the red mixture nearly satisfy all requested relations from Table 2.

When we applied mixtures E we found that the resulting color diagram indeed produces the desired purple, orange and green mixtures, just as well as for mixtures D, while the red mixture is a much more convincing red. The blue is still much lighter than Itten's blue.

For mixtures E, the main issue is that the paint is too thin to easily apply. This could be solved by not adding the paint medium (for which we used "083 Painting medium" from the Royal Talens brand) to the physical red paint mixture, but making the red mixture only 60% effective when creating the so-called secondary mixtures such as red-blue. Effectively, instead of assuming all secondary mixtures to be 50%–50% mixtures as we have done so far in this article, we now allow other percentages. For example, by mixing 60% of 50 gram of red with 50 gram of blue we obtain concentrations of 30/80 and 50/80 (37.5%–62.5% mixture). After applying these paint mixtures (labeled as mixtures F), we indeed found that they result in the same colors as mixtures E, without having the issues with paint applicability. The result is shown in Fig. 4(c).

Based on this result, in the next section we will further explore the possibilities when lifting the constraint that secondary and tertiary mixtures are based on 25-75, 50-50 and 75-25 percentages.

### 3.4. Adapting the red, yellow, and blue primary paints

Starting from mixture A we may also change the formulations for the yellow and red mixtures. For the blue mixture we may now substitute the Cobalt blue pigment by a different blue pigment. Given the analysis on how the  $K$  and  $S$  values for mixture Blue\_A fail to satisfy the relations of Table 2, we look for an alternative blue pigment with larger  $S$  values for 400-500 nm and lower  $K$  values for 500-700 nm as compared to Cobalt blue.

In Table 1, we include values of the  $K$  and  $S$  parameters for an additional series of blue oil paints, including other suppliers. It shows that only two of these paints may qualify: Königsblau (Rembrandt) and Sèvres blue (Van Gogh). However, it is easy to show that whatever quantity we would use it would not be possible to make  $S_{Blue} > K_{Red}$  for 400-500 nm (equation S9a). Instead of changing the properties of the blue mixture it may be better to change the composition of the red mixture, in such a way that the absorption value  $K_{Red}$  is small enough to bring it into the reach of the scattering value  $S_{Blue}$ . However, this implies a large decrease in the value of  $K_{Red}$ , which would require a large addition of white pigment (making the color of the red mixture appear pink) or blue pigment (making the red mixture look purple). Therefore, this approach provides no promising alternatives to the options already discussed.

### 3.5. Allowing mixtures other than 50%–50%

In this section we investigate if our attempts to reproduce Itten's color circle would be more successful if we no longer assume that Itten used only 50%–50% mixtures. Unfortunately, in his book *Kunst der Farbe* [7–9,11,12], Itten does not specify concentrations.

We investigated this by again introducing a virtual paint medium in the calculations. For example, when mixing blue and yellow we assume that the yellow paint is first mixed with optically neutral paint medium, having a fraction  $Q_{Yellow}$  for the actual yellow mixture and  $1-Q_{Yellow}$  for the paint medium. Green mixtures that previously had a concentration  $f$  of yellow and  $(1-f)$  of blue will now contain only  $fQ_{Yellow}$  of yellow and  $(1-f)$  of blue. After renormalization it follows that for example the secondary mixture of green previously had a concentration of yellow of  $f=0.5$ , but now in case of  $Q_{Yellow}=0.6$  we find a concentration of only 0.375 of yellow.

We used this approach to find formulations for the red, yellow and blue primaries that produce convincing secondary colors, using the same calculational procedure presented before. Not surprisingly, we run against the same difficulties that we encountered in our main analysis. For example, the purple secondary color again becomes too dark and too reddish unless we (i) add considerable amounts of Titanium white to the blue and/or the red mixture, or (ii) add a

considerable amount of bluish Quinacridone red to the red mixture, or (iii) do both. In all these cases, the primary colors look considerably different from the color diagram in Itten's book: the red becomes too purplish and/or too bleak, and/or the blue becomes too light.

By changing the amount of virtual paint medium in the primary colors, and even allowing negative amounts of paint medium in the calculations to reach new ranges of concentrations, we found that a variation is possible to the previous results. This resulted in the paint formulations of mixtures G shown in Table 7. Figure 4(c) shows the color circle painted with the formulations of mixtures G, indicating that we succeeded in creating reasonably convincing secondary colors: the orange is close to perfect and the purple is not darker than in Itten's book although its chroma is smaller. This was achieved by introducing virtual paint medium with  $Q_{Red} = 0.5$  and  $Q_{Blue} = 1.5$ . In contrast to our earlier best solution mixtures E and F, the blue primary is now as dark as in Itten's color diagram. But the issues we encountered before also here made us compromise on other aspects. In mixtures G both the blue and the red primaries have to include a substantial amount of Quinacridone red in order to avoid the purple secondary color to become too dark. As a consequence, the blue and red primaries look more purplish than in Itten's color diagram. Also, because of reducing the lightness of the blue primary the green secondary color has now become much darker than in Itten's color diagram.

**Table 7. More alternatives paint recipes for matching the so-called primary colors in Itten's color diagram.<sup>a</sup>**

	Alternatively matching recipes	$L^*$	$a^*$	$b^*$
Red_D	0.35 {Titanium white} + 0.32 {Quinacridone red} + 0.32 {Cadmium red medium}	51.6	48.4	6.6
Yellow_D	0.0071 {Titanium white} + 0.0059 {Ivory black} + 0.76 {Cadmium yellow lemon} + 0.23 {Cadmium yellow medium}	78.8	-2.6	92.0
Blue_D	0.73 {Titanium white} + 0.0016 {Quinacridone red} + 0.0010 {Cadmium red medium} + 0.27 {Cobalt blue}	73.3	-7.1	-30.7
Red_E	0.13 {Titanium white} + 0.13 {Quinacridone red} + 0.33 {Cadmium red medium} + 0.40 {Paint medium}	48.3	54.6	23.0
Yellow_E	0.0071 {Titanium white} + 0.0059 {Ivory black} + 0.76 {Cadmium yellow lemon} + 0.23 {Cadmium yellow medium}	78.8	-2.6	92.0
Blue_E	0.73 {Titanium white} + 0.0016 {Quinacridone red} + 0.0010 {Cadmium red medium} + 0.27 {Cobalt blue}	73.3	-7.1	-30.7
Red_F	0.13 {Titanium white} + 0.13 {Quinacridone red} + 0.33 {Cadmium red medium} + 0.40 {Paint medium}	48.3	54.6	23.0
Yellow_F	0.0071 {Titanium white} + 0.0059 {Ivory black} + 0.76 {Cadmium yellow lemon} + 0.23 {Cadmium yellow medium}	78.8	-2.6	92.0
Blue_F	0.73 {Titanium white} + 0.0016 {Quinacridone red} + 0.0010 {Cadmium red medium} + 0.27 {Cobalt blue}	73.3	-7.1	-30.7
Red_G	0.11 {Titanium white} + 0.11 {Quinacridone red} + 0.28 {Cadmium red medium} + 0.50 {Paint medium}	48.3	54.6	23.0
Yellow_G	0.0071 {Titanium white} + 0.0059 {Ivory black} + 0.76 {Cadmium yellow lemon} + 0.23 {Cadmium yellow medium}	78.8	-2.6	92.0
Blue_G	0.45 {Titanium white} + 0.22 {Quinacridone red} + 0.0031 {Cadmium red medium} + 0.83 {Cobalt blue} - 0.50 {Paint medium}	46.9	10.7	-45.7

<sup>a</sup>The last two columns show predicted color coordinates for the resulting paints. For the additions of (virtual) paint medium in mixtures E, F, and G, see text.

Therefore we conclude that also by allowing secondary colors to deviate from the 50%–50% ratio, the same issues are found as before, making it impossible to reproduce all primary and secondary colors of Itten's color circle.

#### 4. Discussion

The results discussed in the previous sections can be explained as follows. In order to produce the colors in Itten's color diagram we need to satisfy the following conditions (Table 2):

For 400–500 nm, combining relations S9a and S6a shows that we need to have  $K_{Red} < K_{Yellow}$ . This relation is satisfied by both red oil paints and both yellow oil paints in Table 1. Additionally, we need  $K_{Yellow} > S_{Blue}$ , which is also satisfied for all blue and yellow oil paints in Table 1. Finally,  $K_{Red} < S_{Blue}$ . For the red and blue oil paints in Table 1 this is only satisfied if suitable combinations of red and blue oil paints are selected:

- The value of  $S_{Blue}$  needs to be relatively large. This is almost always not the case for blue pigments (Table 1). We already mentioned that almost all blue pigments are blue not because of large scattering at small wavelengths, but because of large absorption for larger wavelengths. This is a key difference between actual blue pigments and the theoretical blue pigment in Table 3 that we used to show that theoretically Itten's color diagram could be reproduced if the optical properties of the pigments could be suitably chosen. When using actual blue pigments, we can create relatively large values of  $S_{Blue}$  either by either using Cerulaeum blue hue (Gamblin) pigment, or by mixing any of the other blue oil paints with a large quantity of Titanium white. More than 60% Titanium white is needed to sufficiently increase the value of  $S_{Blue}$ , so the color of the blue mixture will be much lighter than Itten's dark blue.
- The value of  $K_{Red}$  needs to be relatively small. This is almost always not the case for red pigments (Table 1): their red color is mostly not driven by large scattering values for large wavelengths but by large absorption values for smaller wavelengths. To obtain relatively small values of  $K_{Red}$ , the proportion of Quinacridone red should outweigh Cadmium red medium, and/or Titanium white needs to be included in the mixture. Because of this, the color of the red mixture will become purplish red and/or will become a very bleak red. In both cases, the red mixture will not show the bright red color shown in Itten's diagram.

For 500–600 nm, we need to have  $K_{Blue} < S_{Yellow}$ . For the blue and yellow oil paints in Table 1 it is only possible to satisfy this relation if the blue and yellow mixtures are carefully selected:

- The value of  $K_{Blue}$  needs to be relatively small. This can be realized by using one of four pigments (Königsblau Rembrandt, Sèvres blue from Rembrandt or Van Gogh, Cerulaeum blue from Van Gogh), or by mixing any of the other blue oil paints with a large quantity of white. More than 50% Titanium white is needed to increase the value of  $K_{Blue}$  sufficiently, making the color of the blue mixture much lighter than the dark blue in Itten's color diagram.
- The value of  $S_{Yellow}$  needs to be relatively large. This is possible by mixing in Titanium white, which hardly changes the appearance of the yellow mixture.

For 600–700 nm, we need to have

- Combining relations S6c and S9b it follows that  $S_{Yellow} < S_{Red}$ . Table 1 shows that this is feasible if especially the red mixture is carefully selected. We need a relatively large value of  $S_{Red}$ , which is possible by preferring Cadmium red medium over Quinacridone red pigment and/or by adding Titanium white to the red mixture.
- Additionally,  $K_{Blue} > S_{Yellow}$ . Table 1 shows that this relation is easily satisfied.
- Finally, it is required that  $K_{Blue} < S_{Red}$ . Table 1 shows that with most available blue pigments this is not possible since (contrary to the theoretical blue pigment in Table 1) their blue

color is driven by high absorption for long wavelengths. Therefore this requirement can only be satisfied by adding a substantial amount of Titanium white to the blue pigment(s). This makes the color of the blue mixture much lighter than the dark blue in Itten's color diagram.

This analysis explains why earlier experiments with mixing paints showed that especially the bright violet secondary color from Itten cannot be obtained when using the perceptually pure red and blue "primary colors" prescribed by Itten [2,6,21] (p. 123) [22] (p. 56).

The arguments given here probably also explain why in earlier publications Itten's color circle could not be reproduced with paints when carmine red was combined with ultramarine blue, Prussian blue, or when magenta "red" was combined with cyan "blue", or vermilion (cinnabar) red with ultramarine blue, or vermilion red with cobalt blue (p. 123 of Ref. [6]) (p. 56 in Ref. [22]) (pp. 56, 75, 76 in Ref. [21,22]).

## 5. Conclusion

In this article we investigated if Itten's color circle is useful to teach or illustrate the color mixing of paints. We found that Itten's color circle is misleading since even with large effort it cannot be reproduced. We found no combination of red, yellow and blue paints that after mixing produce the vivid bright secondary colors indicated by Itten's diagram.

Our results were obtained in two steps. First, we mathematically derived that Itten's secondary colors can only be expected to be produced by mixing when the primary colors satisfy certain constraints on their optical parameters (the values of Kubelka-Munk  $K$  and  $S$  parameters). Secondly, we showed that although mathematical solutions to these constraints exist, with actual pigments it is probably impossible to satisfy all constraints because almost all common red and blue pigments derive their color mostly by absorption at non-dominant wavelengths.

Therefore, red pigments often do not satisfy relation (S9a) which requires their absorption to be relatively small for low wavelengths, and most blue pigments do not satisfy relation (S9b) which requires their absorption to be relatively small for large wavelengths. Using the mathematical predictions as a starting point and applying oil paints and spectrophotometer measurement data to test the predictions, we showed that in order to satisfy these relations, the blue and/or red primary colors need to be much lighter and/or much more purplish than in Itten's color circle.

For this reason, Itten's color diagram cannot be regarded as a tool that clarifies how colors mix. When artists mix the red, yellow and blue paints the way Itten instructed them to do, namely by creating, e.g., a blue paint that is not greenish or reddish, it is impossible, or at least very unlikely, that they find Itten's secondary colors. Indeed, this result confirms and provides a theoretical explanation for the results from earlier studies, which either stated the same without further proof (p. 88 in Ref. [5]) or were based on trial and error. Those earlier publications suggested that when following Itten's advice on how to define the paint color primaries, no bright violet is found after mixing red and blue [2,3,6,21] (p. 123) [22] (p. 56), but that a bright violet can be found by using, e.g., a much brighter blue primary than shown in Itten's color diagram [2] (also p. 123 of Ref. [6]), or by using a bluish red like magenta-red [3,22]. Our analysis confirms these earlier findings, and provides a theoretical explanation based on the optical properties of the paint materials. This analysis also confirms and explains the conclusion from earlier publications that Itten's color diagram is not suitable for teaching color theory, since art students are not able to reproduce its mixing rules [3,6].

This analysis is based on the 17 different oil paints listed in Table 1. Other red, yellow and blue pigments may still exist with values of  $K$  and  $S$  parameters that do make it possible to reproduce Itten's diagram with the colors as indicated by Itten. However, after having tried eight different blue pigments we consider it especially unlikely that, when combined with a convincingly red paint, a blue paint is possible with  $K_{Red} < S_{Blue}$  for 400-500 nm and  $K_{Blue} < S_{Red}$  for 600-700 nm,

without making its color much lighter by adding Titanium white and/or much more purplish by adding, e.g., Quinacridone red.

This conclusion is fully in line with earlier publications that gave anecdotal evidence and case studies for the impossibility to experimentally reproduce Itten's color circle using different combinations of red, blue and yellow pigments [3]. These earlier results are now given a theoretical explanation. For an accurate description of color mixing by paints, optical theories such as Kubelka-Munk are needed that take into account contributions from light absorption and scattering by pigments.

Apart from the impossibility to reproduce Itten's color diagram by mixing oil paints, we mentioned and showed that Itten's color diagram also implies several other misconceptions. Itten's color diagram suggests that by viewing the colors of two paints, one can predict what color their mixture would be. In this article we saw that this is not the case (which also implies that the alternative to Itten's color diagram proposed by Wilcox is not correct [3]). Two blue paints that look identical and two yellow paints that look identical may produce very different green mixtures: from very light to very dark, and from very bluish to very yellowish green (this important point is rarely mentioned, a rare example is p. 124 of Ref. [6]). Similarly, mixing the same red and blue paints may lead to black, brown, orange, red or purple mixtures. In order to know what color the mixture will be, the colors of the mixing paints do not provide sufficient information. By determining the optical properties of these paints, such as the Kubelka Munk  $K$  and  $S$  parameters, sufficient information is available to predict the color of the mixture.

The Itten color diagram also reiterates the centuries old fallacy that for mixing paints the three colors red, yellow and blue represent primary colors from which all other colors can be produced by mixing [3,31,32]. This had already been proven wrong in the mid 19th century, by distinguishing additive from subtractive mixing. Itten mentioned additive and subtractive mixing in his book, but his color circle is often used in color theory courses as support for the claim that all colors can be produced from mixing primary colors red, yellow and blue. The incorrectness of this claim is understood by realizing that virtually every painter in history selected multiple red, multiple blue and multiple yellow pigments on their pallettes, preferably even complemented by, e.g., different green pigments, in order to be able to create more colors by mixing the pigments [1,33].

The consequences of the results from this work for color theory as practiced by and taught to artists and designers will be the subject of a next article, which we intend to publish in a journal that has an audience with more artists and designers.

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**Data availability.** Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the author upon reasonable request.

**Supplemental document.** See [Supplement 1](#) for supporting content.

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