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H2SI – AN NEW PERCEPTUAL COLOUR SPACE

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Abstract. In this paper we will gradually develop a new colour space which is equipped with a metric that shares properties with the human perception of colour.

Keywords: Human colour perception, colour matching functions, opponent colour vision, crispening of saturation and brightness, colour distance, transition probability, signal/sensor fusion, image segmentation.

1. Introduction

Colour even nowadays remains a miracle. Some 20000 years ago our ancestors started to liven up their paintings on cave walls by colour, whole industrial imperia developed around it to produce certain rare colours in the past. Colour seems to be very special for beings that can sense them. In this article we try to shade some new light on it which, if you take it literally, does not harm our perception of colour too much, as should become clear later.

Colour, its nature, representation, perception, and geometric properties have been studied by scientists for centuries. The scientific history of work on colour dates back at least to Isaac Newton, who found amongst others that light send through a prism will split into most perceivable colours. Goethe, Young, Maxwell, Hering, Helmholtz, Munsell, Schönfelder, and Schrödinger, to name but a few, worked on the topic, partially opposing each other in their results.

Modern colour science cannot be seen as a unique field of research as it involves many research areas ranging from psychological, biological, medical sciences to more technically oriented areas such as chemistry or computer vision. [15, 6, 7, 3, 2] give an overview on the different aspects in this area.

An excellent and very readable sketch on the history of ideas about colours from arestotele to moden colour science is given in [5].

The more technically oriented work is based on some of this preceding groundwork and deals mainly with the representation of colour and the geometrical and topological relations that colours have. The most widely used explicit definition of a colour representation in a mathematical space - the XYZ colour matching functions space - by the CIE (Commission International de l'Eclairage) in 1931 might be seen as the starting point of a normative process in order to standardize colour for technical/industrial purposes. The original aim was to develop an isotropic colour representation. Isotropic in the sense that perceived colour attributes map to a space where unit differences in one dimension are equal to unit differences in the other dimensions. In loose terms these attributes are given by the colour tone or hue, the chromaticism or saturation and the brightness or lightness or luminance of

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[2]To my father who sparkled the flame of mathematical beauty in me.

colour, but the exact definition of these attributes varies greatly in the literature. Other colour spaces were defined by Musnell (1929) and Mac Adams chromatic value space (1923) cf.[5]

MacAdam showed in the 1940s that the CIE XYZ colour space is not isotropic [10] (Figure 1). Instead of circles of perceived unit differences of colour it contains ellipses which are elongated in chroma with respect to hue and ellipsoids when the brightness dimension is taken into account as well.

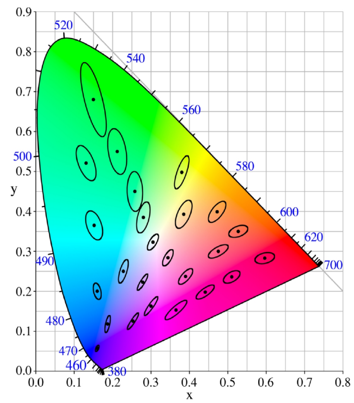

FIGURE 1. MacAdam ellipses plotted on the CIE 1931 xy chromaticity diagram [16]. The ellipses show the location of colours that have a just noticable difference (JND) to the center colour. The ellipses are ten times their actual size, as depicted in MacAdam's paper [10].

His results were strengthened by Judd [4, 5] and Ludwik Silberstein [13, 14] who concluded that no ideal colour space, especially no three-dimensional Euclidean space supports geometrical properties where MacAdam ellipses, i.e. perceived unit differences of colour, can lie on unit circles. Judd termed this property of colour perception 'Super-importance of hue differences' [4].

Judd noted [5]:

> "...; but the implication that gray is both precisely between 5R and 10Y and precisely between 5R and 5BG is rather hard to take. Furthermore, by this model, gray lies precisely between any two hues differing by 25 or more and 50 or less Munsell hue steps."
>
> And
>
> "... There does not seem to be a geometrical model agreeing with Eq. 15; (colour difference formular, authors note) at least I have not been able to think of one."

The existence of this phenomena, at least in human colour perception, has been validated in many psychological experiments over the last century with remarkably small differences from one person to another.

If the 'super-importance of hue' is taken seriously it leads to a simple mathematical consequence. 'Super-importance of hue', as pointed out by Judd, requires a space for colour representations that enables unit circles to have a circumference of roughly 720°, i.e. twice the circumference of a unit circle in Euclidean space. He visualized this as a 'fan crinkled surface' and explained some consequences as cited above.

Newer normalization developments such as the CIElab colour spaces and distance formulas such as ΔE take a different approach and try to map geometrical properties of colour in a non-linear way onto three-dimensional Euclidean spaces. There have been many attempts to overcome the non-linear behaviour of colour distances, such as ΔE, by developing approximations in Euclidean space [12, 8, 9]. But although ΔE allows us to measure local distances of colour under controlled illumination and surrounding conditions reasonably well, it seems to fail when bigger colour differences have to be judged [9].

Recently Bengtsson and Zyczkowski [1] pointed out that a space in which the 'super-importance of hue' property is represented by its geometry would be given by the well-studied mathematical spaces of quantum mechanics and quantum information theory, but without giving an explicit formula. The apparent discrepancy between this and Judd's claim is resolved when the complex nature of these spaces is taken into account.

Later developments took mainly two directions: an industrial and a computational or, to phrase it differently albeit not entirely correct, the direction of absolute and relative colour. Absolute colour refers to the reproduction process of colour in modern technical devices. This is limited, depending on the device. The task is to find good substitutes if a concrete colour is not available. Nowadays this is done via look up tables the so-called ICC profiles that map colour seen by one device such as a camera onto colours producible by anotherdevice which could be a printer or monitor. It is not very enjoyable to see the blue sea on the latest holiday pictures turn purple on the printouts. Therefore colour spaces devised and the error critiria derived, try to model the end users or consumers, i.e. human perception more accurately into newly created colour spaces. The revised colour mappings of the CIE, such as the Luv colour space, and the ΔE distance function on absolute colour errors give a good history on this development. And the state of the art is quite impressive as can be seen in technical processes of colour reproduction such as printing, Tv sets, etc. or in modern computer generated movie renderings.

In other words: the technical process seems to be more or less well handled by the techniques we have and for the purposes they were created for.

Colour as an information source for computational purposes has to be seen different. The ease humans, and most likely other beings that are able to sense a colourful environment, are able to separate scenes into 'important' or dominant parts is puzzling scientists from the beginning. The question whether this is mainly a product of cognitive skills such as learning and context or whether other helpful mechanisms exist is unanswered and this article will not provide a final answer. But with respect to colour, we would like to put a more 'syntactical', better to say mathematical, regime into place that coincides far more with our perception than the ones developed beforeand and has the benefit of being helpful for algorithmic purposes. It might be surprising that something seemingly that simple as a colour edge has no precise definition nowadays and consequently no algorithm to compute it can exist.[1]

[1]Of course there are many algorithms that perform closely related to our perception of a colour edge.

The rest of the paper is organized as follows. In section 2 we will give a new coordinate mapping into a four dimensional complex space. Colours will be uniquely represented as unit vectors lying in a manifold of a complex hyperspere. In section 3 we will calculate the line element in this space and derive the metric tensor. Calculating the Christophel symbols in section 5 enables us to derive a set of partial differantial equations in order to calculate the geodesic between two given colours and we will give some examples showing that Judds implication as cited above are indeed true in the new colour space.

2. The H2SI colour space

To keep our argument simple and avoid all discussions on primary colour matching functions (but see [15, 17]) we take the HSI-colour space as our basic colour space and refer to the literature on colour space conversions, e.g. [5, 3, 17]. Colour is defined in terms of three Variables namely hue, $0 \leq H \leq 2\pi$, saturation, $0 \leq S \leq 1$, and intensity, $0 \leq I \leq 1$.The following set of equations will map the HSI space into the four dimensional complex space, called the $H2SI$ colour space. The acronym stands for either 'hue double, saturation, intensity' or 'Hilbert square, super-importance':

$$
\begin{aligned}
x_1 &= \sqrt{I\left(1-\frac{S}{2}\right)} \\
x_2 &= \sqrt{\frac{S}{2}}\cos(2H)e^{-iH} \\
x_3 &= \sqrt{\frac{S}{2}}\sin(2H)e^{iH} \\
x_4 &= \sqrt{(1-I)\left(1-\frac{S}{2}\right)} \\
\sum_{i=1}^{4}|x_i|^2 &= 1 \qquad (2.1)
\end{aligned}
$$

Equation 2.1 ensures that all colour vectors are normalized and lie on a complex unit hypershere. Therefore formally they can be seen as states of a bipartide quantum system or a four dimensional normalized Hilbert space $\mathcal{H} = \mathbb{C}^2 \otimes \mathbb{C}^2$, which means that concepts developed for these spaces apply, and will be used intensively and systematically in the following sections. The inverse mapping is given by:

$$
\begin{aligned}
S &= 2|x_2|^2 \\
I &= \frac{x_1^2}{(1-S/2)} \\
cos(H) &= Re(x_2) + Im(x_3) \\
sin(H) &= Im(x_3) + Re(x_2) \\
H &= \arctan 2(cos(H), sin(H)) \qquad (2.2)
\end{aligned}
$$

In the next section we will show that the $HS2I$ space contains a unit circle with circumference $\approx 4pi$. A brief introduction to quantum information theory can be found in [11] and the literature mentioned therein. At the same time the $H2SI$ space is an instance of a curved Riemannian space. In sections 3,and 5 we will analyse some geometrical properties such as the line element, the metric tensor and a set of partial differential equations in order to calculate geodesics in this space. We note that the colour co-ordinates as defined in Equation 2.1 are homogenous and may be interpreted as unit bi-quaternions, although we will not make use of these facts. But they indicate that the $H2SI$ space is endowed with a rich mathematical structure.

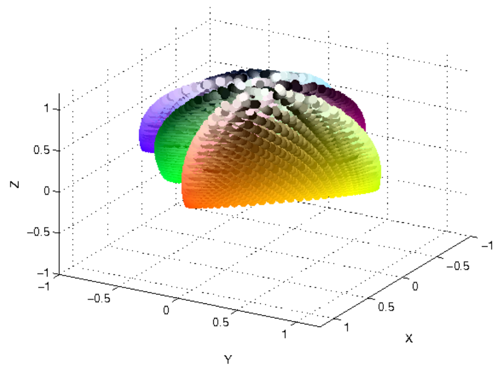

FIGURE 2. Bloch sphere of the partial trace of the $H2SI$ colour space when tracing out system 1

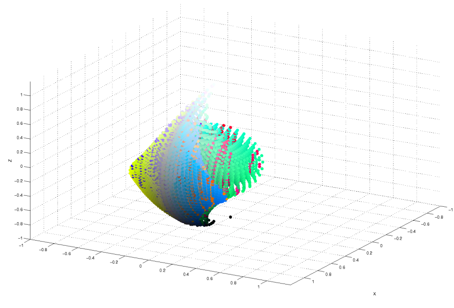

FIGURE 3. Bloch sphere of the partial trace of the $H2SI$ colour space when tracing out system 2

3. The line element and Metric Tensor

$$ds^2 = \sum_{i=1}^{4} |dx_i|^2 \tag{3.1}$$

$$\begin{aligned} dx_{1,4} &= \frac{\partial x_{1,4}}{\partial I}dI + \frac{\partial x_{1,4}}{\partial S}dS \\ dx_{2,3} &= \frac{\partial x_{2,3}}{\partial S}dS + \frac{\partial x_{2,3}}{\partial H}dH \end{aligned} \tag{3.2}$$

$$\begin{aligned} \frac{\partial x_1}{\partial I} &= \frac{\sqrt{1-\frac{S}{2}}}{2\sqrt{I}} \\ \frac{\partial x_1}{\partial S} &= -\frac{\sqrt{I}}{4\sqrt{1-\frac{S}{2}}} \\ \Longrightarrow dx_1 &= \frac{\sqrt{1-\frac{S}{2}}}{2\sqrt{I}}dI - \frac{\sqrt{I}}{4\sqrt{1-\frac{S}{2}}}dS \\ \Longrightarrow dx_1^2 &= \frac{1}{16I\left(1-\frac{S}{2}\right)}\left[(2-S)^2dI^2 + I^2dS^2 - 2I(2-S)dIdS\right] \end{aligned} \tag{3.3}$$

$$\begin{aligned} \frac{\partial x_4}{\partial I} &= -\frac{\sqrt{1-\frac{S}{2}}}{2\sqrt{1-I}} \\ \frac{\partial x_4}{\partial S} &= -\frac{\sqrt{1-I}}{4\sqrt{1-\frac{S}{2}}} \\ \Longrightarrow dx_4 &= -\frac{\sqrt{1-\frac{S}{2}}}{2\sqrt{1-I}}dI - \frac{\sqrt{1-I}}{4\sqrt{1-\frac{s}{2}}}dS \\ \Longrightarrow dx_4^2 &= \frac{1}{16(1-I)\left(1-\frac{S}{2}\right)} \\ &\quad \times\left[(2-S)^2dI^2 + (1-I)^2dS^2 - 2(1-I)(2-S)dIdS\right] \end{aligned} \tag{3.4}$$

$$\begin{aligned} \frac{\partial x_2}{\partial S} &= \frac{\cos(2H)e^{-iH}}{2\sqrt{2S}} \\ \frac{\partial x_2}{\partial H} &= \sqrt{\frac{S}{2}}\left[-2\sin(2H) - i\cos(2H)\right]e^{-iH} \\ \Longrightarrow dx_2 &= \frac{e^{-iH}}{2\sqrt{2S}}\left[2S\left(-2\sin(2H) - i\cos(2H)\right)dH + \cos(2H)dS\right] \\ \Longrightarrow |dx_2|^2 &= \frac{1}{8S}\Big(\cos^2(2H)dS^2 + 4S^2\left[4\sin^2(2H) + \cos^2(2H)\right]dH^2 + \\ &\quad \left[-4S\cos(2H)\sin(2H)\right]dSdH\Big) \end{aligned} \tag{3.5}$$

$$\begin{aligned}
\frac{\partial x_3}{\partial S} &= \frac{\sin(2H)e^{iH}}{2\sqrt{2S}} \\
\frac{\partial x_3}{\partial H} &= \sqrt{\frac{S}{2}}\left[2\cos(2H) + i\sin(2H)\right]e^{iH} \\
\Longrightarrow dx_3 &= \frac{e^{iH}}{2\sqrt{2S}}\left[2S\left(2\cos(2H) + i\sin(2H)\right)dH + \sin(2H)dS\right] \\
\Longrightarrow |dx_3|^2 &= \frac{1}{8S}\Big(\sin^2(2H)dS^2 + 4S^2\left[4\cos^2(2H) + \sin^2(2H)\right]dH^2 + \\
&\quad \left[4S\cos(2H)\sin(2H)\right]dSdH\Big) \qquad (3.6)
\end{aligned}$$

Together this gives the line element:

$$\begin{aligned}
ds^2 = \frac{dS^2}{4S(2-S)} + \frac{2-S}{8I(1-I)}dI^2 + \frac{5}{2}SdH^2 = \\
a\,dS^2 + b\,dI^2 + c\,dH^2 \qquad (3.7)
\end{aligned}$$

defining the quantities a, b and c by

$$a = \frac{1}{4S(2-S)} \;\;,\;\; b = \frac{2-S}{8I(1-I)} \;\;,\;\; c = \frac{5}{2}S \;. \qquad (3.8)$$

As the line element is a function of S and I but independent of H we may visualize it as shown in Figure 4

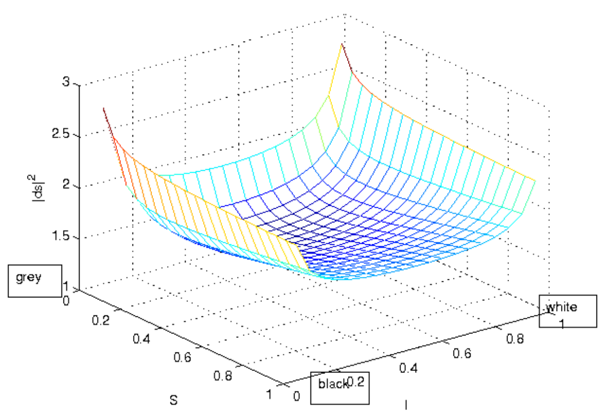

FIGURE 4. Sensitivity of the line element ($\equiv (ds(dS = dI = dH = 1)^2$).The line element is a function of S and I but independent of H. The figure indicates that the metric is most and equally sensetive around black and white, has an enlarged sensitivity towards gray and is allmost constant with respect to S and I, i.e almost Euclidean, when colours are involved.

3 cases can be distinguished:

1) If we assume the quantities H and I to be constant the length of the line element reads

$$s_1 = \left| \int_0^1 \frac{dS}{2\sqrt{S(2-S)}} \right| = \frac{\pi}{4} \quad . \tag{3.9}$$

This is the distance of any saturated colour to the (neutral) grey-point.

2) Assuming S and I to be constant the length of the line element is

$$s_2(S) = \sqrt{\frac{5}{2}S} \int_0^{2\pi} dH = \pi\sqrt{10S} \quad . \tag{3.10}$$

This is the circumference of a colour circle with saturation S. The ratio of the circumference of a fully saturated colour circle $s_2(S=1)$ to its diameter s_1 amounts to

$$\frac{s_2(S=1)}{s_1} = 4\sqrt{10} = 12,65 \approx 4\pi = 720^o \quad . \tag{3.11}$$

which establishes the **super-importance of hue differences** feature of the geometry.
3) Assuming S and H to be constant the length of the line element $s_3(S)$ depends on saturation S:

$$s_3(S) = \frac{\sqrt{2-S}}{2\sqrt{2}} \int_0^1 \frac{dI}{\sqrt{I(1-I)}} = \frac{\pi}{2}\sqrt{1-\frac{S}{2}} \quad . \tag{3.12}$$

If $S = 0$ the length of the line element $s_3(S=0) = \frac{\pi}{2}$ which is the distance from black to white. It is interesting to note that the effect of the super-importance of hue differences vanishes the closer the center of the circles come to the neutral axis.

$$\frac{s_2(S=1)}{s_{3(S=0)}} = 2\sqrt{10} = 6,33 \approx 2\pi = 360^o \quad . \tag{3.13}$$

This was predicted by Silberstein [13, 14].

4. Line distance

The line distance ds is given by the following differential equation (summation convention):

$$\begin{aligned} ds^2 &= g_{\mu\nu} dx^{(\mu)} dx^{(\nu)} \quad , \\ s &= \int_{t_1}^{t_2} \sqrt{g_{\mu\nu} \frac{dx^{(\mu)}}{dt} \frac{dx^{(\nu)}}{dt}} dt \quad , \end{aligned} \tag{4.1}$$

The (diagonal) metric tensor $g_{\mu\nu}$ and the inverse metric tensor $g^{\mu\nu}$ can be read off from Eq. (3.7):

$$g_{\mu\nu} = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix} \quad , \quad g^{\mu\nu} = \begin{pmatrix} 1/a & 0 & 0 \\ 0 & 1/b & 0 \\ 0 & 0 & 1/c \end{pmatrix} \quad . \tag{4.2}$$

$$s = \int_{t_1}^{t_2} \sqrt{a\left(\frac{dx^{(1)}}{dt}\right)^2 + b\left(\frac{dx^{(2)}}{dt}\right)^2 + c\left(\frac{dx^{(3)}}{dt}\right)^2} dt$$

$$= \int_{t_1}^{t_2} \sqrt{a\,u^2 + b\,v^2 + c\,w^2} dt \ , \tag{4.3}$$

where

$$u = \frac{dx^{(1)}}{dt} \ , \ v = \frac{dx^{(2)}}{dt} \ , \ w = \frac{dx^{(3)}}{dt} \ . \tag{4.4}$$

We identify: $x^{(1)} = S$, $x^{(2)} = I$ and $x^{(3)} = H$.

Special cases of Eq. (4.3):

a) S, I constant: $S = 1$, $I = 1/2 \to u = v = 0$
$\to \ ds = \sqrt{c} dH \to s = \sqrt{\frac{5}{2}} \int_0^{2\pi} dH = \pi\sqrt{10}$ (see Eq. (3.10) for $S = 1$).

5. Geodesic line

Differential geometry (Riemann space): Differential equations for minimal distance (geodesic):

$$\frac{d^2x^{(\nu)}}{dt^2} + \Gamma^{\nu}_{\sigma\rho} \frac{dx^{(\sigma)}}{dt} \frac{dx^{(\rho)}}{dt} = 0 \ , \tag{5.1}$$

The Christoffel symbols are defined through

$$\Gamma^{\nu}_{\sigma\rho} = g^{\nu\mu} \frac{1}{2} \left[\frac{\partial g_{\sigma\mu}}{\partial x^{(\rho)}} + \frac{\partial g_{\rho\mu}}{\partial x^{(\sigma)}} - \frac{\partial g_{\sigma\rho}}{\partial x^{(\mu)}} \right] \ . \tag{5.2}$$

The Christoffel symbols follow from Eq. (4.2) and Eq. (3.8):

$$\Gamma^1_{11} = \frac{S-1}{S(2-S)} \ , \ \Gamma^1_{22} = \frac{S(2-S)}{4I(1-I)} \ , \ \Gamma^1_{33} = 5S(S-2) \ , \ \Gamma^2_{22} = \frac{2I-1}{2I(1-I)} \ ,$$

$$\Gamma^2_{12} = \Gamma^2_{21} = \frac{1}{2(S-2)} \ , \ \Gamma^3_{13} = \Gamma^3_{31} = \frac{1}{2S} \ . \tag{5.3}$$

All other Christoffel symbols are zero. Using

$$u = \frac{dx^{(1)}}{dt} = S' \ , \ v = \frac{dx^{(2)}}{dt} = I' \ , \ w = \frac{dx^{(3)}}{dt} = H' \ , \tag{5.4}$$

we obtain from Eq. (5.1) a set of 3 differential equations:

$$u' + \Gamma^1_{11} u^2 + \Gamma^1_{22} v^2 + \Gamma^1_{33} w^2 = 0 \ ,$$
$$v' + \Gamma^2_{22} v^2 + 2\Gamma^2_{12} uv = 0 \ ,$$
$$w' + 2\Gamma^3_{13} uw = 0 \ . \tag{5.5}$$

The last equation can easily be resolved:

$$u = S' = -\frac{1}{2\Gamma^3_{13}} \frac{w'}{w} = -S\frac{w'}{w} \to \frac{w'}{w} = -\frac{S'}{S} \to$$

$$(lnw)' = -(lnS)' \to w = -c\,S \quad , \ c = \text{const} \ . \tag{5.6}$$

Two differential equation for u and v are left over:

$$\begin{aligned} u' + \Gamma^1_{11} u^2 + \Gamma^1_{22} v^2 + (cS)^2 \Gamma^1_{33} = 0 \ \ , \ \ , \\ v' + \Gamma^2_{22} v^2 + 2\Gamma^2_{12} uv = 0 \ \ . \end{aligned} \tag{5.7}$$

6. Special solutions of Eq. (5.7)

a) $S = const$: $\to u = 0$

From Eq. (5.6) we obtain $w = -cS = c_1 \to \ H(t) = c_1 t + c_2$.

Eq. (5.7):

$$v^2 = -\frac{\Gamma^1_{33}}{\Gamma^1_{22}} c_1^2 \ \ , \ \ v' = I'' = -\Gamma^2_{22} v^2 = -10(2I-1)c_1^2 \ \ \to$$

$$I'' + aI + b = 0 \ \ , \ \ a = 20c_1^2 \ \ , \ \ b = -10c_1^2 \ \ \to \ \ I(t) = \sin(\sqrt{a}t) - \frac{b}{a}$$

Substituting $\bar{t} = c_1 t$ (scaling) and defining $\bar{t}_{max} = \frac{\pi}{6\sqrt{20}} = 0,117$ the final result reads

$$\begin{aligned} I(\bar{t}) = \sin(\sqrt{20}\,\bar{t}) + \frac{1}{2} \ \ , \ \ H(\bar{t}) = \bar{t} + c_2 \ \ , \\ 0 \leq I(\bar{t}) \leq 1 \ \ , \ \ -\bar{t}_{max} \leq \bar{t} \leq \bar{t}_{max} \ \ . \end{aligned} \tag{6.1}$$

b) $I = const$: $\to v = 0$

For H =const.

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