USING 2D-FFT TO SEARCH FAST RADIO BURST*

C.H. Niu,[1,2,3] Y.C. Li,[2] F.Q. Wu,[2] Ue-li Pen,[4,2] and X.L. Chen[2]

[1] *Central China Normal University*

Luoyu Road,Wuhan, China
[2] *National Astronomy Observatory , Chinese Academy of Sciences*

Datun(A) Road,No.30,Beijing, China
[3] *University of California Berkeley, Campbell Hall 339, Berkeley CA 94720*
[4] *CITA*

ABSTRACT

Fast Radio Burst have been found from pulsar data for many years. There are several FRB search algorithm like tree algorithm, FDMT et. Here we proposed a different FRB searching algorithm which basically trace a curve in frequency-time image. This algorithm is mainly realized by two dimensional Fast Fourier Transform. We take a 2D FFT on the $f^{-2}(t)$ data map, Then trace the signal along the angle of straight line. In this searching method, it's easier to remove RFI in large scale and will bring a speed up benefit in well-developed 2D FFT library both in CPU and GPU code.

Fast Radio Burst is a high energy radio signal found in the Universe. The first one is found by Lorimer Duncn in 2007, now people always call it as Lormeter burst. Like Pulsar, Its a wide band radio sginal, when it go through the inter stellar or inter galaxy medium, the higher frequency will go faster than lower frequency. When Signal go through dense of ISM The origin of FRB is still unclear, there are lots of theories trying to describe what FRB is.

Corresponding author: Xuelei Chen

* peterniu@nao.cas.cn

1. INTRODUCTION

Fast radio bursts (FRBs) are bright ($\sim$Jy) and short ($\sim$ms) bursts of radio emission that have dispersion measures (DMs) in excess of the line of sight DM contribution expected from the electron distribution of our Galaxy. To date 18 FRBs have been reported — most of them detected at the Parkes telescope (Lorimer et al. 2007; Thornton et al. 2013; Burke-Spolaor & Bannister 2014; Keane et al. 2012; Ravi et al. 2015; Petroff et al. 2015; Keane et al. 2016; Champion et al. 2016; Ravi et al. 2016) and one each at the Arecibo (Spitler et al. 2014) and Green Bank telescopes (Masui et al. 2015).

A plethora of source models have been proposed to explain the properties of FRBs (see e.g. Katz 2016, for a brief review). According to the models, the excess DM for FRBs may be intrinsic to the source, placing it within the Galaxy; it may arise mostly from the intergalactic medium, placing a source of FRBs at cosmological distances ($z \sim 0.2-1$) or it may arise from the host galaxy, placing a source of FRBs at extragalactic, but not necessarily cosmological, distances ($\sim 100\,\mathrm{Mpc}$).

Since the only evidence to claim an extragalactic origin for FRBs has been the anomalously high DM, some models also attempted to explain the excess DM as a part of the model, thus allowing FRBs to be Galactic. All FRBs observed to date have been detected with single dish radio telescopes, for which the localization is of order arcminutes, insufficient to obtain an unambiguous association with any object. To date, no independent information about their redshift, environment, and source could be obtained due to the lack of an accurate localization of FRBs. Keane et al. (2016) attempted to identify the host of FRB 150418 on the basis of a fading radio source in the field that was localized to a $z = 0.492$ galaxy. However, later work identified the radio source as a variable active galactic nucleus (AGN) that may not be related to the source (Williams & Berger 2016; Bassa et al. 2016; Giroletti et al. 2016; Johnston et al. 2017).

Repeated radio bursts were observed from the location of the Arecibo-detected FRB Search Algorithm (Spitler et al. 2016; Scholz et al. 2016), with the same DM as the first detection, indicating a common source. As discussed by Spitler et al. (2016), it is unclear whether the repetition makes FRB Search Algorithm unique among known FRBs, or whether radio telescopes other than Arecibo lack the sensitivity to readily detect repeat bursts from other known FRBs.

Chatterjee et al. (2017) used the Karl G. Jansky Very Large Array (VLA) to directly localize the repeated bursts from FRB Search Algorithm with 100-mas precision and reported an unresolved, persistent radio source and an extended optical counterpart at the location with a chance coincidence probability of $\approx 3 \times 10^{-4}$ — the first unambiguous identification of multi-wavelength counterparts to FRBs. Independently, Marcote et al. (2017) used the European VLBI Network (EVN) to localize the bursts and the persistent source and showed that both are co-located within ~ 12 milliarcseconds.

Here we report a new algorithm to search FRBs .

2. BASICS OF INCOHERENT DEDISPERSION

The dispersion of the electromagnaetic wave pulse cause a delay in arrival time at frequency ν compared with the reference frequency ν_0, which is given by :

$$\Delta t(\nu) = -D(\nu^{-2} - \nu_0^{-2}) \tag{1}$$

where D is the dispersion measure. Thus , We may model a burst with a very short intrinsic width as :

$$I(t,\nu) = I_0(\nu)\delta_D(t - t_s - \frac{D}{\nu^2}) \tag{2}$$

Where δD is the Dirac delta function, t_s marks the signal starting time for infinitely high frequency. If the bandwidth is small, we can approximate

$$\frac{D}{\nu^2} \approx \frac{D}{\nu_0^2}(1 - 2\frac{\nu - \nu_0}{\nu_0})$$

denote $\Delta\nu \equiv \nu - \nu_0$, and assume that the spectrum is not too steep such that within the observing band the signal is constant, then

$$
\begin{aligned}
I(t,\nu) &\approx I_0\delta_D(t - t_s - \frac{D}{\nu_0^2}(1 - 2\frac{\Delta\nu}{\nu_0})) \\
&= I_0\delta_D(t - t_0 + \frac{2D}{\nu_0^3}\Delta\nu)
\end{aligned}
\tag{3}
$$

where t_0 is the arrival time of the signal at thre reference frequency ν_0.

Now consider an integral of this signal between frequency ν_1 and ν_2 , the signal strength would be

$$s = \int d\nu \int dt I(t,\nu) = (\nu_2 - \nu_1)I_0 = I_0 B \tag{4}$$

Where $B = \nu_2 - \nu_1$ is the bandwidth. Now consider the noise. Suppose the data is digitized with time interval δt and frequency channel bandwidth $\delta\nu$. For the incoherent dedispersion, the signal within each time interval and frequency channel is

$$I_n = \frac{2kT_{sys}}{A_{\mathrm{eff}}\sqrt{\delta\nu\delta t}} \tag{5}$$

Suppose we are observing between ν_1,ν_2 with a total of N_ν channels, and processing a time interval $T = N_t\delta t$

where $T \geq \Delta t(\nu_1) - \Delta t(\nu_2)$, i.e. the whole of the dispersed signal is within the data frame.

For incoherent dedispersion , in the absence of the pulse signal, the whole read out of the data frame is given by

$$n = \int d\nu \int dt I_n = \frac{2kT_{sys}}{A_{\text{eff}}} \frac{(\nu_2 - \nu_1)T}{\sqrt{\delta\nu\delta t}}$$
$$= \frac{2kT_{sys}}{A_{\text{eff}}} B^{1/2} T^{1/2} N_\nu^{1/2} N_t^{1/2} \quad (6)$$

So the raw signal to noise ratio is given

$$\text{SNR}_{raw} = \frac{I_0 A_{\text{eff}}}{2kT_{sys}} \left(\frac{B}{N_\nu N_t T} \right)^{1/2} \quad (7)$$

In a perfect incoherent dedispersion , we sum up all the signal, which is still given by s. However, we compare it with the noise in the same dedispersion $\nu - t$ track, not the whole data frame. The noise along the same track is given by

$$n = \int d\nu \int dt I_n \delta_D(t - t_0 + \frac{2D}{\nu_0^3} \Delta\nu) = BI_n \quad (8)$$

Then

$$\text{SNR}_{opt} = \frac{I_0}{I_N} = \frac{I_0 A_{\text{eff}}}{2kT_{sys}} \left(\frac{BT}{N_\nu N_t} \right)^{1/2} \quad (9)$$

Now consider a pulse of finite width. We replace the Dirac δ function by a Gaussian function with the same normalization

$$\delta_D(t - t') \to g(t - t') \equiv \frac{1}{(2\pi)^{1/2}\sigma} \exp[-\frac{(t - t')^2}{2\sigma_t^2}] \quad (10)$$

If the pulse intrinsic width $\sigma > \delta t$, then in a dedispersion along the track only the part of the signal within one time bin would be included, which gives

$$\int_{-\delta t}^{+\delta t} d\Delta t \frac{1}{\sqrt{2\pi}\sigma} e^{-\Delta t^2/2\sigma^2} = erf(\frac{\delta t}{\sqrt{2}\sigma}) \approx \sqrt{\frac{2}{\pi}} \frac{\delta t}{\sigma} \quad (11)$$

Where the last holds for the case $\delta t \ll \sigma$, so in this case

$$s = I_0 B \sqrt{\frac{2}{\pi}} \frac{\delta t}{\sigma} \quad (12)$$

While the noise is still given by Eq.(8), so in this case

$$\text{SNR}_{fin} = \frac{I_0}{I_n} = \frac{I_0 A_{\text{eff}}}{2kT_{sys}} \left(\frac{BT}{N_t N_\nu} \right)^{1/2} \sqrt{\frac{2}{\pi}} \frac{\delta t}{\sigma} \quad (13)$$

3. 2D FFT DEDISPERSION

The usual Fourier transform is :

$$\widetilde{f}(\omega) = \frac{1}{2\pi} \int f(t)e^{-i\omega t}dt$$
$$f(t) = \int \widetilde{f}(\omega)e^{i\omega t}d\omega \quad (14)$$

For $f(t) = \delta_D(t - t_0), \widetilde{\omega} = \frac{1}{2\pi}e^{-i\omega t_0}$. Using the relation :

$$\int d\omega e^{i\omega t_0} = 2\pi\delta_D(t_0) \quad (15)$$

we find the above indeed form a Fourier pair. However , here we want to use ν instead of ω, then the Fourier transform pair are:

$$\widetilde{f}(\nu) = \frac{1}{2\pi} \int f(t)e^{-i2\pi\nu t}dt$$
$$f(t) = 2\pi \int \widetilde{f}(\nu)e^{i2\pi\nu t}d\nu \quad (16)$$

The 2D transform of the signal $I(\nu, t)$ is

$$\widetilde{I}(f, \tau) = \int d\nu \ e^{-2\pi i\nu\tau} \int dt \ e^{-2\pi i ft} I(t, \nu) \quad (17)$$

where we denote the Fourier conjugate variable of ν, t as τ, f to avoid confusion. For the pulse signal given by Eq.(2),

$$\widetilde{I}(f, \tau) = \int d\nu e^{-2\pi i\nu\tau} I_0 e^{-2\pi if(t_0 - \frac{2D}{\nu_0^3}\Delta\nu)}$$
$$= I_0 e^{-i2\pi f(t_0 + \frac{2D}{\nu_0^2})} \delta_D(\tau - \frac{2Df}{\nu_0^3}) \quad (18)$$

Note $\widetilde{I}(\tau, f)$ is non-zero only on the staight line $\tau - \frac{2Df}{\nu_0^2} = 0$, and the value is a complex number whose phase angle gives the arrival time. For the pulse with finite width,

$$\widetilde{I}(f, \tau) = \int d\nu e^{-2\pi i\nu\tau} \int dt \ e^{-2\pi ift} \cdot I_0 \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(t-t')^2}{2\sigma^2}} \quad (19)$$

where $t' = t_0 - \frac{2D}{\nu_0^3} \Delta\nu$.

$$\widetilde{I}(f, \tau) = \int d\nu \ e^{-2\pi i\nu\tau} I_0 e^{-i2\pi ft'} e^{-\frac{(2\pi f\sigma)^2}{2}}$$
$$= I_0 e^{-i2\pi f(t_0 + \frac{2D}{\nu_0^2})} e^{-\frac{(2\pi f\sigma)^2}{2}} \int d\nu \ exp[-i2\pi\nu(\tau - \frac{2Df}{\nu_0^3})]$$
$$= I_0 e^{-i2\pi f(t_0 + \frac{2D}{\nu_0^2})} e^{-\frac{(2\pi f\sigma)^2}{2}} \delta_D(\tau - \frac{2Df}{\nu_0^3}) \quad (20)$$

Note this is similar to Eq(18) except for the factor $e^{-\frac{(2\pi f\sigma)^2}{2}}$, this limits the usable range of f to $|f| < (2\pi\sigma)^{-1}$.

4. TRANSFORM TO POLAR COORDINATES

We can take $\frac{2f}{\nu_0^2}, \tau$ as the x, y in Cartesian coordinates, then the polar cpprdinates ρ, θ can be defined as

$$\rho^2 = \left(\frac{2f}{\nu_0^2}\right)^2 + \tau^2$$
$$\tan\theta = \frac{\tau}{2f/\nu_0^2}$$
(21)

with $\tan\theta = -D$ for the track satisfy Eq.(20). Conversely,

$$f = \frac{\nu_0^2}{2}\rho\cos\theta$$
$$\tau = \rho\sin\theta$$
(22)

Then

$$\widetilde{I}(\rho,\theta) = \frac{I_0}{2\pi}e^{-i2\pi\left(\frac{\nu_0^2 \cdot t}{2} - D\right)\rho\cos\theta}$$
$$\cdot e^{-\frac{\pi^2\sigma^2\nu_0^4}{2}\rho^2\cos^2\theta}\rho^{-1}\delta_D(\theta + \arctan D)$$
(23)

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