Constrain angle resolution by DM smearing

In our algorithm , we use polar coordinates to search signal. However , the grid size should be chosen carefully. Because the different grid size might caused different results. Here , I want to use DM smearing to constrain the angle resolution.

After 2-D FFT , We get a data with a straight line go cross the center. Then we calculate the angle and radius of each pixel. Then we transform the data into polar coordinate with interpolation. There are 2 kinds of interpolate grid : radius grid and angle grid. The angle grid might influenced the result consequently . Along the radius , we will sum it anyway.

Time delay between different frequency comply :

$$\Delta t = 4.15 \times 10^6 ms \cdot DM \times (f_{ref}^{-2} - f_{chan}^{-2}) \tag{1}$$

When we divide the formula above , we could get in each frequency bin , the minimum allowed time delay.

$$t_{DM} = 8.3 \times 10^6 ms \times DM \times \Delta f \times f^{-3} \tag{2}$$

To avoid DM smearing ,

$$t_{dm} \geq t_{samp}$$

We could decide the DM step to avoid DM smearing. When we make the comparison equal, It compromise a limit like:

$$DM_i = 1.205 \times 10^{-7} cm^{-3} pc (i-1) t_{samp} (f^3 / \Delta f) \tag{3}$$

Here we suppose f >> Δf .

When $i = N_{chan}$, we got the diagonal separator. DM below the separator will obey the DM step above. DM beyond the diagonal separator will have bigger DM step.

As the DM has a relationship with Angle. Here we want to use this DM limit to constrain Angle resolution.

From formula (1), we could get the slop of straight line:

$$k_1 = \tan \beta = \frac{f_{chan}^{-2}}{\Delta t} = \frac{1}{C \cdot DM} \tag{4}$$

where $C = 4.15 \times 10^6 ms$, and take reference frequency as infinitely high.

Take advantage of the character of 2D-FFT. After 2D Fourier Transform, It will appear a straight line go cross the center and perpendicular to original straight line. Take horizontal line as original line, we could calculate the angle between the original line note as θ. As mentioned above , $\theta + \beta = 90 ¿$.

so

$$k_2 = \tan\theta = -1/k_1$$

Then we could get the relationship between θ and DM:

$$k_2 = \tan\theta = -C \cdot DM \cdot N_{bins} = \frac{row}{rank} \cdot \frac{row_{rsl}}{rank_{rsl}}$$

where $row_{rsl} = \left(f_{high}^{-2} - f_{low}^{-2}\right)^{-1}$, $rank_{rsl} = (N_t \cdot t_{rsl})^{-1} = T^{-1}$

We can finally got the DM through k2:

$$DM = \frac{row}{rank} \cdot \frac{row_{rsl}}{rank_{rsl}} \cdot \frac{1}{C \cdot N_{bin}}$$

Take differential in both sides:

$$\delta DM = \delta\left(\frac{row}{rank}\right) \cdot \frac{row_{rsl}}{rank_{rsl}} \cdot \frac{1}{C \cdot N_{bin}}$$

To avoid DM smearing combing (3):

$$\delta DM = 1.205 \times 10^{-7} cm^{-3}\, pc \cdot t_{samp} \left(f^3 / \Delta f\right)$$

Take Tianlai parameter: $t_{samp} = 1\, ms,\ f = 800\, MHz,\ \Delta f = 100\, MHz / 1024 = 0.098\, MHz.$

We could get the δDM ≈ 630 $cm^{-3}\, pc$

We could found that δ(row/rank) is also depend on time length.

Conclusion:

I'm not sure whether could we constrain like that, it looks like right. As for radius resolution constrain, Yichao also suggest to keep total pixel constant to make the whole information conservation. It means once we have make choice of angle resolution, we could also get the radius resolution.