

Coherent Emission in Fast Radio Bursts

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(Dated: May 30, 2014)

Abstract

The fast (ms) radio bursts reported by Lorimer, *et al.* [1] and by Thornton, *et al.* [2] have extremely high brightness temperatures if at the inferred cosmological distances. This implies coherent emission by “bunches” of charges. FRB, like the giant pulses of the Crab pulsar, display banded spectra that may be harmonics of plasma frequency emission by plasma turbulence, and are inconsistent with emission by charge distributions moving relativistically. We model the emission region as a screen of half-wave dipole radiators resonant around the frequencies of observation, the maximally bright emission mechanism of nonrelativistic charges, and calculate the implied charge bunching. From this we infer the minimum electron energy required to overcome electrostatic repulsion. If FRB are the counterparts of Galactic events, their Galactic counterparts may be detected from any direction above the horizon by radio telescopes in their far sidelobes or by small arrays of dipoles.

PACS numbers: 95.30.Gv, 95.85.Bh, 98.70.-f

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I. INTRODUCTION

Lorimer, *et al.* [1] discovered a fast (intrinsic duration < 5 ms) radio burst (FRB) in a band 300 MHz wide around $\nu = 1400$ MHz with a chirp indicating a dispersion measure $DM = 375 \text{ pc/cm}^3$ and a fluence $\mathcal{F}_\nu \approx 150 \text{ Jy-ms}$. This dispersion measure is consistent with propagation through the intergalactic medium from redshift $z = 0.3$ and inexplicable as the result of Galactic plasma, but it is not possible to constrain the contribution of plasma local to the emitter.

Thornton, *et al.* [2] discovered four FRB in a band about 400 MHz wide around $\nu = 1400$ MHz, with intrinsic durations $\lesssim 1$ ms (one burst, like that of [1], was temporally resolved, but their widths are explained as multipath dispersion of travel times). Their measured fluences \mathcal{F}_ν were between 0.6 Jy-ms and 8.0 Jy-ms. Observed chirps are explicable as dispersion by intergalactic plasma, indicating $0.5 \lesssim z \lesssim 1.0$. The total energy radiated in the band of observation, assuming isotropy, was (for the most luminous burst) only about 10^{40} ergs, and the corresponding lower bound on luminosity was 10^{43} ergs/s. These energies and powers can be provided by a wide range of processes involving compact objects. The upper bound on duration may be a more significant constraint, but is consistent with the light travel time across neutron stars and stellar mass black holes.

The purpose of this paper is to consider the inferences that can be drawn directly on physical grounds from the observed FRB phenomenology. Unlike [3–7], it is not to develop an astronomical model or to identify source objects.

As was realized long ago for radio pulsars [8], such intense emission from a small source, implied by its short duration, corresponds to a brightness temperature T_b far in excess of any possible equilibrium temperature or even particle energy. A radiation field at a specified frequency interacts with a limited range of particle momenta p . If the particles are uncorrelated and their distribution function $f(p)$ in that range is fitted to an equilibrium distribution at temperature T_{part} , then $T_b \leq T_{part}$ [9]. For a relativistic power law $f(p) \propto p^{-\alpha}$ with $\alpha > -2$

$$k_B T_b \leq k_B T_{part} = \frac{pc}{\alpha + 2}; \quad (1)$$

in general, $k_B T_{part} \simeq pc$. Because a power law distribution is nonequilibrium, thermodynamics permits arbitrarily high T_b . However, unless there is a population inversion (an unprecedented $\alpha < -2$, implying $T_{part} < 0$, in which case there is no bound on T_b) or

coherent emission, T_b is limited by Eq. 1.

A high brightness temperature requires coherent emission by correlated “bunches” of particles [8]. Exponential amplification of a radiation field by an inverted particle distribution function is one process by which particles may be bunched and radiate coherently. Plasma instabilities are another such process, in which bunching is produced by charged particles interacting with each other by near-zone, rather than radiation, fields.

If emission is produced by bunches of charge q with a power law momentum distribution the bunches may be regarded as quasi-particles. For such a nonequilibrium particle distribution function, p in Eq. 1 is replaced by the momentum of the bunch, qp/e :

$$k_B T_b \leq k_B T_{bunch} = \frac{q}{e} \frac{pc}{\alpha + 2}. \quad (2)$$

This upper limit can be approached if coherent bunches survive for the time required for them to equilibrate with the radiation field.

The frequency structure of \mathcal{F}_ν in FRB 110220, comprising bands approximately 100 MHz wide ($\Delta\nu/\nu \approx 0.1$ [2]), is an important clue. It is evidence for the spatial structure of coherent emission, perhaps as the consequence of a collective interaction (plasma instability [10, 11]); the incoherent emission of randomly distributed charges would not show such frequency structure. This frequency structure is also inconsistent with radiation by relativistically moving charges or bunches (synchrotron or curvature radiation) because that produces a broad-band $\Delta\nu/\nu \approx 1$ spectrum [12], even if they are monoenergetic. For this reason we consider radiation by particles moving nonrelativistically in the source frame. However, the source frame may be moving towards us with a Lorentz factor $\Gamma \gg 1$, in analogy to a gamma-ray burst (Thornton, *et al.* [2] argued against observed GRB as sources of FRB on the basis of their event rates and the absence of associations with the observed FRB).

II. THE FAST RADIO BURSTS

Here we apply the brightness temperature argument to the most intense burst, FRB 110220, for which $\mathcal{F}_\nu = 8.0$ Jy-ms and $z = 0.81$. With only an upper bound to the FRB duration, this argument can only set limits, so we ignore an order-of-unity error and take a

static Newtonian universe with the source at a distance $D = 10^{28}$ cm (3 Gpc). For a source of (unmeasured) duration Δt but measured (unpolarized) fluence spectral density \mathcal{F}_ν , the flux density is

$$F_\nu \approx \frac{\mathcal{F}_\nu}{\Delta t} \frac{D^2}{\Delta x^2}, \quad (3)$$

where Δx is the size of the region illuminating the observer and we have assumed isotropic emission at the source. For a static source Δx is its geometrical size, but for a relativistically expanding source

$$\Delta x \approx c\Delta t\Gamma, \quad (4)$$

where the factor of Γ comes from the relativistic beaming of the radiation emitted from a shell of radius $R \approx c\Delta t\Gamma^2$ into an angle $\approx 1/\Gamma$. The brightness temperature in the observer's frame is

$$k_B T_{b,obs} \equiv \frac{1}{2} \frac{F_\nu c^2}{\nu^2} \approx \frac{1}{2} \frac{\mathcal{F}_\nu}{\Delta t} \frac{D^2}{\Delta x^2} \frac{c^2}{\nu^2}. \quad (5)$$

Transforming to the source frame, using the scalings $\mathcal{F}_\nu \propto \Gamma^0$ (because the bandwidth scales with the frequency), $\Delta t \propto \Gamma$, $\nu \propto \Gamma^{-1}$ and substituting (4)

$$k_B T_{b,src} \approx \frac{1}{2} \frac{\mathcal{F}_\nu}{\nu^2} \frac{D^2}{\Delta t^3} \frac{1}{\Gamma} \approx \frac{2 \times 10^{21}}{\Gamma} \Delta t_{-3}^{-3} \text{ ergs}, \quad (6)$$

where ν , \mathcal{F}_ν and Δt are the observed quantities and $\Delta t_{-3} \equiv \Delta t/(1 \text{ ms})$, yields $T_{b,src} > 10^{37}/\Gamma^\circ \text{K}$! It is evident that the radiation must be coherent because particles cannot be accelerated to energies $\mathcal{O}(k_B T_{b,src})$ for any possible Γ . Even if the sources were within the Galactic disc ($D \simeq 100$ pc), the lower bound on $T_{b,src}$ would imply coherent emission.

If the radiation is powered by the dissipation of magnetic energy, we can set a lower bound on the magnetic field:

$$B^2 > \frac{8\pi}{c} \frac{\mathcal{F}_\nu \Delta \nu}{\Delta t} \frac{D^2}{\Delta x^2} \approx \frac{8\pi \mathcal{F}_\nu \Delta \nu D^2}{c^3 \Delta t^3 \Gamma^3} \approx \frac{10^{19}}{\Gamma^3 \Delta t_{-3}^3} \text{ gauss}^2. \quad (7)$$

This suggests magnetic reconnection of neutron star fields, or of white dwarf fields if $\Gamma \gg 1$. The energy flux $B^2 c/8\pi$ is consistent with the upper bound $\mathcal{O}(10^{29} \text{ erg/cm}^2\text{-s})$ on the power density in nonthermal particles set by cascading thermalization into opaque equilibrium pair plasma at higher energy density [13–15].

There are similarities between the FRB and the nanosecond “nanoshots” of the Crab

pulsar [16–19], even though the energy scales differ by a factor $\mathcal{O}(10^{12})$. The inferred brightness temperatures are of similar orders of magnitude, although this is only a very rough comparison because of the likelihood of relativistic expansion (at unknown and different Lorentz factors) towards the observer. More significant is the similarity in spectral structure: FRB and nanoshots both display bands of width $\Delta\nu/\nu \approx 0.1$ [2, 17]. If this width is interpreted as radiation damping, it suggests radiation by impulsively excited oscillations with $Q \approx 10$. In both classes of source the spectral structure may instead be interpreted as harmonic emission (with harmonic index $\mathcal{O}(10)$) of a fundamental frequency $\Delta\nu$, perhaps close to the electron plasma frequency of a strongly turbulent plasma [10].

III. DIPOLE EMISSION MODEL

Radiation by nonrelativistically moving particles may be treated by a multipole expansion, with the dipole term generally dominant [12]. Following the argument in the Introduction, we suggest that it is useful to consider the hypothesis that the emission mechanism in FRB may be described by the coherent emission of non-relativistically moving (in the source frame) clumps of charge.

We model the emission region as a surface covered with half-wave dipole antennas at the observed frequency, and estimate the charge q that must flow in each in order to produce the observed brightness temperature. This is a minimal model of radiation by nonrelativistic accelerated charges [12]. Half-wave (length $L = \lambda/2$, where λ is the wavelength) dipoles are nearly maximally efficient emitters, and lead to the least restrictive demands on the bunching of charges. Because these antennas are approximately impedance-matched to free space they are effective absorbers as well as emitters, so that radiation emitted behind this surface screen is absorbed and is not observed.

The dipoles are not meant as a physical model, but only as a representation of the coupling between source and radiation field that may be applied to generic nonrelativistic radiation mechanisms, not limited to coherent electron plasma wave turbulence [20]. The impedance of an ideal $\lambda/2$ dipole $(73 + 42.5i)\Omega$ is close enough to that of free space (377Ω) that radiating structures of approximately that dimension may have the inferred $Q \approx 10$. Structure on other length scales (in units of the radiated wavelength) radiate inefficiently; it may be present, but almost all the radiation is produced by structure on the scale of $\lambda/2$.

For a broad-spectrum source, the radiation at any wavelength λ is produced by structure (effectively dipoles) with $L \approx \lambda/2$ or spatial structure factor $k \approx \pi/\lambda$. The screen of $\lambda/2$ dipoles is a fair approximation to many turbulent radiation sources.

A single dipole with oscillating charge q and dipole moment $q\lambda/2$ radiates a power

$$P_{dipole} \approx \frac{4\pi^4}{3} \frac{\nu^2 q^2}{c}. \quad (8)$$

We assume that the dipoles are not identical, but that their oscillation frequencies are spread over a bandwidth $\Delta\nu \approx \nu$. A sphere of radius $R = c\Delta t\Gamma^2$ is covered by approximately $16\pi R^2/\lambda^2$ dipoles. Equating the total radiated power to the observed power $4\pi D^2 \mathcal{F}_\nu \nu / \Delta t$ yields

$$q^2 \approx \frac{3}{16\pi^4} \frac{\mathcal{F}_\nu D^2 c}{(\nu \Delta t)^3 \Gamma^4}. \quad (9)$$

The bunching factor

$$\frac{q}{e} \approx 2.7 \times 10^{19} \Gamma^{-2} \Delta t_{-3}^{-3/2}. \quad (10)$$

This result applies to both isotropic and beamed emission, the latter possible if the dipoles are appropriately phased, as might be the case if, for example, the radiating elements are charge bunches in relativistic motion. Although the dipoles are only weakly coupled and radiate approximately independently, the fact that a single burst is observed indicates that they are excited by a common larger scale event.

The total number of electrons radiating, assuming isotropic emission, is

$$N_e = 4\pi \frac{R^2}{(\lambda/2)^2} \frac{q}{e} = \frac{4\sqrt{3}}{\pi} \frac{D^2 \Gamma^2 \sqrt{\mathcal{F}_\nu c \nu \Delta t}}{e} \approx 2.7 \times 10^{33} \Gamma^2 \Delta t_{-3}^{1/2}. \quad (11)$$

The mass of neutralizing protons is only $4 \times 10^9 \Gamma^2 \Delta t_{-3}^{1/2}$ g; alternatively, the radiating plasma may be a pair gas without baryons. The potential associated with the charge q is $V \approx 2q/\lambda$ and the electrostatic energy per electron

$$eV \approx \frac{2eq}{\lambda} \gtrsim 4 \times 10^{11} \Gamma^{-2} \Delta t_{-3}^{-3/2} \text{ eV}. \quad (12)$$

This implies a minimum electron Lorentz factor to permit bunching

$$\gamma = eV/m_e c^2 \gtrsim 10^6 / (\Gamma^2 \Delta t_{-3}^{3/2}). \quad (13)$$

Even $\gamma \gg 1$ need not invalidate the description of the radiation as that of a screen of half-wave dipoles of charge moving nonrelativistically, provided the phase velocities of the coherent charge bunches are nonrelativistic. This is consistent with relativistic energies of individual electrons (a relativistic plasma may support waves with nonrelativistic phase velocities, or bunches may be confined magnetically). A similar argument applies to the nanoshots of the Crab pulsar.

If, however, $\gamma \simeq 1$, then Eq. 13 implies

$$\Gamma \gtrsim 1000 \Delta t_{-3}^{-3/4}. \quad (14)$$

This is larger than values of Γ inferred for gamma-ray bursts, but perhaps by less than an order of magnitude, hinting at but not requiring related processes.

The electrostatic energy in the electrons

$$E_e = N_e e V = \frac{6}{\pi^3} \frac{\mathcal{F}_\nu D^2}{\Delta t} \approx 1.5 \times 10^{33} \Delta t_{-3}^{-1} \text{ erg}. \quad (15)$$

This is about 10^{-7} of the energy implied by the observed fluence. The energy of the radiating electrons must be replenished (for example, by an ongoing plasma instability), or the electrons themselves replaced by others equally energetic and bunched, in about 10^{-7} of the burst duration, a time $< 10^{-10}$ s.

The bunching factor required to explain the inferred $T_{b,src}$ as radiation by a power-law distribution of quasi-particles may be compared to that required for $\lambda/2$ dipole emission. For a power-law distribution of quasiparticle momenta $q/e \approx 2 \times 10^{27} (m_e c/p) \Delta t_{-3}^{-3} \Gamma^{-1}$, taking p to be the value required to explain the observed momentum, which depends on unknown parameters such as the magnetic radius of curvature. This q/e can be much greater than that of Eq. 10, as would also be the corresponding electrostatic energy.

IV. DISCUSSION

The short observed durations of FRB imply either a very small source region or relativistic expansion; $R \approx 3 \times 10^7 \Delta t_{-3} \Gamma^2 \text{ cm}$. In the latter case (14) leads to the estimate $R \approx 3 \times 10^{13} \text{ cm}$, with no obvious astronomical identification. However, Γ and R may be much smaller, provided γ is large (13). This might describe a source confined within a static

magnetosphere, for which $\Gamma = 1$ [21].

Predictions [20, 22, 23] of fast radio bursts do not match the observed FRB, and subsequent explanations [3–7] address the event frequency and energetics but not the coherent emission. We cannot exclude the possibility that if the rate of GRB-like events exceeds (because they are beamed or radiate outside the soft gamma-ray band) the observed rate of GRB by a factor $\gtrsim 10^3$, then FRB may be a GRB epiphenomenon. If so, then $d \ln N_{FRB} / d \ln \mathcal{F}_{FRB} \rightarrow 0$ as $\mathcal{F}_{FRB} \rightarrow 0$ because their sources are discrete and finite in number; they may be detected out to a cosmic horizon. On the other hand, if FRB result from sources, such as stellar flares, that have no intrinsic scale but occur with increasing frequency at small energies, then $d \ln N_{FRB} / d \ln \mathcal{F}_{FRB}$ is determined by the geometry of the Universe and the evolution of the source population, and is $-3/2$ for a non-evolving Newtonian cosmology.

The sources of FRB may also make novel fast (\lesssim ms) events, as yet unobserved, at other frequencies, but the lower sensitivity of quantum-limited detectors and the likely absence of coherent emission at shorter wavelengths may preclude detection. Clumps of net charge q on scales $\mathcal{O}(10 \text{ cm})$ (radiating coherently at 1400 MHz) imply maximum interparticle distances $\mathcal{O}[10(e/q)^{1/3} \text{ cm}] \approx 300 \Gamma^{2/3} \Delta t_{-3}^{1/2} \text{ \AA}$. Spatial coherence on scales $\ll 10 \text{ cm}$, with correspondingly high brightness temperatures at shorter (even visible) λ , requires a total electron density $n_e \gg (2/\lambda)^3$. This may be a more stringent requirement than the $n_e > (2/21 \text{ cm})^3 q/e$ required by the FRB. If structure exists on small enough scales, coherent emission in visible light is possible, but is not implied by the radio observations.

If the sources of FRB are found also in our Galaxy, Galactic events will be $\mathcal{O}(10^{11})$ times brighter than those at cosmological distances and observable outside a telescope’s nominal beam if they are above the horizon. Antenna sidelobes at large ($\gtrsim 30^\circ$) angles are not easily measured, but for the Parkes 64 m telescope used by [1, 2] they are estimated [24] to be suppressed compared to the main beam by no more than 59 dB, while the far sidelobes of the off-axis Green Bank Telescope are suppressed by 78 dB [25]. Search of any observing record (such as pulsar surveys [24, 26]) with ms time resolution and de-dispersing software can detect or set bounds on the frequency of Galactic FRB. In a multi-beam instrument such as that used in the Parkes Multibeam Pulsar Survey out-of-beam events will occur with nearly equal intensity in each beam (unfortunately, a characteristic shared with terrestrial interference). The absence of any credible event in the PMPS [26], which

involved approximately 2000 hours of observing, sets an upper bound of about 10/year on the rate of such Galactic events.

Even a single $\lambda/2$ dipole antenna has a sensitivity $\approx 0.1(\lambda/D)^4 \approx 10^{-11}$ times that of the main beam of an aperture of diameter D , where the numerical estimate applies to the Parkes telescope at 1400 MHz. Galactic events of the same luminosity as those reported in-beam [1, 2] are detectable from any direction above the horizon by arrays of a small number of dipoles, also providing rough directional information. They may be distinguished from terrestrial interference by ms time resolution, processing for plasma dispersion and requiring detection at several widely separated sites, adding long interferometric baselines to constrain localization. An array of 10–100 dipoles tuned to the inferred (≈ 2000 MHz) source frequencies of cosmological FRB could directly test all hypotheses, including that of association with giant SGR outbursts [4], that they are produced by Galactic events that occur at least a few times during the duration of observations. If the radio transient in its source frame extends to frequencies (10–240 MHz) in the LOFAR [27] band, that instrument will either detect them or provide much tighter constraints.

V. NOTE ADDED

Recent papers [28, 29] have constrained the astronomical environment of the sources of FRB.

Acknowledgments

I thank J. Goodman, C. Heiles, T. Piran and K. Postnov for useful discussions.

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- [1] Lorimer, D. R., *et al. Science* **318**, 777 (2007).
 - [2] Thornton, D., *et al. Science* **341**, 53 (2013).
 - [3] Falcke, H. and Rezzolla, L. arXiv:1307.1409 (2013).
 - [4] Popov, S. B. and Postnov, K. A. arXiv:1307.4924 (2013)
 - [5] Totani, T. arXiv:1307.4985 (2013).
 - [6] Kashiyama, K., Ioka, K. and Mészáros, P. arXiv:1307.7708 (2013).

- [7] Loeb, A., Shvartzvald, Y. and Maoz, D. arXiv:1310.2419 (2013).
- [8] Lyne, A. G. and Smith, F. G. *Nature* **218**, 124 (1968).
- [9] Perkins, F. and Salpeter, E. E. *Phys. Rev.* **139A**, 55 (1965).
- [10] Weatherall, J. C. *Ap. J.* **506**, 341 (1998).
- [11] Zheleznyakov, V. V., Zaitsev, V. V. and Zlotnik, E. Ya. *Astron. Lett.* **38**, 589 (2012).
- [12] Jackson, J. D. *Classical Electrodynamics* 3rd ed. (Wiley, New York, 1998).
- [13] Katz, J. I. *Ap. J.* **463**, 305 (1996).
- [14] Katz, J. I. *Ap. J.* **490**, 633 (1997).
- [15] Katz, J. I. arXiv:astro-ph/0603772 (2006).
- [16] Hankins, T. H., Kern, J. S., Weatherall, J. C. and Eilek, J. A. *Nature* **422**, 141 (2003).
- [17] Hankins, T. H. and Eilek, J. A. *Ap. J.* **670**, 693 (2007).
- [18] Lyutikov, M. *Mon. Not. Roy. Astr. Soc.* **381**, 1190 (2007).
- [19] Löhmer, O., Jessner, A., Kramer, M., Wiebelinski, B. and Maron, O. *Astron. Ap.* **480**, 623 (2008).
- [20] Usov, V. V. and Katz, J. I. *Astron. Ap.* **364**, 655 (2000).
- [21] Popov, S. B. and Postnov, K. A. arXiv:0710.2006 (2007).
- [22] Colgate, S. A. and Johnson, M. H. *Phys. Rev. Lett.* **5**, 235 (1960).
- [23] Colgate, S. A. and Noerdlinger, P. D. *Ap. J.* **165**, 509 (1971).
- [24] Burke-Spolaor, S., Bailes, M., Ekers, R., Macquart, J.-P., and Crawford III, F. *Ap. J.* **727**, 18 (2011)
- [25] Boothroyd, A. I., Blagrove, K., Lockman, F. J., Martin, P. G., Pinheiro Gonçalves, D. and Srinanth, S. *Astron. Ap.* **536**, A81 (2011).
- [26] Bagchi, M., Nieves, A. C. and McLaughlin, M. *Mon. Not. Roy. Astr. Soc.* **425**, 2501 (2012).
- [27] van Haarlem, M. P. *et al.* arXiv:1305.3550 (2013).
- [28] Luan, J. and Goldreich, P. arXiv:1401.1795 (2014).
- [29] Kulkarni, S. R., Ofek, E. O., Neill, J. D., Zheng, Z. and Juric, M. arXiv:1402.4766 (2014).