A better market design? Applying "Automated Market Makers" to traditional financial markets*

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Abstract

A new market design, an "Automated Market Maker" (AMM), completely automates trade matching and liquidity provision by leveraging decentralised finance technologies including smart contracts. Can AMMs improve the trading of traditional assets? We derive a model of an AMM's equilibrium liquidity for a given set of asset characteristics. Calibrating the model with 39 million AMM transactions and return and volume of traditional assets, we find that AMMs can make trading more efficient for assets with high volume and low volatility, including foreign exchange and large-cap equities, but are unlikely to be competitive in volatile and thinly traded assets.

Keywords: Market Design, Liquidity, Automated Market Makers, Cryptocurrency, Uniswap

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"This is just odd. Outside of a few products this will never amount to anything."

—Mutual Fund Manager, *on the proposed introduction of ETFs* (Ruggins, 2018)

1 Introduction

Innovations such as ETFs, passive index funds, derivatives or even electronic limit orderbooks are now foundations of modern financial markets, despite being met with initial skepticism from incumbents. Innovation in financial *market design*, in contrast, has been relatively modest.[1] Financial assets — from equities and bonds to futures, have for centuries traded in the same double-sided market structure — whether as forms of auctions or limit orderbooks. This fundamental market design has remained constant through electronification, high-frequency trading and the proliferation of derivative products.[2] That is, until now.

The emergence of cryptocurrencies offers a new frontier in market design,[3] absent any incumbents or vested interests. As they have (historically) lacked regulation these assets have developed in a parallel ecosystem, inventing their own set of trading venues. These venues use market designs to trade cryptoassets that are a radical departure from existing financial markets.

A new market architecture, the Automated Market Maker (AMM), has emerged in the decentralised finance (DeFi) ecosystem, leveraging new technologies including smart contracts and blockchain. These trading systems do away with the conventional wisdom of a double-sided limit orderbook. They abstract away from limit and market orders, do not require continuous updating from market makers, and provide continuous liquidity at all price levels for every minute of every day. All of this logic is encoded in only 378 lines

[1] Budish et al. (2022) proposes a model to demonstrate that private and social incentive divergence may explain a lack of innovation in market design. For an overview of market design, see Roth (2018), Milgrom (2019) and Agarwal and Budish (2021).

[2] There is a large literature on financial market design arising from the field of 'market microstructure'. Much of this focuses on analyzing optimal implementations of different types of double-sided markets, rather than proposing radical new ones. Such as; whether trading should be done through dealers or electronic limit orderbooks (Glosten, 1994), decades long discussions of the optimal tick size (Angel, 1997; Rindi and Werner, 2019), optimal levels of transparency (Boehmer et al., 2005; Benos et al., 2020), and how time should be considered — whether the limit orderbook should be designed as a series of discrete rather than continuous auctions (Budish et al., 2015; Aquilina et al., 2021).

[3] Milgrom (2019) includes section on cryptocurrencies in a recent review of market design innovations.

of static, deterministic code,[4] using a simple formula to determine prices and quantities: $x * y = k$.

But would this new market design improve financial markets? It is not clear that such a simple financial innovation would be desirable, or even possible. An algorithm unable to cancel or re-price resting orders would have quotes which were always stale, leading to "sniping" by arbitrageurs, imposing adverse selection costs on those providing liquidity, (Budish et al., 2015). If prices cannot be revised to reduce accumulated inventory, such liquidity providers become exposed to extreme inventory risks. Such a mechanism would need to ensure these risks are mitigated, or at least adequately compensated. In compensating for these high risks, such a mechanism may enforce costs of trading that were so high that no investor would want to participate in such markets.[5] Alternately, the democratizing force of such a simple market making algorithm could facilitate a broader set of latent liquidity providers — with different cost structures and risk appetites to typical high-frequency traders who provide liquidity in traditional financial markets — possibly providing liquidity at lower costs.

We examine "Uniswap", the largest of the AMMs, using a year of data acquired directly from the public Ethereum blockchain containing over 39 million transactions.[6] Examining the AMMs pricing function, we derive and test the components that govern returns to LPs, reconciling them to established market microstructure concepts. We empirically show that LP returns can be expressed as a function of the underlying prices of the assets in the pool, as well as the balanced and unbalanced order flow. This order flow imbalance is shown to be a significant source of variation in both pool size and profitability.

Using these derived relationships, we are able to test the ability of this new market design to improve the trading process for traditional financial assets — equities, bonds, forex, and commodities. Our results suggest that AMMs can, in many cases, significantly reduce the trading costs incurred by investors in these markets — particularly those characterized by high turnover and low volatility. Figure 1 shows that significant improvements in transactions costs are possible for assets such as very large-capitalization equities, forex

[4]For the core contracts of Uniswap programmed in Solidity, see: github.com/Uniswap/v2-core

[5]Glosten and Milgrom (1985) note that if information asymmetry is too severe, uninformed trading will not occur and the market maker cannot cover the costs of adverse selection, similar to Akerlof (1970).

[6]First, the entire Ethereum blockchain is synchronized. Uniswap transactions and fields are then parsed from the raw blockchain using "The Graph" API.

markets and corporate bonds, whilst smaller market capitalization equities (with higher volatility and lower turnover) do not see such benefits.

[Figure 1 about here.]

With suitable asset classes, our results show that AMMs could — theoretically — reduce the trading costs of investors by significant margins, indicating that this new market design is not only viable, but has the potential to radically reshape the trading landscape in some of finance's largest asset classes.

Despite their atypical design, "Automated Market-Makers" (AMMs) have rapidly become the venue of choice for cryptocurrency traders, executing upwards of $50bn USD per month in digital assets. AMMs overcome adverse selection costs by varying the *amount* of liquidity they provide (pool size) as the equilibrating mechanism, in contrast to traditional market makers that vary the *cost* of liquidity (the spread). In a time-series analysis of pool returns, we observe this equilibrating mechanism, ensuring sustainable liquidity provision in the AMMs. Liquidity flows into more profitable pools, with these inflows reducing fee yields. We also observe equilibrium effects in the cross-section: higher volatility pools are associated with higher fees, and larger pools experience less volatility and reduced order flow toxicity.

This paper reconciles AMMs to the classical theories of market making. These theories derive how MMs can optimally address the three challenges of inventory control — adverse selection risks, inventory holding costs, and explicit fees. Further, they imply that suboptimal solutions to these problems will lead to inefficient liquidity provision that is unlikely to survive more efficient competitors — an implication that potentially spells doom for poorly designed market structures. We contrast the solution to these challenges provided by AMMs with those provided in traditional centralized exchanges.

The first challenge, inventory control, is ensuring that the MM's inventory quickly reverts to the desired level (e.g., a zero or "flat" position in the risky asset) after a trade — holding any inventory (long or short) entails risk. Inventory control models show that MMs can solve this challenge by optimally "shading" quotes down after trading with a seller to attract buyers, and increasing quotes after trading with a buyer to attract sellers, thereby reverting her inventory (e.g., Stoll (1978); Ho and Stoll (1981); Hendershott and Menkveld (2014)). AMMs, similarly have an implicit quote shading mechanism within their pricing formula — following a buy, the AMM's price offered to the next trader increases, incentivizing sellers to help revert the AMMs inventory level, and vice versa. The difference is that rather than having an "optimal" quote shading function, as derived by inventory control models, AMMs have an arbitrary function that moves prices in the right direction but not necessarily by an optimal amount. Instead, the quote shading function in AMMs is designed to ensure the AMM cannot exhaust the inventory of either of the two assets in its pool. It does this by increasing the price of an asset at an increasing rate, as the remaining inventory of that asset falls.

The second challenge faced by such a market making model are adverse selection risks. MMs learn about asset values through order flow, allowing them to avoid making excessive losses to informed traders. Models of asymmetric information (e.g., Kyle (1985); Glosten and Milgrom (1985)) show that a MM facing a mix of informed and uninformed order flow can use Bayesian learning to optimally extract private information from order flow, using it to update their beliefs about an asset's value and adjust prices accordingly. This learning is what drives price discovery — prices converge towards their fundamental values, eventually impounding all private (and public) information. The models imply that impounding private information as quickly as possible is crucial for a MM to minimize their adverse selection losses to informed traders. In reduced form, Bayesian learning results in MMs increasing quotes following buys; and decreasing them following sells, by an amount proportional to the information content of the order flow. We show that the simple deterministic pricing function of AMMs has similar reduced form behavior, with prices adjusting up(down) following buys(sells). Even the amount of the price change following a trade is related to the informativeness of order flow because, as we show, pools tend to be smaller, and thus price impacts larger, for assets with more informed trading and higher volatility profiles. Thus, AMMs function as if they have a mechanism to learn from order flow, albeit a very simple one, allowing price discovery to occur in AMMs. However, whether the rate of implied learning in an AMM makes their price discovery too inefficient for them to be competitive in liquidity provision is an empirical question explored in this paper.

While AMMs incorporate "private information" through trades, as in Kyle (1985); Glosten and Milgrom (1985), more recent models focus on adverse selection arising from "public information". Budish et al. (2015) model a market maker that revises their quotes in response to public information (information that is symmetrically observable) shocks before "snipers" are able trade on them.[7] AMMs differ from these models as they are unable to revise quotes in response to public information, except via trading (i.e., having their stale quotes "sniped"). The inability for AMMs to avoid public adverse selection may result in higher liquidity costs in comparison to traditional MMs. Though, this may be offset by the lack of expenditure by LPs on the "arms race" for speed (as in Budish et al. (2015)) that public adverse selection avoidance requires.

Finally, the market making theories discussed above also derive the level of trading costs (e.g., bid-ask spread) charged by the MM to exactly cover the expected costs of market making, including an inventory holding risk premium and adverse selection costs. Competition drives bid-ask spreads towards these break-even levels such that an inefficient MM that has not minimized these costs (and as such requires a wider bid-ask spread) will lose market share to more competitive MMs. The models imply that equilibrium bid-ask spreads vary across assets because inventory holding risk and adverse selection costs are a function of asset properties, such as volatility and the amount of informed/uninformed trading. We show that the same cross-asset variation in liquidity is true of AMMs, although the equilibrating mechanism is somewhat different. Rather than setting a bid-ask spread, AMMs have a price impact function and a fixed proportional fee per transaction. We show that the fee in AMMs serves the same role as the bid-ask spread in market marking theories — it allows LPs to recoup compensation for adverse selection costs and inventory holding risks. However, unlike bid-ask spreads, the fees in AMMs are fixed. We show that AMMs arrive at an equilibrium level of liquidity by varying the pool size, so that a given amount of fee revenue is shared among fewer liquidity providers (higher fees per provider) when the costs of providing liquidity are higher (e.g., more volatile assets) and result in larger pool sizes for assets with lower liquidity provision costs (e.g., less volatile assets). As a result, AMM liquidity varies across assets, much like what is implied by the market

[7]Others such as Foucault (1999), Foucault et al. (2003), Hoffmann (2014), Biais et al. (2015) also model the market maker's ability to revise stale quotes as an important driver of the cost of liquidity provision. Budish et al. (2020) extend their model to include both public and private information and Aquilina et al. (2021) show that this adverse selection derived from public signals is significant, at 33% of total adverse selection. However, this estimate is from a limit orderbook, where the MM can continuously revise quotes. With the infrequent updates experienced in an AMM, this will be higher. Brogaard et al. (2021) decompose stock returns driven by market-wide, public and private information. They find that the return variance is 37% of public firm-specific information, 24% private firm-specific 8% market-wide and the rest is noise.

making theories, although with a novel mechanism for determining the liquidity level.

The simple pricing function utilized by liquidity providers in AMMs, $xy = k$, combined with a fixed fee and variable pool size, provides AMMs with the necessary mechanisms to address the three key theoretical challenges in market making — inventory control, price discovery (learning from order flow), and setting liquidity costs that vary with asset characteristics. By documenting the dynamic components of AMMs, we are able to empirically observe how AMMs absorb adverse selection, allowing us to determine which types of traditional financial assets AMMs would be well suited to.

Existing research on AMMs is at an early stage, and is currently mostly theoretical. This paper contributes much needed empirical work. Aoyagi (2020) develops a Glosten and Milgrom (1985) style model of liquidity provision in AMMs and demonstrates a stable equilibrium can occur between competitive LPs. Park (2021) focuses on the role of arbitrageurs, showing that the design of constant product AMMs can be improved to reduce front-running on the blockchain network, as is also documented by Daian et al. (2019). Capponi and Jia (2021) model an AMM alongside a centralized market, showing that liquidity provision in AMMs should increase in response to trading volumes, decrease in response to volatility, and impose gas fee externalities on the blockchain network. Empirical work is more scarce, Lehar and Parlour (2021) develop a framework of AMM competition with a limit orderbook, comparing prices and price-impacts of trades. Han et al. (2021) performs a price discovery analysis between Uniswap and a limit orderbook, finding that the orderbook contributes most to price discovery, though, Uniswap's share increases with liquidity. Lehar et al. (2022) examines the liquidity management costs of LPs to AMMs. Barbon and Ranaldo (2021) also derive an equilibrium model of AMM liquidity to compare AMM trading costs to trading costs in centralized cryptocurrency markets. Our paper is the first to estimate the cost of AMMs in a range of traditional asset classes.

[8]

The remainder of the paper is organized as follows. Section 2 explains how AMMs work, Section 3 shows how AMMs can effectively manage adverse selection risks, Section

[8] There are many other papers that examine AMMs from a computer science perspective using theory: Xu et al. (2021) derives adverse selection functions and the conservation function for several competing AMMs, while the function for Uniswap V2 is simple ($x * y = k$) others are more complex. Angeris et al. (2019) formalizes mathematical properties of Uniswap, Angeris et al. (2021) proposes new AMM designs that replicate options payoff structures for LPs and Angeris et al. (2020) demonstrates that the curvature of the AMM conservation function can protect LPs from adverse selection in volatile assets.

4 identifies how well AMMs trend towards equilibrium liquidity, Section 5 estimates and empirically validates the equilibrium pool size, Section 6 empirically calibrates our framework to a variety of traditional financial assets, determining where AMMs may become useful, while Section 7 concludes.

2 How do AMMs Work?

AMMs allow users to buy and sell tokenized assets on a blockchain without the involvement of a centralized exchange such as “Coinbase” or “Binance.”[9] An AMM consists of code written as a smart contract which facilitates trade according to a deterministic algorithm.

Uniswap is comprised of distinct liquidity “pools”, each holding two assets (or tokens). The most popular pool has the tokenized form of USD (USDT) and Ethereum (ETH).[10] Consider x_t and y_t the quantities of USD and ETH in the AMM at time t, respectively.

Pools are created by individual liquidity providers (LPs) who transfer ETH and USDT to the pool’s Ethereum address. LPs provide (or “stake”) a total of x_0 of USD and y_0 of ETH at $t = 0$ when the price of ETH in dollars is P_0. This gives the pool the constant $K = x_0 y_0$. The total USD value of the assets in the AMM at $t = 0$ is V_0, which (as we show later) is equal to $2x_0$ or twice the amount of deposited USD.

This creates resting liquidity which can then be accessed by liquidity demanders (LDs or “traders”) by “swapping” any one of the two assets for the other. A trade is an asset swap in which one asset is added and the other removed in quantities such that the constant K is maintained (by design of constant product AMMs such as Uniswap). Thus, a trade at time t that buys ETH involves removing $\Delta y_t = y_t - y_{t-1}$ from the AMM and adding $\Delta x_t = x_t - x_{t-1}$ such that the constant k is preserved, $x_t y_t = K = x_{t-1} y_{t-1} = (x_{t-1} + \Delta x_t)(y_{t-1} - \Delta y_t)$. Conversely, a trade at time t that sells ETH adds Δy_t to the AMM and removes Δx_t from the AMM, $x_t y_t = (x_{t-1} - \Delta x_t)(y_{t-1} + \Delta y_t)$.

Therefore, the price of the pool is a function of the ratio x and y, but also the size of the trade in relation to the amount of x and y in the pool. This means that the trade

[9] This means AMMs are sometimes referred to as “decentralized exchanges” (DEXs). This is appealing to users as it fulfills the promise of blockchain technology not being reliant on traditional finance institutions.

[10] The mechanics of liquidity provision are the same for any pair. The most popular pool contains Ethereum and USDT, which is a “stable-coin” designed to closely match the value of USD by being redeemable for physical US Dollars . Further information on USDT and the validity of its 1:1 backing can be found in Griffin and Shams (2020).

price deviates from the ratio price as an increasing function of the proportion of the pool's liquidity that is removed.

LPs are incentivized to provide liquidity through a fixed fee of 30 basis points (bps) of the trade value, $f_t = 0.003x_t$. These fees are added to the liquidity pool, slightly increasing the pool's K with each trade.[11]

Importantly, there can be many LPs in any given pool — the largest pools have over 20,000.[12] Additional LPs can join, or existing LPs can increase the amount of liquidity they already stake, increasing the size of the pool — in a process termed "Minting". Minting increases the pool constant K by depositing assets x and y at the pool's current asset ratio of x_t and y_t. They may also withdraw liquidity, reducing the pool's K and withdrawing x_t and y_t at the current ratio of the pool at any time — a process referred to as "Burning". Crucially, this means they are exposed to price changes on their staked assets and thus adverse selection, which we will detail further in the paper. The daily net changes in liquidity to existing pools as a result of minting and burning we refer to as pool "flows".

Pools predominantly consist of unique combinations of cryptocurrency asset pairs. While it is possible for LPs to create new pools for existing pairs, liquidity concentrates on the existing pools for two reasons. The pricing function, as we show below, ensures liquidity demanders pay a premium to transact on less liquid pools which in turn discourages uninformed trading interest. Secondly, the Uniswap interface[13] has a drop-down menu for the largest 45 currencies. Users can enter specific pool identifiers, but must enter a 42 character string. Consequently, in the sample of the 200 largest pools used in this paper, there are no pools with duplicate asset pairs.

2.1 AMM Properties

we set out various properties for how an AMM functions for traders buying and selling ETH (y), with USD (x) being the unit of account. A derivation of each is given in Appendix C. We assume that T is a sufficiently short period such that no mints or burns occur during

[11] For tractability, we assume fees accrue to a separate account (which assumes LPs withdraw fees). Over the time horizons in question, this assumption makes little difference to the overall results.

[12] This is ascertained from the number of unique blockchain "addresses". See Wang et al. (2021) for more statistics on Uniswap LP addresses.

[13] Accessible via app.uniswap.org

the period from $t = 0$ to $t = T$.

Property 1: Ignoring fees, the trade price to buy a quantity of ETH Δy_t is given as follows:

$$P(\Delta y_t) = \frac{\Delta x_t}{\Delta y_t} = \frac{x_{t-1}}{y_{t-1} - \Delta y_t} \tag{1}$$

with a trade to sell $\Delta y_t'$ units of ETH being the same but with the last term changing from $-\Delta y_t$ to $+\Delta y_t$. This price function is used to generate the AMM's bonding curve with Figure 2 demonstrating the impact on the price of larger quantities traded.

[Figure 2 about here.]

Property 2: The "midpoint" price of ETH in the pool measured in USD (the price of an infinitesimally small trade that has negligible price impact) is purely a function of the quantities of the two assets in the pool at the time of the trade: $P_{t,MID} = x_t/y_t$.

Property 3: Ignoring fees, the sequence in which trades occur does not matter for the final outcome (state) of the AMM — its pool quantities and midpoint price.

Property 4: Ignoring fees, a round-trip trade reverts both price and quantities back to their initial state, with the trader making neither profit nor loss.

Property 5: Two unidirectional trades of quantity x will have an equivalent impact as one trade of $2x$ in the same direction (same end price in the AMM, same end state in terms of quantities in the AMM, and same cost to the trader).

Property 6: If the AMM receives a series of trades, only the buy/sell imbalance quantity of the series of trades is needed to measure the impact on the AMM's state. The balanced volume (buy volume equal to sell volume) has no impact (irrespective of what combination of trades generates the balanced volume), nor does the sequence of trades matter. The AMM is "memory-less" in that the impact of a trade depends only on the current state of the AMM and not the history of trades.

Property 7: Assuming the presence of arbitrageurs, the value of each of the two assets staked by the LP are equal, when measured in one unit of account, for example, the USD value of x_t is always equal to the USD value of y_t. This is true at every point, including minting, burning and swapping.

3 Can AMMs Effectively Manage Adverse Selection Risks?

For full market automation to be possible, liquidity providers must be adequately compensated for the risks they bear. The two main risks liquidity providers face in financial markets are: inventory risk and adverse selection risk. Inventory risks arise out of the necessity for an LP to hold assets to meet buying or selling demands, exposing them to changes in asset values. In modern financial markets, LPs are able to minimize inventory risks using algorithms that minimize holding times and hedge inventory exposures in correlated assets. AMMs, by contrast, require an LP to hold or provide fully collateralized positions in both assets. The fees an AMM LP earns are a linear function of the amount of inventory they hold, whereas the fees earned by an LP in traditional markets may bear little or no relationship to their inventory.

Adverse selection risks arise from information asymmetry between the liquidity provider and demanders of that liquidity. In traditional markets, the LP provides liquidity at prices based on all publicly available information, experiencing adverse selection from more informed liquidity demanders with additional private information, (Glosten and Milgrom, 1985). In AMM settings, LPs are unable to set prices incorporating even public information. As a result, they bear all the costs associated with public as well as private information being impounded into prices. However, in modern markets LPs now bear a significant proportion of public information adverse selection, as shown in Aquilina et al. (2021). The economics of market making, at its core, is a problem of balancing profits from uninformed traders against the losses incurred to informed traders (adverse selection costs). Traditional market making and liquidity provision in an AMM are similar in that they both profit from uninformed order flow — balanced (or round-trip) trading, and incur losses from informed, directional trading. Thus, all else equal, in both mechanisms, the more balanced — and the less directional — order flow (more uninformed traders and less informed traders), the more profitable is liquidity provision.

Where the two mechanisms differ is that in traditional markets the profits from uninformed, balanced flow are modulated by adjusting the bid-ask spread. If the bid-ask spread is too wide, the market-maker (MM) will earn profits in excess of their adverse selection costs, encouraging competing liquidity providers to undercut the MM's quotes, reducing the spread towards its equilibrium (or efficient) level. If the spread is too narrow, MM's make losses, leading to a widening of spreads and/or a departure of competing liquidity providers. Thus the liquidity of a stock in a traditional MM model (bid-ask spread and quoted depth) is a function of the relative amount of informed and uninformed trading.

In AMMs, there is no explicit bid-ask spread set by the LPs, rather the fee paid by liquidity demanders is fixed at 30 basis points,[14] but varies in percentage terms for each contributing LP (and is modulated by) the quantity of staked assets in the liquidity pools. This results in a slightly different equilibrating mechanism to traditional market-making, although both are driven by similar underlying principles.

If a pool is highly profitable, with fee yield exceeding adverse selection costs, the pool will attract flows from new LPs staking the pools assets. Assuming both informed and uninformed traders' actions remain unchanged (as is traditional in models of market making), a larger quantity of staked assets will reduce the fee yield component $FY_T = \frac{F_T}{V_0}$ by increasing the value staked in denominator, V_0, without impacting the numerator — total pool fees. With adverse selection unchanged, LP's total profits will reduce. Conversely, if the LP is making losses due to adverse selection costs that exceed the fee yield, LPs will withdraw assets from the AMM, reducing the total staked value and increasing the fee yield — driving up LP profitability to a break-even level.

Thus, AMMs are expected to equilibrate their level of liquidity to match the levels of informed and uninformed trading, such that LPs do not earn excess profits. Unlike traditional market making, the mechanism is the modulation of the pool size, rather than adjustments to the bid-ask spread. Ultimately, the effects on liquidity are similar — if the spread is increased to cover a high adverse selection cost, this increases the cost of trading, much like decreasing the pool size to increase yield also increases price impact due to the lower liquidity constant, K. Larger price impacts of trading are economically similar to paying a higher bid-ask spread. Ultimately, the relation between the level of liquidity and the underlying properties of the asset and its traders (mix of informed and uninformed, fundamental value volatility) are similar for both AMMs and traditional market-makers, despite mechanical differences in their design.

[14] This refers to Uniswap Version 2. The launch of Uniswap V3 allows for separate pools to set fees at 1, 5, 30 and 100 basis points. These three options are still far from dynamic spreads in traditional markets.

3.1 Return Components for the LP in the AMM

In the previous section we set out the mechanism by which AMMs can manage liquidity risks in equilibrium. In this section we derive the return components to liquidity provision so that we can empirically assess AMM feasibility for assets with different characteristics.

Assume that there are no mints or burns between $t = 0$ and $t = T$. An LP that deposits assets at $t = 0$ and withdraws them at $t = T$, earns a *Total Return* of:

$$R_{TOTAL} = \frac{V_T + F_T}{V_0} - 1 \tag{2}$$

where $V_0 = x_0 + y_0 P_0 = x_0 + y_0 \frac{x_0}{y_0} = 2x_0$ is the USD value of the initial staked assets at $t = 0$, $V_T = x_T + y_T P_T = 2x_T$ is the value of the staked assets in the pool at $t = T$ (after T trades) excluding fees, and $F_T = \sum_{t=1}^{t=T} f_t = 0.003 \sum_{t=1}^{t=T} |\Delta x_t|$ is the sum of the accrued fees on the T trades. The LP's total return in (2) can be rewritten such that it is broken down into three components:

$$R_{TOTAL} = \underbrace{\left(\frac{V_{T,FIXED}}{V_0} - 1\right)}_{\text{Inventory Holding Return}} + \underbrace{\left[\left(\frac{V_T}{V_0} - 1\right) - \left(\frac{V_{T,FIXED}}{V_0} - 1\right)\right]}_{\text{Adverse Selection Cost}} + \underbrace{\frac{F_T}{V_0}}_{\text{Fee Yield}} \tag{3}$$

where $V_{T,FIXED} = x_0 + y_0 P_T$ is what the staked assets would be worth (in USD) if the LP had held them passively outside of the AMM — keeping the initial quantities fixed at x_0 and y_0.

Each of these three return components has an economic interpretation that maps to the market microstructure models of liquidity provision by market makers in traditional financial markets:

The first component, Inventory Holding Return: $IHR_T = \frac{V_{T,FIXED}}{V_0} - 1$, is an increase or decrease in the value of the LP's inventory, purely from asset price changes, and not from changes in the quantities held (in contrast, adverse selection costs play out through the quantities changing in adverse ways). In traditional models of market making, the MM typically holds inventory close to zero by being allowed to short sell and buy on margin, but temporarily takes on non-zero positions while intermediating between buys and sells until such time as she can revert her inventory back to zero. While holding a non-zero inventory, the MM faces the risk that the asset price adversely appreciates or falls, which gives rise to inventory holding risk. If the MM is risk-averse, the inventory holding risk is undesirable and MM's factor it into their bid-ask spread so that they are compensated for bearing such risk.[15] In contrast, in the AMM, the LP cannot hold an inventory close to zero (unless they hedge their staked liquidity positions with external contracts) as they physically have to post a positive quantity of both assets into the AMM pool. Thus, inventory holding risk is a material consideration for an AMM LP, and the IHR_T reflects the profit or loss (as a return) on the inventory the LP holds.

The second component, Adverse Selection Cost: $ASC_T = (\frac{V_T}{V_0} - 1) - (\frac{V_{T,FIXED}}{V_0} - 1)$, is how much worse off the LP is from staking both assets to the AMM pool compared to the benchmark of holding the same initial asset quantities outside of the AMM. It results from adverse changes to the quantities of the two assets in the AMM pool as traders use the AMM to swap one asset for another. When the value of the risky asset falls, and the AMM price has not yet adjusted, arbitrageurs will buy the asset at a low price in external markets and sell it to the AMM at the AMM's (stale) high price, making the AMM overpay for the asset. Similarly, when the value of the risky asset increases arbitrageurs will buy the asset at the (stale) low price in the AMM and sell it at a higher price in an external market, resulting in the AMM selling the asset at a price below fundamental. Consequently, the quantities of the assets within the AMM change in an adverse way, resulting in an adverse selection cost akin to the classic adverse selection cost in traditional models of market making (for example, Glosten and Milgrom (1985); Kyle (1985)). As we will show, this return component is strictly less than or equal to zero, consistent with it being a true "cost" to the LP of providing traders the option to buy or sell assets from the AMM and having arbitrageurs do so, on average, in a way that "exploits" stale prices in the AMM. We will also show that this component is largely a function of the asset price changes — irrespective of whether the asset price increases or decreases, the LP will incur an adverse selection cost, ASC_T.

The third component, Fee Yield: $FY_T = \frac{F_T}{V_0}$ is expressed as a yield on the staked asset value earned by the LP. This component of total return is strictly positive as long as there are trades with the AMM. The ability to earn fee revenue is what incentivizes LPs to stake their assets knowing that they face adverse selection costs — thus, the basic

[15] See classic inventory management models such as Stoll (1978), Ho and Stoll (1980), Ho and Stoll (1981), Stoll (1989), and more recently Hendershott and Menkveld (2014).

proposition of an AMM is to try and earn a sufficient fee yield to at least cover the adverse selection costs. Ideally fees will also compensate the LP for the inventory holding risk. The fee yield also maps very closely to traditional market microstructure models in which MMs earn profits from the round-trip trades of uninformed traders and incur losses to the directional trades — similar to the cost of trading with informed traders. In traditional market making models, the MM adjusts the bid-ask spread that she charges per round-trip trade so that she earns just enough from the uninformed traders (in a competitive market) to cover the losses made to the informed traders. Similarly in AMMs, round-trip trades accrue fee yield without imposing any adverse selection or inventory holding costs, whereas directional trades cause price changes that impose adverse selection costs on the LP.

While the *Total Return* is the sum of each of these three components and reflects the overall returns to liquidity providers, the Staking Return is the net sum of the *Adverse Selection Cost* and the *Fee Yield*. It is thus the net economic impact of the choice to invest or provide liquidity to the AMM by "staking" assets, rather than holding them outside of the AMM. Therefore, we use it as a key measure in assessing the profitability of liquidity provision to AMMs.

Figure 3 plots *Total Return* and each of the three components against the return of the underlying pool assets, expressed as the ratio $R = \frac{R_{t=1}}{R_{t=0}}$ of changes in price over the time period t. *Total Return* is bounded at -100% (with no fee revenue) and -70% with 30% assumed fee revenue in this Figure. For example, a pool holding assets ETH and USDT would have -100% return if the price of ETH goes to zero. This is because arbitrageurs swap ETH into the pool in exchange for USDT, leaving the pool holding only ETH, a now worthless asset. The *Inventory Holding Return* (IHR) is bounded at -50% when the ETH price goes to zero. Notice that the IHR is larger than the *Total Return*. As the ETH price increases, arbitrageurs swap ETH out of the pool in exchange for USDT, limiting the total returns the LP receives from ETH's appreciation. *Adverse Selection Costs* is bounded at -50% because if the price of ETH goes to zero, the LP loses all the of the USDT they deposited but they would have lost the ETH regardless of investing in the pool, so just the loss in USDT is considered. At the other end of the curve, losses can exceed 100% if the price of ETH increases significantly such that the opportunity cost of holding ETH outside the pool exceeds the value of the ETH deposited.

[Figure 3 about here.]

3.2 Adverse Selection Costs and Staking Returns in the Data

Next, we empirically assess whether AMMs sufficiently reward LPs for the significant risks they take as automated market-makers. The AMM receives little protection from the risks involved in liquidity provision. It cannot reprice stale quotes when market prices of the assets in the pool change (adverse selection risks), and it cannot actively manage inventory risks by reducing inventory holdings (inventory holding risks). The LP bears the totality of these risks with the hope that the fees they earn will offset them. In practice, this is mostly the case. The value-weighted average fee revenue, net of adverse selection costs, is positive. We refer to this measure as the "Staking Return" as it is the net impact on returns of an LP transferring their tokens to a Uniswap pool, termed "staking" in blockchain terminology.

Table 1 sets out the descriptive statistics of Uniswap pool characteristics and LP returns for the sample of the top 200 pools by size.[16] *Pool Size* is highly skewed, with the largest 1% of pools having more than a quarter of a billion dollars in tokenized assets, with a mean *Pool Size* of \$8.5 million. LPs do not frequently update their positions, with the median pool-date experiencing no flows, with the largest 1% of *Flows* being 34% of pool value and the lowest 1% being -24%.

Fee Yield averages 12 basis points per pool-date, or 44% per year when annualized. *Adverse Selection Costs* (ASC) are either zero or negative by construction. The pool-date mean *ASC* is -17bps per pool-date, whilst the weighted mean is -4bps (annualized -62% and -15%, respectively). This sets the sum of the two — the *Staking Return* — below 0% for the unweighted mean, but positive 2bps (7% annualized) for the pool-size weighted mean. This implies that AMM liquidity provision is profitable for an LP that invests proportionate to the size of the top 200 pools, though this profitability is not evenly distributed across pools.

Given the overall increase in cryptocurrency prices over the sample period one would

[16] We also require the pool to have traded for more than 8 months of the 1 year sample, and remove the first 7 days for each pool (as pool balances take time to grow to meaningful sizes). We also remove all pools that contain "re-balancing tokens". These are tokens that modify their balances according to predefined rules. These tokens create problems for assessing profitability. We obtain a list of re-balancing tokens from "Coingecko". While Uniswap V2 starts on May 19th, 2020, we remove the first 3 weeks of the sample and begin on the 1st of June, 2020 as pool sizes take until this time to become meaningful.

expect both *Total Returns* and *Inventory Holding Returns*/IHR, which are positively related to the returns in the underlying assets, to be highly positive. The mean pool-date return is 6 (total) and 11 basis points (IHR), but the median is negative. The majority of pools in the sample are "Token-Token" pools, however, which means that price changes can be driven by changes in pricing relationships between two tokens, rather than a general increase in overall cryptocurrency prices. Order flow is highly balanced, with a weighted mean *Uninformed Order flow* of 92%. This highly balanced order flow helps explain the viability of AMMs. As a comparison, the equivalent measure for the S&P500 Futures contract is 77.43% in Easley et al. (2012).

[Table 1 about here.]

Figure 4 presents a histogram of ASC_T and $StakingReturn_T$ which is $ASC_T + FeeYield_T$ for each pool-date in the sample. *ASC* is always negative or zero, whilst the addition of *Fee Yield* shifts the distribution to the right to outweigh *ASC* to be positive for 58.8% of pool-date observations. This demonstrates that the fees earned by LPs are sufficiently large to compensate LPs for adverse selection costs they bear, for most pool-dates in the sample. It also provides evidence that liquidity provision in Uniswap AMMs may approach a rational equilibrium. Small changes in pool size (while maintaining constant order flow) could be sufficient to generate positive average returns to LPs.

[Figure 4 about here.]

Panel B of Figure 5 reports pool-size weighted average *Adverse Selection Costs* and *Staking Return* across various pool characteristics: pool-type, *Volatility* and *Pool Size.* "Token-Stable" pools have the highest returns, averaging 4.6bps per day, due to the large *Fee Yield* of 8.2bps. "Stable-Stable" pools return 3bps per day, almost all of which is fee yield due to the lack of adverse selection costs associated with stable USD pegged tokens. "Token-Token" pools experience the most *Adverse Selection Cost*, at 5.7bps per day with insufficient fees (4.9bps) to recoup these losses. While it is possible for "Token-Token" pools to be profitable ("SUSHI-WETH" has the second highest total cumulative return in Figure 5), the large number of token-token pools in the sample means that there is likely a bias towards higher *Volatility* tokens in this category.

Higher *Volatility* pools are less profitable, with only the lowest quartile of pools in Panel B being profitable. While this may seem intuitive, it is possible for higher volatility pools to be profitable: negative profits could drive outflows from the pool, reducing *Pool Size* and equilibrating *Fee Yields*. However, if the pool size decreases such that it becomes prohibitively expensive to execute due to the AMM bonding curve slope, volumes may also endogenously decrease so that *Fee Yield* does not increase. This might explain the positive relationship between pool size and profitability observed in Panel C. Only the largest quartile of pools are found to be profitable.

[Figure 5 about here.]

3.3 Deriving Determinants of LP Returns

Liquidity providers to AMMs receive *Total Returns* that can be decomposed into three components: *IHR*, *ASC* and *Fee Yield*. These components are either driven by price changes in the underlying pool assets, the volatility of these assets, the nature of order flow in the AMM or the amount of liquidity staked in the AMM. We present theoretical relationships for each component.

The IHR can be expressed as a function of returns:

$$IHR_T = \frac{1}{2}\left(\frac{P_T}{P_0} - 1\right) = \frac{1}{2} r_y \tag{4}$$

and as returns are a function of unbalanced order flow, it can be expressed as a function of the Relative Order Imbalance ($ROIB$), where $ROIB_T = OIB_T/V_0$. OIB_T is the cumulative net order flow $\sum_{t=1}^{t=T} \Delta x_t$ and V_0 is the pool value:

$$IHR_T = 2ROIB_T + 2ROIB_T^2 \tag{4B}$$

so that the inventory holding return is an increasing function of the order imbalance, expressed as a proportion of the total pool value in USD.

Adverse Selection Costs can also be expressed as a function of returns,

$$ASC_T = \sqrt{R_T} - \frac{1}{2}\left(R_T + 1\right) \leq 0 \tag{5}$$

where $R_T = \frac{P_T}{P_0}$, which shows that ASC are always less than or equal to zero, are minimized when prices do not change, and increase from returns in either direction (positive or negative returns). ASC_T can also be expressed as a function of the order imbalance.

$$ASC_T = -2\left(\frac{\Delta x_T}{V_0}\right)^2 = -2\,(ROIB_T)^2 \leq 0 \tag{5B}$$

Which shows that balanced or round-trip trades do not contribute to ASC_T, when $ROIB_T = 0$ and $ASC_T = 0$ and when the order imbalance increases in either direction, it increases ASC_T at a quadratic rate.

The *Fee Yield* is unrelated to returns and is instead a function of the total traded volume Q_T, irrespective of either the trade direction or whether the flow is balanced or unbalanced:

$$FY_T = \frac{F_T}{V_0} = \frac{0.003\sum_{t=1}^{t=T}|\Delta x_t|}{V_0} = \frac{0.003Q_T}{V_0} = 0.003TURN_T \tag{6}$$

Thus, higher traded volumes are expected to increase *Fee Yield*. Also, lower pool values (lower V_0) will increase the *Fee Yield*. So, ultimately, *Fee Yield* is an increasing function of trading volume normalized by pool size, which we refer to as pool turnover, $TURN_T = Q_T/V_0$.

The three LP return components can be re-combined to examine the drivers of *Total Returns*:

In terms of price changes:

$$TotalReturn_T = \sqrt{\frac{P_T}{P_0}} - 1 + 0.003\frac{Q_T}{V_0} \tag{7}$$

In terms of order flow:

$$TotalReturn_T = 2ROIB_T + 0.003\,|ROIB_T| + 0.003RBAL_T \tag{8}$$

Interestingly, while the adverse selection costs are increasing in the absolute amount of order imbalance in either direction (excess buying or excess selling), the *Total Return* is directionally increasing in the volume of buys (decreasing in the volume of sells) due to the strong effects of the *IHR* component (Formula 8). The same effect can be seen in terms of returns: while the *ASC* increases irrespective of the direction of returns as long as the asset price changes, the total return is positively related to the return to the risky asset (Formula 7).

4 Equilibrium Effects: Is Liquidity Allocated Where it is Needed?

In any market, there needs to be a mechanism to allocate capital to its most productive use. The sensitivity of fund inflows and outflows to fund performance is of key interest in the investment funds management literature. In Uniswap AMMs, flows are crucial in determining LP returns and pushing pools towards equilibrium. In this section we search for evidence that pools trend towards equilibrium by varying their size.

HYPOTHESIS 1: Liquidity should inflow to pools that have higher *Total Return*.

To examine if flows chase returns we first compute correlations between current period pool flows and 30 lags and leads of fee yield in the time-series. Figure 6 shows that fee yield is positively correlated with lagged flows but negatively correlated with future flows. This demonstrates the equilibrating effect of flows on fee yield — flows chase yields but also act to depress them.

[Figure 6 about here.]

Table 2 regresses pool *Flow* on lagged measures of each pool return component, as well as lagged pool *Flow*, similar to a structural VAR model. Consistent with the correlogram, fees are positively related to flows. A one percent increase in fees in the previous day results in a 1.1% flow into the pool. However, as the mean *Fee Yield* in the sample is 12bps, there is a muted reaction by flows to such an increase, which perhaps explains why the cross-section of AMMs is not in full equilibrium, with some pools exhibiting both positive and negative *Total Returns*. Capponi and Jia (2021) also find this positive relationship between flows and fee yield in a contemporaneous model at the weekly level.

A positive relationship between *IHR* and *Total Returns* is also found, however their coefficients are not meaningfully large. *ASC* exhibits either no, or a very small, relationship with *Flow*. This implies that LPs may make investment decisions over longer horizons rather than the daily horizon used in this specification. As *ASC* forms a key component of staking returns, perhaps the same explanation can be offered for the lack of a significant relationship between *Staking Return* and *Flow*. Fee yields are readily visible and easy to calculate, perhaps identifying why these are the largest drivers of LP flows.

The regression includes dummy variables "*Sushiswap Attack*" and "*Uniswap Incentive*". These variables take the value of one during the period a rival AMM and Uniswap provide incentives to LPs in certain pools. Date fixed effects likely subsume most of the significance of these variables, so we see little significance in these coefficients.

[Table 2 about here.]

While positive returns may attract inflows (and negative returns outflows), it is also possible that staking flows by LPs themselves impact pool returns. This occurs as the explicit fees charged on trading (the primary revenue of the LPs) are independent of pool size. Larger pools spread these revenues out amongst a wider base of LPs, reducing overall pool return. To test for these effects, we propose the following:

Hypothesis 2: Following liquidity inflows (/outflows) to/from an AMM pool, the *Total Return* of the LPs in that pool should be lower (/higher), as should be *Fee Yield*.

Table 3 regresses each return component on lagged flows, as well as lagged pool returns, similar to a structural VAR model. *Fee Yield* equilibrates as predicted. An increase in pool *Flow* of 10% in the previous day results in a decrease in the daily *Fee Yield* of 0.6 basis points. As the weighted mean *Fee Yield* averages 6 basis points, this represents a similar reduction of 10 percent.

Perhaps unsurprisingly, the external incentives provided by Uniswap to encourage liquidity provision are associated with a reduction in *Fee Yield* and *Staking Return*. This is because LPs received the newly created "Uniswap Token" during this period and were thus willing to accept lower pool yields in return for these additional incentives. The "*Sushi Attack*" period has the opposite impact, as LPs were incentivized to leave Uniswap to instead stake on their competitor, SushiSwap, during this period. While they also impact *Total Returns* and *IHR*, this is likely confounded by price changes to major assets such as Ethereum during the period.

In summary, this section provides evidence that pool size works to equilibrate fee yields through inflows and outflows. LPs direct flows to high yield pools which then dilutes the percentage *Fee Yield* received by all LPs in the pool.

[Table 3 about here.]

5 Deriving the Equilibrium Pool Size

In this section we will derive the theoretical equilibrium pool size for an AMM. We start with our expression for the total return for an LP, Formula 3. Consider the marginal LP that holds each of the two tokens and is considering whether to commit them to the AMM or not.[17] Assuming risk neutrality and competition that drives profits down to the opportunity cost of capital R_c (e.g., the interest rate that could be earned by risk-free lending of the capital), the LP will be indifferent (and hence this is the equilibrium condition) when the Adverse Selection Cost and Fee Yield components together equal R_c:

$$\underbrace{\left[\left(\frac{V_T}{V_0}-1\right)-\left(\frac{V_{T,FIXED}}{V_0}-1\right)\right]}_{\text{Adverse Selection Cost}<0}+\underbrace{\frac{F_T}{V_0}}_{\text{Fee Yield }>0}=R_c \tag{9}$$

Redefine the ASC to be a positive number reflecting the magnitude of the adverse selection cost, i.e., $ASC = -\left[\left(\frac{V_T}{V_0}-1\right)-\left(\frac{V_{T,FIXED}}{V_0}-1\right)\right] > 0$. As shown previously, the ASC is determined primarily by the price changes of the tokens, and is therefore an increasing function of the token price volatility (σ) :

$$ASC = \frac{1}{2}\left(\frac{P_T}{P_0}+1\right)-\sqrt{\frac{P_T}{P_0}} > 0 = f(\sigma) \text{ such that } \frac{dASC}{d\sigma} > 0 \tag{10}$$

Yet the fee yield is a function of the total trading volume (Q_T) and the pool size (V_0):

$$\underbrace{\frac{F_T}{V_0}}_{\text{Fee Yield}} = 0.003\frac{Q_T}{V_0} \tag{11}$$

Assuming at least some traders (LDs) are sensitive to the costs of trading, total trading volume is an increasing function of the pool size, because larger pools have lower slippage and therefore lower trading costs:

$$Q_T = f\left(V_0\right) \text{ such that } \frac{dQ_T}{dV_0} > 0 \tag{12}$$

[17] This is likely to be the marginal liquidity provider as they will not require a premium for IHR —- other potential LPs that do not hold the tokens would be in a similar position — either assuming they can hedge the token prices with futures or are risk neutral.

Putting the pieces together (substitute into the equilibrium condition (9)), the equilibrium pool size (V_0^*) is:

$$0.003\frac{Q_T\left(V_0^*\right)}{V_0^*} - \text{ASC}(\sigma) = R_c \tag{13}$$

$$V_0^* = 0.003\frac{Q_T\left(V_0^*\right)}{R_c + \text{ASC}(\sigma)} \tag{14}$$

This equilibrium condition implies that, all else equal, the equilibrium AMM liquidity is:

1. Decreasing with higher asset volatility, σ;
2. Decreasing with higher opportunity cost of capital, R_c (e.g., over-collateralized lending yields of the tokens);
3. Increasing with higher trading volume, $Q_T(V_0^*)$;
4. Increasing with the AMM fee (e.g., 30bps rather than 1bp)[18]

Therefore, we expect to see particularly deep/liquid AMMs for assets that have low volatility (like stable-stable pairs), low interest rates, and high trading volumes. But there is more to this — the equilibrium pool size depends critically on how trading volume in the AMM responds to trading costs and hence to pool size, i.e., the elasticity of trading demands. A slight reorganization of (14) makes this clear:

$$V_0^* = \left[\frac{0.003}{R_c + ASC(\sigma)}\right] Q_T\left(V_0^*\right) \tag{15}$$

The term $\left[\frac{0.003}{R_c+ASC(\sigma)}\right]$ is independent of trading volume, and can be thought of as a 'constant' or exogenous parameter with respect to the equilibrating process between pool size and trading volume. Its interpretation is that it tells you the equilibrium pool size as a multiple (or fraction) of the AMM's trading volume. The term tells us that, all else equal, for a given trading volume, pools will be smaller if the asset has a higher ASC or opportunity cost of capital, and the pool will be larger if the AMM charges a higher fee.

[18]While Uniswap V2 has a fixed fee of 30bps, Uniswap V3, which is currently in operation, allows for 100, 30, 5 and 1 basis point fee tiers. We examine an even wider theoretical fee schedule in our calibration in the next section.

For example, suppose annual lending interest rates are 3% p.a., ASC is 5% p.a., and the AMM charges a 30bps fee, we get $\left[\frac{0.003}{R_c+\text{ASC}(\sigma)}\right] = \frac{0.003}{0.03+0.05} = 0.0375$, i.e., for these given exogenous parameters, the equilibrium pool size will be 3.75% of the annual AMM trading volume. It is not clear that is a feasible/viable pool size — e.g., suppose we start with a $1mil pool size and $1mil of annual trading. That is too big a pool size for equilibrium, being 100% rather than 3.75% of the trading volume — the model tells us the fee yield will not be sufficient to cover the ASC at that pool size. So, what would happen is the pool size would shrink (LPs withdraw as they realize $ASC > FY$). If there was no change to trading volume, the pool size would just shrink down to $37.5k and we would have an equilibrium. But where it gets tricky is that volume would also shrink as the pool size shrinks as trading becomes more costly due to slippage. The lower volume then leads to an even lower equilibrium pool size, and so on, potentially never finding a stable point (i.e., equilibrium size could be zero and the AMM is infeasible). Whether it is feasible or not depends critically on how sensitive volume is to trading costs and thus to pool size.

Figure 7 illustrates the equilibrium between pool size and trading volume and the role of the exogenous constant in determining (i) the optimal pool size and thus AMM liquidity, and (ii) whether the AMM is feasible in the first place:

[Figure 7 about here.]

The Volume-Pool Size relation is assumed to be monotonically increasing (corresponds to assuming traders are cost-sensitive such that more traders choose to use the AMM when it is deeper/cheaper) but at a diminishing marginal rate (because there is only a finite amount of trade that would take place in the asset even as slippage costs tend towards zero.

5.1 Empirical Validation of Equilibrium Pool Size Derivation

To empirically validate our derivation of the equilibrium pool size, we regress the actual pool size on the predicted equilibrium pool size in our sample. The three inputs necessary for deriving the equilibrium size for a given pool are adverse selection costs, the value traded, and the cost of capital. We have the first two measures from our existing analysis, but obtain the estimates of cost of capital (R_c) from daily lending rates from the on-chain lending platform 'Aave'. This is a suitable proxy for the opportunity cost as holders of tokens can derive returns from lending through the Aave platform. We match the constituent tokens of pool pairs and average the lending rates. Where rates are not available, we apply the average rate for the same token-type ('stable' or 'token').[19]

We calculate V^* using the monthly means of daily observations of ASC, Q and R_c. We regress the last observation of the pool-size for a given month $log(V_0)$ on the predicted $log(V_0^*)$. This means the regression compares the actual pool-size for a month to the components that determine it over the previous month, i.e. a 1-month look-back period.[20]

Table 4 performs four separate panel regressions across the top 200 pools by size in our sample. First, a pooled regression with no fixed effects, second; a panel regression with pool fixed effects, then month, and then pool-month fixed effects. Overall, the predicted equilibrium pool-size performs well, with an overall R^2 of 77%. A clear result is that the model is less successful at explaining variations in pool-size across the time-series dimension, as opposed to the cross-section. This may imply that pools are more successful at equilibrating in the cross-section than over time, but it is more likely that this is driven by a tendency for both Q and ASC to be more variable on a time-series basis.

[Table 4 about here.]

6 Calibration of AMMs in Traditional Assets

We now seek to apply AMMs to traditional financial markets, aiming to understand whether the AMM market structure could provide benefits over existing structures. While AMMs are implemented mostly on the Ethereum blockchain, they could also conceivably be implemented as a piece of code on NYSE or NASDAQ's matching engine. This could be done as a separate orderbook that trades alongside the traditional continuous limit orderbook, or as a new market entrant alongside NYSE/NASDAQ, without necessitating any blockchain.

[19] We winsorize these rates at 99% by token to lessen the impact of extreme outlier dates.

[20] We find this time-period gives us the largest R^2, though a weekly model also performs well, with overall R^2 of 72% rather than 77%. While we do find the average holding period of liquidity providers is around 7 days, (see Appendix Table 6) our regressions suggest the equilibration process may take longer than a week. We drop observations prior to and including the 'Sushiswap attack' and Uniswap incentive periods up to the 21st of November, 2020 as they provide additional incentives that are not factored into our model.

We seek to determine which asset classes AMMs would be best suited to by calibrating our equilibrium pool size model to the attributes of several asset classes. In doing so, we suppose a theoretical AMM that mirrors the existing trading volume and volatility characteristics of the entire existing market for a given security.

To implement this calibration, we obtain the volume, returns and opportunity costs of capital for a variety of traditional assets. We then examine what the theoretical pool size would be, for a given fee level. We further compute what the trading costs (in basis points) would be for a 'typical' order size in each asset class.[21] We also obtain the 'free float' value of the asset to allow comparisons with the predicted equilibrium pool size.[22]

To compare our AMM transaction cost estimates to those in traditional assets, the 'implementation shortfall' (IS) is the most appropriate measure. In most markets, trading must be undertaken in discrete chunks (called 'child orders') that make up the total desired trade package ('parent order'). This is to allow an orderbook time to replenish so that the order does not need to 'walk the book' along multiple price levels of liquidity and thus pay a larger spread. However, while parent order executions are taking place, 'information leakage' can occur, which changes orderbook prices. The IS captures these costs by measuring trading costs across an entire parent order. In contrast, the design of an AMM allows for an entire parent order to be executed in one trade, as there are limited advantages to order-splitting. Indeed, this ability of AMMs to absorb orders larger than a traditional limit orderbook is one of the key advantages such a market structure can generate.

Measures of IS from traditional markets are as follows. In equities, Frazzini et al. (2018) provide an IS estimate of 9.84bps for large-cap equities and 19.93 for small-cap in a global sample of parent orders from 2006 to 2016. Van Kervel and Menkveld (2019) and Neumeier et al. (2023) also provide estimates of 8.27-13.01bps for large, U.S. equities. In FX, Melvin et al. (2020) estimates transaction costs of 1.29bps for EUR-USD (G10 currency) and

[21] The trade sizes are informed by the following papers. We take the largest credible sizes for conservatism (as it biases AMM transaction costs to be larger). For equities: Van Kervel and Menkveld (2019) and Neumeier et al. (2023) have mean parent orders of \$2.2m USD and \$1.2m USD, while Frazzini et al. (2018) has a smaller size of \$0.6m due to their sample being just one hedge fund. We take \$2.2m as an upper bound for large caps and \$0.5m as a lower-bound for small-caps. For FX, Melvin et al. (2020) estimates parent order sizes using data from FX clearer 'CLS'. While they estimate average sizes as \$1.5m, they estimate and use an upper bound of \$25m in their analysis.

[22] This is the market capitalization for equities, the M3 money supply for FX, value on issue for bonds, open interest for commodity futures and the total issuance value for tokens.

26bps for ILS-USD (exotic currency). Barbon and Ranaldo (2022) provide estimates of transaction costs on centralized exchanges in cryptocurrency markets of between 0.77-3.46bps.[23]

Table 5 shows our results for U.S. equities (mega, large and small capitalizations), forex, bonds, commodities and cryptocurrency transactions. In most cases, the optimal fixed fee is found to be between 1-5bps, in line with the typical 'quoted spread' observed in traditional limit order markets. AMMs appear to be least suitable (or most costly) in low-liquidity, high-volatility assets, such as the small-cap U.S. equities — generating minimum transactions costs of 0.5-2.8% for a modest $500,000 trade. Interestingly, AMMs appear to be 'best suited' (in the sense of creating low transactions costs) to low-volatility, high-volume assets, such as the forex, commodities and large-cap equities markets. Our estimates for the forex markets are lower than current estimates for traditional transactions costs in this market — our estimate of 1bps for EUR/USD is 22% smaller than the estimate of 1.29bps from Melvin et al. (2020), and is around 3x smaller for exotic currency pairs such as ILS/USD or BRL/USD (7.1-9bps vs 26bps). Our minimum transactions costs in mega-cap U.S. equities measure 3.6-5bps — around half of current academic estimates of IS of 8.27-13.01bps.

The minimum transactions costs for our other major asset classes fall within reasonable bounds: Commodities transactions cost 1.9-25.3bps, as compared to the 8-12bps in gold markets (Hauptfleisch et al. (2016)); U.S. Corporate Bonds cost 6.6-15.5bps, as compared to the 20-50bps reported by O'Hara and Zhou (2021); and US Treasuries cost 2bps-5bps which is around 70% larger than estimates for the most liquid securities.[24]

While it may seem implausible for AMMs to provide cheaper trading than traditional market structures, there are many key features of AMMs that could explain such a result. As discussed above, they do not incur information leakage from order-splitting. Also, while trades in AMMs incur slippage (and a fixed fee), they do not have any concept of a quoted spread or a tick size. Each AMM trade effectively trades at a modified midpoint price, analogous to midpoint dark-pool trading.[25] This 'saves' the trade paying a traditional

[23]These are bid-ask spread estimates for a $10,000 trade on Binance.

[24]There is limited research on the implementation shortfall of U.S. Treasury Bonds. Dobrev and Meldrum (2020) find bid-ask spreads are most often at minimum tick sizes of 0.78-1.56bps. Given our AMM costs are at the exchange-level (i.e. they include explicit and implicit costs) whilst our comparisons don't factor in fees, AMMs may still be competitive with current transactions costs in these markets.

[25]For further discussion on midpoint dark pool trading, see Foley and Putniņš (2016).

quoted spread. Further, for a large enough pool size, the slippage on a large trade can be infinitesimally small. As shown by Dyhrberg et al. (2019), small tick sizes lead to additional costs for traders due to issues such as undercutting. Large pools are likely to generate slippage (or spread) far smaller than would be feasible as a tick size in a limit orderbook. AMMs also do not need to recover the cost of capital and operational expenses that are factored into the 'spread' charged by a market maker or dealer such as Citadel and Virtu. These unusual features of a novel market design provide a promising future for their integration in traditional financial assets, should market regulation allow for such new innovations.

[Table 5 about here.]

7 Conclusion

Centralized limit orderbooks have held a stranglehold on trading in financial assets for centuries. Remarkably, it has recently become possible to replace them with a simple set of code. This innovation, borne out of the regulation-free cryptocurrency markets, effectively democratizes liquidity provision, allowing unsophisticated investors with no comparative advantage in market making to directly participate in what is typically a highly competitive, and specialized, profession. While many have shown that these AMMs are able to maintain prices that closely mirror those found in centralized exchanges, few have sought to understand the dynamics of how such mechanisms deal with inventory and adverse selection. We show that the architecture created by these Automated Market Makers is able to efficiently provide liquidity by charging fees to compensate "liquidity providers". While these fees are fixed for those trading with the AMM, capital flows to discrete AMM "pools" equilibrate the return on capital to appropriately compensate the liquidity providers for their adverse selection risks. We characterize and derive the components of these mechanisms, relating them to the traditional conceptual costs existent in traditional theory models of market making. providing evidence for this equilibrium which explains the viability of this new market design.

Our evidence suggests that AMM liquidity pools have difficulty reaching equilibrium, with negative staking returns in high volatility pairs (which are often smaller pools) as well as in most "token-token" pairs. To resolve these negative staking returns, some liquidity providers must endogenously depart, reducing the size of these pools and consequently increasing the proportional fee revenue. Such an equilibrating mechanism necessarily reduces the capacity of the AMM to support large levels of liquidity. In traditional markets, by contrast, market-makers can vary both the *price* of liquidity (by charging a higher spread), as well as varying the *amount* of liquidity (by varying the size of their limit orders). In this way, the amount of liquidity, or the market depth, can remain constant. This suggests that a dynamic (as opposed to a fixed) fee for liquidity providers would improve overall liquidity in the AMM. Indeed, the latest iterations of AMMs are experimenting with the availability of multiple pools for each pair of assets, where each of the pools carry different fixed fees for liquidity provision. This theoretically provides LPs with an additional mechanism for equilibration — the ability to vary the "price" of liquidity without varying the quantity.

Utilizing our theoretical model and the empirical features of the various asset pairs (volatility, volume, and the cost of capital) we are able to show that our model performs remarkably well at predicting the observed pool sizes, further validating the relevance of our theoretical contribution. Finally, we take our theoretical data to other non-crypto assets, including bonds, equities and foreign exchange. We then ask a simple question: given the observed characteristics of these asset classes, what would happen if we moved all trading to an AMM? To answer this question, we estimate both the total size of the AMM pool that would be required, as well as the transactions costs on average sized trades. Our results suggest that AMMs perform exceptionally well in low-volatility, high-volume assets such as Forex, generating transactions costs rivaling those currently seen in traditional marketplaces. High-volume medium-volatility assets such as large-cap equities perform reasonably, whilst low-volume high-volatility assets such as small-mid cap equities generate transactions costs so large that they are clearly infeasible.

Our results suggest that the innovative market design AMMs provide may be more than just a curiosity of the cryptocurrency market. Indeed, they may be optimal for several traditional financial assets, with cryptocurrencies merely serving as a regulation-free sandbox in which these innovative structures were free to evolve. As new assets such as Central Bank Digital Currencies are introduced, new market frameworks will be required to facilitate things such as cross-border trading. Our results show that new market designs such as AMMs should be considered as first-order priorities in such developments. It is remarkable that even at such an early stage, AMMs offer tangible benefits in comparison to centralized markets, which have had a few centuries head start. There are still several limitations, but rapid iterations to the design of the AMM (absent typical regulatory barriers) may quickly close this gap. While modifying the AMM is as simple as modifying the code, the economic incentives and equilibrating mechanisms implicit in such changes require careful consideration.

Appendices

A LP Holding Times

Examining the duration at which LPs provide liquidity, or 'resting times' helps inform the time horizon of our analysis.[26] Resting times are measured as the time interval between the mint and burn events of a given LP. Table 6 shows that LPs provide liquidity for 5.6 days on average, with over 50% of LPs providing liquidity for over 2 days. This shows that AMM liquidity is provided over longer time horizons than centralized exchanges, which may contribute towards resiliency. This proportion is even higher when LPs that never cancel are included in the sample, where we assume the holding time is until the end of the sample. The mean holding time increases to 35.7 days with over 50% holding more than 7.7 days.

[Table 6 about here.]

B Framework of AMM Properties and LP Returns

Consider an AMM for the asset pair USD and ETH, where x_t and y_t are the quantities of USD and ETH in the AMM at time t, respectively.[27]

Liquidity providers (LPs) stake a total of x_0 of USD and y_0 of ETH at $t = 0$ when the price of ETH in dollars is P_0. This gives the pool the constant $K = x_0 y_0$. The total USD value of the assets in the AMM at $t = 0$ is V_0, which (as we show later) is equal to $2x_0$ or twice the amount of USD deposited in the AMM, because the design of the AMM ensures that the value of both assets in the AMM is equal at the time of staking assets and at any subsequent time.

The AMM facilitates trades by liquidity demanders (LDs or "traders"). Trades with the AMM occur in discrete time at $t = 1, 2, 3, \ldots T$ where $t = T$ is the time at which the

[26] Resting times also inform the appropriate time horizon over which to assess LP profitability

[27] We use USD and ETH throughout an example of two assets staked in the AMM because this is the most popular combination of assets in AMMs. The derivations and mechanics of liquidity provision are the same for any pair. One asset is the unit of account (or numeraire), in this case USD is considered the unit of account, and the other is denoted as the "risky asset" as its price can fluctuate relative to the unit of account. While it is natural to think of USD as "stable" and ETH being a risky asset that fluctuates in price, ETH can also be considered the unit of account without affecting the derivations in this paper.

LP ceases liquidity provision and exits the game with the assets that can be withdrawn from the AMM at the time.

A trade is an asset swap in which one asset is added and the other removed in quantities such that the constant K is maintained (by design of constant product AMMs). Thus, a trade at time t that buys ETH involves removing $\Delta y_t = y_t - y_{t-1}$ from the AMM and adding in $\Delta x_t = x_t - x_{t-1}$ to the AMM such that the constant is preserved, $x_t y_t = K = x_{t-1}y_{t-1} = (x_{t-1} - \Delta x_t)(y_{t-1} + \Delta y_t)$. Conversely, a trade at time t that sells ETH by adding Δy_t to the AMM and removing Δx_t from the AMM, $x_t y_t = (x_{t-1} - \Delta x_t)(y_{t-1} + \Delta y_t)$.

Each trade also pays a fee equal to 30 basis points (bps) of the trade value, $f_t = 0.003x_t$. While fees would usually be added to the liquidity pool thereby slightly increasing the pool's K with each trade, for expositional clarity and tractability, we assume that fees accrue to a separate account, which is equivalent to assuming that LPs withdraw the fees from the AMM to maintain their staked amount of liquidity constant and rather than reinvest the fees to increase the provided liquidity. Over the time horizons in question, this simplifying assumption makes little difference to the overall results. If (as we will show later) there is an equilibrium level of liquidity (value of staked assets), then maintaining that equilibrium would require an LP to withdraw fees, consistent with the approach we use in the derivations below.

While the amount of liquidity staked in an AMM can be varied through time by LPs "minting" (staking more of both assets) and "burning" (redeeming or withdrawing assets), we assume that T is a sufficiently short period such that no mints or burns occur during the period from $t = 0$ to $t = T$. Allowing for mints or burns during this staking horizon would not qualitatively change the key results, it merely complicates the calculations.

Defining some trading volumes that will be used in later calculations, during the staking period $t = 0$ to $t = T$, the total dollar volume of LD buys is $BUYS_T = \sum_{t=1}^{t=T} max[0, \Delta x_t]$, i.e., the total amount of USD added to the AMM, and similarly the total dollar volume of LD sells is $SELLS_T = \sum_{t=1}^{t=T} max[0, -\Delta x_t]$, that is, the total amount of USD removed from the AMM. The total dollar volume of trading is $Q_t = BUYS_T + SELLS_T = \sum_{t=1}^{t=T} |\Delta x_t|$. The balanced (roundtrip) order flow is $BAL_T = 2 * min[BUYS_T, SELLS_T]$ and the order imbalance between buys and sells is $OIB_T = BUYS_T - SELLS_T = \sum_{t=1}^{t=T} \Delta x_t$, such that the total dollar volume is the sum of the balanced volume and the absolute value of the order imbalance $Q_T = BAL_T + |OIB_T|$. It is convenient to also normalize the trade volumes by the total pool value, V_0, such that the turnover in horizon T is $TURN_T = Q_T/V_0$, the balanced turnover ("relative" balanced volume) is $RBAL_T = BAL_T/V_0$, and the relative order imbalance is $ROIB_T = OIB_T/V_0$.

C AMM Properties

PROPERTY 1: Ignoring fees, the trade prices in this AMM ($ per ETH) for any quantity of ETH Δy_t are given as follows:

- Consider a trade to buy $\Delta y_t > 0$ units of ETH in exchange for paying $\Delta x_t > 0$ To retain the constant product, we must have $(x_{t-1} + \Delta x_t)(y_{t-1} - \Delta y_t) = K = x_t y_t$. Rearranging, the price of the swap (how many dollars are spent per unit ETH in the swap) is:

$$P(\Delta y_t) = \frac{\Delta x_t}{\Delta y_t} = \frac{x_{t-1}}{y_{t-1} - \Delta y_t} \tag{1}$$

- Similarly, a trade to sell $\Delta y'_t$ units of ETH and receive $\Delta x'_t$ will occur at the price (how many dollars are received by the trader per unit ETH in the swap):

$$P(\Delta y_t) = \frac{\Delta x_t}{\Delta y_t} = \frac{x_{t-1}}{y_{t-1} + \Delta y_t}$$

  which is the same as 1, just denoting the sell quantity as a negative value, $\Delta y_t = -\Delta y_t^{'} < 0$, that is, the AMM is governed by the price function (1) in which we have $\Delta y_t > 0$ for buys and $\Delta y_t < 0$ for sells of ETH.

- This price function gives the AMM's bonding curve in Figure 2.

PROPERTY 2: The 'midpoint' ETH price of the pool in USD (the price of an infinitesimally small trade that has negligible price impact) is purely a function of the two asset quantities in the pool at the time: $P_{0,MID} = x_0/y_0$ or more generally $P_{t,MID} = x_t/y_t$.

- To see this, consider an infinitesimally small swap to buy Δy_t units of ETH and pay Δx_t in the pricing function (1) above, that is, as $\Delta y_t \to 0, P_(\Delta y_t) \to \frac{x_t}{y_t}$.

PROPERTY 3: Ignoring fees, the sequence in which trades occur does not matter for the final outcome (state) of the AMM, being its pool quantities and midpoint price.

- To see this, consider a trade to buy or sell $\Delta y_t^{'}$ units of ETH ($\Delta y_t > 0$ implies a buy, and $\Delta y_t < 0$ implies a sell), followed by a trade to buy or sell $\Delta y_t''$ units of ETH ('first scenario') and compare that with the two trades occurring in the reverse sequence ($\Delta y_t''$ first and $\Delta y_t'$ second, 'second scenario'). Let the \$ paid or received in each of the trades be $\Delta x_t^{'}$ and $\Delta x_t''$ in Scenario 1 and Δy_t^{**} and Δy_t^{*} in scenario 2 (when buying ETH $\Delta x > 0$ is the amount of \$ paid by the trader, and when selling ETH $\Delta x < 0$ is the amount received).

- To retain the constant product, in Scenario 1 we must have: $(x_0 + \Delta x')(y_0 - \Delta y') = K$, giving pool quantities $x_1 = x_0 + \Delta x'$ and $y_1 = y_0 - \Delta y'$ after 1^{st} trade

  $(x_1 + \Delta x'')(y_1 - \Delta y'') = K$, giving pool quantities $x_2 = x_1 + \Delta x''$ and $y_2 = y_1 - \Delta y''$ after 2^{nd} trade

  and thus, $x_2 = x_0 + \Delta x' + \Delta x''$ and $y_2 = y_0 + \Delta y' + \Delta y''$ and

  $(x_0 + \Delta x' + \Delta x'')(y_0 - \Delta y' - \Delta y'') = K$

- Similarly, in Scenario 2 (reverse order) we must have:

  $(x_0 + \Delta x^{**})(y_0 - \Delta y'') = K$, giving pool quantities $x_1 = x_0 + \Delta x^{**}$ and $y_1 = y_0 - \Delta y''$ after 1^{st} trade

  $(x_1 + \Delta x^{*})(y_1 - \Delta y') = K$, giving pool quantities $x_2 = x_1 + \Delta x^{*}$ and $y_2 = y_1 - \Delta y'$ after 2^{nd} trade

  and thus, $x_2 = x_0 + \Delta x^{*} + \Delta x^{**}$ and $y_2 = y_0 + \Delta y' + \Delta y''$ and

  $(x_0 + \Delta x^{*} + \Delta x^{**})(y_0 - \Delta y' - \Delta y'') = K$

- Equating the two final equations in each scenario, we get:

$$\left(x_0 + \Delta x' + \Delta x''\right)\left(y_0 - \Delta y' - \Delta y''\right) = \left(x_0 + \Delta x^{*} + \Delta x^{**}\right)\left(y_0 - \Delta y' - \Delta y''\right)$$

  $\Delta x' + \Delta x'' = \Delta x^{*} + \Delta x^{**}$ implying x_2 is the same under both scenarios and so is y_2

- Therefore, under both trade sequences, the final quantities of the two assets in the AMM are the same, and so too must be the final 'midpoint' prices of the AMM, so the sequence in which trades occur does not matter for the final outcome (state) of the AMM.

Property 4: Still ignoring fees, a roundtrip trade reverts the price back to the original, reverts the pool quantities back to the original, and the trader breaks even (receives the same amount of \$ as she paid).

- To see this, consider a trade to buy Δy units of ETH in exchange for paying Δx (pay \$$\Delta x$) and then selling the same Δy units of ETH back to receive $\Delta x'$. To retain the constant product, we must have

$$\left(x_0 + \Delta x - \Delta x'\right)(y_0 - \Delta y + \Delta y) = K = x_0 y_0$$
$$\left(x_0 + \Delta x - \Delta x'\right)(y_0) = x_0 y_0$$
$$x_0 + \Delta x - \Delta x' = x_0$$
$$\Delta x - \Delta x' = 0$$
$$\Delta x = \Delta x'$$

- Therefore, the dollars paid to the pool equal the dollars received from the pool, so the pool quantities all revert back to their original, so too must the price.

Property 5: Two small trades in the same direction are equivalent to one larger trade in the same direction with quantity equal to the sum of the two smaller quantities (same end price in the AMM, same end state in terms of quantities in the AMM, and same cost to the trader). In other words, there is no advantage from trade slicing as the outcomes are the same.

- To see this, consider a trade to buy Δy units of ETH, then buy another $\Delta y'$ units of ETH (first scenario) and compare that with a trade to buy $\Delta y'' = \Delta y + \Delta y'$ units of ETH (second scenario). Let the \$ paid in each of the trades be $\Delta x, \Delta x'$, and $\Delta x''$.

- To retain the constant product, we must have

$$\left(x_0 + \Delta x + \Delta x'\right)\left(y_0 - \Delta y - \Delta y'\right) = K \text{ (Scenario 1)}$$
$$\left(x_0 + \Delta x''\right)\left(y_0 - \Delta y''\right) = K \text{ (Scenario 2)}$$

  Thus the two left hand sides must be equal,

$$\left(x_0 + \Delta x + \Delta x'\right)\left(y_0 - \Delta y - \Delta y'\right) = \left(x_0 + \Delta x''\right)\left(y_0 - \Delta y''\right)$$

and recognizing that $\Delta y'' = \Delta y + \Delta y'$, we get:

$$\Delta x'' = \Delta x + \Delta x'$$

which says that the trader would pay just as much in \$ for the one big purchase of ETH vs the two small buys that sum to the big trade's volume. Therefore, also the quantities of both assets left in the pool are the same under the two scenarios, and thus so too is the ending "midpoint" price in the pool (even though the trade prices are different).

PROPERTY 6: If the AMM receives a series of trades, only the buy/sell imbalance quantity of the series of trades is needed to work out the impact on the AMM's state (change in asset quantities in the pool and the pool's midpoint price), that is, the balanced part of volume (buy volumes equal to sell volumes) has no impact (irrespective of what combination of trades makes the balanced volume), the sequence of trades does not matter, and the AMM is "memoryless" in that the impact of a trade depends only on the current state of the AMM and not the history of trades.

- To see this, exploit Property 3 saying the sequence does not matter and Property 5 saying we can sum trade quantities together, and sum all the ETH buy quantities to the aggregate quantity $\Delta y_{BUY} = \sum_{i \in BUYS} [\Delta y_i] \geq 0$ and sum all the ETH sell quantities to the aggregate quantity $\Delta y_{SELL} = \sum_{i \in SELLS} [-\Delta y_i] \geq 0$ (note here we am defining the sell quantity as a positive value, but in the pricing function (1) we would have $\Delta y = -\Delta y_{SELL}$). Let the aggregate \$ spent by the trader on the buys be $\Delta x_{BUY} \geq 0$ and the aggregate \$ received by the trader on the sells be $\Delta x_{SELL} \geq 0$. The total trade volumes can be broken up into the order imbalance, $\Delta y_{OIB} = \Delta y_{BUY} - \Delta y_{SELL}$ and the roundtrip trades, $\Delta y_{ROUNDTRIP} = 2 \times \min [\Delta y_{BUY}, \Delta y_{SELL}]$. For example, if we have buys that sum to 2ETH and sells that sum to 5ETH then the 2 ETH buys are perfectly offset by 2ETH in sells (total roundtrip volume of 4ETH) but the additional 3ETH in sells is not offset so the order imbalance is -3.
- From Property 4 we know that the roundtrip trades have no effect on the state of the AMM, so only the imbalance matters. Thus, any sequence of any number

of trades $t = 1, \ldots N$ in any directions, can be summarized in a sufficient statistic, $\Delta y_{OIB} = \Delta y_{BUY} - \Delta y_{SELL}$, which determines the change in the AMM asset quantities and midpoint price from x_0, y_0, P_0 to: $y_N = y_0 - \Delta y_{OIB}, x_N = x_0 + \frac{x_0 \Delta y_{OIB}}{y_0 - \Delta y_{OIB}}$ (drawing on equation (1)), and

$$P_{N,MID} = \frac{x_N}{y_N} = \frac{x_0 + \frac{x_0 \Delta y_{OIB}}{y_0 - \Delta y_{OIB}}}{y_0 - \Delta y_{OIB}} = \frac{x_0}{y_0 - \Delta y_{OIB}} + \frac{x_0 \Delta y_{OIB}}{(y_0 - \Delta y_{OIB})^2}$$
$$= \frac{x_0}{y_0 - \Delta y_{OIB}} \left(1 + \frac{\Delta y_{OIB}}{y_0 - \Delta y_{OIB}}\right)$$

- The midpoint price change in the AMM, expressed as a return, is:

$$R - 1 = \frac{P_{N,MID}}{P_{0,MID}} - 1 = \left(\frac{y_0}{x_0}\right)\left(\frac{x_0}{y_0 - \Delta y_{OIB}}\right)\left(1 + \frac{\Delta y_{OIB}}{y_0 - \Delta y_{OIB}}\right)$$
$$= \frac{y_0}{y_0 - \Delta y_{OIB}}\left(1 + \frac{\Delta y_{OIB}}{y_0 - \Delta y_{OIB}}\right) - 1 \quad (16)$$

  that is, in any series of trades, the change in the price of ETH in the AMM is purely a function of the order imbalance quantity (how much more ETH was bought than sold) and the initial ETH quantity in the pool, which determines the pool depth.

PROPERTY 7: At every point in time, including when assets are staked and redeemed, assuming the 'midpoint' ETH price of the pool is approximately equal to the value of ETH (arbitrage has driven the AMM to the 'correct' price) the value of each of the two assets staked by the LP are equal, when measured in one unit of account, for example, the USD value of x_t is always equal to the USD value of y_t

- To see this, convert the quantity of asset y into an equivalent value in currency x using the midpoint price implied by the AMM at the time: $y_t P_{t,MID} = y_t \frac{x_t}{y_t} = x_t$
- It follows that an LP stakes assets in equal value weights, and those equal value weights are maintained by the AMM pricing function even as trades occur.

D Effects of fees

Fees, if left in the AMM (default option), result in an increase in the pool's assets by the amount of the fee each time a transaction occurs, effectively slowly increasing the constant

K through time due to reinvestment by the LPs. In such cases, some of the properties above will not hold exactly, but will be a reasonable approximation. For example, consider a buy followed by an equal size sell. The second trade, the sell, will have a slightly smaller price impact due to the slight increase in K from the first trade. Thus, the AMM midpoint price will not return exactly to the same price following the roundtrip trade (but it will return to a very close value), so Properties 3-6 only hold to an approximation. In practice, the approximation will be very close, irrespective of whether fees are left in the pool as reinvested liquidity or withdrawn, given fees are 0.30% of a transaction value and typical staking horizons are only a matter of a few days. Treating the fees as if they accrue to a separate account (equivalent to withdrawing the fees) maintains tractable solutions to the various derivations below. We therefore adopt this approach throughout the rest of the document, treating fees as if they accrue to an account but are not 'recycled' back into the liquidity pools.

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Figure 1: Costs of Trading in AMMs in Comparison to Traditional Markets
This figure plots estimated trading costs for theoretical AMMs implemented across various traditional assets, as a percentage of typical trading costs for each asset class. Points below 100% (dashed red line) represent assets that would be cheaper to trade using an AMM. To calculate, we determine the equilibrium AMM pool size (as in Formula 14) which is a function of weekly mean ASC and traded values calculated from a distribution of returns in 2021-2022. Pool size is estimated under 5 different Liquidity Provider fee levels: 100bps, 30bps (the current fee level of a Uniswap AMM), 5bps, 1bps and 0.1bps. Trading costs are plotted for an AMM at the optimal pool size for an average institutional sized 'parent order' (see trade sizes in Table 5).

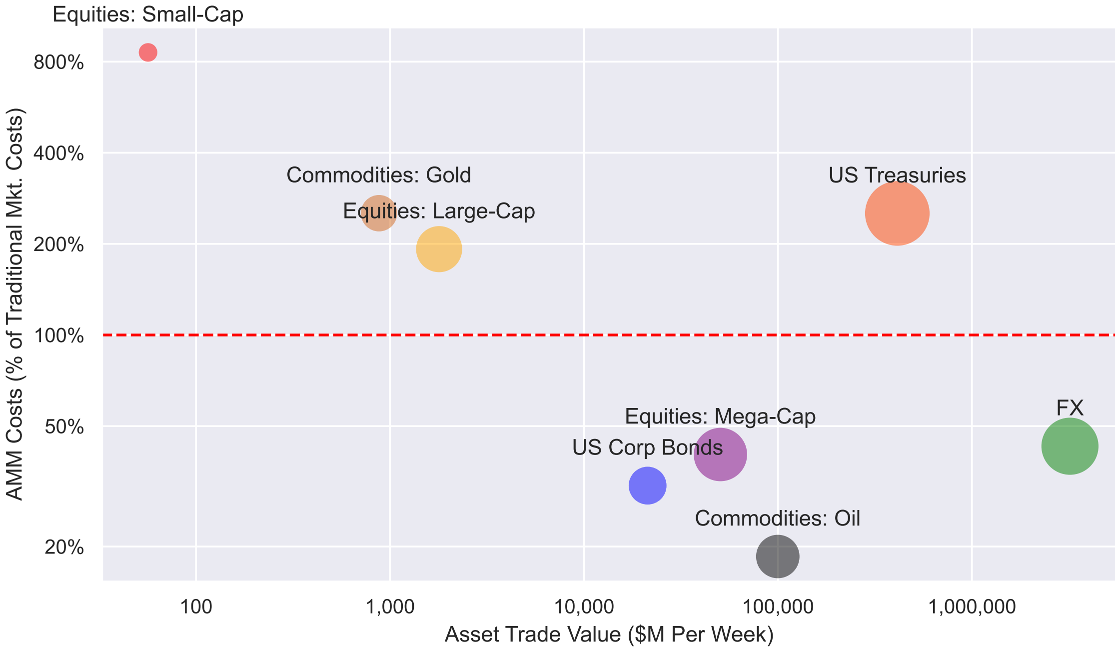

Figure 2: AMM Price Bonding Curve for Different Pool Sizes

This plot illustrates the price paid or received ($ per ETH) when buying (+) or selling (-) ETH in an AMM containing ETH and USD for different pool sizes. It assumes the initial price of the pool is $P_0 = \$1,000$. The less liquid market of $K = 1,000$ corresponds to 1 ETH and $1,000 staked, the more liquid market of $K = 4,000$ corresponds to 2 ETH and $2,000 staked and the highly liquid market of $K = 400,000$ corresponds to 20 ETH and $20,000 staked.

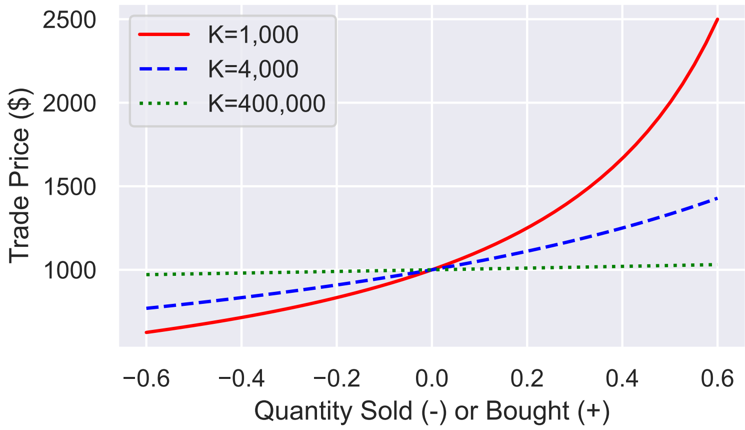

Figure 3: Return Components to Liquidity Providers (LPs) to AMMs
Panel A of this figure sets out the components of returns to Uniswap liquidity provision in response to price changes in the pool, calculated as the log of the price ratio $R+1$. The *Inventory Holding Return* is the return a liquidity provider would have obtained from holding the token assets outside of the pool, calculated as: $(R-1)/2$. *Adverse Selection Costs* (ASC) are calculated as the Total Returns less the Inventory Holding Return, measuring returns that an LP would have otherwise obtained by not providing Uniswap liquidity. This figure assumes *Fee Yield* of 30% over the time period. *Staking Return* is the net of *ASC* and *Fee Yield*. *Total Return* is the change in the value of the pool assets. Panel B plots the actual return components against the daily change in price for each pool-date in the sample. The *ASC* and *IHR* curves match those in Panel A, varying only with R on the x-axis. *Total Return* and *Staking Return* vary with R but also upwards along the y-axis when fee-yield is non-zero. This is in contrast to Panel A, where a fixed fee of 30% is assumed.

(a) Theoretical LP Return Components

(b) Actual LP Return Components

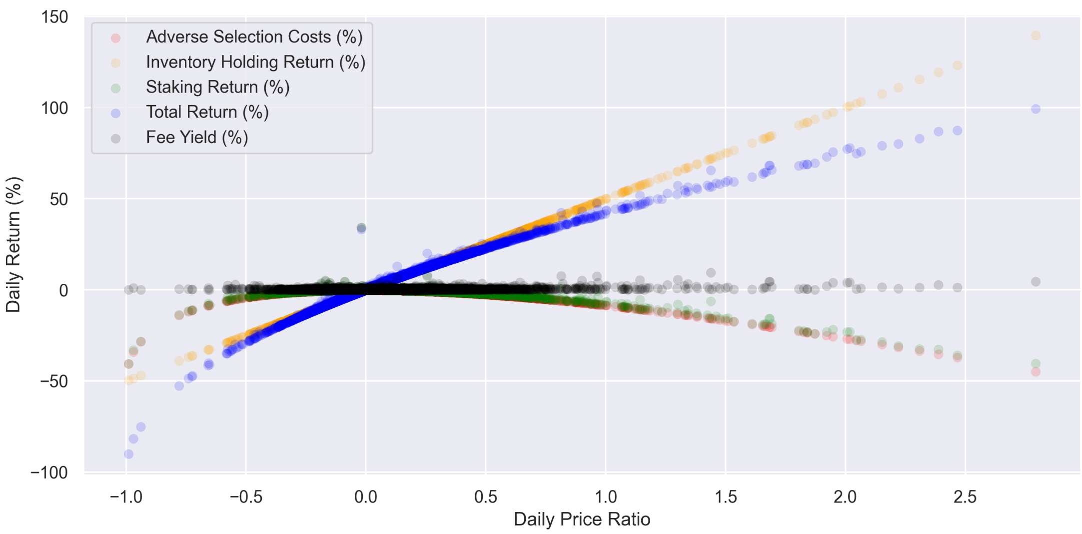

Figure 4: Histogram of Adverse Selection Costs and Staking Return for Each Pool-Date

This figure presents a histogram of ASC_T and $StakingReturn_T$ (constructed as $ASC_T + FeeYield_T$) for each pool-date in the sample. The distribution is winsorized at 1% and 99% cutoffs. Pool-date observations. 58.86% of observations of the Staking Return are above zero.

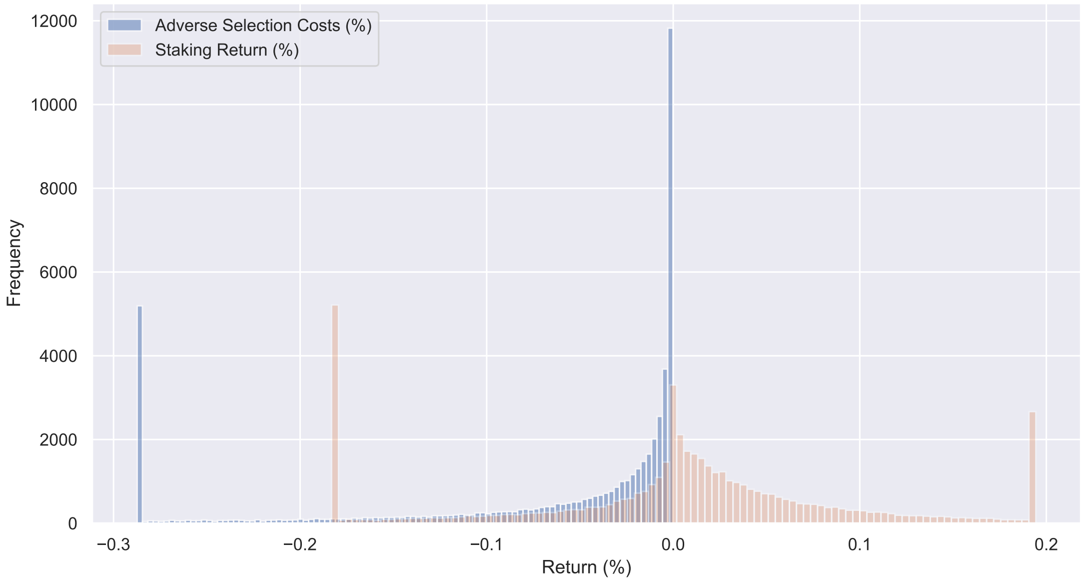

Figure 5: Adverse Selection Costs and Fees by Pool-Type, Volatility & Size
This figure sets out the *Adverse Selection Costs* (ASC) and *Staking Return* (ASC + Fee Yield) for the sample of the largest 200 pools by size. *ASC* and *Fee Yield* are calculated daily assuming a theoretical LP that invests at the start of the day and redeems at the end. Panel A reports the value-weighted average of each measure by *Pool Type*, "T-T" refers to "Token-Token" pools, "T-S" refers to "Token-Stable" pools and "S-S" refers to "Stable-Stable" pools. There are 181, 15 and 4 of each in the sample. Panel B reports the value-weighted average of each measure by *Volatility Quintile*. Quintiles are constructed by calculating the rolling 30-day standard deviation of returns for each pair over the sample, and then ranking pair means into quintiles.

(a) by Pool-Type **(b)** by Volatility **(c)** by Pool Size

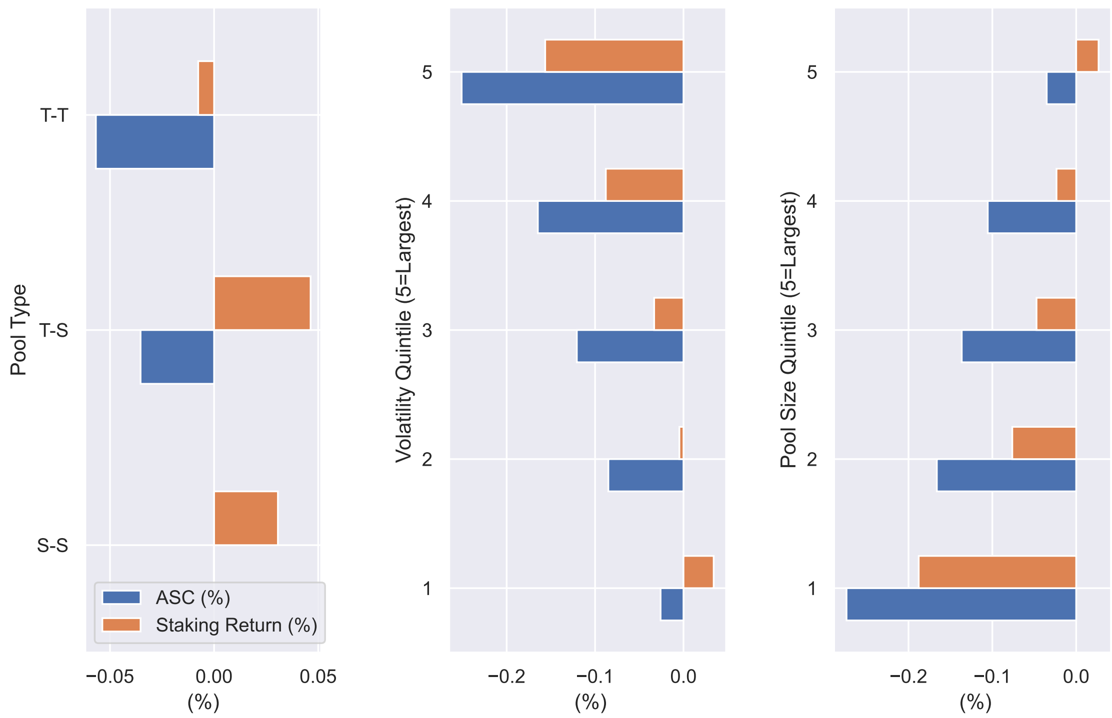

Figure 6: Correlogram of Current Period Flows Against 30 Day Lag/Lead Fee Yield
Pearson correlation coefficients are constructed between current period flows and the respective lag/lead Fee Yield first within a given pool. Means are then constructed across all pools.

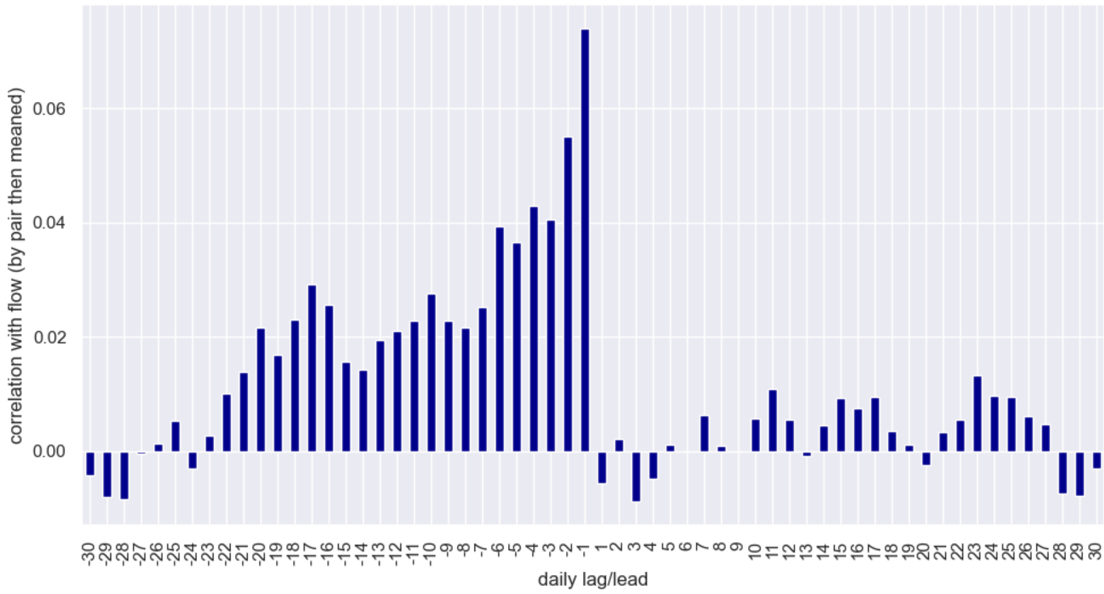

Figure 7: Theoretical Equilibrium Pool Sizes in Relation to Volume
This figure plots the pool sizes of theoretical AMM pools on the x-axis and the trading volumes they receive on the y-axis. In this figure, volume endogenously relates to pool size wherein traders are attracted to larger (more liquid) pools, but at a diminishing marginal rate. In this figure, the other factors that determine the size of pools are considered exogenous: adverse selection costs (ASC) and the cost of capital (R_c). These factors form the pool "constants" and are illustrated as three separate curves from largest to smallest, with $V*$ denoting the equilibrium pool-sizes on each of these curves.

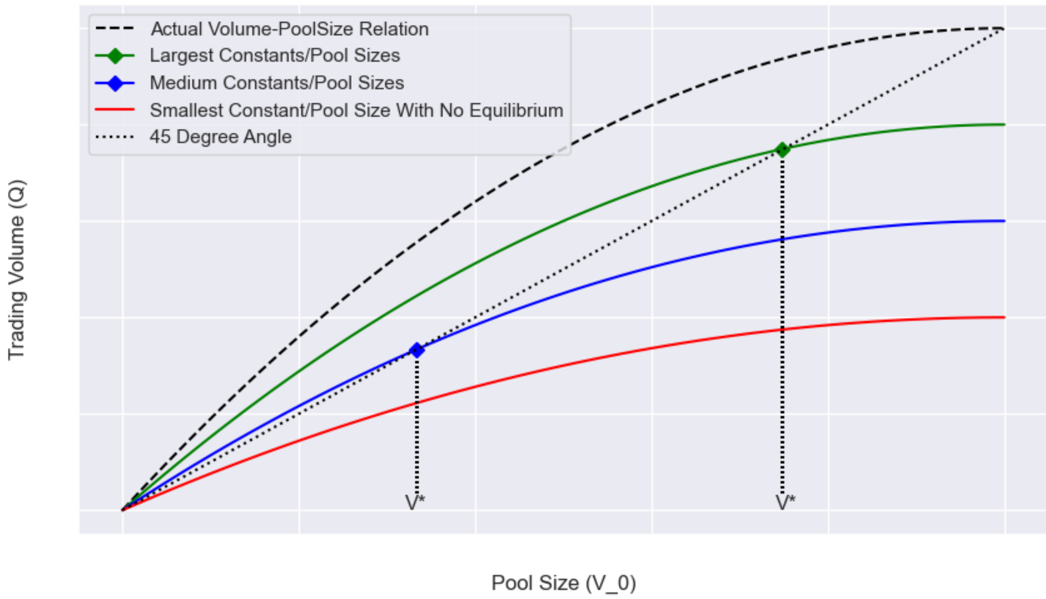

Table 1: Descriptive Statistics of Pools and Returns

This table sets out descriptive statistics of pool-date observations for the sample of the largest 200 pools. There are 51,901 pool-date observations in the sample period from 1st of June 2020 to the 19th of May 2021. Pool-size weighted means are reported alongside means, due to the highly positively skewed distribution of pool size in the sample. We report 1%, Median and 99% distribution cutoffs for each variable as well as % of pool-date observations that are above 0. *Pool Size*, V_0, is calculated as the value of the two pool assets in USD at the end of the day. *Swap Value*, Q_t, is the total daily gross swap value in USD. Conversions to USD use 'Coinmarketcap' closing prices. *Flow* is the total net mint and burn amounts divided by V_0. *Total Return* is calculated as in Formula 7, *IHR* (Inventory Holding Return) as in 4, *ASC* (Adverse Selection Cost) as in 5, *Fee Yield* as in 6, *Staking Return* is the net sum of Fee Yield and ASC, *RBAL* (proportion of balanced order flow) is $BAL_T = 2*min[BUYS_T, SELLS_T]$ divided by the pool value V_0. *ROIB* (proportion of order imbalance) is calculated as $ROIB_T = OIB_T/V_0$ where OIB_T is the cumulative net order flow $\sum_{t=1}^{t=T} \Delta x_t$. *Return* is $\frac{P_t}{P_0} - 1$, *Turnover* is Q_t/V_0. *Volatility* is calculated as the standard deviation of returns at the rolling horizon of 30 days prior to each pool-date observation. *Toxic Order flow* is OIB_T on Q_t and *Uninformed Order flow* is BAL_T on Q_t.

	Mean	Wtd. Mean	Stddev	1%	50%	99%	% >0
Pool Size ($ Millions)	8.56		46.46	0.01	0.71	251.42	100.00
Swap Value ($ Millions)	1.90	44.31	16.36	0.00	0.11	43.59	100.00
Flow (% of Pool Size)	3.09	-0.08	343.62	-23.69	0.00	33.84	29.62
Total Return (%)	0.06	0.16	7.67	-11.92	-0.37	19.16	43.42
IHR (%)	0.11	0.14	6.76	-11.34	-0.42	20.48	42.55
ASC (%)	-0.17	-0.04	1.71	-2.05	-0.02	0.00	0.00
Fee Yield (%)	0.12	0.06	4.87	0.00	0.05	0.77	100.00
Staking Return (%)	-0.05	0.02	5.11	-1.74	0.01	0.48	59.76
RBAL (%)	35.56	21.28	1,146.55	0.00	13.81	253.52	100.00
ROIB (%)	-1.85	0.08	477.95	-6.00	-0.18	9.81	43.45
Return (%)	0.22	0.27	13.51	-22.68	-0.83	40.96	42.55
Turnover (%)	39.61	22.24	1,621.89	0.16	15.31	260.34	100.00
Volatility (Std Deviations)	10.18	5.40	5.61	0.28	9.20	33.46	100.00
Toxic Order flow (%)	16.92	8.35	23.73	0.09	7.24	100.00	100.00
Uninformed Order flow (%)	83.08	91.65	23.73	0.00	92.76	99.91	100.00

Table 2: Time Series Regression of Flows on LP Return Components
This table reports regressions of pool flows on lagged return components. *Flow* is the daily net mint and burn value divided by the pool value, in percent. The following are in basis points: *Total Return* as in Formula 7, *Fee Yield* as in Formula 6, *ASC* as in 5, *IHR* as in 4 and *StakingReturn* is the net of *ASC* and *Fee Yield*. *'SushiAttack'* equals 1 for period AMM rival Sushiswap provided LP incentives. *'UniIncentive'* equals 1 for time period of Uniswap LP Incentives. Pool-date observations with absolute value of *Flows* above the 99th percentile are removed.

	(1) Flow	(2) Flow	(3) Flow	(4) Flow	(5) Flow
SushiAttack	3.72	3.63	3.66	3.71	3.67
	(1.49)	(1.46)	(1.47)	(1.49)	(1.47)
UniIncentive	0.10	0.25	0.09	0.09	0.08
	(0.22)	(0.55)	(0.19)	(0.21)	(0.19)
$Flow_{t-1}$	0.04***	0.03***	0.04***	0.04***	0.04***
	(3.67)	(3.39)	(3.76)	(3.70)	(3.76)
$Flow_{t-2}$	0.03***	0.03***	0.03***	0.03***	0.03***
	(3.48)	(3.33)	(3.54)	(3.50)	(3.53)
$Flow_{t-3}$	0.02***	0.02***	0.02***	0.02***	0.02***
	(3.15)	(3.04)	(3.15)	(3.16)	(3.15)
$TotalReturn_{t-1}$	0.01***				
	(3.13)				
$TotalReturn_{t-2}$	0.01***				
	(3.79)				
$TotalReturn_{t-3}$	0.01*				
	(1.82)				
$FeeYield_{t-1}$		1.06***			
		(2.62)			
$FeeYield_{t-2}$		0.36			
		(1.44)			
$FeeYield_{t-3}$		0.28			
		(1.44)			
ASC_{t-1}			-0.03*		
			(-1.69)		
ASC_{t-2}			-0.02		
			(-1.39)		
ASC_{t-3}			-0.01		
			(-0.55)		
IHR_{t-1}				0.01***	
				(2.86)	
IHR_{t-2}				0.01***	
				(3.37)	
IHR_{t-3}				0.01	
				(1.53)	
$StakingReturn_{t-1}$					-0.69
					(-0.62)
$StakingReturn_{t-2}$					-1.28
					(-0.97)
$StakingReturn_{t-3}$					0.59
					(0.56)
Constant	-0.00	-0.16***	-0.01	-0.00	-0.00
	(-0.09)	(-4.54)	(-0.45)	(-0.17)	(-0.10)
Obs.	50,019	50,019	50,019	50,019	50,019
R^2 (%)	0.406	0.667	0.363	0.396	0.351
Number of pairid	200	200	200	200	200
Pool FE:	Yes	Yes	Yes	Yes	Yes
Date FE:	Yes	Yes	Yes	Yes	Yes

SE clustered by pool and date
T-statistics in parentheses. *p<.05; **p<.01; ***p<.001

Table 3: Time Series Regression of LP Return Components on Flows

This table reports regressions of pool return components on lagged flows. *Flow* is calculated as the daily net of mint and burn values divided by the pool value, expressed as a percentage. The return components are expressed in basis points. See previous Table 2 for variable descriptions. Pool-date observations with absolute value of *Flows* above the 99th percentile are removed.

	(1) Total Return	(2) Fee Yield	(3) ASC	(4) IHR	(5) StakingReturn
SushiAttack	-116.63**	1.95**	-3.18*	-114.83**	-0.00
	(-2.24)	(2.01)	(-1.67)	(-2.25)	(-0.19)
UniIncentive	-25.38*	-4.98***	1.80***	-17.60	-0.07***
	(-1.93)	(-7.69)	(5.59)	(-1.33)	(-15.87)
$Flow_{t-1}$	-1.26**	-0.06**	0.14	-1.47**	0.00
	(-2.16)	(-2.17)	(0.93)	(-2.18)	(1.24)
$Flow_{t-2}$	0.53	-0.02	-0.08*	0.57	-0.00
	(1.21)	(-0.76)	(-1.90)	(1.27)	(-1.25)
$Flow_{t-3}$	0.00	0.01	0.14	-0.17	0.00**
	(0.01)	(0.31)	(1.48)	(-0.34)	(1.97)
$TotalReturn_{t-1}$	0.00				
	(0.04)				
$TotalReturn_{t-2}$	-0.01				
	(-1.26)				
$TotalReturn_{t-3}$	0.01				
	(0.95)				
$FeeYield_{t-1}$		0.30***			
		(3.45)			
$FeeYield_{t-2}$		0.09			
		(1.62)			
$FeeYield_{t-3}$		0.08**			
		(2.12)			
ASC_{t-1}			0.04**		
			(2.19)		
ASC_{t-2}			0.01		
			(0.99)		
ASC_{t-3}			0.00		
			(0.54)		
IHR_{t-1}				0.01	
				(0.44)	
IHR_{t-2}				-0.01	
				(-1.02)	
IHR_{t-3}				0.01	
				(1.05)	
$StakingReturn_{t-1}$					0.02*
					(1.73)
$StakingReturn_{t-2}$					0.01
					(0.77)
$StakingReturn_{t-3}$					0.00
					(0.26)
Constant	1.74	4.80***	-15.71***	9.13***	-0.07***
	(0.67)	(9.01)	(-17.05)	(3.10)	(-9.56)
Obs.	50,019	50,019	50,019	50,019	50,019
R^2 (%)	0.036	14.781	0.168	0.036	0.075
Number of pairid	200	200	200	200	200
Pool FE:	Yes	Yes	Yes	Yes	Yes
Date FE:	Yes	Yes	Yes	Yes	Yes

SE clustered by pool and date

T-statistics in parentheses. *p<.05; **p<.01; ***p<.001

Table 4: Regression of Pool Size on Predicted Pool Size
This table regresses the last observation of pool size (V_0) in a given month on the predicted equilibrium pool-size (V^*) which is calculated from the mean of daily observations that determine it (ASC, Q and R_C) over that month. The sample size is pool-month dates of the top 200 pools in our sample, excluding the early period of Uniswap V2 and the incentive period up to and including 21st of November, 2020.

	(1) $log(V_0)$	(2) $log(V_0)$	(3) $log(V_0)$	(4) $log(V_0)$
$log(V_0^*)$	0.85***	0.55***	0.85***	0.51***
	(44.74)	(7.61)	(44.47)	97.50)
Constant	1.66***	3.68***	1.66***	3.92***
	(10.98)	(7.41)	(12.63)	(8.75)
No. Observations	997	997	997	997
R-Squared (Within)	0.31	0.45	0.31	0.44
R-Squared (Between)	0.94	0.81	0.94	0.78
R-Squared (Overall)	0.92	0.80	0.92	0.77
Estimator	Pooled	Panel	Panel	Panel
Fixed Effects	None	Pool	Month	Pool-Month

Standard errors clustered by pool and month.
T-Statistics in parentheses. *p<.05; **p<.01; ***p<.001

Table 5: Calibration of Trading Costs of AMMs in Traditional Asset Classes
This table presents the theoretical trading costs in basis points if an AMM were implemented in various traditional asset classes at the optimal pool size: 'AMM Optimal Cost'. First, the equilibrium pool size of the asset is determined (as in Formula 14) by computing weekly mean ASC from a distribution of returns and traded values over the period 2021-2022. This is estimated under 5 different fee conditions: 100bps, 30bps (the current fee level of a Uniswap AMM), 5bps, 1bps and 0.1bps. Trading costs are then calculated for the average institutional sized 'parent order' (this is reported under 'input trade size' in USD) for each fee condition. 'AMM Optimal Cost' reports the minimum of these trading costs and 'AMM Optimal Pool Size' and 'AMM Optimal LP Fee' report the corresponding pool-size and LP fee at this minimum. Traditional market trading costs for each asset class are reported in 'Trad. Cost Comp.' for comparison. The size of the traditional markets are also reported for each instrument in 'Trad. Free Float Comp.' for comparison with the optimal pool size.

Category	Ticker	AMM Optimal Cost (bps)	Trad. Cost Comp. (bps)	AMM Optimal Pool Size (USD $M)	Trad. Free Float Comp. (USD $M)	AMM Optimal LP Fee (bps)	Input Trade Size ($)
Equities Meg.Cap	AAPL	3.6	8.27-13.01	17,141	2,260,000	1	2,200,000
Equities Meg.Cap	MSFT	5.0	8.27-13.01	11,076	1,810,000	1	2,200,000
Equities Lrg.Cap	JNJ	15.3	8.27-13.01	4,280	423,555	5	2,200,000
Equities Lrg.Cap	KO	21.7	8.27-13.01	2,635	262,240	5	2,200,000
Equities Lrg.Cap	XOM	24.4	8.27-13.01	2,266	467,640	5	2,200,000
Equities Sml.Cap	TREE	52.1	19.93	451	493	30	500,000
Equities Sml.Cap	POWW	127.9	19.93	101	287	30	500,000
Equities Sml.Cap	BOOM	148.7	19.93	205	412	100	500,000
Equities Sml.Cap	LAW	242.3	19.93	69	458	100	500,000
Equities Sml.Cap	CATO	284.1	19.93	53	201	100	500,000
FX G10	EUR_USD	1.0	1.29	578,190	16,562,578	0.1	25,000,000
FX Exotic	BRL_USD	7.1	26	238,049	1,275,614	5	25,000,000
FX Exotic	ILS_USD	9.0	26	123,528	176,321	5	25,000,000
FX Exotic	THB_USD	9.0	26	124,019	748,830	5	25,000,000
US Corp Bonds	MSFT	6.6	20-50	11,724	53,847	5	959,000
US Corp Bonds	AAPL	9.1	20-50	4,631	110,572	5	959,000
US Corp Bonds	JNJ	13.2	20-50	2,334	30,102	5	959,000
US Corp Bonds	XOM	11.2	20-50	3,093	41,203	5	959,000
US Corp Bonds	KO	15.5	20-50	1,833	36,700	5	959,000
US Treasury Bonds	US10	2.0	0.78-1.56	243,500	24,108,577	1	12,000,000
US Treasury Bonds	US30	5.0	0.78-1.56	60,699	24,108,577	1	12,000,000
US Treasury Notes	US02	1.9	0.78-1.56	262,673	24,108,577	1	12,000,000
Commodities Oil	LCOc1	1.9	8-12	20,230	206,175	1	864,000
Commodities Gold	GCc1	25.3	8-12	2,082	38,400	5	2,111,625
Token T-S	USDC_WETH	6.8	0.77	34	235	5	3,058
Token T-T	WBTC_WETH	10.0	1.03	11	110	5	2,676
Token S-S	USDC_USDT	20.0	3.47	9	441	5	6,697

Table 6: Liquidity Provider Resting Times

This table presents liquidity provider resting times, measured as the time interval between the mint (submission) and burn (cancellation) events of a discrete LP address. Only burn events which remove more than 90% of an LP's resting liquidity are considered. Statistics are then reported by LP for the top 200 pools by value in the sample.

	Mint and	Resting Time (Days)	
	Burn Counts	w/ Never Cancels	No Never Cancels
Mean	5.6	35.67	20.32
Median	2.0	7.65	5.29
Std. Dev	55.4	61.57	34.51
p10	1.0	0.21	0.16
p25	2.0	1.24	0.99
p75	5.0	39.48	24.49
p90	10.0	111.84	60.20