Draft: **Alternatives to Einstein's General Relativity Theory**

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Referee Report

Dear editor. I advance the final conclusions of this report.

Final comments. The wording of the review follows a pattern that, on the one hand, can be considered typical of a work of excellence given its magnitude, and on the other, it reveals the high degree of understanding by the author of the treated themes as well as its currentness. In conclusion, the text represents an excellent contribution to the discussion of relevant topics in the field of gravitation and astrophysics. That is why I am pleased to recommend its publication. There are a few small typos identified. The physical-mathematical formulation of the text is very accurate and sophisticated. The text is well written and does not need any further revision by this referee.

Please find in the following the complete report.

Focus of the review. The author focuses his attention in this review on alternative theories to general relativity, restricting however its content on classical models/theories, comprising rather algebraic extensions of the standard theory. Proposed quantum mechanical extensions were barely discussed on this review. More specifically, the author addresses the topics: algebraic extensions of the space-time coordinates and metric extensions, the pseudo-complex General Relativity, the canonical transformation formalism, the MOND model and Quintessence, and attempts to quantize the standard theory. In the introduction of the review, the author focuses his attention to algebraic extension to GR, i.e., to theories mapping real coordinates, as used in GR, to another type with different algebraic properties, as for example complex coordinates, pseudo-complex, quaternions and hyper- quaternions. The historical review part of the text has a large formal scope, although restricted to the upon cited algebraic extensions of GR, highlighting important features and issues as for instance the elimination of the singularity at the center of a black hole, or how to eliminate the event horizon, recurrent themes in current research. The final part of the text, about the pseudo-complex General Relativity, is the main core of the review. The author expresses here some of the main motivations for the pc-GR theory, from the physical and the philosophical points of view, the most important being the avoidance of the event horizon. In this part of the text the author presents a rigorous technical approach to pc-GR as well as an important series of theoretical predictions. In chapter 6, predictions about a gravastar (gravitational vacuum star) and about Bose-Einstein condensation of scalar fields (boson stars) are also briefly discussed. Gravastars were included, because also there dark energy is responsible for the stability of a star, though the observation of gravitational waves seem to exclude this theory. Chapter 7 is devoted to canonical transformations of General Relativity. Chapter 8 is devoted to the MOND, one of few extensions of gravitational theories with many observational predictions and the conformal gravity approach. The MOND theory was presented briefly, emphasizing that it avoids the need of dark matter through modifying Newton's law of gravity. In chapter 9, the author discusses Scalar Field Gravity. And in chapter 10, the author mention theories not considered in this review which might be important in quantizing Gravity. In chapter 11, the conclusions are presented.

Emphasis of the review. The emphasis of the review is on the following topics: 1. A historical overview of attempts to extend GR, for different reasons, as to avoid an event-horizon or a

singularity at the center of a black hole. 2. One principle emphasis is on how to make observational predictions.

Theoretical models discussed or briefly mentioned. The theoretical models discussed or cited in this review are: General Relativity; String Theory; Quantum Loop Gravity; MOND; Newton Theory; Unimodular Gravity; M. Born Theory (for unifying Gravity and Quantum Mechanics); Pauli-Villars Regularization Scheme; A. Einstein EGR Theory (for unification of Electrodynamics with Gravitation); Maxwell equations; G. Kunstatter and R. Yates Hermitian Gravity; Algebraic Extensions to GR; Non-Symmetric Gravitational Theory (NGT); E.R. Caianiello retook of M. Born proposal; Anti-Hermitian Gravity; pc-complexified GR; pseudo-complex General Relativity (pc-GR); Kähler manifold; P. Finsler metric; Gravstar Model; Bosons stars; Covariant Canonical Gauge Theory of Gravitation (CCGG); ΛCDM; Conformal Gravity Approach; Scalar Field Gravity; Cold Dark Matter;

The data being used or cited. Data from: Event Horizon Telescope Collaboration (EHT), Large Hadron Collider (LHC), HiRes collaboration, Auger collaboration,

Main results and predictions. Below in the final part of the text, the main results and predictions are outlined.

General Comments. One of the most fundamental dilemmas of alternative classical theories to gravity is that they need at the same time to explain the large-scale structure of the universe, and still reproduce the successful predictions of general relativity, without taking into account the puzzling effects of dark matter and dark energy. Additionally, classical approaches shall seek to overcome the limitations of general relativity on the strong gravity and high curvature regime. Quantum theories, in particular, face additional dilemmas, that is, to make their predictions on quantum scales compatible with the successful predictions at the local domain of general relativity, and at the same time, encapsulate the unification of quantum mechanics with general relativity. Black holes and neutron stars collisions, the detection of gravitational waves, the observation of fast pulsar systems among other notable recent observations, let astronomers to probe gravity on a new scale and with higher precision, making feasible, as never before, the development of more consistent alternative models for general relativity and gravity.

In this context, the topics chosen by the author are very current and timely. Even the restrictions to classical approaches is adequate, since it is necessary to seek to exhaust the classical alternatives, although it is also opportune to seek for other non-classical alternatives to overcome the limitations of the standard classical theory of gravity. For this reason, it seems clear to this referee, that the author was careful on his decision do not relegated to address, despite in a briefer mention, --- otherwise it would lead too far the content of this review ---, attempts to quantize the theory. Further reading of the text reveals that the author's fundamental motivation was to provide a broad presentation of possible extensions of the theory of General Relativity (GR). Safeguarding that GR is well tested, the author rightly emphasizes that there are still properties and observations which cannot be accounted for by the standard theory, leaving this way room for improvements. The basic argument stressed by the author for discussing possible extensions of GR in the vicinity of very strong gravitational fields, --- the low resolution of the EHT observations ---, is very strong since it opens new windows for theoretical extensions and interpretations. In particular, the view of the author that phenomenological models are essential in this process seems perfectly adequate and current in these times of a new emergent astronomy.

In synthesis, by focusing his attention primarily on classical alternative models to GR, the author demonstrates the due concern in exhausting the alternatives of this type of formulation in the search to overcome the limitations of the standard formulation. This concern is adequate and its historical revisiting of primordial models can bring new interpretations regarding data and calculations performed in the past, thus enriching the arsenal of perspectives to approach current problems. One example in this contextualization is the proposal by M. Born that indeed still deserves to be recognized as an important approach line for the unification of QM with Gravity.

More Specific Topics. Gravitational-wave astronomy has enabled extraordinary tests of the nature of gravity and about the properties of black holes. The (several) reasons to search for extensions to GR presented by the author are fundamental for a better understanding of the evolutionary universe: for example, the search for alternatives to the intriguing presence of dark energy and its role in acceleration of the universe; the singularity at the center of a black hole and the information paradox; classical theories which potentially are able to provide some insight into the quantization of gravity, among others. The introduction of the review sounds canonical but with some daring points, especially when the author proposes to discuss the issue of accessibility of information in the vicinity of a black hole, a relevant question that needs a certain amount of boldness combined with audacity to elucidate such a controversial and at the same time intriguing point.

The reading of the historical section sounds very elucidative. In particular, as an example, with respect to “revisiting” powerful formulations and conceptions a little forgotten in time as the proposal by M. Born and the A. Einstein’s unsuccessful attempted formulation to unify GR with Electrodynamics. Both formulations have simple but very clear descriptions of their neuralgic aspects. Moreover, in this part of the review, the author provides a broad view about technical aspects associated with the different means of extending GR algebraically, either by redefining the metric or the coordinates. A special mention to the pseudo-complex extension part of the text, field in which the author is one of the most outstanding and respected researchers. In particular the author emphasizes a strong argument in favor of pc-GR, as the only algebraic extension of the coordinates, which does not suffer from ghost and/or tachyon solutions. In the following chapter the author delves deeper into the theme of pc-GR and presents, in addition to a thorough synthesis of its formal aspects, an important series of theoretical predictions.

The pseudo-complex General Relativity. Chapter 5 is the main core of the review. The author expresses here some of the main motivation for this theory, from physical and philosophical points of view. The main motivations for the theory are convincingly argued, in special the one related to the information paradox, which seems to be unchallenged. Although the other arguments have their importance very well exposed in the text. One of the most relevant aspects of pc-GR is that it provides observable predictions in the near future, in other words, it is verifiable.

Results and predictions. There are many results and predictions that would be worth mentioning. In order not to lengthen this report too much, we limit this description to some of them, emphasizing its most relevant aspects.

For instance, the results and predictions of the theory about the dependence of the vacuum fluctuations as a function on the radial distance implies that mass not only deforms space-time but also changes the vacuum properties in its vicinity, which might be helpful for the search of a quantized version of gravity.

Another important result: in the pc-GR, a relevant mechanism for the exclusion of an event-horizon is presented on basis of the metric for the pc-Kerr case.

Some important additional consequences of the pc-GR are also highlighted. As for instance: a best comprehension of the structure of light emission of accretion discs around a black hole; the predictions about the orbital frequency of a point particle in a circular orbit around a Kerr-black hole and the position of the innermost stable circular orbit; the predicted light emission from an accretion disc; simulation of the accretion disc with a resolution of 5μas, while resolution reported by the EHT is of 24μas; the investigation of Quasi-Periodic Objects - quasi-periodic brightenings observed in accretion discs and an alternative explanation for the corresponding discrepancies observed in the orbiting distances of binary systems; vacuum-fluctuations dependence of the mass density; pc-GR applied to cosmological models to find an alternative explanation to dark matter and its consequences to the acceleration of the universe; the proposal to search for highly redshifted optical partners of observed gravitational wave events – observation of coincidence of a gravitational wave event of two black holes with an optical partner at large high redshifts would be a sign in favor of the corresponding predictions of pc-GR.

In chapter 6, predictions about a gravastar (gravitational vacuum star) and about its distinction to a black hole are presented, despite skeptical arguments on its realization based on the observation of gravitational waves; Bose-Einstein condensation of scalar fields (boson stars) are also mentioned.

Chapter 7 is devoted to canonical transformation descriptions of General Relativity. Again, a rigorous theoretical approach is presented. The author describes in this chapter important topics: canonical gauge transformations and applications to the acceleration of the universe; the author concludes that the acceleration of the universe as described within the CCGG is in good agreement with the data.

In Chapter 8 the MOND - Modified Newtonian Dynamics is reviewed. Some virtues of the MOND are highlighted as for instance the links of the luminosity to the velocity dispersion of the stars in elliptical galaxies.

In chapter 9, the author discusses Scalar Field Gravity. And in chapter 10, the author mention theories not considered in this review which might be important in quantizing Gravity. Conclusions are presented in chapter 11.

The review has an appendix and a list of bibliographical citations very useful.

Final comments. The wording of the review follows a pattern that, on the one hand, can be considered typical of a work of excellence given its magnitude, on the other hand, it reveals the high degree of understanding by the author of the treated themes as well as its currentness. In conclusion, the text represents an excellent contribution to the discussion of relevant topics in the field of gravitation and astrophysics. That is why I am pleased to recommend its publication. There are a few small typos identified. The physical-mathematical formulation of the text is very accurate and sophisticated. The text is well written and does not need any further revision by this referee.

Below please find the indication of typos.

Typo in the title: Alternatives (Aternatives), page 1.

Typos: replacing test by test is not better in the first line of the introduction? Line 5: on the introduction: there is "the" and "a" before black hole. Line 8: "…there is no arguments…" should be replaced by "…there are no arguments…" Line 44: after Also there is a comma. Line six on page 6: should be a comma after "on" (before "of") – "…later on of importance…."

In page 6: "(M. Born uses the signature (+ - - -)" A parenthesis is missing. On line 134, replace "blochs" by blocks. On the last line on page 6: "…momentum part.),". I think this dot inside the parenthesis should be eliminated. Line 149, page 7: "right-hand" instead of "right hand".

Page 11, table 1: "pseudo-compelex" should be replaced by "pseudo-complex". Line 241: replace "an skew-symmetric" by "a skew-symmetric". Line 244: "matters" instead of "matter". Line 364: (detailed calculations are presented chapter 2 of [26]). "in" is missing before "chapter 2". Equation (50) is divided in two pages. Line 415: "to avoid de…" should replace "de" by "the".

Line 548: "The Riemann-, Ricci-tensor and Riemann scalar", change "Riemann scalar" to "Riemann-scalar". Below equation (68): "co-variant" should be replaced by "covariant".

Line 572: "(67), with (68)…" – something missing here? Line 577: something missing here? Line 591: inidcates; Line 593: "…the density of the vacuum fluctuations increase…" should be "increases".

Line 598: It is expressed: "The finding is, that the collapse may be halted when the collapse is slow enough." Question: The internal pressure does not play the most important role to halt the collapse? Slow enough means that the different stages of internal compression / decompression of the star occur in continuous processes of weakly unstable quasi-equilibrium states?

Line 600: must be "background metric ". Line 747: correct the word – "Quasi-Periodic Objetcs".