

# 1 Correctness and Performance Charts

This version of the document is dated 2023-06-13.

The following charts show the correctness of many of the algorithms in “**Bernoulli Factory Algorithms**<sup>1</sup>” and show their performance in terms of the number of bits they use on average. For each algorithm, and for each of 100  $\lambda$  values evenly spaced from 0.0001 to 0.9999:

- 500 runs of the algorithm were done. Then...
- The number of bits used by the runs were averaged, as were the return values of the runs (since the return value is either 0 or 1, the mean return value will be in the interval [0, 1]). The number of bits used included the number of bits used to produce each coin flip, assuming the coin flip procedure for  $\lambda$  was generated using the `Bernoulli#coin()` method in *bernoulli.py*, which produces that probability in an optimal or near-optimal way.

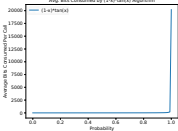
For each algorithm, if a single run was detected to use more than 5000 bits for a given  $\lambda$ , the entire data point for that  $\lambda$  was suppressed in the charts below.

In addition, for each algorithm, a chart appears showing the minimum number of input coin flips that any fast Bernoulli factory algorithm will need on average to simulate the given function, based on work by Mendo (2019)[^1]. Note that some functions require a growing number of coin flips as  $\lambda$  approaches 0 or 1. Note that for the 2014, 2016, and 2019 algorithms—

- an  $\epsilon$  of  $1 - (x + c) * 1.001$  was used (or 0.0001 if  $\epsilon$  would be greater than 1), and
- an  $\epsilon$  of  $(x - c) * 0.9995$  for the subtraction variants.

Points with invalid  $\epsilon$  values were suppressed. For the low-mean algorithm, an  $m$  of  $\max(0.49999, xc1.02)$  was used unless noted otherwise.

## 1.1 The Charts

Algorithm	Simulated Mean	Average Bits Consumed	Coin Flips
(1-x)*tan(x)			

$$(1-x)/\cos(x)$$

$$(1/3)*x/(1+(1/3)*x)$$

$$(2/3)*x/(1+(2/3)*x)$$

$$(3/2)*x/(1+(3/2)*x)$$

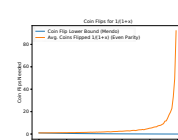
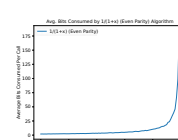
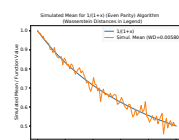
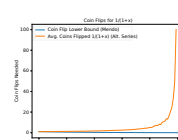
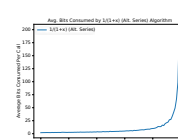
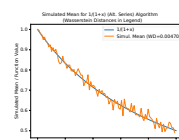
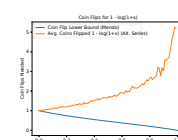
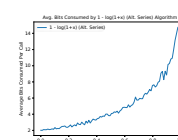
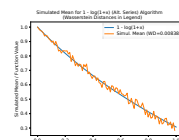
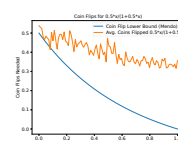
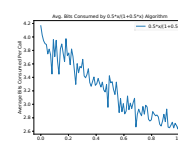
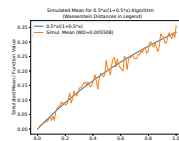
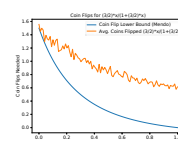
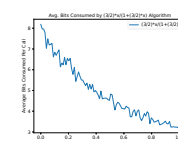
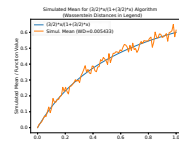
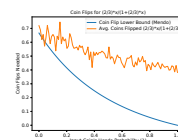
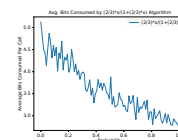
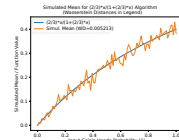
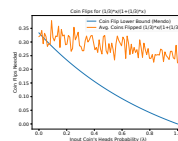
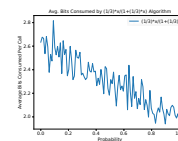
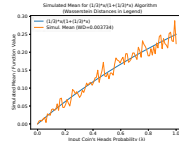
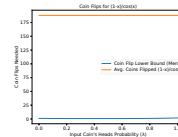
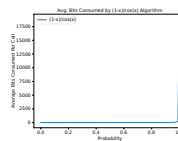
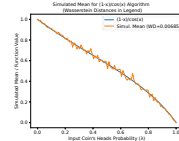
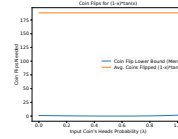
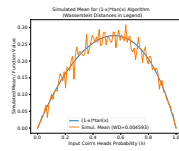
$$0.5*x/(1+0.5*x)$$

$$1 - \ln(1+x) \text{ (Alt. Series)}$$

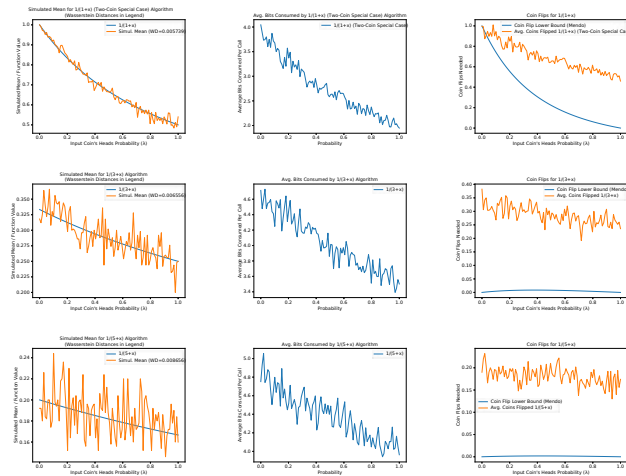
$$1/(1+x) \text{ (Alt. Series)}$$

$$1/(1+x) \text{ (Even Parity)}$$

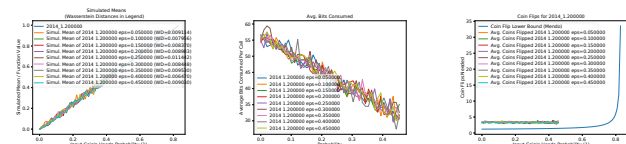
$$1/(1+x) \text{ (Two-Coin Special)}$$



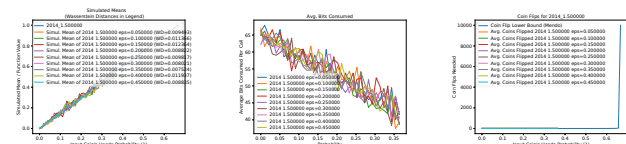
Case)


$$1/(3+x)$$
$$1/(5+x)$$

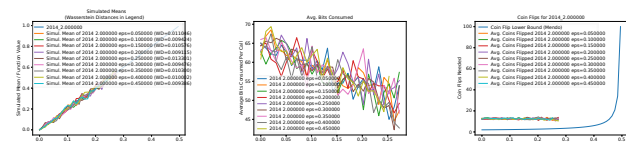
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eps=0.050000



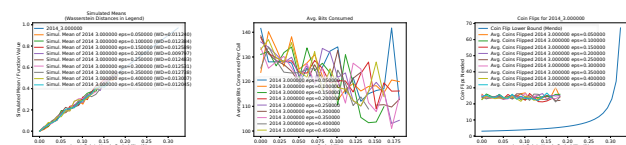
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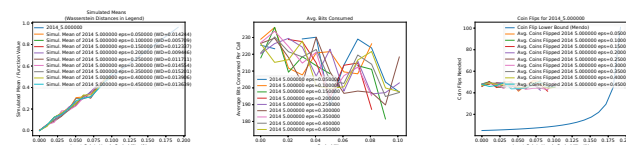
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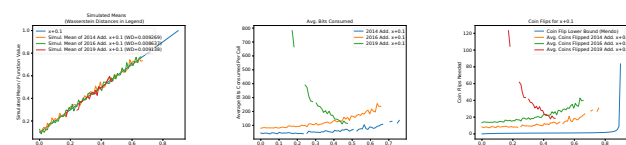
```
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```
2014 5.000000
eps=0.050000
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2014 Add. x+0.1



2014 Add. x+0.2

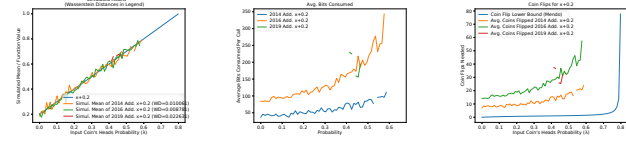


Figure 1 consists of three subplots comparing the proposed model with the baseline model. The left subplot shows the 'Simulated Mean of Log10(Reads)' on the y-axis (ranging from 0 to 1.5) versus 'Log10(Reads)' on the x-axis (ranging from 0 to 1). It includes a blue line for 'Simulated Reads' and a green line for 'Simulated Mean of 2018-ABIS', with a linear regression line. The middle subplot shows 'Avg. Bq. Consumed' on the y-axis (ranging from 0 to 300) versus 'Log10(Reads)' on the x-axis (ranging from 0 to 1). It includes a blue line for '2018-ABIS' and an orange line for '2018-ABIS + v=0.3'. The right subplot shows 'Cost (Bq. Consumed)' on the y-axis (ranging from 0 to 25) versus 'Log10(Reads)' on the x-axis (ranging from 0 to 1). It includes a blue line for 'Cost (Bq. Consumed)' and an orange line for 'Cost (Bq. Consumed) + v=0.3'.

Figure 1 consists of three subplots comparing the proposed method with the baseline. The left subplot, titled "Unconstrained Results", shows the "Simulated Mean Error (kPa)" on the y-axis (ranging from -0.4 to 1.6) versus the "Avg. Size (mm)" on the x-axis (ranging from 0.0 to 0.6). It includes four data series: "2014 ASD, n=0.5" (blue line with circles), "2014 ASD, n=1" (orange line with circles), "2016 ASD, n=0.5" (green line with circles), and "2016 ASD, n=1" (red line with circles). The middle subplot, titled "Avg. Size Constraint", shows the "Simulated Mean Error (kPa)" on the y-axis (ranging from -0.4 to 1.6) versus the "Avg. Size (mm)" on the x-axis (ranging from 0.0 to 0.6). It includes four data series: "2014 ASD, n=0.5" (blue line with circles), "2014 ASD, n=1" (orange line with circles), "2016 ASD, n=0.5" (green line with circles), and "2016 ASD, n=1" (red line with circles). The right subplot, titled "Cost Ratio for n=0.5", shows the "Cost Ratio (kPa)" on the y-axis (ranging from 0 to 100) versus the "Avg. Size (mm)" on the x-axis (ranging from 0.0 to 0.6). It includes four data series: "Cost Ratio, 2014 ASD, n=0.5" (blue line with circles), "Cost Ratio, 2014 ASD, n=1" (orange line with circles), "Cost Ratio, 2016 ASD, n=0.5" (green line with circles), and "Cost Ratio, 2016 ASD, n=1" (red line with circles). The plots show that the proposed method (n=0.5) generally results in lower mean errors and lower cost ratios compared to the baseline (n=1) across the range of average sizes.

Figure 1 consists of four subplots comparing the proposed model with the baseline model. The top-left plot, titled 'Structural Features', shows 'Days of Infection (Log Scale)' on the y-axis (0.0 to 1.0) versus 'Days of Infection (Baseline)' on the x-axis (0.0 to 1.0). It includes data for 'Proposed' (red line with circles), 'Baseline' (blue line with circles), and 'Proposed with Structural Features' (green line with circles). The top-right plot, titled 'Avg. Size Compared', shows 'Avg. Size Compared' on the y-axis (0 to 400) versus 'Days of Infection' on the x-axis (0 to 30). It includes data for 'Proposed' (red line with circles), 'Baseline' (blue line with circles), 'Proposed with Structural Features' (green line with circles), 'Proposed with Structural Features and Temporal Features' (orange line with circles), 'Proposed with Structural Features and Spatial Features' (purple line with circles), and 'Proposed with Structural Features and Temporal and Spatial Features' (brown line with circles). The bottom-left plot, titled 'Avg. Size Compared', shows 'Avg. Size Compared' on the y-axis (0 to 400) versus 'Days of Infection' on the x-axis (0 to 30). It includes data for 'Proposed' (red line with circles), 'Baseline' (blue line with circles), 'Proposed with Structural Features' (green line with circles), 'Proposed with Structural Features and Temporal Features' (orange line with circles), 'Proposed with Structural Features and Spatial Features' (purple line with circles), and 'Proposed with Structural Features and Temporal and Spatial Features' (brown line with circles). The bottom-right plot, titled 'Cost Ratio Results', shows 'Cost Ratio Results' on the y-axis (0 to 140) versus 'Days of Infection' on the x-axis (0 to 30). It includes data for 'Proposed' (red line with circles), 'Baseline' (blue line with circles), 'Proposed with Structural Features' (green line with circles), 'Proposed with Structural Features and Temporal Features' (orange line with circles), 'Proposed with Structural Features and Spatial Features' (purple line with circles), and 'Proposed with Structural Features and Temporal and Spatial Features' (brown line with circles).

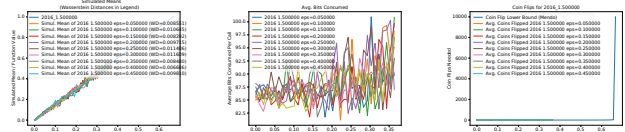
Figure 1 consists of three subplots. The left subplot, titled 'Structural Similarity', shows the relationship between 'Structural Similarity (SSIM)' on the y-axis and 'Average Structural Similarity (Avg. SSIM)' on the x-axis. It includes a legend with the following entries:  $\gamma=1.5$ , 'Exact Mean of 2014-Late v1.5 (ME=0.07532)', 'Exact Mean of 2014-Late v1.5 (ME=0.07532)', 'Exact Mean of 2014-Late v1.5 (ME=0.11344)', 'Exact Mean of 2014-Late v1.5 (ME=0.11344)', and 'Exact Mean of 2014-Late v1.5 (ME=0.11344)'. The right subplot, titled 'Average Edge Count', shows 'Average Edge Count' on the y-axis versus 'Average Edge Count' on the x-axis. The legend includes: '2014-Late v1.5', '2014-Late v1.5', '2014-Late v1.5', '2014-Late v1.5', '2014-Late v1.5', and '2014-Late v1.5'. The left subplot also includes a legend for 'Count per Method' with entries: 'Count per Method: 0.0099', 'Count per Method: 0.0099', 'Count per Method: 0.0099', 'Count per Method: 0.0099', 'Count per Method: 0.0099', and 'Count per Method: 0.0099'.

Figure 1 consists of three subplots. Subplot (a) is a scatter plot titled 'Structural Weaves' showing 'Observed Mean (log2 scale)' on the y-axis versus 'Estimated Mean (log2 scale)' on the x-axis. It includes data points and regression lines for several datasets: 'Equal. Means of 2544 coils,  $p=2$  (0.0042512)', 'Equal. Means of 2544 coils,  $p=2$  (0.0042512)', 'Equal. Means of 2544 coils,  $p=2$  (0.0042512)', 'Equal. Means of 2544 coils,  $p=2$  (0.0042512)', 'Equal. Means of 2544 coils,  $p=2$  (0.0042512)', 'Equal. Means of 2544 coils,  $p=2$  (0.0042512)', 'Equal. Means of 2544 coils,  $p=2$  (0.0042512)', 'Equal. Means of 2544 coils,  $p=2$  (0.0042512)', 'Equal. Means of 2544 coils,  $p=2$  (0.0042512)', 'Equal. Means of 2544 coils,  $p=2$  (0.0042512)'. Subplot (b) is a line graph titled 'Avg. Bits Consumed' showing 'Avg. Bits Consumed' on the y-axis versus 'Iteration' on the x-axis. It compares the proposed model (blue line) with other models (orange, green, red, purple lines). Subplot (c) is a line graph titled 'Cost Bits for  $p=2$ ' showing 'Cost Bits' on the y-axis versus 'Iteration' on the x-axis. It compares the proposed model (blue line) with other models (orange, green, red, purple lines).

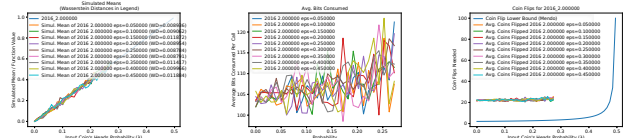
[illegible][illegible]

Figure 1 consists of three subplots. Subplot (a) is an ROC curve for the proposed model, showing the True Positive Rate (TPR) on the y-axis (0.0 to 1.0) versus the False Positive Rate (FPR) on the x-axis (0.0 to 1.0). The curve is a solid blue line, indicating good performance. Subplot (b) is an average ROC curve for the proposed model, showing the Average True Positive Rate (ATPR) on the y-axis (0.0 to 1.0) versus the Average False Positive Rate (AFPR) on the x-axis (0.0 to 1.0). The curve is a solid blue line, indicating good performance. Subplot (c) is a confusion matrix for the proposed model, showing the True Positive Rate (TPR) on the y-axis (0.0 to 1.0) versus the False Positive Rate (FPR) on the x-axis (0.0 to 1.0). The matrix is a 2x2 grid with values: TP=0.85, FP=0.15, FN=0.15, and TN=0.85.

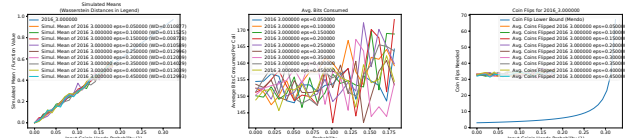
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eps=0.050000



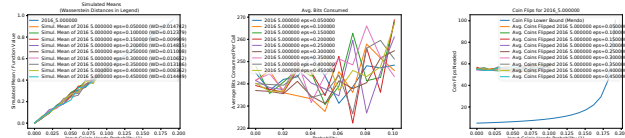
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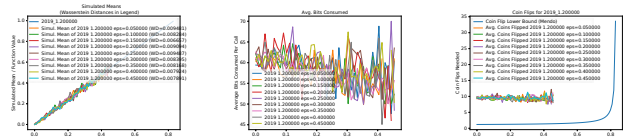
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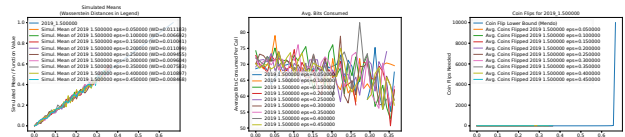
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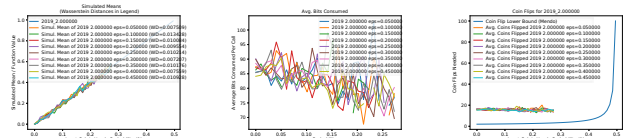
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2019 1.200000
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```



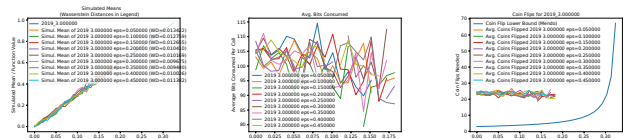
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eps=0.050000



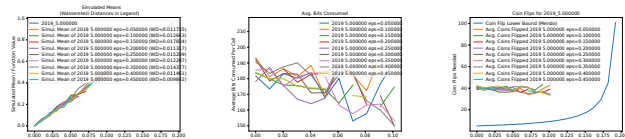
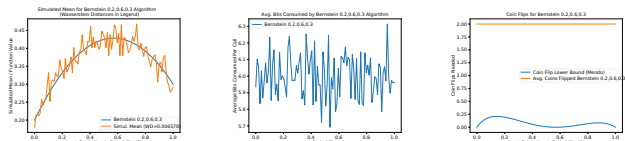
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eps=0.050000
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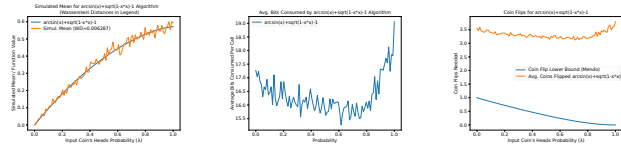
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eps=0.050000
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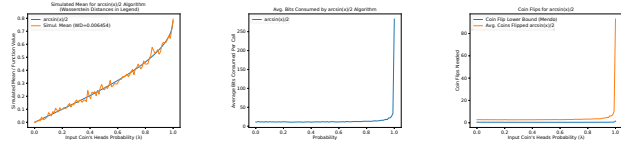
```
2019 5.000000
eps=0.050000
```

Bernstein  
0.2,0.6,0.3

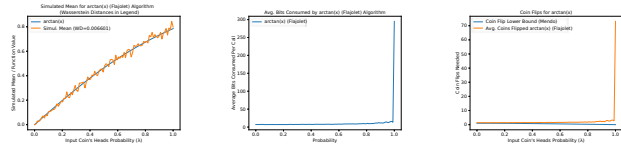
$\arcsin(x) + \sqrt{1-x^2} - 1$



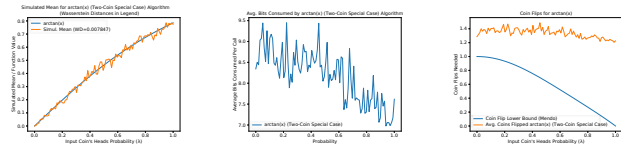
$\arcsin(x)/2$



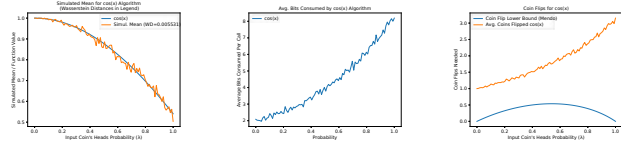
$\arctan(x)$   
(Flajolet)



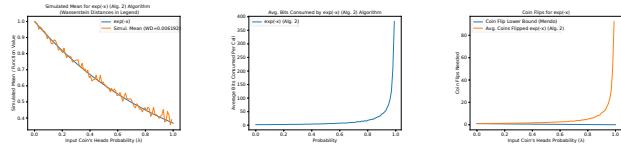
$\arctan(x)$  (Two-Coin Special Case)



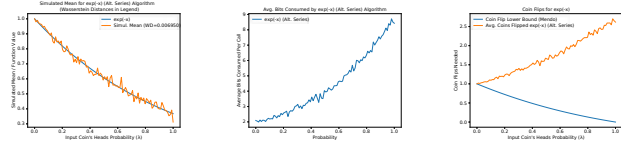
$\cos(x)$



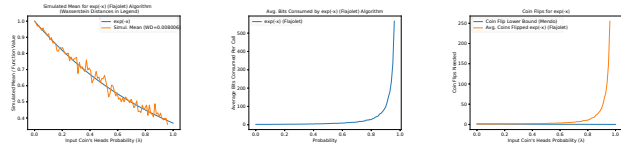
$\exp(-x)$  (Alg. 2)



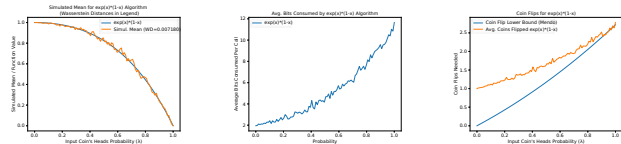
$\exp(-x)$  (Alt. Series)



$\exp(-x)$  (Flajolet)



$\exp(x) * (1-x)$



$\ln(1+x)$  (Flajolet)

$\ln(1+x)$  (Two-Coin Special Case)

$\text{pow}(x, 1/3)$

$\text{pow}(x, 2/1)$

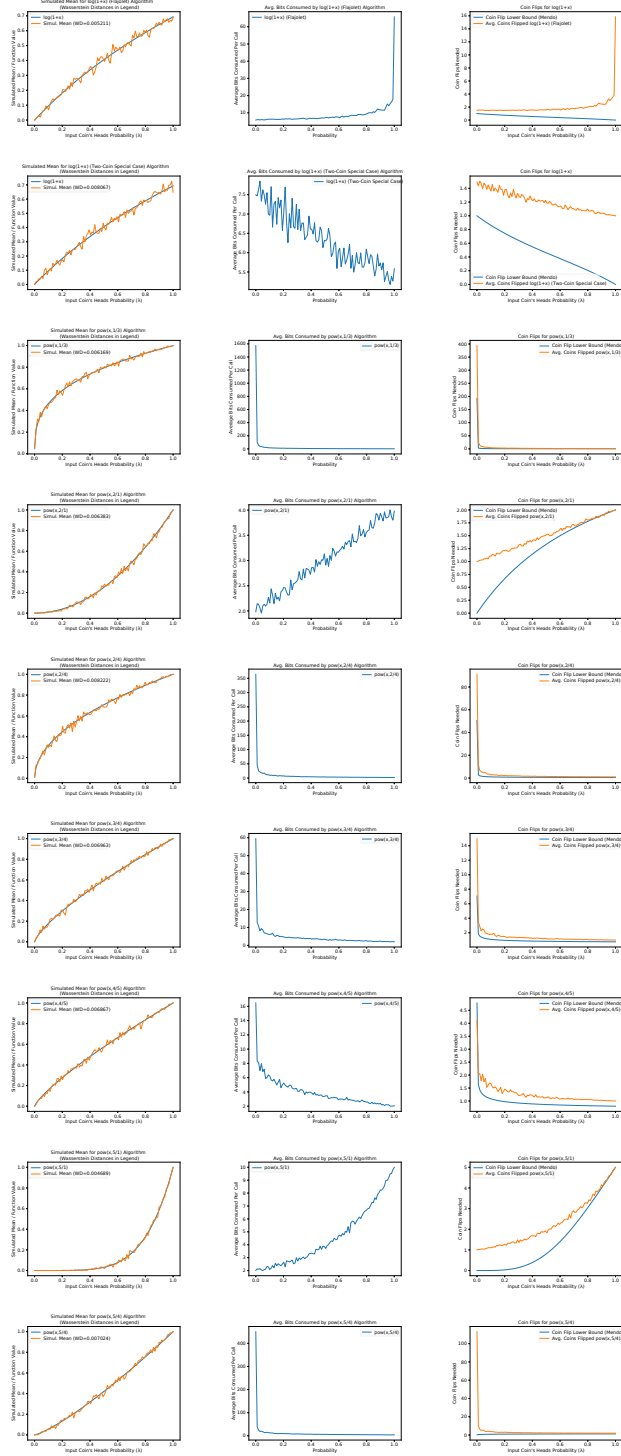
$\text{pow}(x, 2/4)$

$\text{pow}(x, 3/4)$

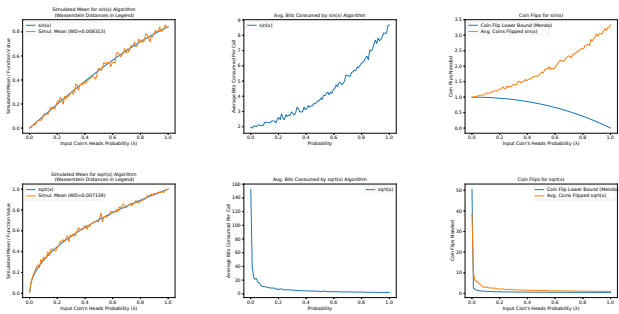
$\text{pow}(x, 4/5)$

$\text{pow}(x, 5/1)$

$\text{pow}(x, 5/4)$



sin(x)



sqrt(x)

1. <https://peteroupc.github.io/bernoulli.md>