

Correctness and Performance Charts

This version of the document is dated 2022-11-07.

The following charts show the correctness of many of the algorithms in "**Bernoulli Factory Algorithms**" and show their performance in terms of the number of bits they use on average. For each algorithm, and for each of 100 λ values evenly spaced from 0.0001 to 0.9999:

- 500 runs of the algorithm were done. Then...
- The number of bits used by the runs were averaged, as were the return values of the runs (since the return value is either 0 or 1, the mean return value will be in the interval $[0, 1]$). The number of bits used included the number of bits used to produce each coin flip, assuming the coin flip procedure for λ was generated using the `Bernoulli#coin()` method in *bernoulli.py*, which produces that probability in an optimal or near-optimal way.

For each algorithm, if a single run was detected to use more than 5000 bits for a given λ , the entire data point for that λ was suppressed in the charts below.

In addition, for each algorithm, a chart appears showing the minimum number of input coin flips that any fast Bernoulli factory algorithm will need on average to simulate the given function, based on work by Mendo (2019)[¹]. Note that some functions require a growing number of coin flips as λ approaches 0 or 1. Note that for the 2014, 2016, and 2019 algorithms—

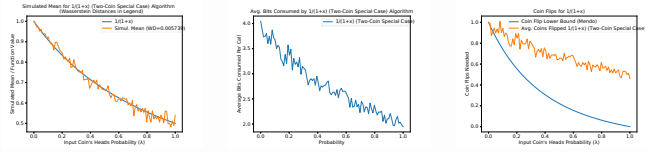
- an ϵ of $1 - (x + c) * 1.001$ was used (or 0.0001 if ϵ would be greater than 1), and
- an ϵ of $(x - c) * 0.9995$ for the subtraction variants.

Points with invalid ϵ values were suppressed. For the low-mean algorithm, an m of $\max(0.49999, x*c*1.02)$ was used unless noted otherwise.

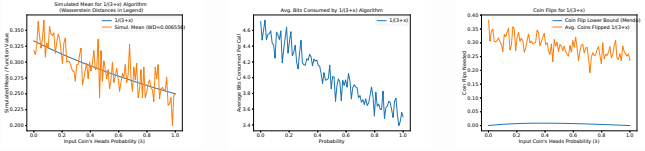
0.1 The Charts

Algorithm	Simulated Mean	Average Bits Consumed	Coin Flips
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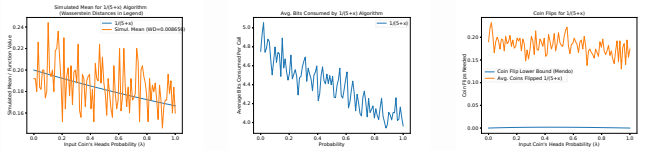
1/(1+x) (Two-Coin Special Case)



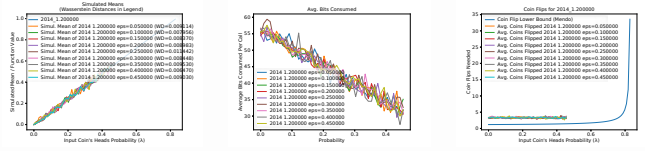
1/(3+x)



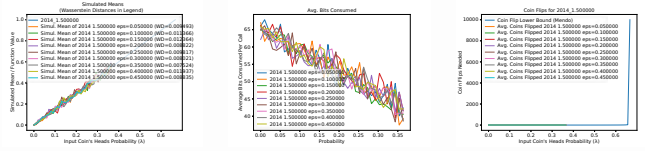
1/(5+x)



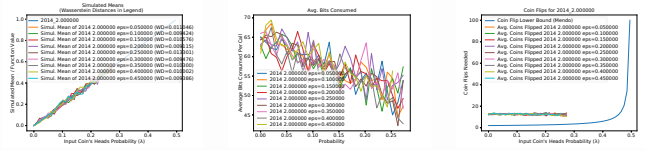
2014 1.200000
eps=0.050000



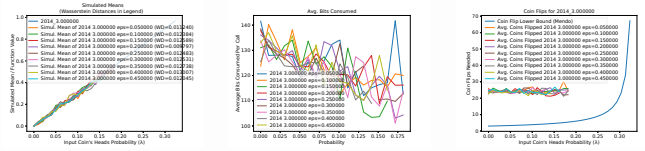
2014 1.500000
eps=0.050000



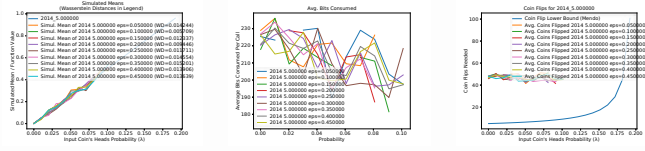
2014 2.000000
eps=0.050000



2014 3.000000
eps=0.050000



2014 5.000000
eps=0.050000



2014 Add. x+0.1

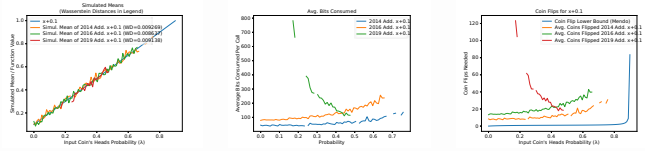


Figure 1 consists of three subplots. The left plot, titled 'Dispersed Values (Observations minus Regression)', shows True Values (mm) on the y-axis (0 to 1.5) versus Predicted Values (mm) on the x-axis (0.5 to 1.5). It includes data for 2014-2016 (green dots) and 2017-2018 (red dots), with a blue line representing the regression. The middle plot, titled 'Avg. RMSE (mm) vs. Normalized Error', shows Average RMSE (mm) on the y-axis (0 to 10) versus Normalized Error on the x-axis (0.5 to 1.5). It includes data for 2014-2016 (green line) and 2017-2018 (red line). The right plot, titled 'RMSE (mm) vs. Normalized Error', shows RMSE (mm) on the y-axis (0 to 80) versus Normalized Error on the x-axis (0.5 to 1.5). It includes data for 2014-2016 (green line) and 2017-2018 (red line).

[illegible]

Figure 1 consists of three subplots labeled (a), (b), and (c), each showing performance metrics for the proposed model across different input class probabilities (0.0 to 0.6).

(a) ROC Curve for v1.5: The y-axis is 'Real AUC (Proposed Model)' and the x-axis is 'Input Class Probabilities (x)'. The plot shows a diagonal line from (0,0) to (1,1) with several colored lines representing different models. The legend lists:

- Green line: Proposed Model for v1.5
- Red line: Proposed Model for v1.5 (v1.5)
- Blue line: Proposed Model for v1.5 (v1.5)
- Orange line: Proposed Model for v1.5 (v1.5)
- Purple line: Proposed Model for v1.5 (v1.5)
- Light Blue line: Proposed Model for v1.5 (v1.5)
- Dark Blue line: Proposed Model for v1.5 (v1.5)
- Light Green line: Proposed Model for v1.5 (v1.5)
- Dark Green line: Proposed Model for v1.5 (v1.5)
- Light Orange line: Proposed Model for v1.5 (v1.5)
- Dark Orange line: Proposed Model for v1.5 (v1.5)
- Light Purple line: Proposed Model for v1.5 (v1.5)
- Dark Purple line: Proposed Model for v1.5 (v1.5)
- Light Blue line: Proposed Model for v1.5 (v1.5)
- Dark Blue line: Proposed Model for v1.5 (v1.5)
- Light Green line: Proposed Model for v1.5 (v1.5)
- Dark Green line: Proposed Model for v1.5 (v1.5)
- Light Orange line: Proposed Model for v1.5 (v1.5)
- Dark Orange line: Proposed Model for v1.5 (v1.5)
- Light Purple line: Proposed Model for v1.5 (v1.5)
- Dark Purple line: Proposed Model for v1.5 (v1.5)

(b) Average ROC Curve for v1.5: The y-axis is 'Average ROC Curve for v1.5' and the x-axis is 'Input Class Probabilities (x)'. The plot shows a diagonal line from (0,0) to (1,1) with several colored lines representing different models. The legend lists:

- Green line: Proposed Model for v1.5
- Red line: Proposed Model for v1.5 (v1.5)
- Blue line: Proposed Model for v1.5 (v1.5)
- Orange line: Proposed Model for v1.5 (v1.5)
- Purple line: Proposed Model for v1.5 (v1.5)
- Light Blue line: Proposed Model for v1.5 (v1.5)
- Dark Blue line: Proposed Model for v1.5 (v1.5)
- Light Green line: Proposed Model for v1.5 (v1.5)
- Dark Green line: Proposed Model for v1.5 (v1.5)
- Light Orange line: Proposed Model for v1.5 (v1.5)
- Dark Orange line: Proposed Model for v1.5 (v1.5)
- Light Purple line: Proposed Model for v1.5 (v1.5)
- Dark Purple line: Proposed Model for v1.5 (v1.5)
- Light Blue line: Proposed Model for v1.5 (v1.5)
- Dark Blue line: Proposed Model for v1.5 (v1.5)
- Light Green line: Proposed Model for v1.5 (v1.5)
- Dark Green line: Proposed Model for v1.5 (v1.5)
- Light Orange line: Proposed Model for v1.5 (v1.5)
- Dark Orange line: Proposed Model for v1.5 (v1.5)
- Light Purple line: Proposed Model for v1.5 (v1.5)
- Dark Purple line: Proposed Model for v1.5 (v1.5)

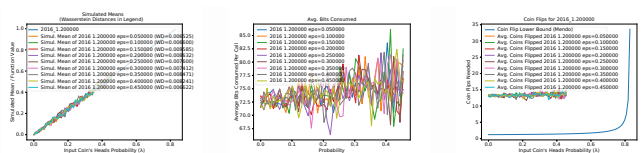
(c) Cost-Peak Ratio for v1.5: The y-axis is 'Cost-Peak Ratio' and the x-axis is 'Input Class Probabilities (x)'. The plot shows a diagonal line from (0,0) to (1,1) with several colored lines representing different models. The legend lists:

- Green line: Proposed Model for v1.5
- Red line: Proposed Model for v1.5 (v1.5)
- Blue line: Proposed Model for v1.5 (v1.5)
- Orange line: Proposed Model for v1.5 (v1.5)
- Purple line: Proposed Model for v1.5 (v1.5)
- Light Blue line: Proposed Model for v1.5 (v1.5)
- Dark Blue line: Proposed Model for v1.5 (v1.5)
- Light Green line: Proposed Model for v1.5 (v1.5)
- Dark Green line: Proposed Model for v1.5 (v1.5)
- Light Orange line: Proposed Model for v1.5 (v1.5)
- Dark Orange line: Proposed Model for v1.5 (v1.5)
- Light Purple line: Proposed Model for v1.5 (v1.5)
- Dark Purple line: Proposed Model for v1.5 (v1.5)
- Light Blue line: Proposed Model for v1.5 (v1.5)
- Dark Blue line: Proposed Model for v1.5 (v1.5)
- Light Green line: Proposed Model for v1.5 (v1.5)
- Dark Green line: Proposed Model for v1.5 (v1.5)
- Light Orange line: Proposed Model for v1.5 (v1.5)
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- Dark Purple line: Proposed Model for v1.5 (v1.5)

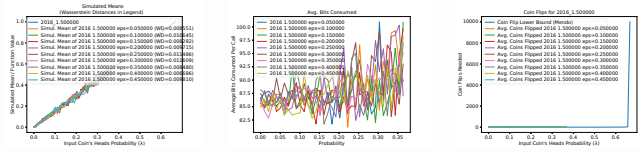
Figure 1 consists of three subplots. The left subplot, titled 'Unobserved Means', shows the relationship between input and output means for various datasets. The x-axis is 'Input Cost's Mean (Cost Ratio X1)' and the y-axis is 'Output Cost's Mean (Cost Ratio X2)'. A red line represents the proposed model's performance, which closely follows the diagonal line. The middle subplot, titled 'Avg. RMSE Concentrated', shows the average root mean square error (RMSE) for different datasets. The x-axis is 'Probability' and the y-axis is 'Average RMSE (Cost Ratio X2)'. A red line represents the proposed model's performance, which is significantly lower than the other datasets. The right subplot, titled 'Cost Ratio for X2 vs X1', shows the cost ratio for different datasets. The x-axis is 'Input Cost's Mean (Cost Ratio X1)' and the y-axis is 'Output Cost's Mean (Cost Ratio X2)'. A red line represents the proposed model's performance, which is significantly lower than the other datasets.

[illegible][illegible][illegible]

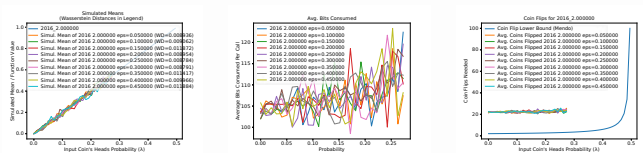
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2016 1.200000
eps=0.050000
```



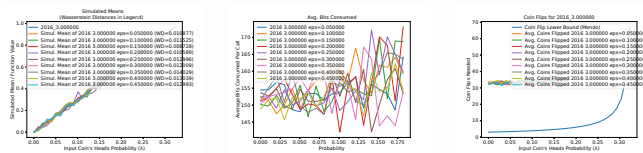
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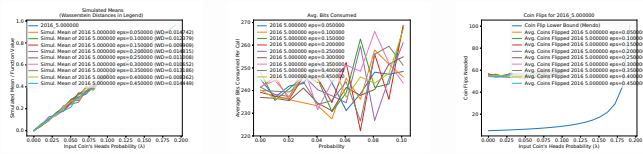
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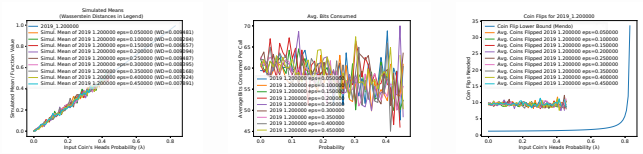
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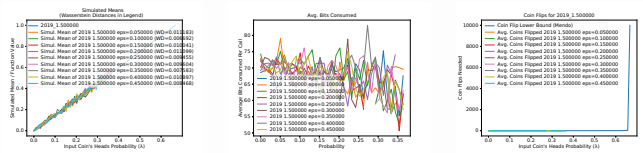
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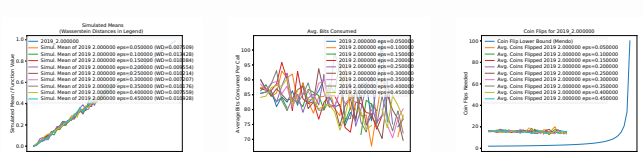
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2019 1.200000
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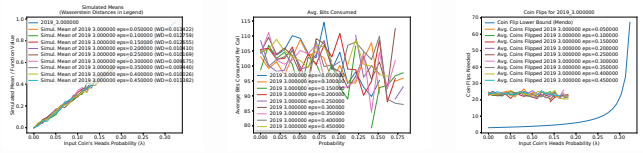
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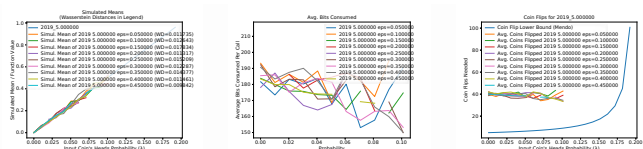
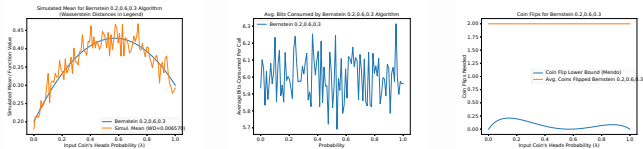
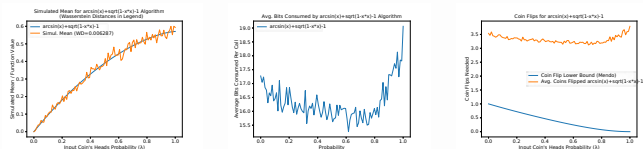
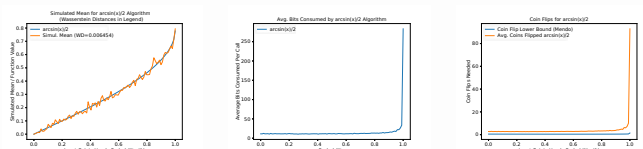
2019 2.000000



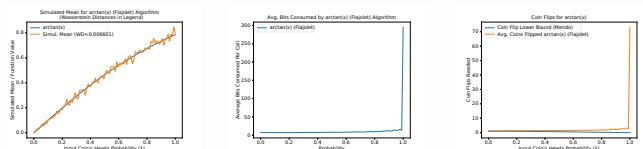
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2019 3.000000
eps=0.050000
```



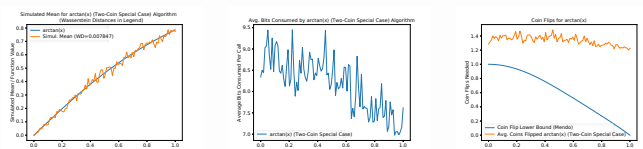
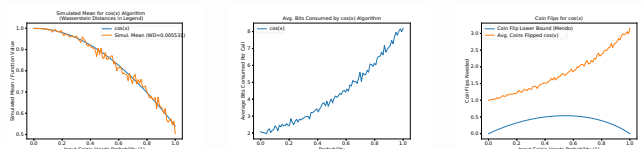
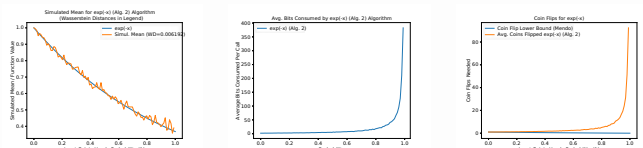
```
2019 5.000000
eps=0.050000
```

Bernstein
0.2,0.6,0.3
$$\arcsin(x) + \sqrt{1-x^2} - 1$$
 $\arcsin(x)/2$ 

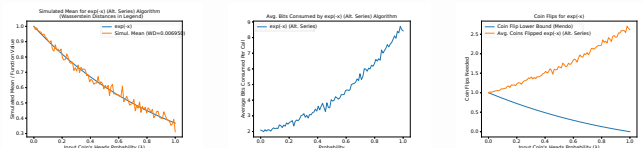
arctan(x)
(Flajolet)



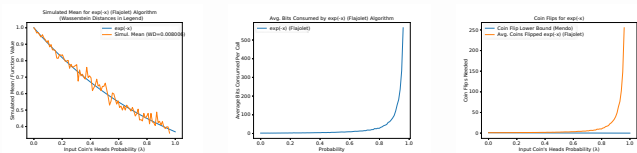
arctan(x) (Two-Coin Special Case)

 $\cos(x)$  $\exp(-x)$ (Alg. 2)

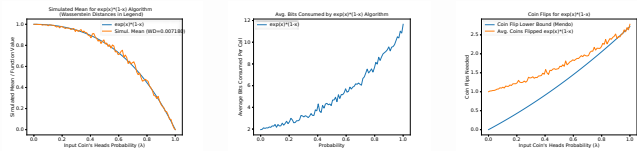
exp(-x) (Alt.
Series)



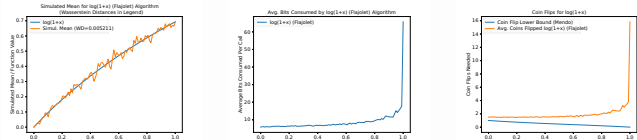
$\exp(-x)$ (Flajolet)



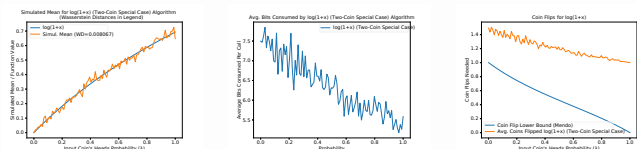
$\exp(x)*(1-x)$



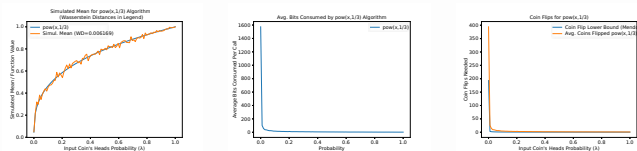
$\ln(1+x)$
(Flajolet)



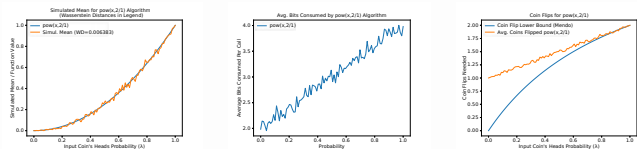
$\ln(1+x)$ (Two-Coin Special Case)



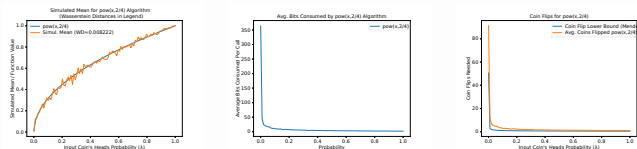
$\text{pow}(x, 1/3)$



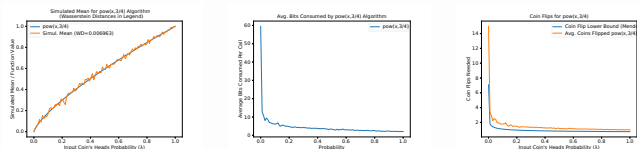
$\text{pow}(x, 2/1)$



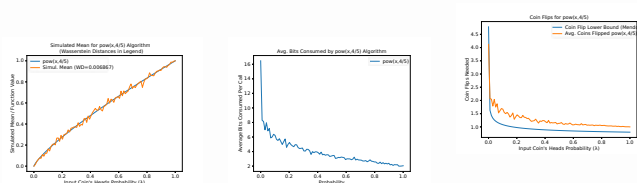
$\text{pow}(x, 2/4)$



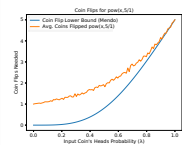
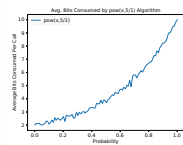
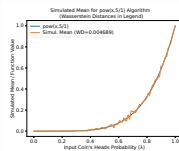
$\text{pow}(x, 3/4)$



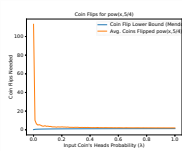
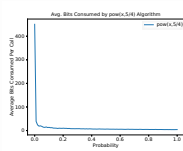
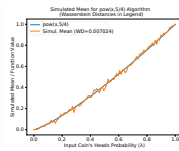
$\text{pow}(x, 4/5)$



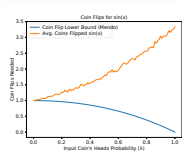
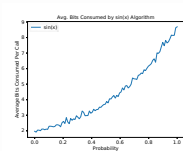
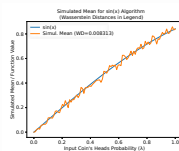
$\text{pow}(x, 5/1)$



$\text{pow}(x, 5/4)$



$\sin(x)$



$\text{sqrt}(x)$

