Untitled

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Proposition: The sequence $c/2^n$ is bounded from above by the sequence $p(1-p)^n$ (where $n \geq 0$ is an integer) when c is 0.4 and p is 0.49.

Proof: This proof relies on finding the values of p that satisfy the following inequality:

$$C/2^{(n+b)r} \leq p(1-p)^n, \tag{ineq}$$

where $n > 0$ and $b > 0$ are integers, and $c > 0$ and $r > 0$ are real numbers. Taking the logarithm of both sides, rewrite them to:

$$\ln c + \ln 1/2((n+b)r) \leq \ln p + \ln 1 - pn.$$

Now observe that both functions are linear in n. Now consider—

$$g(n,p) = \ln c + \ln 1/2((n+b)r) - \ln p - \ln 1 - pn.$$

Proving the statement thus boils down to finding out the areas where $h(p)$, the function that solves the equation $g(n,p) = 0$ for n, is 0 or less. $h(p)$ is:

$$h(p) = \frac{-br \ln 2 + \ln c - \ln p}{r \ln 2 + \ln 1 - p}.$$

All values of p for which $-\infty < h(p) \leq 0$ will satisfy (ineq). Given $b = 0$, $r = 1$, $c = 0.4$, and $p = 0.49$, $h(p)$ is less than 0, which completes the proof.