McGill University

Faculty of Science

Department of Mathematics and Statistics

Part A Examination

Statistics: Methodology Paper

Date: 8th May 2015 Time: 1pm-5pm

Instructions

- Answer only **two** questions from Section L. If you answer more than two questions, then only the **FIRST TWO questions will be marked.**
- Answer only **two** questions from Section G. If you answer more than two questions, then only the **FIRST TWO questions will be marked.**

Questions	Marks
L1	
L2	
L3	
G1	
G2	
G3	

This exam comprises the cover page and fourteen pages of questions.

Notation: In Section L, the following notation will be used: for $i = 1, \ldots, n$, y_i is the observed response; Y_i is the random variable version of the response; $\mathbf{y}$ and $\mathbf{Y}$ are the $n \times 1$ vector versions of the responses; $\mathbf{x}_i$ is the row vector of predictor values, $\mathbf{X}$ is the matrix of predictor values; $\widehat{y}_i$, $\widehat{Y}_i$, $\widehat{\mathbf{y}}$ and $\widehat{\mathbf{Y}}$ are the fitted or predicted response values or vectors arising from a given model; β is the vector of regression coefficients; $\widehat{\beta}$ is the vector of estimates or estimators. Furthermore, $\mathbf{0}_n$ is the n-dimensional vector of zeros, and $\mathbf{I}_n$ is the n-dimensional identity matrix.

L1. (a) Consider the linear regression model specified (in vector form) by the equation

$$\mathbf{Y} = \mathbf{X}\beta + \varepsilon$$

for random error vector ε, where β is a $p \times 1$ vector of regression coefficients, and where

$$\mathbb{E}[\varepsilon|\mathbf{X}] = \mathbf{0}_n \qquad \text{Var}[\varepsilon|\mathbf{X}] = \sigma^2\mathbf{I}_n.$$

(i) Derive the form of the ordinary least squares (OLS) estimator of β, and show that it is unbiased.

4 MARKS

(ii) Show that for the linear regression model above, the vector of residuals

$$\mathbf{e} = \mathbf{y} - \widehat{\mathbf{y}}$$

is orthogonal to each of the columns of $\mathbf{X}$, and is also orthogonal to the vector of fitted values, $\widehat{\mathbf{y}}$.

4 MARKS

(iii) Show that the (ANOVA) sums of squares decomposition for the model may be written

$$\mathbf{Y}^\top(\mathbf{I}_n - \mathbf{H}_1)\mathbf{Y} = \mathbf{Y}^\top(\mathbf{I}_n - \mathbf{H})\mathbf{Y} + \mathbf{Y}^\top(\mathbf{H} - \mathbf{H}_1)\mathbf{Y}$$

for matrices $\mathbf{H}$ and $\mathbf{H}_1$ to be defined.

4 MARKS

(b) Suppose now that a random sample of pairs of random variables $(X_i, Y_i), i = 1, \ldots, n$ is considered, and that a bivariate Normal distribution is presumed, that is

$$\begin{pmatrix} X_i \\ Y_i \end{pmatrix} \sim \text{Normal}_2(\boldsymbol{\mu}, \Sigma) \qquad i = 1, \ldots, n$$

where

$$\boldsymbol{\mu} = \begin{pmatrix} \mu_X \\ \mu_Y \end{pmatrix} \qquad \Sigma = \begin{bmatrix} \sigma_X^2 & \sigma_{XY} \\ \sigma_{XY} & \sigma_Y^2 \end{bmatrix}.$$

The conditional model for Y_i given $X_i = x_i$ is to be considered.

Derive the maximum likelihood estimators of the parameters in the conditional model, and compare them with the estimators obtained using a regression approach and ordinary least squares.

Recall that, for the bivariate normal model, the conditional distribution of Y_i given $X_i = x_i$ is Normal,

$$\text{Normal}\left(\mu_Y + \rho\frac{\sigma_Y}{\sigma_X}(x_i - \mu_X), \sigma_Y^2(1 - \rho^2)\right)$$

where

$$\rho = \frac{\sigma_{XY}}{\sigma_X\sigma_Y}.$$

8 MARKS

L2. The following data set relates to a small clinical study of three cholesterol lowering drugs (variable D, with levels denoted $1, 2, 3$); each patient in the study was classified into one of two subgroups (variable G, with levels denoted $1, 2$) according to a metabolic classification scheme, and then given a course of treatment using one allocated drug. The outcome of the study was recorded as the cholesterol reduction (in millimoles per litre, mmol/l) in standard blood samples taken before and after the course of treatment.

D	1	1	1	1	1	1	2	2	2	2	3	3	3	3	3
G	1	2	1	2	2	2	2	1	2	1	1	1	1	2	2
Y	0.63	0.99	1.31	3.10	1.99	1.55	1.70	-0.10	1.76	0.29	0.76	0.55	-0.16	0.09	-0.52

(a) Derive an expression for the bias in the OLS estimators for the linear model parameters derived from the fit of an additive model $D+G$ if in fact the true (data generating) model includes both main effects and an interaction term between D and G. 8 MARKS

(b) The following R output relates to a partial analysis of the data:

```
> summary(lm(Y ~ D))
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   1.5617     0.2277   6.858 1.75e-05 ***
D2           -0.9942     0.3600  -2.761 0.017241 *
D3           -1.4657     0.3378  -4.339 0.000963 ***
---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.5578 on 12 degrees of freedom
Multiple R-squared:  0.6232,    Adjusted R-squared:  0.5604
F-statistic: 9.922 on 2 and 12 DF,  p-value: 0.002863

> anova(lm(Y ~ G+D))
Analysis of Variance Table

Response: Y
          Df Sum Sq Mean Sq F value   Pr(>F)
G          1 2.1090 2.10901  7.9474 0.016694 *
D          2 4.8796 2.43978  9.1938 0.004495 **
Residuals 11 2.9191 0.26537
---
```

Summarize the evidence available in this analysis to support the research hypothesis that the metabolic subgroups have a significantly different expected response to each other.

Note that the 0.95 quantile of the $F_{1,11} \equiv \texttt{Fisher}(1, 11)$ distribution is 4.844.

8 MARKS

(c) Compute the R^2 statistic for the model $G + D$. 4 MARKS

L3. The following data relate a study of state by state crime statistics in the United States. The variables included in the data frame `cdata` are

- state id (`sid`),
- state name (`state`),
- violent crimes per 100,000 people (`crime`),
- murders per 1,000,000 (`murder`),
- the percent of the population living in metropolitan areas (`pctmetro`),
- the percent of the population that is white (`pctwhite`),
- percent of population with a high school education or above (`pcths`),
- percent of population living under poverty line (`poverty`),
- percent of population that are single parents (`single`).

There are 51 data points. The objective of the study was to understand whether the per capita rates of violent crime were related to the other measured predictors.

(a) Summarize the conclusions to be made from the following analysis in `R`:

```
> fit.full<-lm(crime ~ (pctmetro+pctwhite+pcths+poverty+single)^2,data=cdata)
> anova(fit.full)
Analysis of Variance Table

Response: crime
                  Df  Sum Sq Mean Sq  F value    Pr(>F)
pctmetro           1 2879417 2879417 149.8809 3.313e-14 ***
pctwhite           1 2675621 2675621 139.2728 9.381e-14 ***
pcths              1   48679   48679   2.5339   0.12042
poverty            1 1725561 1725561  89.8198 3.389e-11 ***
single             1  938579  938579  48.8554 3.940e-08 ***
pctmetro:pctwhite  1  434360  434360  22.6096 3.356e-05 ***
pctmetro:pcths     1   48597   48597   2.5296   0.12072
pctmetro:poverty   1   33940   33940   1.7666   0.19240
pctmetro:single    1   12314   12314   0.6410   0.42876
pctwhite:pcths     1    4911    4911   0.2556   0.61631
pctwhite:poverty   1    1256    1256   0.0654   0.79968
pctwhite:single    1   54613   54613   2.8427   0.10068
pcths:poverty      1    8078    8078   0.4205   0.52094
pcths:single       1   87040   87040   4.5307   0.04041 *
poverty:single     1  103111  103111   5.3672   0.02649 *
Residuals         35  672398   19211
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Question L3 continues on the next page.

```
> fit.1<-lm(crime~pctmetro+pctwhite+pcths+poverty+single,data=cdata)
> anova(fit.full,fit.1,test='F')
Analysis of Variance Table

Model 1: crime ~ (pctmetro + pctwhite + pcths + poverty + single)^2
Model 2: crime ~ pctmetro + pctwhite + pcths + poverty + single
  Res.Df     RSS  Df Sum of Sq      F    Pr(>F)
1     35  672398
2     45 1460618 -10   -788221 4.1029 0.0008645 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

4 MARKS

(b) A second phase of analysis was then considered:

```
> anova(fit1,fit2,fit3,fit4,fit5)
Analysis of Variance Table

Model 1: crime ~ pctmetro + pctwhite + pcths + poverty + single
Model 2: crime ~ pctmetro * pctwhite + pcths + poverty + single
Model 3: crime ~ pctmetro + pctwhite + pcths + poverty * single
Model 4: crime ~ pctmetro * pctwhite + pcths + poverty * single
Model 5: crime ~ pctmetro * pctwhite + poverty * single
  Res.Df     RSS Df Sum of Sq       F    Pr(>F)
1     45 1460618
2     44 1026258  1    434360 19.3773 6.978e-05 ***
3     44 1443119  0   -416861
4     43  963884  1    479235 21.3792 3.435e-05 ***
5     44  968243 -1     -4359  0.1945    0.6614
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
> drop1(fit5,test='F')
Single term deletions

Model:
crime ~ pctmetro * pctwhite + poverty * single
                  Df Sum of Sq     RSS    AIC F value    Pr(>F)
<none>                          968243 516.42
pctmetro:pctwhite  1    498431 1466674 535.60 22.6503 2.126e-05 ***
poverty:single     1     58529 1026772 517.42  2.6597    0.1101
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Summarize the conclusions to be made from this output. 5 MARKS

Question L3 continues on the next page.

(c) A third analysis was carried out:

```
> fit6<-lm(crime~murder+pctmetro*pctwhite+poverty*single,data=cdata)
> coef(summary(fit6))
                       Estimate  Std. Error   t value     Pr(>|t|)
(Intercept)       -3114.5252651 830.1133288 -3.751928 0.0005206568
murder               22.6502533   7.1452314  3.169982 0.0028083836
pctmetro             24.1719303   9.3092552  2.596548 0.0128367490
pctwhite             12.4397412   6.9541255  1.788829 0.0806885140
poverty              85.9493355  23.3236529  3.685072 0.0006358045
single              164.3391613  34.0881907  4.821000 0.0000181525
pctmetro:pctwhite    -0.2061182   0.1031445 -1.998344 0.0520279415
poverty:single       -6.5010010   2.0484230 -3.173661 0.0027797707

> anova(fit5,fit6)
Analysis of Variance Table

Model 1: crime ~ pctmetro * pctwhite + poverty * single
Model 2: crime ~ murder + pctmetro * pctwhite + poverty * single
  Res.Df    RSS Df Sum of Sq      F   Pr(>F)
1     44 968243
2     43 784833  1    183410 10.049 0.002808 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
> fit7<-lm(murder~pctmetro*pctwhite+poverty*single,data=cdata)
> summary(fit7)
Coefficients:
                    Estimate Std. Error t value Pr(>|t|)
(Intercept)       -49.082179  15.874574  -3.092 0.003447 **
pctmetro            0.860490   0.147479   5.835 5.90e-07 ***
pctwhite            0.558241   0.120188   4.645 3.09e-05 ***
poverty            -1.290914   0.451983  -2.856 0.006522 **
single             -0.187994   0.718661  -0.262 0.794859
pctmetro:pctwhite  -0.009216   0.001675  -5.502 1.81e-06 ***
poverty:single      0.148482   0.036971   4.016 0.000227 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.85 on 44 degrees of freedom
Multiple R-squared:  0.9378,    Adjusted R-squared:  0.9293
F-statistic: 110.5 on 6 and 44 DF,  p-value: < 2.2e-16
```

Comment on the utility of the predictor `murder` in predicting per capita crime rate.

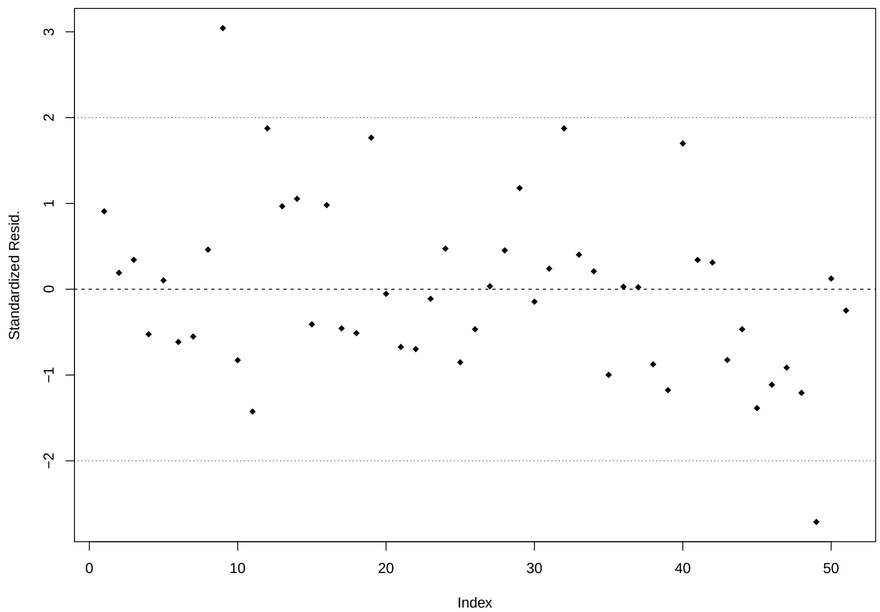
4 MARKS

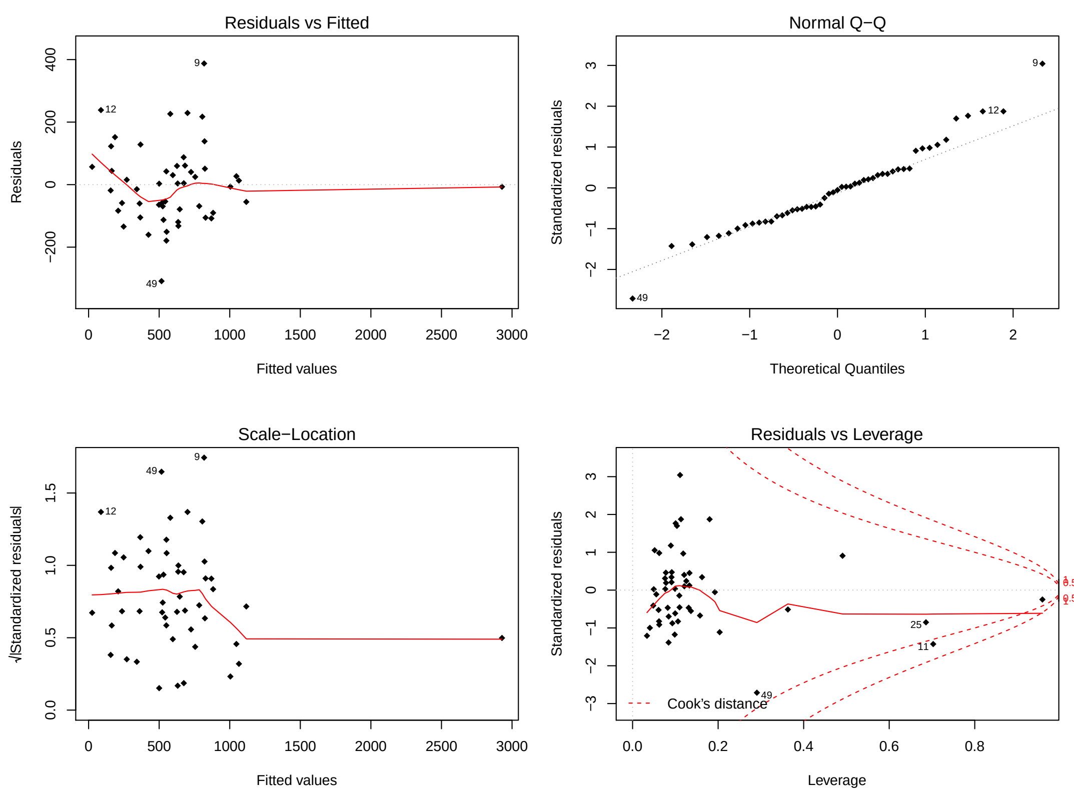
Question L3 continues on the next page.

(d) Define the *standardized* and *studentized* residuals that are typically examined when considering the fit of a regression model, and explain why each is preferable to the usual residual quantity $e_i = y_i - \widehat{y}_i$. 2 MARKS

(d) Define the *leverage* of a data point that may be computed for the fit of a regression model. 2 MARKS

(f) The following residual plots were obtained from the model `fit6`:





Summarize the conclusions that should be drawn from these plots. 3 MARKS

G1. The Bradley-Terry model is often used to rank individuals or objects subject to repeated comparisons amongst pairs. Take, for example, a competitive tennis league run by a local tennis club. For each match in league play, the two players' names and the winner of the match are recorded. Assume that there are K total players. Let π_{ab} be the probability that player a defeats player b in a match, where $a \neq b$. Suppose that $\pi_{ab} + \pi_{ba} = 1$ (which in our context implies that there are no ties). The Bradley-Terry model assumes that:

$$\log\left(\frac{\pi_{ab}}{\pi_{ba}}\right) = \beta_a - \beta_b.$$

For $a < b$, let N_{ab} denote the number of matches between players a and b, with player a winning n_{ab} times and player b winning n_{ba} times.

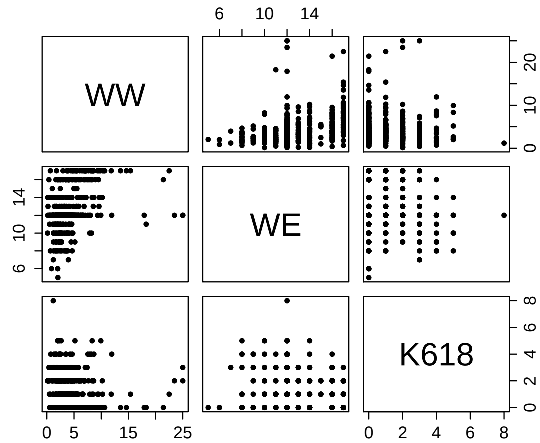
(a) Treating the quantities $\{n_{ab} : a < b\}$ as realizations of independent binomial variates, show that the Bradley-Terry model can be formulated as a generalized linear model. Clearly define the design matrix, the link, and the response functions. 4 MARKS

(b) Using your answer to part (a) or otherwise, find the minimal sufficient statistics for the model parameters in terms of the match data, $\{n_{ab} : a < b\}$. 4 MARKS

(c) Describe how you would test, using the Bradley-Terry model, the hypothesis that player a is more likely to win against player b in a match than she is to lose. Clearly state the null and alternative hypotheses, how you would construct the test statistic, and the rejection region for your test. 8 MARKS

(d) There is some concern that the players on the east side of the court (i.e. the side facing the sunset) are at a disadvantage because they face the setting sun. Assume that all courts are outdoors and that this potential disadvantage is the same across all matches. Describe how to modify your model to estimate the extent of the disadvantage. 4 MARKS

G2. Cross-sectional data were collected on 428 women in order to model the association between women's wages (`WW`, in dollars/10,000) and two covariates: the number of years of education (`WE`) and the number of children they have between the ages of 6 and 18 (`K618`). The figure below shows pairwise scatterplots of the three variable of interest.



(a) Researchers first fit the regression model contained in the output below. Summarize the conclusions from the following initial analysis in `R`. 5 MARKS

```
> G2.M1 = glm(WW ~ WE+K618,data=wagedata)
> summary(G2.M1)
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -1.84788    0.87958  -2.101   0.0362 *
WE           0.48888    0.06622   7.382 8.28e-13 ***
K618        -0.12074    0.11501  -1.050   0.2944
---
(Dispersion parameter for gaussian family taken to be 9.697015)

    Null deviance: 4679.1  on 427  degrees of freedom
Residual deviance: 4121.2  on 425  degrees of freedom
AIC: 2191.9

> anova(G2.M1)
Analysis of Deviance Table
Model: gaussian, link: identity
Response: WW
Terms added sequentially (first to last)

     Df Deviance Resid. Df Resid. Dev
NULL                   427     4679.1
WE    1   547.13       426     4131.9
K618  1    10.69       425     4121.2
> sum(resid(G2.M1,"pearson")^2)
[1] 4121.232
```

Question G2 continues on the next page.

(b) Based on the residual diagnostic plots below and the output above, do you think that the model in part (a) is appropriate? Explain your reasoning clearly. 3 MARKS

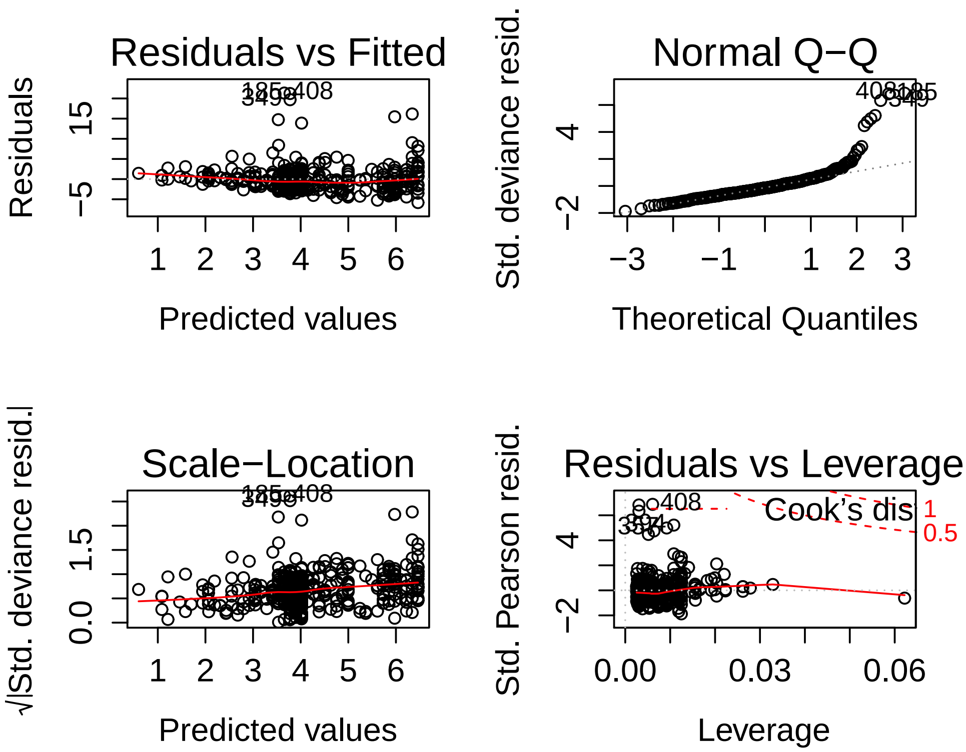


Figure 1: Residual Diagnostic Plots for model `G2.M1`

Question G2 continues on the next page.

(c) A second regression model was fit using the code below. Summarize the conclusions drawn from this second analysis. Clearly define the model that is fit by the code below and interpret the parameter estimates obtained from the model. 6 MARKS

```
> G2.M2 = glm(WW ~ WE+K618,data=wagedata,family=Gamma(link=log))
> summary(G2.M2)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.04125    0.21335   0.193    0.847
WE           0.10902    0.01606   6.787 3.87e-11 ***
K618        -0.01900    0.02790  -0.681    0.496
---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for Gamma family taken to be 0.5705232)

    Null deviance: 205.08  on 427  degrees of freedom
Residual deviance: 175.87  on 425  degrees of freedom
AIC: 1889.9
Number of Fisher Scoring iterations: 5

> anova(G2.M2)
Analysis of Deviance Table

Model: Gamma, link: log
Response: WW
Terms added sequentially (first to last)

     Df Deviance Resid. Df Resid. Dev
NULL                   427     205.08
WE    1  28.9431       426     176.14
K618  1   0.2701       425     175.87

> sum(resid(G2.M2,"pearson")^2)
[1] 242.4723
```

(d) Compute the estimated mean wages for a woman with 15 years of education and 3 children aged 6-18 under both models `G1.M1` and `G2.M2`. 2 MARKS

Question G2 continues on the next page.

(e) Based on the residual diagnostic plots below and the output above, which of the two models (`G1.M1` and `G2.M2`) do you believe is most appropriate? Explain your reasoning clearly.

4 MARKS

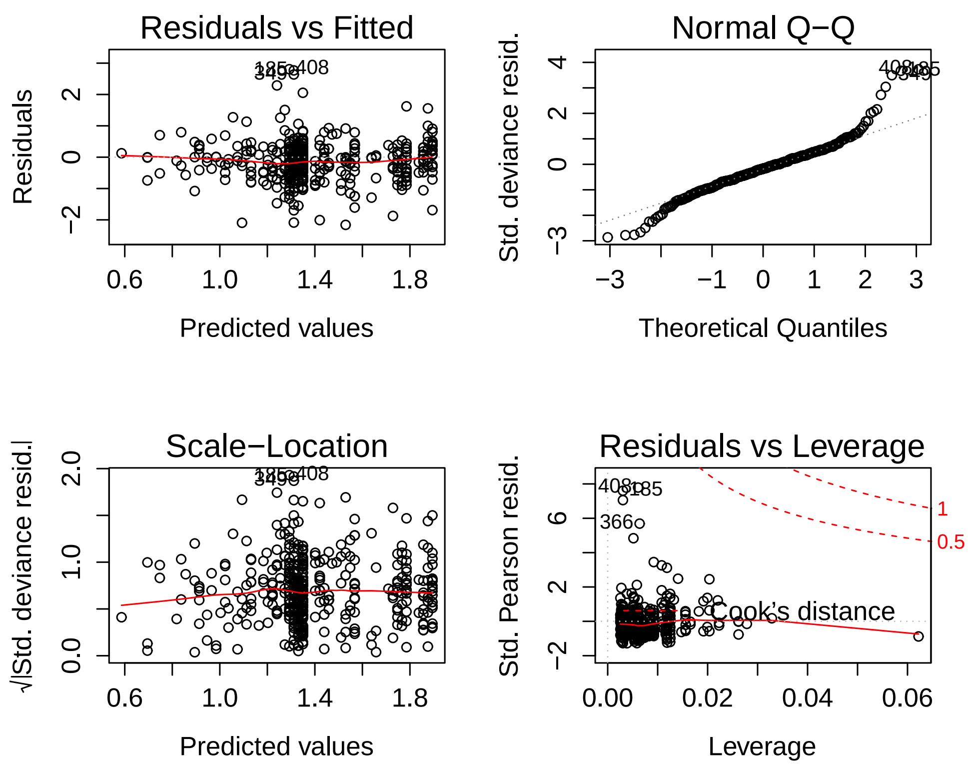


Figure 2: Residual Diagnostic Plots for model `G2.M2`

G3. The following four questions were asked in a social science survey of a random sample of adults.

1. How would you classify yourself on the political spectrum: Conservative, Moderate, or Liberal? (coded by variable `pol` in the R code that follows)
2. Do you believe that teenagers should have access to birth control (yes/no)? (`bc`)
3. Do you frequently attend religious services (yes/no)? (`re`)
4. Do you believe that it is appropriate for unmarried people to have sexual relations before marriage (yes/no)? (`pr`)

The data are summarized in the contingency table below:

Prem. Sex	No				Yes			
Att. Relig.	No		Yes		No		Yes	
Birth Control	No	Yes	No	Yes	No	Yes	No	Yes
Pol: Cons	73	25	99	15	24	22	8	4
Pol: Mod	87	37	73	20	50	60	20	13
Pol: Lib	51	36	51	19	33	88	6	12

(a) For the following questions, consider the analyses in the R code on page 14 involving only the questions relating to birth control, religious services attendance, and premarital sex.

(i) For each of the four models (`G3.M1`, `G3.M2`, and `G3.M3`, describe what each of the models assume in terms of the dependence structure amongst the three variables. 4 MARKS

(ii) For model `G3.M4`, interpret the values of the parameter estimates for the coefficient `bcYes:reYes` with respect to conditional odds ratios. 2 MARKS

(iii) Choose which of the four models presented is most appropriate for inference. Explain your answers using the output. 4 MARKS

(b) For the following questions, consider the analyses in the R code on page 15 (two pages following) involving all four survey questions.

(i) What restriction is placed upon all pairwise associations between variables by the model `G3.M5`? 2 MARKS

(ii) Is there evidence that a more complex model is required than `G3.M5`? Explain your answer clearly. 3 MARKS

(iii) Is there evidence that any of the terms in model `G3.M5` can be dropped? If so, indicate which term can be dropped and describe how that changes the interpretation of the conditional associations between the variables. 5 MARKS

Question G3 continues on the next page.

```
> G3.M1 = glm(counts~bc+re+pr,family="poisson");coef(summary(G3.M1))
              Estimate Std. Error   z value     Pr(>|z|)
(Intercept)  4.3406489 0.05470167 79.351308 0.000000e+00
bcYes       -0.4935838 0.06773576 -7.286902 3.171633e-13
reYes       -0.5443742 0.06817375 -7.985099 1.404092e-15
prYes       -0.5443742 0.06817375 -7.985099 1.404090e-15
> deviance(G3.M1)
[1] 290.7711
> G3.M2 = glm(counts~bc*re+pr,family="poisson");coef(summary(G3.M2))
              Estimate Std. Error   z value     Pr(>|z|)
(Intercept)  4.2058846 0.06141030 68.488260 0.000000e+00
bcYes       -0.1710644 0.08292163 -2.062965 3.911598e-02
reYes       -0.2129753 0.08387910 -2.539075 1.111461e-02
prYes       -0.5443742 0.06817375 -7.985099 1.404096e-15
bcYes:reYes -0.9591711 0.15104714 -6.350144 2.151138e-10
> deviance(G3.M2)
[1] 247.8169
> min(fitted(G3.M2))
[1] 10.15839
> G3.M3 = glm(counts~bc*re+pr*re,family="poisson");coef(summary(G3.M3))
              Estimate Std. Error   z value     Pr(>|z|)
(Intercept)  4.0234606 0.06836970 58.848595 0.000000e+00
bcYes       -0.1710644 0.08292163 -2.062965 3.911598e-02
reYes        0.2220751 0.09609577  2.310977 2.083414e-02
prYes       -0.1093238 0.08274270 -1.321250 1.864181e-01
bcYes:reYes -0.9591711 0.15104715 -6.350144 2.151140e-10
reYes:prYes -1.3715590 0.16226361 -8.452659 2.847412e-17
> deviance(G3.M3)
[1] 166.8142
> min(fitted(G3.M3))
[1] 5.126471
> G3.M4 = glm(counts~bc*re+pr*re + bc*pr,family="poisson")
> coef(summary(G3.M4))
               Estimate Std. Error    z value     Pr(>|z|)
(Intercept)  4.25442961 0.06739717 63.1247490 0.000000e+00
bcYes       -0.77062791 0.11424445 -6.7454296 1.525749e-11
reYes        0.05300852 0.09217738  0.5750708 5.652434e-01
prYes       -0.68255152 0.11137037 -6.1286635 8.862035e-10
bcYes:reYes -0.64182095 0.16009147 -4.0090891 6.095343e-05
reYes:prYes -1.18981459 0.16818668 -7.0743689 1.501305e-12
bcYes:prYes  1.23740502 0.14971409  8.2651206 1.395536e-16
> deviance(G3.M4)
[1] 96.64615
> min(fitted(G3.M4))
[1] 9.583359
```

Question G3 continues on the next page.

```
> G3.M5 = glm(counts~bc*re+bc*pol+bc*pr +
        re*pol + re*pr + pol*pr,family="poisson")
> summary(G3.M5)
Coefficients:
             Estimate Std. Error z value Pr(>|z|)
(Intercept)   4.33602    0.10469  41.419  < 2e-16 ***
bcYes        -1.18446    0.16976  -6.977 3.01e-12 ***
reYes         0.22899    0.13268   1.726 0.084379 .
polLib       -0.39337    0.15248  -2.580 0.009885 **
polMod        0.08541    0.13825   0.618 0.536694
prYes        -1.22124    0.17507  -6.976 3.04e-12 ***
bcYes:reYes  -0.59793    0.16255  -3.678 0.000235 ***
bcYes:polLib  0.92882    0.19362   4.797 1.61e-06 ***
bcYes:polMod  0.30484    0.18933   1.610 0.107377
bcYes:prYes   1.14683    0.15315   7.488 6.99e-14 ***
reYes:polLib -0.34408    0.18830  -1.827 0.067658 .
reYes:polMod -0.25827    0.17291  -1.494 0.135261
reYes:prYes  -1.14593    0.16980  -6.749 1.49e-11 ***
polLib:prYes  0.80176    0.20306   3.948 7.87e-05 ***
polMod:prYes  0.71995    0.19521   3.688 0.000226 ***
---
(Dispersion parameter for poisson family taken to be 1)

    Null deviance: 477.7846  on 23  degrees of freedom
Residual deviance:   6.9631  on  9  degrees of freedom
AIC: 161.4

> min(fitted(G3.M5))
[1] 4.769702

> drop1(G3.M5)
Single term deletions
Model:
counts ~ bc * re + bc * pol + bc * pr + re * pol + re * pr +
    pol * pr
       Df Deviance    AIC
<none>       6.963 161.40
bc:re   1   20.725 173.16
bc:pol  2   32.950 183.39
bc:pr   1   64.064 216.50
re:pol  2   10.701 161.14
re:pr   1   56.124 208.56
pol:pr  2   25.866 176.31
```