Math 152 – Python Lab 7

Directions: Use Python to solve each problem, unless the question states otherwise. (Template link)

1. Given $a_n = \left(\dfrac{7}{8}\right)^n$:

   (a) Show that $a_n \to 0$.

   (b) Plot the first 50 terms of the sequence (starting with $n = 0$) and the first 50 partial sums on the same graph.

   (c) Find the exact and approximate sum of the series.

2. Given the series $\displaystyle\sum_{n=2}^{\infty} n^2 e^{-n}$ and the function $f(x) = x^2 e^{-x}$:

   (a) Compute $\displaystyle\int f(x)dx$ and $\displaystyle\int_1^{\infty} f(x)dx$.

   (b) In a print statement, state your conclusion about the convergence or divergence of the series based on your answer to (a).

   (c) Compute s_{10}, s_{50}, s_{100}, and s. Give approximate answers for the partial sums.

   (d) Use the Remainder Estimate for the Integral Test to estimate $s - s_{100}$. Compare the actual value of $s - s_{100}$. Does the actual error fall in the expected range?

   (e) According to the Remainder Estimate, how many terms are needed to sum the series to within 10^{-10}? Compute the sum to confirm $|s - s_N| < 10^{-10}$. (NOTE: to expedite the computation, convert the terms to floating point before summing. Also, I highly recommend using **sp.nsolve**.)

3. Given the series $\displaystyle\sum_{n=1}^{\infty} \frac{5n^2 - 1}{3n^4 + 5n + 1}$:

   (a) Let $a_n = \dfrac{5n^2 - 1}{3n^4 + 5n + 1}$. Define a series b_n with which to compare it.

   (b) Plot the first 50 terms of a_n and b_n on the same graph to determine which is larger. If the graph is not clear, use the logical test $a_n < b_n$ to test the logical value comparing each term.

   (Parts C and D are on the back!)

(c) Determine whether $\sum_{n=1}^{\infty} b_n$ converges or not, and state your conclusion about the convergence of $\sum_{n=1}^{\infty} a_n$ as a result.

(d) If (c) is inconclusive, determine whether $\frac{a_n}{b_n}$ converges or not, and state your conclusion about the convergence of $\sum_{n=1}^{\infty} a_n$. (NOTE: If you still cannot conclude anything, start over with a different b_n!)