Math 152 – Python Lab 9

Directions: Use Python to solve each problem, unless the question states otherwise. (Template link)

1. Given the power series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (n+1) 9^{n+1} (x+3)^{n+2}}{5^{n+3}}$:

   (a) Find the Ratio Test limit.

   (b) State the radius of convergence and the endpoints. If applicable, substitute to show whether each endpoint is in the interval of convergence or not.

   (c) It can be shown that the series converges to $f(x) = \dfrac{9(x+3)^2}{(9x+32)^2}$ on its interval of convergence. To illustrate this, find s_5, s_{10}, s_{15}. Then, plot these three polynomials and f on the same set of axes with the window $x \in [-4, -2]$, $y \in [-4, 4]$.

2. The power series $J_1(x) = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{n!(n+1)!2^{2n+1}}$ is called the *Bessel function* of order 1. The Bessel function measures the radial part of the vibration of a circular drumhead.

   (a) What is the radius of convergence of the series?

   (b) Graph the first 5 partial sums and the first order Bessel function on the same plot with domain $x \in [0, 5]$ and range $y \in [-0.6, 0.6]$, to demonstrate the partial sums' convergence to J_1. (The command for the Bessel function in SymPy is **sp.besselj(n,x)** where n is the order of the curve and x is the variable.)

   (c) Plot the first 5 orders of Bessel functions (use the command for the Bessel function of order n given in Part B).

(*Problem 3 is on the back!*)

3. Recall that the **Taylor series** of a function f (centered at $x = a$) is given by

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n,$$

where $f^{(n)}(a)$ is the nth derivative of f evaluated at $x = a$. In SymPy, the nth derivative of a function f can be computed using the command **sp.diff(f,x,n)**, then by substituting $x = a$ we get $f^{(n)}(a)$.

For each of the following functions, use a **for** loop to compute the 10th degree **Taylor polynomial** (in other words, the partial sum of the Taylor series containing only the terms up to $n = 10$.)

(a) $f(x) = \sin(x)$, centered at $x = 0$

(b) $f(x) = \tan(x)$, centered at $x = 0$

(c) $f(x) = e^x$, centered at $x = 0$

(d) $f(x) = \sin(x)$, centered at $x = \pi/2$