Math 152 – Python Lab 8

Directions: Use Python to solve each problem, unless the question states otherwise. (Template link)

1. Given the series $\sum_{n=1}^{\infty}(-1)^n \frac{n^3}{e^{4n-2}}$:

   (a) Using the Remainder Estimate for the Alternating Series Test for N terms, plot the upper bound (function) in the window and the line $y = 0.00005$. Use your graph to determine how many terms are needed to sum the series to within 0.00005.

   (b) Use **sp.nsolve** to confirm your graphical answer from part (a).

   (c) Find the sum of the series to within 0.00005.

2. Given $\sum_{n=0}^{\infty} \frac{(1000)^n}{n!}$:

   (a) Print the first 11 terms of the series (from a_0 to a_{10}). Based on your output, do you expect the series to converge or diverge?

   (b) Apply the Ratio Test to the series, i.e., compute $\left|\frac{a_{n+1}}{a_n}\right|$ and $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$.

   (c) What does your answer to part (b) tell you about the series?

3. Given the power series $\sum_{n=0}^{\infty}(-1)^n \frac{x^{6n+5}}{2n+1}$:

   (a) Simplify $\left|\frac{a_{n+1}}{a_n}\right|$ and compute the limit $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$.

   (b) State the interval of convergence for this power series. Remember to check the endpoints.

   (c) Find the partial sums s_1, s_3, and s_5 for this power series. You can use the **sp.summation** command for this.

   (d) It can be shown that the series converges to $f(x) = x^2 \arctan(x^3)$. Plot s_1, s_3, s_5, and f on the same graph to illustrate this. Use the interval of convergence found in part (b) as your plot's x-domain.