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Final Project

Discussion of the Augmented Apollo Powered Descent Guidance Paper by Ping Lu

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Contents

I. Introduction

In 1969, the ability to safely guide a manned spacecraft for landing on another gravitational body was demonstrated for the first time. The mission used the Apollo Powered Descent Guidance (APDG) to calculate the thrust input needed to get from the initial in-orbit position to the desired landing site. Since space missions usually involve high monetary investments, mission operators tend to stick to flight-proven technology, to keep the risk of failure low. However, the technological advancements since the Apollo program and the progress in research led to more advanced guidance laws that handle the landing task more efficiently. The paper *Augmented Apollo Powered Descent and Guidance* (A^2PDG) [1] discusses a closed-loop guidance approach that leads to a near fuel-optimal landing that is automatically triggered at the ideal powered descent initiation (PDI) conditions. The developed guidance system can be used for future automated Moon or Mars missions, to make them more fuel-optimal, while still mostly using flight-proven technology for the critical guidance.

The heritage E-Guidance law assumes a linear function of time for the acceleration profile, with two constant parameters that have to be determined. Integrating this acceleration profile leads to a squared velocity and a cubed trajectory profile. By using those equations and the initial position/velocity of the spacecraft, the desired final position/velocity at the landing site, as well as the maneuver duration (t_{go}), the two constants can be eliminated. This leads to a time-dependent thrust acceleration profile that brings the lander from its current position to the desired landing site. Although this approach will generate the correct thrust profiles to land the spacecraft for the given conditions, those solutions are in general not the most optimal trajectories. The major flaw is that the PDI condition and initial time-to-go have to be predetermined, although they significantly influence the trajectory. Conventionally, the control input is solved for a variety of initial conditions and different initial time-to-go's, to find the most optimal solution for the mission requirements. However, this approach requires pre-calculations of many possible entry trajectories and takes versatility from the automated onboard system. Also, these guidance systems do not include minimum and maximum thrust levels, potentially leading to unfeasible solutions.

The Tunable Apollo Powered Descent Guidance (TAPDG) law presented in the paper is an implicit guidance law that tries to recreate a given desired trajectory but has a controllable gain that determines how closely the law follows the desired trajectory. It is shown that the E-Guidance and APDG guidance laws are two special cases of the more general TAPDG. Moreover, [2] shows that the solution of the E-guidance law is usually close to the fuel-optimal solution. APDG, on the other hand, generally uses more fuel but has a steeper trajectory that leads to an almost vertical attitude at landing, preferred for a soft touchdown. Conveniently, the tunable gain in the TAPDG law can be used to trade of between the fuel-optimal and trajectory-optimal solution, providing a way to match this parameter to the mission requirements. Although this generalizes the guidance law and gives more control over the true trajectory of the solution, it introduces yet another parameter while still relying on the PDI and initial time-to-go condition.

This leaves the task open, to find the ideal initial conditions and time-to-go at which the powered descent should be triggered. In the field of space propulsion *ideal* usually means minimum fuel consumption, since more fuel results in either higher launch mass or decreased payload mass. Thus, an algorithm is needed that triggers the PDI with a time-to-go that will guide the spacecraft to the desired location with minimal fuel consumption. The paper makes use of the so-called Universal Powered Guidance (UPG) algorithm presented in [3] that finds the fuel-optimal time-to-go for a given initial condition. It is used with a soft landing condition that forces the velocity to go to zero but only requires the position to be somewhere on the surface of the body. Therefore, during the unpowered descent of the spacecraft, the UPG algorithm is repeatably triggered to find the optimal time-to-go and final position for the current state of the spacecraft. As long as the distance to the UPG calculated optimal landing position is smaller than the distance to the desired landing spot, the PDI is not triggered. However, when those two distances lie within a certain threshold, the powered descent is initiated with the optimal time-to-go and the pinpoint guidance of the TAPDG. This will ensure a near fuel-optimal descent.

Lastly, a gravity turn guidance will be added to control the final vertical descent of the lander. This is required because the TAPDG trajectories lead to the desired final position and velocity but not necessarily the desired attitude. Especially those trajectories that are close to the E-Guidance law reach the final position with a shallow angle. To prevent the lander from tumbling over, the paper defines the desired final position for the TAPDG at a certain height above the surface. The lander will reach this position with a certain angle to the vertical. Now, the gravity turn guidance takes over to bring this angle to zero and land with a final attitude of zero with the desired terminal velocity. A gravity turn is characterized by continuously firing the thrusters opposite to the spacecraft's velocity vector.

In summary, the complete closed loop guidance system can be divided into four parts. First, the spacecraft comes in on a ballistic (or lifting) unpowered trajectory that is governed by gravity (and optionally atmospheric lift). At a certain point an algorithm is repeatably triggered that uses UPG to calculate the optimal time-to-go and final position for a soft landing initiated at that position. Once the optimal soft landing position and the desired landing position overlap, the powered descent is initiated and the TAPDG law takes over (using the current position and optimal time-to-go). As a last step, the gravity turn guidance law brings the lander down to the ground in a vertical orientation and with the desired terminal velocity.

II. Mathematical Framework

A. Tunable Apollo Powered Descent Guidance

Let us explore the E-Guidance law as one special case of the more general TAPDG system first. During the powered descent, the spacecraft dynamics are ruled by the following differential equations, when we assume a constant gravitational acceleration $\boldsymbol{g}$ and no other forces (like aerodynamic drag) acting on the vehicle:

$$\dot{\boldsymbol{r}}(t) = \boldsymbol{v}(t) \tag{1}$$

$$\dot{\boldsymbol{v}}(t) = \boldsymbol{g} + \boldsymbol{a}_T(t) \tag{2}$$

Here, $\boldsymbol{r}$ is the position and $\boldsymbol{v}$ the velocity of the spacecraft, while $\boldsymbol{a}_T$ is the control input (acceleration) performed by the spacecraft's thrusters. In the E-Guidance framework we assume this control input to be a linear function of time ($t \in [t_0, t_f]$) so that:

$$\boldsymbol{a}_T(t) = \boldsymbol{\alpha}_0 + \boldsymbol{\alpha}_1 t - \boldsymbol{g} \tag{3}$$

Additionally, we require that the spacecraft reaches a given final position denoted with $\boldsymbol{r}_f$ at a final velocity $\boldsymbol{v}_f$. Using this, let us integrate equations (1) and (2) from time t to the final time t_f and insert the conditions from above.

$$\boldsymbol{v}(t_f) = \boldsymbol{v}_f = \int_t^{t_f} \boldsymbol{a}_T(\tau) + \boldsymbol{g}\, d\tau = \boldsymbol{v}(t) + \boldsymbol{\alpha}_0 t_{go} + \frac{\boldsymbol{\alpha}_1}{2} t_{go}^2 \tag{4}$$

$$\boldsymbol{r}(t_f) = \boldsymbol{r}_f = \int_t^{t_f} \boldsymbol{v}(\tau)\, d\tau = \boldsymbol{r}(t) + \boldsymbol{v}(t) t_{go} + \frac{\boldsymbol{\alpha}_0}{2} t_{go}^2 + \frac{\boldsymbol{\alpha}_1}{6} t_{go}^3 \tag{5}$$

Here, the time to go is defined as $t_{go} = t_f - t$. With some simple algebra, those two equations can be solved for $\boldsymbol{\alpha}_0$ and $\boldsymbol{\alpha}_1$ which can then be inserted into equation (3), leading to the solved E-Guidance law:

$$\boldsymbol{a}_T(t) = -\frac{2}{t_{go}}[\boldsymbol{v}_f - \boldsymbol{v}(t)] + 6t_{go}^2[\boldsymbol{r}_f - \boldsymbol{r}(t) - \boldsymbol{v}(t)t_{go}] - \boldsymbol{g} \tag{6}$$

This thrust profile is explicitly time-dependent, through t_{go}, as well as implicitly through the current position $\boldsymbol{r}(t)$ and velocity $\boldsymbol{v}(t)$. From this, the full trajectory, velocity and thrust profiles can be determined by numerically integrating the dynamics equations, beginning with the initial conditions at the PDI.

It can be shown that the linear thrust-profile of the E-Guidance approach minimizes the performance index J_1 (note that t_f is a predetermined constant in this case). For many cases, the solution that minimizes this index lies close to the fuel-optimal case.

$$J_1 = \int_{t_0}^{t_f} \|\boldsymbol{a}_T(\tau)\|^2 \, d\tau \tag{7}$$

Instead of assuming a linear acceleration, the same derivation can be performed with a quadratic thrust profile, while also enforcing an additional condition on the final thrust vector $\boldsymbol{a}(t_f) = \boldsymbol{a}_f$. This leads to the APDG law that is generally less fuel efficient when compared to the E-Guidance law under the same conditions, but has the advantage of forcing a final acceleration vector that correlates to the final attitude of the lander. This way, forcing a final acceleration perpendicular to the surface will make sure that the spacecraft comes in vertically.

$$\boldsymbol{a}_T(t) = -\frac{6}{t_{go}}[\boldsymbol{v}_f - \boldsymbol{v}(t)] + 12t_{go}^2[\boldsymbol{r}_f - \boldsymbol{r}(t) - \boldsymbol{v}(t)t_{go}] + \boldsymbol{a}_f \tag{8}$$

The TAPDG law can be understood as a generalization and combination of the two presented guidance laws. Here, two tunable parameters are added that describe how well the actual acceleration profile follows a desired profile. The desired profile is assumed to be linear in time (equivalent to E-Guidance) with an unknown jolt parameter $\boldsymbol{n}$. Additionally, a final acceleration $\boldsymbol{a}_f$ is enforced (equivalent to APDG):

$$\boldsymbol{a}_d(t) = \boldsymbol{a}_f + \boldsymbol{n}t \tag{9}$$

Unlike the two special cases derived above, the commanded thrust profile does not have to directly follow the desired trajectory. Instead, the gain parameters k_v and k_r control how close the true trajectory will lie to the desired one.

$$\boldsymbol{a}_T(t) = \boldsymbol{a}_d - \frac{k_v}{t_{go}}[\boldsymbol{v}(t) - \boldsymbol{v}_d] - \frac{k_r}{t_{go}^2}[\boldsymbol{r}(t) - \boldsymbol{r}_d] \tag{10}$$

Like in the derivation above, we can integrate the desired thrust and use the initial conditions to find expressions for the desired position and velocity. After inserting these relations back into equation (10), we get a relation that is dependent on the initial conditions, k_v, k_r, and $\boldsymbol{n}$. Since it is not clear what explicit choice should be made for $\boldsymbol{n}$, the values for k_v and k_r are chosen in such a way that the $\boldsymbol{n}$ cancels out of the equation. This does not only eliminate $\boldsymbol{n}$ as a parameter but also forces a dependency on the relation of the tunable gain parameters:

$$k_v = \frac{1}{3}k_r + 2 \tag{11}$$

With this all set, we can write out the final form of the thrust profile for the TAPDG law as follows:

$$\boldsymbol{a}_T(t) = \frac{2}{t_{go}}\left(\frac{1}{3}k_r + 2\right)[\boldsymbol{v}_f - \boldsymbol{v}(t)] + \frac{k_r}{t_{go}^2}[\boldsymbol{r}_f - \boldsymbol{r}(t) - \boldsymbol{v}(t)t_{go}] + \frac{1}{6}(k_r - 6)\boldsymbol{a}_f - \frac{1}{6}(12 - k_r)\boldsymbol{g} \tag{12}$$

Comparing this to the E-Guidance and APDG law, it can be found that those two laws emerge from this equation with $k_r = 6$ and $k_r = 12$ respectively. Thus, by tuning k_r in the interval $[6, 12]$, a trade-off between fuel efficiency and trajectory shape can be made.

B. PDI and time-to-go Determination

For a fuel-optimal landing, we have to make sure that the TAPDG trajectory is initiated for the ideal initial conditions and time-to-go. To achieve that, the paper proposes the use of the UPG algorithm. It is a powerful tool to find the fuel-optimal solution to a landing problem for an initial state of the spacecraft but without a predetermined time-to-go. This is achieved by minimizing the performance index

$$J_2 = \kappa \|\boldsymbol{r}(t_f) - \boldsymbol{r}_f\| + \int_{t_0}^{t_f} \frac{\|\boldsymbol{T}(\tau)\|}{v_{ex}} \, d\tau, \tag{13}$$

where $\kappa \geq 0$ and $\|...\|$ is the Euclidean norm. This index has two main parts to it. The first part (scaled by κ) penalizes a trajectory by the distance of the final position of the solution to the actual desired landing position. The second part represents the total propellant consumption for the given maneuver, where t_f is a parameter and no predetermined value. By minimizing this performance index, optimal solutions to a variety of landing problems can be found.

For this paper, the UPG is used to solve to so-called soft landing problem. It differs only slightly from the pinpoint landing we enforced before for the derivation of the TAPDG. The new constraints

$$\boldsymbol{r}(t_f) = \|\boldsymbol{r}_f\| \tag{14}$$

$$\boldsymbol{v}(t_f) = \boldsymbol{v}_f \tag{15}$$

still enforce a final velocity $\boldsymbol{v}_f$ for a soft touch down but weaken the requirement for the final position of the lander. Instead of a precise landing position, the final position of the soft landing approach only has to lie somewhere on the surface of the gravitational body. Solving for the root of a seven parameter system defined by the UPG algorithm leads to the optimal time-to-go and final position of the spacecraft. In this case, the performance index J_2 is used with $\kappa = 0$ since we only require a soft landing and do not want to penalize the distance from the desired landing position.

Building on the capabilities of the UPG system, an algorithm is established that triggers the TAPDG for the ideal initial conditions (see Table 1). This algorithm is activated during the unpowered descent of the lander when it crosses a certain threshold distance from the desired landing position.

C. Gravity Turn

A gravity turn is the most fuel efficient way to bring the lander from an angled incoming trajectory to a soft, vertical landing. For this maneuver, the acceleration vector always points along the opposite direction of the vehicle's velocity. This way the height h and the horizontal velocity can be brought to zero, while the desired vertical velocity $\dot{h}_f$ is reached. Assume a linear profile for the horizontal velocity

$$\dot{h}(\tau) = \dot{h}(t_{\text{start}}) + a_H(\tau - t), \tag{16}$$

where t_{start} is the time at which the gravity turn is initiated and τ is the time since the start of the gravity turn. By integrating the vertical acceleration and velocity from t_{start} to τ_f, we get two equations

$$\dot{h}(\tau_f) = \dot{h}_f = \dot{h}_{\text{start}} + a_H \tau_{go} \tag{17}$$

$$h(\tau_f) = h_f = h + \dot{h}_{\text{start}} \tau_{go} + \frac{a_H}{2} \tau_{go}^2, \tag{18}$$

where $\tau_{go} = \tau_f - t_{\text{start}}$. These equations only have the two unknowns, so we can solve for the burn time and horizontal acceleration simultaneously:

$$a_H = \frac{\dot{h}_f^2 - \dot{h}_{\text{start}}^2}{2(h_f - h_{\text{start}})} \tag{19}$$

$$\tau_{go} = \frac{2(h_f - h_{\text{start}})}{\dot{h}_f + \dot{h}_{\text{start}}} \tag{20}$$

To get from the desired horizontal acceleration profile to the true required thrust commands for the lander, we need to take gravity and the angle between the velocity vector and the horizontal γ into account. Simple geometry leads to the final results

$$\boldsymbol{a}_T = \frac{1}{\sin \gamma} \left[\frac{\dot{h}_f^2 - \dot{h}_{\text{start}}^2}{2(h_f - h_{\text{start}})} + g \right] \hat{\boldsymbol{v}}, \tag{21}$$

where $\hat{\boldsymbol{v}}$ is the unit vector in the direction of the spacecraft's velocity.

III. Analysis Techniques

The guidance laws introduced in section II. can be solved by numerical integration starting from the initial conditions. Thus, the error strongly depends on the accuracy of the chosen method of integration. First order accuracy can be achieved by simply evaluating the first derivative at the current state and propagating it forward by one time step. More advanced algorithms like the RK45 method proposed by Fehlberg in 1969 [4] use multiple evaluations of the derivative around the current state to achieve up to fourth-order accuracy. In any case, attention has to be paid to the singularity at $t_{go} = 0$ that can lead to numerical problems.

An alternative to the integration approach would be to include an intermediate step between solving for the parameters $\boldsymbol{\alpha}_0$ and $\boldsymbol{\alpha}_1$ and inserting them in the acceleration equation. Using the initial conditions, numerical values for the two constants could be obtained that are time-independent. Those could then be plugged into the analytical solutions for $\boldsymbol{r}(t)$, $\boldsymbol{v}(t)$, and $\boldsymbol{a}_T(t)$, possibly leading to a higher order accuracy solution since there is no accumulation of numerical errors. However, the integration method was chosen because it makes a compact formulation of the guidance law possible and solves all three trajectories simultaneously. It would also allow for the easy addition of a varying gravitational acceleration.

It should also be mentioned that the paper uses the rather complex UPG algorithm presented in [3]. This algorithm is based on solving a seven variable root finding problem which leads to the fuel-optimal time-to-go and final position of the spacecraft. Since the detailed description of this algorithm lies outside the scope of the discussed paper, it is not elaborated on in more detailed and not implemented in the simulation.

IV. Confirmation of Results

This section will study the behavior of the derived guidance laws building on the mathematical framework established in the previous sections. For this, a realistic mission profile is used that involves a powered landing on another gravitational body. Since NASA's Artemis mission plans to bring humans back to the moon within this decade [5], and several private companies are already launching their own moon landers [6, 7], the importance of guided moon landings is bigger than ever. Many of those missions will be fully automated, requiring robust guidance systems that can adjust to different conditions and errors. Thus, the simulations in this chapter will examine the trajectories of a guided moon landing initiated from a circular moon orbit with an orbital height of 15 km[1]. All simulations are performed in the orbital plane with $z = 0$.

For all simulations, the TAPDG and gravity turn guidance laws are combined to form one merged guidance system. For the TAPDG, a final vertical velocity of -5.0 m/s and a position 10 m above the ground is desired, while the terminal vertical velocity at landing is set to 0.5 m/s. The addition of the gravity turn guidance is important to make the steep and shallow trajectories comparable to each other. The shallower trajectories might be more fuel efficient, but they could not land with their final attitude. Adding the gravity turn gives a penalty to those trajectories, since more thrust is required to bring them into the desired orientation.

First, the impact of the tunable gain parameter k_r in the TAPDG law on the trajectory shapes and total fuel consumption will be examined. For this, nine equally spaced parameters in the range [6,12] are used to calculate the full guidance dynamics for each case. The descent is initiated 10° before reaching the landing site (in the azimuthal position). For all cases the trajectory and the total control effort (Δv) are plotted and compared to each other (see Figure 2). The results confirm the expectations from the theoretical description: For the boundary conditions, the TAPDG law resembles the linear or quadratic thrust profiles of the E-Guidance or APDG laws (see Figure 4). Now, the parameter k_r can be tuned to reach trajectories that lie in between those special cases, where the E-Guidance has the shallowest trajectory with the lowest Δv and the APDG the steepest trajectory with the highest Δv. A closer look at the landing site reveals how the gravity turn guidance is successful in landing the spacecraft vertically for all TAPDG trajectories.

Second, the ideal PDI condition and time-to-go for the most fuel efficient TAPDG trajectory (k_r) are determined. For this, an optimizer is used to numerically minimize the total control effort and find the ideal initiation angle and time-to-go. Figure 5 shows the ideal trajectory and the respective velocity and acceleration profile. While the data regarding the influence of k_r was simulated with an initial angle of 10° and a maneuver duration time of 120 s, the optimal initiation angle and burn time are determined to be 84.78° and 183.33 s. The difference in control effort is immense, ranging from 5.3 km/s to the optimal 1742.38 m/s.

The simulations show how the tunable parameter in the TAPDG law can be used to achieve a trade-off between trajectory shape and fuel consumption. Moreover, the traditional approach to determine the ideal PDI conditions and time-to-go is portrayed for a noiseless system. This shows that a pre-flight trajectory optimization can be used to initiate the guidance system in the right moment. However, any errors or derivations from the mission profile will cause other these optimal conditions to change. Thus, this optimization would have to be performed for a variety of initial conditions and landing sites. This further highlights the advantages the introduced PDI condition determination algorithm creates.

[1] The full simulation code and all plots are available in a shared one drive folder: click here.

V. Discussion

The paper successfully designs and simulates an end-to-end closed-loop guidance system that allows for an autonomous landing from a lifting or ballistic trajectory. Starting from the heritage guidance systems used for the moon landings, the paper identifies weaknesses and tries to account for them with modified or added guidance schemes. At the same time, it is successful in preserving the core element of the guidance system as a flight proven algorithm. The thorough Monte-Carlo simulations of a lifting Mars re-entry and landing show that the system is resilient to external disturbances, reaching the desired landing position with a very high precision.

A disadvantage of keeping the flight-proven TAPDG law as the core guidance protocol is that the newly presented system also inherits all the assumptions that belong to it. This means drag forces are neglected that would be present during an atmospheric descent. Moreover, the solution still relies on the approximation that the gravitational acceleration $\boldsymbol{g}$ is constant along the trajectory. Small adjustments to the numerical integration could introduce a variable $\boldsymbol{g}$ and would correct the true thrust input. However, this solution will not be fuel-optimal anymore.

Additionally, the whole system relies on accurate position, velocity, and time measurements for the PDI. Since the spacecraft will probably not have any kind of external positioning system available, it would need to keep close track of all forces acting on the vehicle to infer its velocity and position from that. This will accumulate increasing errors once star or sun trackers can't provide a precise measurement anymore. The same is true for the time measurement that is crucial for performing the correct thrust profile at the right time.

The different steps of the here-presented guidance approach are known and well explored. However, the combination of the different steps to a fully closed-loop guidance system that safely guides a lander from the unpowered descent to the desired position is novel. Since the main part of the system still comes with a lot of simplifying assumptions, the system can not guide a fully autonomous landing. Outgoing from the presented guidance law, further development and integration into the whole control system could account for those problems and produce a near fuel-optimal guidance strategy.

A Figures and Tables

Table 1: Decision logic for triggering the PDI during unpowered descent.

1. Use UPG with the current initial conditions to calculate the distance to the ideal soft landing position.
2. Compare the distance for this soft landing with the distance to the desired landing position. If it is within a certain threshold, trigger the PDI with the ideal time-to-go that was calculated. Otherwise, continue.
3. If the distance to the desired landing location is below a certain threshold, trigger the PDI with the last calculated ideal time-to-go.
4. Continue with the unpowered flight and return to step one after a given wait period.

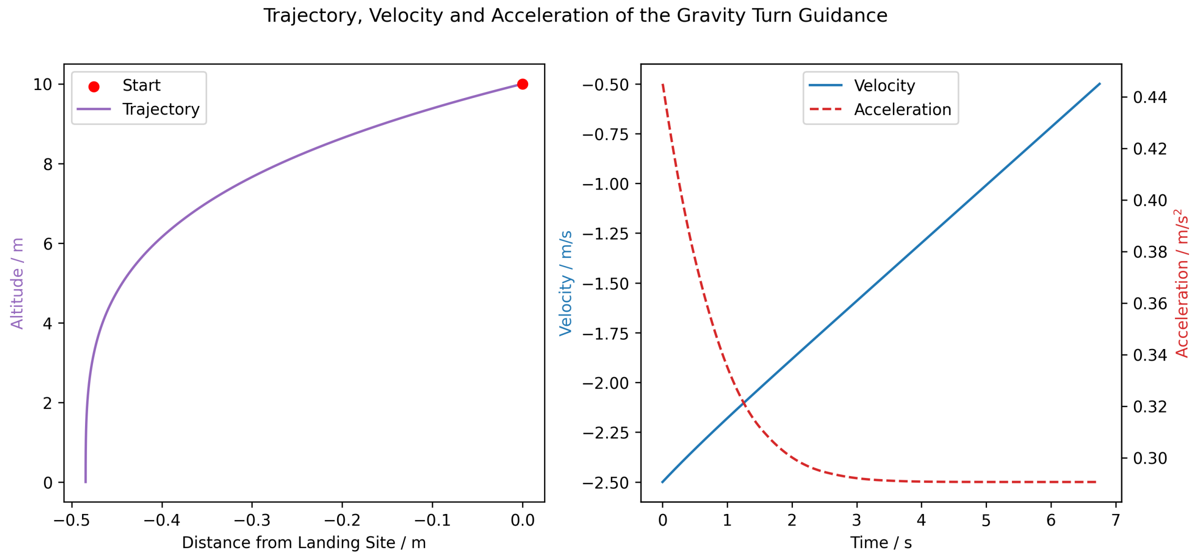

Figure 1: These plots show the trajectory, velocity, and acceleration of only the gravity turn guidance law. The initial conditions are chosen close to the worst case that was determined by running simulations of the TAPDG law on its own ($h = 10\,\mathrm{m}$, $|\boldsymbol{v}| = 2.5\,\mathrm{m/s}$ and at an angle of $80°$ to the horizontal). On the trajectory plot one can see that the spacecraft lands in a distance of $\approx 0.5\,\mathrm{m}$ to the desired landing position. This is due to the TAPDG law delivering the spacecraft to a position directly above the landing site. Since it takes some time to get rid of the horizontal velocity, the deviation occurs. To account for this, the gravity turn guidance could be called shortly before landing, simulate the expected final position and take over as soon as this position is within a threshold to the desired landing position. However, this is the worst case scenario (the TAPDG trajectories later show that the spacecraft usually comes in steeper and slower), so the deviation to the desired landing position is tolerated without further adjustments to the guidance system.

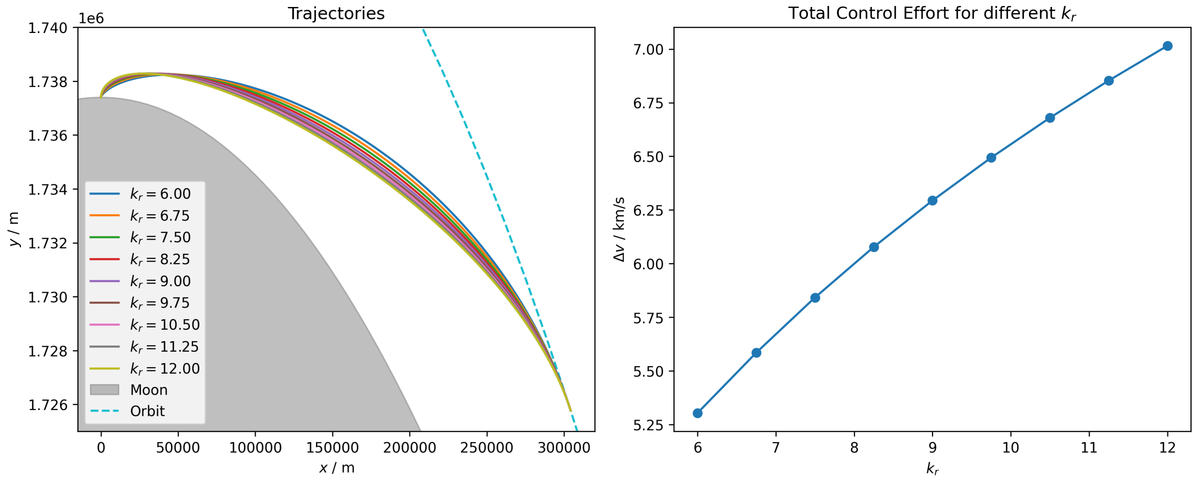

Figure 2: This plot shows the trajectories and total Δv for the combined TAPDG and gravity turn guidance system. As discussed in the theoretical sections, the E-Guidance law is the most fuel-efficient, with the lowest Δv consumption. The trajectories show that the lander comes in at a steeper angle for higher k_r, which is advantageous for the attitude in the final seconds of the burn.

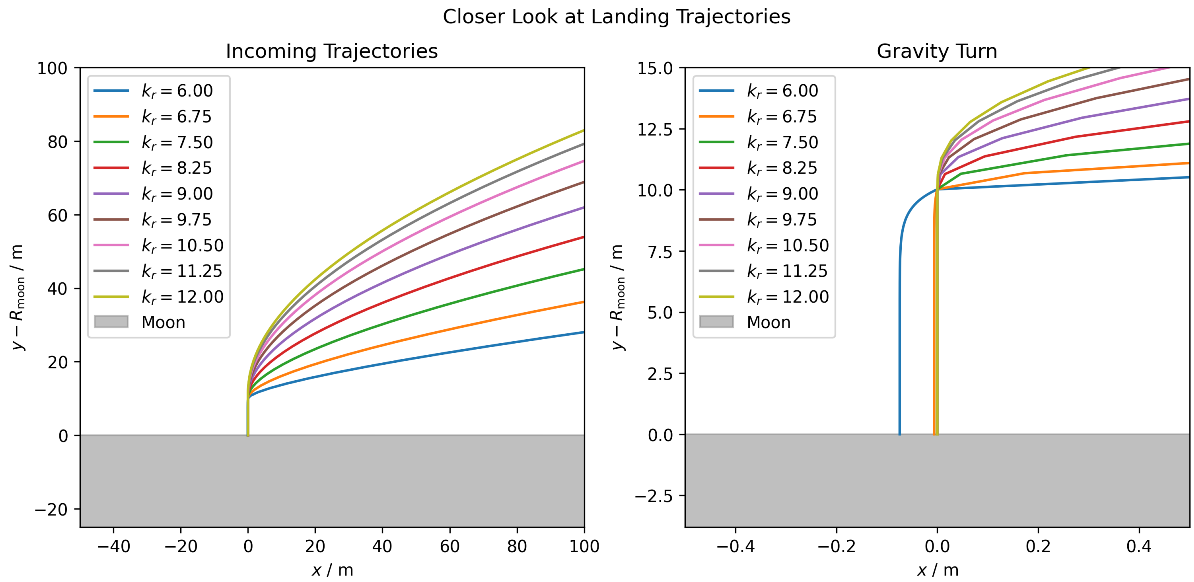

Figure 3: These two plots give a closer look at the final landing trajectory of the combined guidance law. On the left, the different incoming trajectories dependent on the gain parameter k_r can be nicely distinguished. The higher the value for k_r, the steeper the trajectories come in and the more important is the gravity turn. On the right the gravity turn dynamics are visualized, and it can be observed that the horizontal error is very small for most trajectories. However, the distance to the projected landing site is significant for the E-Guidance law ($k_r = 6$).

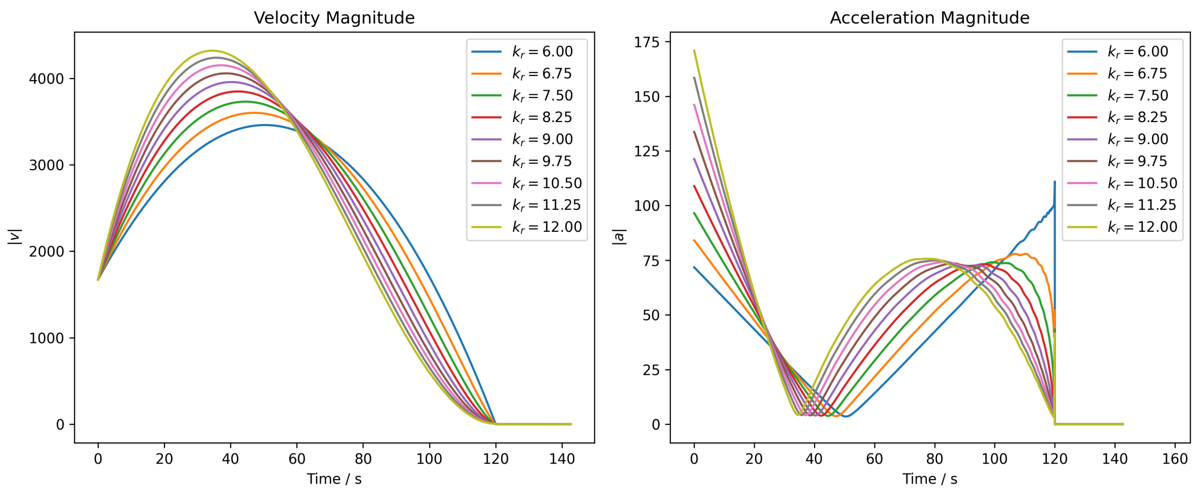

Figure 4: This Plot shows the magnitude of the velocity and acceleration vector during the descent. For the E-Guidance law ($k_r = 6$), the acceleration profile is linear (because the norm is plotted the sign change is not visible). On the opposite end ($k_r = 12$) the profile is parabolic. Approaching the final time of the TAPDG law, the acceleration profiles begin to show a ripple and spikes close to the final time t_f. This is due to the singularity at $t_{go} = 0$ that is approached.

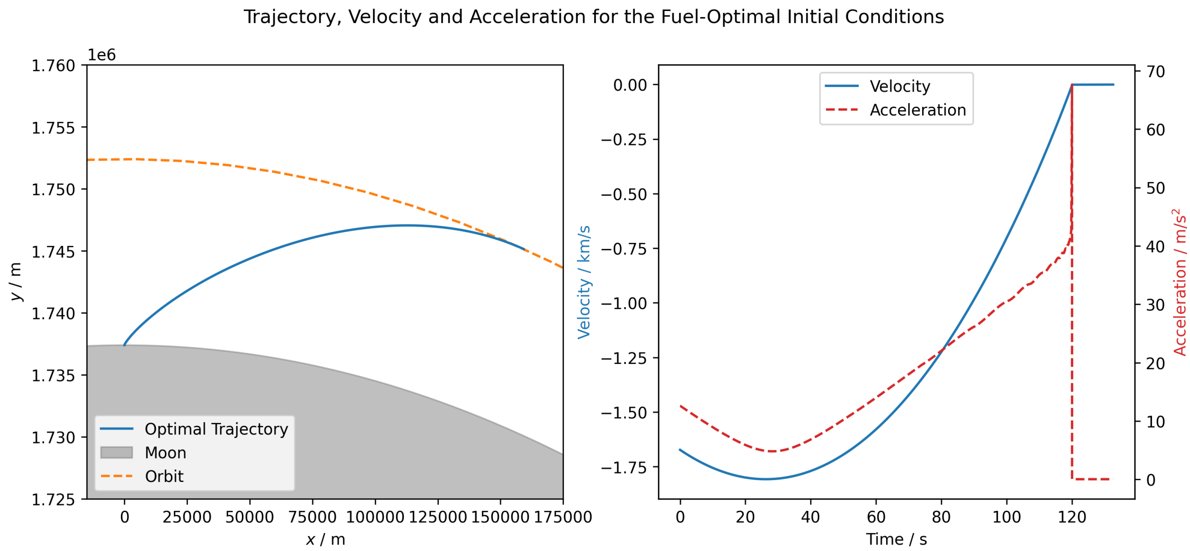

Figure 5: These two plots show the landing trajectory and the related velocity and acceleration profile for the fuel-optimal PDI conditions and time-to-go that was determined by a Δv minimizing function. The optimal parameters were determined to be $\theta = 84.78°$ and $t_{go} = 183.33$ s, leading to a total Δv of 1742.38 m/s. The spike at the end of the thrust profile is due to the singularity at $t_{go} = 0$ and is only a numerical error. In reality, the thrust should continue linearly.

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