second edition

PRINCIPLES OF CONCURRENT AND DISTRIBUTED PROGRAMMING

M. BEN-ARI

ADDISON-WESLEY

Principles of Concurrent and Distributed Programming

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Principles of Concurrent and Distributed Programming

Second Edition

M. Ben-Ari

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Visit **www.pearsoned.co.uk/ben-ari** to find valuable online resources

Companion Website for students
- Source code for all the algorithms in the book
- Links to sites where software for studying concurrency may be downloaded.

For instructors
- PDF slides of all diagrams, algorithms and scenarios (with LaTeX source)
- Answers to exercises

For more information please contact your local Pearson Education sales representative or visit **www.pearsoned.co.uk/ben-ari**

Preface

Concurrent and distributed programming are no longer the esoteric subjects for graduate students that they were years ago. Programs today are inherently concurrent or distributed, from event-based implementations of graphical user interfaces to operating and real-time systems to Internet applications like multiuser games, chats and ecommerce. Modern programming languages and systems (including Java, the system most widely used in education) support concurrent and distributed programming within their standard libraries. These subjects certainly deserve a central place in computer science education.

What has not changed over time is that concurrent and distributed programs cannot be "hacked." Formal methods *must* be used in their specification and verification, making the subject an ideal vehicle to introduce students to formal methods. Precisely for this reason I find concurrency still intriguing even after forty years' experience writing programs; I hope you will too.

I have been very gratified by the favorable response to my previous books *Principles of Concurrent Programming* and the first edition of *Principles of Concurrent and Distributed Programming*. Several developments have made it advisable to write a new edition. Surprisingly, the main reason is not any revolution in the *principles* of this subject. While the superficial technology may change, basic concepts like *interleaving*, *mutual exclusion*, *safety* and *liveness* remain with us, as have the basic constructs used to write concurrent programs like *semaphores*, *monitors*, *channels* and *messages*. The central problems we try to solve have also remained with us: *critical section*, *producer–consumer*, *readers and writers* and *consensus*. What has changed is that concurrent programming has become ubiquitous, and this has affected the choice of language and software technology.

Language: I see no point in presenting the details of any particular language or system, details that in any case are likely to obscure the *principles*. For that reason, I have decided not to translate the Ada programs from the first edition into Java programs, but instead to present the algorithms in pseudocode. I believe that the high-level pseudocode makes it easier to study the algorithms. For example, in the

Byzantine Generals algorithm, the pseudocode line:

```
for all other nodes
```

is much easier to understand than the Java lines:

```
for (int i = 0; i < numberOfNodes; i++)
   if (i != myID)
```

and yet no precision is lost.

In addition, I am returning to the concept of *Principles of Concurrent Programming*, where concurrency simulators, not concurrent programming languages, are the preferred tool for teaching and learning. There is simply no way that extreme scenarios—like the one you are asked to construct in Exercise 2.3—can be demonstrated without using a simulator.

Along with the language-independent development of models and algorithms, explanations have been provided on concurrency in five languages: the Pascal and C dialects supported by the BACI concurrency simulator, Ada[1] and Java, and Promela, the language of the model checker Spin. Language-dependent sections are marked by ^{L}. Implementations of algorithms in these languages are supplied in the accompanying software archive.

A word on the Ada language that was used in the first edition of this book. I believe that—despite being overwhelmed by languages like C++ and Java—Ada is still the best language for developing complex systems. Its support for concurrent and real-time programming is excellent, in particular when compared with the trials and tribulations associated with the concurrency constructs in Java. Certainly, the protected object and rendezvous are elegant constructs for concurrency, and I have explained them in the language-independent pseudocode.

Model checking: A truly new development in concurrency that justifies writing a revised edition is the widespread use of *model checkers* for verifying concurrent and distributed programs. The concept of a state diagram and its use in checking correctness claims is explained from the very start. Deductive proofs continue to be used, but receive less emphasis than in the first edition. A central place has been given to the Spin model checker, and I warmly recommend that you use it in your study of concurrency. Nevertheless, I have refrained from using Spin and its language Promela exclusively because I realize that many instructors may prefer to use a mainstream programming language.

[1] All references to Ada in this book are to Ada 95.

I have chosen to present the Spin model checker because, on the one hand, it is a widely-used industrial-strength tool that students are likely to encounter as software engineers, but on the other hand, it is very "friendly." The installation is trivial and programs are written in a simple programming language that can be easily learned. I have made a point of using Spin to verify all the algorithms in the book, and I have found this to be extremely effective in increasing my understanding of the algorithms.

An outline of the book: After an introductory chapter, Chapter 2 describes the abstraction that is used: the interleaved execution of atomic statements, where the simplest atomic statement is a single access to a memory location. Short introductions are given to the various possibilities for studying concurrent programming: using a concurrency simulator, writing programs in languages that directly support concurrency, and working with a model checker. Chapter 3 is the core of an introduction to concurrent programming. The critical-section problem is the central problem in concurrent programming, and algorithms designed to solve the problem demonstrate in detail the wide range of pathological behaviors that a concurrent program can exhibit. The chapter also presents elementary verification techniques that are used to prove correctness.

More advanced material on verification and on algorithms for the critical-section problem can be found in Chapters 4 and 5, respectively. For Dekker's algorithm, we give a proof of freedom from starvation as an example of deductive reasoning with temporal logic (Section 4.5). Assertional proofs of Lamport's fast mutual exclusion algorithm (Section 5.4), and Barz's simulation of general semaphores by binary semaphores (Section 6.10) are given in full detail; Lamport gave a proof that is partially operational and Barz's is fully operational and difficult to follow. Studying assertional proofs is a good way for students to appreciate the care required to develop concurrent algorithms.

Chapter 6 on semaphores and Chapter 7 on monitors discuss these classical concurrent programming primitives. The chapter on monitors carefully compares the original construct with similar constructs that are implemented in the programming languages Ada and Java.

Chapter 8 presents synchronous communication by channels, and generalizations to rendezvous and remote procedure calls. An entirely different approach discussed in Chapter 9 uses logically-global data structures called spaces; this was pioneered in the Linda model, and implemented within Java by JavaSpaces.

The chapters on distributed systems focus on algorithms: the critical-section problem (Chapter 10), determining the global properties of termination and snapshots (Chapter 11), and achieving consensus (Chapter 12). The final Chapter 13 gives an overview of concurrency in real-time systems. Integrated within this chapter

are descriptions of software defects in spacecraft caused by problems with concurrency. They are particularly instructive because they emphasize that some software really does demand precise specification and formal verification.

A summary of the pseudocode notation is given in Appendix A. Appendix B reviews the elementary mathematical logic needed for verification of concurrent programs. Appendix C gives a list of well-known problems for concurrent programming. Appendix D describes tools that can be used for studying concurrency: the BACI concurrency simulator; Spin, a model checker for simulating and verifying concurrent programs; and DAJ, a tool for constructing scenarios of distributed algorithms. Appendix E contains pointers to more advanced textbooks, as well as references to books and articles on specialized topics and systems; it also contains a list of websites for locating the languages and systems discussed in the book.

Audience: The intended audience includes advanced undergraduate and beginning graduate students, as well as practicing software engineers interested in obtaining a scientific background in this field. We have taught concurrency successfully to high-school students, and the subject is particularly suited to non-specialists because the basic principles can be explained by adding a very few constructs to a simple language and running programs on a concurrency simulator. While there are no specific prerequisites and the book is reasonably self-contained, a student should be fluent in one or more programming languages and have a basic knowledge of data structures and computer architecture or operating systems.

Advanced topics are marked by ^{A}. This includes material that requires a degree of mathematical maturity.

Chapters 1 through 3 and the non-^{A} parts of Chapter 4 form the introductory core that should be part of any course. I would also expect a course in concurrency to present semaphores and monitors (Chapters 6 and 7); monitors are particularly important because concurrency constructs in modern programming languages are based upon the monitor concept. The other chapters can be studied more or less independently of each other.

Exercises: The exercises following each chapter are technical exercises intended to clarify and expand on the models, the algorithms and their proofs. Classical problems with names like the *sleeping barber* and the *cigarette smoker* appear in Appendix C because they need not be associated with any particular construct for synchronization.

Supporting material: The companion website contains an archive with the source code in various languages for the algorithms appearing in the book. Its address is:

`http://www.pearsoned.co.uk/ben-ari.`

Lecturers will find slides of the algorithms, diagrams and scenarios, both in ready-to-display PDF files and in LaTeX source for modification. The site includes instructions for obtaining the answers to the exercises.

Acknowledgements: I would like to thank:

- Yifat Ben-David Kolikant for six years of collaboration during which we learned together how to really teach concurrency;
- Pieter Hartel for translating the examples of the first edition into Promela, eventually tempting me into learning Spin and emphasizing it in the new edition;
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- The Helsinki University of Technology for inviting me for a sabbatical during which this book was completed.

M. Ben-Ari
Rehovot and Espoo, 2005

1 What is Concurrent Programming?

1.1 Introduction

An "ordinary" program consists of data declarations and assignment and control-flow statements in a programming language. Modern languages include structures such as procedures and modules for organizing large software systems through abstraction and encapsulation, but the statements that are actually executed are still the elementary statements that compute expressions, move data and change the flow of control. In fact, these are precisely the instructions that appear in the machine code that results from compilation. These machine instructions are executed *sequentially* on a computer and access data stored in the main or secondary memories.

A *concurrent program* is a set of sequential programs that can be executed in parallel. We use the word *process* for the sequential programs that comprise a concurrent program and save the term *program* for this set of processes.

Traditionally, the word *parallel* is used for systems in which the executions of several programs overlap in time by running them on separate processors. The word *concurrent* is reserved for potential parallelism, in which the executions may, but need not, overlap; instead, the parallelism may only be apparent since it may be implemented by sharing the resources of a small number of processors, often only one. Concurrency is an extremely useful *abstraction* because we can better understand such a program by pretending that all processes are being executed in parallel. Conversely, even if the processes of a concurrent program are actually executed in parallel on several processors, understanding its behavior is greatly facilitated if we impose an order on the instructions that is compatible with shared execution on a single processor. Like any abstraction, concurrent programming is important because the behavior of a wide range of real systems can be modeled and studied without unnecessary detail.

In this book we will define formal models of concurrent programs and study algorithms written in these formalisms. Because the processes that comprise a concur-

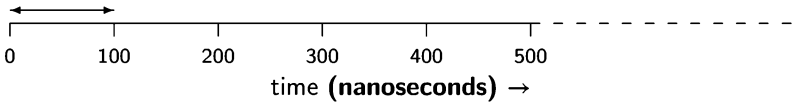
rent program may interact, it is exceedingly difficult to write a correct program for even the simplest problem. New tools are needed to specify, program and verify these programs. Unless these are understood, a programmer used to writing and testing sequential programs will be totally mystified by the bizarre behavior that a concurrent program can exhibit.

Concurrent programming arose from problems encountered in creating real systems. To motivate the concurrency abstraction, we present a series of examples of real-world concurrency.

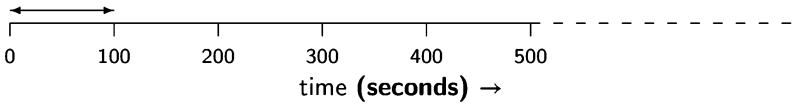
1.2 Concurrency as abstract parallelism

It is difficult to intuitively grasp the speed of electronic devices. The fingers of a fast typist seem to fly across the keyboard, to say nothing of the impression of speed given by a printer that is capable of producing a page with thousands of characters every few seconds. Yet these rates are extremely slow compared to the time required by a computer to process each character.

As I write, the *clock speed* of the central processing unit (CPU) of a personal computer is of the order of magnitude of one gigahertz (one billion times a second). That is, every nanosecond (one-billionth of a second), the hardware clock ticks and the circuitry of the CPU performs some operation. Let us roughly estimate that it takes ten clock ticks to execute one machine language instruction, and ten instructions to process a character, so the computer can process the character you typed in one hundred nanoseconds, that is 0.0000001 of a second:



To get an intuitive idea of how much effort is required on the part of the CPU, let us pretend that we are processing the character by hand. Clearly, we do not consciously perform operations on the scale of nanoseconds, so we will multiply the time scale by one billion so that every clock tick becomes a second:

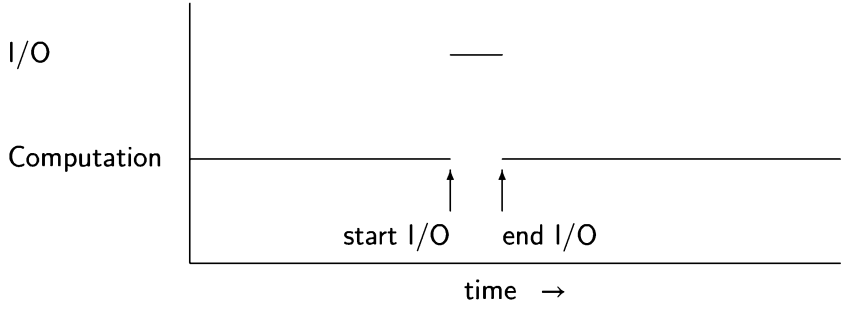


Thus we need to perform 100 seconds of work out of every *billion* seconds. How much is a billion seconds? Since there are $60 \times 60 \times 24 = 86{,}400$ seconds in a

day, a billion seconds is 1,000,000,000/86,400 = 11,574 days or about 32 years. You would have to invest 100 seconds every 32 years to process a character, and a 3,000-character page would require only $(3{,}000 \times 100)/(60 \times 60) \approx 83$ hours over half a lifetime. This is hardly a strenuous job!

The tremendous gap between the speeds of human and mechanical processing on the one hand and the speed of electronic devices on the other led to the development of operating systems which allow I/O operations to proceed "in parallel" with computation. On a *single* CPU, like the one in a personal computer, the processing required for each character typed on a keyboard cannot really be done in parallel with another computation, but it is possible to "steal" from the other computation the fraction of a microsecond needed to process the character. As can be seen from the numbers in the previous paragraph, the degradation in performance will not be noticeable, even when the overhead of switching between the two computations is included.

What is the connection between concurrency and operating systems that overlap I/O with other computations? It would theoretically be possible for every program to include code that would periodically sample the keyboard and the printer to see if they need to be serviced, but this would be an intolerable burden on programmers by forcing them to be fully conversant with the details of the operating system. Instead, I/O devices are designed to interrupt the CPU, causing it to jump to the code to process a character. Although the processing is sequential, it is conceptually simpler to work with an abstraction in which the I/O processing performed as the result of the interrupt is a separate process, executed concurrently with a process doing another computation. The following diagram shows the assignment of the CPU to the two processes for computation and I/O.



1.3 Multitasking

Multitasking is a simple generalization from the concept of overlapping I/O with a computation to overlapping the computation of one program with that of another. Multitasking is the central function of the *kernel* of all modern operating systems. A *scheduler* program is run by the operating system to determine which process should be allowed to run for the next interval of time. The scheduler can take into account priority considerations, and usually implements *time-slicing*, where computations are periodically interrupted to allow a fair sharing of the computational resources, in particular, of the CPU. You are intimately familiar with multitasking; it enables you to write a document on a word processor while printing another document and simultaneously downloading a file.

Multitasking has become so useful that modern programming languages support it *within* programs by providing constructs for *multithreading*. Threads enable the programmer to write concurrent (conceptually parallel) computations within a single program. For example, interactive programs contain a separate thread for handling events associated with the user interface that is run concurrently with the main thread of the computation. It is multithreading that enables you to move the mouse cursor while a program is performing a computation.

1.4 The terminology of concurrency

The term *process* is used in the theory of concurrency, while the term *thread* is commonly used in programming languages. A technical distinction is often made between the two terms: a process runs in its own address space managed by the operating system, while a thread runs within the address space of a single process and may be managed by a multithreading kernel within the process. The term *thread* was popularized by *pthreads* (*POSIX threads*), a specification of concurrency constructs that has been widely implemented, especially on UNIX systems. The differences between processes and threads are not relevant for the study of the synchronization constructs and algorithms, so the term *process* will be used throughout, except when discussing threads in the Java language.

The term *task* is used in the Ada language for what we call a process, and we will use that term in discussions of the language. The term is also used to denote small units of work; this usage appears in Chapter 9, as well as in Chapter 13 on real-time systems where *task* is the preferred term to denote units of work that are to be scheduled.

1.5 Multiple computers

The days of one large computer serving an entire organization are long gone. Today, computers hide in unforeseen places like automobiles and cameras. In fact, your personal "computer" (in the singular) contains more than one processor: the graphics processor is a computer specialized for the task of taking information from the computer's memory and rendering it on the display screen. I/O and communications interfaces are also likely to have their own specialized processors. Thus, in addition to the multitasking performed by the operating systems kernel, parallel processing is being carried out by these specialized processors.

The use of multiple computers is also essential when the computational task requires more processing than is possible on one computer. Perhaps you have seen pictures of the "server farms" containing tens or hundreds of computers that are used by Internet companies to provide service to millions of customers. In fact, the entire Internet can be considered to be one *distributed system* working to disseminate information in the form of email and web pages.

Somewhat less familiar than distributed systems are *multiprocessors*, which are systems designed to bring the computing power of several processors to work in concert on a single computationally-intensive problem. Multiprocessors are extensively used in scientific and engineering simulation, for example, in simulating the atmosphere for weather forecasting and studying climate.

1.6 The challenge of concurrent programming

The challenge in concurrent programming comes from the need to *synchronize* the execution of different processes and to enable them to *communicate*. If the processes were totally independent, the implementation of concurrency would only require a simple scheduler to allocate resources among them. But if an I/O process accepts a character typed on a keyboard, it must somehow communicate it to the process running the word processor, and if there are multiple windows on a display, processes must somehow synchronize access to the display so that images are sent to the window with the current focus.

It turns out to be extremely difficult to implement safe and efficient synchronization and communication. When your personal computer "freezes up" or when using one application causes another application to "crash," the cause is generally an error in synchronization or communication. Since such problems are time- and situation-dependent, they are difficult to reproduce, diagnose and correct.

The aim of this book is to introduce you to the constructs, algorithms and systems that are used to obtain correct behavior of concurrent and distributed programs. The choice of construct, algorithm or system depends critically on assumptions concerning the requirements of the software being developed and the architecture of the system that will be used to execute it. This book presents a survey of the main ideas that have been proposed over the years; we hope that it will enable you to analyze, evaluate and employ specific tools that you will encounter in the future.

Transition

We have defined concurrent programming informally, based upon your experience with computer systems. Our goal is to study concurrency abstractly, rather than a particular implementation in a specific programming language or operating system. We have to carefully specify the abstraction that describe the allowable data structures and operations. In the next chapter, we will define the concurrent programming abstraction and justify its relevance. We will also survey languages and systems that can be used to write concurrent programs.

2 The Concurrent Programming Abstraction

2.1 The role of abstraction

Scientific descriptions of the world are based on abstractions. A living animal is a system constructed of organs, bones and so on. These organs are composed of cells, which in turn are composed of molecules, which in turn are composed of atoms, which in turn are composed of elementary particles. Scientists find it convenient (and in fact necessary) to limit their investigations to one level, or maybe two levels, and to "abstract away" from lower levels. Thus your physician will listen to your heart or look into your eyes, but he will not generally think about the molecules from which they are composed. There are other specialists, pharmacologists and biochemists, who study that level of abstraction, in turn abstracting away from the quantum theory that describes the structure and behavior of the molecules.

In computer science, abstractions are just as important. Software engineers generally deal with at most three levels of abstraction:

Systems and libraries Operating systems and libraries—often called Application Program Interfaces (API)—define computational resources that are available to the programmer. You can open a file or send a message by invoking the proper procedure or function call, without knowing how the resource is implemented.

Programming languages A programming language enables you to employ the computational power of a computer, while abstracting away from the details of specific architectures.

Instruction sets Most computer manufacturers design and build families of CPUs which execute the same instruction set as seen by the assembly language programmer or compiler writer. The members of a family may be implemented in totally different ways—emulating some instructions in software or using memory for registers—but a programmer can write a compiler for that instruction set without knowing the details of the implementation.

Of course, the list of abstractions can be continued to include logic gates and their implementation by semiconductors, but software engineers rarely, if ever, need to work at those levels. Certainly, you would never describe the semantics of an assignment statement like x←y+z in terms of the behavior of the electrons within the chip implementing the instruction set into which the statement was compiled.

Two of the most important tools for software abstraction are encapsulation and concurrency.

Encapsulation achieves abstraction by dividing a software module into a public specification and a hidden implementation. The specification describes the available operations on a data structure or real-world model. The detailed implementation of the structure or model is written within a separate module that is not accessible from the outside. Thus changes in the internal data representation and algorithm can be made without affecting the programming of the rest of the system. Modern programming languages directly support encapsulation.

Concurrency is an abstraction that is designed to make it possible to reason about the dynamic behavior of programs. This abstraction will be carefully explained in the rest of this chapter. First we will define the abstraction and then show how to relate it to various computer architectures. For readers who are familiar with machine-language programming, Sections 2.8–2.9 relate the abstraction to machine instructions; the conclusion is that there are no important concepts of concurrency that cannot be explained at the higher level of abstraction, so these sections can be skipped if desired. The chapter concludes with an introduction to concurrent programming in various languages and a supplemental section on a puzzle that may help you understand the concept of state and state diagram.

2.2 Concurrent execution as interleaving of atomic statements

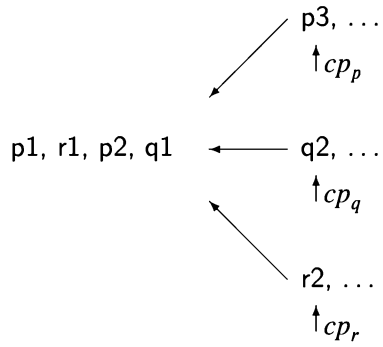
We now define the concurrent programming abstraction that we will study in this textbook. The abstraction is based upon the concept of a *(sequential) process*, which we will not formally define. Consider it as a "normal" program fragment written in a programming language. You will not be misled if you think of a process as a fancy name for a procedure or method in an ordinary programming language.

Definition 2.1 A *concurrent program* consists of a finite set of (sequential) processes. The processes are written using a finite set of *atomic statements*. The execution of a concurrent program proceeds by executing a sequence of the atomic

statements obtained by *arbitrarily interleaving* the atomic statements from the processes. A *computation* is an execution sequence that can occur as a result of the interleaving. Computations are also called *scenarios*. ▮

Definition 2.2 During a computation the *control pointer* of a process indicates the next statement that can be executed by that process.[1] Each process has its own control pointer. ▮

Computations are created by interleaving, which merges several statement streams. At each step during the execution of the current program, the next statement to be executed will be "chosen" from the statements pointed to by the control pointers *cp* of the processes.



Suppose that we have two processes, p composed of statements p1 followed by p2 and q composed of statements q1 followed by q2, and that the execution is started with the control pointers of the two processes pointing to p1 and q1. Assuming that the statements are assignment statements that do not transfer control, the possible scenarios are:

p1→q1→p2→q2,
p1→q1→q2→p2,
p1→p2→q1→q2,
q1→p1→q2→p2,
q1→p1→p2→q2,
q1→q2→p1→p2.

Note that p2→p1→q1→q2 is *not* a scenario, because we respect the sequential execution of each individual process, so that p2 cannot be executed before p1.

[1] Alternate terms for this concept are *instruction pointer* and *location counter*.

We will present concurrent programs in a language-independent form, because the concepts are universal, whether they are implemented as operating systems calls, directly in programming languages like Ada or Java,[2] or in a model specification language like Promela. The notation is demonstrated by the following trivial two-process concurrent algorithm:

Algorithm 2.1: Trivial concurrent program	
integer n ← 0	
p	**q**
integer k1 ← 1 p1: n ← k1	integer k2 ← 2 q1: n ← k2

The program is given a title, followed by declarations of global variables, followed by two columns, one for each of the two processes, which by convention are named process p and process q. Each process may have declarations of local variables, followed by the statements of the process. We use the following convention:

Each labeled line represents an atomic statement.

A description of the pseudocode is given in Appendix A.

States

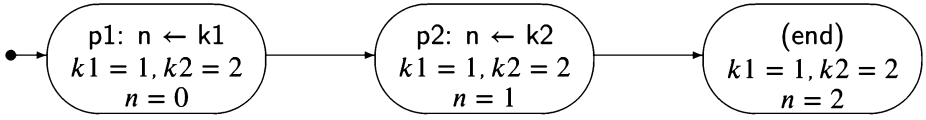
The execution of a concurrent program is defined by states and transitions between states. Let us first look at these concepts in a sequential version of the above algorithm:

Algorithm 2.2: Trivial sequential program
integer n ← 0
integer k1 ← 1 integer k2 ← 2 p1: n ← k1 p2: n ← k2

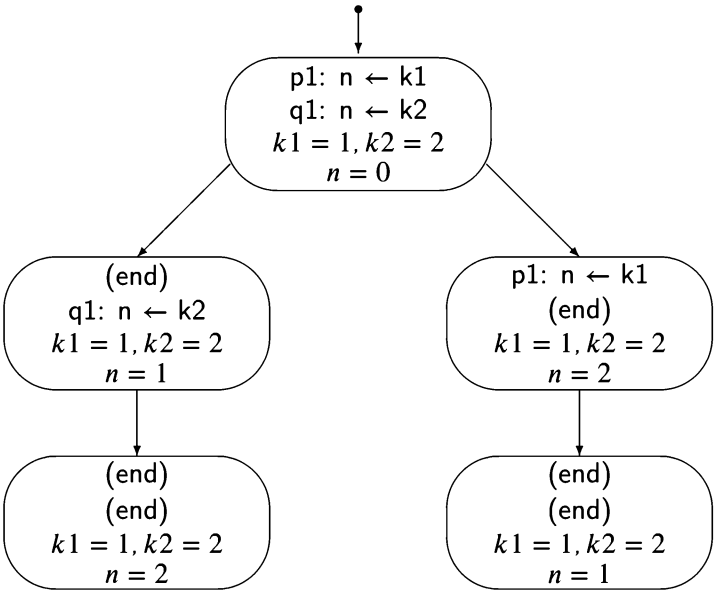
At any time during the execution of this program, it must be in a state defined by the value of the control pointer and the values of the three variables. Executing a statement corresponds to making a transition from one state to another. It is clear that this program can be in one of three states: an initial state and two other states

[2]The word Java will be used as an abbreviation for the Java programming language.

obtained by executing the two statements. This is shown in the following diagram, where a node represents a state, arrows represent the transitions, and the initial state is pointed to by the short arrow on the left:



Consider now the trivial concurrent program Algorithm 2.1. There are two processes, so the state must include the control pointers of both processes. Furthermore, in the initial state there is a choice as to which statement to execute, so there are two transitions from the initial state.



The lefthand states correspond to executing p1 followed by q1, while the righthand states correspond to executing q1 followed by p1. Note that the computation can terminate in two different states (with different values of n), depending on the interleaving of the statements.

Definition 2.3 The *state* of a (concurrent) algorithm is a tuple[3] consisting of one element for each process that is a label from that process, and one element for each global or local variable that is a value whose type is the same as the type of the variable. ∎

[3]The word *tuple* is a generalization of the sequence *pair*, *triple*, *quadruple*, etc. It means an ordered sequence of values of any fixed length.

The number of possible states—the number of tuples—is quite large, but in an execution of a program, not all possible states can occur. For example, since no values are assigned to the variables k1 and k2 in Algorithm 2.1, no state can occur in which these variables have values that are different from their initial values.

Definition 2.4 Let s_1 and s_2 be states. There is a *transition* between s_1 and s_2 if executing a statement in state s_1 changes the state to s_2. The statement executed must be one of those pointed to by a control pointer in s_1. ▮

Definition 2.5 A *state diagram* is a graph defined inductively. The initial state diagram contains a single node labeled with the initial state. If state s_1 labels a node in the state diagram, and if there is a transition from s_1 to s_2, then there is a node labeled s_2 in the state diagram and a directed edge from s_1 to s_2.

For each state, there is only one node labeled with that state.

The set of *reachable states* is the set of states in a state diagram. ▮

It follows from the definitions that a computation (scenario) of a concurrent program is represented by a directed path through the state diagram starting from the initial state, and that all computations can be so represented. Cycles in the state diagram represent the possibility of infinite computations in a finite graph.

The state diagram for Algorithm 2.1 shows that there are two different scenarios, each of which contains three of the five reachable states.

Before proceeding, you may wish to read the supplementary Section 2.14, which describes the state diagram for an interesting puzzle.

Scenarios

A scenario is defined by a sequence of states. Since diagrams can be hard to draw, especially for large programs, it is convenient to use a tabular representation of scenarios. This is done simply by listing the sequence of states in a table; the columns for the control pointers are labeled with the processes and the columns for the variable values with the variable names. The following table shows the scenario of Algorithm 2.1 corresponding to the lefthand path:

Process p	Process q	n	k1	k2
p1: n←k1	q1: n←k2	0	1	2
(end)	**q1: n←k2**	1	1	2
(end)	(end)	2	1	2

In a state, there may be more than one statement that can be executed. We use bold font to denote the statement that was executed to get to the state in the following row.

> Rows represent *states*. If the statement executed is an assignment statement, the new value that is assigned to the variable is a component of the *next state* in the scenario, which is found in the *next row*.

At first this may be confusing, but you will soon get used to it.

2.3 Justification of the abstraction

Clearly, it doesn't make sense to talk of the global state of a computer system, or of coordination between computers at the level of individual instructions. The electrical signals in a computer travel at the speed of light, about 2×10^8 m/sec,[4] and the clock cycles of modern CPUs are at least one gigahertz, so information cannot travel more than $2 \times 10^8 \cdot 10^{-9} = 0.2$ m during a clock cycle of a CPU. There is simply not enough time to coordinate individual instructions of more than one CPU.

Nevertheless, that is precisely the abstraction that we will use! We will assume that we have a "bird's-eye" view of the global state of the system, and that a statement of one process executes by itself and to completion, before the execution of a statement of another process commences.

It is a convenient fiction to regard the execution of a concurrent program as being carried out by a global entity who at each step selects the process from which the next statement will be executed. The term interleaving comes from this image: just as you might interleave the cards from several decks of playing cards by selecting cards one by one from the decks, so we regard this entity as interleaving statements by selecting them one by one from the processes. The interleaving is arbitrary, that is—with one exception to be discussed in Section 2.7—we do *not* restrict the choice of the process from which the next statement is taken.

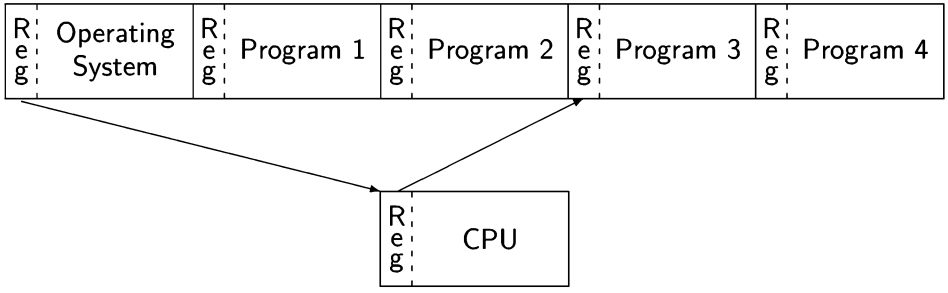
The abstraction defined is highly artificial, so we will spend some time justifying it for various possible computer architectures.

[4]The speed of light in a metal like copper is much less than it is in a vacuum.

Multitasking systems

Consider the case of a concurrent program that is being executed by multitasking, that is, by sharing the resources of one computer. Obviously, with a single CPU there is no question of the simultaneous execution of several instructions. The selection of the next instruction to execute is carried out by the CPU and the operating system. Normally, the next instruction is taken from the same process from which the current instruction was executed; occasionally, interrupts from I/O devices or internal timers will cause the execution to be interrupted. A new process called an *interrupt handler* will be executed, and upon its completion, an operating system function called the scheduler may be invoked to select a new process to execute.

This mechanism is called a *context switch*. The diagram below shows the memory divided into five segments, one for the operating system code and data, and four for the code and data of the programs that are running concurrently:

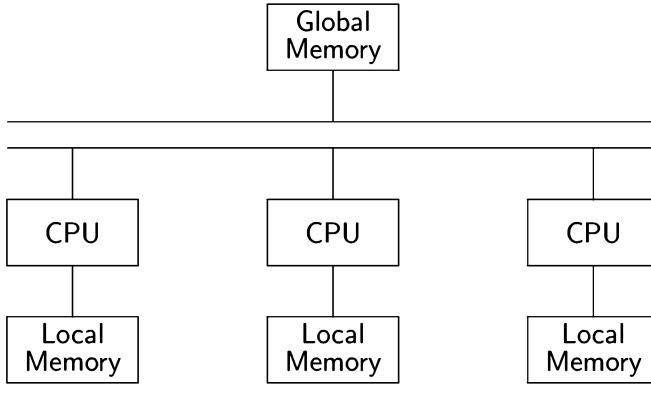


When the execution is interrupted, the registers in the CPU (not only the registers used for computation, but also the control pointer and other registers that point to the memory segment used by the program) are saved into a prespecified area in the program's memory. Then the register contents required to execute the interrupt handler are loaded into the CPU. At the conclusion of the interrupt processing, the symmetric context switch is performed, storing the interrupt handler registers and loading the registers for the program. The end of interrupt processing is a convenient time to invoke the operating system scheduler, which may decide to perform the context switch with another program, not the one that was interrupted.

In a multitasking system, the non-intuitive aspect of the abstraction is not the interleaving of atomic statements (that actually occurs), but the requirement that any *arbitrary* interleaving is acceptable. After all, the operating system scheduler may only be called every few milliseconds, so many thousands of instructions will be executed from each process before any instructions are interleaved from another. We defer a discussion of this important point to Section 2.4.

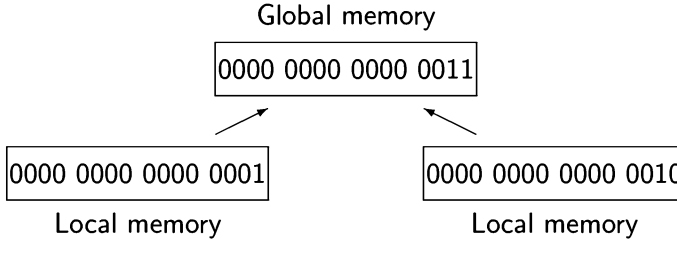
Multiprocessor computers

A multiprocessor computer is a computer with more than one CPU. The memory is physically divided into banks of *local memory*, each of which can be accessed only by one CPU, and *global memory*, which can be accessed by all CPUs:



If we have a sufficient number of CPUs, we can assign each process to its own CPU. The interleaving assumption no longer corresponds to reality, since each CPU is executing its instructions independently. Nevertheless, the abstraction is useful here.

As long as there is no *contention*, that is, as long as two CPUs do not attempt to access the same resource (in this case, the global memory), the computations defined by interleaving will be indistinguishable from those of truly parallel execution. With contention, however, there is a potential problem. The memory of a computer is divided into a large number of cells that store data which is read and written by the CPU. Eight-bit cells are called *bytes* and larger cells are called *words*, but the size of a cell is not important for our purposes. We want to ask what might happen if two processors try to read or write a cell simultaneously so that the operations overlap. The following diagram indicates the problem:



It shows 16-bit cells of local memory associated with two processors; one cell contains the value $0 \cdots 01$ and one contains $0 \cdots 10 = 2$. If both processors write to the cell of global memory at the same time, the value might be undefined; for example, it might be the value $0 \cdots 11 = 3$ obtained by or'ing together the bit representations of 1 and 2.

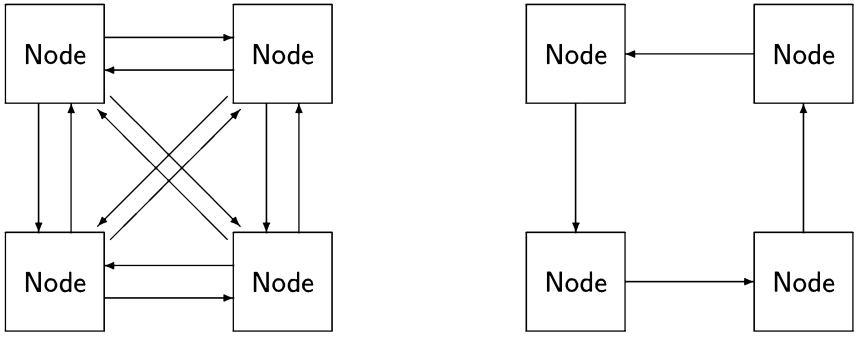
In practice, this problem does not occur because memory hardware is designed so that (for some size memory cell) one access completes before the other commences. Therefore, we can assume that if two CPUs attempt to read or write the same cell in global memory, the result is the same *as if* the two instructions were executed in either order. In effect, atomicity and interleaving are performed by the hardware.

Other less restrictive abstractions have been studied; we will give one example of an algorithm that works under the assumption that if a read of a memory cell overlaps a write of the same cell, the read may return an arbitrary value (Section 5.3).

The requirement to allow arbitrary interleaving makes a lot of sense in the case of a multiprocessor; because there is no central scheduler, any computation resulting from interleaving may certainly occur.

Distributed systems

A distributed system is composed of several computers that have no global resources; instead, they are connected by communications *channels* enabling them to send messages to each other. The language of graph theory is used in discussing distributed systems; each computer is a *node* and the nodes are connected by *(directed) edges*. The following diagram shows two possible schemes for interconnecting nodes: on the left, the nodes are fully connected while on the right they are connected in a ring:



In a distributed system, the abstraction of interleaving is, of course, totally false, since it is impossible to coordinate each node in a geographically distributed system. Nevertheless, interleaving is a very useful fiction, because as far as each node is concerned, it only sees discrete events: it is either executing one of its own statements, sending a message or receiving a message. *Any* interleaving of all the events of all the nodes can be used for reasoning about the system, as long as the interleaving is consistent with the statement sequences of each individual node and with the requirement that a message be sent before it is received.

Distributed systems are considered to be distinct from concurrent systems. In a concurrent system implemented by multitasking or multiprocessing, the global memory is accessible to all processes and each one can access the memory efficiently. In a distributed system, the nodes may be geographically distant from each other, so we cannot assume that each node can send a message *directly* to all other nodes. In other words, we have to consider the *topology* or *connectedness* of the system, and the quality of an algorithm (its simplicity or efficiency) may be dependent on a specific topology. A fully connected topology is extremely efficient in that any node can send a message directly to any other node, but it is extremely expensive, because for n nodes, we need $n \cdot (n-1) \approx n^2$ communications channels. The ring topology has minimal cost in that any node has only one communications line associated with it, but it is inefficient, because to send a message from one arbitrary node to another we may need to have it relayed through up to $n-2$ other nodes.

A further difference between concurrent and distributed systems is that the behavior of systems in the presence of faults is usually studied within distributed systems. In a multitasking system, hardware failure is usually catastrophic since it affects all processes, while a software failure may be relatively innocuous (if the process simply stops working), though it can be catastrophic (if it gets stuck in an infinite loop at high priority). In a distributed system, while failures can be catastrophic for single nodes, it is usually possible to diagnose and work around a faulty node, because messages may be relayed through alternate communication paths. In fact, the success of the Internet can be attributed to the robustness of its protocols when individual nodes or communications channels fail.

2.4 Arbitrary interleaving

We have to justify the use of *arbitrary* interleavings in the abstraction. What this means, in effect, is that we ignore *time* in our analysis of concurrent programs. For example, the hardware of our system may be such that an interrupt can occur only once every millisecond. Therefore, we are tempted to assume that several

thousand statements are executed from a single process before any statements are executed from another. Instead, we are going to assume that after the execution of *any* statement, the next statement may come from any process. What is the justification for this abstraction?

The abstraction of arbitrary interleaving makes concurrent programs amenable to formal analysis, and as we shall see, formal analysis is necessary to ensure the correctness of concurrent programs. Arbitrary interleaving ensures that we only have to deal with finite or countable sequences of statements $a_1, a_2, a_3, \ldots$, and need not analyze the actual time intervals between the statements. The only relation between the statements is that a_i precedes or follows (or immediately precedes or follows) a_j. Remember that we did not specify what the atomic statements are, so you can choose the atomic statements to be as coarse-grained or as fine-grained as you wish. You can initially write an algorithm and prove its correctness under the assumption that each function call is atomic, and then refine the algorithm to assume only that each statement is atomic.

The second reason for using the arbitrary interleaving abstraction is that it enables us to build systems that are robust to modification of their hardware and software. Systems are always being upgraded with faster components and faster algorithms. If the correctness of a concurrent program depended on assumptions about time of execution, every modification to the hardware or software would require that the system be rechecked for correctness (see [62] for an example). For example, suppose that an operating system had been proved correct under the assumption that characters are being typed in at no more than 10 characters per terminal per second. That is a conservative assumption for a human typist, but it would become invalidated if the input were changed to come from a communications channel.

The third reason is that it is difficult, if not impossible, to precisely repeat the execution of a concurrent program. This is certainly true in the case of systems that accept input from humans, but even in a fully automated system, there will always be some *jitter*, that is some unevenness in the timing of events. A concurrent program cannot be "debugged" in the familiar sense of diagnosing a problem, correcting the source code, recompiling and rerunning the program to check if the bug still exists. Rerunning the program may just cause it to execute a different scenario than the one where the bug occurred. The solution is to develop programming and verification techniques that ensure that a program is correct under *all* interleavings.

2.5 Atomic statements

The concurrent programming abstraction has been defined in terms of the interleaving of *atomic* statements. What this means is that an atomic statement is executed to completion without the possibility of interleaving statements from another process. An important property of atomic statements is that if two are executed "simultaneously," the result is the same as if they had been executed sequentially (in either order). The inconsistent memory store shown on page 15 will not occur.

It is important to specify the atomic statements precisely, because the correctness of an algorithm depends on this specification. We start with a demonstration of the effect of atomicity on correctness, and then present the specification used in this book.

Recall that in our algorithms, each labeled line represents an atomic statement. Consider the following trivial algorithm:

Algorithm 2.3: Atomic assignment statements	
integer n ← 0	
p	**q**
p1: n ← n + 1	q1: n ← n + 1

There are two possible scenarios:

Process p	Process q	n
p1: n←n+1	q1: n←n+1	0
(end)	**q1: n←n+1**	1
(end)	(end)	2

Process p	Process q	n
p1: n←n+1	**q1: n←n+1**	0
p1: n←n+1	(end)	1
(end)	(end)	2

In both scenarios, the final value of the global variable n is 2, and the algorithm is a correct concurrent algorithm with respect to the postcondition $n = 2$.

Now consider a modification of the algorithm, in which each atomic statement references the global variable n at most once:

Algorithm 2.4: Assignment statements with one global reference	
integer n ← 0	
p	**q**
integer temp	integer temp
p1: temp ← n	q1: temp ← n
p2: n ← temp + 1	q2: n ← temp + 1

There are scenarios of the algorithm that are also correct with respect to the post-condition $n = 2$:

Process p	Process q	n	p.temp	q.temp
p1: temp←n	q1: temp←n	0	?	?
p2: n←temp+1	q1: temp←n	0	0	?
(end)	**q1: temp←n**	1		?
(end)	**q2: n←temp+1**	1		1
(end)	(end)	2		

As long as p1 and p2 are executed immediately one after the other, and similarly for q1 and q2, the result will be the same as before, because we are simulating the execution of n←n+1 with two statements. However, other scenarios are possible in which the statements from the two processes are interleaved:

Process p	Process q	n	p.temp	q.temp
p1: temp←n	q1: temp←n	0	?	?
p2: n←temp+1	**q1: temp←n**	0	0	?
p2: n←temp+1	q2: n←temp+1	0	0	0
(end)	**q2: n←temp+1**	1		0
(end)	(end)	1		

Clearly, Algorithm 2.4 is *not* correct with respect to the postcondition $n = 2$.[5]

We learn from this simple example that the correctness of a concurrent program is relative to the specification of the atomic statements. The convention in the book is that:

> Assignment statements are atomic statements, as are evaluations of boolean conditions in control statements.

This assumption is not at all realistic, because computers do not execute source code assignment and control statements; rather, they execute machine-code instructions that are defined at a much lower level. Nevertheless, by the simple expedient of defining local variables as we did above, we can use this simple model to demonstrate the same behaviors of concurrent programs that occur when machine-code instructions are interleaved. The source code programs might look artificial, but the convention spares us the necessity of using machine code during the study of concurrency. For completeness, Section 2.8 provides more detail on the interleaving of machine-code instructions.

[5]Unexpected results that are caused by interleaving are sometimes called *race conditions*.

2.6 Correctness

In sequential programs, rerunning a program with the same input will always give the same result, so it makes sense to "debug" a program: run and rerun the program with breakpoints until a problem is diagnosed; fix the source code; rerun to check if the output is not correct. In a concurrent program, some scenarios may give the correct output while others do not. You cannot debug a concurrent program in the normal way, because each time you run the program, you will likely get a different scenario. The fact that you obtain the correct answer may just be a fluke of the particular scenario and not the result of fixing a bug.

In concurrent programming, we are interested in problems—like the problem with Algorithm 2.4—that occur *as a result of interleaving*. Of course, concurrent programs can have ordinary bugs like incorrect bounds of loops or indices of arrays, but these present no difficulties that were not already present in sequential programming. The computations in examples are typically trivial such as incrementing a single variable, and in many cases, we will not even specify the actual computations that are done and simply abstract away their details, leaving just the name of a procedure, such as "critical section." Do not let this fool you into thinking that concurrent programs are toy programs; we will use these simple programs to develop algorithms and techniques that can be added to otherwise correct programs (of any complexity) to ensure that correctness properties are fulfilled in the presence of interleaving.

For sequential programs, the concept of *correctness* is so familiar that the formal definition is often neglected. (For reference, the definition is given in Appendix B.) Correctness of (non-terminating) concurrent programs is defined in terms of properties of computations, rather than in terms of computing a functional result. There are two types of correctness properties:

Safety properties The property must *always* be true.

Liveness properties The property must *eventually* become true.

More precisely, for a safety property P to hold, it must be true that in *every* state of *every* computation, P is true. For example, we might require as a safety property of the user interface of an operating system: *Always, a mouse cursor is displayed.* If we can prove this property, we can be assured that no customer will ever complain that the mouse cursor disappears, no matter what programs are running on the system.

For a liveness property P to hold, it must be true that in *every* computation there is *some* state in which P is true. For example, a liveness property of an operating

system might be: *If you click on a mouse button, eventually the mouse cursor will change shape*. This specification allows the system not to respond immediately to the click, but it does ensure that the click will not be ignored indefinitely.

It is very easy to write a program that will satisfy a safety property. For example, the following program for an operating system satisfies the safety property *Always, a mouse cursor is displayed*:

```
while true
    display the mouse cursor
```

I seriously doubt if you would find users for an operating system whose only feature is to display a mouse cursor. Furthermore, safety properties often take the form of *Always, something "bad" is not true*, and this property is trivially satisfied by an empty program that does nothing at all. The challenge is to write concurrent programs that do useful things—thus satisfying the liveness properties—without violating the safety properties.

Safety and liveness properties are *duals* of each other. This means that the negation of a safety property is a liveness property and vice versa. Suppose that we want to prove the safety property *Always, a mouse cursor is displayed.* The negation of this property is a liveness property: *Eventually, no mouse cursor will be displayed.* The safety property will be true if and only if the liveness property is false. Similarly, the negation of the liveness property *If you click on a mouse button, eventually the cursor will change shape*, can be expressed as *Once a button has been clicked, always, the cursor will not change its shape*. The liveness property is true if this safety property is false. One of the forms will be more natural to use in a specification, though the dual form may be easier to prove.

Because correctness of concurrent programs is defined on *all* scenarios, it is impossible to demonstrate the correctness of a program by testing it. Formal methods have been developed to verify concurrent programs, and these are extensively used in critical systems.

Linear and branching temporal logics[A]

The formalism we use in this book for expressing correctness properties and verifying programs is called *linear temporal logic (LTL)*. LTL expresses properties that must be true at a state in a single (but arbitrary) scenario. *Branching temporal logic* is an alternate formalism; in these logics, you can express that for a property to be true at state, it must be true in *some* or *all* scenarios starting from the state. CTL [24] is a branching temporal logic that is widely used especially in the verification

of computer hardware. Model checkers for CTL are SMV and NuSMV. LTL is used in this book both because it is simpler and because it is more appropriate for reasoning about software algorithms.

2.7 Fairness

There is one exception to the requirement that any arbitrary interleaving is a valid execution of a concurrent program. Recall that the concurrent programming abstraction is intended to represent a collection of independent computers whose instructions are interleaved. While we clearly stated that we did not wish to assume anything about the absolute speeds at which the various processors are executing, it does not make sense to assume that statements from any specific process are *never* selected in the interleaving.

Definition 2.6 A scenario is *(weakly) fair* if at any state in the scenario, a statement that is continually enabled eventually appears in the scenario. ∎

If after constructing a scenario up to the ith state $s_0, s_1, \ldots, s_i$, the control pointer of a process p points to a statement pj that is continually enabled, then pj will appear in the scenario as s_k for $k > i$. Assignment and control statements are continually enabled. Later we will encounter statements that may be disabled, as well as other (stronger) forms of fairness.

Consider the following algorithm:

Algorithm 2.5: Stop the loop A	
integer n ← 0 boolean flag ← false	
p	**q**
p1: while flag = false p2: n ← 1 − n	q1: flag ← true q2:

Let us ask the question: does this algorithm *necessarily* halt? That is, does the algorithm halt for all scenarios? Clearly, the answer is no, because one scenario is p1, p2, p1, p2, ..., in which p1 and then p2 are always chosen, and q1 is never chosen. Of course this is not what was intended. Process q is continually ready to run because there is no impediment to executing the assignment to flag, so the non-terminating scenario is not fair. If we allow only fair scenarios, then eventually an execution of q1 must be included in every scenario. This causes process q to terminate immediately, and process p to terminate after executing at most two more

statements. We will say that under the assumption of weak fairness, the algorithm is correct with respect to the claim that it always terminates.

2.8 Machine-code instructions[A]

Algorithm 2.4 may seem artificial, but it a faithful representation of the actual implementation of computer programs. Programs written in a programming language like Ada or Java are compiled into machine code. In some cases, the code is for a specific processor, while in other cases, the code is for a virtual machine like the *Java Virtual Machine (JVM)*. Code for a virtual machine is then interpreted or a further compilation step is used to obtain code for a specific processor. While there are many different computer architectures—both real and virtual—they have much in common and typically fall into one of two categories.

Register machines

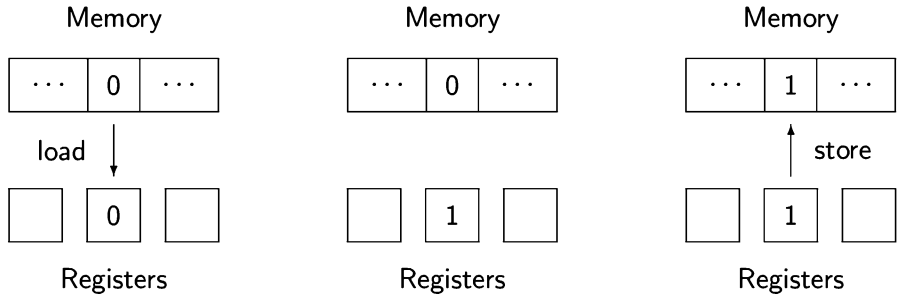
A *register machine* performs all its computations in a small amount of high-speed memory called registers that are an integral part of the CPU. The source code of the program and the data used by the program are stored in large banks of memory, so that much of the machine code of a program consists of load instructions, which move data from memory to a register, and store instructions, which move data from a register to memory. load and store of a memory cell (byte or word) is atomic.

The following algorithm shows the code that would be obtained by compiling Algorithm 2.3 for a register machine:

Algorithm 2.6: Assignment statement for a register machine	
integer n ← 0	
p	**q**
p1: load R1,n	q1: load R1,n
p2: add R1,#1	q2: add R1,#1
p3: store R1,n	q3: store R1,n

The notation add R1,#1 means that the value 1 is added to the contents of register R1, rather than the contents of the memory cell whose address is 1.

The following diagram shows the execution of the three instructions:



First, the value stored in the memory cell for n is loaded into one of the registers; second, the value is incremented within the register; and third, the value is stored back into the memory cell.

Ostensibly, both processes are using the same register R1, but in fact, each process keeps its own copy of the registers. This is true not only on a multiprocessor or distributed system where each CPU has its own set of registers, but even on a multitasking single-CPU system, as described in Section 2.3. The context switch mechanism enables each process to run within its own context consisting of the current data in the computational registers and other registers such as the control pointer. Thus we can look upon the registers as analogous to the local variables temp in Algorithm 2.4 and a bad scenario exists that is analogous to the bad scenario for that algorithm:

Process p	Process q	n	p.R1	q.R1
p1: load R1,n	q1: load R1,n	0	?	?
p2: add R1,#1	**q1: load R1,n**	0	0	?
p2: add R1,#1	q2: add R1,#1	0	0	0
p3: store R1,n	**q2: add R1,#1**	0	1	0
p3: store R1,n	q3: store R1,n	0	1	1
(end)	**q3: store R1,n**	1		1
(end)	(end)	1		

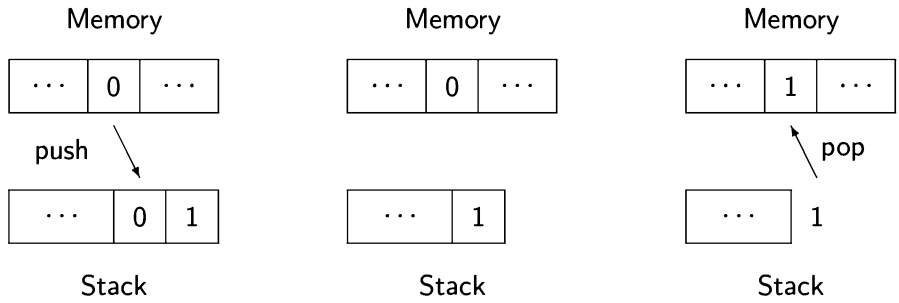
Stack machines

The other type of machine architecture is the *stack machine*. In this architecture, data is held not in registers but on a stack, and computations are implicitly performed on the top elements of a stack. The atomic instructions include push and

pop, as well as instructions that perform arithmetical, logical and control operations on elements of the stack. In the register machine, the instruction add R1,#1 explicitly mentions its operands, while in a stack machine the instruction would simply be written add, and it would add the values in the top two positions in the stack, leaving the result on the top in place of the two operands:

Algorithm 2.7: Assignment statement for a stack machine	
integer n ← 0	
p	**q**
p1: push n	q1: push n
p2: push #1	q2: push #1
p3: add	q3: add
p4: pop n	q4: pop n

The following diagram shows the execution of these instructions on a stack machine:



Initially, the value of the memory cell for n is pushed onto the stack, along with the constant 1. Then the two top elements of the stack are added and replaced by one element with the result. Finally (on the right), the result is popped off the stack and stored in the memory cell. Each process has its own stack, so the top of the stack, where the computation takes place, is analogous to a local variable.

It is easier to write code for a stack machine, because all computation takes place in one place, whereas in a register machine with more than one computational register you have to deal with the allocation of registers to the various operands. This is a non-trivial task that is discussed in textbooks on compilation. Furthermore, since different machines have different numbers and types of registers, code for register machines is not portable. On the other hand, operations with registers are extremely fast, whereas if a stack is implemented in main memory access to operands will be much slower.

For these reasons, stack architectures are common in virtual machines where simplicity and portability are most important. When efficiency is needed, code for the virtual machine can be compiled and optimized for a specific real machine. For our purposes the difference is not great, as long as we have the concept of memory that may be either global to all processes or local to a single process. Since global and local variables exist in high-level programming languages, we do not need to discuss machine code at all, as long as we understand that some local variables are being used as surrogates for their machine-language equivalents.

Source statements and machine instructions

We have specified that source statements like

```
n ← n + 1
```

are atomic. As shown in Algorithm 2.6 and Algorithm 2.7, such source statements must be compiled into a sequence of machine language instructions. (On some computers n←n+1 can be compiled into an atomic increment instruction, but this is not true for a general assignment statement.) This increases the number of interleavings and leads to incorrect results that would not occur if the source statement were really executed atomically.

However, we can study concurrency at the level of source statements by decomposing atomic statements into a sequence of simpler source statements. This is what we did in Algorithm 2.4, where the above atomic statement was decomposed into the pair of statements:

```
temp ← n + 1
n ← temp + 1
```

The concept of a "simple" source statement can be formally defined:

Definition 2.7 An occurrence of a variable v is defined to be *critical reference*: (a) if it is assigned to in one process and has an occurrence in another process, or (b) if it has an occurrence in an expression in one process and is assigned to in another.

A program satisfies the *limited-critical-reference (LCR)* restriction if each statement contains at most one critical reference. ∎

Consider the first occurrence of n in n←n+1. It is assigned to in process p and has (two) occurrences in process q, so it is critical by (a). The second occurrence of n in n←n+1 is critical by (b) because it appears in the expression n+1 in p

and is also assigned to in q. Consider now the version of the statements that uses local variables. Again, the occurrences of n are critical, but the occurrences of temp are *not*. Therefore, the program satisfies the LCR restriction. Concurrent programs that satisfy the LCR restriction yield the same set of behaviors whether the statements are considered atomic or are compiled to a machine architecture with atomic load and store. See [50, Section 2.2] for more details.

The more advanced algorithms in this book do not satisfy the LCR restriction, but can easily be transformed into programs that do satisfy it. The transformed programs may require additional synchronization to prevent incorrect scenarios, but the additions are simple and do not contribute to understanding the concepts of the advanced algorithms.

2.9 Volatile and non-atomic variables^A

There are further complications that arise from the compilation of high-level languages into machine code. Consider for example, the following algorithm:

Algorithm 2.8: Volatile variables	
integer n ← 0	
p	**q**
integer local1, local2	integer local
p1: n ← *some expression*	q1: local ← n + 6
p2: *computation not using n*	q2:
p3: local1 ← (n + 5) ∗ 7	q3:
p4: local2 ← n + 5	q4:
p5: n ← local1 * local2	q5:

The single statement in process q can be interleaved at any place during the execution of the statements of p. Because of optimization during compilation, the computation in q may not use the *most recent* value of n. The value of n may be maintained in a register from the assignment in p1 through the computations in p2, p3 and p4, and only actually stored back into n at statement p5. Furthermore, the compiler may re-order p3 and 4 to take advantage of the fact that the value of n+5 needed in p3 is computed in p4.

These optimizations have no semantic effect on sequential programs, but they do in concurrent programs that use global variables, so they can cause programs to be incorrect. Specifying a variable as *volatile* instructs the compiler to load and store the value of the variable at each use, rather than attempt to optimize away these loads and stores.

Concurrency may also affect computations with multiword variables. A load or store of a full-word variable (32 bits on most computers) is accomplished atomically, but if you need to load or store longer variables (like higher precision numbers), the operation might be carried out non-atomically. A load from another process might be interleaved between storing the lower half and the upper half of a 64-bit variable. If the processor is not able to ensure atomicity for multiword variables, it can be implemented using a synchronization mechanism such as those to be discussed throughout the book. However, these mechanisms can block processes, which may not be acceptable in a real-time system; a non-blocking algorithm for consistent access to multiword variables is presented in Section 13.9.

2.10 The BACI concurrency simulator^L

In Section 2.3, we argued that the proper abstraction for concurrent programs is the arbitrary interleaving of the atomic statements of sequential processes. The main reason why this abstraction is appropriate is that we have no control over the relative rate at which processes execute or over the occurrence of external stimuli that drive reactive programs. For this reason, the normal execution of a program on a computer is *not* the best way to study concurrency. Later in this section, we discuss the implementation of concurrency in two programming languages, because eventually you will wish to write concurrent programs using the constructs available in real languages, but for studying concurrency there is a better way.

A *concurrency simulator* is a software tool that interprets a concurrent program under the fine-grained control of the user. Rather than executing or interpreting the program as fast as possible, a concurrency simulator allows the user to control the interleaving at the level of atomic statements. The simulator maintains the data structures required to emulate concurrent execution: separate stacks and registers for each process. After interpreting a single statement of the program, the simulator updates the display of the state of the program, and then optionally halts to enable the user to choose the process from which the next statement will be taken. This fine-grained control is essential if you want to create potentially incorrect scenarios, such as the one shown in Sections 2.5 and 2.8. For this reason, we recommend that you use a concurrency simulator in your initial study of the topic.

This textbook is intended to be used with one of two concurrency simulators: Spin or BACI. The Spin system is described later in Section 4.6 and Appendix D.2. Spin is primarily a professional verification tool called a model checker, but it can also be used as a concurrency simulator. BACI is a concurrency simulator designed as an educational tool and is easier to use than Spin; you may wish to begin your studies using BACI and then move on to Spin as your studies progress. In this

section, we will explain how to translate concurrent algorithms into programs for the BACI system.

BACI (*Ben-Ari Concurrency Interpreter*) consists of a compiler and an interpreter that simulates concurrency. BACI and the user interface jBACI are described in Appendix D.1. We now describe how to write the following algorithm in the dialects of Pascal and C supported by the BACI concurrency simulator:

Algorithm 2.9: Concurrent counting algorithm	
integer n ← 0	
p	**q**
integer temp	integer temp
p1: do 10 times	q1: do 10 times
p2: temp ← n	q2: temp ← n
p3: n ← temp + 1	q3: n ← temp + 1

The algorithm simply increments a global variable twenty times, ten times in each of two processes.

The Pascal version of the counting algorithm is shown in Listing 2.1.[6] The main difference between this program and a normal Pascal program is the **cobegin** ... **coend** compound statement in line 22. Syntactically, the only statements that may appear within this compound statement are procedure calls. Semantically, upon executing this statement, the procedures named in the calls are initialized as concurrent processes (in addition to the main program which is also considered to be a process). When the **coend** statement is reached, the processes are allowed to execute concurrently, and the main process is blocked until all of the new processes have terminated. Of course, if one or more processes do not terminate, the main process will be permanently blocked.

The C version of the counting algorithm is shown in Listing 2.2. The compound statement **cobegin** { } in line 17 is used to create concurrent processes as described above for Pascal. One major difference between the BACI dialect of C and standard C is the use of I/O statements taken from C++.

The BACI dialects support other constructs for implementing concurrent programs, and these will be described in due course. In addition, there are a few minor differences between the standard languages and these dialects that are described in the BACI documentation.

[6]By convention, to execute a loop n times, indices in Pascal and Ada range from 1 to n, while indices in C and Java range from 0 to $n - 1$.

```
1  program count;
2  var n: integer := 0;
3  procedure p;
4  var temp, i : integer;
5  begin
6    for i := 1 to 10 do
7      begin
8      temp := n;
9      n := temp + 1
10     end
11 end;
12 procedure q;
13 var temp, i : integer;
14 begin
15   for i := 1 to 10 do
16     begin
17     temp := n;
18     n := temp + 1
19     end
20 end;
21 begin
22   cobegin p; q coend;
23   writeln('The value of n is ', n)
24 end.
```

Listing 2.1: A Pascal program for the counting algorithm

2.11 Concurrency in Ada^L

The Ada programming language was one of the first languages to include support for concurrent programming as part of the standard language definition, rather than as part of a library or operating system. Listing 2.3 shows an Ada program implementing Algorithm 2.9. The Ada term for process is *task*. A task is declared with a specification and a body, though in this case the specification is empty. Tasks are declared as objects of the task type in the declarative part of a subprogram. It is activated at the subsequent **begin** of the subprogram body. A subprogram will not terminate until all tasks that it activated terminate. In this program, we wish to print out the final value of the variable N, so we use an inner block (**declare** ... **begin** ... **end**) which will not terminate until both tasks P and Q have terminated. When the block terminates, we can print the final value of N. If we did not need to print this value, the task objects could be global and the main

```
int n = 0;
void p() {
  int temp, i;
  for (i = 0; i < 10; i++) {
    temp = n;
    n = temp + 1;
  }
}
void q() {
  int temp, i;
  for (i = 0; i < 10; i++) {
    temp = n;
    n = temp + 1;
  }
}
void main() {
  cobegin { p(); q(); }
  cout << "The value of n is " << n << "\n";
}
```

Listing 2.2: A C program for the counting algorithm

program would simply be **null**; after executing the null statement, the program waits for the task to terminate:

```
procedure Count is
    task type Count_Task;
    task body Count_Task is ...end Count_Task;
    P, Q: Count_Task;
begin
    null;
end Count;
```

Of course, if you run this program, you will almost certainly get the answer 20, because it is highly unlikely that there will be a context switch from one task to another during the loop. We can artificially introduce context switches by writing the statement **delay** 0.0 between the two statements in the loop. When I ran this version of the program, it consistently printed 10.

```
1  with Ada.Text_IO; use Ada.Text_IO;
2  procedure Count is
3     N: Integer  := 0;
4     pragma Volatile(N);
5     task type Count_Task;
6     task body Count_Task is
7        Temp: Integer;
8     begin
9        for I in 1..10 loop
10          Temp := N;
11          N := Temp + 1;
12       end loop;
13    end Count_Task;
14 begin
15    declare
16       P, Q: Count_Task;
17    begin
18       null;
19    end;
20    Put_Line("The value of N is " & Integer'Image(N));
21 end Count;
```

Listing 2.3: An Ada program for the counting algorithm

Volatile and atomic

A type or variable can be declared as volatile or atomic (Section 2.9) using **pragma** Volatile or **pragma** Atomic:[7]

```
N: Integer  := 0;
pragma Volatile(N);
```

pragma Volatile_Components and **pragma** Atomic_Components are used for arrays to ensure that the components of an array are volatile or atomic. An atomic type or variable is also volatile. It is implementation-dependent which types can be atomic.

[7]These constructs are described in the Systems Programming Annex (C.6) of the Ada Language Reference Manual.

```
1  class Count extends Thread {
2      static volatile int n = 0;
3      public void run() {
4        int temp;
5        for (int i = 0; i < 10; i++) {
6          temp = n;
7          n = temp + 1;
8        }
9      }
10     public static void main(String[] args) {
11       Count p = new Count();
12       Count q = new Count();
13       p.start ();
14       q.start ();
15       try {
16         p.join ();
17         q.join ();
18       }
19       catch (InterruptedException e) { }
20       System.out.println ("The value of n is " + n);
21     }
22 }
```

Listing 2.4: A Java program for the counting algorithm

2.12 Concurrency in Java^L

The Java language has supported concurrency from the very start. However, the concurrency constructs have undergone modification; in particular, several dangerous constructs have been *deprecated* (dropped from the language). The latest version (1.5) has an extensive library of concurrency constructs java.util.concurrent, designed by a team led by Doug Lea. For more information on this library see the Java documentation and [44]. Lea's book explains how to integrate concurrency within the framework of object-oriented design and programming—a topic not covered in this book.

The Java language supports concurrency through objects of type Thread. You can write your own classes that extend this type as shown in Listing 2.4. Within a class that extends Thread, you have to define a method called run that contains the code to be run by this process. Once the class has been defined, you can define fields of this type and assign to them objects created by allocators (lines 11–12). However,

allocation and construction of a Thread object do not cause the thread to run. You must explicitly call the start method on the object (lines 13–14), which in turn will call the run method.

The global variable n is declared to be **static** so that we will have one copy that is shared by all of the objects declared to be of this class. The main method of the class is used to allocate and initiate the two processes; we then use the join method to wait for termination of the processes so that we can print out the value of n. Invocations of most thread methods require that you catch (or throw) InterruptedException .

If you run this program, you will almost certainly get the answer 20, because it is highly unlikely that there will be a context switch from one task to another during the loop. You can artificially introduce context switches between between the two assignment statements of the loop: static method Thread.yield () causes the currently executing thread to temporarily cease execution, thus allowing other threads to execute.

Since Java does not have multiple inheritance, it is usually not a good idea to extend the class Thread as this will prevent extending any other class. Instead, any class can contain a thread by implementing the interface Runnable:

```
class Count extends JFrame implements Runnable {
   public void run() { ... }
}
```

Threads are created from Runnable objects as follows:

```
Count count = new Count();
Thread t1 = new Thread(count);
```

or simply:

```
Thread t1 = new Thread(new Count());
```

Volatile

A variable can be declared as volatile (Section 2.9):

```
volatile int n = 0;
```

Variables of primitive types except **long** and **double** are atomic, and **long** and **double** are also atomic if they are declared to be **volatile**. A reference variable is

```
1  #include "for.h"
2  #define TIMES 10
3  byte    n = 0;
4  proctype P() {
5      byte temp;
6      for (i,1, TIMES)
7          temp = n;
8          n = temp + 1
9      rof (i)
10 }
11 init {
12     atomic {
13         run P();
14         run P()
15     }
16     (_nr_pr == 1);
17     printf("MSC: The value is %d\n", n)
18 }
```

Listing 2.5: A Promela program for the counting algorithm

atomic, but that does not mean that the object pointed to is atomic. Similarly, if a reference variable is declared as **volatile** it does not follow that the object pointed to is volatile. Arrays can be declared as volatile, but not components of an array.

2.13 Writing concurrent programs in Promela^L

Promela is the model specification language of the model checker Spin, which will be described in Chapter 4. Here we describe enough of the syntax and semantics of the Promela language to write a program for Algorithm 2.9.

The syntax and semantics of declarations and expressions are similar to those of C, but the control structures might be unfamiliar, because they use the syntax and semantics of *guarded commands*, which is a notation commonly used in theoretical computer science. A Promela program for the counting algorithm is shown in Listing 2.5, where we have used macros defined in the file for.h to write a familiar for-loop.

The process type P is declared using the keyword **proctype**; this only declares a type so (anonymous) instances of the type must be activated using **run** P(). If you declare a process with the reserved name **init**, it is the first process that is run.

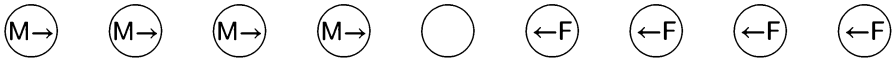
Promela does not have a construct similar to **cobegin**, but we can simulate it using other statements, **run** and **atomic**. The **run** P() statements activate the two processes. **atomic** is used to ensure that both activations are completed before the new processes begin; this prevents meaningless scenarios where one or more processes are never executed.

The join (waiting for both processes to finish before continuing) is simulated by waiting for _nr_pr to have the value 1; this is a predefined variable counting the number of active processes (including **init**).

When the two processes have terminated, the final value of n is printed. The prefix MSC is a convention that is used by postprocessors to display the output of the program.

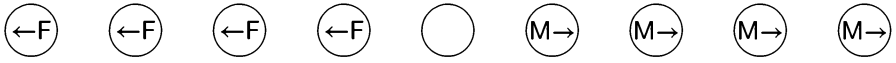
2.14 Supplement: the state diagram for the frog puzzle

This section demonstrates state diagrams using an interesting puzzle. Consider a system consisting of $2n+1$ stones set in a row in a swamp. On the leftmost n stones are male frogs who face right and on the rightmost n stones are female frogs who face left; the middle stone is empty:



Frogs may move in the direction they are facing: jumping to the adjacent stone if it is empty, or if not, jumping over the frog to the second adjacent stone if that stone is empty. For example, the following configuration will result, if the female frog on the seventh stone jumps over the frog on the sixth stone, landing on the fifth stone:

Let us now ask if there is a sequence of moves that will exchange the positions of the male and female frogs:



This puzzle faithfully represents a concurrent program under the model of interleaving of atomic operations. There are $2n$ processes, and one or more of them

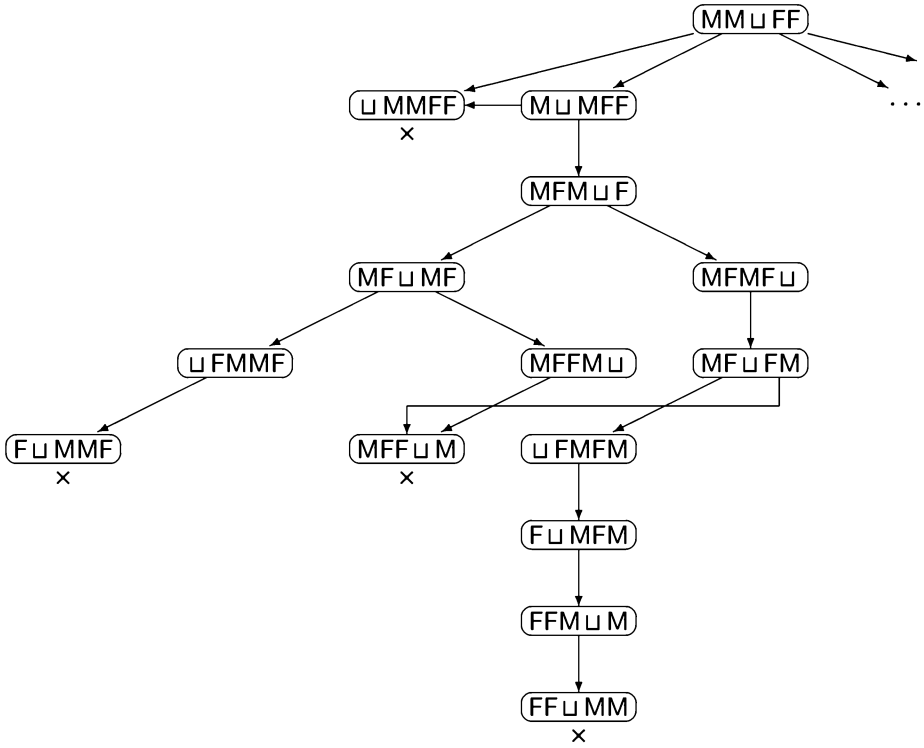


Figure 2.1: State diagram for the frog puzzle

may be able to make a move at any time, although only one of the possible moves is selected. The configuration of the frogs on the stones represents a state, and the transitions are the allowable moves.

It is easy to put an upper bound on the number of possible states: there are $2n$ frogs and one empty place that can be placed one after another giving $(2n+1)!/(n! \cdot n!)$ possible states. The divisor $n! \cdot n!$ comes from the fact that all the male frogs and all the female frogs are identical, so permutations among frogs of the same gender result in identical states. For $n = 2$, there are 30 states, and for $n = 3$, there are 140 states. However, not all states will be reachable in an actual computation.

To build a state diagram, start with the initial state and for each possible move draw the new state; if the new state already exists, draw an arrow to the existing state. For $n = 2$, Figure 2.1 shows all possible states that can occur if the computation starts with a move by a male frog. Note that the state ⊔MMFF can be reached in either one or two moves, and that from this state no further moves are possible as indicated by the symbol ×.

From the diagram we can read off interesting correctness assertions. For example, there is a computation leading to the target state FF ⊔ MM; no computation from state MF ⊔ MF leads to the target state; all computations terminate (trivial, but still nice to know).

Transition

This chapter has presented and justified the concurrent program abstraction as the interleaved execution of atomic statements. The specification of atomic statements is a basic task in the definition of a concurrent system; they can be machine instructions or higher-order statements that are executed atomically. We have also shown how concurrent programs can be written and executed using a concurrency simulator (Pascal and C in BACI), a programming language (Ada and Java) or simulated with a model checker (Promela in Spin). But we have not actually solved any problems in concurrent programming. The next three chapters will focus on the critical section problem, which is the most basic and important problem to be solved in concurrent programming.

Exercises

1. How many computations are there for Algorithm 2.7.
2. Construct a scenario for Algorithm 2.9 in which the final value of n is 10.
3. (Ben-Ari and Burns [10]) Construct a scenario for Algorithm 2.9 in which the final value of n is 2.
4. For positive values of K, what are the possible final values of n in the following algorithm?

Algorithm 2.10: Incrementing and decrementing	
integer n ← 0	
p	**q**
integer temp	integer temp
p1: do K times	q1: do K times
p2: temp ← n	q2: temp ← n
p3: n ← temp + 1	q3: n ← temp − 1

5. (Apt and Olderog [3]) Assume that for the function f, there is some integer value i for which $f(i) = 0$. Here are five concurrent algorithms that search for i. An algorithm is correct if for all scenarios, *both* processes terminate after one of them has found the zero. For each algorithm, show that it is correct or find a scenario that is a counterexample.

Algorithm 2.11: Zero A	
boolean found	
p	**q**
integer i ← 0	integer j ← 1
p1: found ← false	q1: found ← false
p2: while not found	q2: while not found
p3: i ← i + 1	q3: j ← j − 1
p4: found ← f(i) = 0	q4: found ← f(j) = 0

Algorithm 2.12: Zero B	
boolean found ← false	
p	**q**
integer i ← 0	integer j ← 1
p1: while not found	q1: while not found
p2: i ← i + 1	q2: j ← j − 1
p3: found ← f(i) = 0	q3: found ← f(j) = 0

Algorithm 2.13: Zero C	
boolean found ← false	
p	**q**
integer i ← 0	integer j ← 1
p1: while not found	q1: while not found
p2: i ← i + 1	q2: j ← j − 1
p3: if f(i) = 0	q3: if f(j) = 0
p4: found ← true	q4: found ← true

In process p of the following two algorithms, await turn = 1 and turn ← 2 are executed as a one atomic statement when the value of *turn* is 1; similarly, await turn = 2 and turn ← 1 are executed atomically in process q.

Algorithm 2.14: Zero D	
boolean found ← false integer turn ← 1	
p	**q**
integer i ← 0	integer j ← 1
p1: while not found	q1: while not found
p2: await turn = 1	q2: await turn = 2
turn ← 2	turn ← 1
p3: i ← i + 1	q3: j ← j − 1
p4: if f(i) = 0	q4: if f(j) = 0
p5: found ← true	q5: found ← true

Algorithm 2.15: Zero E	
boolean found ← false integer turn ← 1	
p	**q**
integer i ← 0	integer j ← 1
p1: while not found	q1: while not found
p2: await turn = 1	q2: await turn = 2
turn ← 2	turn ← 1
p3: i ← i + 1	q3: j ← j − 1
p4: if f(i) = 0	q4: if f(j) = 0
p5: found ← true	q5: found ← true
p6: turn ← 2	q6: turn ← 1

6. Consider the following algorithm where each of ten processes executes the statements with i set to a different number in 1, . . . , 10:

Algorithm 2.16: Concurrent algorithm A
integer array[1..10] C ← ten *distinct* initial values integer array[1..10] D
integer myNumber, count p1: myNumber ← C[i] p2: count ← number of elements of C less than myNumber p3: D[count + 1] ← myNumber

(a) What does the algorithm do?

(b) What would happen if C in line p3 were replaced by D?

(c) What would happen if the array C were initialized with values that are not all distinct? Correct the algorithm to take care of this case.

7. Consider the following algorithm:

Algorithm 2.17: Concurrent algorithm B	
integer n ← 0	
p	**q**
p1: while n < 2 p2: write(n)	q1: n ← n + 1 q2: n ← n + 1

(a) Construct scenarios that give the output sequences: 012, 002, 02.

(b) Must the value 2 appear in the output?

(c) How many times can the value 2 appear in the output?

(d) How many times can the value 1 appear in the output?

8. Consider the following algorithm:

Algorithm 2.18: Concurrent algorithm C	
integer n ← 1	
p	**q**
p1: while n < 1 p2: n ← n + 1	q1: while n ≥ 0 q2: n ← n − 1

(a) Construct a scenario in which the loop in p executes exactly once.

(b) Construct a scenario in which the loop in p executes exactly three times.

(c) Construct a scenario in which both loops execute infinitely often.

9. Consider the following algorithm:

Algorithm 2.19: Stop the loop B	
integer n ← 0 boolean flag ← false	
p	**q**
p1: while flag = false p2: n ← 1 − n p3:	q1: while flag = false q2: if n = 0 q3: flag ← true

(a) Construct a scenario for which the program terminates.

(b) What are the possible values of n when the program terminates?

(c) Does the program terminate for all scenarios?

(d) Does the program terminate for all fair scenarios?

10. Consider the following algorithm:

Algorithm 2.20: Stop the loop C	
integer n ← 0 boolean flag ← false	
p	**q**
p1: while flag = false p2: n ← 1 − n	q1: while n = 0 // Do nothing q2: flag ← true

(a) Construct a scenario for which the program terminates.

(b) What are the possible values of n when the program terminates?

(c) Does the program terminate for all scenarios?

(d) Does the program terminate for all fair scenarios?

11. Complete Figure 2.1 with all possible states that can occur if the computation starts with a move by a female frog. Make sure not to create duplicate states.

12. (The welfare crook problem, Feijen [3, Section 7.5]) Let a, b, c be three ordered arrays of integer elements; it is known that some element appears in each of the three arrays. Here is an outline of a sequential algorithm to find the smallest indices i, j, k, for which $a[i] = b[j] = c[k]$:

Algorithm 2.21: Welfare crook problem	
integer array[0..N] a, b, c ← ... (as required) integer i ← 0, j ← 0, k ← 0	
loop	
p1: if condition-1	
p2: i ← i + 1	
p3: else if condition-2	
p4: j ← j + 1	
p5: else if condition-3	
p6: k ← k + 1	
else exit loop	

(a) Write conditional expressions that make the algorithm correct.

(b) Develop a concurrent algorithm for this problem.

3 The Critical Section Problem

3.1 Introduction

This chapter presents a sequence of attempts to solve the critical section problem for two processes, culminating in Dekker's algorithm. The synchronization mechanisms will be built without the use of atomic statements other than atomic load and store. Following this sequence of algorithms, we will briefly present solutions to the critical section problem that use more complex atomic statements. In the next chapter, we will present algorithms for solving the critical section problem for N processes.

Algorithms like Dekker's algorithm are rarely used in practice, primarily because real systems support higher synchronization primitives like those to be discussed in subsequent chapters (though see the discussion of "fast" algorithms in Section 5.4). Nevertheless, this chapter is the heart of the book, because each incorrect algorithm demonstrates some pathological behavior that is typical of concurrent algorithms. Studying these behaviors within the framework of these elementary algorithms will prepare you to identify and correct them in real systems.

The proofs of the correctness of the algorithms will be based upon the explicit construction of state diagrams in which all scenarios are represented. Some of the proofs are left to the next chapter, because they require more complex tools, in particular the use of temporal logic to specify and verify correctness properties.

3.2 The definition of the problem

We start with a specification of the structure of the critical section problem and the assumptions under which it must be solved:

- Each of N processes is executing in a infinite loop a sequence of statements that can be divided into two subsequences: the *critical section* and the *non-critical section*.
- The correctness specifications required of any solution are:

Mutual exclusion Statements from the critical sections of two or more processes must not be interleaved.

Freedom from deadlock If *some* processes are trying to enter their critical sections, then *one* of them must eventually succeed.

Freedom from (individual) starvation If *any* process tries to enter its critical section, then *that* process must eventually succeed.

- A *synchronization mechanism* must be provided to ensure that the correctness requirements are met. The synchronization mechanism consists of additional statements that are placed before and after the critical section. The statements placed before the critical section are called the *preprotocol* and those after it are called the *postprotocol*. Thus, the structure of a solution for two processes is as follows:

Algorithm 3.1: Critical section problem	
global variables	
p	**q**

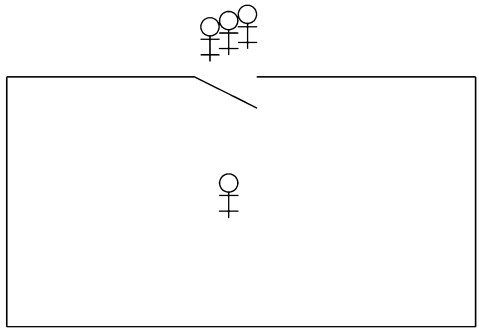
local variables
loop forever
  non-critical section
  preprotocol
  critical section
  postprotocol
	local variables
	loop forever
	  non-critical section
	  preprotocol
	  critical section
	  postprotocol

- The protocols may require local or global variables, but we assume that no variables used in the critical and non-critical sections are used in the protocols, and vice versa.

- The critical section must *progress*, that is, once a process starts to execute the statements of its critical section, it must eventually finish executing those statements.

- The non-critical section need not progress, that is, if the control pointer of a process is at or in its non-critical section, the process may terminate or enter an infinite loop and not leave the non-critical section.

The following diagram will help visualize the critical section problem:



The stick figures represent processes and the box is a critical region in which at most one process at a time may be executing the statements that form its critical section. The solution to the problem is given by the protocols for opening and closing the door to the critical region in such a manner that the correctness properties are satisfied.

The critical section problem is intended to model a system that performs complex computation, but occasionally needs to access data or hardware that is shared by several processes. It is unreasonable to expect that individual processes will never terminate, or that nothing bad will ever occur in the programs of the system. For example, a check-in kiosk at an airport waits for a passenger to swipe his credit card or touch the screen. When that occurs, the program in the kiosk will access a central database of passengers, and it is important that only one of these programs update the database at a time. The critical section is intended to model this operation only, while the non-critical section models all the computation of the program except for the actual update of the database.

It would not make sense to require that the program actually participate in the update of the database by programs in other kiosks, so we allow it to wait indefinitely for input, and we take into account that the program could malfunction. In other words, we do not require that the non-critical section *progress*. On the other hand, we do require that the critical section progress so that it eventually terminates. The reason is that the process executing a critical section typically holds a "lock" or "permission resource," and the lock or resource must eventually be released so that other processes can enter their critical sections. The requirement for progress is reasonable, because critical sections are usually very short and their progress can be formally verified.

Obviously, deadlock ("my computer froze up") must be avoided, because systems are intended to provide a service. Even if there is local progress within the protocols as the processes set and check the protocol variables, if no process ever

succeeds in making the transition from the preprotocol to the critical section, the program is deadlocked.

Freedom from starvation is a strong requirement in that it must be shown that *no* possible execution sequence of the program, no matter how improbable, can cause starvation of *any* process. This requirement can be weakened; we will see one example in the algorithm of Section 5.4.

A good solution to the critical section problem will also be efficient, in the sense that the pre- and postprotocols will use as little time and memory as possible. In particular, if only a single process wishes to enter its critical section it will succeed in doing so almost immediately.

3.3 First attempt

Here is a first attempt at solving the critical section problem for two processes:

Algorithm 3.2: First attempt			
integer turn ← 1			
p		**q**	
	loop forever		loop forever
p1:	non-critical section	q1:	non-critical section
p2:	await turn = 1	q2:	await turn = 2
p3:	critical section	q3:	critical section
p4:	turn ← 2	q4:	turn ← 1

The statement await turn=1 is an implementation-independent notation for a statement that waits until the condition turn=1 becomes true. This can be implemented (albeit inefficiently) by a *busy-wait loop* that does nothing until the condition is true.

The global variable turn can take the two values 1 and 2, and is initially set to the arbitrary value 1. The intended meaning of the variable is that it indicates whose "turn" it is to enter the critical section. A process wishing to enter the critical section will execute a preprotocol consisting of a statement that waits until the value of turn indicates that its turn has arrived. Upon exiting the critical section, the process sets the value of turn to the number of the other process.

We want to prove that this algorithm satisfies the three properties required of a solution to the critical section problem. To that end we first explain the construction of state diagrams for concurrent programs, a concept that was introduced in Section 2.2.

3.4 Proving correctness with state diagrams

States

You do not need to know the *history* of the execution of an algorithm in order to predict the result of the next step to be executed. Let us first examine this claim for sequential algorithms and then consider how it has to be modified for concurrent algorithms. Consider the following algorithm:

Algorithm 3.3: History in a sequential algorithm
integer a ← 1, b ← 2
p1: Millions of statements p2: a ← (a+b)*5 p3: ...

Suppose now that the computation has reached statement p2, and the values of a and b are 10 and 20, respectively. You can now predict with absolute certainty that as a result of the execution of statement p2, the value of a will become 150, while the value of b will remain 20; furthermore, the control pointer of the computer will now contain p3. The history of the computation—how exactly it got to statement p2 with those values of the variables—is irrelevant.

For a concurrent algorithm, the situation is similar:

Algorithm 3.4: History in a concurrent algorithm	
integer a ← 1, b ← 2	
p	**q**
p1: Millions of statements p2: a ← (a+b)*5 p3: ...	q1: Millions of statements q2: b ← (a+b)*5 q3: ...

Suppose that the execution has reached the point where the first process has reached statement p2 and the second has reached q2, and the values of a and b are again 10 and 20, respectively. Because of the interleaving, we cannot predict whether the next statement to be executed will come from process p or from process q; but we can predict that it will be from either one or the other, and we can specify what the outcome will be in either case.

In the sequential Algorithm 3.3, states are triples such as $s_i = (\text{p2}, 10, 20)$. From the semantics of assignment statements, we can predict that executing the statement p2 in state s_i will cause the state of the computation to change to $s_{i+1} = (\text{p3}, 150, 20)$. Thus, given the initial state of the computation $s_0 = (\text{p1}, 1, 2)$, we

can predict the result of the computation. (If there are input statements, the values placed into the input variables are also part of the state.) It is precisely this property that makes *debugging* practical: if you find an error, you can set a breakpoint and restart a computation in the same initial state, confident that the state of the computation at the breakpoint is the same as it was in the erroneous computation.

In the concurrent Algorithm 3.4, states are quadruples such as $s_i = (\mathsf{p2}, \mathsf{q2}, 10, 20)$. We cannot predict what the next state will be, but we can be sure that it is either $s^p_{i+1} = (\mathsf{p3}, \mathsf{q2}, 150, 20)$ or $s^q_{i+1} = (\mathsf{p2}, \mathsf{q3}, 10, 150)$ depending on whether the next state taken is from process p or process q, respectively. While we cannot predict which states will appear in any particular execution of a concurrent algorithm, the set of reachable states (Definition 2.5) are the only states that can appear in any computation. In the example, starting from state s_i, the next state will be an element of the set $\{s^p_{i+1}, s^q_{i+1}\}$. From each of these states, there are possibly two new states that can be reached, and so on. To check correctness properties, it is only necessary to examine the set of reachable states and the transitions among them; these are represented in the state diagram.

For the first attempt, Algorithm 3.2, states are triples of the form $(\mathsf{p}_i, \mathsf{q}_j, \mathit{turn})$, where *turn* denotes the value of the variable turn. Remember that we are assuming that any variables used in the critical and non-critical sections are distinct from the variables used in the protocols and so cannot affect the correctness of the solution. Therefore, we leave them out of the description of the state. The mutual exclusion correctness property holds if the set of all accessible states does *not* contain a state of the form $(\mathsf{p3}, \mathsf{q3}, \mathit{turn})$ for some value of *turn*, because p3 and q3 are the labels of the critical sections.

State diagrams

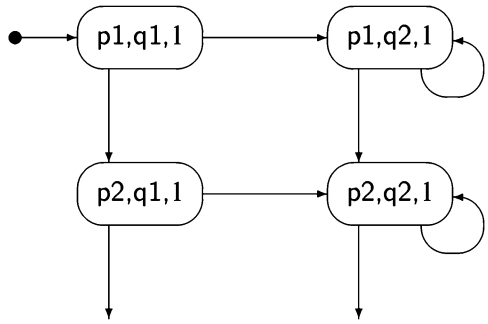
How many states can there be in a state diagram? Suppose that the algorithm has N processes with n_i statements in process i, and M variables where variable j has m_j possible values. The number of possible states is the number of tuples that can be formed from these values, and we can choose each element of the tuple independently, so the total number of states is $n_1 \times \cdots \times n_N \times m_1 \cdots \times m_M$. For the first attempt, the number of states is $n_1 \times n_2 \times m_1 = 4 \times 4 \times 2 = 32$, since it is clear from the text of the algorithm that the variable turn can only have two values, 1 and 2. In general, variables may have as many values as their representation can hold, for example, 2^{32} for a 32-bit integer variable.

However, it is possible that not all states can actually occur, that is, it is possible that some states do not appear in any scenario starting from the initial state $s_0 = (\mathsf{p1}, \mathsf{q1}, 1)$. In fact, we hope so! We hope that the states $(\mathsf{p3}, \mathsf{q3}, 1)$ and $(\mathsf{p3}, \mathsf{q3}, 2)$,

which violate the correctness requirement of mutual exclusion, are not accessible from the initial state.

To prove a property of a concurrent algorithm, we construct a state diagram and then analyze if the property holds in the diagram. The diagram is constructed *incrementally*, starting with the initial state and considering what the potential next states are. If a potential next state has already been constructed, then we can connect to it, thus obtaining a finite presentation of an unbounded execution of the algorithm. By the nature of the incremental construction, *turn* will only have the values 1 and 2, because these are the only values that are ever assigned by the algorithm.

The following diagram shows the first four steps of the incremental construction of the state diagram for Algorithm 3.2:



The initial state is (p1, q1, 1). If we execute p1 from process p, the next state is (p2, q1, 1): the first element of the tuple changes because we executed a statement of the first process, the second element does not change because we did *not* execute a statement of that process, and the third element does not change since—by assumption—the non-critical section does not change the value of the variable turn. If, however, we execute q1 from process q, the next state is (p1, q2, 1). From this state, if we try to execute another statement, q2, from process q, we remain in the same state. The statement is await turn=2, but *turn* = *1*. We do not draw another instance of (p1, q2, 1); instead, the arrow representing the transition points to the existing state.

The incremental construction terminates after 16 of the 32 possible states have been constructed, as shown in Figure 3.1. You may (or may not!) want to check if the construction has been carried out correctly.

A quick check shows that neither of the states (p3, q3, 1) nor (p3, q3, 2) occurs; thus we have proved that the mutual exclusion property holds for the first attempt.

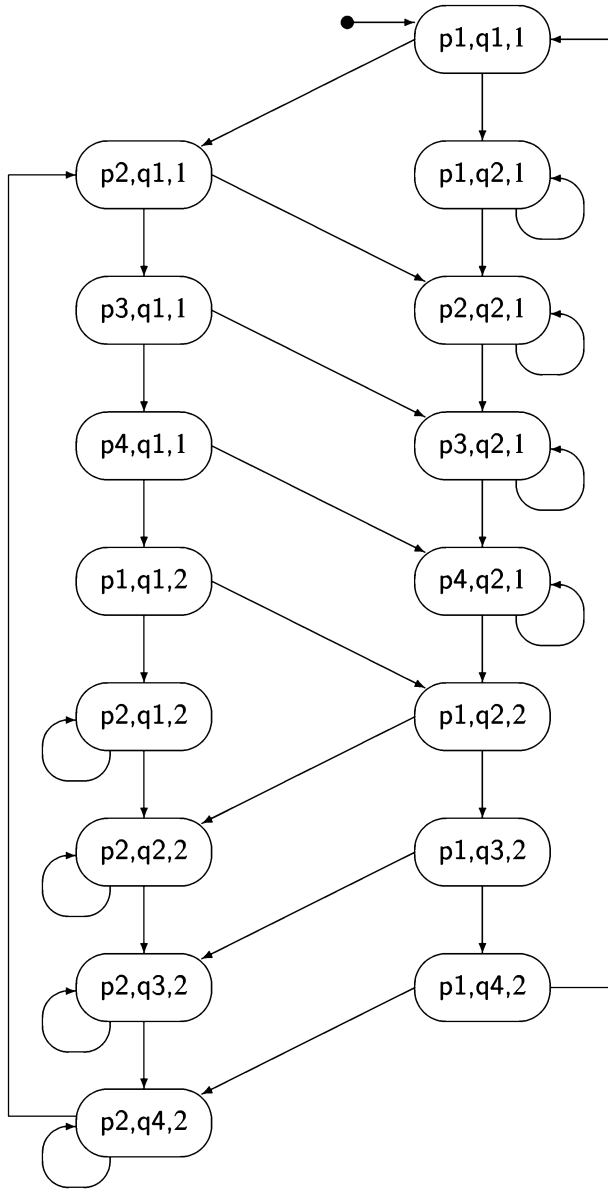


Figure 3.1: State diagram for the first attempt

Abbreviating the state diagram

Clearly, the state diagram of the simple algorithm in the first attempt is unwieldy. When state diagrams are built, it is important to minimize the number of states that must be constructed. In this case, we can reduce the number of accessible states from 16 to 4, by reducing the algorithm to the one shown below, where we have removed the two statements for the critical and non-critical sections.

Algorithm 3.5: First attempt (abbreviated)	
integer turn ← 1	
p	**q**
loop forever	loop forever
p1: await turn = 1	q1: await turn = 2
p2: turn ← 2	q2: turn ← 1

Admittedly, this is suspicious. The whole point of the algorithm is to ensure that mutual exclusion holds during the execution of the critical sections, so how can we simply erase these statements? The answer is that whatever statements are executed by the critical section are totally irrelevant to the correctness of the synchronization algorithm.

If in the first attempt, we replace the statement p3: critical section by the comment p3: // critical section, the specification of the correctness properties remain unchanged, for example, we must show that we cannot be in either of the states (p3, q3, 1) or (p3, q3, 2). But if we are at p3 and that is a comment, it is just *as if* we were at p4, so we can remove the comment; similar reasoning holds for p1. In the abbreviated first attempt, the critical sections have been "swallowed up" by turn ← 2 and turn ← 1, and the non-critical sections have been "swallowed up" by await turn=1 and await turn=2. The state diagram for this algorithm is shown in Figure 3.2, where for clarity we have given the actual statements and variables names instead of just labels.

3.5 Correctness of the first attempt

We are now in a position to try to prove the correctness of the first attempt. As noted above, the proof that mutual exclusion holds is immediate from an examination of the state diagram.

Next we have to prove that the algorithm is free from deadlock. Recall that this means that if *some* processes are trying to enter their critical section then *one* of them must eventually succeed. In the abbreviated algorithm, a process is trying to

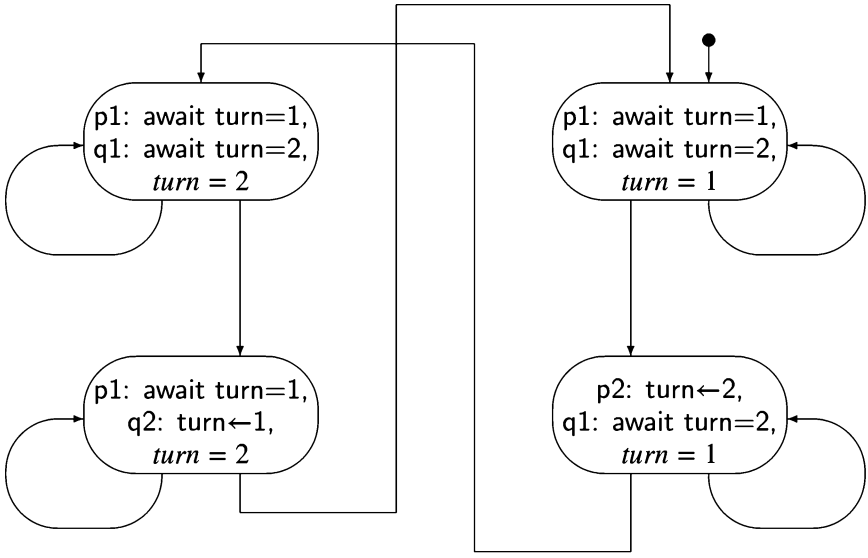


Figure 3.2: State diagram for the abbreviated first attempt

enter its critical section if it is trying to execute its await statement. We have to check this for the four states; since the two left states are symmetrical with the two right states, it suffices to check one of the pairs.

Consider the upper left state (await turn=1, await turn=2, *turn* = 2). Both processes are trying to execute their critical sections; if process q tries to execute await turn=2, it will succeed and enter its critical section.

Consider now the lower left state (await turn=1, turn ← 1, *turn* = 2). Process p may try to execute await turn=1, but since *turn* = 2, it does not change the state. By the assumption of progress on the critical section and the assignment statement, process q will eventually execute turn ← 1, leading to the upper right state. Now, process p can enter its critical section. Therefore, the property of freedom from deadlock is satisfied.

Finally, we have to check that the algorithm is free from starvation, meaning that if *any* process is about to execute its preprotocol (thereby indicating its intention to enter its critical section), then eventually it will succeed in entering its critical section. The problem will be easier to understand if we consider the state diagram for the unabbreviated algorithm; the appropriate fragment of the diagram is shown in Figure 3.3, where NCS denotes the non-critical section. Consider the state at the lower left. Process p is trying to enter its critical section by executing p2: await turn=1, but since *turn* = 2, the process will loop indefinitely in its await state-

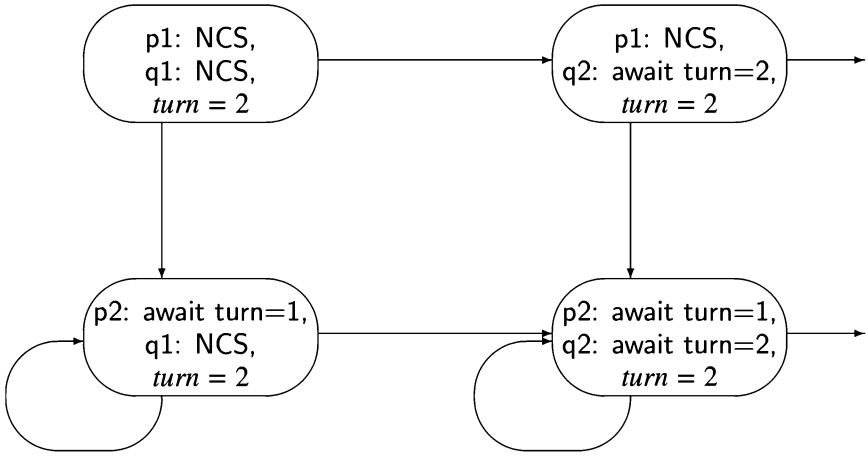


Figure 3.3: Fragment of the state diagram for the first attempt

ment until process q executes q4: turn←1. But process q is at q1: NCS and there is *no* assumption of progress for non-critical sections. Therefore, starvation has occurred: process p is continually checking the value of turn trying to enter its critical section, but process q need never leave its non-critical section, which is necessary if p is to enter its critical section.

Informally, turn serves as a permission resource to enter the critical section, with its value indicating which process holds the resource. There is always *some* process holding the permission resource, so *some* process can always enter the critical section, ensuring that there is no deadlock. However, if the process holding the permission resource remains indefinitely in its non-critical section—as allowed by the assumptions of the critical section problem—the other process will never receive the resource and will never enter its critical section. In our next attempt, we will ensure that a process in its non-critical section cannot prevent another one from entering its critical section.

3.6 Second attempt

The first attempt was incorrect because both processes set and tested a single global variable. If one process dies, the other is blocked. Let us try to solve the critical section problem by providing each process with its own variable (Algorithm 3.6). The intended interpretation of the variables is that wanti is true from the step where process i *wants* to enter its critical section until it leaves the critical section. await statements ensure that a process does not enter its critical section while the other process has its flag set. This solution does not suffer from the problem of the

Algorithm 3.6: Second attempt	
boolean wantp ← false, wantq ← false	
p	**q**
loop forever	loop forever
p1: non-critical section	q1: non-critical section
p2: await wantq = false	q2: await wantp = false
p3: wantp ← true	q3: wantq ← true
p4: critical section	q4: critical section
p5: wantp ← false	q5: wantq ← false

previous one: if a process halts in its non-critical section, the value of its variable want will remain false and the other process will always succeed in immediately entering its critical section.

Let us construct a state diagram for the abbreviated algorithm:

Algorithm 3.7: Second attempt (abbreviated)	
boolean wantp ← false, wantq ← false	
p	**q**
loop forever	loop forever
p1: await wantq = false	q1: await wantp = false
p2: wantp ← true	q2: wantq ← true
p3: wantp ← false	q3: wantq ← false

Unfortunately, when we start to incrementally construct the state diagram (Figure 3.4), we quickly find the state (p3: wantp←false, q3: wantq←false, *true*, *true*), showing that the mutual exclusion property is not satisfied. (await !want is used instead of await want=false because of space constraints in the node.)

Paths in the state diagram correspond to scenarios; this portion of the scenario can also be displayed in tabular form:

Process p	Process q	wantp	wantq
p1: await wantq=false	q1: await wantp=false	*false*	*false*
p2: wantp←true	**q1: await wantp=false**	*false*	*false*
p2: wantp←true	q2: wantq←true	*false*	*false*
p3: wantp←false	**q2: wantq←true**	*true*	*false*
p3: wantp←false	q3: wantq←false	*true*	*true*

To prove that mutual exclusion holds, we must check that no forbidden state appears in *any* scenario. So if mutual exclusion does in fact hold, we need to construct

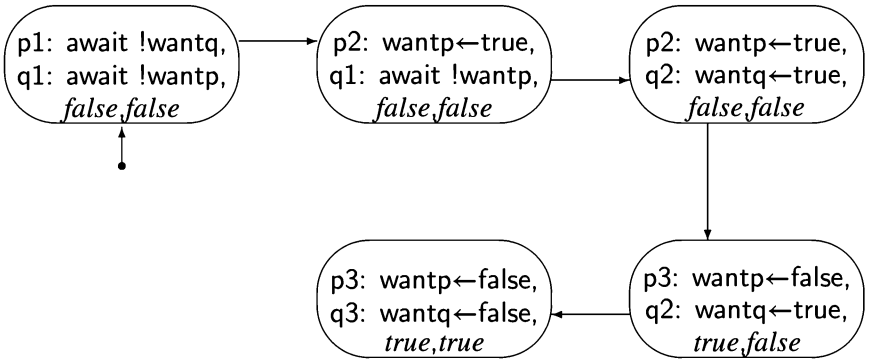


Figure 3.4: Fragment of the state diagram for the second attempt

the full state diagram for the algorithm, because every path in the diagram is a scenario; then we must examine every state to make sure that it is not a forbidden state. When constructing the diagram incrementally, we check each state as it is created, so that we can stop the construction if a forbidden state is encountered, as it was in this case. It follows that if the incremental construction is completed (that is, there are no more reachable states), then we know that no forbidden states occur.

3.7 Third attempt

In the second attempt, we introduced the variables want that are intended to indicate when a process is in its critical section. However, once a process has successfully completed its await statement, it cannot be prevented from entering its critical section. The state reached after the await but before the assignment to want is effectively part of the critical section, but the value of want does not indicate this fact. The third attempt (Algorithm 3.8) recognizes that the await statement

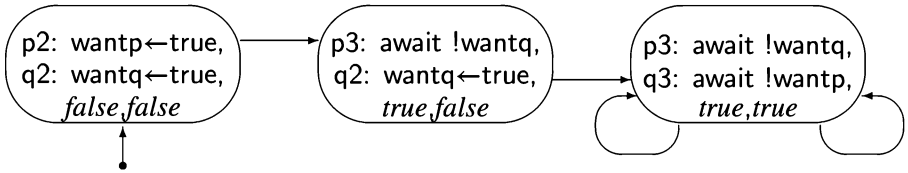
Algorithm 3.8: Third attempt	
boolean wantp ← false, wantq ← false	
p	**q**
loop forever	loop forever
p1: non-critical section	q1: non-critical section
p2: wantp ← true	q2: wantq ← true
p3: await wantq = false	q3: await wantp = false
p4: critical section	q4: critical section
p5: wantp ← false	q5: wantq ← false

should be considered part of the critical section by moving the assignment to want to before the await. We leave it as an exercise to construct the state diagram for this algorithm, and to show that no state in the diagram violates mutual exclusion. In Section 4.2, we will use logic to prove deductively that the algorithm satisfies the mutual exclusion property.

Unfortunately, the algorithm can deadlock as shown by the following scenario:

Process p	Process q	wantp	wantq
p1: non-critical section	q1: non-critical section	*false*	*false*
p2: wantp←true	**q1: non-critical section**	*false*	*false*
p2: wantp←true	q2: wantq←true	*false*	*false*
p3: await wantq=false	**q2: wantq←true**	*true*	*false*
p3: await wantq=false	q3: await wantp=false	*true*	*true*

This can also be seen in the following fragment of the state diagram:



In the rightmost state, both process are trying to enter the critical section, but neither will ever do so, which is our definition of deadlock.

The term *deadlock* is usually associated with a "frozen" computation where nothing whatsoever is being computed; the situation here—a scenario where several processes are actively executing statements, but nothing useful gets done—is also called *livelock*.

3.8 Fourth attempt

In the third attempt, when a process sets the variable want to true, not only does it indicate its intention to enter its critical section, but also *insists* on its right to do so. Deadlock occurs when both processes simultaneously insist on entering their critical sections. The fourth attempt to solve the critical section problem tries to remedy this problem by requiring a process to give up its intention to enter its critical section if it discovers that it is contending with the other process (Algorithm 3.9). The assignment p4: wantp←false followed immediately by an assignment to the same variable p5: wantp←true would not be meaningful in a sequential algorithm, but

Algorithm 3.9: Fourth attempt	
boolean wantp ← false, wantq ← false	
p	**q**
loop forever	loop forever
p1: non-critical section	q1: non-critical section
p2: wantp ← true	q2: wantq ← true
p3: while wantq	q3: while wantp
p4: wantp ← false	q4: wantq ← false
p5: wantp ← true	q5: wantq ← true
p6: critical section	q6: critical section
p7: wantp ← false	q7: wantq ← false

it can be useful in a concurrent algorithm. Since we allow arbitrary interleaving of statements from the two processes, the second process may execute an arbitrary number of statements between the two assignments to wantp. When process p relinquishes the attempt to enter the critical section by resetting wantp to false, process q may now execute the await statement and succeed in entering its critical section.

A state diagram or deductive proof will show that the mutual exclusion property is satisfied and that there is no deadlock. However, a scenario for starvation exists as shown by the cycle in Figure 3.5. The interleaving is "perfect" in the sense that the execution of a statement by process q is always followed immediately by the execution of the equivalently-numbered statement in process p. In this scenario, both processes are starved.

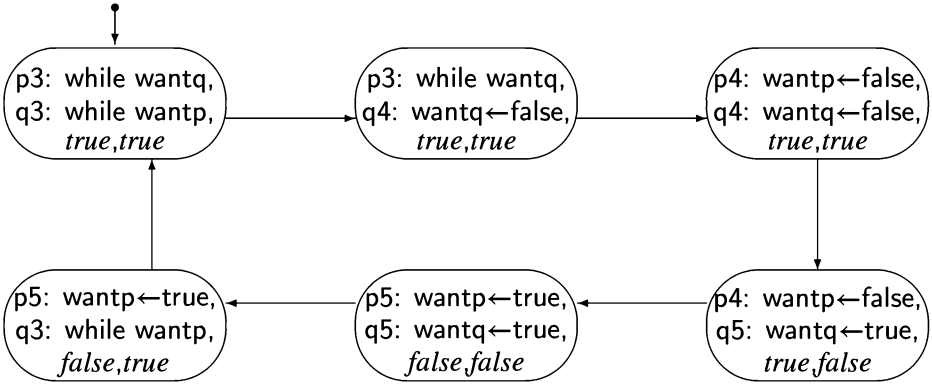


Figure 3.5: Cycle in a state diagram for the fourth attempt

At this point, most people object that this is not a "realistic" scenario; we can hardly expect that whatever is causing the interleaving can indefinitely ensure that exactly two statements are executed by process q followed by exactly two statements from p. But our model of concurrency does not take probability into account. Unlikely scenarios have a nasty way of occurring precisely when a bug would have the most dangerous and costly effects. Therefore, we reject this solution, because it does not fully satisfy the correctness requirements.

3.9 Dekker's algorithm

Dekker's algorithm for solving the critical section problem is a combination of the first and fourth attempts:

Algorithm 3.10: Dekker's algorithm	
boolean wantp ← false, wantq ← false integer turn ← 1	
p	**q**
loop forever	loop forever
p1: non-critical section	q1: non-critical section
p2: wantp ← true	q2: wantq ← true
p3: while wantq	q3: while wantp
p4: if turn = 2	q4: if turn = 1
p5: wantp ← false	q5: wantq ← false
p6: await turn = 1	q6: await turn = 2
p7: wantp ← true	q7: wantq ← true
p8: critical section	q8: critical section
p9: turn ← 2	q9: turn ← 1
p10: wantp ← false	q10: wantq ← false

Recall that in the first attempt we explicitly passed the right to enter the critical section between the processes. Unfortunately, this caused the processes to be too closely coupled and prevented correct behavior in the absence of contention. In the fourth attempt, each process had its own variable which prevented problems in the absence of contention, but in the presence of contention both processes insist on entering their critical sections.

Dekker's algorithm is like the fourth attempted solution, except that the *right to insist on entering*, rather than the *right to enter*, is explicitly passed between the processes. The individual variables wantp and wantq ensure mutual exclusion.

Suppose now that process p detects contention by finding wantq to be true at statement p3: while wantq. Process p will now consult the global variable turn to see if it is its turn to insist upon entering its critical section ($turn \neq 2$, that is, $turn = 1$). If so, it executes the loop at p3 and p4 until process q resets wantq to false, either by terminating its critical section at q10 or by deferring in q5. If not, process p will reset wantp to false and defer to process q, waiting until that process changes the value of turn after executing its critical section.

Dekker's algorithm is correct: it satisfies the mutual exclusion property and it is free from deadlock and starvation. The correctness properties by can be proved by constructing and analyzing a state diagram, but instead we will give a deductive proof in Section 4.5.

3.10 Complex atomic statements

We have found that it is difficult to solve the critical section problem using just atomic load and store statements. The difficulty disappears if an atomic statement can both load and store. The solutions given here are shown for two processes, but they are correct for any number of processes.

The *test-and-set* statement is an *atomic statement* defined as the execution of the following two statements with no possible interleaving between them:

```
test –and–set(common, local) is
   local  ← common
   common ← 1
```

Algorithm 3.11: Critical section problem with test-and-set	
integer common ← 0	
p	**q**
integer local1	integer local2
loop forever	loop forever
p1: non-critical section	q1: non-critical section
repeat	repeat
p2: test-and-set(	q2: test-and-set(
common, local1)	common, local2)
p3: until local1 = 0	q3: until local2 = 0
p4: critical section	q4: critical section
p5: common ← 0	q5: common ← 0

Algorithm 3.12: Critical section problem with exchange	
integer common ← 1	
p	**q**
integer local1 ← 0	integer local2 ← 0
loop forever	loop forever
p1: non-critical section	q1: non-critical section
repeat	repeat
p2: exchange(common, local1)	q2: exchange(common, local2)
p3: until local1 = 1	q3: until local2 = 1
p4: critical section	q4: critical section
p5: exchange(common, local1)	q5: exchange(common, local2)

Algorithm 3.11 is the simple solution to the critical section problem using test-and-set. We leave it as an exercise to prove that this solution satisfies the mutual exclusion property.

Note: A commonly used notation for specifying atomicity is to put angled brackets around the group of statements: ⟨local ← common; common ← 1⟩. In Promela, the keyword **atomic** is used.

The *exchange* statement is an *atomic statement* defined as the atomic swap of the values of two variables:

```
exchange(a, b) is
   integer  temp
   temp ← a
   a ← b
   b ← temp
```

The solution to the critical section problem with exchange is also very simple (Algorithm 3.12). Here too, the proof of correctness is left as an exercise.

We conclude with two other atomic statements that have been implemented in computers; in the exercises you are asked to solve the critical section problem using these statements.

The *fetch-and-add* statement is defined as the atomic execution of the following statements:

```
fetch−and−add(common, local, x) is
   local  ← common
   common ← common + x
```

The *compare-and-swap* statement is defined as the atomic execution of the following statements:

```
compare–and–swap(common, old, new) is
   integer  temp
   temp ← common
   if  common = old
      common ← new
   return  temp
```

These solutions use busy-wait loops, but this is not a disadvantage in a multiprocessor, especially if the contention is low.

Transition

This chapter introduced the basic concepts of writing a concurrent program to solve a problem. Even in the simplest model of atomic load and store of shared memory, Dekker's algorithm can solve the critical section problem, although it leaves much to be desired in terms of simplicity and efficiency. Along the journey to Dekker's algorithm, we have encountered numerous ways in which concurrent programs can be faulty. Since it is impossible to check every possible scenario for faults, state diagrams are an invaluable tool for verifying the correctness of concurrent algorithms.

The next chapter will explore verification of concurrent programming in depth, first using invariant assertions which are relatively easy to understand and use, and then using deductive proofs in temporal logic which are more sophisticated. Finally, we show how programs can be verified using the Spin model checker.

Exercises

1. Complete the state diagram for the abbreviated version of the second attempt (Figure 3.4).

2. Construct the state diagram for (an abbreviated version of) the third attempt (Algorithm 3.8). Use it to show that mutual exclusion holds for the algorithm.

3. For the fourth attempt (Algorithm 3.9):

   (a) Construct the tabular form of the scenario for starvation shown in Figure 3.5.

   (b) Explain why this scenario is considered to be starvation and not livelock as in the third attempt.

   (c) Construct a scenario in which one process is starved while the other enters its critical section infinitely often.

4. Peterson's algorithm for the critical section problem is based upon Dekker's algorithm but is more concise because it collapses two await statements into one with a compound condition. Prove the correctness of Peterson's algorithm by constructing a state diagram for an abbreviated version of the algorithm.

Algorithm 3.13: Peterson's algorithm	
boolean wantp ← false, wantq ← false integer last ← 1	
p	**q**
loop forever	loop forever
p1: non-critical section	q1: non-critical section
p2: wantp ← true	q2: wantq ← true
p3: last ← 1	q3: last ← 2
p4: await wantq = false or last = 2	q4: await wantp = false or last = 1
p5: critical section	q5: critical section
p6: wantp ← false	q6: wantq ← false

(In our presentation, the compound condition is atomic because it is in one labeled line; the algorithm also works if the two conditions are evaluated non-atomically.)

5. (Buhr and Haridi [16]) Compare the use of the variable turn in Dekker's algorithm with the use of the variable last in Peterson's algorithm. What advantage does Dekker's algorithm have? Does it matter if the order of the assignments in the postprotocol is switched?

6. (Manna and Pnueli [51, p. 156]) Prove that mutual exclusion is satisfied by the following algorithm for the critical section problem:

Algorithm 3.14: Manna–Pnueli algorithm	
integer wantp ← 0, wantq ← 0	
p	**q**
loop forever	loop forever
p1: non-critical section	q1: non-critical section
p2: if wantq = −1	q2: if wantp = −1
wantp ← −1	wantq ← 1
else wantp ← 1	else wantq ← −1
p3: await wantq ≠ wantp	q3: await wantp ≠ − wantq
p4: critical section	q4: critical section
p5: wantp ← 0	q5: wantq ← 0

Note that the if statement is atomic. Find a scenario showing that the algorithm is not correct if the if statement is not atomic.

7. (Doran and Thomas [26]) Prove the correctness of the following variant of Dekker's algorithm:

Algorithm 3.15: Doran–Thomas algorithm	
boolean wantp ← false, wantq ← false integer turn ← 1	
p	**q**
loop forever	loop forever
p1: non-critical section	q1: non-critical section
p2: wantp ← true	q2: wantq ← true
p3: if wantq	q3: if wantp
p4: if turn = 2	q4: if turn = 1
p5: wantp ← false	q5: wantq ← false
p6: await turn = 1	q6: await turn = 2
p7: wantp ← true	q7: wantq ← true
p8: await wantq = false	q8: await wantp = false
p9: critical section	q9: critical section
p10: wantp ← false	q10: wantq ← false
p11: turn ← 2	q11: turn ← 1

8. (Lamport [38]) Consider possible solutions of the critical section problem with one bit per process. (Assume that the initial value of all bits is 0.) Show that there are no solutions of the following forms:

   (a) Bit i is set to 1 only by process i and returns spontaneously to 0.

   (b) Bit i is set to 1 only by process i and reset to 0 only by processes other than i.

9. Prove the correctness of Algorithm 3.11 for the critical section problem using test-and-set.

10. Prove the correctness of Algorithm 3.12 for the critical section problem using exchange.

11. Solve the critical section problem using fetch-and-add.

12. Solve the critical section problem using compare-and-swap.

4 Verification of Concurrent Programs

The previous chapters have shown that concurrent programs can have subtle errors, that the errors cannot be discovered by debugging and that corrections cannot be checked by testing. For this reason, formal specification and verification of correctness properties are far more important for concurrent programs than they are for sequential programs. In this chapter we will explore several such methods.

We begin with a short section on the specification of correctness properties and then present the most important technique for proving properties, inductive proofs of invariants. This technique is used to prove that mutual exclusion holds for the third attempt. Inductive proofs are easy to carry out once you know what the invariants are, but they can be quite difficult to discover.

We have already discussed the construction of state diagrams and their use for proving correctness properties. Unfortunately, the full state diagram of even the simplest concurrent program is quite large, so it is not practical to manually construct such diagrams. In fact, for algorithms after the first attempt, we did not bother to construct full diagrams, and limited ourselves to displaying interesting fragments. Even if we could display a large diagram, there remains the task of reasoning with the diagram, that is, of finding forbidden states, cycles that lead to starvation of a process, and so on.

However, we can use a computer program not only to construct the diagram, but also to simultaneously check a correctness property. Such a program is called a *model checker*, because it checks if the state diagram of the concurrent program is a model (satisfying interpretation) of a formula specifying a correctness property of the program. Model checkers have become practical tools for verifying concurrent programs, capable of checking properties of programs that have billions of states. They use advanced algorithms and data structures to deal efficiently with large numbers of states.

Before discussing model checking in general and the Spin tool in particular, we present a system of temporal logic. Temporal logics have proved to be the most effective formalism for specifying and verifying concurrent programs. They can

be used deductively to verify programs as will be shown in Section 4.5; temporal logics are also used to specify correctness properties for model checkers, and as such must be mastered by every student of concurrency. We have (more or less arbitrarily) divided the presentation of temporal logic into two sections: Section 4.3 should suffice for readers interested in using temporal logic to specify elementary correctness properties in Spin, while Section 4.4 covers more advanced aspects of the logic.

For further study of temporal logics, consult [9, 50, 51]. You may want to examine the STeP system, [12] which provides computer support for the construction of deductive proofs, and the *Temporal Logic of Actions (TLA)* [41], developed by Leslie Lamport for specifying concurrent programs.

4.1 Logical specification of correctness properties

We assume that you understand the basic concepts of the propositional calculus, as summarized in Appendix B.

The atomic propositions will be propositions about the values of the variables and the control pointers during an execution of a concurrent program. Given a variable of boolean type such as `wantp`, the atomic proposition *wantp* is true in a state if and only if the value of the variable `wantp` is true in that state. These two fonts will be used to denote the program variables and the associated symbols in logic. Similarly, boolean-valued relations on arithmetic variables are atomic propositions: $turn \neq 2$ is true in a state if and only if the value of the variable `turn` in that state is not 2.

Each label of a statement of a process will be used as an atomic proposition whose intended interpretation is "the control pointer of that process is currently at that label." We again use fonts to distinguish between the label `p1` and the proposition $p1$ that may be true or false, depending on the state of the computation. Of course, since the control pointer of a single process can only point to one statement at a time, if p_i is true, then p_j is false for all $j \neq i$; we will use this fact implicitly in our proofs.

To give a concrete example, let use write some formulas related to Algorithm 3.8, the third attempt. The formula

$$p1 \wedge q1 \wedge \neg wantp \wedge \neg wantq$$

is true in exactly one state, (`p1`,`q1`*false*,*false*), where the control pointers of both processes are at the initial statements and the values of both variables are *false*. The

formula is certainly true in the initial state of any computation (by construction of the program), and just as certainly is false in the second state of any computation (because either $p1$ or $q1$ will become false), unless, of course, both processes halt in their non-critical sections, in which case the computation remains in this state indefinitely.

A more interesting formula is

$$p4 \wedge q4,$$

which is true if the control pointers of both processes point to critical section; in other words, if this formula is true, the mutual exclusion property is not satisfied. Therefore, the program satisfies the mutual exclusion property in states in which

$$\neg(p4 \wedge q4),$$

the negation of $p4 \wedge q4$, is true.

$p1 \wedge q1 \wedge \neg\, wantp \wedge \neg\, wantp$ is an example of a formula that is true in some states (the initial state of the computation), but false in (most) other states, in particular, in any state in which the control pointers of the processes are at statements other than p1 and q1. For the program to satisfy the mutual exclusion property, the formula $\neg(p4 \wedge q4)$ must be true *in all possible states of all possible computations*. The next section shows how to prove such claims.

4.2 Inductive proofs of invariants

The formula $\neg(p4 \wedge q4)$ is called an *invariant* because it must invariably be true at any point of any computation. Invariants are proved using *induction*, not over the natural numbers, but over the states of all computations. (You may want to review Appendix B.2 on arithmetical induction before reading on.)

(a) Prove that A holds in the initial state. This is called the *base case*.

(b) Assume that A is true in all states up to the current one, and prove that A is true in the next state. This is called the *inductive step*, and the assumption that A is true in the current and previous states is called the *inductive hypothesis*. In a concurrent program, there may be more than one possible successor to the current state, so the inductive step must be proved for each one of them.

If (a) and (b) can be proved, we can conclude that A is true for all states of all computations.

Given an invariant, a proof by induction over all states is easier than it sounds. First, it is only necessary to take into account the effect of statements that can potentially change the truth of the atomic propositions in the formula. For example, it is trivial to prove the invariance of $(turn = 1) \vee (turn = 2)$ in the first attempt or in Dekker's algorithm. The initial value of the variable turn is 1. The only two statements that can affect the truth of the formula are the two assignments to that variable; but one assigns the value 1 and the other assigns the value 2, so the inductive step is trivial to prove.

Second, many correctness claims are implications of the form $p4 \rightarrow wantp$, that is, if a control pointer is at a certain statement, then a formula on the values of the variables is true. By the properties of material implication, if the formula is assumed true by the inductive hypothesis, it can be falsified in only two very limited ways; see Appendix B.3 for a review of this property of material implication. We now show how to prove that the third attempt satisfies the mutual exclusion property.

Proof of mutual exclusion for the third attempt

Algorithm 3.8, the third attempt at solving the critical section problem, is repeated here for convenience:

Algorithm 4.1: Third attempt	
boolean wantp ← false, wantq ← false	
p	**q**
loop forever	loop forever
p1: non-critical section	q1: non-critical section
p2: wantp ← true	q2: wantq ← true
p3: await wantq = false	q3: await wantp = false
p4: critical section	q4: critical section
p5: wantp ← false	q5: wantq ← false

Lemma 4.1 The formula $A = p3..5 \rightarrow wantp$ is invariant.

A disjunction of consecutive locations $pi \vee \cdots \vee pj$ is abbreviated $pi..j$.

Proof: The base case is trivial because $p1$ is true initially so $p3..5$ is false.

Let us prove the inductive step, assuming the truth of A. Trivially, the execution of any statement by process q cannot change the truth of A, because A is expressed in terms of locations in process p that process q cannot change, and a variable wantp whose value is changed only by statements of p. Trivially, executing p1: non-critical section cannot change the truth of A. Only slightly less trivially, executing

p3 or p4 does not change the truth of A: these statements do not assign to wantp and executing p3 (respectively, p4) moves the control pointer to p4 (respectively p5), maintaining the truth of $p3..5$.

So, the only two statements that can possibly falsify A are p2 and p5. Normally, of course, we wouldn't go through such a long list of trivialities; we would just begin the proof by saying: the only statements that need to be checked are p2 and p5.

Executing p2 makes $p3..5$ true, but it also makes *wantp* true, so A remains true. Executing p5 makes $p3..5$ false, so by the properties of material implication A remains true, regardless of what happens to *wantp*. Therefore, we have proved by induction that $A = p3..5 \rightarrow wantp$ is true in all states of all computations. ∎

The converse of formula A is also easily proved:

Lemma 4.2 The formula $B = wantp \rightarrow p3..5$ is invariant.

Proof: The base case is trivial because *wantp* is false initially. Again, the only statements that can falsify B are p2 and p5. p2 makes *wantp* "suddenly" become true, but it also makes $p3..5$ true, preserving the truth of B. p5 makes $p3..5$ false, but it also falsifies *wantp*. ∎

Combining the two lemmas and symmetrical proofs for process q, we have:

Lemma 4.3 $p3..5 \leftrightarrow wantp$ and $q3..5 \leftrightarrow wantq$ are invariant.

We are now ready to prove that the third attempt satisfies mutual exclusion.

Theorem 4.4 The formula $\neg(p4 \wedge q4)$ is invariant.

Proof: The proof will be easier to understand if, rather than show that $\neg(p4 \wedge q4)$ is an invariant, we show that $p4 \wedge q4$ is false in every state. Clearly, $p4 \wedge q4$ is false in the initial state. So assume that $p4 \wedge q4$ is false; what statements might make it "suddenly" become true? There are only two: (successfully) executing p3: await wantq=false when $q4$ is true, and (successfully) executing q3: await wantp=false when $p4$ is true. Since the program is symmetrical, it is sufficient to consider one of them.

p3: await wantq=false can be successfully executed only if *wantq* is false; but by Lemma 4.3, if process q is at q4 then *wantq* is true. We conclude that the await statement in p3 cannot be successfully executed when $q4$ is true, so $p4 \wedge q4$ cannot become true. ∎

4.3 Basic concepts of temporal logic

In the usual study of mathematical logic, an assignment of truth values to atomic propositions is used to give an interpretation to a formula; in the interpretation, the formula will be either *true* or *false*. But within the context of the execution of a program, the assignment may change from state to state. For example, if A is the proposition $turn = 1$, then the proposition is true in the initial state of Dekker's algorithm, but it will become false in any state that immediately follows the execution of the statement turn $\leftarrow$ 2. In other words, $turn = 1$ is sometimes true and sometimes false, and a new system of logic is needed to deal with such situations.

Temporal logic is a system of formal logic obtained by adding temporal operators to propositional or predicate logic. In this section, we will informally present a logic called (propositional) *linear temporal logic (LTL)*.

Definition 4.5 A computation is an infinite sequence of states $\{s_0, s_1, \ldots\}$. ▮

Atomic propositions like $turn = 1$ can be either true or false in a state s of a computation. The temporal operators will also be defined as being true or false in a state s of a computation, but their truth will depend on other states of the computation, not just on s.

Always and eventually

Definition 4.6 The temporal operator $\Box$ is read *box* or *always*. The formula $\Box A$ is true in a state s_i of a computation if and only if the formula A is true in *all* states s_j, $j \geq i$, of the computation. ▮

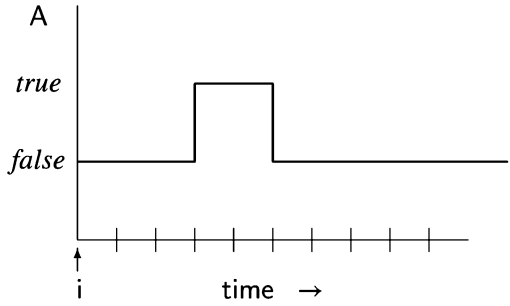
The meaning of $\Box A$ is shown in the following diagram, where time flows in discrete units from left to right on the x-axis, while the y-axis has two levels for the two possible values of the formula A, *true* and *false*:

The origin of the x-axis is the index i of the state s_i at which the truth of the formula is being evaluated; this is not necessarily the beginning of the computation.

In Section 2.6, correctness properties were divided into safety and liveness properties. $\Box A$ is a safety property because it specifies what must always be true. For example, the specification of the mutual exclusion property discussed previously can be expressed as $\Box \neg(p4 \wedge q4)$. Read this as *always* $\neg(p4 \wedge q4)$, or in *every* state $\neg(p4 \wedge q4)$ is true.

Definition 4.7 The temporal logic operator $\Diamond$ is read *diamond* or *eventually*. The formula $\Diamond A$ is true in a state s_i of a computation if and only if the formula A is true in *some* state s_j, $j \geq i$, of the computation. ▮

The meaning of $\Diamond A$ is shown in the following diagram:



Note that if $\Diamond A$ is true in s_i because A is true in s_j ($j \geq i$), then A may subsequently become false in a later state s_k, $k > j$. In the example, $\Diamond A$ is true in state s_i because A is true in state s_{i+3} (and in state s_{i+4}), although it subsequently becomes false in state s_{i+5} and remains false in subsequent states.

The eventually operator is used to specify liveness properties, for example, the freedom from starvation for the third attempt. Since the algorithm is symmetric, it is sufficient to specify for one process, say p, that it must enter its critical section. The formula $p2 \rightarrow \Diamond p4$ means that *if* the computation is at a state in which the control pointer of process p is at statement p2 (indicating that it wishes to enter the critical section), then *eventually* the control pointer will be at p4. More precisely, freedom from starvation should be specified as $\Box(p2 \rightarrow \Diamond p4)$, so that we require that *any* state in which $p2$ is true is followed by a state in which $p4$ is true.

The meaning of the operator $\Diamond$ is consistent with our model of concurrency, in that it only specifies that eventually some proposition must be true, without specifying if that will happen immediately, soon, or after many millions of execution steps.

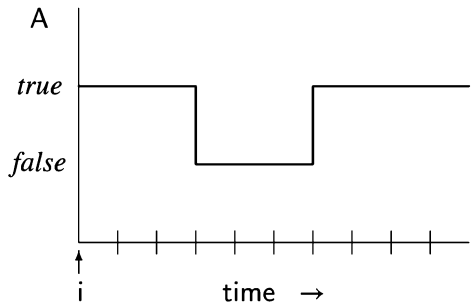
Both $\Box A$ and $\Diamond A$ are interpreted reflexively: if $\Box A$ is true in a state s, then A must be true in s, and if A is true in s, then $\Diamond A$ is true. In words, "always" and "eventually" include "now."

Duality

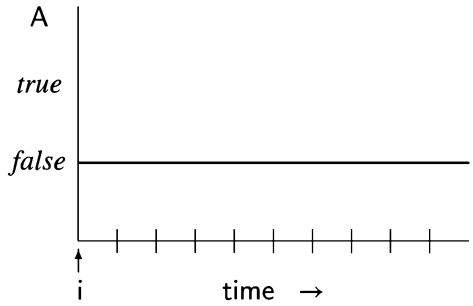
There is a duality to temporal operators, similar to the duality of deMorgan's laws:

$$\neg(A \wedge B) \leftrightarrow (\neg A \vee \neg B),$$
$$\neg(A \vee B) \leftrightarrow (\neg A \wedge \neg B).$$

$\neg \Box A$ (it is false that A is always true) is equivalent to $\Diamond \neg A$ (eventually A is false):

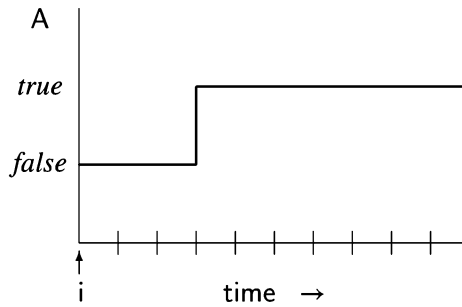


$\neg \Diamond A$ (it is false that A is eventually true) is equivalent to $\Box \neg A$ (A is always false):



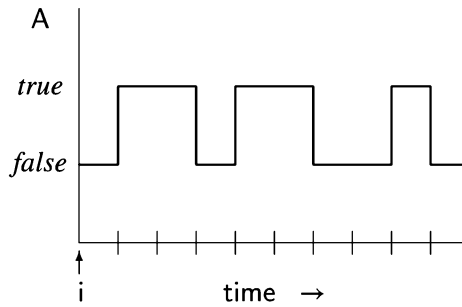
Sequences of operators

The meaning of the formula $\Diamond \Box A$ with a compound temporal operator is shown in the following diagram:



Eventually A will be come true and remain true; in the diagram, A is false for the first three units of time, but then it becomes and *remains* true.

The meaning of $\Box\Diamond A$ is different:



At every point in time, there is a time in the future when A becomes true; later, it may return to have the value false, but if so, it must become true again.

4.4 Advanced concepts of temporal logic^A

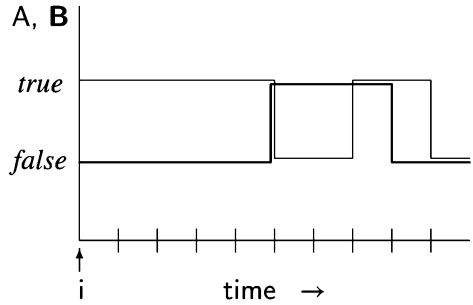
Until

The always and eventually operators have limited expressive power, because they are applied to only one formula. Many specifications of correctness properties involve two or more propositions. For example, we may want to specify that process p1 enters its critical section at most once before process p2 enters its critical section.

The *until* operator $\mathcal{U}$ is a binary temporal operator. $A\,\mathcal{U}\,B$ means that eventually B becomes true and that A is true until that happens:

Definition 4.8 $A\,\mathcal{U}\,B$ is true is a state s_i if and only if B is true in some state s_j, $j \geq i$, and A is true in all states s_k, $i \leq k < j$. ∎

This is shown in the diagram below, where the truth value of A is represented by the thin line and the truth value of B by the thick line:



At the fifth state from the origin, B becomes true, and *until* then, A is true. What happens afterwards is not relevant to the truth of $A \mathcal{U} B$. Note that A is not required to remain true when B becomes true. By reflexivity, $A \mathcal{U} B$ is true if B is true at the initial state; in this case the requirement on A is vacuous.

There is also an operator called *weak until*, denoted $\mathcal{W}$. For the weak operator, the formula B is not required to become true eventually, though if it does not, A must remain true indefinitely.

Next

The semantics of linear temporal logic are defined over the sequence of states of a computation over time. Thus, in any state, it makes sense to talk of the *next* state. The corresponding unary temporal operator is denoted $\mathcal{X}$ or $\bigcirc$.

Definition 4.9 $\bigcirc A$ is true is a state s_i if and only if A is true in state s_{i+1}. ∎

The next operator is rarely used in specifications because the interleaving semantics of concurrent computation are not sensitive to the precise sequence of states, only to their relative order. For example, given the following statements in a process:

```
integer x ← 0
x ← 1
x ← 2
```

it makes sense to claim that $(x = 1) \rightarrow \Diamond(x = 2)$ (assuming that the computation is fair), but not to claim $(x = 1) \rightarrow \bigcirc(x = 2)$. We don't know how many states may occur between the execution of the first assignment statement and the second, nor do we usually care.

If the precise behavior of the system in the interval between the execution of the statements is important, we would use an *until* formula because we want to specify something about *all* intervening states and not just the next one. For example, $(x = 1)\mathcal{U}(x = 2)$ claims that $x = 1$ remains true in all states between the execution of these two statements.

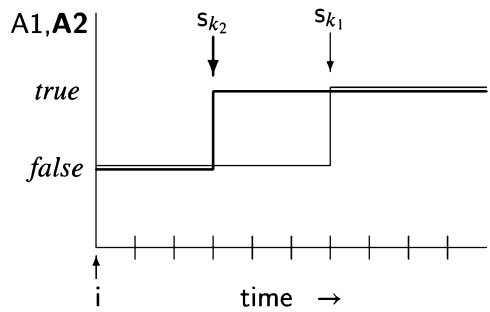
Deduction with temporal operators

Temporal logic is a formal system of deductive logic with its own axioms and rules of inference [9]. Similarly, the semantics of concurrent programs can be formalized within temporal logic, and then the logic can be used to rigorously prove correctness properties of programs. Here we will show how semi-formal reasoning can be used to prove formulas of temporal logic. This type of reasoning will be used in the next section to prove freedom from starvation in Dekker's algorithm.

Here is a formula that is a theorem of temporal logic:

$$(\Diamond\Box A1 \wedge \Diamond\Box A2) \rightarrow \Diamond\Box(A1 \wedge A2).$$

To prove an implication, we have to prove that if the antecedent is true, then so is the consequent. The antecedent is a conjunction, so we assume that both its subformulas are true. Let us now use the definitions of the temporal operators. $\Diamond\Box A1$ is true if there is some state s_{k_1} such that $\Box A1$ is true in s_{k_1}; this holds if $A1$ is true for all states s_j such that $j \geq k_1$. For $\Diamond\Box A2$ to be true, there must be some s_{k_2} with a similar property. The following diagram shows what it means for the antecedent to be true.

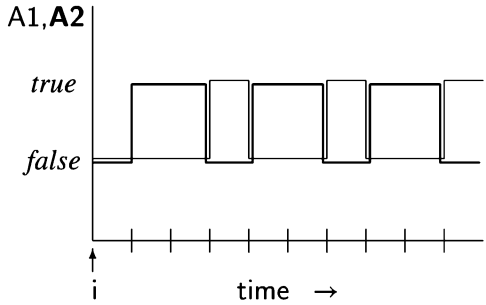


Let k be the larger of k_1 and k_2. Clearly, $A1 \wedge A2$ is true in all states s_j such that $j \geq k$. By definition of $\Box$, $\Box(A1 \wedge A2)$ is true in s_k, and then by definition of $\Diamond$, $\Diamond\Box(A1 \wedge A2)$ is true in the initial state s_i.

Let us now consider the similar formula

$$(\Box\Diamond A1 \wedge \Box\Diamond A2) \rightarrow \Box\Diamond(A1 \wedge A2).$$

Suppose that $A1$ is true at $s_3, s_6, s_9, \ldots$ and that $A2$ is true at $s_1, s_2, s_4, s_5, s_7, s_8, \ldots$, as shown in the following diagram:



Clearly, the antecedent is true and the consequent is false, so we have falsified the formula, showing that it is *not* a theorem of temporal logic.

In the exercises, you are asked to prove or disprove other important formulas of temporal logic.

Specifying overtaking

Consider the following scenario:

$$try_p,\ \underbrace{try_q,\ cs_q,\ \ldots,\ try_q,\ cs_q,}_{1000\text{ times}}\ cs_p.$$

Process p tries to enter its critical section, but does so only after process q tries and successfully enters its critical section one thousand times. The scenario is *not* an example of starvation, because it remains true that *eventually* p enters its critical section. This scenario shows that freedom from starvation is a very weak property.

To ensure that a process enters its critical section within a reasonable amount of time, we can specify that an algorithm satisfy the property of *k-bounded overtaking*, meaning that from the time a process p attempts to enter its critical section, another process can enter at most k times before p does:

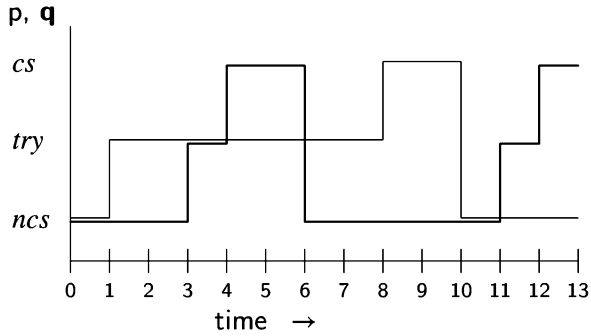
$$try_p,\ \underbrace{try_q,\ cs_q,\ try_q,\ cs_q,\ try_q,\ cs_q,}_{3\text{-}overtaking}\ cs_p.$$

The bakery algorithm for the critical section problem to be discussed in Chapter 5 satisfies the property of 1-bounded overtaking: if process p tries to enter its critical

section, any other process can enter its critical section at most once before p does. This property can be expressed using the *weak until* operator $\mathcal{W}$:

$$try_p \rightarrow (\neg\, cs_q)\ \mathcal{W}\ (cs_q)\ \mathcal{W}\ (\neg\, cs_q)\ \mathcal{W}\ (cs_p). \tag{4.1}$$

Let us interpret this formula on the following diagram, where the execution of p is represented by thin lines and that of q by thick lines:



We want the formula to be true at time 1 when try_p is true. Between 1 and 4, $\neg\, cs_q$ is true, so we have to check the truth of $(cs_q)\ \mathcal{W}\ (\neg\, cs_q)\ \mathcal{W}\ (cs_p)$ at time 4. Now cs_q is true from time 4 to time 6, so it remains to check if $(\neg\, cs_q)\ \mathcal{W}\ (cs_p)$ is true at time 6. And, in fact, $\neg\, cs_q$ remains true until cs_p becomes true at time 8.

Note that the formula does not specify freedom from starvation, because the $\mathcal{W}$ operator does not require that its right operand ever becomes true. Therefore, the formula is true in an execution in which both cs_p and cs_q are always false. In the exercises, we ask you to modify the formula so that it also specifies freedom from starvation.

4.5 A deductive proof of Dekker's algorithm^A

In this section we give a full deductive proof of the correctness of Algorithm 3.10, Dekker's algorithm, repeated as Algorithm 4.2 for convenience. The following lemma is a direct result of the structure of the program and its proof is very similar to that of Lemma 4.3.

Lemma 4.10 The following formulas are invariant:

$$turn = 1 \vee turn = 2, \tag{4.2}$$

$$p3..5 \vee p8..10 \leftrightarrow wantp, \tag{4.3}$$

$$q3..5 \vee q8..10 \leftrightarrow wantq. \tag{4.4}$$

Algorithm 4.2: Dekker's algorithm	
boolean wantp ← false, wantq ← false integer turn ← 1	
p	**q**
loop forever	loop forever
p1: non-critical section	q1: non-critical section
p2: wantp ← true	q2: wantq ← true
p3: while wantq	q3: while wantp
p4: if turn = 2	q4: if turn = 1
p5: wantp ← false	q5: wantq ← false
p6: await turn = 1	q6: await turn = 2
p7: wantp ← true	q7: wantq ← true
p8: critical section	q8: critical section
p9: turn ← 2	q9: turn ← 1
p10: wantp ← false	q10: wantq ← false

The statement and proof that mutual exclusion holds for Dekker's algorithm is similar to that of Theorem 4.4 and is left as an exercise.

Reasoning about progress

Invariants can be used to specify and verify safety properties like mutual exclusion, but they cannot be used for proving liveness properties like freedom from starvation. To prove a liveness property, we must reason about *progress*, that is, if the computation is in a state satisfying A, it must progress to another state in which B is true. We will now explain how to reason about progress and then use the techniques to prove the freedom from starvation of Dekker's algorithm.

We will assume that all computations are weakly fair (Section 2.7). In terms of progress, this means that if a statement of a process can be executed, then eventually it will be executed. Weak fairness is needed to rule out trivial counterexamples; if a process is never allowed to execute, then of course it may be starved. We are only interested in scenarios for counterexamples that are the result of a lack of synchronization among processes.

Assignment statements: Clearly, if the control pointer of a process p points to an assignment statement var←expression and if (by weak fairness) p is eventually allowed to execute, then the execution eventually completes. Therefore, progress holds for assignment statements, meaning that in any such computation, there are states s_i and s_{i+1}, such that s_{i+1} is obtained from s_i by changing the control pointer of p—it is incremented to the next statement—and changing the value of *var* to be the value of *expression* as evaluated in s_i.

Critical and non-critical sections: By assumption (Section 3.2), critical sections progress while non-critical sections need not. In Dekker's algorithm, progress for the critical section means that $p8 \rightarrow \Diamond p9$ must always be true, or equivalently, $\Box(p8 \rightarrow \Diamond p9)$ must be true. For the non-critical section, we *cannot* claim $p1 \rightarrow \Diamond p2$, because it is possible that $\Diamond\Box p1$, if, from some point in time, process p remains at p1 indefinitely.

Control statements: The situation is more complicated in the case of control statements with conditions like if, while and await statements. Consider for example, statement p4: if turn = 2. Neither of the following formulas is true:

$$p4 \wedge (turn = 2) \rightarrow \Diamond p5,$$
$$p4 \wedge \neg(turn = 2) \rightarrow \Diamond p3.$$

Suppose that the control pointer of process p is actually at p4 and that the value of the variable turn is actually 2. It does not follow that the process p eventually reaches p5. This does happen in interleavings in which the next step is taken from process p, but there are interleavings in which process q executes an arbitrary number of statements, among which might be q9: turn←1. When p is next allowed to execute, it will continue to statement p3, not to p5.

To prove that a *specific branch* of a conditional statement is taken, we must first prove that the condition is held at a certain value indefinitely:

$$p4 \wedge \Box(turn = 2) \rightarrow \Diamond p5.$$

This may seem to be too strong a requirement, because $turn = 2$ may not be true forever, but it is important that it be true indefinitely, until process p is allowed to execute the if statement and continue to statement p5.

A proof rule for progress: Frequently, we will have a pair of lemmas of the form: (a) $\Box A \rightarrow \Diamond B$, meaning that if (from now) A remains true then B is eventually true, and (b) $\Diamond\Box A$, meaning that eventually A remains true indefinitely. We would like to conclude $\Diamond B$. If (b) had been $\Box A$, this would follow immediately from (a) by modus ponens. Nevertheless, the inference is sound, as we ask you to show in an exercise.

We now turn to the proof of freedom from starvation of Dekker's algorithm. Semi-formal reasoning will be used about progress. When we say that a formula $\Diamond B$ becomes true "by progress," we mean that the statements of the program and formulas of the form $\Box A$ that have already been proved constrain the program to execute statements making B eventually true.

A proof of freedom from starvation

We will prove freedom from starvation under the assumption that q always progresses out of its critical section: $\Box\Diamond\neg q1$; this will be called *the progress assumption on* q. The full proof follows from additional invariants that you are asked to prove in the exercises.

The intended interpretation of the following lemma is that if p insists on entering its critical section then eventually q will defer to it. Read it as: if *wantp* and *turn* = 1 are always true, then eventually *wantq* will be indefinitely false.

Lemma 4.11 $\Box wantp \wedge \Box turn = 1 \rightarrow \Diamond\Box\neg wantq$.

Proof: If the antecedent $\Box wantp \wedge \Box turn = 1$ is true, by the progress assumption on q and the progress of the other statements of q, process q will eventually reach q6: await turn=2, and it will remain there because $\Box turn = 1$. Invariant (4.4) implies that *wantq* eventually becomes false and remains false, making the consequent true. ∎

Theorem 4.12 $p2 \rightarrow \Diamond p8$.

Proof: To prove the theorem, we assume its negation $p2 \wedge \neg\Diamond p8$ and derive a contradiction.

If $\Box turn = 2$, then $p2 \wedge \neg\Diamond p8$ implies by progress that $\Diamond\Box p6$ (eventually process p remains p6: await turn=1). By invariant (4.3), this implies $\Diamond\Box\neg wantp$. By the progress assumption for q and progress of the other statements in process q, $\Box turn = 2$ and $\Diamond\Box\neg wantp$ imply that eventually q9: turn $\leftarrow$ 1 is executed, so $\Diamond turn = 1$. This contradicts the assumption $\Box turn = 2$, establishing the truth of $\neg\Box turn = 2$, which is equivalent to $\Diamond turn = 1$ by invariant (4.2).

By assumption, $p2 \wedge \neg\Diamond p8$, so process p will never execute p9: turn $\leftarrow$ 2; therefore, $\Diamond turn = 1$ implies $\Diamond\Box turn = 1$. By progress, it follows that process p is eventually looping at the two statements p3: while wantq and p4: if turn=2. Invariant (4.3) then implies $\Box wantp$, and from Lemma 4.11 it follows that $\Diamond\Box\neg wantq$, contradicting the claim that p repeatedly executes p3 without entering the critical section. ∎

4.6 Model checking

If the number of reachable states in a program is not too large, it is possible to construct a diagram of the states and transitions, and to check correctness properties by examining the diagram. Clearly, if we use variables of types like integer or floating point, the number of possible states is astronomical. Nevertheless, constructing state diagrams is a practical tool for verification for several reasons.

First, algorithms for concurrency typically use variables which take on only a few values or which can be *modeled* by just a few values. If we want to verify a communications protocol, for example, we need not include the rich structure of the messages themselves, as it is sufficient to represent any message by a small number representing its type.

Second, the construction of the state diagram can be incremental, starting with the initial state. The structure of the program often limits the number of states that can actually be accessible in an execution. Furthermore, we can perform *model checking*: checking the truth of a correctness specification as the incremental diagram is constructed, so that if a falsifying state is found, the construction need not be carried further.

Third, we do not actually have to display the state diagram. The diagram is just a data structure which can be constructed and stored by a computer program. It is a directed graph whose nodes contain tuples of values, and the size of a tuple (the number of bits needed to store it) is fixed for any particular program.

In this book we will explain how to perform model checking using Spin (Appendix D.2). Spin is a model checker that has become very popular in both academic research and in industrial software development, because it is extremely efficient and yet easy to use. There are many other model checkers intended for use with different languages, models and logics. We draw your attention in particular to Java PathFinder (JPF) [69] and to the tools developed by the SAnToS research group. They are intended to be used for the verification of *programs* written in Java, in contrast with Spin which is intended for use in the modeling and design of concurrent and distributed systems.

4.7 Spin and the Promela modeling language^L

In this section we present the Promela language that is used in Spin to write concurrent programs. Promela is a programming language with syntax and semantics like any other language, but it is called a modeling language, because it contains a

```
1  bool wantp = false, wantq = false;
2  byte turn = 1;
3
4  active proctype p() {
5     do :: wantp = true;
6           do :: !wantq -> break;
7              :: else ->
8                    if :: (turn == 1)
9                       :: (turn == 2) ->
10                            wantp = false;
11                            (turn == 1);
12                            wantp = true
13                   fi
14          od;
15          printf("MSC: p in CS\n") ;
16          turn = 2;
17          wantp = false
18    od
19 }
20
21 active proctype q() { /* similar */ }
```

Listing 4.1: Dekker's algorithm in Promela

limited number of constructs that are intended to be used to build models of concurrent systems. Designing such a language is a delicate balancing act: a larger language is more expressive and easier to use for writing algorithms and programs, but a smaller language facilitates efficient model checking. We will not give the details of the syntax and semantics of Promela, as these are readily available in the Spin online documentation and in the reference manual [33]. Instead, we will describe how it can be used to model and verify the algorithms that are used in this textbook.

The description of the language will refer to Listing 4.1 which shows Dekker's algorithm in Promela. The syntax and semantics of declarations and expressions are similar to those of C. The control structures might be unfamiliar, as they use the syntax and semantics of *guarded commands*, frequently used in theoretical computer science. To execute an **if** statement, the guards are evaluated; these are the first expressions following the :: in each alternative. If some of them evaluate to true, one of them is (nondeterministically) selected for execution; if none evaluate to true, the statement *blocks*. In the algorithm, the guards of the **if** statement:

```
if
  :: (turn == 1)
  :: (turn == 2) -> ...
fi
```

are exhaustive and mutually exclusive (because the value of turn is either 1 or 2), so there is no blocking and no nondeterminism. A **do** statement is executed like an **if** statement, except that after executing the statements of one alternative, the execution loops back to its beginning. A **break** statement is needed to exit a loop.

An **else** guard is used to obtain the semantics of a non-blocking **if** or **do** statement as shown in line 7. The meaning of this guard is that if all other guards evaluate to false, then this alternative is selected.

Of course, the concept of blocking the execution of statement makes sense only within the context of a concurrent program. So writing:

```
if :: (turn == 1) -> /* Do nothing */
fi
```

makes sense. There is only one guard turn==1 and if it is false, the statement will block until some other process assigns 1 to turn, making the statement executable. In fact, in Promela, *any* boolean expression will simply block until it evaluates to true, so the above **if** statement can be expressed simply as:

```
(turn == 1)
```

as shown in line 9.

A process is declared as a **proctype**, that is, a process declaration is a type and you can instantiate several processes from the process type. We have used **active** as a shortcut to both declare the types, and instantiate and activate a procedure of each type.

An array-like syntax can be used to activate several procedures of the same type:

```
#define NPROCS 3
active [NPROCS] proctype p() { ... }
```

Note the use of the C-language preprocessor to parameterize the program. Alternatively, processes can be declared without **active** and then explicitly activated within a special initialization process:

```
proctype p(byte N) {
  ...
}

init {
  int n = ... ;
  atomic {
      run p(2*n);
      run p(3*n);
  }
}
```

This is useful if you want to compute initial parameters and pass their values to the processes. (See, for example, the algorithm in Section 8.3.) **atomic** was discussed in Section 2.13.

Within each process, the predefined identifier _pid gives a unique identifier for each process, and can be used, for example, in output statements or to index arrays:

```
bool want[NPROCS] = false;
  ...
want[_pid] = true;
printf ("MSC: process %d in critical section ", _pid);
```

4.8 Correctness specifications in Spin^L

In Section 4.3, logical formulas expressing correctness specifications used atomic propositions like $p1$ that are true if and only if the control pointer of a process is at that label. In Spin correctness specifications are expressed using *auxiliary variables*. For example, to prove that Dekker's algorithm satisfies mutual exclusion, we add an auxiliary variable critical that is incremented and then decremented when the processes are in their critical sections:

```
byte   critical  = 0;
active proctype p() {
   /* preprotocol */
   critical ++;
   printf ("MSC: p in CS\n") ;
   critical --;
   /* postprotocol */
}
```

The values of these variables are not otherwise used in the computation (and you will receive a warning message to that effect when you compile the program), so they cannot affect the correctness of the algorithm. However, it is clear that the boolean-valued expression *critical* ≤ 1 is true if and only if at most one process is in its critical section. A state in which *critical* > 1 is a state in which mutual exclusion is violated.

The **printf** statements are not used in the proof of correctness; they serve only to display information during simulation.

We are now ready to ask Spin to verify that Dekker's algorithm satisfies mutual exclusion. The simplest way to do this is to attach an *assertion* to each critical section claiming that at most one process is there in any state:

```
critical ++;
assert(critical <= 1);
critical --;
```

Clearly, if mutual exclusion is violated then it is violated when the two processes are in their critical sections, so it is sufficient to check *within* the critical sections that the property is always true. Running a Spin verification in Safety mode will return the answer that there are no errors. (Verification of Promela models in Spin is described in greater detail in Appendix D.2.)

An alternative way of proving that the algorithm satisfies mutual exclusion is to use a formula in temporal logic. Atomic propositions in Spin must be identifiers, so we start by defining an identifier that is true when mutual exclusion holds:

```
#define mutex (critical <= 1)
```

We can now execute a verification run of Spin in Acceptance mode with the temporal logic formula `[]mutex`. The two-character symbol `[]` is used for the temporal operator *always* $\Box$, and the two-character symbol `<>` is used for the temporal operator *eventually* $\Diamond$. Spin will report that there are no errors.

Let us introduce an error into the program; for example, suppose that we forget to write the not symbol `!` in the guard in line 6. Running a verification will cause Spin to report an error: `claim violated`. To assist the user in diagnosing the error, Spin writes a *trail*, which is a representation of a scenario that leads to the claim being violated. You can now run Spin in simulation mode on the trail to examine this scenario, and you will in fact discover that the execution reaches a state in which *critical* $= 2$, falsifying the proposition `mutex`.

We can use Spin to verify that Dekker's algorithm is free from starvation. Since the algorithm is symmetric, it is sufficient to prove this for one of the processes,

say process p. First we define a proposition that is true if p is in its critical section:

```
bool PinCS = false;
#define nostarve PinCS

active proctype p() {
   /* preprotocol */
   critical ++;
   PinCS = true;
   PinCS = false;
   critical --;
   /* postprotocol */
}
```

The property we want to verify is given by the following temporal logic formula:

```
[]<> nostarve
```

and a verification run shows that this holds. By default, Spin will check all scenarios, even those that are not weakly fair (Section 2.7). To prove freedom from starvation of Dekker's algorithm, you must specify that only weakly fair scenarios are to be checked.

4.9 Choosing a verification technique[A]

Verification of concurrent and distributed programs is a dynamic field. A central goal of this introductory book is to prepare you for further study of verification by introducing a variety of techniques and tools, instead of choosing and presenting a unified approach. A few words are in order to compare the various approaches, and you may wish to return to this discussion again as you progress in your studies.

The basic concept in defining the semantics of any program is that of the state, consisting of the control pointers and the contents of the memory. Executing a statement of a program simply transforms one state into another. The correctness specification of a sequential program is straightforward: it relates the final state at termination to the initial state. (See Appendix B.4 for an example.) Concurrent programs are much more difficult to verify, both because of the nondeterministic nature of the computation, and also because of the complexity of the correctness specifications. There are basically two classes of specifications: safety properties and liveness properties.

A safety property must be true at *all* states, so—needless to say—you have to check that it is true at all states. There are two ways of doing this: generate all possible states and check that each one satisfies the property, or show that any state that may be generated satisfies the property. The former technique is used by model checkers and the latter by deductive systems. Model checking is basically a "mindless" mechanical enumeration of all the possible states of the computation. Practical model checkers like Spin are thus characterized by the clever and sophisticated techniques they employ for incrementally and efficiently constructing and checking the set of states, not by any insight they provide into the structure of a concurrent algorithm. They are primarily used for debugging high-level system specifications, because if a verification run fails, the model check will provide the detailed scenario of a counterexample to the property. Receiving the message errors: 0 is a cause for rejoicing, but it doesn't help you understand why an algorithm is correct.

Deductive systems are similar to those used in mathematics. A safety property in the form of an invariant is proved by induction over the structure of the algorithm: if it is true initially and if its truth is preserved by all statements in the program, then it will be true in any possible state. This we know without actually creating any states. The first advantage a deductive proof has over model checking is that it can handle state spaces of indefinite size that might overwhelm a model checker; since no states are actually created, there is no problem of computational resources. The second advantage is the insight that you get from working through the steps of the induction. The are two disadvantages of deductive proofs relative to model checking. First, it can require some ingenuity to develop the proof, so your failure to prove an algorithm may just be the result of failing to find the right invariant. Second, although mechanical systems are available to help develop deductive proofs [12], most proofs are done by hand and thus are prone to human error, unlike a well-debugged model checker which does not make mistakes.

Deductive proofs and model checking complement each other, which is why they are both included in this book. For example, while it is easy and important to verify Barz's algorithm by model checking in Spin, the complex deductive proof in Section 6.10 gives insight into how the algorithm works.

A liveness property claims that a state satisfying a property will inevitably occur. It is not sufficient to check states one by one; rather, all possible scenarios must be checked. This requires more complex theory and software techniques. More resources are required for checking liveness properties and this limits the complexity of models that can be checked. In deductive proofs, it is not sufficient to check the inductive steps of individual statements; the proof rules needed are much more sophisticated, as shown by the proof of the freedom from starvation of Dekker's algorithm.

Transition

The proof of safety properties of concurrent programs like mutual exclusion is usually straightforward. The difficulty is to discover the right invariants, but once the invariants are specified, checking them is relatively easy. Deductive proofs of liveness properties require complex reasoning in temporal logic. A practical alternative is to use computer programs called model checkers to conduct a systematic search for counterexamples to the correctness assertions. The Spin model checker and its language Promela were described.

By now you must be totally bored solving the critical section problem within the model of atomic load and store to global memory. If so, skip to Chapter 6 to continue your study with new models and new problems. Eventually, you will want to return to Chapter 5 to study more advanced algorithms for the critical section problem.

Exercises

1. Give invariants for the first attempt (Algorithm 3.2) and show that mutual exclusion holds.

2. Prove Lemma 4.10 and use it to prove that Dekker's algorithm satisfies the mutual exclusion property.

3. What is the difficulty in proving freedeom from starvation in Dekker's algorithm for the case where process q may terminate in its critical section? Prove the following invariants and use them to prove freedom from starvation:

$$p6 \wedge (turn = 2) \rightarrow q2..9, \tag{4.5}$$
$$q6 \wedge (turn = 1) \rightarrow p2..9, \tag{4.6}$$
$$q10 \rightarrow turn = 1, \tag{4.7}$$
$$p10 \rightarrow turn = 2. \tag{4.8}$$

4. Prove the proof rule for progress: $\Box A \rightarrow \Diamond B$ and $\Diamond\Box A$ imply $\Diamond B$.

5. Express $\Box$ and $\Diamond$ in terms of $\mathcal{U}$.

6. Prove that $\Box$ distributes over conjunction $(\Box A \wedge \Box B) \leftrightarrow \Box(A \wedge B)$ and that $\Diamond$ distributes over disjunction $(\Diamond A \vee \Diamond B) \leftrightarrow \Diamond(A \vee B)$. Prove or disprove the corresponding formulas for distributing $\Box$ over disjunction and $\Diamond$ over conjunction.

7. Prove:

$$\Box\Box A \leftrightarrow \Box A, \tag{4.9}$$
$$\Diamond\Diamond A \leftrightarrow \Diamond A, \tag{4.10}$$
$$\Diamond\Box\Diamond A \leftrightarrow \Box\Diamond A, \tag{4.11}$$
$$\Box\Diamond\Box A \leftrightarrow \Diamond\Box A. \tag{4.12}$$

It follows that strings of unary temporal operators "collapse" to a single operator or to a pair of distinct operators.

8. Using the operator $\mathcal{U}$, modify Equation 4.1 (page 79) so that it also specifies freedom from starvation for process p.

9. The temporal operator *leads to*, denoted $\rightsquigarrow$, is defined as: $A \rightsquigarrow B$ is true in a state s_i if and only if for all states s_j, $j \geq i$, if A is true s_j, then B is true in some state s_k, $k \geq j$. Express $\rightsquigarrow$ in terms of the other temporal operators.

10. Prove the correctness of Peterson's algorithm, repeated here for convenience:

Algorithm 4.3: Peterson's algorithm	
boolean wantp ← false, wantq ← false integer last ← 1	
p	**q**
loop forever	loop forever
p1: non-critical section	q1: non-critical section
p2: wantp ← true	q2: wantq ← true
p3: last ← 1	q3: last ← 2
p4: await wantq = false or last = 2	q4: await wantp = false or last = 1
p5: critical section	q5: critical section
p6: wantp ← false	q6: wantq ← false

First show that

$$(p4 \wedge q5) \rightarrow (wantq \wedge last = 1), \tag{4.13}$$
$$(p5 \wedge q4) \rightarrow (wantp \wedge last = 2) \tag{4.14}$$

are invariant, and then use them to prove that mutual exclusion holds. To prove liveness for process p, prove the following formulas:

$$p4 \wedge \Box\neg p5 \rightarrow \Box\Diamond(wantq \wedge (last \neq 2)), \tag{4.15}$$
$$\Diamond\Box(\neg\, wantq) \vee \Diamond(last = 2), \tag{4.16}$$
$$p4 \wedge \Box\neg p5 \wedge \Diamond(last = 2) \rightarrow \Diamond\Box(last = 2), \tag{4.17}$$

and deduce a contradiction.

11. Show that Peterson's algorithm does not satisfy the LCR restriction (page 27). Write a Promela program for the algorithm that does satisfy the restriction.

12. Write a Promela program for the frog puzzle of Section 2.14 with six frogs and use Spin to find a solution. Hint: use atomic to ensure that each move of a frog is an atomic operation.

13. In the Promela program for Dekker's algorithm, is it sufficient to add the assertion to just one of the processes p and q, or should they be added to both?

14. Modify the Promela program for Dekker's algorithm to prove that $\Box\Diamond p8$.

15. (Ruys [56]) To check a safety property P of a Promela program, we use the LTL formula `[]P`. Alternatively, we could add an additional process that is always enabled and checks the property as an assertion:

```
active proctype monitor() {
    assert(P)
}
```

Discuss the advantages and disadvantages of the two approaches. Similarly, discuss the following alternatives for the monitor process:

```
active proctype monitor() {
    !P -> assert(P)
}
```

```
active proctype monitor() {
    do :: assert(P) od
}
```

5 Advanced Algorithms for the Critical Section Problem[A]

In this chapter we present two advanced algorithms for the critical section problem. These algorithms are important for two reasons: first, they work for an arbitrary number of processes, and second, they raise additional issues concerning the concurrency abstraction and the specification of the critical section problem. Both algorithms were developed by Leslie Lamport.

5.1 The bakery algorithm

In the bakery algorithm, a process wishing to enter its critical section is required to take a numbered *ticket*, whose value is greater than the values of all outstanding tickets. The process waits until its ticket has the lowest value of all outstanding tickets and then enters its critical section. The name of the algorithm is taken from ticket dispensers used at the entrance of bakeries and similar institutions that wish to serve customers on a first-come, first-served basis. Before looking at the full bakery algorithm, let us examine a simplified version for two processes:

Algorithm 5.1: Bakery algorithm (two processes)			
integer np $\leftarrow$ 0, nq $\leftarrow$ 0			
p		**q**	
	loop forever		loop forever
p1:	non-critical section	q1:	non-critical section
p2:	np $\leftarrow$ nq + 1	q2:	nq $\leftarrow$ np + 1
p3:	await nq = 0 or np $\leq$ nq	q3:	await np = 0 or nq < np
p4:	critical section	q4:	critical section
p5:	np $\leftarrow$ 0	q5:	nq $\leftarrow$ 0

np and nq hold the ticket numbers of the two processes. A value of 0 indicates that the process does not want to enter its critical section, while a positive value represents an implicit queue of the processes that do want to enter, with lower ticket numbers denoting closeness to the head of the queue. If the ticket numbers are equal, we arbitrarily assign precedence to process p.

Lemma 5.1 The following formulas are invariant:

$$\begin{aligned} np = 0 &\leftrightarrow p1 \vee p2, \\ nq = 0 &\leftrightarrow q1 \vee q2, \\ p4 &\rightarrow (nq = 0) \vee (np \leq nq), \\ q4 &\rightarrow (np = 0) \vee (nq < np). \end{aligned}$$

Proof: The first two invariants follow trivially from the structure of the program, since np is only assigned to in process p and nq is only assigned to in process q.

Let us prove the third invariant as the proof of the fourth is symmetric. Clearly, $p4 \rightarrow (nq = 0) \vee (np \leq nq)$ is true initially. Suppose that it is true because both the antecedent and consequent are false; can it be falsified by the antecedent "suddenly" becoming true? This will occur if process p executes p3, successfully passing the await statement. But, by the condition, $nq = 0 \vee (np \leq nq)$ must be true, so the consequent cannot have been false.

Suppose now that the formula is true because both the antecedent and consequent are true; can it be falsified by the consequent "suddenly" becoming false? For the antecedent to remain true, process p must remain at p4, so the values of the variables can be changed only by process q. The only possibility is to change the value of nq, either in statement q2 or in statement q5. But q2 makes $np \leq nq$ true and q5 makes $nq = 0$ true, so the consequent remains true. ∎

Theorem 5.2 The bakery algorithm for two processes satisfies the mutual exclusion property.

Proof: Combining the third and fourth invariants from Lemma 5.1, we get:

$$(p4 \wedge q4) \rightarrow ((nq = 0) \vee (np \leq nq)) \wedge ((np = 0) \vee (nq < np)).$$

By the first two invariants of the lemma, if $p4 \wedge q4$ is true, then neither $np = 0$ nor $nq = 0$ is true, so the formula simplifies to:

$$(p4 \wedge q4) \rightarrow (np \leq nq) \wedge (nq < np).$$

The consequent is clearly false by the properties of arithmetic, so we can deduce that $p4 \wedge q4$ is always false, and therefore $\neg(p4 \wedge q4)$ is invariant, proving that the mutual exclusion property holds. ∎

Theorem 5.3 The two-process bakery algorithm is free from starvation.

Proof: Let us prove that process p is not starved: $p2 \rightarrow \Diamond p4$. Suppose to the contrary that $p2$ and $\neg \Diamond p4$ are true. By progress of the assignment statement p2,

$p3$ becomes true; together with $\neg\Diamond p4$, this implies $\Diamond\Box p3$. By the first invariant of Lemma 5.1, $\Diamond\Box p3$ implies $\Diamond\Box(np = k)$ for some $k > 0$. Furthermore, $\Diamond\Box p3$ is true only if eventually the await statement at p3 is never executed successfully. By weak fairness, p must attempt to execute p3 infinitely often only to find the condition false. Therefore,

$$\Box\Diamond\neg(nq = 0 \vee np \leq nq)$$

must be true. From deMorgan's laws and the distributive laws of temporal logic, specifically:

$$\Box(A \wedge B) \rightarrow (\Box A \wedge \Box B) \text{ and } \Diamond(A \wedge B) \rightarrow (\Diamond A \wedge \Diamond B),$$

this formula implies

$$\Box\Diamond(nq \neq 0) \wedge \Box\Diamond(nq < np).$$

$\Box\Diamond(nq \neq 0)$ means that process q leaves the critical section infinitely often, so by progress, q executes q2: nq←np+1 infinitely often. By $\Diamond\Box(np = k)$, $\Diamond\Box(nq = k + 1)$, for $k > 0$, contradicting $\Box\Diamond(nq < np)$. ∎

We have assumed that the assignments at statements p2 and q2 are atomic. This can be unrealistic, because each involves computing an expression with the value of one variable and assigning the result to another. In the exercises you are asked to find a scenario that violates mutual exclusion when the normal load and store model is used, and to prove the correctness of another version of the algorithm.

5.2 The bakery algorithm for N processes

Here is the bakery algorithm for N processes:

Algorithm 5.2: Bakery algorithm (N processes)	
	integer array[1..n] number ← [0,. . . ,0]
	loop forever
p1:	non-critical section
p2:	number[i] ← 1 + max(number)
p3:	for all *other* processes j
p4:	await (number[j] = 0) or (number[i] ≪ number[j])
p5:	critical section
p6:	number[i] ← 0

Each of the N processes executes the same algorithm, except that i is set to a different constant in the range 1 to N, called the ID number of the process.

The statement for all *other* processes j is an abbreviation for

```
for j from 1 to N
   if j ≠ i
```

The notation (number[i] ≪ number[j]) is an abbreviation for:

```
(number[i] < number[j]) or
((number[i] = number[j]) and (i < j))
```

that is, either the first ticket number is lower than the second, or they are equal and the first process ID number is lower than the second.

Each process chooses a number that is greater than the maximum of all outstanding ticket numbers. A process is allowed to enter its critical section when it has a lower ticket number than all other processes who want to enter their critical sections. In case of a tie in comparing ticket numbers, the lower numbered process is arbitrarily given precedence. Again we are making an unrealistic assumption, namely, that computing the maximum of the values of an array is atomic. As shown in the next section, these assumptions can be removed at the cost of complicating the algorithm.

The bakery algorithm is elegant because it has the property that no variable is both read and written by more than one process (unlike the variable turn in Dekker's algorithm). Despite this elegance, the bakery algorithm is impractical for two reasons. First, the ticket numbers will be unbounded if some process is always in the critical section. Second, each process must query every other process for the value of its ticket number, and the entire loop must be executed even if no other process wants to enter its critical section.

See [48, Section 10.7] for a proof of the correctness of the bakery algorithm. The ideas used in the algorithm are also used in the Ricart–Agrawala algorithm for distributed mutual exclusion (Section 10.3).

5.3 Less restrictive models of concurrency

The model of concurrency that we use throughout this book is that of interleaved execution of atomic statements, where each access to a variable in memory (load or store) is atomic. In a sense, we are "passing the buck," because we are depending on a hardware-level synchronization mechanism to ensure mutual exclusion of memory accesses. Here is the original bakery algorithm for N processes, which is correct under weaker assumptions than the atomicity of load and store to global variables:

Algorithm 5.3: Bakery algorithm without atomic assignment

```
        boolean array[1..n] choosing ← [false,...,false]
        integer array[1..n] number ← [0,...,0]
    loop forever
p1:     non-critical section
p2:     choosing[i] ← true
p3:     number[i] ← 1 + max(number)
p4:     choosing[i] ← false
p5:     for all other processes j
p6:         await choosing[j] = false
p7:         await (number[j] = 0) or (number[i] ≪ number[j])
p8:     critical section
p9:     number[i] ← 0
```

Algorithm 5.3 differs from Algorithm 5.2 by the addition of a boolean array choosing. If choosing[i] is true, a process is in the act of choosing a ticket number, and other processes must wait for the choice to be made before comparing numbers.

Each global variable is written to by exactly one process, so there are no cases in which multiple write operations to the same variable can overlap. It is also reasonable to assume that if a set of read operations to a variable does not overlap a write operation to that variable, then they all return the correct value. However, if read operations overlap a write operation, it is possible that inconsistent values may be obtained. Lamport has shown that the bakery algorithm is correct even if such read operations return arbitrary values [36].

5.4 Fast algorithms

In the bakery algorithm, a process wishing to enter its critical section must read the values of all the elements of the array number to compute the maximum ticket number, and it must execute a loop that contains await statements that read the values of all the elements of the two arrays. If there are dozens of processes in the system, this is going to be extremely inefficient. The overhead may be unavoidable: if, in general, processes attempt to enter their critical sections at short intervals, there will be a lot of contention, and a process really will have to query the state of all the other processes. If, however, contention is low, it makes sense to search for an algorithm for the critical section problem that is very efficient, in the sense that only if there is contention does the process incur the significant overhead of querying the other processes.

An algorithm for the critical section problem is *fast*, if in the absence of contention a process can access its critical section by executing pre- and postprotocols consisting of a *fixed* (and small) number of statements, none of which are await statements. The first fast algorithm for mutual exclusion was given by Lamport in [40], and this paper initiated extensive research into algorithms that are efficient under various assumptions about the characteristics of a system, such as the amount of contention and the behavior of the system under errors. Here we will describe and prove Lamport's algorithm restricted to two processes and then note the changes needed for an arbitrary number of processes.

Outline of the fast algorithm

Here is the outline of the algorithm, where we assume that the process identifiers p and q are encoded as nonzero integer constants so that their values can be assigned to variables and compared with the values of these variables:

Algorithm 5.4: Fast algorithm for two processes (outline)	
integer gate1 ← 0, gate2 ← 0	
p	**q**
loop forever	loop forever
non-critical section	non-critical section
p1: gate1 ← p	q1: gate1 ← q
p2: if gate2 ≠ 0 goto p1	q2: if gate2 ≠ 0 goto q1
p3: gate2 ← p	q3: gate2 ← q
p4: if gate1 ≠ p	q4: if gate1 ≠ q
p5: if gate2 ≠ p goto p1	q5: if gate2 ≠ q goto q1
critical section	critical section
p6: gate2 ← 0	q6: gate2 ← 0

(To simplify the proof of correctness, we have given an abbreviated algorithm as was done in the previous chapters.)

The concept of the algorithm is displayed graphically in Figures 5.1–5.3. A process is represented by a stick figure. The non-critical section is at the left of each diagram; a process must pass through two gates to reach the critical section on the right side of the diagram. Figure 5.1 shows what happens in the absence of contention. The process enters the first gate, writing its ID number on the gate [p1, (a)]. (The notation correlates the line number p1 in the algorithm with diagram (a) in the figure.) It then looks at the second gate [p2, (b)]; in the absence of contention, this gate will not have an ID written upon it, so the process writes its ID on that gate also [p3, (c)]. It then looks back over its shoulder at the first gate to

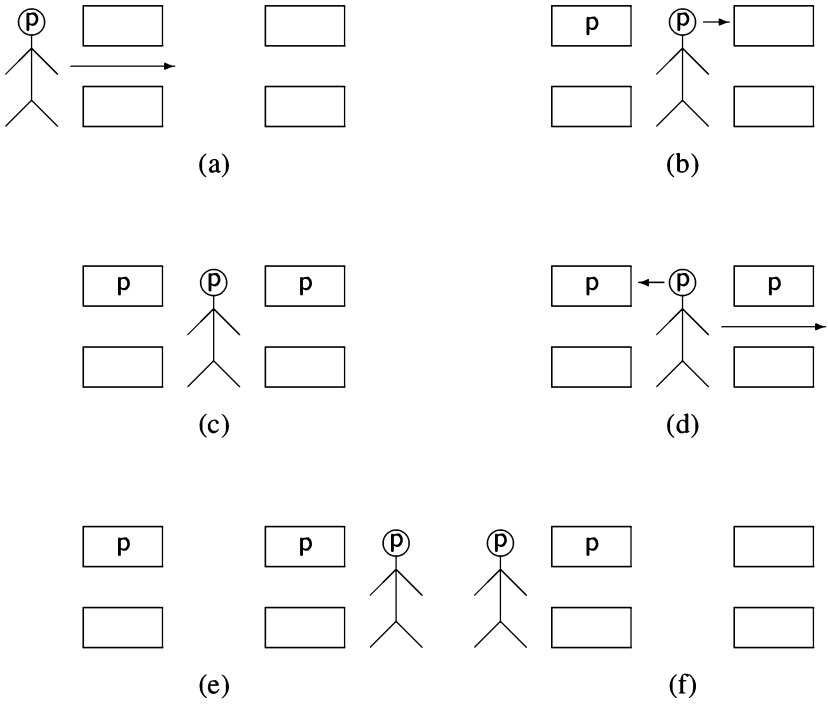


Figure 5.1: Fast algorithm—no contention

check if its ID is still written there [p4, (d)]. If so, it enters the critical section [p6, (e)]. Upon leaving the critical section and returning to the non-critical section, it erases its ID from the second gate [p6, (f)].

In the absence of contention, the algorithm is very efficient, because a process can access its critical section at the cost of three statements that assign a constant to a global variable (p1, p3, p6), and two if statements that check that the value of a global variable is not equal to a constant (p2, p4).

The case of contention at the second gate is shown in Figure 5.2. The process enters the first gate [p1, (a)], but when it checks the second gate [p2, (b)], it sees that the other process q has already written its ID, preparing to enter the critical section. Since q has gotten through both gates sooner, process p returns to before the first gate [p2, (c)] to try again [p1, (d)].

In the case of contention at the first gate, the algorithm is a bit more complex (Figure 5.3). Initially, the algorithm proceeds as before [p1, (a)], [p2, (b)], [p3, (c)], until looking back over its shoulder, process p perceives that process q has entered the first gate and written its ID [p4, (d)]. There are now two possibilities:

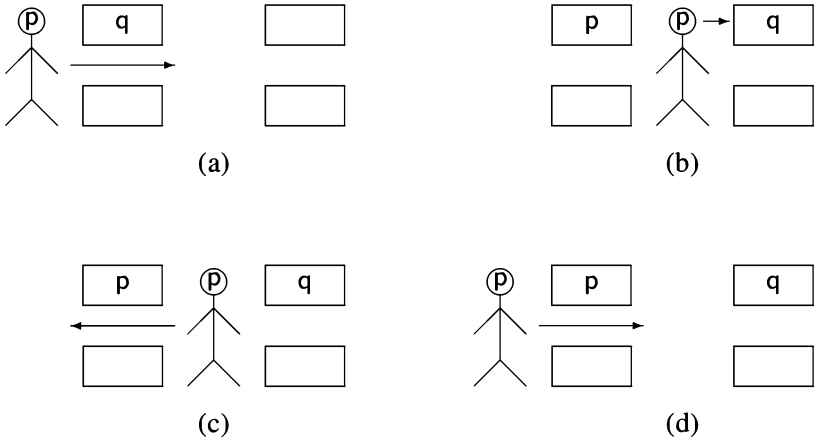


Figure 5.2: Fast algorithm—contention at gate 2

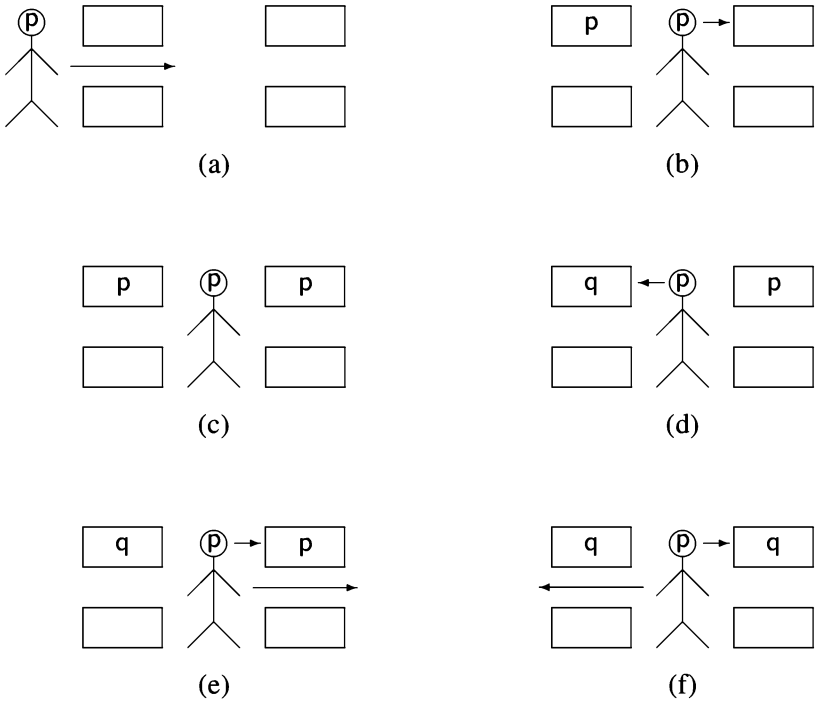


Figure 5.3: Fast algorithm—contention at gate 1

the ID of process p is still written on the second gate; if so, it continues into the critical section [p5, (e)]. Or, process q has passed process p and written its ID on the second gate; if so, the new process defers and returns to the first gate [p5, (f)].

The outline of the algorithm is not correct; in the exercises you are asked to find a scenario in which mutual exclusion is not satisfied. We will now partially prove the outline of the algorithm and then modify the algorithm to ensure correctness.

Partial proof of the algorithm

To save turning pages, here is the algorithm again:

Algorithm 5.5: Fast algorithm for two processes (outline)			
integer gate1 ← 0, gate2 ← 0			
p		**q**	
	loop forever		loop forever
	non-critical section		non-critical section
p1:	gate1 ← p	q1:	gate1 ← q
p2:	if gate2 ≠ 0 goto p1	q2:	if gate2 ≠ 0 goto q1
p3:	gate2 ← p	q3:	gate2 ← q
p4:	if gate1 ≠ p	q4:	if gate1 ≠ q
p5:	if gate2 ≠ p goto p1	q5:	if gate2 ≠ q goto q1
	critical section		critical section
p6:	gate2 ← 0	q6:	gate2 ← 0

Lemma 5.4 The following formulas are invariant:

$$p5 \wedge gate2 = p \quad \rightarrow \quad \neg(q3 \vee q4 \vee q6) \tag{5.1}$$

$$q5 \wedge gate2 = q \quad \rightarrow \quad \neg(p3 \vee p4 \vee p6) \tag{5.2}$$

We *assume* the truth of this lemma and use it to prove the following lemma:

Lemma 5.5 The following formulas are invariant:

$$p4 \wedge gate1 = p \quad \rightarrow \quad gate2 \neq 0 \tag{5.3}$$

$$p6 \quad \rightarrow \quad gate2 \neq 0 \wedge \neg q6 \wedge (q3 \vee q4 \rightarrow gate1 \neq q) \tag{5.4}$$

$$q4 \wedge gate1 = q \quad \rightarrow \quad gate2 \neq 0 \tag{5.5}$$

$$q6 \quad \rightarrow \quad gate2 \neq 0 \wedge \neg p6 \wedge (p3 \vee p4 \rightarrow gate1 \neq p) \tag{5.6}$$

Mutual exclusion follows immediately from invariants (5.4) and (5.6).

Proof: By symmetry of the algorithm, it is sufficient to prove that (5.3) and (5.4) are invariant. Trivially, both formulas are true initially.

Proof of 5.3: Executing p3 makes $p4$, the first conjunct of the antecedent, true, but it also makes $gate2 = p$, so the consequent is true. Executing statement p1 makes the second conjunct of the antecedent $gate1 = p$ true, but $p4$, the first conjunct, remains false. Executing q6 while process p is at p4 can make the consequent false, but by the inductive hypothesis, (5.6) is true, so $p3 \vee p4 \rightarrow gate1 \neq p$, and therefore the antecedent remains false.

Proof of 5.4: There are two transitions that can make the antecedent $p6$ true—executing p4 or p5 when the conditions are false:

p4 to p6: By the if statement, $gate1 \neq p$ is false, so $gate1 = p$ is true; therefore, by the inductive hypothesis for (5.3), $gate2 \neq 0$, proving the truth of the first conjunct of the consequent. Since $gate1 = p$ is true, $gate1 \neq q$, so the third conjunct is true. It remains to show $\neg q6$. Suppose to the contrary that $q6$ is true. By the inductive hypothesis for (5.6), if both $q6$ and $p4$ are true, then so is $gate1 \neq p$, contradicting $gate1 = p$.

p5 to p6: By the if statement, $gate2 = p$, so $gate2 \neq 0$, proving the truth of the first conjunct. By the assumed invariance of (5.1), $\neg(q3 \vee q4 \vee q6)$, so $\neg q6$, which is the second conjunct, is true, as is the third conjunct since its antecedent is false.

Assume now that the antecedent is true; there are five transitions of process q that can possibly make the consequent false:

q6: This statement cannot be executed since $\neg q6$ by the inductive hypothesis.

q4 to q6: This statement cannot be executed because $q3 \vee q4 \rightarrow gate1 \neq q$ by the inductive hypothesis.

q5 to q6: This statement cannot be executed. It can only be executed if $gate2 \neq q$ is false, that is, if $gate2 = q$. By the assumed invariance of (5.2) this can be true only if $\neg(p3 \vee p4 \vee p6)$, but $\neg p6$ contradicts the truth of the antecedent $p6$.

q1: This can falsify $gate1 \neq q$, but trivially, $q3 \vee q4$ will also be false.

q2 to q3: This can falsify the third conjunct if $gate1 \neq q$ is false. But the statement will be executed only if $gate2 = 0$, contradicting the inductive hypothesis of the first conjunct. ∎

Lemma 5.4 is not true for Algorithm 5.5. The algorithm must be modified so that (5.1) and (5.2) are invariant without invalidating the invariants we have already proved. This is done by adding local variables wantp and wantq with the usual meaning of wanting to enter the critical section (Algorithm 5.6). Clearly, these additions do not invalidate the proof of Lemma 5.5 because the additional variables

Algorithm 5.6: Fast algorithm for two processes

```
integer gate1 ← 0, gate2 ← 0
boolean wantp ←false, wantq ← false
```

p	q
loop forever	loop forever
non-critical section	non-critical section
p1: gate1 ← p	q1: gate1 ← q
wantp ← true	wantq ← true
p2: if gate2 ≠ 0	q2: if gate2 ≠ 0
wantp ← false	wantq ← false
goto p1	goto q1
p3: gate2 ← p	q3: gate2 ← q
p4: if gate1 ≠ p	q4: if gate1 ≠ q
wantp ← false	wantq ← false
await wantq = false	await wantp = false
p5: if gate2 ≠ p goto p1	q5: if gate2 ≠ q goto q1
else wantp ← true	else wantq ← true
critical section	critical section
p6: gate2 ← 0	q6: gate2 ← 0
wantp ← false	wantq ← false

are not referenced in the proof, and because await statements can affect the liveness of the algorithm, but not the truth of the invariants.

Let use now prove that 5.1 is invariant in Algorithm 5.6.

Lemma 5.6 $p5 \wedge gate2 = p \rightarrow \neg(q3 \vee q4 \vee q6)$ is invariant.

Proof: Suppose that the antecedent is true; then $gate2 = p$ prevents the execution of both q2 to q3 and q5 to q6, so the consequent cannot be falsified.

Suppose now that the consequent is false; can the antecedent become true? The conjunct $gate2 = p$ in the antecedent cannot become true, because gate2 is not assigned the value p when executing p4 (making $p5$ true) nor by any statement in process q.

Consider now executing p4 to p5 to make $p5$ true. By assumption, the consequent $\neg(q3 \vee q4 \vee q6)$ is false, so the control pointer of process q is at q3 or q4 or q6. It follows trivially from the structure of the program that $wantq \leftrightarrow (q2 \vee q3 \vee q4 \vee q6)$ is invariant. Therefore, await wantq = false will not allow the control pointer of process p to reach p5. ∎

Generalization to N processes

To generalize Algorithm 5.6 to an arbitrary number of processes, use an array of boolean variables want so that each process writes to its own variable. Replace the statement await wantq = false by a loop that waits in turn for each want variable to become false, meaning that that process has left its critical section:

```
for all other processes j
    await want[j] = false
```

The N-process algorithm is still a fast algorithm, because in the absence of contention it executes a fixed number of statements, none of which is an await-statement. Only if there is contention will the loop of await statements be executed. This algorithm does not prevent starvation of an individual process (though there are other algorithms that do satisfy this requirement). However, since starvation scenarios typically involve a high rate of contention, if (as assumed) little contention occurs, a process will not be indefinitely starved.

5.5 Implementations in Promela^L

A complete verification of the bakery algorithm cannot be done in Spin because the ticket numbers are unbounded, so either variables will overflow or the number of states will overflow the memory allocated to Spin. The depth of search can be increased to some large value (say, pan -m1000); then, you can claim that if mutual exclusion is satisfied to that depth, there is unlikely to be a counterexample.

Transition

This chapter has presented advanced algorithms for the critical section problem that improve on earlier algorithms in two areas. First, algorithms have been developed for models that are weaker than atomic load and store to global memory. While this model is adequate to describe multitasking on a single CPU, the required level of atomicity cannot be guaranteed for multiprocessor architectures. Second, early algorithms for solving the critical section problem required that a process wishing to enter its critical section interrogate every other process. This can be highly inefficient if the number of processes is large, and newer algorithms can reduce this inefficiency under certain assumptions.

In most systems, higher-level synchronization constructs are used to simplify the development of concurrent programs. The next chapter introduces the semaphore, the first such construct.

Exercises

1. The correctness proof of the two-process bakery algorithm (Algorithm 5.1) assumed that the assignment statements np←nq+1 and nq←np+1 are atomic. Find a scenario in which mutual exclusion does not hold when np←nq+1 is replaced by temp ← nq followed by np ← temp + 1, and similarly for nq←np+1.

2. Show that Algorithm 5.1 as modified in the previous exercise is correct if the statement np←1 is added before np←nq+1 in process p and similarly for q.

3. Construct a scenario for the bakery algorithm showing that the ticket numbers can be unbounded.

4. (Fisher, cited in [40]) Show that the following algorithm solves the critical section problem for n processes provided that delay is sufficiently long:

Algorithm 5.7: Fisher's algorithm

```
        integer gate ← 0
    loop forever
        non-critical section
        loop
p1:         await gate = 0
p2:         gate ← i
p3:         delay
p4:     until gate = i
        critical section
p5:     gate ← 0
```

5. Show that mutual exclusion does not hold for Algorithm 5.4.

6. Show that Algorithm 5.6 is free from deadlock.

7. Construct a scenario that leads to starvation in Algorithm 5.6.

8. (Lamport [39]) Show that mutual exclusion and freedom from deadlock hold for the following algorithm:

Algorithm 5.8: Lamport's one-bit algorithm	
	boolean array[1..n] want ← [false,. . . ,false]
	loop forever
	non-critical section
p1:	want[i] ← true
p2:	for all processes j < i
p3:	if want[j]
p4:	want[i] ← false
p5:	await not want[j]
	goto p1
p6:	for all processes j > i
p7:	await not want[j]
	critical section
p8:	want[i] ← false

Show that starvation is possible. Prove that mutual exclusion is satisfied even if a process i terminates arbitrarily, provided that it resets want[i] to false.

9. (Manna and Pnueli [51, p. 232]) Show that mutual exclusion and freedom from deadlock (but not starvation) hold for the following algorithm with a server process and n client processes:

Algorithm 5.9: Manna–Pnueli central server algorithm	
	integer request ← 0, respond ← 0
	client process i
	loop forever
	non-critical section
p1:	while respond ≠ i
p2:	request ← i
	critical section
p3:	respond ← 0
	server process
	loop forever
p4:	await request ≠ 0
p5:	respond ← request
p6:	await respond = 0
p7:	request ← 0

6 Semaphores

The algorithms for the critical section problem described in the previous chapters can be run on a *bare machine*, that is, they use only the machine language instructions that the computer provides. However, these instructions are too low-level to be used efficiently and reliably. In this chapter, we will study the *semaphore*, which provides a concurrent programming construct on a higher level than machine instructions. Semaphores are usually implemented by an underlying operating system, but we will investigate them by defining the required behavior and assuming that this behavior can be efficiently implemented.

Semaphores are a simple, but successful and widely used, construct, and thus worth exploring in great detail. We start with a section on the concept of process state. Then we define the semaphore construct and show how it trivially solves the critical section problem that we worked so hard to solve in the previous chapters. Section 6.4 defines invariants on semaphores that can be used to prove correctness properties. The next two sections present new problems, where the requirement is not to achieve mutual exclusion but rather cooperation between processes.

Semaphores can be defined in various ways, as discussed in Section 6.8. Section 6.9 introduces Dijkstra's famous problem of the dining philosophers; the problem is of little practical importance, but it is an excellent framework in which to study concurrent programming techniques. The next two sections present advanced algorithms by Barz and Udding that explore the relative strength of different definitions of semaphores. The chapter ends with a discussion of semaphores in various programming languages.

6.1 Process states

A multiprocessor system may have more processors than there are processes in a program. In that case, every process is always *running* on some processor. In a multitasking system (and similarly in a multiprocessor system in which there are more processes than processors), several processes will have to share the computing resources of a single CPU. A process that "wants to" run, that is, a process that

can execute its next statement, is called a *ready* process. At any one time, there will be one running process and any number of ready processes. (If no processes are ready to run, an *idle* process will be run.)

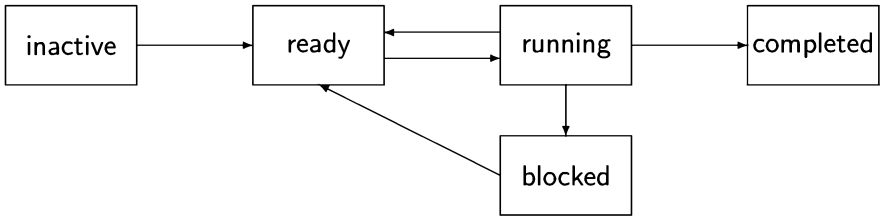
A system program called a *scheduler* is responsible for deciding which of the ready processes should be run, and performing the context switch, replacing a running process with a ready process and changing the state of the running process to ready. Our model of concurrency by interleaving of atomic statements makes no assumptions about the behavior of a scheduler; arbitrary interleaving simply means that the scheduler may perform a context switch at any time, even after executing a single statement of a running process. The reader is referred to textbooks on operating systems [60, 63] for a discussion of the design of schedulers.

Within our model, we do not explicitly mention the concept of a scheduler. However, it is convenient to assume that every process p has associated with it an attribute called its *state*, denoted p.state. Assignment to this attribute will be used to indicate a change in the state of a process. For example, if p.state = running and q.state = ready, then a context switch between them can be denoted by:

```
p.state ← ready
q.state ← running
```

The synchronization constructs that we will define assume that there exists another process state called *blocked*. When a process is blocked, it is not ready so it is not a candidate for becoming the running process. A blocked process can be *unblocked* or *awakened* or *released* only if an external action changes the process state from blocked to ready. At this point the unblocked process becomes a candidate for execution along with all the current ready processes; with one exception (Section 7.5), an unblocked process will not receive any special treatment that would make it the new running process. Again, within our model we do not describe the implementation of the actions of blocking and unblocking, and simply assume that they happen as defined by the synchronization primitives. Note that the await statement does not block a process; it is merely a shorthand for a process that is always ready, and may be running and checking the truth of a boolean expression.

The following diagram summarizes the possible state changes of a process:



Initially, a process is *inactive*. At some point it is activated and its state becomes ready. When a concurrent program begins executing, one process is running and the others are ready. While executing, it may encounter a situation that requires the process to cease execution and to become blocked until it is unblocked and returns to the ready state. When a process executes its final statement it becomes *completed*.

In the BACI concurrency simulator, activation of a process occurs when the coend statement is executed. The activation of concurrent processes in languages such as Ada and Java is somewhat more complicated, because it must take into account the dependency of one process on another.

6.2 Definition of the semaphore type

A semaphore S is a compound data type with two fields, S.V of type non-negative integer, and S.L of type set of processes. We are using the familiar dotted notation of records and structures from programming languages. A semaphore S must be initialized with a value $k \geq 0$ for S.V and with the empty set $\emptyset$ for the S.L:

```
semaphore S ← (k, ∅)
```

There are two *atomic* operations defined on a semaphore S (where p denotes the process that is executing the statement):[1]

wait(S)

```
if  S.V > 0
    S.V ← S.V − 1
else
    S.L ← S.L ∪ p
    p.state ← blocked
```

If the value of the integer component is nonzero, decrement its value (and the process p can continue its execution); otherwise—it is zero—process p is added to the set component and the state of p becomes blocked. We say that p is blocked *on* the semaphore S.

[1]The original notation is `P(S)` for `Wait(S)` and `V(S)` for `signal(S)`, the letters P and V taken by Dijkstra from words in Dutch. This notation is still used in research papers and some systems.

signal(S)

```
if S.L = ∅
   S.V ← S.V + 1
else
   let q be an arbitrary element of S.L
   S.L ← S.L − {q}
   q.state ← ready
```

If S.L is empty, increment the value of the integer component; otherwise—S.L is nonempty—unblock q, an arbitrary element of the set of processes blocked *on* S.L. The state of process p does not change.

A semaphore whose integer component can take arbitrary non-negative values is called a *general semaphore*. A semaphore whose integer component takes only the values 0 and 1 is called a *binary semaphore*. A binary semaphore is initialized with (0, ∅) or (1, ∅). The wait(S) instruction is unchanged, but signal(S) is now defined by:

```
if S.V = 1
   // undefined
else if S.L = ∅
   S.V ← 1
else // (as above)
   let q be an arbitrary element of S.L
   S.L ← S.L − {q}
   q.state ← ready
```

A binary semaphore is sometimes called a *mutex*. This term is used in pthreads and in java.util.concurrent.

Once we have become familiar with semaphores, there will be no need to explicitly write the set of blocked processes S.L.

6.3 The critical section problem for two processes

Using semaphores, the solution of the critical section problem for two processes is trivial (Algorithm 6.1). A process, say p, that wishes to enter its critical section executes a preprotocol that consists only of the wait(S) statement. If $S.V = 1$ then S.V is decremented and p enters its critical section. When p exits its critical section and executes the postprotocol consisting only of the signal(S) statement, the value of S.V will once more be set to 1.

Algorithm 6.1: Critical section with semaphores (two processes)			
binary semaphore S ← (1, ∅)			
p		q	
	loop forever		loop forever
p1:	non-critical section	q1:	non-critical section
p2:	wait(S)	q2:	wait(S)
p3:	critical section	q3:	critical section
p4:	signal(S)	q4:	signal(S)

If q attempts to enter its critical section by executing wait(S) before p has left, $S.V = 0$ and q will become blocked on S. S, the value of the semaphore S, will become $(0, \{q\})$. When p leaves the critical section and executes signal(S), the "arbitrary" process in the set S.L = $\{q\}$ will be q, so that process will be unblocked and can continue into its critical section.

The solution is similar to Algorithm 3.6, the second attempt, except that the definition of the semaphore operations as atomic statements prevents interleaving between the test of S.V and the assignment to S.V.

To prove the correctness of the solution, consider the abbreviated algorithm where the non-critical section and critical section statements have been absorbed into the following wait(S) and signal(S) statements, respectively.

Algorithm 6.2: Critical section with semaphores (two proc., abbrev.)			
binary semaphore S ← (1, ∅)			
p		q	
	loop forever		loop forever
p1:	wait(S)	q1:	wait(S)
p2:	signal(S)	q2:	signal(S)

The state diagram for this algorithm is shown in Figure 6.1. Look at the state in the top center, which is reached if initially process p executes its wait statement, successfully entering the critical section. If process q now executes its wait statement, it finds $S.V = 0$, so the process is added to $S.L$ as shown in the top right state. Since q is blocked, there is no outgoing arrow for q, and we have labeled the only arrow by p to emphasize that fact. Similarly, in the bottom right state, p is blocked and only process q has an outgoing arrow.

A violation of the mutual exclusion requirement would be a state of the form (p2: signal(S), q2: signal(S), ...). We immediately see that no such state exists, so we conclude that the solution satisfies this requirement. Similarly, there is no deadlock, since there are no states in which both processes are blocked. Finally,

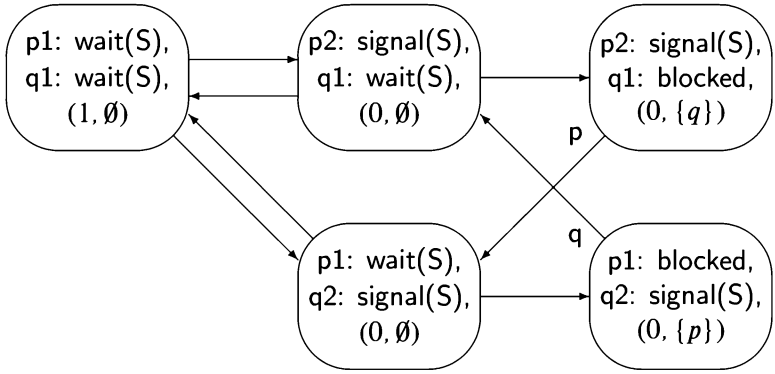


Figure 6.1: State diagram for the semaphore solution

the algorithm is free from starvation, since if a process executes its wait statement (thus leaving the non-critical section), it enters either the state with the signal statement (and the critical section) or it enters a state in which it is blocked. But the only way out of a blocked state is into a state in which the blocked process continues with its signal statement.

6.4 Semaphore invariants

Let k be the initial value of the integer component of the semaphore, $\#signal(S)$ the number of signal(S) statements that have been executed and $\#wait(S)$ the number of wait(S) statements that have been executed. (A process that is blocked when executing wait(S) is *not* considered to have successfully executed the statement.)

Theorem 6.1 A semaphore S satisfies the following invariants:

$$S.V \geq 0, \tag{6.1}$$

$$S.V = k + \#signal(S) - \#wait(S), \tag{6.2}$$

Proof: By definition of a semaphore, $k \geq 0$, so 6.1 is initially true, as is 6.2 since $\#signal(S)$ and $\#wait(S)$ are zero. By assumption, non-semaphore operations cannot change the value of (either component of) a semaphore, so we only need consider the effect of the wait and signal operations. Clearly, 6.1 is invariant under both operations. If either operation changes $S.V$, the truth of 6.2 is preserved: If a wait operation decrements $S.V$, then $\#wait(S)$ is incremented, and similarly a signal operation that increments $S.V$ also increments $\#signal(S)$. If a signal operation unblocks a blocked process, $S.V$ is not changed, but both $\#signal(S)$ and $\#wait(S)$ increase by one so the righthand side of 6.2 is not changed. ∎

Theorem 6.2 The semaphore solution for the critical section problem is correct: there is mutual exclusion in the access to the critical section, and the program is free from deadlock and starvation.

Proof: Let $\#CS$ be the number of processes in their critical sections. We will prove that

$$\#CS + S.V = 1 \tag{6.3}$$

is invariant. By a trivial induction on the structure of the program it follows that

$$\#CS = \#wait(S) - \#signal(S) \tag{6.4}$$

is invariant. But invariant (6.2) for the initial value $k = 1$ can be written $\#wait(S) - \#signal(S) = 1 - S.V$, so it follows that $\#CS = 1 - S.V$ is invariant, and this can be written as $\#CS + S.V = 1$.

Since $S.V \geq 0$ by invariant (6.1), simple arithmetic shows that $\#CS \leq 1$, which proves the mutual exclusion property.

For the algorithm to enter deadlock, both processes must be blocked on the wait statement and $S.V = 0$. Since neither process is in the critical section, $\#CS = 0$. Therefore $\#CS + S.V = 0$, which contradicts invariant (6.3).

Suppose that some process, say p, is starved. It must be indefinitely blocked on the semaphore so $S = (0, S.L)$, where p is in the list $S.L$. By invariant (6.3), $\#CS = 1 - S.V = 1 - 0 = 1$, so the other process q is in the critical section. By progress, q will complete the critical section and execute signal(S). Since there are only two processes in the program, $S.L$ must *equal* the singleton set $\{p\}$, so p will be unblocked and enter its critical section. ∎

6.5 The critical section problem for N processes

Using semaphores, the same algorithm gives a solution for the critical section problem for N processes:

Algorithm 6.3: Critical section with semaphores (N proc.)	
	binary semaphore S ← $(1, \emptyset)$
	loop forever
p1:	non-critical section
p2:	wait(S)
p3:	critical section
p4:	signal(S)

The proofs in Theorem 6.2 remain unchanged for N processes as far as mutual exclusion and freedom from deadlock are concerned. Unfortunately, freedeom from starvation does not hold. For the abbreviated algorithm:

Algorithm 6.4: Critical section with semaphores (N proc., abbrev.)	
	binary semaphore S $\leftarrow (1, \emptyset)$
	loop forever
p1:	wait(S)
p2:	signal(S)

consider the following scenario for three processes, where the statement numbers for processes q and r correspond to those for p in Algorithm 6.4:

n	Process p	Process q	Process r	S
1	**p1: wait(S)**	q1: wait(S)	r1: wait(S)	$(1, \emptyset)$
2	p2: signal(S)	**q1: wait(S)**	r1: wait(S)	$(0, \emptyset)$
3	p2: signal(S)	q1: blocked	**r1: wait(S)**	$(0, \{q\})$
4	**p1: signal(S)**	q1: blocked	r1: blocked	$(0, \{q, r\})$
5	**p1: wait(S)**	q1: blocked	r2: signal(S)	$(0, \{q\})$
6	p1: blocked	q1: blocked	**r2: signal(S)**	$(0, \{p, q\})$
7	p2: signal(S)	q1: blocked	**r1: wait(S)**	$(0, \{q\})$

Line 7 is the same as line 3 and the scenario can continue indefinitely in this loop of states. The two processes p and r "conspire" to starve process q.

When there were only two processes, we used the fact that $S.L$ is the singleton set $\{p\}$ to claim that process p must be unblocked, but with more than two processes this claim no longer holds. Starvation is caused by the fact that, in our definition of the semaphore type, a signal operation may unblock an *arbitrary* element of S.L. Freedom from starvation does not occur if the definition of the semaphore type is changed so that S.L is a queue not a set. (See Section 6.8 for a comparison of semaphore definitions.)

In the exercises, you are asked to show that if the initial value of S.V is k, then at most k processes can be in the critical section at any time.

6.6 Order of execution problems

The critical section problem is an abstraction of synchronization problems that occur when several processes compete for the same resource. Synchronization

problems are also common when processes must coordinate the *order of execution* of operations of different processes. For example, consider the problem of the concurrent execution of one step of the mergesort algorithm, where an array is divided into two halves, the two halves are sorted concurrently and the results are merged together. Thus, to mergesort the array [5, 1, 10, 7, 4, 3, 12, 8], we divide it into two halves [5, 1, 10, 7] and [4, 3, 12, 8], sort them obtaining [1, 5, 7, 10] and [3, 4, 8, 12], respectively, and then merge the two sorted arrays to obtain [1, 3, 4, 5, 7, 8, 10, 12].

To perform this algorithm concurrently, we will use three processes, two for sorting and one for merging. Clearly, the two sort processes work on totally independent data and need no synchronization, while the merge process must not execute its statements until the two other have completed. Here is a solution using two binary semaphores:

Algorithm 6.5: Mergesort		
integer array A binary semaphore S1 ← (0, ∅) binary semaphore S2 ← (0, ∅)		
sort1	**sort2**	**merge**
p1: sort 1st half of A p2: signal(S1) p3:	q1: sort 2nd half of A q2: signal(S2) q3:	r1: wait(S1) r2: wait(S2) r3: merge halves of A

The integer components of the semaphores are initialized to zero, so process merge is initially blocked on S1. Suppose that process sort1 completes before process sort2; then sort1 signals S1 enabling merge to proceed, but it is then blocked on semaphore S2. Only when sort2 completes will merge be unblocked and begin executing its algorithm. If sort2 completes before sort1, it will execute signal(S2), but merge will still be blocked on S1 until sort1 completes.

6.7 The producer–consumer problem

The producer–consumer problem is an example of an order-of-execution problem. There are two types of processes in this problem:

Producers A producer process executes a statement produce to create a data element and then sends this element to the consumer processes.

Consumers Upon receipt of a data element from the producer processes, a consumer process executes a statement consume with the data element as a parameter.

As with the critical section and non-critical section statements, the produce and consume statements are assumed not to affect the additional variables used for synchronization and they can be omitted in abbreviated versions of the algorithms.

Producers and consumers are ubiquitous in computing systems:

Producer	**Consumer**
Communications line	Web browser
Web browser	Communications line
Keyboard	Operating system
Word processor	Printer
Joystick	Game program
Game program	Display screen
Pilot controls	Control program
Control program	Aircraft control surfaces

When a data element must be sent from one process to another, the communications can be *synchronous*, that is, communications cannot take place until both the producer and the consumer are ready to do so. Synchronous communication is discussed in detail in Chapter 8.

More common, however, is *asynchronous communications* in which the communications channel itself has some capacity for storing data elements. This store, which is a queue of data elements, is called a *buffer*. The producer executes an append operation to place a data element on the tail of the queue, and the consumer executes a take operation to remove a data element from the head of the queue.

The use of a buffer allows processes of similar average speeds to proceed smoothly in spite of differences in transient performance. It is important to understand that a buffer is of no use if the average speeds of the two processes are very different. For example, if your web browser continually produces and sends more data than your communications line can carry, the buffer will always be full and there is no advantage to using one. A buffer can also be used to resolve a clash in the structure of data. For example, when you download a compressed archive (a *zip* file), the files inside cannot be accessed until the entire archive has been read.

There are two synchronization issues that arise in the producer–consumer problem: first, a consumer cannot take a data element from an empty buffer, and second, since the size of any buffer is finite, a producer cannot append a data element to a full buffer. Obviously, there is no such thing as an infinite buffer, but if the buffer is very large compared to the rate at which data is produced, you might want to avoid the extra overhead required by a finite buffer algorithm and risk an occasional loss of data. Loss of data will occur when the producer tries to append a data element to

a full buffer; either the operation will not succeed, in which case that element is lost, or it will overwrite one of the existing elements in the buffer (see Section 13.6).

Infinite buffers

If there is an infinite buffer, there is only one interaction that must be synchronized: the consumer must not attempt a take operation from an empty buffer. This is an order-of-execution problem like the mergesort algorithm; the difference is that it happens repeatedly within a loop.

Algorithm 6.6: producer–consumer (infinite buffer)	
infinite queue of dataType buffer ← empty queue semaphore notEmpty ← (0, ∅)	
producer	**consumer**
dataType d loop forever p1: d ← produce p2: append(d, buffer) p3: signal(notEmpty)	dataType d loop forever q1: wait(notEmpty) q2: d ← take(buffer) q3: consume(d)

Since the buffer is infinite, it is impossible to construct a finite state diagram for the algorithm. A *partial* state diagram for the abbreviated algorithm:

Algorithm 6.7: producer–consumer (infinite buffer, abbreviated)	
infinite queue of dataType buffer ← empty queue semaphore notEmpty ← (0, ∅)	
producer	**consumer**
dataType d loop forever p1: append(d, buffer) p2: signal(notEmpty)	dataType d loop forever q1: wait(notEmpty) q2: d ← take(buffer)

is shown in Figure 6.2. In the diagram, the value of the buffer is written with square brackets and a buffer element is denoted by x; the consumer process is denoted by *con*. The horizontal arrows indicate execution of operations by the producer, while the vertical arrows are for the consumer. Note that the left two states in the lower row have no arrows for the consumer because it is blocked.

Assume now that the two statements of each process form one atomic statement each. (You are asked to remove this restriction in an exercise.) The following invariant holds:

$$notEmpty.V = \#buffer,$$

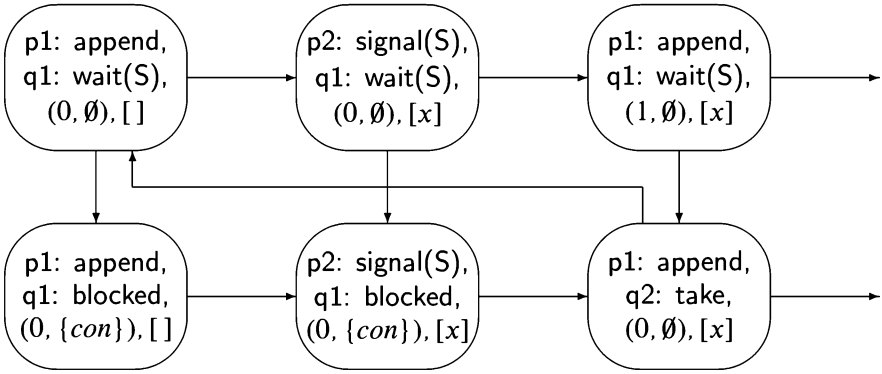


Figure 6.2: Partial state diagram for producer–consumer with infinite buffer

where *#buffer* is the number of elements in buffer. The formula is true initially since there are no elements in the buffer and the value of the integer component of the semaphore is zero. Each execution of the statements of the producer appends an element to the buffer and also increments the value of *notEmpty.V*. Each execution of the statements of the consumer takes an element from the buffer and also decrements the value of *notEmpty.V*. From the invariant, it easily follows that the consumer does not remove an element from an empty buffer.

The algorithm is free from deadlock because as long as the producer continues to produce data elements, it will execute signal(notEmpty) operations and unblock the consumer. Since there is only one possible blocked process, the algorithm is also free from starvation.

Bounded buffers

The algorithm for the producer–consumer problem with an infinite buffer can be easily extended to one with a finite buffer by noticing that the producer "takes" empty places from the buffer, just as the consumer takes data elements from the buffer (Algorithm 6.8). We can use a similar synchronization mechanism with a semaphore notFull that is initialized to N, the number of (initially empty) places in the finite buffer. We leave the proof of the correctness of this algorithm as an exercise.

Algorithm 6.8: producer–consumer (finite buffer, semaphores)	
finite queue of dataType buffer ← empty queue semaphore notEmpty ← $(0, \emptyset)$ semaphore notFull ← $(N, \emptyset)$	
producer	**consumer**
dataType d loop forever p1: d ← produce p2: wait(notFull) p3: append(d, buffer) p4: signal(notEmpty)	dataType d loop forever q1: wait(notEmpty) q2: d ← take(buffer) q3: signal(notFull) q4: consume(d)

Split semaphores

Algorithm 6.8 uses a technique called *split semaphores*. This is not a new semaphore type, but simply a term used to describe a synchronization mechanism built from semaphores. A split semaphore is a group of two or more semaphores satisfying an invariant that the sum of their values is at most equal to a fixed number N. In the case of the bounded buffer, the invariant:

$$notEmpty + notFull = N$$

holds at the beginning of the loop bodies.

In the case that $N = 1$, the construct is called a *split binary semaphore*. Split binary semaphores were used in Algorithm 6.5 for mergesort.

Compare the use of a split semaphore with the use of a semaphore to solve the critical section problem. Mutual exclusion was achieved by performing the wait and signal operations on a semaphore within a single process. In the bounded buffer algorithm, the wait and signal operations on each semaphore are in different processes. Split semaphores enable one process to wait for the completion of an event in another.

6.8 Definitions of semaphores

There are several different definitions of the semaphore type and it is important to be able to distinguish between them. The differences have to do with the specification of liveness properties; they *do not* affect the safety properties that follow from the semaphore invariants, so the solution we gave for the critical-section problem satisfies the mutual exclusion requirement whatever definition we use.

Strong semaphores

The semaphore we defined is called a *weak semaphore* and can be compared with a *strong semaphore*. The difference is that S.L, the set of processes blocked on the semaphore S, is replaced by a queue:

wait(S)

```
if S.V > 0
   S.V ← S.V − 1
else
   S.L ← append(S.L, p)
   p.state ← blocked
```

signal(S)

```
if S.L = ∅
   S.V ← S.V + 1
else
   q ← head(S.L)
   S.L ← tail (S.L)
   q.state ← ready
```

Recall (page 114) that we gave a scenario showing that starvation is possible in the solution for the critical section problem with more than two processes. For a strong semaphore, starvation is impossible for any number N of processes. Suppose p is blocked on S, that is, p appears in the queue S.L. There are at most $N-1$ processes in S.L, so there are at most $N-2$ processes ahead of p on the queue. After at most $N-2$ signal(S) operations, p will be at the head of the queue S.L so it will unblocked by the next signal(S) operation.

Busy-wait semaphores

A *busy-wait semaphore* does not have a component S.L, so we will identify S with S.V. The operations are defined as follows:

wait(S)

```
await S > 0
S ← S − 1
```

signal(S)

```
S ← S + 1
```

Semaphore operations are *atomic* so there is no interleaving between the two statements implementing the wait(S) operation.

With busy-wait semaphores you cannot ensure that a process enters its critical section even in the two-process solution, as shown by the following scenario:

n	Process p	Process q	S
1	**p1: wait(S)**	q1: wait(S)	1
2	p2: signal(S)	**q1: wait(S)**	0
3	**p2: signal(S)**	q1: wait(S)	0
4	**p1: wait(S)**	q1: wait(S)	1

Line 4 is identical to line 1 so these states can repeat indefinitely. The scenario is fair, in the technical sense of Section 2.7, because process q is selected infinitely often in the interleaving, but because it is always selected when $S = 0$, it never gets a chance to successfully complete the wait(S) operation.

Busy-wait semaphores are appropriate in a multiprocessor system where the waiting process has its own processor and is not wasting CPU time that could be used for other computation. They would also be appropriate in a system with little contention so that the waiting process would not waste too much CPU time.

Abstract definitions of semaphores[A]

The definitions given above are intended to be behavioral and not to specify data structures for implementation. The set of the weak semaphore is a mathematical entity and is only intended to mean that the unblocked process is chosen arbitrarily. The queue of the strong semaphore is intended to mean that processes are unblocked in the order they were blocked (although it can be difficult to specify what order means in a concurrent system [37]). A busy-wait semaphore simply allows interleaving of other processes between the signal and the unblocking of a blocked process. How these three behaviors are implemented is unimportant.

It is possible to give abstract definitions of *strongly fair* and *weakly fair* semaphores, similar to the definitions of scheduling fairness given in Section 2.7 (see [58, Section 10.1]). However, these definitions are not very useful, and any real implementation will almost certainly provide one of the behaviors we have given.

6.9 The problem of the dining philosophers

The problem of the dining philosophers is a classical problem in the field of concurrent programming. It offers an entertaining vehicle for comparing various formalisms for writing and proving concurrent programs, because it is sufficiently simple to be tractable yet subtle enough to be challenging.

The problem is set in a secluded community of five philosophers who engage in only two activities—*thinking* and *eating*:

Algorithm 6.9: Dining philosophers (outline)

```
    loop forever
p1:     think
p2:     preprotocol
p3:     eat
p4:     postprotocol
```

Meals are taken communally at a table set with five plates and five forks (Figure 6.3). In the center of the table is a bowl of spaghetti that is endlessly replenished. Unfortunately, the spaghetti is hopelessly tangled and a philosopher needs two forks in order to eat.[2] Each philosopher may pick up the forks on his left and right, but only one at a time. The problem is to design pre- and postprotocols to ensure that a philosopher only eats if she has two forks. The solution should also satisfy the correctness properties that we described in the chapter on mutual exclusion.

The correctness properties are:

- A philosopher eats only if she has two forks.
- Mutual exclusion: no two philosophers may hold the same fork simultaneously.

[2]It is part of the folklore of computer science to point out that the story would be more believable if the bowl contained rice and the utensils were chopsticks.

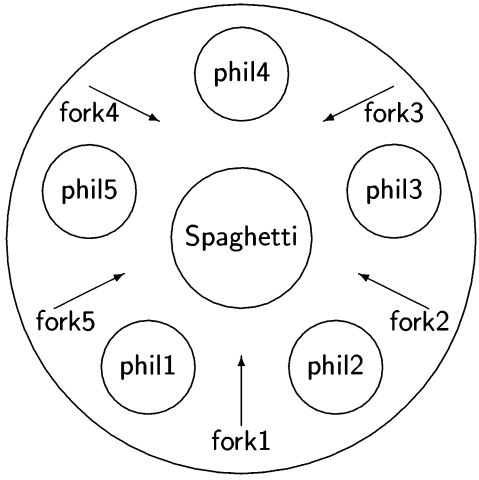


Figure 6.3: The dining philosophers

- Freedom from deadlock.
- Freedom from starvation (pun!).
- Efficient behavior in the absence of contention.

Here is a first attempt at a solution, where we assume that each philosopher is initialized with its index i, and that addition is implicitly modulo 5.

Algorithm 6.10: Dining philosophers (first attempt)	
	semaphore array[0..4] fork ← [1,1,1,1,1]
	loop forever
p1:	think
p2:	wait(fork[i])
p3:	wait(fork[i+1])
p4:	eat
p5:	signal(fork[i])
p6:	signal(fork[i+1])

Each fork is modeled as a semaphore: wait corresponds to taking a fork and signal corresponds to putting down a fork. Clearly, a philosopher holds both forks before eating.

Theorem 6.3 No fork is ever held by two philosophers.

Proof: Let $\#P_i$ be the number of philosophers holding fork i, so that $\#P_i = \#wait(fork[i]) - \#signal(fork[i])$. By the semaphore invariant (6.2), $fork[i] = 1 + (-\#P_i)$, or $\#P_i = 1 - fork[i]$. Since the value of a semaphore is non-negative (6.1), we conclude that $\#P_i \leq 1$. ∎

Unfortunately, this solution deadlocks under an interleaving that has all philosophers pick up their left forks—execute wait(fork[i])—before any of them tries to pick up a right fork. Now they are all waiting on their right forks, but no process will ever execute a signal operation.

One way of ensuring liveness in a solution to the dining philosophers problem is to limit the number of philosophers entering the dining room to four:

Algorithm 6.11: Dining philosophers (second attempt)

```
                semaphore array[0..4] fork ← [1,1,1,1,1]
                semaphore room ← 4
    loop forever
p1:   think
p2:   wait(room)
p3:   wait(fork[i])
p4:   wait(fork[i+1])
p5:   eat
p6:   signal(fork[i])
p7:   signal(fork[i+1])
p8:   signal(room)
```

The addition of the semaphore room obviously does not affect the correctness of the safety properties we have proved.

Theorem 6.4 The algorithm is free from starvation.

Proof: We must assume that the semaphore room is a blocked-queue semaphore so that any philosopher waiting to enter the room will eventually do so. The fork semaphores need only be blocked-set semaphores since only two philosophers use each one. If philosopher i is starved, she is blocked forever on a semaphore. There are three cases depending on whether she is blocked on fork[i], fork[i+1] or room.

Case 1: Philosopher i is blocked on her left fork. Then philosopher $i - 1$ holds fork[i] as her right fork, that is, philosopher $i - 1$ has successfully executed both her wait statements and is either eating, or about to signal her left or right fork

semaphores. By progress and the fact that there are only two processes executing operations on a fork semaphore, eventually she will release philosopher i.

Case 2: Philosopher i is blocked on her right fork. This means that philosopher $i + 1$ has successfully taken her left fork (fork[i+1]) and will never release it. By progress of eating and signaling, philosopher $i + 1$ must be blocked forever on her right fork. By induction, it follows that if i is blocked on her right fork, then so must all the other philosophers: $i + j, 1 \leq j \leq 4$. However, by the semaphore invariant on room, for some j, philosopher $i + j$ is not in the room, thus obviously not blocked on a fork semaphore.

Case 3: We assume that the room semaphore is a blocked-queue semaphore, so philosopher i can be blocked on room only if its value is zero indefinitely. By the semaphore invariant, there are at most four philosophers executing the statements between wait(room) and signal(room). The previous cases showed that these philosophers are never blocked indefinitely, so one of them will eventually execute signal(room). ∎

Another solution that is free from starvation is an asymmetric algorithm, which has the first four philosophers execute the original solution, but the fifth philosopher waits first for the *right* fork and then for the *left* fork:

Algorithm 6.12: Dining philosophers (third attempt)	
	semaphore array[0..4] fork ← [1,1,1,1,1]
	philosopher 4
	loop forever
p1:	think
p2:	wait(fork[0])
p3:	wait(fork[4])
p4:	eat
p5:	signal(fork[0])
p6:	signal(fork[4])

Again it is obvious that the correctness properties on eating are satisfied. Proofs of freedom from deadlock and freedom from starvation are similar to those of the previous algorithm and are left as an exercise.

A solution to the dining philosophers problem by Lehmann and Rabin uses random numbers. Each philosopher "flips a coin" to decide whether to take her left or right fork first. It can be shown that with probability one no philosopher starves [48, Section 11.4].

6.10 Barz's simulation of general semaphores[A]

In this section we present Hans W. Barz's simulation of a general semaphore by a pair of binary semaphores and an integer variable [6]. (See [67] for a survey of the attempts to solve this problem.) The simulation will be presented within the

Algorithm 6.13: Barz's algorithm for simulating general semaphores

```
                    binary semaphore S ← 1
                    binary semaphore gate ← 1
                    integer count ← k
loop forever
   non-critical section

p1:   wait(gate)                    // Simulated wait
p2:   wait(S)
p3:   count ← count − 1
p4:   if count > 0 then
p5:      signal(gate)
p6:   signal(S)

      critical section

p7:   wait(S)                       // Simulated signal
p8:   count ← count + 1
p9:   if count = 1 then
p10:     signal(gate)
p11:  signal(S)
```

context of a solution of the critical section problem that allows $k > 0$ processes simultaneously in the critical section (Algorithm 6.13).

k is the initial value of the general semaphore, statements p1..6 simulate the wait statement, and statements p7..11 simulate the signal statement. The binary semaphore gate is used to block and unblock processes, while the variable count holds the value of the integer component of the simulated general semaphore. The second binary semaphore S is used to ensure mutual exclusion when accessing count. Since we will be proving only the safety property that mutual exclusion holds, it will be convenient to consider the binary semaphores as busy-wait semaphores, and to write the value of the integer component of the semaphore gate as *gate*, rather than *gate.V*.

It is clear from the structure of the algorithm that S is a binary semaphore; it is less clear that gate is one. Therefore, we will not assume anything about the value of gate other than that it is an integer and prove (Lemma 6.5(6), below) that it only takes on the values 0 and 1.

Let us start with an informal description of the algorithm. Since gate is initialized to 1, the first process attempting to execute a simulated wait statement will succeed in passing p1: wait(gate), but additional processes will block. The first process and each subsequent process, up to a total of $k - 1$, will execute p5: signal(gate), releasing additional processes to successfully complete p1: wait(gate). The if statement at p4 prevents the kth process from executing p5: signal(gate), so further processes will be blocked at p1: wait(gate).

When $count = 0$, a single simulated signal operation will increment count and execute p11: signal(gate), unblocking one of the processes blocked at p1: wait(gate); this process will promptly decrement count back to zero. A sequence of simulated signal operations, however, will cause count to have a positive value, although the value of *gate* remains 1 since it is only signaled once. Once count has a positive value, one or more processes can now successfully execute the simulated wait.

An inductive proof of the algorithm is quite complex, but it is worth studying because so many incorrect algorithms have been proposed for this problem.

In the proof, we will reduce the number of steps in the induction to three. The binary semaphore S prevents the statements p2..6 from interleaving with the statements p7..11. p1 can interleave with statements p2..6 or p7..11, but cannot affect their execution, so the effect is the same as if *all* the statements p2..6 or p7..11 were executed before p1.[3]

We will use the notation *entering* for *p*2..6 and *inCS* for *p*7. We also denote by *#entering*, respectively *#inCS*, the number of processes for which *entering*, respectively *inCS*, is true.

Lemma 6.5 The conjunction of the following formulas is invariant:

(1) $entering \rightarrow (gate = 0)$,

(2) $entering \rightarrow (count > 0)$,

(3) $\#entering \leq 1$,

(4) $((gate = 0) \wedge \neg\, entering) \rightarrow (count = 0)$,

(5) $(count \leq 0) \rightarrow (gate = 0)$,

(6) $(gate = 0) \vee (gate = 1)$.

[3]Formally, wait statements are *right movers* [46].

Proof: The phrase "by (n), A holds" will mean: by the inductive hypothesis on formula (n), A must be true before executing this statement. The presentation is somewhat terse, so before reading further, make sure that you understand how material implications can be falsified (Appendix B.3).

Initially:
(1) All processes start at $p1$, so the antecedent is false.
(2) As for (1).
(3) $\#entering = 0$.
(4) $gate = 1$ so the antecedent is false.
(5) $count = k > 0$ so the antecedent is false.
(6) $gate = 1$ so the formula is true.

Executing p1:
(1) By (6), $gate \leq 1$, so p1: wait(gate) can successfully execute only if $gate = 1$, making the consequent $gate = 0$ true.
(2) *entering* becomes true, so the formula can be falsified only if the consequent is false because $count \leq 0$. But p1 does not change the value of count, so we must assume $count \leq 0$ before executing the statement. By (5), $gate = 0$, so the p1: wait(gate) cannot be executed.
(3) This can be falsified only if *entering* is true before executing p1. By (1), if *entering* is true, then $gate = 0$, so the p1: wait(gate) cannot be executed.
(4) *entering* becomes true, so the antecedent $\neg$ *entering* becomes false.
(5) As for (1).
(6) As for (1).

Executing p2..6:
(1) Some process must be at p2 to execute this statement, so *entering* is true, and by (3), $\#entering = 1$. Therefore, the antecedent *entering* becomes false.
(2) As for (1).
(3) As for (1).
(4) By (1), $gate = 0$, and by (2), $count > 0$. If $count = 1$, the consequent $count = 0$ becomes true. If $count > 1$, p3, p4 and p5 will be executed, so that *gate* becomes 1, falsifying the antecedent.
(5) By (1), $gate = 0$, and by (2), $count > 0$. If $count > 1$, after decrementing its value in p3, $count > 0$ will become true, falsifying the antecedent. If $count = 1$, the antecedent becomes true, but p5 will not be executed, so $gate = 0$ remains true.
(6) By (1), $gate = 0$ and it can be incremented only once, by p5.

Executing p7..11:
(1) The value of *entering* does not change. If it was true, by (2) $count > 0$, so that *count* becomes greater than 1 by p8, ensuring by the if statement at p9 that the value of *gate* does not change in p10.

(2) The value of *entering* does not change, and the value of *count* can only increase.
(3) Trivially, the value of *#entering* is not changed.
(4) Suppose that the consequent $count = 0$ were true before executing the statements and that it becomes false. By the if statement at p9, statement p10: signal(gate) is executed, so the antecedent $gate = 0$ is also falsified. Suppose now that both the antecedent and the consequent were false; then the antecedent cannot become true, because the value of *entering* does not change, and if $gate = 0$ is false, it certainly cannot become true by executing p10: signal(gate).
(5) The consequent can be falsified only if $gate = 0$ were true before executing the statements and p10 was executed. By the if statement at p9, that can happen only if $count = 0$. Then $count = 1$ after executing the statements, falsifying the antecedent. If the antecedent were false so that $count > 0$, it certainly remains false after incrementing count in p8.
(6) The formula can be falsified only if $gate = 1$ before executing the statements and p10: signal(gate) is executed. But that happens only if $count = 0$, which implies $gate = 0$ by (5). ∎

Lemma 6.6 The formula $count = k - \#inCS$ is invariant.

Proof: The formula is initially true by the initialization of count and the fact that all processes are initially at their non-critical sections. Executing p1 does not change any term in the formula. Executing p2..6 increments *#inCS* and decrements *count*, preserving the invariant, as does executing p7..11, which decrements *#inCS* and increments *count*. ∎

Theorem 6.7 Mutual exclusion holds for Algorithm 6.13, that is, $\#inCS \leq k$ is invariant.

Proof: Initially, $\#inCS \leq k$ is true since $k > 0$. The only step that can falsify the formula is p2..6 executed in a state in which $\#inCS = k$. By Lemma 6.6, in this state $count = 0$, but *entering* is also true in that state, contradicting (2) of Lemma 6.5. ∎

The implementation of Barz's algorithm in Promela is discussed in Section 6.15.

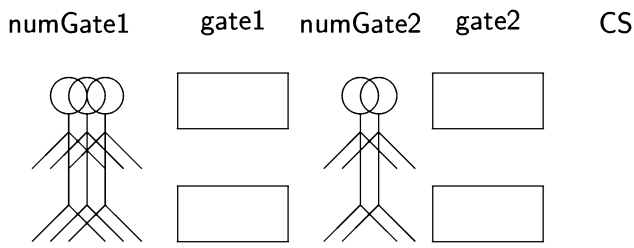
6.11 Udding's starvation-free algorithm[A]

There are solutions to the critical-section problem using weak semaphores that are free of starvation. We give without proof a solution developed by Jan T. Udding [68]. Algorithm 6.14 is similar to the fast algorithms described in Section 5.4, with two *gates* that must be passed in order to enter the critical section.

Algorithm 6.14: Udding's starvation-free algorithm

```
                    semaphore gate1 ← 1, gate2 ← 0
                    integer numGate1 ← 0, numGate2 ← 0
    loop forever
      non-critical section
p1:   wait(gate1)
p2:   numGate1 ← numGate1 + 1
p3:   signal(gate1)

p4:   wait(gate1)                    // First gate
p5:   numGate2 ← numGate2 + 1
p6:   if numGate1 > 0
p7:     signal(gate1)
p8:   else signal(gate2)

p9:   wait(gate2)                    // Second gate
p10:  numGate2 ← numGate2 − 1

      critical section
p11:  if numGate2 > 0
p12:    signal(gate2)
p13:  else signal(gate1)
```

The two semaphores constitute a split binary semaphore satisfying $0 \leq gate1 + gate2 \leq 1$, and the integer variables count the number of processes about to enter or trying to enter each gate.



The semaphore gate1 is reused to provided mutual exclusion during the initial increment of numGate1. In our model, where each line is an atomic operation, this protection is not needed, but freedom from starvation depends on it so we have included it.

The idea behind the algorithm is that processes successfully pass through a gate (complete the wait operation) as a group, without interleaving wait operations from the other processes. Thus a process p passing through gate2 into the critical section may quickly rejoin the group of processes in front of gate1, but all of the colleagues from its group will enter the critical section before p passes the first gate again. This ensures that the algorithm is free from starvation.

The attempted solution still suffers from the possibility of starvation, as shown by the following outline of a scenario:

n	Process p	Process q	gate1	gate2	nGate1	nGate2
1	**p4: wait(g1)**	q4: wait(g1)	1	0	2	0
2	**p9: wait(g2)**	q9: wait(g2)	0	1	0	2
3	**CS**	q9: wait(g2)	0	0	0	1
4	**p12: signal(g2)**	q9: wait(g2)	0	0	0	1
5	p1: wait(g1)	**CS**	0	0	0	0
6	**p1: wait(g1)**	q13: signal(g1)	0	0	0	0
7	p1: blocked	**q13: signal(g1)**	0	0	0	0
8	p4: wait(g1)	**q1: wait(g1)**	1	0	1	0
9	**p4: wait(g1)**	q4: wait(g1)	1	0	2	0

Two processes are denoted p and q with appropriate line numbers; the third process r is omitted because it remains blocked at r1: wait(g1) throughout the scenario. Variable names have been shortened: gate to g and numGate to nGate.

The scenario starts with processes p and q at gates p4 and q4, respectively, while the third process r has yet to pass the initial wait operation at r1. The key line in the scenario is line 7, where q unblocks his old colleague p waiting again at p1 instead of r who has been waiting at r1 for quite a long time.

By adding an additional semaphore, we can ensure that at most one process is at the second gate at any one time. The semaphore onlyOne is initialized to 1, a wait(onlyOne) operation is added before p4 and a signal(onlyOne) operation is added before p9. This ensures that at most one process will be blocked at gate1.

6.12 Semaphores in BACI^L

Types semaphore and binarysem with the operations **wait** and **signal** are defined in both the C and Pascal dialects of BACI. The implementation is of weak semaphores: when a **signal** statement is executed, the list of processes blocked on this semaphore is searched starting from a random position in the list.

6.13 Semaphores in Ada^L

The Ada language does not include a semaphore type, but it is easily implemented using protected types:[4]

```
protected type Semaphore(Initial: Natural) is
  entry Wait;
  procedure Signal;
private
  Count: Natural := Initial ;
end Semaphore;

protected body Semaphore is
  entry Wait when Count > 0 is
  begin
    Count := Count - 1 ;
  end Wait;

  procedure Signal is
  begin
    Count := Count + 1 ;
    end Signal;
end Semaphore;
```

The private variable Count is used to store the integer component of the semaphore. The queue of the entry Wait is the queue of the processes blocked on the semaphore, and its barrier **when** Count > 0 ensures that the wait operation can be executed only if the value of Count is positive. Since entry queues in Ada are defined to be FIFO queues, this code implements a strong semaphore.

To use this semaphore, declare a variable with a value for the discriminant Initial and then invoke the protected operations:

```
S: Semaphore(1);
...
S.wait ;
-- critical section
S.signal ;
```

[4]Protected types and objects are explained in Sections 7.10–7.10.

6.14 Semaphores in Java^L

The Semaphore class is defined within the package java.util.concurrent. The terminology is slightly different from the one we have been using. The integer component of an object of class Semaphore is called the number of *permits* of the object, and unusually, it can be initialized to be negative. The wait and signal operations are called acquire and release, respectively. The acquire operation can throw the exception InterruptedException which must be caught or thrown.

Here is a Java program for the counting algorithm with a semaphore added to ensure that the two assignment statements are executed with no interleaving.

```
1  import java.util .concurrent. Semaphore;
2  class CountSem extends Thread {
3    static volatile int n = 0;
4    static Semaphore s = new Semaphore(1);
5
6    public void run() {
7      int temp;
8      for (int i = 0; i < 10; i++) {
9        try {
10         s. acquire();
11       }
12       catch (InterruptedException e) {}
13       temp = n;
14       n = temp + 1;
15       s. release ();
16     }
17   }
18
19   public static void main(String[] args) {
20       // (as before)
21   }
22 }
```

The constructor for class Semaphore includes an extra boolean parameter used to specify if the semaphore is fair or not. A fair semaphore is what we have called a strong semaphore, but an unfair semaphore is more like a busy-wait semaphore than a weak semaphore, because the release method simply unblocks a process but does not acquire the lock associated with the semaphore. (See Section 7.11 for more details on synchronization in Java.)

6.15 Semaphores in Promela^L

In Promela it is easy to implement a busy-wait semaphore:

```
#define wait(s)    atomic { s > 0 ; s-- }
#define signal(s)  s++
```

Statements within **atomic** are executed without interleaving, except that if a statement blocks, statements from other processes will be executed. In wait(s), either the two instructions will be executed as one atomic statement, or the process will block because $s = 0$; when it can finally be executed because $s > 0$, the two statements will be executed atomically.

To implement a weak semaphore, we must explicitly store S.L, the set of blocked processes. We declare a semaphore type definition containing an integer field for the V component of the semaphore and a boolean array field for the L component with an element for each of NPROCS processes:

```
typedef Semaphore {
    byte count;
    bool blocked[NPROCS];
};
```

Inline definitions are used for the semaphore operations, beginning with an operation to initialize the semaphore:

```
inline  initSem(S, n) { S.count = n }
```

If the count is positive, the wait operation decrements the count; otherwise, the process is blocked waiting for this variable to become false:

```
inline  wait(S) {
   atomic {
     if
     :: S.count >= 1 -> S.count--
     :: else ->
          S.blocked[_pid-1] = true;
          !S.blocked[_pid-1]
     fi
   }
}
```

The semaphore is initialized in the **init** process whose _pid is zero, so the other processes have _pids starting from one.

The signal operation nondeterministically sets one of the blocked processes to be unblocked. If there are no blocked processes, the count is incremented. The following code assumes that there are three processes:

```
inline signal (S) {
  atomic {
    if
    :: S.blocked[0] -> S.blocked[0] = false
    :: S.blocked[1] -> S.blocked[1] = false
    :: S.blocked[2] -> S.blocked[2] = false
    :: else -> S.count++
    fi
  }
}
```

The code for an arbitrary number of processes is more complicated:

```
inline signal (S) {
  atomic {
    S.i = 0;
    S.choice = 255;
    do
    :: (S.i == NPROCS) -> break
    :: (S.i < NPROCS) && !S.blocked[S.i] -> S.i++
    :: else ->
        if
        :: (S.choice == 255) -> S.choice = S.i
        :: (S.choice != 255) -> S.choice = S.i
        :: (S.choice != 255) ->
        fi ;
        S.i ++
    od;
    if
    :: S.choice == 255 -> S.count++
    :: else -> S.blocked[S.choice] = false
    fi
  }
}
```

Two additional variables are added to the **typedef**: i to loop through the array of blocked processes, and choice to record the choice of a process to unblock, where

255 indicates that a process has yet to be chosen. If some process is blocked—indicated by S.blocked[S.i] being true—we nondeterministically choose either to select it by assigning its index to S.choice or to ignore it.

The implementation of strong semaphores using channels is left as an exercise. We also present an implementation of weak semaphores using channels, that is not restricted to a fixed number of processes.

Proving Barz's algorithm in Spin[A]

A Promela program for Algorithm 6.13, Barz's algorithm, is shown in Listing 6.1. The semaphore gate is implemented as a general semaphore; then we *prove* that it is a binary semaphore by defining a symbol bingate as (gate <= 1) and proving the temporal logic formula [] bingate. Mutual exclusion is proved by incrementing and decrementing the variable critical and checking assert (critical<=K).

It is more difficult to check the invariants of Lemma 6.5. The reason is that some of them are not true at every statement, for example, between p8: count←count+1 and p10: signalB(gate). Promela enables us to do what we did in the proof: consider all the statements p2..6 and p7..11 as single statements. **d_step**, short for *deterministic step*, specifies that the included statements are to be executed as a single statement. **d_step** differs from **atomic** in that it cannot contain any statement that blocks; this is true in Listing 6.1 because within the scope of the **d_step**s there are no blocking expressions like gate>0, and because the **if** statements include an **else** alternative to ensure that they don't block.

Transition

Semaphores are an elegant and efficient construct for solving problems in concurrent programming. Semaphores are a popular construct, implemented in many systems and easily simulated in others. However, before using semaphores you must find out the precise definition that has been implemented, because the liveness of a program can depend on the specific semaphore semantics.

Despite their popularity, semaphores are usually relegated to low-level programming because they are not structured. Variants of the monitor construct described in the next chapter are widely used because they enable encapsulation of synchronization.

```
#define NPROCS 3
#define K       2
byte gate = 1;
int  count = K;
byte critical   = 0;
active [NPROCS] proctype P () {
  do ::
    atomic { gate > 0; gate--; }
    d_step {
      count--;
      if
      :: count > 0 -> gate++
      :: else
      fi
    }
    critical  ++;
    assert (critical   <= 1);
    critical  --;
    d_step {
      count++;
      if
      :: count == 1 -> gate++
      :: else
      fi
    }
  od
}
```

Listing 6.1: Barz's algorithm in Promela

Exercises

1. Consider the following algorithm:

Algorithm 6.15: Semaphore algorithm A	
semaphore S ← 1, T ← 0	
p	**q**
p1: wait(S) p2: write("p") p3: signal(T)	q1: wait(T) q2: write("q") q3: signal(S)

   (a) What are the possible outputs of this algorithm?

   (b) What are the possible outputs if we erase the statement wait(S)?

   (c) What are the possible outputs if we erase the statement wait(T)?

2. What are the possible outputs of the following algorithm?

Algorithm 6.16: Semaphore algorithm B		
semaphore S1 ← 0, S2 ← 0		
p	**q**	**r**
p1: write("p") p2: signal(S1) p3: signal(S2)	q1: wait(S1) q2: write("q") q3:	r1: wait(S2) r2: write("r") r3:

3. What are the possible outputs of the following algorithm?

Algorithm 6.17: Semaphore algorithm with a loop	
semaphore S ← 1 boolean B ← false	
p	**q**
p1: wait(S) p2: B ← true p3: signal(S) p4:	q1: wait(S) q2: while not B q3: write("*") q4: signal(S)

4. Show that if the initial value of S.V in Algorithm 6.3 is k, at most k processes can be in the critical section at any time.

5. The following algorithm attempts to use binary semaphores to solve the critical section problem with at most k out of N processes in the critical section:

Algorithm 6.18: Critical section problem (k out of N processes)

```
                    binary semaphore S ← 1, delay ← 0
                    integer count ← k

    integer m
    loop forever
p1:     non-critical section
p2:     wait(S)
p3:     count ← count − 1
p4:     m ← count
p5:     signal(S)
p6:     if m ≤ −1
p7:         wait(delay)
p8:     critical section
p9:     wait(S)
p10:    count ← count + 1
p11:    if count ≤ 0
p12:        signal(delay)
p13:    signal(S)
```

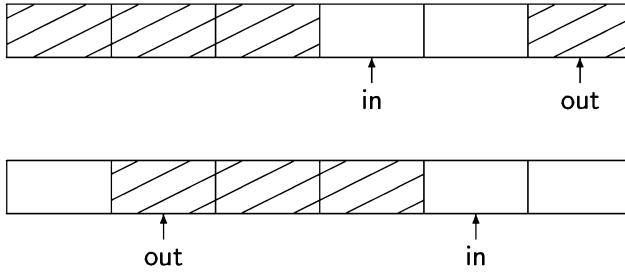
For $N = 4$ and $k = 2$, construct a scenario in which $delay = 2$, contrary to the definition of a binary semaphore. Show this also for $N = 3$ and $k = 2$.

6. Modify Algorithm 6.5 to use a single general semaphore.

7. Prove the correctness of Algorithm 6.7 without the assumption of atomicity.

8. Develop an abbreviated version of Algorithm 6.8. Assumimg atomicity of the statements of each process, show that $notFull.V = N - \#Buffer$ is invariant. What correctness property can you conclude from this invariant?

9. A bounded buffer is frequently implemented using a circular buffer, which is an array that is indexed modulo its length:



One variable, in, contains the index of the first empty space and another, out, the index of the first full space. If $in > out$, there is data in buffer[out..in-1]; if $in < out$, there is data in buffer[out..N] and buffer[0..in-1]; if $in = out$, the buffer is empty. Prove the correctness of the following algorithm for the producer–consumer problem with a circular buffer:

Algorithm 6.19: producer–consumer (circular buffer)

```
dataType array[0..N] buffer
integer in, out ← 0
semaphore notEmpty ← (0, ∅)
semaphore notFull ← (N, ∅)
```

	producer		**consumer**
	dataType d		dataType d
	loop forever		loop forever
p1:	d ← produce	q1:	wait(notEmpty)
p2:	wait(notFull)	q2:	d ← buffer[out]
p3:	buffer[in] ← d	q3:	out ← (out+1) modulo N
p4:	in ← (in+1) modulo N	q4:	signal(notFull)
p5:	signal(notEmpty)	q5:	consume(d)

10. Prove the correctness of the asymmetric solution to the dining philosophers problem (Algorithm 6.12).

11. A partial scenario for starvation in Udding's algorithm was given on page 131; complete the scenario.

12. Prove or disprove the correctness of Algorithm 6.20 for implementing a general semaphore by binary semaphores.

13. Implement a strong semaphore in Promela.

Algorithm 6.20: Simulating general semaphores	
	binary semaphore S ← 1, gate ← 0 integer count ← 0
	wait
p1:	wait(S)
p2:	count ← count − 1
p3:	if count < 0
p4:	signal(S)
p5:	wait(gate)
p6:	else signal(S)
	signal
p7:	wait(S)
p8:	count ← count + 1
p9:	if count ≤ 0
p10:	signal(gate)
p11:	signal(S)

14. Here is a version of the implementation of the signal operation of a weak semaphore in Promela that does not use the variable choice. Is the program correct?

```
inline signal (S) {
  atomic {
    S.i = 0;
    do
    :: S.i == NPROCS -> break
    :: (S.i < NPROCS) && S.blocked[S.i] -> break
    :: (S.i < NPROCS) && S.blocked[S.i] -> S.i++
    :: (S.i < NPROCS) && !S.blocked[S.i] -> S.i++
    od;
    if
    :: S.i == NPROCS -> S.count++
    :: else -> S.blocked[S.i] = false
    fi
  }
}
```

15. Weak semaphores can be implemented in Promela by storing the blocked processes in a channel:

```
chan ch = [NPROCS] of { pid };
```

Explain the following implementations of the wait and signal operations:[5]

```
inline wait(S) {
  atomic {
    if :: S.count >= 1 -> S.count--;
       :: else ->
            S.ch ! _pid;
            !( S.ch ?? [eval( _pid)])
    fi
  }
}
```

```
inline signal (S) {
  atomic {
    S.i = len(S.ch);
    if :: S.i == 0 -> S.count++
       :: else ->
          do :: S.i == 1 -> S.ch ? _; break
             :: else ->
                  S.i --;
                  S.ch ? S.temp;
                  if
                  :: break
                  :: S.ch ! S.temp
                  fi
          od
    fi
  }
}
```

[5]My thanks to Gerard Holzmann for his assistance in developing this implementation.

16. [2, Section 4.4][6] Algorithm 6.21 is a solution of the problem of the readers and writers using semaphores. SignalProcess is a subprogram that is executed following each of the start and end operations.

    (a) Prove that entry, readerSem and writerSem form a split binary semaphore, that is, prove that

    $$0 \leq entry + readerSem + writerSem \leq 1.$$

    Prove the correctness of the algorithm by showing the invariance of:

    $$(writers > 0) \rightarrow (writers = 1) \wedge (readers = 0).$$

    (b) The subprogram SignalProcess is uniform for all the operations. Simplify the algorithm by writing separate code for each of the operations.

    (c) Show that readers are given precedence over writers.

    (d) Modify the algorithm to give writers precedence over readers.

    (e) Modify the algorithm to give alternate precedence to readers and writers.

[6]This exercise should be attempted only after studying the description of the problem and monitor solution in Section 7.6.

Algorithm 6.21: Readers and writers with semaphores

```
semaphore readerSem ← 0, writerSem ← 0
integer delayedReaders ← 0, delayedWriters ← 0
semaphore entry ← 1
integer readers ← 0, writers ← 0
```

SignalProcess

```
if writers = 0 or delayedReaders > 0
   delayedReaders ← delayedReaders − 1
   signal(readerSem)
else if readers = 0 and writers = 0 and delayedWriters > 0
   delayedWriters ← delayedWriters − 1
   signal(writerSem)
else signal(entry)
```

StartRead

```
p1:  wait(entry)
p2:  if writers > 0
p3:     delayedReaders ← delayedReaders + 1
p4:     signal(entry)
p5:     wait(readerSem)
p6:  readers ← readers + 1
p7:  SignalProcess
```

EndRead

```
p8:  wait(entry)
p9:  readers ← readers − 1
p10: SignalProcess
```

StartWrite

```
p11: wait(entry)
p12: if writers > 0 or readers > 0
p13:    delayedWriters ← delayedWriters + 1
p14:    signal(entry)
p15:    wait(writerSem)
p16: writers ← writers + 1
p17: SignalProcess
```

EndWrite

```
p18: wait(entry)
p19: writers ← writers − 1
p20: SignalProcess
```

7 Monitors

7.1 Introduction

The semaphore was introduced to provide a synchronization primitive that does not require busy waiting. Using semaphores we have given solutions to common concurrent programming problems. However, the semaphore is a low-level primitive because it is unstructured. If we were to build a large system using semaphores alone, the responsibility for the correct use of the semaphores would be diffused among all the implementers of the system. If one of them forgets to call signal(S) after a critical section, the program can deadlock and the cause of the failure will be difficult to isolate.

Monitors provide a structured concurrent programming primitive that concentrates the responsibility for correctness into modules. Monitors are a generalization of the *kernel* or *supervisor* found in operating systems, where critical sections such as the allocation of memory are centralized in a privileged program. Applications programs request services which are performed by the kernel. Kernels are run in a hardware mode that ensures that they cannot be interfered with by applications programs.

The monitors discussed in this chapter are decentralized versions of the monolithic kernel. Rather than have one system program handle all requests for services involving shared devices or data, we define a separate monitor for each object or related group of objects that requires synchronization. If operations of the same monitor are called by more than one process, the implementation ensures that these are executed under mutual exclusion. If operations of different monitors are called, their executions can be interleaved.

Monitors have become an extremely important synchronization mechanism because they are a natural generalization of the *object* of object-oriented programming, which encapsulates data and operation declarations within a *class*.[1] At runtime, objects of this class can be allocated, and the operations of the class invoked

[1] In Java, the same unit, the *class*, is used for both physical and logical encapsulation. In Ada, the physical encapsulation unit is the *package*, which may contain more than one declaration of a *type* and its operations.

on the fields of the object. The monitor adds the requirement that only one process can execute an operation on an object at any one time. Furthermore, while the fields of an object may be declared either public (directly accessible outside the class) or private (accessible only by operations declared within the class), the fields of a monitor are all private. Together with the requirement that only one process at a time can execute an operation, this ensures that the fields of a monitor are accessed consistently.

Actual implementations of monitors in programming languages and systems are quite different from one another. We begin the chapter by describing the classical monitor, a version of which is implemented in the BACI concurrency simulator. Then we will discuss protected objects of Ada, followed by synchronized methods in Java which can be used to implement monitors. It is extremely important that you learn how to analyze different implementations: their advantages, their disadvantages and the programming paradigms appropriate to each one.

7.2 Declaring and using monitors

In Section 2.5, we discussed the importance of atomic statements. Even the trivial Algorithm 2.4 showed that interleaving a pair of statements executed by each of two processes could lead to unexpected scenarios. The following algorithm shows the same two statements encapsulated within a monitor:

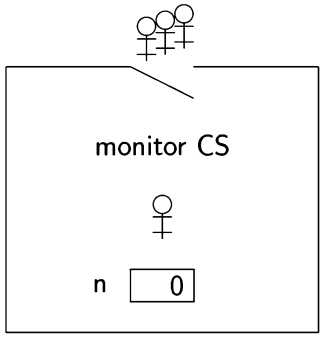
Algorithm 7.1: Atomicity of monitor operations

```
monitor CS
   integer n ← 0

   operation increment
      integer temp
      temp ← n
      n ← temp + 1
```

p	q
p1: CS.increment	q1: CS.increment

The monitor CS contains one variable n and one operation increment; two statements are contained within this operation, together with the declaration of the local variable. The variable n is not accessible outside the monitor. Two processes, p and q, each call the monitor operation CS.increment. Since by definition only one process at a time can execute a monitor operation, we are ensured mutual exclusion in

access to the variable, so the only possible result of executing this algorithm is that n receives the value 2. The following diagram shows how one process is executing statements of a monitor operation while other processes wait outside:



This algorithm also solves the critical section problem, as can be seen by substituting an arbitrary critical section statement for the assignment statements. Compare this solution with the solution to the critical section problem using semaphores given in Section 6.3. The statements of the critical section are encapsulated in the monitor rather than replicated in each process. The synchronization is implicit and does not require the programmers to correctly place wait and signal statements.

The monitor is a static entity, not a dynamic process. It is just a set of operations that "sit there" waiting for a process to invoke one of them. There is an implicit lock on the "door" to the monitor, ensuring only one process is "inside" the monitor at any time. A process must open the lock to enter the monitor; the lock is then closed and remains closed until the process leaves the monitor.

As with semaphores, if there are several processes attempting to enter a monitor, only one of them will succeed. *There is no explicit queue associated with the monitor entry, so starvation is possible.*

In our examples, we will declare single monitors, but in real programming languages, a monitor would be declared as a type or a class and you can allocate as many objects of the type as you need.

7.3 Condition variables

A monitor implicitly enforces mutual exclusion to its variables, but many problems in concurrent programming have explicit synchronization requirements. For example, in the solution of the producer–consumer problem with a bounded buffer (Section 6.7), the producer must be blocked if the buffer is full and the consumer

must be blocked if the buffer is empty. There are two approaches to providing synchronization in monitors. In one approach, the required condition is named by an explicit *condition variable* (sometimes called an *event*). Ordinary boolean expressions are used to test the condition, blocking on the condition variable if necessary; a separate statement is used to unblock a process when the condition becomes true. The alternate approach is to block directly on the expression and let the implementation implicitly unblock a process when the expression is true. The classical monitor uses the first approach, while the second approach is used by protected objects (Section 7.10). Condition variables are one of the synchronization constructs in pthreads, although they are not part of an encapsulating structure like monitors.

Simulating semaphores

Let us see how condition variables work by simulating a solution to the critical section problem using semaphores:

Algorithm 7.2: Semaphore simulated with a monitor

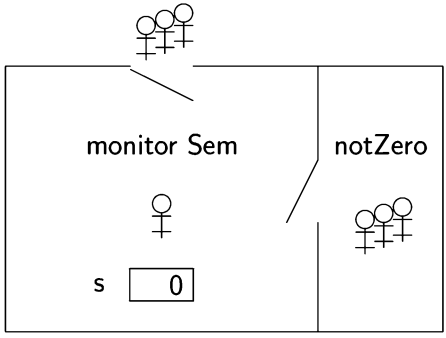
```
monitor Sem
   integer s ← k
   condition notZero

   operation wait
      if s = 0
         waitC(notZero)
      s ← s − 1

   operation signal
      s ← s + 1
      signalC(notZero)
```

p	q
loop forever	loop forever
non-critical section	non-critical section
p1: Sem.wait	q1: Sem.wait
critical section	critical section
p2: Sem.signal	q2: Sem.signal

We refrain from giving line numbers for the statements in the monitor because each operation is executed as a single atomic operation.

The monitor implementation follows directly from the definition of semaphores. The integer component of the semaphore is stored in the variable s and the condition variable cond implements the queue of blocked processes. By convention, condition variables are named with the condition you want to be true. waitC(notZero) is read as "wait for notZero to be true," and signalC(notZero) is read as "signal that notZero is true." The names waitC and signalC are intended to reduce the possibility of confusion with the similarly-named semaphore statements.

If the value of s is zero, the process executing Sem.wait—the simulated semaphore operation—executes the monitor statement waitC(notZero). The process is said to be blocked *on the condition*:



A process executing waitC blocks unconditionally, because we assume that the condition has been tested for in a preceding if statement; since the entire monitor operation is atomic, the value of the condition cannot change between testing its value and executing waitC. When a process executes a Sem.signal operation, it unblocks the first process (if any) blocked on that condition.

Operations on condition variables

With each condition variable is associated a *FIFO queue of blocked processes*. Here is the definition of the execution of the *atomic* operations on condition variables by an arbitrary process p:

waitC(cond)

```
append p to cond
p.state ← blocked
monitor.lock ← release
```

Process p is blocked on the queue cond. Process p leaves the monitor, releasing the lock that ensures mutual exclusion in accessing the monitor.

signalC(cond)

```
if cond ≠ empty
    remove head of cond and assign to q
    q.state ← ready
```

If the queue cond is nonempty, the process at the head of the queue is unblocked.

There is also an operation that checks if the queue is empty:

empty(cond)

```
return cond = empty
```

Since the state of the process executing the monitor operation waitC becomes blocked, it is clear that it must release the lock to enable another process to enter the monitor (and eventually signal the condition).[2] In the case of the signalC operation, the unblocked process becomes ready and there is no obvious requirement for the signaling operation to leave the monitor (see Section 7.5).

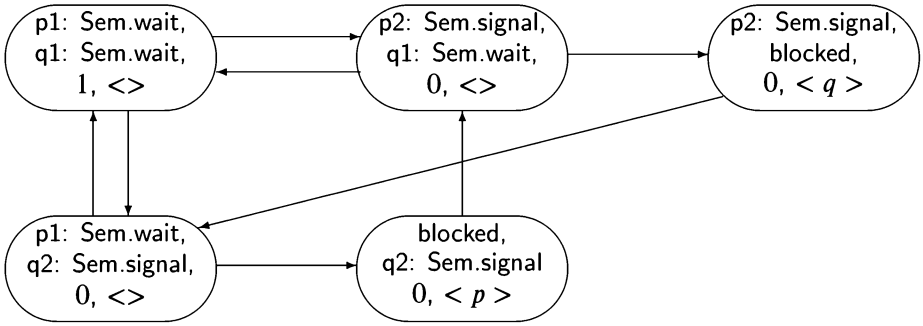
The following table summarizes the differences between the monitor operations and the similarly-named semaphore operations:

Semaphore	Monitor
wait may or may not block	waitC always blocks
signal always has an effect	signalC has no effect if queue is empty
signal unblocks an arbitrary blocked process	signalC unblocks the process at the head of the queue
a process unblocked by signal can resume execution immediately	a process unblocked by signalC must wait for the signaling process to leave monitor

Correctness of the semaphore simulation

When constructing a state diagram for a program with monitors, *all* the statements of a monitor operation can be considered to be a single step because they are executed under mutual exclusion and no interleaving is possible between the statements. In Algorithm 7.2, each state has four components: the control pointers of the two processes p and q, the value of the monitor variable s and the queue associated with the condition variable notZero. Here is the state diagram assuming that the value of s has been initialized to 1:

[2] In pthreads, there is no implicit lock as there is for a monitor, so a mutex must be explicitly supplied to the operation pthread_cond_wait.



Consider, first, the transition from the state (p2: Sem.signal, q1: Sem.wait, 0, <>) at the top center of the diagram to the state (p2: Sem.signal, blocked, 0, $< q >$) at the upper right. Process q executes the Sem.wait operation, finds that the value of s is 0 and executes the waitC operation; it is then blocked on the queue for the condition variable notZero.

Consider, now, the transition from (p2: Sem.signal, blocked, 0, $< q >$) to the state (p1: Sem.wait, q2: Sem.signal, 0, <>) at the lower left of the diagram. Process p executes the Sem.signal operation, incrementing s, and then signalC will unblock process q. As we shall discuss in Section 7.5, process q *immediately resumes* the execution of its Sem.wait operation, so this can be considered as part of the same atomic statement. q will decrement s back to zero and exit the monitor. Since signalC(nonZero) is the last statement of the Sem.signal operation executed by process p, we may also consider that that process exits the monitor as part of the atomic statement. We end up in a state where process q is in its critical section, denoted in the abbreviated algorithm by the control pointer indicating the next invocation of Sem.signal, while process p is outside its critical section, with its control pointer indicating the next invocation of Sem.wait.

There is no state of the form (p2: Sem.signal, q2: Sem.signal, $\cdots$, $\cdots$) in the diagram, so the mutual exclusion requirement is satisfied.

The state diagram for a monitor can be relatively simple, because the internal transitions of the monitor statements can be grouped into a single transition.

7.4 The producer–consumer problem

Algorithm 7.3 is a solution for the producer–consumer problem with a finite buffer using a monitor. Two condition variables are used and the conditions are explicitly checked to see if a process needs to be suspended. The entire processing of the buffer is encapsulated within the monitor and the buffer data structure is not visible to the producer and consumer processes.

Algorithm 7.3: producer–consumer (finite buffer, monitor)

```
monitor PC
   bufferType buffer ← empty
   condition notEmpty
   condition notFull

   operation append(datatype v)
      if buffer is full
         waitC(notFull)
      append(v, buffer)
      signalC(notEmpty)

   operation take()
      datatype w
      if buffer is empty
         waitC(notEmpty)
      w ← head(buffer)
      signalC(notFull)
      return w
```

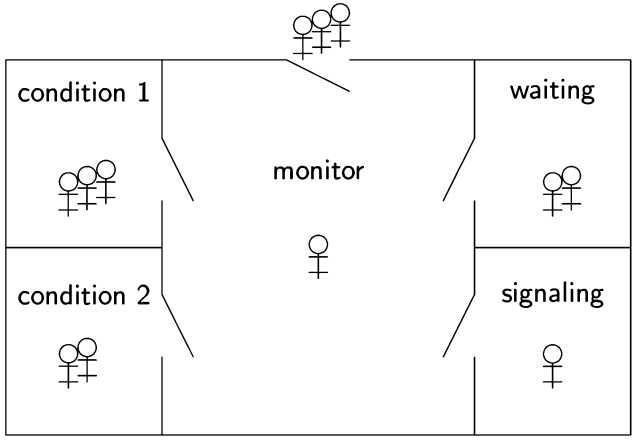
producer	consumer
datatype d	datatype d
loop forever	loop forever
p1: d ← produce	q1: d ← PC.take
p2: PC.append(d)	q2: consume(d)

7.5 The immediate resumption requirement

The definition of signalC(cond) requires that it unblock the first process blocked on the queue for cond. When this occurs, the signaling process (say p) can now continue to execute the next statement after signalC(cond), while the unblocked process (say q) can execute the next statement after the waitC(cond) that caused it to block. But this is not a valid state, since the specification of a monitor requires that at most one process at a time can be executing statements of a monitor operation. Either p or q will have to be blocked until the other completes its monitor operation. That is, we have to specify if signaling processes are given precedence over waiting processes, or vice versa, or perhaps the selection of the next process is arbitrary. There may also be processes blocked on the entry to the monitor, and the specification of precedence has to take them into account.

Note that the terminology is rather confusing, because the waiting processes are processes that have just been *released* from being blocked, rather than processes that are waiting because they are blocked.

The following diagram shows the states that processes can be in: waiting to enter the monitor, executing within the monitor (but only one), blocked on condition queues, on a queue of processes just released from waiting on a condition, and on a queue of processes that have just completed a signal operation:



Let us denote the precedence of the signaling processes by S, that of the waiting processes by W and that of the processes blocked on the entry by E. There are thirteen different ways of assigning precedence, but many of them make no sense. For example, it does not make sense to have E greater than either W or S, because that would quickly cause starvation as new processes enter the monitor before earlier ones have left. The classical monitor specifies that $E < S < W$ and later we will discuss the specification $E = W < S$ that is implemented in Java. See [15] for an analysis of monitor definitions.

The specification $E < S < W$ is called the *immediate resumption requirement (IRR)*, or *signal and urgent wait*. It means that when a process blocked on a condition variable is signaled, it immediately begins executing ahead of the signaling process. It is easy to see the rationale behind IRR. Presumably, the signaling process has changed the state of the monitor (modified its variables) so that the condition now holds; if the waiting process resumes immediately, it can assume the condition does hold and continue with the statements in its operation. For example, look again at the monitor that simulates a semaphore (Algorithm 7.2). The Sem.signal operation increments s just before executing the signalC(notZero) statement; if the Sem.wait operation resumes immediately, it will find s to be nonzero, the condition it was waiting for.

Similarly, in Algorithm 7.3, the monitor solution for the producer–consumer problem, append(v, Buffer) immediately precedes signalC(notEmpty), ensuring that the buffer is not empty when a consumer process is unblocked, and the statement w ← head(Buffer) immediately precedes signalC(notFull), ensuring that the buffer is not full when a producer process is unblocked. With immediate resumption there is no need to recheck the status of the buffer.

Without the IRR, it would be possible for the signaling process to continue its execution, causing the condition to again become false. Waiting processes would have to recheck the boolean expression for the condition in a while loop:

```
while s = 0
   waitC(notZero)
s ← s − 1
```

The disadvantage of the IRR is that the signaling process might be unnecessarily delayed, resulting in less concurrency than would otherwise be possible. However, if—as in our examples—signalC(cond) is the last statement in a monitor operation, there is no need to block the execution of the process that invoked the operation.

7.6 The problem of the readers and writers

The problem of the readers and writers is similar to the mutual exclusion problem in that several processes are competing for access to a critical section. In this problem, however, we divide the processes into two classes:

Readers Processes which are required to exclude writers but not other readers.

Writers Processes which are required to exclude both readers and other writers.

The problem is an abstraction of access to databases, where there is no danger in having several processes read data concurrently, but writing or modifying data must be done under mutual exclusion to ensure consistency of the data. The concept database must be understood in the widest possible sense; even a couple of memory words in a real-time system can be considered a database that needs mutual exclusion when being written to, while several processes can read it simultaneously.

Algorithm 7.4 is a solution to the problem using monitors. We will use the term "reader" for any process executing the statements of reader and "writer" for any process executing the statements of writer.

Algorithm 7.4: Readers and writers with a monitor

```
monitor RW
   integer readers ← 0
   integer writers ← 0
   condition OKtoRead, OKtoWrite

   operation StartRead
      if writers ≠ 0 or not empty(OKtoWrite)
         waitC(OKtoRead)
      readers ← readers + 1
      signalC(OKtoRead)

   operation EndRead
      readers ← readers − 1
      if readers = 0
         signalC(OKtoWrite)

   operation StartWrite
      if writers ≠ 0 or readers ≠ 0
         waitC(OKtoWrite)
      writers ← writers + 1

   operation EndWrite
      writers ← writers − 1
      if empty(OKtoRead)
         then signalC(OKtoWrite)
         else signalC(OKtoRead)
```

reader	writer
p1: RW.StartRead	q1: RW.StartWrite
p2: read the database	q2: write to the database
p3: RW.EndRead	q3: RW.EndWrite

The monitor uses four variables:

readers The number of readers currently reading the database after successfully executing StartRead but before executing EndRead.

writers The number of writers currently writing to the database after successfully executing StartWrite but before executing EndWrite.

OKtoRead A condition variable for blocking readers until it is "OK to read."

OKtoWrite A condition variable for blocking writers until it is "OK to write."

The general form of the monitor code is not difficult to follow. The variables readers and writers are incremented in the Start operations and decremented in the End operations to reflect the natural invariants expressed in the definitions above. At the beginning of the Start operations, a boolean expression is checked to see if the process should be blocked, and at the end of the End operations, another boolean expression is checked to see if some condition should be signaled to unblock a process.

A reader is suspended if some process is currently writing ($writers \neq 0$) or if some process is waiting to write ($\neg empty(OKtoWrite)$). The first condition is obviously required by the specification of the problem. The second condition is a decision to give the first blocked writer precedence over waiting readers. A writer is blocked only if there are processes currently reading ($readers \neq 0$) or writing ($writers \neq 0$). StartWrite does not check the condition queues.

EndRead executes signalC(OKtoWrite) if there are no more readers. If there are blocked writers, one will be unblocked and allowed to complete StartWrite. EndWrite gives precedence to the first blocked reader, if any; otherwise it unblocks a blocked writer, if any.

Finally, what is the function of signalC(OKtoRead) in the operation StartRead? This statement performs a *cascaded unblocking* of the blocked readers. Upon termination of a writer, the algorithm gives precedence to unblocking a blocked reader over a blocked writer. However, if one reader is allowed to commence reading, we might as well unblock them all. When the first reader completes StartRead, it will signal the next reader and so on, until this cascade of signals unblocks all the blocked readers.

What about readers that attempt to start reading during the cascaded unblocking? Will they have precedence over blocked writers? By the immediate resumption requirement, the cascaded unblocking will run to completion before any new reader is allowed to commence execution of a monitor operation. When the last signalC(OKtoRead) is executed (and does nothing because the condition queue is empty), the monitor will be released and a new reader may enter. However, it is

subject to the check in StartRead that will cause it to block if there are waiting writers.

These rules ensure that there is no starvation of either readers or writers. If there are blocked writers, a new reader is required to wait until the termination of (at least) the first write. If there are blocked readers, they will (all) be unblocked before the next write.

7.7 Correctness of the readers and writers algorithm^A

Proving monitors can be relatively succinct, because monitor invariants are only required to hold outside the monitor procedures themselves. Furthermore, the immediate resumption requirement allows us to infer that what was true before executing a signal is true when a process blocked on that condition becomes unblocked.

Let R be the number of readers currently reading and W the number of writers currently writing. The proof of the following lemma is immediate from the structure of the program.

Lemma 7.1 The following formulas are invariant:

$$\begin{aligned} R &\geq 0, \\ W &\geq 0, \\ R &= \mathit{readers}, \\ W &= \mathit{writers}. \end{aligned}$$

We implicitly use these invariants to identify changes in the values of the program variables readers and writers with changes in the variables R and W that count processes. The safety of the algorithm is expressed using the variables R and W:

Theorem 7.2 The following formula is invariant:

$$(R > 0 \rightarrow W = 0) \;\wedge\; (W \leq 1) \;\wedge\; (W = 1 \rightarrow R = 0).$$

In words, if a reader is reading then no writer is writing; at most one writer is writing; and if a writer is writing then no reader is reading.

Proof: Clearly, the formula is initially true. Because of the properties of the monitor, there are eight atomic statements: execution of the four monitor operations from start to completion, and execution of the four monitor operations that include signalC statements unblocking a waiting process.

Consider first executing a monitor operation from start to completion:

- StartRead: This operation increments R and could falsify $R > 0 \rightarrow W = 0$, but the if statement ensures that the operation completes only if $W = 0$ is true. The operation could falsify $R = 0$ and hence $W = 1 \rightarrow R = 0$, but again the if statement ensures that the operation completes only if $W = 1$ is false.
- EndRead: This operation decrements R, possibly making $R > 0$ false and $R = 0$ true, but these changes cannot falsify the invariant.
- StartWrite: This operation increments W and could falsify $W \leq 1$, but the if statement ensures that the operation completes only if $W = 0$ is true. The operation could falsify $W = 0$ and hence $R > 0 \rightarrow W = 0$, but the if statement ensures that $R > 0$ is false.
- EndWrite: This operation decrements W, so $W = 0$ and $W \leq 1$ could both become true while $W = 1$ becomes false; these changes cannot falsify the invariant.

Consider now the monitor operations that result in unblocking a process from the queue of a condition variable.

- signal(OKtoRead) in StartRead: The execution of StartRead including signal(OKtoRead) can cause a reader r to become unblocked. But the if statement ensures that this happens only if $W = 0$, and the IRR ensures that r continues its execution immediately so that $W = 0$ remains true.
- signal(OKtoRead) in EndWrite: By the inductive hypothesis, $W \leq 1$, so $W = 0$ when the reader resumes, preserving the truth of the invariant.
- signal(OKtoWrite) in EndRead: The signal is executed only if $R = 1$ upon starting the operation; upon completion $R = 0$ becomes true, preserving the truth of the first and third subformulas. In addition, if $R = 1$, by the inductive hypothesis we also have that $W = 0$ was true. By the IRR for StartWrite of the unblocked writer, $W = 1$ becomes true, preserving the truth of the second subformula.
- signal(OKtoWrite) in EndWrite: By the inductive hypothesis $W \leq 1$, the writer executing EndWrite must be the only writer that is writing so that $W = 1$. By the IRR, the unblocked writer will complete StartWrite, preserving the truth of $W = 1$ and hence $W \leq 1$. The value of R is, of course, not affected, so the truth of the other subformulas is preserved. ∎

The expressions not empty(OKtoWrite) in StartRead and empty(OKtoRead) in EndWrite were not used in the proof because they are not relevant to the safety of the algorithm.

We now prove that readers are not starved and leave the proof that writers are not starved as an exercise. It is important to note that freedom from starvation holds only for processes that start executing StartRead or StartWrite, because starvation is possible at the entrance to a monitor. Furthermore, both reading and writing are assumed to progress like critical sections, so starvation can occur only if a process is blocked indefinitely on a condition variable.

Let us start with a lemma relating the condition variables to the number of processes reading and writing:

Lemma 7.3 The following formulas are invariant:

$$\neg\, empty(OKtoRead) \quad \rightarrow \quad (W \neq 0) \vee \neg\, empty(OKtoWrite),$$
$$\neg\, empty(OKtoWrite) \quad \rightarrow \quad (R \neq 0) \vee (W \neq 0).$$

Proof: Clearly, both formulas are true initially since condition variables are initialized as empty queues.

The antecedent of the first formula can only become true by executing StartRead, but the if statement ensures that the consequent is also true. Can the consequent of the formula become false while the antecedent is true? One way this can happen is by executing EndWrite for the last writer. By the assumption of the truth of the antecedent $\neg\, empty(OKtoRead)$, signalC(OKtoRead) is executed. By the IRR, this leads to a cascade of signals that form part of the atomic monitor operation, and the final signal falsifies $\neg\, empty(OKtoRead)$. Alternatively, EndRead could be executed for the last reader, causing signalC(OKtoWrite) to falsify $\neg\, empty(OKtoWrite)$. But this causes $W \neq 0$ to become true, so the consequent remains true.

The truth of the second formula is preserved when executing StartWrite by the condition of the if statement. Can the consequent of the formula become false while the antecedent is true? Executing EndRead for the last reader falsifies $R \neq 0$, but since $\neg\, empty(OKtoWrite)$ by assumption, signalC(OKtoWrite) makes $W \neq 0$ true. Executing EndWrite for the last (only) writer could falsify $W \neq 0$, but one of the signalC statements will make either $R \neq 0$ or $W \neq 0$ true. ∎

Theorem 7.4 Algorithm 7.4 is free from starvation of readers.

Proof: If a reader is starved is must be blocked indefinitely on OKtoRead. We will show that a signal(OKtoRead) statement must eventually be executed, unblocking

the first reader on the queue. Since the condition queues are assumed to be FIFO, it follows by (numerical) induction on the position of a reader in the queue that it will eventually be unblocked and allowed to read.

To show

$$\neg\, empty(OKtoRead) \rightarrow \Diamond signalC(OKtoRead),$$

assume to the contrary that

$$\neg\, empty(OKtoRead) \wedge \Box \neg\, signalC(OKtoRead).$$

By the first invariant of Lemma 7.3, $(W \neq 0) \vee \neg\, empty(OKtoWrite)$. If $W \neq 0$, by progress of the writer, eventually EndWrite is executed; signalC(OKtoRead) will be executed since by assumption $\neg\, empty(OKtoRead)$ is true (and it remains true until some signalC(OKtoRead) statement is executed), a contradiction.

If $\neg\, empty(OKtoWrite)$ is true, by the second invariant of Lemma 7.3, $(R \neq 0) \vee (W \neq 0)$. We have just shown that $W \neq 0$ leads to a contradiction, so consider the case that $R \neq 0$. By progress of readers, if no new readers execute StartRead, all readers eventually execute EndReader, and the last one will execute signalC(OKtoWrite); since we assumed that $\neg\, empty(OKtoWrite)$ is true, a writer will be unblocked making $W \neq 0$ true, and reducing this case to the previous one. The assumption that no new readers successfully execute StartRead holds because $\neg\, empty(OKtoWrite)$ will cause a new reader to be blocked on OKtoRead. ∎

7.8 A monitor solution for the dining philosophers

Algorithm 6.10, an attempt at solving the problem of the dining philosophers with semaphores, suffered from deadlock because there is no way to test the value of two fork semaphores simultaneously. With monitors, the test can be encapsulated, so that a philosopher waits until both forks are free before taking them.

Algorithm 7.5 is a solution using a monitor. The monitor maintains an array fork which counts the number of free forks *available to* each philosopher. The takeForks operation waits on a condition variable until two forks are available. Before leaving the monitor with these two forks, it decrements the number of forks available to its neighbors. After eating, a philosopher calls releaseForks, which, in addition to updating the array fork, checks if freeing these forks makes it possible to signal a neighbor.

Let $eating[i]$ be true if philosopher i is eating, that is, if she has successfully executed takeForks(i) and has not yet executed releaseForks(i). We leave it as an exercise to show that $eating[i] \leftrightarrow (forks[i] = 2)$ is invariant. This formula expresses the requirement that a philosopher eats only if she has two forks.

Algorithm 7.5: Dining philosophers with a monitor

```
monitor ForkMonitor
   integer array[0..4] fork ← [2, ..., 2]
   condition array[0..4] OKtoEat
   operation takeForks(integer i)
      if fork[i] ≠ 2
         waitC(OKtoEat[i])
      fork[i+1] ← fork[i+1] − 1
      fork[i−1] ← fork[i−1] − 1
   operation releaseForks(integer i)
      fork[i+1] ← fork[i+1] + 1
      fork[i−1] ← fork[i−1] + 1
      if fork[i+1] = 2
         signalC(OKtoEat[i+1])
      if fork[i−1] = 2
         signalC(OKtoEat[i−1])
```

philosopher i

```
     loop forever
p1:     think
p2:     takeForks(i)
p3:     eat
p4:     releaseForks(i)
```

Theorem 7.5 Algorithm 7.5 is free from deadlock.

Proof: Let E be the number of philosophers who are eating, In the exercises, we ask you to show that the following formulas are invariant:

$$\neg\, empty(OKtoEat[i]) \;\rightarrow\; (fork[i] < 2), \tag{7.1}$$

$$\sum_{i=0}^{4} fork[i] = 10 - 2 * E. \tag{7.2}$$

Deadlock implies $E = 0$ and all philosophers are enqueued on OKtoEat. If no philosophers are eating, from (7.2) we conclude $\sum fork[i] = 10$. If they are all enqueued waiting to eat, from (7.1) we conclude $\sum fork[i] \leq 5$ which contradicts the previous formula. ∎

Starvation is possible in Algorithm 7.5. Two philosophers can conspire to starve their mutual neighbor as shown in the following scenario (with the obvious abbreviations):

n	phil1	phil2	phil3	$f0$	$f1$	$f2$	$f3$	$f4$
1	**take(1)**	take(2)	take(3)	2	2	2	2	2
2	release(1)	take(2)	**take(3)**	1	2	1	2	2
3	release(1)	**take(2)** to **waitC(OK[2])**	release(3)	1	2	0	2	1
4	**release(1)**	(blocked)	release(3)	1	2	0	2	1
5	**take(1)**	(blocked)	release(3)	2	2	1	2	1
6	release(1)	(blocked)	**release(3)**	1	2	0	2	1
7	release(1)	(blocked)	**take(3)**	1	2	1	2	2

Philosophers 1 and 3 both need a fork shared with philosopher 2. In lines 1 and 2, they each take a pair of forks, so philosopher 2 is blocked when trying to take forks in line 3. In lines 4 and 5 philosopher 1 releases her forks and then immediately takes them again; since *forks*[*2*] does not receive the value 2, philosopher 2 is not signaled. Similarly, in lines 6 and 7, philosopher 3 releases her forks and then immediately takes them again. Lines 4 through 7 of the scenario can be continued indefinitely, demonstrating starvation of philosopher 2.

7.9 Monitors in BACI^L

The monitor of the C implementation of Algorithm 7.4 is shown in Listing 7.1. The program is written in the dialect supported by BACI. The syntax and the semantics are very similar to those of Algorithm 7.4, except that the condition queues are not FIFO, and boolean variables are not supported. A Pascal program is also available in the archive.

7.10 Protected objects

In the classical monitor, tests, assignment statements and the associated statements waitC, signalC and emptyC must be explicitly programmed. The protected object simplifies the programming of monitors by encapsulating the manipulation of the queues together with the evaluation of the expressions. This also enables significant optimization of the implementation. Protected objects were defined and implemented in the language Ada, though the basic concepts have been around since the beginning of research on monitors. The construct is so elegant that it is worthwhile studying it in our language-independent notation, even if you don't use Ada in your programming.

```
monitor RW {
  int readers = 0, writing = 1;
  condition OKtoRead, OKtoWrite;

  void StartRead() {
    if (writing || !empty(OKtoWrite))
        waitc(OKtoRead);
    readers = readers + 1;
    signalc(OKtoRead);
  }

  void EndRead() {
    readers = readers - 1;
    if (readers == 0)
        signalc(OKtoWrite);
  }

  void StartWrite() {
    if (writing || (readers != 0))
        waitc(OKtoWrite);
    writing = 1;
  }

  void EndWrite() {
    writing = 0;
    if (empty(OKtoRead))
        signalc(OKtoWrite);
    else
        signalc(OKtoRead);
  }
}
```

Listing 7.1: A C solution for the problem of the readers and writers

Algorithm 7.6: Readers and writers with a protected object

```
protected object RW
   integer readers ← 0
   boolean writing ← false

   operation StartRead when not writing
      readers ← readers + 1

   operation EndRead
      readers ← readers − 1

   operation StartWrite when not writing and readers = 0
      writing ← true

   operation EndWrite
      writing ← false
```

reader		writer	
	loop forever		loop forever
p1:	RW.StartRead	q1:	RW.StartWrite
p2:	read the database	q2:	write to the database
p3:	RW.EndRead	q3:	RW.EndWrite

Algorithm 7.6 is a solution to the problem of the readers and writers that uses a protected object. As with monitors, execution of the operations of a protected object is done under mutual exclusion. Within the operations, however, the code consists only of the trivial statements that modify the variables.

Synchronization is performed upon entry to and exit from an operation. An operation may have a suffix when expression, called a *barrier*. The operation may start executing only when the expression of the barrier is true. If the barrier is false, the process is blocked on a FIFO queue; there is a separate queue associated with each entry. (By a slight abuse of language, we will say that the barrier is evaluated and its value is true or false.) In Algorithm 7.6, StartRead can be executed only if no process is writing and StartWrite can be executed only if no process is either reading or writing.

When can a process become unblocked? Assuming that the barrier refers only to variables of the protected object, their values can be changed only by executing an operation of the protected object. So, when the execution of an operation has been completed, all barriers are re-evaluated; if one of them is now true, the process at

the head of the FIFO queue associated with that barrier is unblocked. Since the process evaluating the barrier is terminating its operation, there is no failure of mutual exclusion when a process is unblocked.

In the terminology of Section 7.5, the precedence specification of protected objects is $E < W$. Signaling is done implicitly upon the completion of a protected action so there are no signaling processes. Servicing entry queues is given precedence over new processes trying to enter the protected objects.

Starvation is possible in Algorithm 7.6; we leave it as an exercise to develop algorithms that are free from starvation.

Protected objects are simpler than monitors because the barriers are used implicitly to carry out what was done explicitly with condition variables. In addition, protected objects enable a more efficient implementation. Consider the following outline of a scenario of Algorithm 7.4, the monitor solution to the readers and writers problem:

Process reader	Process writer
waitC(OKtoRead)	operation EndWrite
(blocked)	**writing ← false**
(blocked)	**signalC(OKtoRead)**
readers ← readers + 1	return from EndWrite
signalC(OKtoRead)	return from EndWrite
read the data	**return from EndWrite**
read the data	...

When the writer executes signalC(OKtoRead), by the IRR it must be blocked to let a reader process be unblocked. But as soon as the process exits its monitor operation, the blocked signaling process is unblocked, only to exit its monitor operation. These context switches are denoted by the breaks in the table. Even with the optimization of combining signalC with the monitor exit, there will still be a context switch from the signaling process to the waiting process.

Consider now a similar scenario for Algorithm 7.6, the solution that uses a protected object:

Process reader	Process writer
when not writing	operation EndWrite
(blocked)	**writing ← false**
(blocked)	**when not writing**
(blocked)	**readers ← readers + 1**
read the data	...

The variables of the protected object are accessible only from within the object itself. When the writer resets the variable writing, it is executing under mutual exclusion, so it might as well evaluate the barrier for reader and even execute the statements of reader's entry! When it is finished, both processes can continue executing other statements. Thus a *protected action* can include not just the execution of the body of an operation, but also the evaluation of the barriers and the execution of the bodies of other operations. This reduces the number of context switches that must be done; as a context switch is usually much more time-consuming than simple statements, protected objects can be more efficient than monitors.

To enable this optimization, it is forbidden for barriers to depend on parameters of the entries. That way there need be only one queue per *entry*, not one queue per *entry call*, and the data needed to evaluate the barrier are globally accessible to all operations, not just locally in the entry itself. In order to block a process on a queue that depends on data in the call, the entry call first uses a barrier that does not depend on this data; then, within the protected operation, the data are checked and the call is *requeued* on another entry. A discussion of this topic is beyond the scope of this book.

Protected objects in Ada^L

If you only want to protect a small amount of data (and freedom from starvation is not required), the problem of the readers and writers can be solved simply by using procedures and functions of protected objects:

```
1  protected RW is
2    procedure Write(I: Integer );
3    function Read return Integer;
4  private
5    N: Integer  := 0;
6  end RW;
7
8  protected body RW is
9    procedure Write(I: Integer ) is
10   begin
11     N := I;
12   end Write;
13   function Read return Integer  is
14   begin
15     return N;
16   end Read;
17 end RW;
```

The operations are declared in the public part of the specification, while variables that are global to the operations are declared in the private part. These "monitor variables" are only accessible in the bodies of the operations declared in the body of the protected object and not by its clients. *procedures* are operations that are never blocked except for the ordinary mutual exclusion of access to a protected object. *functions* are limited to read-only access of the variables and more than one task may call a function concurrently. As discussed in the previous section, an *entry* is guarded by a barrier.

Listing 7.2 shows the implementation of Algorithm 7.6 in Ada. Full explanations of protected objects in Ada can be found in [7, 18].

7.11 Monitors in Java^L

In Java, there is no special construct for a monitor; instead, *every* object has a lock that can be used for synchronizing access to fields of the object. Listing 7.3 shows a class for the producer–consumer problem. Once the class is defined, we can declare objects of this class:

```
PCMonitor monitor = new PCMonitor();
```

and then invoke its methods:

```
monitor.Append(5);
int n = monitor.Take();
```

The addition of the **synchronized** specifier to the declaration of a method means that a process must acquire the lock for that object before executing the method. From within a **synchronized** method, another **synchronized** method of the same object can be invoked while the process retains the lock. Upon return from the last synchronized method invoked on an object, the lock is released.

There are no fairness specifications in Java, so if there is contention to call synchronized methods of an object an arbitrary process will obtain the lock.

A class all of whose methods are declared **synchronized** can be called a Java monitor. This is just a convention because—unlike the case with protected objects in Ada—there is no syntactical requirement that all of the methods of an object be declared as **synchronized**. It is up to the programmer to ensure that no synchronization errors occur when calling an unsynchronized method.

```
1   protected RW is
2      entry StartRead;
3      procedure EndRead;
4      entry StartWrite;
5      procedure EndWrite;
6   private
7      Readers: Natural :=0;
8      Writing: Boolean := false;
9   end RW;
10
11  protected body RW is
12     entry StartRead
13       when not Writing is
14     begin
15        Readers := Readers + 1;
16     end StartRead;
17
18     procedure EndRead is
19     begin
20        Readers := Readers - 1;
21     end EndRead;
22
23     entry StartWrite
24       when not Writing and Readers = 0 is
25     begin
26        Writing := true;
27     end StartWrite;
28
29     procedure EndWrite is
30     begin
31        Writing := false;
32     end EndWrite;
33  end RW;
```

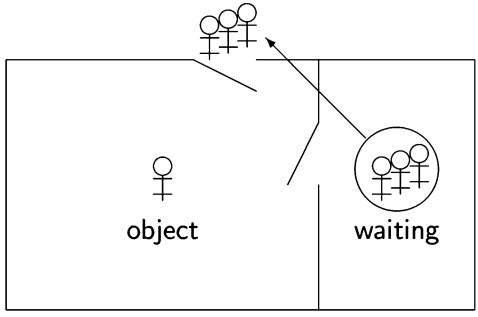
Listing 7.2: Solution for the problem of the readers and writers in Ada

```
class PCMonitor {
  final int N = 5;
  int Oldest = 0, Newest = 0;
  volatile int Count = 0;
  int Buffer[] = new int[N];

  synchronized void Append(int V) {
    while (Count == N)
      try {
          wait();
      } catch (InterruptedException e) {}
    Buffer[Newest] = V;
    Newest = (Newest + 1) % N;
    Count = Count + 1;
    notifyAll ();
  }

  synchronized int Take() {
    int temp;
    while (Count == 0)
      try {
          wait();
      } catch (InterruptedException e) {}
    temp = Buffer[Oldest];
    Oldest = (Oldest + 1) % N;
    Count = Count - 1;
    notifyAll ();
    return temp;
  }
}
```

Listing 7.3: A Java monitor for the producer–consumer problem

If a process needs to block itself—for example, to avoid appending to a full buffer or taking from an empty buffer—it can invoke the method wait. (This method throws InterruptedException which must be thrown or caught.) The method notify is similar to the monitor statement signalC and will unblock one process, while notifyAll will unblock all processes blocked on this object.

The following diagram shows the structure of a Java monitor:



There is a lock which controls the door to the monitor and only one process at a time is allowed to hold the lock and to execute a synchronized method. A process executing wait blocks on this object; the process joins the *wait set* associated with the object shown on the righthand side of the diagram. When a process joins the wait set, it also releases the lock so that another process can enter the monitor. notify will release one process from the wait set, while notifyAll will release all of them, as symbolized by the circle and arrow.

A process must hold the synchronization lock in order to execute either notify or notifyAll, and it continues to hold the lock until it exits the method or blocks on a wait. The unblocked process or processes must reacquire the lock (only one at a time, of course) in order to execute, whereas with the IRR in the classical monitor, the lock is passed from the signaling process to the unblocked process. Furthermore, the unblocked processes receive no precedence over other processes trying to acquire the lock. In the terminology of Section 7.5, the precedence specification of Java monitors is $E = W < S$.

An unblocked process cannot assume that the condition it is waiting for is true, so it must recheck it in a while loop before continuing:[3]

```
synchronized method1() {
  while (! booleanExpression)
      wait();
  // Assume booleanExpression is true
}
synchronized method2() {
  // Make booleanExpression true
  notifyAll ()
}
```

[3]We have left out the exception block for clarity.

If the expression in the loop is false, the process joins the wait set until it is unblocked. When it acquires the lock again it will recheck the expression; if it is true, it remains true because the process holds the lock of the monitor and no other process can falsify it. On the other hand, if the expression is again false, the process will rejoin the wait set.

When notify is invoked *an arbitrary process may be unblocked* so starvation is possible in a Java monitor. Starvation-free algorithms may be written using the concurrency constructs defined in the library java.util.concurrent.

A Java monitor has no condition variables or entries, so that it is impossible to wait on a specific condition.[4] Suppose that there are two processes waiting for conditions to become true, and suppose that process p has called method1 and process q has called method2, and that both have invoked wait after testing for a condition. We want to be able to unblock the process whose condition is now true:

```
synchronized method1() {
  if (x == 0)
    wait();
}
synchronized method2() {
  if (y == 0)
    wait();
}
synchronized method3( ... ) {
  if ( ... ) {
      x = 10;
      notify (someProcessBlockedInMethod1); // Not legal!!
  }
  else {
      y = 20;
      notify (someProcessBlockedInMethod2); // Not legal!!
  }
}
```

Since an arbitrary process will be unblocked, this use of notify is not correct, because it could unblock the wrong process. Instead, we have to use notifyAll which unblocks *all* processes; these processes now compete to enter the monitor again. A process must wait within a loop to check if its condition is now true:

[4]Condition variables can be defined using the interface java.util.concurrent.locks.Condition.

```
synchronized method1() {
  while (x == 0)
    wait();
}
synchronized method2() {
  while (y == 0)
    wait();
}
synchronized method3( ... ) {
  if ( ... )
    x = 10;
  else
    y = 20;
  notifyAll ();
}
```

If the wrong process is unblocked it will return itself to the wait set.

However, if only one process is waiting for a lock, or if all the processes waiting are waiting for the same condition, then notify can be used, at a significant saving in context switching.

A Java solution for the problem of the readers and writers is shown in Listing 7.4. The program contains no provision for preventing starvation.

Synchronized blocks

We have shown a programming style using synchronized methods to build monitors in Java. But **synchronized** is more versatile because it can be applied to any object and to any sequence of statements, not just to a method. For example, we can obtain the equivalent of a critical section protected by a binary semaphore simply by declaring an empty object and then synchronizing on its lock. Given the declaration:

```
Object obj = new Object();
```

the statements within any synchronized block on this object in any process are executed under mutual exclusion:

```
synchronized (obj) {
    // critical section
}
```

```
1  class RWMonitor {
2    volatile int readers = 0;
3    volatile boolean writing = false;
4
5    synchronized void StartRead() {
6      while (writing )
7        try {
8          wait();
9        } catch (InterruptedException e) {}
10     readers = readers + 1;
11     notifyAll ();
12   }
13   synchronized void EndRead() {
14     readers = readers - 1;
15     if (readers == 0)
16         notifyAll ();
17   }
18   synchronized void StartWrite() {
19     while (writing || (readers != 0))
20       try {
21         wait();
22       } catch (InterruptedException e) {}
23     writing = true;
24   }
25   synchronized void EndWrite() {
26     writing = false;
27     notifyAll ();
28   }
29 }
```

Listing 7.4: A Java monitor for the problem of the readers and the writers

There is no guarantee of liveness, however, because in the presence of contention for a specific lock, an arbitrary process succeeds in acquiring the lock.

7.12 Simulating monitors in Promela^L

Promela is not an appropriate formalism for studying monitors, since monitors are a formalism for encapsulation, which is not supported in Promela. We will show how to implement the synchronization constructs of monitors in Promela.

A monitor has a lock associated with it; the code of each procedure of the monitor must be enclosed within enterMon and leaveMon:

```
bool lock = false;
inline enterMon() {
    atomic {
        !lock;
        lock = true;
    }
}
inline leaveMon() {
    lock = false;
}
```

The variable lock is true when the lock is held by a process, so enterMon simply blocks until the lock is false.

Each simulated condition variable contains a field gate for blocking the process and a count of processing waiting on the condition:

```
typedef Condition {
  bool gate;
  byte waiting;
}
```

The field waiting is used only to implement emptyC:

```
#define emptyC(C) (C.waiting == 0)
```

The name has been changed since **empty** is a reserved word in Promela.

The operation waitC sets lock to false to release the monitor and then waits for C.gate to become true:

```
inline waitC(C) {
   atomic {
     C.waiting++;
     lock = false;      /* Exit monitor */
     C.gate;            /* Wait for gate */
     C.gate = false;    /* Reset gate */
     C.waiting--;
   }
}
```

signalC sets C.gate to true so that the blocked process may continue:

```
inline signalC(C) {
  atomic {
    if
    :: (C.waiting > 0) -> /* Signal only if waiting */
        C.gate = true;    /* Signal the gate */
        !lock;            /* IRR, wait for lock */
        lock = true;      /* Take lock again */
    :: else
    fi ;
  }
}
```

The signalC operation does not release the lock, so the waitC operation will be able to execute under the IRR. When the waiting process finally leaves the monitor, it will reset the lock, allowing the signaling process to continue.[5]

We leave the implementation of FIFO condition queues as an exercise.

Transition

Monitor-like constructs are the most popular form of synchronization found in programming languages. These constructs are structured and integrate well with the techniques of object-oriented programming. The original formulation by Hoare and Brinch Hansen has served as inspiration for similar constructs, in particular the synchronized methods of Java, and the very elegant protected objects of Ada. As with semaphores, it is important to understand the precise semantics of the construct that you are using, because the style of programming as well as the correctness and efficiency of the resulting programs depend on the details of the implementation.

Both the semaphore and the monitor are highly centralized constructs, blocking and unblocking processes, maintaining queues of blocked processes and encapsulating data. As multiprocessing and distributed architectures become more popular, there is a need for synchronization constructs that are less centralized. These constructs are based upon communications rather than upon sharing. The next chapter presents several models that achieve synchronization by using communications between sending processes and receiving processes.

[5]The implementation does not give precendence to signaling processes over entering processes.

Exercises

1. Develop a simulation of monitors by semaphores.

2. Show that in Algorithm 7.4, the integer variable writers can be replaced by a boolean variable writing that is equivalent to $writers > 0$.

3. Prove that there is no starvation of writers in Algorithm 7.4.

4. Modify the solution to the problem of the readers and the writers so as to implement each of the following rules:

   (a) If there are reading processes, a new reader may commence reading even if there are waiting writers.

   (b) If there are waiting writers, they receive precedence over all waiting readers.

   (c) If there are waiting readers, no more than two writers will write before a process is allowed to read.

5. Prove the invariant $eating[i] \leftrightarrow (forks[i] = 2)$ from Section 7.8 on the problem of the dining philosophers. Prove also that formulas (7.1) and (7.2) are invariant.

6. Here is the declaration of a protected object in Ada for a starvation-free solution of the problem of the readers and writers. The solution uses an extra pair of private entries to distinguish between groups of processes that have been waiting to read or write, and new processes that we want to block until the previous group has succeeded. Write the body of the protected object and prove the correctness of the program.

```
protected RW is
    entry StartRead;
    procedure EndRead;
    entry StartWrite;
    procedure EndWrite;
private
    entry ReadGate;
    entry WriteGate;
    Readers: Natural := 0;
    Writing: Boolean := false;
end RW;
```

7. Here is the declaration of a protected object in Ada for a starvation-free solution of the problem of the readers and writers. The solution uses an extra variable WaitingToRead to record the group of waiting readers. Write the body of the protected object and prove the correctness of the program.[6]

```
protected RW is
    entry StartRead;
    procedure EndRead;
    entry StartWrite;
    procedure EndWrite;
private
    WaitingToRead: Integer := 0;
    Readers: Natural := 0;
    Writing: Boolean := false;
end RW;
```

8. In Java, what does **synchronized** (**this**) mean? What is the difference, if any, between:

```
void p() {
  synchronized (this) {
    // statements
  }
}
```

and

```
synchronized void p() {
  // statements
}
```

9. Suppose that we are performing computations with variables of type **int** in Java, and that these variables are likely to be accessed by other processes. Should you declare the variables to be **volatile** or should you use a **synchronized** block around each access to the variables?

10. Implement FIFO condition queues in Promela.

[6]My thanks to Andy Wellings and Alan Burns for providing this solution.

11. In his original paper on monitors, Hoare allowed the waitC statement to have an additional integer parameter called a priority:

```
waitC(cond, priority )
```

Processes are stored on the queue associated with the condition variable in ascending order of the priority given in the waitC statement. Show how this can be used to implement a discrete event simulation, where each process executes a computation and then determines the next time at which it is to be executed.

8 Channels

The concurrent programming constructs that we have studied use shared memory, whether directly or through a shared service of an operating system. Now we turn to constructs that are based upon *communications*, in which processes send and receive *messages* to and from each other. We will study synchronization in progressively looser modes of communications: in this chapter, we discuss synchronization mechanisms appropriate for tightly coupled systems (channels, rendezvous and remote procedure call); then we discuss spaces which provide communications with persistence; and finally we will study algorithms that were developed for fully distributed systems. As always, we will be dealing with an abstraction and not with the underlying implementation. The interleaving model will continue to be used; absolute time will not be considered, only the relative order in which messages are sent and received by a process.

The chapter begins with an analysis of alternatives in the design of constructs for synchronization by communications. Sections 8.2–8.4 present algorithms using channels, the main topic of the chapter. The implementation of channels in Promela is discussed in Section 8.5. The rendezvous construct and its implementation in Ada is presented in Section 8.6. The final section gives a brief survey of remote procedure calls.

8.1 Models for communications

Synchronous vs. asynchronous communications

Communications by its nature requires two processes—one to send a message and one to receive it—so we must specify the degree of cooperation required between the two processes. In *synchronous communications*, the exchange of a message is an *atomic* action requiring the participation of both the sending process, called the *sender*, and the receiving process, called the *receiver*. If the sender is ready to send but the receiver is not ready to receive, the sender is blocked, and similarly, if the receiver is ready to receive before the sender is ready to send, the receiver is blocked. The act of communications synchronizes the execution sequences of

the two processes. Alternatively, the sender can be allowed to send a message and continue without blocking. Communications are then *asynchronous* because there is no temporal dependence between the execution sequences of the two processes. The receiver could be executing any statement when a message is sent, and only later check the communications channel for messages.

Deciding on a scheme is based upon the capacity of the communications channel to buffer messages, and on the need for synchronization. In asynchronous communications, the sender may send many messages without the receiver removing them from the channel, so the channel must be able to buffer a potentially unlimited number of messages. Since any buffer is finite, eventually the sender will be blocked or messages will be discarded. Synchronous and asynchronous communications are familiar from telephone calls and email messages. A telephone call synchronizes the activities of the caller and the person who answers. If the persons cannot synchronize, busy signals and unanswered calls result. On the other hand, any number of messages may be sent by email, and the receiver may choose to check the incoming mail at any time. Of course, the capacity of an electronic mailbox is limited, and email is useless if you need immediate synchronization between the sender and the receiver.

The choice between synchronous and asynchronous communications also depends on the level of the implementation. Asynchronous communications requires a buffer for messages sent but not received, and this memory must be allocated somewhere, as must the code for managing the buffers. Therefore, asynchronous communications is usually implemented in software, rather than in hardware. Synchronous communications requires no support other than send and receive instructions that can be implemented in hardware (see Section 8.3).

Addressing

In order to originate a telephone call, the caller must know the telephone number, the *address*, of the receiver of the call. Addressing is asymmetric, because the receiver does not know the telephone number of the caller. (Caller identification is possible, though the caller may choose to block this option.) Email messages use symmetric addressing, because every message contains the address of the sender.

Symmetric addressing is preferred when the processes are cooperating on the solution of a problem, because the addresses can be fixed at compile time or during a configuration stage, leading to easier programming and more efficient execution. This type of addressing can also be implemented by the use of *channels*: rather than naming the processes, named channels are declared for use by a pair or a group of processes.

Asymmetric addressing is preferred for programming *client–server systems*. The client has to know the name of the service it requests, while the server can be programmed without knowledge of its future clients. If needed, the client identification must be passed dynamically as part of the message.

Finally, it is possible to pass messages without any addressing whatsoever! Matching on the message structure is used in place of addresses. In Chapter 9, we will study spaces in which senders broadcast messages with no address; the messages can be received by any process, even by processes that were not in existence when the message was sent.

Data flow

A single act of communications can have data flowing in one direction or two. An email message causes data to flow in one direction only, so a reply requires a separate act of communications. A telephone call allows two-way communications.

Asynchronous communications necessarily uses one-way data flow: the sender sends the message and then continues without waiting for the receiver to accept the message. In synchronous communications, either one- or two-way data flow is possible. Passing a message on a one-way channel is extremely efficient, because the sender need not be blocked while the receiver processes the message. However, if the message does in fact need a reply, it may be more efficient to block the sender, rather than release it and later go through the overhead of synchronizing the two processes for the reply message.

CSP and occam

Channels were introduced by C.A.R. Hoare in the CSP formalism [32], which has been extremely influential in the development of concurrent programming [55, 58]. The programming language occam is directly based upon CSP, as is much of the Promela language that we use in this book. For more information on CSP, occam and their software tools for programming and verification, see the websites listed in Appendix E.

8.2 Channels

A *channel* connects a sending process with a receiving process. Channels are *typed*, meaning that you must declare the type of the messages that can be sent

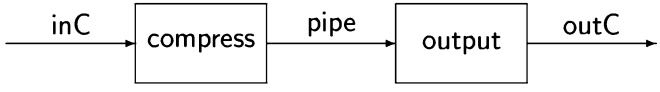
on the channel. In this section, we discuss synchronous channels. The following algorithm shows how a producer and a consumer can be synchronized using a channel:

Algorithm 8.1: Producer–consumer (channels)	
channel of integer ch	
producer	**consumer**
integer x	integer y
loop forever	loop forever
p1: x ← produce	q1: ch ⇒ y
p2: ch ⇐ x	q2: consume(y)

The notation ch⇐x means that the value of x is sent on the channel ch, and similarly, ch⇒y means that the value of the message received from the channel is assigned to the variable y. The producer will attempt to send a message at p2, while the consumer will attempt to receive a message at q1. The data transfer will take place only after the control pointers of the two processes reach those points. Synchronous execution of the send and receive operations is considered to be a single change in state. So if $x = v$ in process p, then executing ch⇐x in p and ch⇒y in process q leads to a state in which $y = v$. Channels in operating systems are called *pipes*; they enable programs to be constructed by connecting an existing set of programs.

Pipelined computation using channels can be demonstrated using a variant of *Conway's problem* (Algorithm 8.2). The input to the algorithm is a sequence of characters sent by an environment process to an input channel; the output is the same sequence sent to an environment process on another channel after performing two transformations: (a) runs of $2 \leq n \leq 9$ occurrences of the same character are replaced by the digit corresponding to n and the character; (b) a newline character is appended following every Kth character of the transformed sequence.

The algorithm is quite easy to follow because the two transformations are implemented in separate processes: the compress process replaces runs as required and sends the characters one by one on the pipe channel, while the output process takes care of inserting the newline characters:



Channels are efficient and easy to use but they lack flexibility. In particular, it is difficult to program a *server* using channels, because it is not possible for a server simply to export the interface of the services that it is offering; it must also export

Algorithm 8.2: Conway's problem

```
constant integer MAX ← 9
constant integer K ← 4
channel of integer inC, pipe, outC
```

compress		output	
	char c, previous ← 0		char c
	integer n ← 0		integer m ← 0
	inC ⇒ previous		
	loop forever		loop forever
p1:	inC ⇒ c	q1:	pipe ⇒ c
p2:	if (c = previous) and (n < MAX−1)	q2:	outC ⇐ c
p3:	n ← n + 1	q3:	m ← m + 1
	else		
p4:	if n > 0	q4:	if m >= K
p5:	pipe ⇐ intToChar(n+1)	q5:	outC ⇐ newline
p6:	n ← 0	q6:	m ← 0
p7:	pipe ⇐ previous	q7:	
p8:	previous ← c	q8:	

a specific set of channels. In other words, programs that use channels must be configured, either at compile time or when the program is initiated. In Section 8.6, we describe the rendezvous which is more flexible and appropriate for writing servers.

8.3 Parallel matrix multiplication

One-way channels are extremely efficient, and can be implemented in hardware. There used to be a computing element called a *transputer*, which contained a CPU, memory and four pairs of one-way channels on a single chip. Synchronous communications was implemented through channel statements which could be directly compiled to machine-language instructions of the transputer. This facilitated the construction of arrays of processors that could be programmed to execute truly parallel programs. While the transputer no longer exists, its architecture has influenced that of modern chips, especially digital signal processors. Here we will demonstrate an algorithm that can be efficiently implemented on such systems.

Consider the problem of matrix multiplication:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 6 \\ 10 & 5 & 18 \\ 16 & 8 & 30 \end{bmatrix}$$

Each element of the resulting matrix can be computed independently of the other elements. For example, the element 30 in the bottom right corner of the matrix is obtained by the following computation, which can be performed before, after or simultaneously with the computation of other elements:

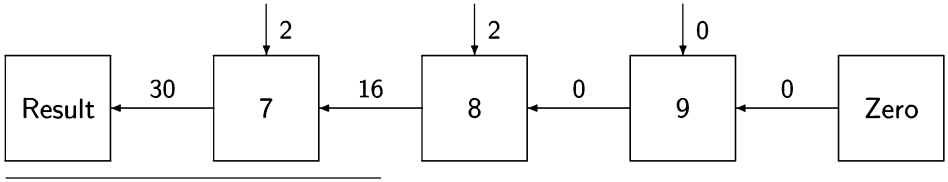
$$[7, 8, 9] \times \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} = 7 \cdot 2 + 8 \cdot 2 + 9 \cdot 0 = 14 + 16 + 0 = 30.$$

We will implement matrix multiplication for 3×3 matrices using small numbers, although this algorithm is practical only for large matrices of large values, because of the overhead involved in the communications.

Figure 8.1 shows 21 processors connected so as to compute the matrix multiplication.[1] Each processor will execute a single process. The one-way channels are denoted by arrows. We will use geographical directions to denote the various neighbors of each processor.

The central array of 3×3 processes multiplies pairs of elements and computes the partial sums. One process is assigned to each element of the first matrix and initialized with the value of that element. The Source processes are initialized with the row vectors of the second matrix and the elements are sent one by one to the Multiplier processes to the south; they in turn pass on the elements until they are finally caught in the Sink processes. (The Sink processes exist so that the programming of all the Multiplier processes can be uniform without a special case for the bottom row of the array.) A Multiplier process receives a partial sum from the east, and adds to it the result of multiplying its element and the value received from the north. The partial sums are initialized by the Zero processes and they are finally sent to the Result processes on the west.

Let us study the computation of one element of the result:



[1] In the general case, $n^2 + 4n$ processors are needed for multiplication of matrices of size $n \times n$.

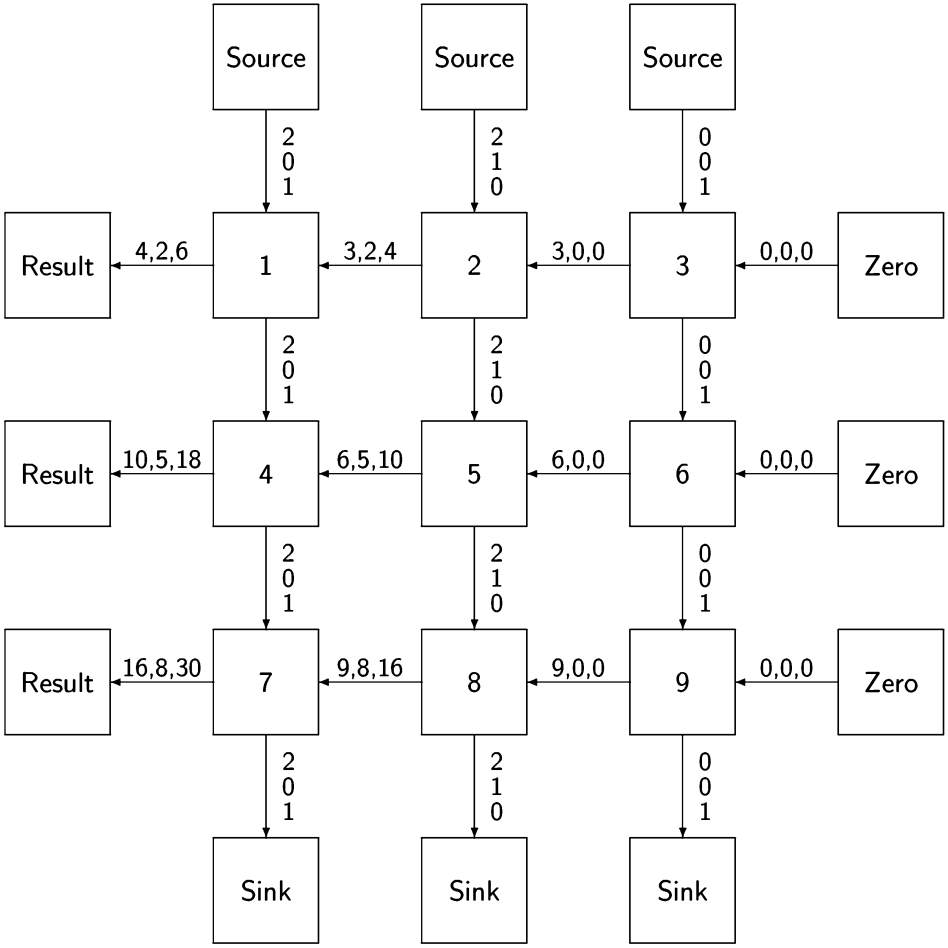


Figure 8.1: Process array for matrix multiplication

This is the computation of the corner element that we showed above as a mathematical equation. Since each process waits for its two input values before computing an output, we can read the diagram from right to left: first, $9 \cdot 0 + 0 = 0$, where the partial sum of 0 comes from the Zero process; then, $8 \cdot 2 + 0 = 16$, but now the the partial sum of 0 comes from the Multiplier process labeled 9; finally, using this partial sum of 16, we get $7 \cdot 2 + 16 = 30$ which is passed to the Result process.

Except for the Multiplier processes, the algorithms for the other processes are trivial. A Zero process executes West⇐0 three times to initialize the Sum variables of a Multiplier process; a Source process executes South⇐Vector[i] for each of the three elements of the rows of the second matrix; a Sink process executes

North⇒dummy three times and ignores the values received; a Result process executes East⇒result three times and prints or otherwise uses the values received. A Result process knows which row and column the result belongs to: the row is determined by its position in the structure and the column by the order in which the values are received.

Here is the algorithm for a Multiplier process:

Algorithm 8.3: Multiplier process with channels	
	integer FirstElement channel of integer North, East, South, West integer Sum, integer SecondElement
	loop forever
p1:	North ⇒ SecondElement
p2:	East ⇒ Sum
p3:	Sum ← Sum + FirstElement · SecondElement
p4:	South ⇐ SecondElement
p5:	West ⇐ Sum

Each process must be initialized with five parameters: the element FirstElement and channels that are appropriate for its position in the structure. This configuration can be quite language-specific and we do not consider it further here.

Selective input

In all our examples so far, an input statement ch⇒var is for a single channel, and the statement blocks until a process executes an output statement on the corresponding channel. Languages with synchronous communication like CSP, Promela and Ada allow selective input, in which a process can attempt to communicate with more than one channel at the same time:

```
either
   ch1 ⇒ var1
or
   ch2 ⇒ var2
or
   ch3 ⇒ var3
```

Exactly one alternative of a selective input statement can succeed. When the statement is executed, if communications can take place on one or more channels, one of them is selected nondeterministically. Otherwise, the process blocks until the communications can succeed on one of the alternatives.

The matrix multiplication program can use selective input to take advantage of additional parallelism in the algorithm. Instead of waiting for input first from North and then from East, we can wait for the first available input and then wait for the other one:

Algorithm 8.4: Multiplier with channels and selective input	
	integer FirstElement channel of integer North, East, South, West integer Sum, integer SecondElement
	loop forever either
p1:	North ⇒ SecondElement
p2:	East ⇒ Sum
	or
p3:	East ⇒ Sum
p4:	North ⇒ SecondElement
p5:	South ⇐ SecondElement
p6:	Sum ← Sum + FirstElement · SecondElement
p7:	West ⇐ Sum

Once one alternative has been selected, the rest of the statements in the alternative are executed normally, in this case, an input statement from the other channel. The use of selective input ensures that the processor to the east of this Multiplier is not blocked unnecessarily.

8.4 The dining philosophers with channels

A natural solution for the dining philosophers problem is obtained by letting each fork be a process that is connected by a channel to the philosophers on its left and right.[2] There will be five philosopher processes and five fork processes, shown in the left and right columns of Algorithm 8.5, respectively. The philosopher processes start off by waiting for input on the two adjacent forks channels. Eventually, the fork processes will output values on their channels. The values are of no interest, so we use boolean values and just ignore them upon input. Note that once a fork process has output a value it is blocked until it receives input from the channel. This will only occur when the philosopher process that previously received a value on the channel returns a value after eating.

[2]This works in Promela because a channel can be used by more than one pair of processes.

Algorithm 8.5: Dining philosophers with channels	
channel of boolean forks[5]	
philosopher i	**fork i**
boolean dummy	boolean dummy
loop forever	loop forever
p1: think	q1: forks[i] ⇐ true
p2: forks[i] ⇒ dummy	q2: forks[i] ⇒ dummy
p3: forks[i+1] ⇒ dummy	q3:
p4: eat	q4:
p5: forks[i] ⇐ true	q5:
p6: forks[i+1] ⇐ true	q6:

8.5 Channels in Promela^L

The support for channels in Promela is extensive. In addition to synchronous channels, asynchronous channels can be declared with any fixed capacity, although channels with a large capacity will give rise to an extremely large number of states, making it difficult to verify an algorithm. The message type of the channel can be declared to be of any type or sequence of types, and arrays of channels can be declared. Here is an example of the declaration of a single synchronous channel whose messages are integers, and an array of four asynchronous channels with capacity ten and a message type that is a sequence of an integer and a boolean value:

```
chan ch = [0] of { int }
chan charray[4] = [10] of { int, bool }
```

ch ? v1, v2, ... denotes receiving a value from the channel ch and storing it in the variables v1, v2, ..., and ch ! val1, val2, ... denotes sending these values on the channel. A receive operation can be performed only if the message matches, that is, if the number and types of parameters match those of the message.

Listing 8.1 shows a solution to Conway's problem in Promela. Two additional processes are not shown: process Generator supplies input on channel inC, and process Printer displays the output from channel outC.

```
1  #define N 9
2  #define K 4
3  chan inC, pipe, outC = [0] of { byte };
4  active proctype Compress() {
5    byte previous, c, count = 0;
6    inC ? previous;
7    do
8    :: inC ? c ->
9       if
10      :: (c == previous) && (count < N-1) -> count++
11      :: else ->
12         if
13         :: count > 0 ->
14            pipe ! count+1;
15            count = 0
16         :: else
17         fi;
18         pipe ! previous;
19         previous = c;
20      fi
21   od
22 }
23 active proctype Output() {
24   byte c, count = 0;
25   do
26   :: pipe ? c;
27      outC ! c;
28      count++;
29      if
30      :: count >= K ->
31         outC ! '\n';
32         count = 0
33      :: else
34      fi
35   od
36 }
```

Listing 8.1: A solution to Conway's problem in Promela

Here is the Multiplier process from Algorithm 8.4 written in Promela:

```
proctype Multiplier (byte Coeff;
        chan North; chan East; chan South; chan West) {
    byte Sum, X;
    do ::
        if  ::  North ? X -> East ? Sum;
            ::  East  ? Sum -> North ? X;
        fi ;
        South ! X;
        Sum = Sum + X*Coeff;
        West  !  Sum;
    od
}
```

An **init** process is required to invoke **run** for each process with the appropriate channels. The semantics of the **if** statement are that it blocks until one of the guards becomes true, that is, until it is possible to complete a receive operation from either the North or East channel. Once the **if** statement commits to one of these options, it executes the receive statement for the other channel.

Promela also includes statements for examining if a channel is full or empty, for examining elements in the channel without removing them, and for sending and receiving messages in other than FIFO order.

8.6 Rendezvous

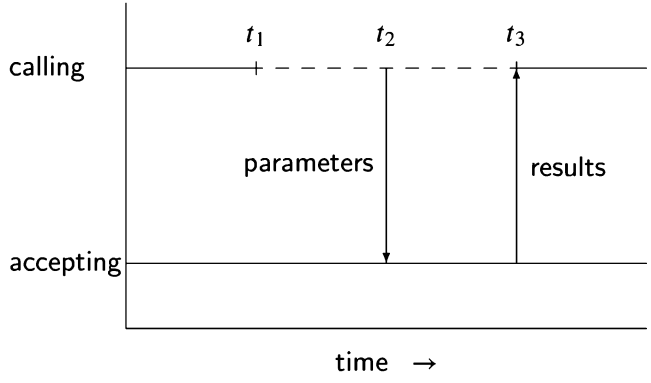
The name rendezvous invokes the image of two people who choose a place to meet; the first one to arrive must wait for the arrival of the second. In the metaphor, the two people are symmetric and the rendezvous place is neutral and passive. However, in the synchronization construct, the location of the rendezvous belongs to one of the processes, called the *accepting* process. The other process, the *calling* process, must know the identity of the accepting process and the identity of the rendezvous which is called an *entry* (the same term used in protected objects; see Section 7.10). The accepting process does not know and does not need to know the identity of the calling process, so the rendezvous is appropriate for implementing servers that export their services to all potential clients.

Algorithm 8.6 shows how a rendezvous is used to implement a client–server application. The client process calls the service entry of the server process supplying some parameters; it must now block until the server process is ready to accept the

Algorithm 8.6: Rendezvous	
client	**server**
integer parm, result	integer p, r
loop forever	loop forever
p1: parm ← ...	q1:
p2: server.service(parm, result)	q2: accept service(p, r)
p3: use(result)	q3: r ← do the service(p)

call. Then the rendezvous is performed using the parameters and possibly producing a result. Upon completion of the rendezvous, the result is returned to the client process. At the completion of the rendezvous, the client process is unblocked and the server process remains, of course, unblocked.

The semantics of a rendezvous are illustrated in the following diagram:



At time t_1, the calling process is blocked pending acceptance of the call which occurs at t_2. At this time the parameters are transferred to the accepting process. The execution of the statements of the accept block by the accepting process (the interval t_2–t_3) is called the execution of the rendezvous. At time t_3, the rendezvous is complete, the results are returned to the calling process and both processes may continue executing. We leave it as an exercise to draw the timing diagram for the case where the accepting task tries to execute the accept statement before the calling task has called its entry.

The rendezvous in Ada^L

The rendezvous is one of the main synchronization constructs in the Ada language. Here we describe the construct using the standard example, the bounded buffer:

```
1  task body Buffer is
2    B: Buffer_Array;
3    In_Ptr, Out_Ptr, Count: Index := 0;
4  begin
5    loop
6      select
7        when Count < Index'Last =>
8          accept Append(I: in Integer ) do
9              B(In_Ptr) := I;
10         end Append;
11       Count := Count + 1; In_Ptr := In_Ptr + 1;
12     or
13       when Count > 0 =>
14         accept Take(I: out Integer ) do
15             I := B(Out_Ptr);
16         end Take;
17       Count := Count - 1; Out_Ptr := Out_Ptr + 1;
18     or
19         terminate;
20     end select;
21   end loop;
22 end Buffer;
```

The select statement enables the buffer to choose nondeterministically between two *guarded* alternatives. The guards are boolean expressions prefixed the accept statements. If the expression evaluates to true, the alternative an *open* alternative and a rendezvous with the accept statement is permitted. If the expression evaluates to false, the alternative is *closed* and rendezvous is not permitted.

In the example, if the buffer is empty ($Count = 0$), the only open alternative is the Append entry, while if the buffer is full ($Count = N$), the only open alternative is Take. It is required that there always be at least one open alternative; in the example this requirement holds because for a buffer of positive length it is impossible that $Count = 0$ and $Count = N$ simultaneously.

If $0 < Count < N$, both alternatives are open. If there are calling tasks waiting on both entries, the accepting task will choose arbitrarily between the entries and commence a rendezvous with the first task on the queue associated with the chosen entry. If only one entry queue is nonempty, the rendezvous will be with the first calling task on that queue. If both queues are empty, the accepting task will wait for the first task that calls an entry.

An Ada programmer would normally prefer to use a protected object rather than a task with rendezvous to implement a bounded buffer, because protected objects are passive and their statements are executed by the producer and consumer tasks that exist anyway. Here, there is an extra task and thus the extra overhead of context switches. This design would be appropriate if, for example, the task had complex processing to perform on the buffer such as writing parts of it to a disk.

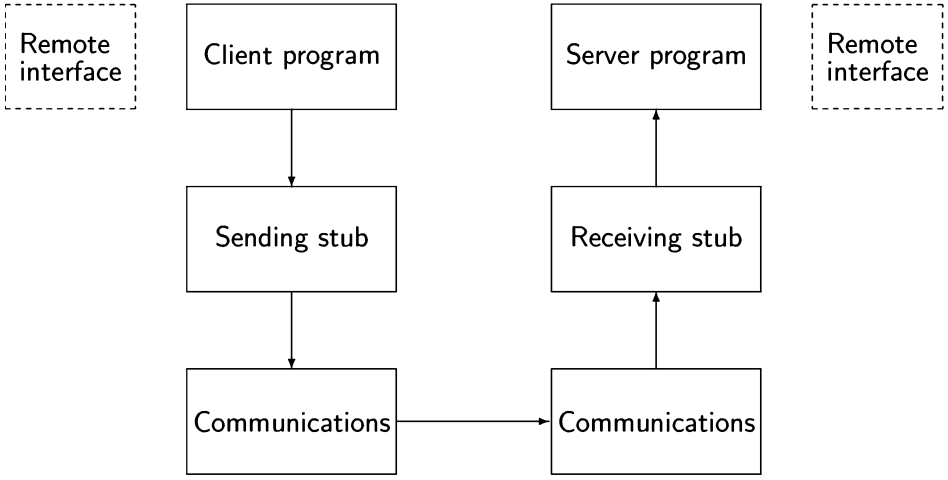
What happens to the buffer task if all producer and consumer tasks that could potentially call it terminate? The answer is that the program would deadlock with the select statement indefinitely waiting for an entry call. The terminate alternative (line 19) enables graceful shutdown of systems containing server tasks of this form. See [7] or [18] for details of this feature, as well as of other features that enable conditional rendezvous and timeouts.

8.7 Remote procedure calls^A

Remote procedure call (RPC) is a construct that enables a client to request a service from a server that may be located on a different processor. The client calls a server in a manner no different from an ordinary procedure call; then, a process is created to handle the invocation. The process may be created on the same processor or on another one, but this is transparent to the client which invokes the procedure and waits for it to return. RPC is different from a rendezvous because the latter involves the active participation of two processes in synchronous communications.

RPC is supported in Ada by the constructs in the *Distributed Systems Annex* and in Java by the *Remote Method Invocation (RMI)* library. Alternatively, a system can implement a language-independent specification called *Common Object Request Broker Architecture (CORBA)*. Writing software for distributed systems using RPC is not too difficult, but it is language-specific and quite delicate. We will just give the underlying concepts here and refer you to language textbooks for the details.

To implement RPC, both the client and the server processes must be compiled with a *remote interface* containing common type and procedure declarations; the client process will invoke the procedures that are implemented in the server process. In Java, a remote interface is created by extending the library interface java.rmi.Remote. In Ada, packages are declared with **pragma** Remote_Types and **pragma** Remote_Call_Interface. The following diagram shows what happens when the client calls a procedure that is implemented in the server:



Since the procedure does not actually exist in the client process, the call is processed by a *stub*. The stub *marshals* the parameters, that is, it transforms the parameters from their internal format into a sequence of data elements that can be sent through the communications channel. When the remote call is received by the server, it is sent to another stub which *unmarshals* the parameters, transforming them back into the internal format expected by the language. Then it performs the call to the server program on behalf of the client. If there are return values, similar processing is performed: the values are marshaled, sent back to the client and unmarshaled.

To execute a program using RPC, processes must be assigned to processors, and the clients must be able to locate the services offered by the servers. In Java this is performed by a system called a *registry*; servers call the registry to bind the services they offer and the clients call the registry to lookup the services they need. In Ada, a configuration tool is used to allocate programs to *partitions* on the available processors. Again, the details are somewhat complex and we refer you to the language documentation.

Transition

Channels enable us to construct decentralized concurrent programs that do not necessarily share the same address space. Synchronous communication, where the sender and receiver wait for each other, is the basic form of synchronization, as it does not require design decisions about buffering. More complex forms of communication, the rendezvous in Ada and the remote procedure call implemented in

many systems, are used for higher-level synchronization, and are especially suited to client–server architectures.

What is common to all the synchronization constructs studied so far is that they envision a set of processes executing more or less simultaneously, so it makes sense to talk about one process blocking while waiting for the execution of a statement in another process. The Linda model for concurrency discussed in the next chapter enables highly flexible programs to be written by replacing process-based synchronization with data-based synchronization.

Exercises

1. Develop an algorithm for pipeline sort. There are n processes and n numbers are fed into the input channel of the first process. When the program terminates, the numbers are stored in ascending order in the processes:

2. Develop a solution for the dining philosophers problem under the restriction that a channel must be connected to exactly one sender and one receiver.

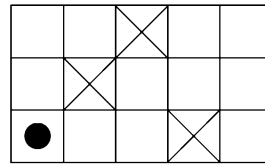
3. Develop an algorithm to merge two sequences of data. A process receives data on two input channels and interleaves the data on one output channel. Try to implement a *fair merge* that is free from starvation of both input channels.

4. Develop an algorithm to simulate a digital logic circuit. Each gate in the circuit will be represented by a process and wires between the gates will be represented by channels. You need to decide how to handle fan-out, when a single wire leading from one gate is connected to several other gates.

5. (Hamming's problem) Develop an algorithm whose output is the sequence of all multiples of 2, 3 and 5 in ascending order. The first elements of the sequence are: $0, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15$. There will be four processes: one to multiply by each of the three factors and a fourth process to merge the results.

6. (Hoare [32]) Let $x_1, x_2, \ldots$ and $y_1, y_2, \ldots$ be two sequences of numbers. Develop an algorithm to compute the sequence $2x_1 + 3y_1, 2x_2 + 3y_2, \ldots$. The multiplications must be performed in parallel. Modify the program to compute $2x_1 + 3x_1, 2x_2 + 3x_2, \ldots$ by splitting the input sequence $x_1, x_2, \ldots$ into two identical sequences.

7. (Hoare [32]) Develop an algorithm to simulate the following game. A counter is placed on the lower lefthand square of a rectangular board, some of whose squares are blocked. The counter is required to move to the upper righthand square of the board by a sequence of moves either upward or to the right:



8. Draw a timing diagram similar to the one in Section 8.6 for the case where the accepting task in a rendezvous tries to execute the accept statement before the calling task has called its entry.

9. Suppose that an exception (runtime error) occurs during the execution of the server (accepting) process in a rendezvous. What is a reasonable action to take and why?

10. Compare the monitor solution of the producer–consumer problem in Java given in Section 7.11, with the rendezvous solution in Ada given in Section 8.6. In which solution is there a greater potential for parallel execution?

9 Spaces

Synchronization primitives that are based upon shared memory or synchronous communications have the disadvantage of tight *coupling*, both in time and in space. To communicate using synchronous channels, both the sending and the receiving processes must exist simultaneously, and the identity of the channel must be accessible to both. The Linda model decouples the processes involved in synchronization. It also enables shared data to be *persistent*, that is, data can exist after the termination of the process that created them and used by processes that are activated only later. The properties of loose coupling and persistence exist in the file systems of operating systems, but Linda provides an abstract model that can be implemented in various ways.

Sections 9.1–9.4 present the Linda model. The model is particularly appropriate for the master–worker paradigm, described in Section 9.5. The chapter concludes with a discussion of implementations of the Linda model.

9.1 The Linda model

The model defines a global data structure, for which will we use the metaphor of a giant bulletin board called a *space* on which *notes* can be posted.[1] A note is a sequence of typed elements; for example, ('a',27,false) is a note whose three elements are of types character, integer and boolean, respectively. The first element of a note is a character literal that conveys the intended meaning of a note.

The atomic statements of Linda are:

postnote(v1, v2, ...) This statement creates a note from the values of the parameters and posts it in the space. If there are processes blocked waiting for a note matching this parameter signature, an arbitrary one of them is unblocked.

[1]This text will use terminology consistent with that used in the jBACI concurrency simulator. The original terminology of Linda is described in Section 9.5.

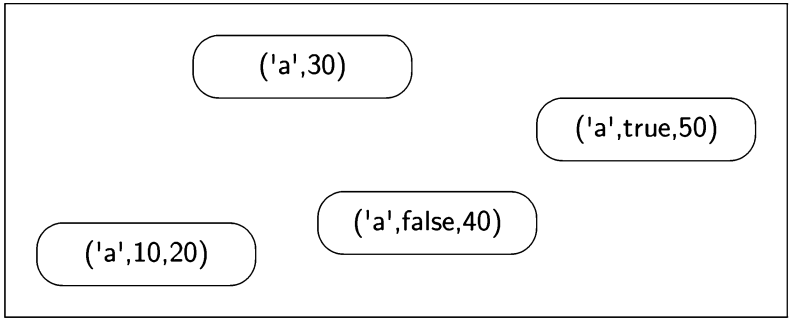
removenote(x1, x2, ...) The parameters must be variables. The statement removes a note that matches the parameter signature from the space and assigns its values to the parameters. If no matching note exists, the process is blocked. If there is more than one matching note, an arbitrary one is removed.

readnote(x1, x2, ...) Like removenote, but it leaves the note in the space.

Suppose that the following statements are executed:

```
postnote('a', 10, 20);
postnote('a', 30);
postnote('a', false , 40);
postnote('a', true, 50);
```

The content of the space is shown in the following diagram:



We must define what it means for the parameter signature of an operation and a node to *match*. The two are defined to match if the following two conditions hold: (a) The sequence of types of the parameters equals the sequence of types of the note elements. (This implies, of course, that the number of parameters is the same as the number of elements of a note.) (b) If a parameter is not a variable, its value must equal the value of the element of the note in the same position. For the space shown in the diagram above, let us trace the result of executing the following statements:

```
    integer m, integer n, boolean b
(1) removenote('b', m, n)
(2) removenote('a', m, n)
(3) readnote('a', m)
(4) removenote('a', n)
(5) removenote('a', m, n)
(6) removenote('a', b, n)
(7) removenote('a', b, m)
(8) postnote('b', 60, 70)
```

The process executing statement (1) blocks, because there is no note in the space matching the first parameter of the statement. Statement (2) matches the note ('a',10,20) and removes it from the space; the value of the second element 10 is assigned to the variable m and 20 is assigned to n. Statement (3) matches, but does not remove, the note ('a',30), assigning 30 to m. Since the note remains in the space it also matches statement (4); the note is removed and 30 is assigned to n. Statement (6) matches both ('a',false,40) and ('a',true,50); one of them is removed, assigning either *false* to b and 40 to n or *true* to b and 50 to n. The other note remains in the space and is removed during the execution of statement (7). Finally, statement (8) posts a note that matches the blocked call in statement (1). The process that executed (1) is unblocked and the note is removed from the space, assigning the values 60 to m and 70 to n.

9.2 Expressiveness of the Linda model

The Linda model combines both data and synchronization in its statements, making it more expressive than semaphores. To implement a general semaphore in Linda, initialize the space by posting a number of notes equal to the initial value of the semaphore:

```
do K times
  postnote('s')
```

removenote is now equivalent to wait and postnote to signal, as can be seen in the following algorithm for the critical section problem:

Algorithm 9.1: Critical section problem in Linda

```
    loop forever
p1:     non-critical section
p2:     removenote('s')
p3:     critical section
p4:     postnote('s')
```

The Linda model only specifies the concepts of spaces, notes and the atomic statements, so there is not much point in asking how to simulate a monitor. The encapsulation aspects of the monitor would have to be supplied by the language in which Linda is embedded.

To simulate sending a value on a channel c, simply post a note with the channel name and the value—postnote('c',value); to simulate receiving the value, remove the note using the same channel name and a variable—removenote('c',var).

What may not be immediately apparent from these examples is the loose coupling and persistence of the system. One process could initialize the system with a few notes and terminate, and only later could another process be initiated and executed to remove the notes and to use the values in the notes in its computation.

9.3 Formal parameters

To motivate the need for formal parameters, let us consider the outline of a client–server algorithm in Linda:

Algorithm 9.2: Client–server algorithm in Linda

client	server
constant integer me ← ...	integer client
serviceType service	serviceType s
dataType result, parm	dataType r, p
p1: service ← // Service requested	q1: removenote('S', client, s, p)
p2: postnote('S', me, service, parm)	q2: r ← do (s, p)
p3: removenote('R', me, result)	q3: postnote('R', client, r)

The client process posts a note 'S' containing its ID, the service requested, and the parameters for the service; then it waits to remove a result note 'R' matching its ID and containing the result. The server process waits to remove an 'S' note, performs the service and then posts the result in an 'R' note. It saves the ID of the client to

ensure that the result returns to the client that requested the service. This algorithm works well as long as all servers are able to perform all the services that will be requested. More often, however, different servers provide different services.

The Linda statements described so far are somewhat limited in the way they match parameter signatures with notes; a parameter whose value is a constant expression like a literal matches only that value, while a variable parameter matches *any value* of the same type. More flexibility is obtained with a *formal parameter*, which is a variable parameter that only matches notes containing the *current value* of the variable. We will denote this by appending the symbol = to the variable name.

Suppose that a server can provide one service only; Algorithm 9.2 can be changed so that a server removes only notes requesting that service:

Algorithm 9.3: Specific service

	client		server
	constant integer me ← ...		integer client
	serviceType service		serviceType s
	dataType result, parm		dataType r, p
p1:	service ← // Service requested	q1:	s ← // Service provided
p2:	postnote('S', me, service, parm)	q2:	removenote('S', client, s=, p)
p3:		q3:	r ← do (s, p)
p4:	removenote('R', me, result)	q4:	postnote('R', client, r)

In the server process, the variable s is specified as a formal parameter so removenote will only match notes whose third element is equal to the current value of the variable.

Formal parameters can be used to implement an infinite buffer:

Algorithm 9.4: Buffering in a space

	producer		consumer
	integer count ← 0		integer count ← 0
	dataType d		dataType d
	loop forever		loop forever
p1:	d ← produce	q1:	removenote('B', count=, d)
p2:	postnote('B', count, d)	q2:	consume(d)
p3:	count ← count + 1	q3:	count ← count + 1

The formal parameter in the consumer ensures that it consumes the values in the order they are produced. We leave it as an exercise to implement a bounded buffer in Linda.

Here is a simulation of the matrix multiplication algorithm with channels:

Algorithm 9.5: Multiplier process with channels in Linda	
	integer FirstElement integer North, East, South, West integer Sum, integer SecondElement
	loop forever
p1:	removenote('E', North=, SecondElement)
p2:	removenote('S', East=, Sum)
p3:	Sum ← Sum + FirstElement · SecondElement
p4:	postnote('E', South, SecondElement)
p5:	postnote('S', West, Sum)

Notes identified as 'E' contain the *E*lements passed from north to south, while notes identified as 'S' contain the partial *S*ums passed from east to west.

9.4 The master–worker paradigm

The channel algorithm for matrix multiplication has a processor structure that is very rigid, with exactly one processor assigned to each matrix element. It makes no sense to remove one processor for repair or to replace a few processors by faster ones. In Linda, we can write a program that is flexible so that it adapts itself to the amount of processing power available. This is called *load balancing* because we ensure that each processor performs as much computation as possible, regardless of its speed. One process called a *master* posts task notes in the space; other processes called *workers* take task notes from the space and perform the required computation. The master–worker paradigm is very useful in concurrent programming, and is particularly easy to implement with spaces because of their freedom from temporal and spatial constraints.

We demonstrate the master–worker paradigm with the matrix multiplication problem. The master process first initializes the space with the n row vectors and the n column vectors:

```
postnote('A', 1, (1,2,3))
postnote('A', 2, (4,5,6))
postnote('A', 3, (7,8,9))
postnote('B', 1, (1,0,1))
postnote('B', 2, (0,1,0))
postnote('B', 3, (2,2,0))
```

Then it posts n^2 notes of the form ('T', i, j), one for each task. Finally, it waits for the n^2 result notes of the form ('R', i, j, result). A worker process removes a task note, performs the (vector) multiplication and posts a note with the result:

Algorithm 9.6: Matrix multiplication in Linda

constant integer n ← ...

master	worker
integer i, j, result	integer r, c, result
integer r, c	integer array[1..n] vec1, vec2
	loop forever
p1: for i from 1 to n	q1: removenote('T', r, c)
p2: for j from 1 to n	q2: readnote('A', r=, vec1)
p3: postnote('T', i, j)	q3: readnote('B', c=, vec2)
p4: for i from 1 to n	q4: result ← vec1 · vec2
p5: for j from 1 to n	q5: postnote('R', r, c, result)
p6: removenote('R',r,c,result)	q6:
p7: print r, c, result	q7:

Note the use of variable parameters r= and c= in the worker processes to ensure that the correct vectors are read. In the master process, statement p6 uses variables r and c so that the loop indices i and j are not overwritten; the notes with the results are removed in an arbitrary order as they are posted.

The code of the algorithm is totally independent of the number of worker processes. As long as a process successfully removes a task note, it contributes to the ongoing computation. Nor is the algorithm sensitive to the relative speeds at which the worker processes are executed. A worker executed by a fast processor will simply complete more tasks during a time period than a worker executed by a slow processor. Furthermore, computation can be dynamically speeded up if the computer architecture enables the addition of processors during the execution of the program; the new processors can begin executing the algorithm for the worker process on tasks that are still posted in the space. Conversely, if a specific processor needs to be removed from the system, the worker process it is executing can be stopped at any time after completing the body of the loop.

Granularity

The master–worker paradigm is quite flexible because we can specify the *granularity* of the task to suit the relative performances of the processors and the communications system. The above algorithm uses a very small granularity where one

processor is responsible for computing one result at a time, so the communications overhead is relatively high.

It is easy to modify the algorithm so that it works at any level of granularity. We have added a constant chunk and posted task notes only every chunk'th column. (Assume that n is divisible by chunk.) For example, if n is 100 and *chunk* is 10, then the task notes posted will be: ('T',1,1), ('T',1,11), ..., ('T',1,91), ('T',2,1), ..., ('T',2,91), ..., ('T',100,1), ..., ('T',100,91). The following algorithm shows the changes that must be made in the worker processes:

Algorithm 9.7: Matrix multiplication in Linda with granularity

```
constant integer n ← ...
constant integer chunk ← ...
```

master	worker
integer i, j, result	integer r, c, k, result
integer r, c	integer array[1..n] vec1, vec2
	loop forever
p1: for i from 1 to n	q1: removenote('T', r, k)
p2: for j from 1 to n step by chunk	q2: readnote('A', r=, vec1)
p3: postnote('T', i, j)	q3: for c from k to k+chunk-1
p4: for i from 1 to n	q4: readnote('B', c=, vec2)
p5: for j from 1 to n	q5: result ← vec1 · vec2
p6: removenote('R',r,c,result)	q6: postnote('R', r, c, result)
p7: print r, c, result	q7:

The row vector is read (once per chunk) and then a loop internal to the process reads the column vectors, performs the multiplications and posts the results.

9.5 Implementations of spaces^L

The Linda model has been implemented in both research and commercial settings. For students of concurrency, it is easier to work with implementations that are embedded within one of the languages used in this book. The presentation of the model in this chapter is consistent with its implementation in both the C and Pascal dialects of the jBACI concurrency simulator, and with a package written in Ada. A tuple can contain at most three elements, of which the first is a character, and the other two are integer expressions or variables.

In this section, we briefly describe two commercial systems, C-Linda and JavaSpaces, and then discuss implementations of the model in Java and Promela.

C-Linda

The first implementation of Linda embedded the model in the C language. The resulting system, which has evolved into a commercial product, is called C-Linda. We will briefly review the original presentation of Linda so that you will be able to read the literature on this topic.

A note is called a *tuple* and a space is called a *tuple space*. The names of the statements are out for postnote, in for removenote and rd for readnote. There is an additional statement, eval, that is used primarily for activating processes. There are also non-blocking versions of in and rd, which allow a process to remove or read a tuple and to continue executing if a matching tuple does not exist in the tuple space.

There is no limit on the number or types of parameters; by convention, the first parameter is a string giving the type of the tuple. The notation for formal parameters is ?var.

JavaSpaces

In JavaSpaces, notes are called *entries*. Each type of note is declared in a separate class that implements the Entry interface. For the task notes of the matrix multiplication example, this would be:

```
public class Task implements Entry {
    public Integer row;
    public Integer col;
    public Task() {
    }
    public Task(int r, int c) {
        row = new Integer(r);
        col = new Integer(c);
    }
}
```

For technical reasons, an entry must have a public constructor with no arguments, even if it has other constructors. All fields must be public so that the JavaSpaces system can access them when comparing entries.

JavaSpaces implements write, take and read operations on the space (or spaces—there may be more than one space in a system). An operation is given a template entry that must match an entry in the space. **null** fields can match any value.

Since an entry is an object of a class, the class may have methods declared within it. A remote processor reading or taking an entry from a space can invoke these methods on the object.

JavaSpaces is integrated into Sun's Jini Network Technology, and it provides features that were not part of the original Linda model. *Leases* enable you to specify a period of time that an entry is to remain in the space; when the time expires, the entry is removed by the space itself. *Distributed events* enable you to specify listener methods that are called when certain events occur in a space. *Transactions* are used to make a system more fault tolerant by grouping space operations so that they either all execute to completion or they leave the space unchanged.

Java

The software archive contains an implementation of the Linda primitives in Java that demonstrates how Linda can be embedded into object-oriented programming.

Here is the outline of the definition of a note:

```
1  public class Note {
2      public String id;
3      public Object[] p;
4
5      // Constructor for an array of objects
6      public Note (String id, Object[] p) {
7          this.id = id;
8          if (p != null) this.p = p.clone();
9      }
10     // Constructor for a single integer
11     public Note (String id, int p1) {
12         this(id, new Object[]{new Integer(p1)});
13     }
14
15     // Accessor for a single integer value
16     public int get(int i) {
17         return ((Integer)p[i]).intValue();
18     }
19 }
```

A note consists of a String value for its type, together with an array of elements of type Object. To make it easy to use in simple examples, constructors taking parameters of type **int** are included; these must be placed within objects of the

wrapper class Integer in order to store them as elements of type Object. For convenience, the method get retrieves a value of type **int** from the wrapper.

The class Space is not shown here. It is implemented as a Java Vector into which notes may be added and removed. The method that searches for a matching note for removenote or readnote checks that the id fields are equal. A match is then declared if either of the arrays of elements of the two notes is **null**. Otherwise, after ensuring that the arrays are of the same length, the elements are matched one by one with a **null** element matching any value. If a matching note is not found, the thread executes wait; any call to postnote executes **notifyAll** and the blocked threads perform the search again in case the new note matches.

For the matrix multiplication example, the class Worker is declared as a private inner class derived from Thread:

```
1  private class Worker extends Thread {
2    public void run() {
3      Note task = new Note("task");
4      while (true) {
5        Note t = space.removenote(task);
6        int row = t.get(0);
7        int col = t.get(1);
8        Note r = space.readnote(match("a", row));
9        Note c = space.readnote(match("b", col));
10       int sum = 0;
11       for (int i = 1; i <= SIZE; i++)
12         sum = sum + r.get(i)*c.get(i );
13         space. postnote(new Note("result",row,col , sum));
14     }
15   }
16 }
```

The code is straightforward except for the calls to the method match. The call match("a",row) creates a note

```
{ "a", new Integer(row), null, null, null }
```

that will match precisely the "a" note whose first element is the value of row. The entire note is then returned by the method readnote.

```
chan space = [25] of {byte, short, short, short, short};

active[WORKERS] proctype Worker() {
    short row, col, sum, r1, r2, r3, c1, c2, c3;
    do
    :: space ?? 't', row, col, _, _;
       space ?? <'a', eval(row), r1, r2, r3>;
       space ?? <'b', eval(col), c1, c2, c3>;
       sum = r1*c1 + r2*c2 + r3*c3;
       space ! 'r', row, col, sum, 0;
    od;
}
```

Listing 9.1: A Promela program for matrix multiplication

Promela

It is interesting to look at a Linda-like program for matrix multiplication (Listing 9.1), if only because it shows some of the features of programming with channels in Promela. The notes are stored in a space implemented as a channel. Normally, the send operation ch ! values and receive operation ch ? variables treat a channel as a FIFO data structure; the send operation places the data on the tail of the queue, while the receive operation removes the data from the head of the queue. This is reasonable since most communications channels are in fact FIFO structures. Here we use the receive operation ch ?? message which removes a message from the channel only if it matches the template given in the operation. Therefore, if the message template contains some constants, only a message with those values will be removed by the receive operation.

In the program, each message in the channel consists of five fields, a byte field and four short integers. The first receive statement is:

```
space ?? 't', row, col, _, _
```

Since the first element is the constant 't', the operation will remove from the channel only those messages whose first element is 't'; these are the notes containing the tasks to be performed. The other fields are variables so they will be filled in with the values that are in the message. In this case, only the first two fields are of interest; the other two are thrown away by reading them into the anonymous variable _.

The next two operations read the row and column vectors, where notes with 'a' are the row vectors and notes with 'b' are the column vectors:

```
space ?? <'a', eval(row), r1, r2, r3>;
```

The word **eval** is used to indicate that we don't want row to be considered as a variable, but rather as a constant whose value is the current value of the variable. This ensures that the correct row is read. The angled brackets < > are used to indicate that the values of the messages should be read, while the message itself remains on the channel. Of course, this is exactly the functionality needed for readnote. After computing the inner product of the vectors, the result note is posted to the space by sending it to the channel.

The obstacles to implementing Linda programs in Promela include the lack of true data structuring and even more so the restriction that a channel can hold only one data type.

Transition

The constructs of the Linda model are simple, yet they enable the construction of elegant and flexible programs. Even so, implementations of the model are efficient and are used in distributed systems, especially in those attempting to achieve shorter execution times by dividing large computations into many smaller tasks.

With this chapter we leave the presentation of different models of concurrency and focus on algorithms appropriate for distributed systems that communicate by sending and receiving messages.

Exercises

1. Implement a general semaphore in Linda using just one note.
2. Implement an array in Linda.
3. Suppose that the space contains a large number of notes of the form ('v',n). Develop an algorithm in Linda that prints out the maximum value of n in the space.
4. Implement a bounded buffer in Linda.
5. Modify Algorithm 9.6 so that the worker processes terminate.

6. Compare the following algorithm for matrix multiplication with Algorithm 9.6:

Algorithm 9.8: Matrix multiplication in Linda (exercise)	
constant integer n ← ...	
master	**worker**
integer i, j, result	integer i, r, c, result
integer r, c	integer array[1..n] vec1, vec2
	loop forever
p1: postnote('T', 0)	q1: removenote('T' i)
p2:	q2: if i < (n · n) − 1
p3:	q3: postnote('T', i+1)
p4:	q4: r ← (i / n) + 1
p5:	q5: c ← (i modulo n) + 1
p6: for i from 1 to n	q6: readnote('A', r=, vec1)
p7: for j from 1 to n	q7: readnote('B', c=, vec2)
p8: removenote('R',r,c,result)	q8: result ← vec1 · vec2
p9: print r, c, result	q9: postnote('R', r, c, result)

7. Suppose that one of the worker processes in Algorithm 9.6 or Algorithm 9.8 terminates prematurely. How does that affect the computation?

8. Introduce additional parallelism into the Linda algorithm for matrix multiplication by dividing the computation of the inner products result ← vec1 · vec2 into several tasks.

9. An $n \times n$ *sparse matrix* is one in which the number of nonzero elements is significantly smaller than n^2. Develop an algorithm in Linda for multiplication of sparse matrices.

10. Write the initializing postnote statements so that Algorithm 9.5 will compute the matrix multiplication on the example in Figure 8.1.

10 Distributed Algorithms

In this chapter and the next two, we present algorithms designed for loosely-connected distributed systems that communicate by sending and receiving messages over a communications network.

The algorithms in this chapter are for the critical section problem. (Traditionally, the problem is called distributed mutual exclusion, although mutual exclusion is just one of the correctness requirements.) The first algorithm is the Ricart–Agrawala algorithm, which is based upon the concept of *permissions*: a node wishing to enter its critical section must obtain permission from each of the other nodes. Algorithms based upon *token-passing* can be more efficient, because permission to enter the critical section resides in a token can that easily be passed from one node to another. We present a second algorithm by Ricart and Agrawala that uses token-passing, followed by a more efficient algorithm by Neilsen and Mizuno.

The variety of models studied in distributed systems is large, because you can independently vary assumptions concerning the topology of the network, and the reliability of the channels and the nodes. Therefore, we start with a section defining our model for distributed systems.

10.1 The distributed systems model

We distinguish between *nodes* and *processes*: A node is intended to represent a physically identifiable object like a computer, while the individual computers may be running multiple processes, either by sharing a single processor or on multiple processors. We assume that the internal synchronization among processes in a node is accomplished using shared-memory primitives, while processes in different nodes can communicate only by sending and receiving messages.

In Chapters 10 and 11, we assume that the nodes do not fail. Actually, a somewhat weaker assumption will suffice: A node may fail in the sense of not performing its "own" computation, as long as it continues to execute the processes that send and receive messages as required by the algorithms in these chapters. In Chapter 12 we will present algorithms that are robust under failure of individual nodes.

Communications channels

There are additional assumptions on the communications channels between the nodes:

- Each node has a two-way channel connecting it with each other node.
- The channels deliver the messages without error, although (except for the CL algorithm in Section 11.4) not necessarily in the order they were sent.
- The transit times of messages are finite but arbitrary.

The design of an algorithm must take into account the topology of the network. The algorithms presented here assume a fully-connected topology. In one sense this is arbitrary because algorithms for other topologies could also be presented, but in another sense it is a reasonable choice, because one node can usually send a message to any other node, although it may have to be relayed through intermediate nodes before reaching its destination.

The assumption that communications are error-free is an abstraction, but again, a reasonable abstraction. There is a large body of knowledge on techniques for ensuring reliable communications in the presence of errors, and we assume that the underlying network implements these techniques. The algorithms studied here are intended for higher-level software built upon the network. Finally, the assumption of finite but arbitrary transit times for messages is consistent with the type of models that we have used before: we do not want our algorithms to be sensitive to changes in the relative speeds of the channels and the processors at the nodes, so correctness will never depend on absolute times.

The algorithm for each node will be parameterized by a unique identification number for the node:

```
constant integer myID ← ...
```

To save space in the algorithms, this declaration will not be explicitly written.

Sending and receiving messages

There are two statements for communications:

send(MessageType, Destination[, Parameters]) MessageType identifies the type of the message which is sent from this node to a Destination node with optional arbitrary Parameters.

receive(MessageType[, Parameters]) A message of type MessageType with optional arbitrary Parameters is received by this node.

Here is an example, where node 5 sends a message of type request to node 3, together with two integer parameters whose values are 20 and 30:

When the message is received at node 3, these values are transferred to the variables m and n declared at that node.

There is an asymmetry between the two statements. In our algorithms, the data (message type and parameters) sent to each destination node will usually depend on the destination node. Therefore, the algorithm of the sending node must pass the destination ID to the underlying communications protocol so that the protocol can send the message to the correct node. Neither the ID of the source node nor the ID of the destination node need be sent within the message; the latter because the destination (receiving) node knows its own ID and does not have to be told what it is by the protocol, and the former because this information is not needed in most algorithms.

If the source node does wish to make its ID known to the destination node, for example to receive a reply, it must include myID as an explicit parameter:

The mechanics of formatting parameters into a form suitable for a communications protocol are abstracted away. A process that executes a receive statement:

```
receive (message, parameters)
```

blocks until a message of the proper type is received; values are copied into the variable parameters and the process is awakened. We have thus abstracted away the processes executing the communications protocol, in particular, the process responsible for identifying the message type and unblocking processes waiting for such messages.

Concurrency within the nodes

The model of distributed computation is intended to represent a network of nodes that are independent computers, synchronizing and communicating by sending and receiving messages. Clearly, the programs running at each node can be composed of concurrent processes that are synchronized using constructs such as semaphores and monitors. In the following chapters, the algorithms given for a node may be divided up into several concurrent processes.

> We assume atomicity of the algorithms at each process within a node. This does not apply to the critical and non-critical sections in a solution of the critical section problem, only to the pre- and postprotocols.

In particular, when a message is received, the handling of the message is considered part of the same atomic statement, and interleaving with other processes of the same node is prevented. The reason for this assumption is that we are interested in analyzing synchronization by message passing *between* nodes, and therefore ignore the synchronization within a node. The latter can be easily solved using constructs such as semaphores that will be familiar by now.

The algorithms in Chapters 11–12 are intended to perform a function for an *underlying computation* that is the real work being done at the node. The algorithms are grafted onto the underlying computations as explained in each case.

Studying distributed algorithms

There are several possibilities available for studying the execution of distributed algorithms. Many modern languages like Ada and Java include libraries for implementing distributed systems; in fact, you can even write distributed programs on a single personal computer by using a technique called *loopback*, where messages sent by the computer are received by the same computer without actually traversing a network. However, these libraries are complex and it takes some effort to learn how to use them, but more importantly, mastering the mechanics of distributed *programming* will tend to obscure the concepts of the distributed *algorithms*. You are likely to encounter many languages and systems over time, but the algorithmic principles will not change.

The real difficulty in understanding distributing algorithms comes from the "bookkeeping": at each step, you must remember the state of each node in the network, its local data and which messages have been sent to and received from the other nodes. To facilitate learning distributed algorithms, I have developed a software

tool called DAJ for constructing scenarios step-by-step (see Appendix D.3 and [8]). At each step the tool displays the data structures at the nodes and optionally prompts for algorithm-specific messages that remain to be sent. Visualizations of virtual global data structures are also provided for some algorithms: the global queue of the Ricart–Agrawala algorithm, the spanning tree of the Dijkstra–Scholten algorithm and the knowledge trees of the Byzantine Generals algorithm.

10.2 Implementations

The model that we have described is appropriate for a network of computers connected by a protocol that enables reliable point-to-point communications. The *Transmission Control Protocol (TCP)* is the most commonly used protocol for these purposes, especially in local area networks and on the Internet. TCP is built upon a lower-level protocol called the *Internet Protocol (IP)* which is used to send packets of data. The assembling of packets into messages, together with recovery from lost or damaged packets, is the responsibility of TCP.

Implementing a distributed system, however, involves more than just sending and receiving messages. The nodes have to learn the network topology which may change dynamically as new nodes are added and as others are removed, perhaps as a result of faults. Processes have to be assigned to nodes, and the location of data must be specified. If the nodes are *heterogeneous*, that is, if they use different computers, operating systems and languages, there will be problems adapting the data in the messages to the nodes. On a heterogeneous network, some nodes may be more appropriate for executing a certain process than others, and the assignment of processes to nodes must take this into account.

The subject of the implementation of distributed systems is beyond the scope of this book. We have already discussed two types of systems. Remote procedure calls, as implemented by the Java Remote Method Invocation and Ada, integrate extremely well with the object-oriented constructs of those languages. Implementations of Linda and JavaSpaces distribute the data—the space—over the nodes of the network, so that programming an application is relatively simple because transferring data from one node to another is automatic.

There are two important and very popular implementations of distributed systems that present the programmer with an abstract view of the underlying network: the *Parallel Virtual Machine (PVM)* [27] and the *Message Passing Interface (MPI)* [29].

PVM is a software system that presents the programmer with a virtual distributed machine. This means that, regardless of the actual network configuration, the pro-

grammer sees a set of nodes and can freely assign processes to nodes. The architecture of the virtual machine can be dynamically changed by any of the nodes in the system, so that PVM can be used to implement fault-tolerant systems. PVM is portable over a wide range of architectures, and furthermore, it is designed for *interoperability*, that is, the same program can run on a node of any type and it can send messages to and receive message from a node of any type.

While PVM is a specific software system that you can download and use, MPI is a *specification* that has been implemented for many computers and systems. It was originally designed to provide a common interface to multiprocessors so that applications programs written to execute on one computer could be easily ported to execute on another.

MPI was designed to emphasize performance by letting each implementation use the most efficient constructs on the target computer, while the designers of PVM emphasized interoperability at the expense of performance (an approach more suited to a network of computers than to a multiprocessor). While there has been some convergence between the two systems, there are significant differences that have to be taken into account when choosing a platform for implementing a distributed system. Detailed comparisons of PVM and MPI can be found in articles posted on the websites of the two systems listed in Appendix E.

10.3 Distributed mutual exclusion

Recall that a process in the critical section problem is structured as an infinite loop consisting of the non-critical section, the preprotocol, the critical section and the postprotocol. As before, we are not concerned with the computation within the non-critical or critical sections; the critical section might consist of the modification of a database accessible from any node within the network.

The algorithm we describe was proposed by Glenn Ricart and Ashok K. Agrawala, and it is based upon ticket numbers as in the bakery algorithm (Section 5.2). Nodes choose ticket numbers and compare them; the lowest number is allowed to enter the critical section, while other numbers must wait until that node has left the critical section. In a distributed system, the numbers cannot be directly compared, so they must be sent in messages.

Initial development of the algorithm

We will develop the algorithm in stages. Algorithm 10.1 is an initial outline. The algorithm is structured as two concurrent processes: a Main process that contains

Algorithm 10.1: Ricart–Agrawala algorithm (outline)

```
                    integer myNum ← 0
                    set of node IDs deferred ← empty set
Main
    loop forever
p1:     non-critical section
p2:     myNum ← chooseNumber
p3:     for all other nodes N
p4:         send(request, N, myID, myNum)
p5:     await reply's from all other nodes
p6:     critical section
p7:     for all nodes N in deferred
p8:         remove N from deferred
p9:         send(reply, N, myID)
Receive
    integer source, requestedNum
    loop forever
p10:    receive(request, source, requestedNum)
p11:    if requestedNum < myNum
p12:        send(reply, source, myID)
p13:    else add source to deferred
```

the familiar elements of the critical section problem, while the second process Receive executes a portion of the algorithm upon receipt of a request message.

The preprotocol begins with the selection of an arbitrary ticket number, which is then sent in a request message to all other processes. The process then waits until it has received reply messages from all these processes, at which point it may enter its critical section.

Before discussing the postprotocol, let us consider what happens in the Receive process. That process receives the request message and compares the requestedNum with the number myNum chosen by the node. (For now we assume that the numbers chosen are distinct.) If requestedNum is less than myNum, this means that the sending node has taken a *lower* ticket number, so the receiving node agrees to let it enter the critical section by sending a reply message to the node that was the source of the request message. The node that has the lowest ticket number will receive replies from all other nodes and enter its critical section.

If, however, requestedNum is greater than myNum, the receiving node has the lower ticket number, so it notifies the sending node *not* to enter the critical section. Cleverly, it does this by simply not sending a reply message! Since a node must

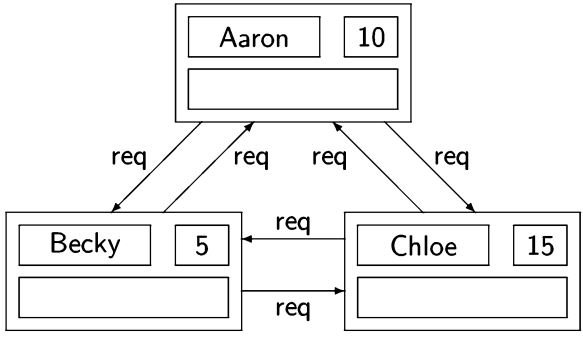
receive replies from *all* other nodes in order to enter its critical section, the absence of a reply is sufficient to prevent it from prematurely entering its critical section.

A set of processes called deferred is maintained. This set contains the IDs of nodes that sent request messages with higher ticket numbers than the number chosen by the node. In the postprotocol that is executed upon completion of the critical section, these nodes are finally sent their reply messages.

The scenario of an example

We now demonstrate a scenario using diagrams to show the states that occur in the scenario. Rather than use numbers, we personify the nodes by giving them names for IDs. Nodes are shown as rectangles containing the data structures: myID, the identity of the node, in the upper left corner; myNum, the number chosen by the node, in the upper right corner; and deferred, the set of deferred nodes, in the lower part of the rectangle. Arrows show the messages sent from one node to another.

Here is the state of the system after all nodes have chosen ticket numbers and sent request messages (abbreviated req) to all other nodes:



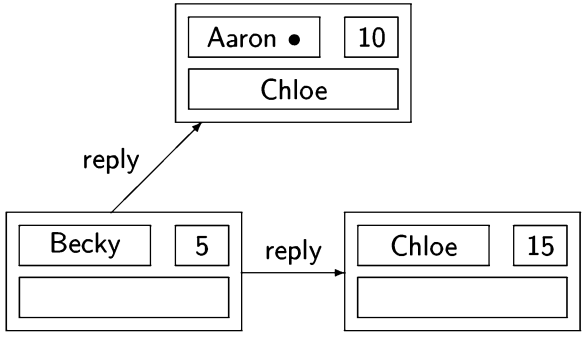
The following diagram shows the result of the execution of the loop body for Receive at each node:

Chloe sends reply messages to both Aaron and Becky, because both have lower numbers than she does. Becky does not send any reply messages, instead adding Aaron and Chloe to her set of deferred nodes, because both processes have higher numbers than she does. Aaron's number is in between the other two, so he sends a reply to Becky while appending Chloe to his deferred set.

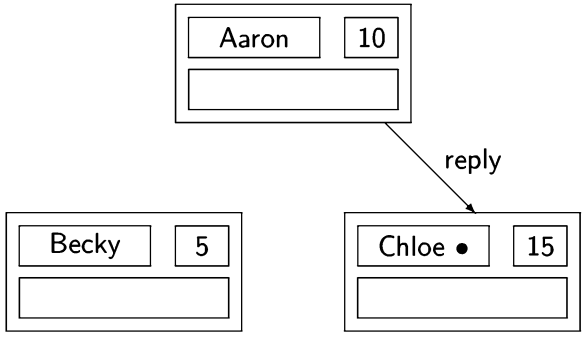
At this point, the three nodes have constructed a *virtual queue*:

The queue is virtual because it does not actually exist as a data structure in any node, but the effect of the messages is to order the nodes as if in a queue.

Becky can now execute her critical section, denoted by the symbol • next to her name. When she completes her critical section, Becky sends the two deferred reply messages to Aaron and Chloe:



Aaron has now received both replies and can enter his critical section. Upon completion, he sends a reply message to Chloe who can now enter her critical section:



There are many more possible scenarios even with this choice of ticket numbers. In our scenario, all request messages were sent before any reply messages, but it is also possible that a node immediately replies to a received request message.

Equal ticket numbers

Since the system is distributed it is impossible to coordinate the choice of ticket numbers, so several processes can choose the same number. In a truly symmetric algorithm where all nodes execute exactly the same program, there is no way to break ties, but here we can use the symmetry-violating assumption that each process has a distinct ID. The comparison of numbers will use the IDs to break ties, by replacing line p11 in the Receive process by:

```
if    (requestedNum < myNum) or
    ( (requestedNum = myNum) and (source < myID) )
```

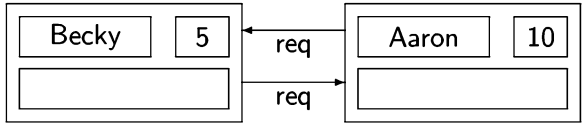
As with the bakery algorithm (Section 5.2), it is convenient to introduce a new notation for this comparison:

```
if  requestedNum ≪ myNum
```

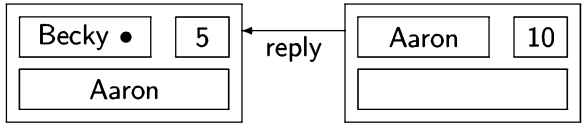
Choosing ticket numbers

We have not specified the manner in which ticket numbers are chosen, but have left it as an arbitrary function chooseNumber. If the choice is in fact arbitrary, it is not hard to find a scenario even with just two nodes that leads to a violation of mutual exclusion.

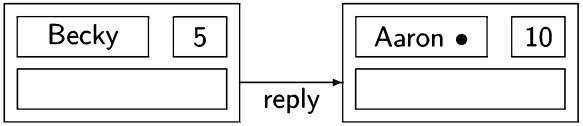
Let Aaron and Becky choose the same ticket numbers as before and send requests to each other:



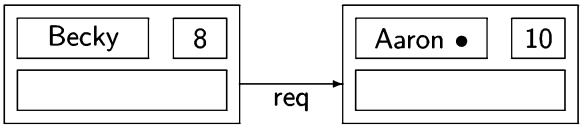
Aaron sends a reply enabling Becky to enter her critical section:



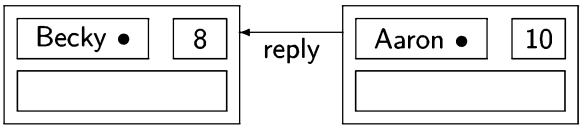
Eventually, Becky completes her critical section and sends a reply to the deferred node Aaron:



Suppose now that Becky quickly re-executes her loop body, choosing a new ticket number 8 and sending the request to Aaron:



Aaron will compare ticket numbers and send a reply to Becky, enabling her to enter her critical section before Aaron leaves his:



To prevent this violation of mutual exclusion, we must ensure (as in a real bakery with physical tickets) that the numbers chosen are monotonic, in the sense that a node will choose a number that is higher than all other ticket numbers *it knows about*. To each node we add a variable highestNum, which stores the highest number received in any request message so far. p2: myNum ← chooseNumber is implemented to assign a new ticket number that is larger than highestNum:

```
myNum ← highestNum + 1
```

In process Receive the following statement is inserted after p10: receive(...):

```
highestNum ← max(highestNum, requestedNum)
```

Quiescent nodes

There is a problem associated with the possibility that a node is not required by the specification of the critical section problem to actually attempt to enter its critical section. In our model of a distributed system, all that we require is that the Receive

Algorithm 10.2: Ricart–Agrawala algorithm

```
                    integer myNum ← 0
                    set of node IDs deferred ← empty set
                    integer highestNum ← 0
                    boolean requestCS ← false
Main
    loop forever
p1:     non-critical section
p2:     requestCS ← true
p3:     myNum ← highestNum + 1
p4:     for all other nodes N
p5:         send(request, N, myID, myNum)
p6:     await reply's from all other nodes
p7:     critical section
p8:     requestCS ← false
p9:     for all nodes N in deferred
p10:        remove N from deferred
p11:        send(reply, N, myID)
Receive
    integer source, requestedNum
    loop forever
p12:    receive(request, source, requestedNum)
p13:    highestNum ← max(highestNum, requestedNum)
p14:    if not requestCS or requestedNum ≪ myNum
p15:        send(reply, source, myID)
p16:    else add source to deferred
```

process continues to receive requests and send replies even if the Main process has terminated in its non-critical section.

However, Receive will only send a reply if myNum is greater than requestedNum. Initially, myNum is zero so the node will *not* send a reply. Similarly, if the node remains inactive for a long period while other nodes attempt to enter their critical sections, the selection of ever-increasing ticket numbers will cause them to become larger than the current value of myNum at the quiescent node.

To solve this problem, we add an additional flag requestCS which the Main process sets before choosing a ticket number and resets upon exit from its critical section. If this flag is not set, the Receive process will immediately send a reply; otherwise, it will compare ticket numbers to decide if a reply should be sent or deferred.

The complete RA algorithm is shown as Algorithm 10.2.

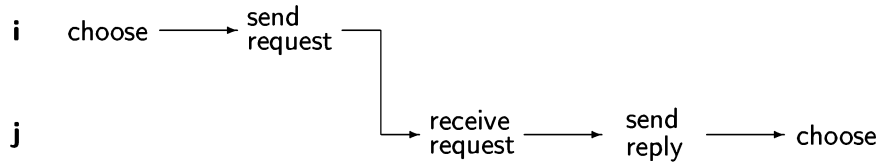
10.4 Correctness of the Ricart–Agrawala algorithm

The RA algorithm satisfies the mutual exclusion property, does not deadlock and does not starve individual nodes.

Theorem 10.1 Mutual exclusion holds in the Ricart–Agrawala algorithm.

Proof: To prove mutual exclusion, suppose to the contrary that two nodes i and j are in the critical section. By the structure of the algorithm, there must have been a last time when each node chose a ticket number, $myNum_i$ and $myNum_j$, respectively. There are three cases to consider.

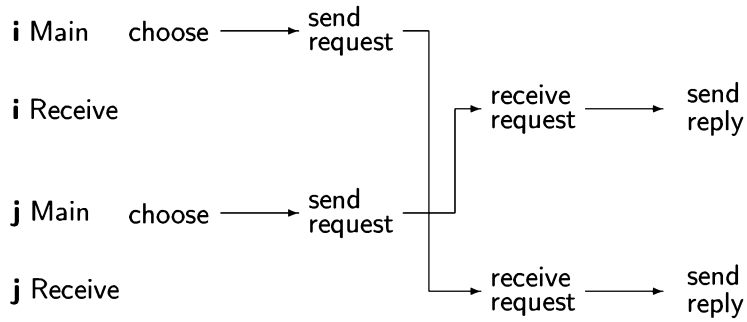
Case 1: Node j chose $myNum_j$ *after* it sent its reply to node i. The arrows in the following diagram show the precedence relationships between the statements of the two processes.



The horizontal arrows follow either from the sequential nature of the processes at the nodes or from the assumption of this case. The precedence represented by the vertical arrow from send request in node i to receive request in node j follows because a message cannot be received until after it is sent. Node j sends the reply only after updating (in Receive) highestNum to a value greater than or equal to $myNum_i$, and therefore it will choose a number $myNum_j$ greater than $myNum_i$. When node i receives this request it will not send a reply as long as it is in its critical section. We conclude that in this case both nodes cannot be in their critical sections at the same time.

Case 2: Node i chose $myNum_i$ *after* it sent its reply to node j. This case is symmetric with Case1.

Case 3: Nodes i and j chose their ticket numbers $myNum_i$ and $myNum_j$, respectively, *before* sending reply messages to each other. The diagram for this case is somewhat more complicated, because we have to take into account the two concurrent processes at each node, shown one under another. No relative timing information should be read from the diagram, only the precedence information derived from the structure of the algorithm and the fact that a message is received after it is sent.



Consider now what happens when the nodes i and j decide to reply. For the entire period between choosing ticket numbers and making this decision, they both want to enter the critical section so that $requestCS = true$, and the ticket numbers that are being compared are the same ticket numbers that they have chosen. So i will reply if and only if $myNum_j \ll myNum_i$ and j will reply if and only if $myNum_i \ll myNum_j$. By the definition of $\ll$ both expressions cannot be true, so one of these expressions will evaluate to false, causing that node to defer its reply. We conclude that in this case too both nodes cannot be in their critical sections at the same time. ∎

Theorem 10.2 The Ricart–Agrawala algorithm is free from starvation and therefore free from deadlock.

Proof: Suppose that node i requests entry to the critical section by setting requestCS, choosing a ticket number $myNum_i$ and sending request messages to all other nodes. Can node i be blocked forever waiting for reply messages at p6? At some point in time t, these request messages will have arrived at all the other nodes, and their variables highestNum will be set to values greater than or equal to $myNum_i$. So from t onwards, *any* process attempting to enter its critical section will choose a ticket number higher than $myNum_i$.

Let $aheadOf(i)$ be the set of nodes that at time t are requesting to enter the critical section and have chosen a ticket numbers lower than $myNum_i$. We have shown that processes can only leave $aheadOf(i)$, never join it. If we can show that eventually some node will leave $aheadOf(i)$, it follows by (numerical) induction that the set is eventually empty, enabling node i to enter its critical section.

Since the ticket numbers form a monotonic sequence under $\ll$, there must be a node k in $aheadOf(i)$ with a minimal number. By the assumptions on sending and receiving messages, the request messages from node k must eventually be sent and received; since k has the minimal ticket number, all other nodes will send reply messages that are eventually received. Therefore, node k will eventually enter and exit its critical section, and leave the set $aheadOf(i)$. ∎

10.5 The RA algorithm in Promela^L

Promela is well-suited for modeling distributed systems. Although we have emphasized the distinction between the distributed message passing among the nodes and the shared-memory synchronization of concurrent processes at individual nodes, for modeling purposes a single program can do both. In the model of the RA algorithm, for each of the *NPROC* nodes there will be two processes, a Main process and a Receive process; each pair is parameterized by a node number that we have been denoting as myID. The global variables *within the nodes* are declared as arrays, one element for each node:

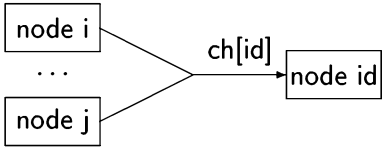
```
byte myNum[NPROCS];
byte highestNum[NPROCS];
bool requestCS[NPROCS];
chan deferred[NPROCS] = [NPROCS] of { byte };
```

The declaration of the sets deferred as channels is explained below.

The channels between the nodes are modeled, of course, by channels:

```
mtype = { request, reply } ;
chan ch[NPROCS] = [NPROCS] of { mtype, byte, byte } ;
```

The intended interpretation is that ch[id] is the channel *to* the receiving node id:



We are using the fact that a Promela channel can be used by more than one process. The channel capacity is defined to be NPROCS in order to model out-of-order delivery of messages.

The Main process is shown in Listing 10.1. **atomic** is used to prevent interleaving between setting requestCS and choosing a number. The symbol ?? in

```
ch[myID] ?? reply, _ , _;
```

means to remove any arbitrary reply message from the channel, without regard to the FIFO order normally inherent in the definition of a channel.

To model the set of deferred nodes, we could have defined an array of ID numbers, but it is simpler to use a channel as a queue. The Receive process (below) sends

```
proctype Main( byte myID ) {
   do ::
      atomic {
         requestCS[myID] = true ;
         myNum[myID] = highestNum[myID] + 1 ;
      }

      for (J, 0, NPROCS-1)
         if
         :: J != myID ->
               ch[J] ! request, myID, myNum[myID];
         :: else
         fi
      rof (J);

      for (K, 0, NPROCS-2)
         ch[myID] ?? reply, _ , _;
      rof (K);

      critical_section ();
      requestCS[myID] = false;

      byte N;
      do
         :: empty(deferred[myID]) -> break;
         :: deferred [myID] ? N -> ch[N] ! reply, 0, 0
      od
   od
}
```

Listing 10.1: Main process for Ricart–Agrawala algorithm

deferred processes to the channel and the Main process receives node IDs until the channel is empty. These IDs are then used as indices to send reply messages.

The Receive process is shown in Listing 10.2. **atomic** is used to prevent interleaving with the Main process, and ?? is used to receive an arbitrary request message.

```
proctype Receive( byte myID ) {
    byte reqNum, source;
    do ::
        ch[myID] ?? request, source, reqNum;

        highestNum[myID] =
            ((reqNum > highestNum[myID]) ->
                reqNum : highestNum[myID]);

        atomic {
            if
            :: requestCS[myID] &&
                ( (myNum[myID] < reqNum) ||
                ( (myNum[myID] == reqNum) &&
                        (myID < source)
                ) ) ->
                    deferred [myID] ! source
            :: else ->
                ch[source] ! reply, 0, 0
            fi
        }
    od
}
```

Listing 10.2: Receive process for Ricart–Agrawala algorithm

10.6 Token-passing algorithms

The problem with a permission-based algorithm is that it can be inefficient if there are a large number of nodes. Furthermore, the algorithm does not show improved performance in the absence of contention; a node wishing to enter its critical section must always send and receive messages from all other nodes.

In token-based algorithms, permission to enter the critical section is associated with the possession of an object called a *token*. Mutual exclusion is trivially satisfied by token-based algorithms, because their structure clearly shows this association. The algorithms are also efficient: only one message is needed to transfer the token and its associated permission from one node to another, and a node possessing the token can enter its critical section any number of times without sending or receiving any messages. The challenge is to construct an algorithm for passing the token that ensures freedom from deadlock and starvation.

Algorithm 10.3: Ricart–Agrawala token-passing algorithm

```
        boolean haveToken ← true in node 0, false in others
        integer array[NODES] requested ← [0,...,0]
        integer array[NODES] granted ← [0,...,0]
        integer myNum ← 0
        boolean inCS ← false
sendToken
    if exists N such that requested[N] > granted[N]
        for some such N
            send(token, N, granted)
            haveToken ← false
Main
    loop forever
p1:     non-critical section
p2:     if not haveToken
p3:         myNum ← myNum + 1
p4:         for all other nodes N
p5:             send(request, N, myID, myNum)
p6:         receive(token, granted)
p7:         haveToken ← true
p8:     inCS ← true
p9:     critical section
p10:    granted[myID] ← myNum
p11:    inCS ← false
p12:    sendToken
Receive
    integer source, reqNum
    loop forever
p13:    receive(request, source, reqNum)
p14:    requested[source] ← max(requested[source], reqNum)
p15:    if haveToken and not inCS
p16:        sendToken
```

Algorithm 10.3 was developed by Ricart and Agrawala as a token-passing version of their original algorithm.[1]

Look first at the algorithm without considering the content of the token message sent in sendToken (called from p12 and p16), and received in p6. Unlike the permissions in the RA algorithm, the passing of the permission by a token is contingent; a token will not be passed unless it is needed. As long as it is not needed, a node may hold the token as indicated by the boolean variable haveToken; the if statement at p2 enables the node to repeatedly enter its critical section.

Two data structures are used by the algorithm to decide if there are outstanding requests that require a node to give up the token it holds. The token message includes an array granted whose elements are the ticket numbers held by each node the *last* time it was granted permission to enter its critical section. In addition, each node stores in the array requested the ticket numbers accompanying the *last* request messages from the other nodes. While each node may have different data in requested depending on when request messages are received, only the copy of granted maintained by the node with the token is meaningful, and it is passed from one node to another as part of the token. It enables the algorithm to decide unambiguously what outstanding request messages have not been satisfied.

Here is an example of these two data structures maintained by the node Chloe in a five-node system:

	Aaron	Becky	Chloe	Danielle	Evan
requested	4	3	0	5	1
granted	4	2	2	4	1

request messages have been received at node Chloe from Becky and Danielle that were sent *after* the last time they were granted permission to enter their critical sections. Therefore, if Chloe holds the token and is not in her critical section, she must send it to one of them. If she is in the critical section, Chloe sends the token upon leaving, thereby preventing starvation that would occur if she immediately re-entered her critical section. However, if Chloe has not received a request message from another process, she can retain the token and re-enter her critical section.

The rest of the algorithm simply maintains these data structures. The current ticket number of a node is incremented in p3 and then sent in request messages to enable

[1] The algorithm appears in the *Authors' Response* to a *Technical Correspondence* by Carvalho and Roucairol [21]. A similar algorithm was discovered independently by Suzuki and Kasami [64].

the array requested to be updated in the other nodes. granted is updated when a node completes its critical section.

It is easy to show (exercise) that there is only a single token and that this ensures that mutual exclusion is satisfied.

Theorem 10.3 The algorithm does not deadlock.

Proof: If some nodes wish to enter their critical sections and cannot, they must be blocked indefinitely waiting at receive(token, granted). For all such nodes i, requested[i] is eventually greater than granted[i] in the node holding the token, so at p16 a token message will be sent to some node when its request is received, unless the node with the token is in its critical section. By the assumed progress of critical sections, the token will eventually be sent in p12. ∎

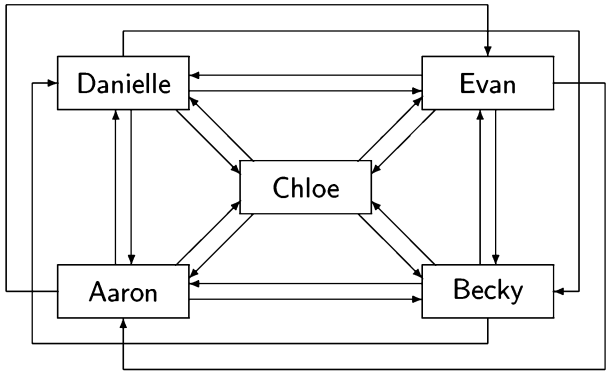
Starvation is possible because of the arbitrary selection of a requesting process for some such N in the algorithm for sendToken. In the exercises, you are asked to modify the algorithm to prevent starvation.

The original RA algorithm required that a node entering its critical section send $N - 1$ request messages and receive $N - 1$ reply messages. These messages are short, containing only a few values of fixed size. In the token-passing algorithm, a node need not send messages if it possesses the token; otherwise, it needs to send $N - 1$ request messages as before, but it only need receive one token message. However, the token message is long, containing N ticket numbers for the other processes. If N is large, this can be inefficient, but in practice, the token-passing algorithm should be more efficient than the RA algorithm because there is a large overhead to sending a message, regardless of its size.

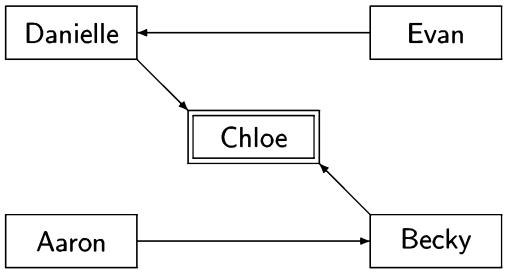
10.7 Tokens in virtual trees^A

The problem with the RA token-passing algorithm is that the queue of waiting processes is carried along with the token message. In this section we describe the Neilsen–Mizuno algorithm for distributed mutual exclusion [54] that is based upon passing a small token in a set of virtual trees that is implicitly constructed by the algorithm. It is recommended that you first study the concept of virtual data structures in the context of the Dijkstra–Scholten algorithm (Section 11.1) before studying this algorithm.

Before presenting the algorithm, let us work out an example on a five-node distributed system, where we assume that the nodes are fully connected, that is, that any node can send a message directly to any other node:



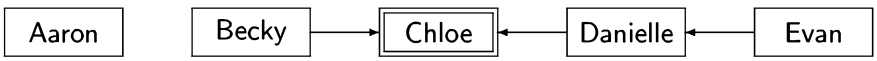
Let us now suppose that the system is initialized so that a set of edges is selected that define an arbitrary spanning tree with the directed edges pointing to the root, for example:



It will be convenient to draw the nodes in a horizontal line:

The node at the root of the tree possesses the token and is possibly in its critical section. The node possessing the token, in this case Chloe, is drawn with double lines. As in the RA token-passing algorithm, Chloe can enter her critical section repeatedly as long as she does not receive any request messages.

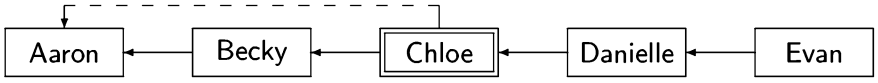
Suppose now that Aaron wishes to enter his critical section. He sends to his parent node, Becky, a message (request, Aaron, Aaron); the first parameter is the ID of the *sender* of the message, while the second parameter is the ID of the *originator* of the request. In the first message sent by a node wishing to enter its critical section, the two parameters are, of course, the same. After sending the message, Aaron zeroes out his parent field, indicating that he is the root of a new tree:



The intent is that Aaron will eventually become the root with the token, and enter his critical section; he receives requests from other nodes so that he can pass the token to another node when he leaves his critical section.

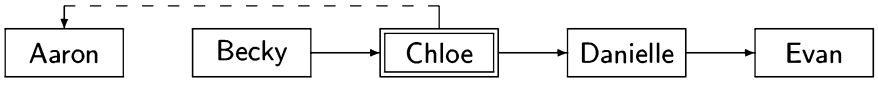
Becky will now relay the request to the root by sending the message (request, Becky, Aaron). Becky is sending a message on behalf of Aaron who wishes to enter his critical section. Becky also changes her parent field, but this time to Aaron, the sender of the message she is relaying:

The node receiving this message, Chloe, possesses the token; however, by the assumption in this scenario, she is in her critical section and must therefore defer sending the token. Chloe sets a field deferred to the value of the originator parameter in the message. This is indicated by the dashed arrow in the following diagram:

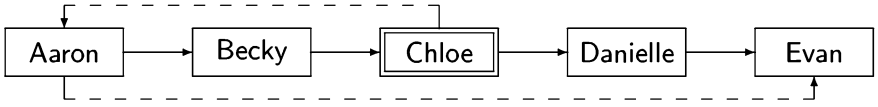


Chloe also sets her parent field to the sender of the message so that she can relay other messages.

Suppose now that Evan concurrently originates a request to enter his critical section, and suppose that his request is received by Chloe *after* the request from Aaron. Chloe is no longer a root node, so she will simply relay the message as an ordinary node, setting her parent to be Danielle. The chain of relays will continue until the message (request, Becky, Evan) arrives at the root, Aaron:

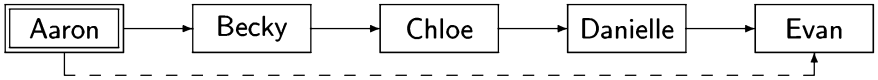


Aaron is a *root node without the token*, so he knows that he must appear in the deferred field of some other node. (In fact, he is in the deferred field of Chloe who holds the token.) Therefore, Aaron places the originator he has just received in his deferred field and as usual sets his parent field to the sender:

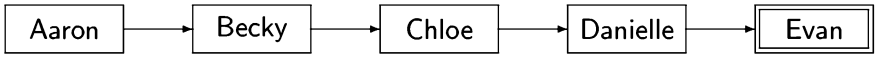


The deferred fields implicitly define a queue of processes waiting to enter their critical sections.

When Chloe finally leaves her critical section, she sends a token message to the node Aaron listed in her deferred field, enabling Aaron to enter his critical section:



When Aaron leaves his critical section, and then sends the token to Evan who enters his critical section:



The state of the computation returns to a quiescent state with one node possessing the token.

Algorithm 10.4, the Neilsen–Mizuno algorithm, is very memory-efficient. Only three variables are needed at each node: the boolean flag holding, which is used to indicate that a root node holds the token but is not in its critical section, and the node numbers for parent and deferred. The messages are also very small: the request message has two integer-valued parameters and the token message has no parameters. We leave it as an exercise to write the statements that initialize the parent fields.

The time that elapses between sending a request and receiving the token will vary, depending on the topology of the virtual trees. In Algorithm 10.3, a request message was sent to *every* node, so the token, if available, could be directly sent to the next requesting process; here the request message might have to be relayed through many nodes, although the token itself will be sent directly to the node whose ID is passed along in the originator field.

A proof of the correctness of the algorithm can be found in [54].

Transition

We have shown several algorithms for solving the critical section problem, which is the basic problem of concurrent programming. In a certain sense, the critical section problem is not typical of distributed algorithms, because it assumes that there is a centralized resource that needs to be protected from being accessed by several nodes. The next chapter poses problems that go to the heart of distributed programming: implementing cooperation among a set of independent nodes.

Algorithm 10.4: Neilsen–Mizuno token-passing algorithm

```
            integer parent ← (initialized to form a tree)
            integer deferred ← 0
            boolean holding ← true in the root, false in others
Main
    loop forever
p1:     non-critical section
p2:     if not holding
p3:         send(request, parent, myID, myID)
p4:         parent ← 0
p5:         receive(token)
p6:     holding ← false
p7:     critical section
p8:     if deferred ≠ 0
p9:         send(token, deferred)
p10:        deferred ← 0
p11:    else holding ← true
Receive
    integer source, originator
    loop forever
p12:    receive(request, source, originator)
p13:    if parent = 0
p14:        if holding
p15:            send(token, originator)
p16:            holding ← false
p17:        else deferred ← originator
p18:    else send(request, parent, myID, originator)
p19:    parent ← source
```

Exercises

Ricart–Agrawala

1. What is the total number of messages sent in a scenario in which all nodes enter their critical sections once?

2. (a) Construct a scenario in which the ticket numbers are unbounded.

   (b) What is the maximum difference between two ticket numbers?

3. What would happen if several nodes had the same value for myID?

4. (a) Can the deferred lists of all the nodes be nonempty?

   (b) What is the maximum number of entries in a single deferred list?

   (c) What is the maximum number of entries in all the deferred lists together?

The following four questions refer to Algorithm 10.2 and do not assume that the processes in each node are executed atomically.

5. Suppose that we exchanged the lines p8 and p9-p11. Would the algorithm still be correct?

6. Why don't we have to add highestNum ← max(highestNum, myNum) after statement p3: myNum ← highestNum + 1?

7. Can the statement p13: highestNum ← max(highestNum, requestNum) be replaced by p13: highestNum ← requestNum?

8. Construct a faulty scenario in which there is interleaving between the choosing statements p2-p3 and the statements that make the decision to defer p13-16.

9. Modify the Promela program for the RA algorithm to implement the deferred list using an array instead of a channel.

Ricart–Agrawala token-passing algorithm

10. Prove that mutual exclusion holds.

11. In node i, can requested[j] be less than granted[j] for $j \neq i$?

12. Show that the algorithm is not correct if the pre- and postprotocols and the processing of a request message in Receive are not executed under mutual exclusion.

13. Construct a scenario leading to starvation. Modify the algorithm so that it is free from starvation. You can either modify the statement for some such N in SendToken (Ricart–Agrawala) or modify the data structure granted (Suzuki–Kasami).

Neilsen–Mizuno algorithm

14. Develop a distributed algorithm to initialize the virtual tree in Algorithm 10.4.

11 Global Properties

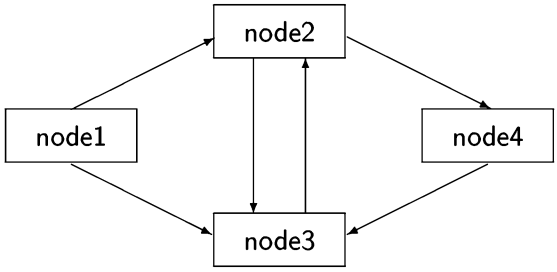
Almost by definition, there is no meaning to a global property of a distributed system. There is a parallel with Albert Einstein's theory of relativity: since information takes a finite amount of time to travel from one node to another, by the time you collect information on the global state of a system in a single node, it is "out of date." As with relativity, even the concept of time is problematic, and a central problem in the field of distributed systems is that of defining time and synchronizing clocks ([37], [48, Chapter 18]).

In this chapter, we present algorithms for two problems that can be characterized as detecting and recording global properties of distributed systems. The central concept is not *simultaneity*, but *consistency*: an unambiguous accounting of the state of the system. The first problem is to determine if the computations at each node in the system have terminated. This problem has no counterpart in concurrent systems that have shared resources, because each process can simply set a flag in shared memory to indicate that it has terminated. The second problem is to construct a snapshot of a distributed system. We would like to know where every message actually is at a "certain time," but since each node has its own clock and there are transmission delays, it is not feasible to talk about a "certain time." It does make sense, however, to compute a consistent snapshot, in the sense that every message is unambiguously attributed to a specific node or to a specific channel.

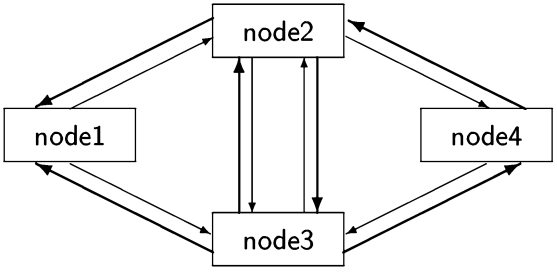
11.1 Distributed termination

A concurrent program terminates when all of its processes have terminated, and similarly a distributed system terminates when the processes at all of its nodes have terminated. The problem posed in this section is to detect a state in which all nodes have terminated. A survey of algorithms for distributed termination can be found in [52]. In this section, we describe an algorithm by Edsger W. Dijkstra and Carel S. Scholten for detecting termination in a distributed system. The presentation follows that in [25], where a preliminary algorithm is developed that is live but not safe, and is then extended to the full Dijkstra–Scholten (DS) algorithm which is also safe.

The algorithm collects information from a set of nodes over a period of time in order to decide whether termination has occurred or not. We make one change from the model of distributed computation described in Section 10.1: we do not assume that each node is connected to each other node, only that the set of nodes form a connected directed graph. We do assume the existence of a unique *environment node* which has no incoming edges and from which every node is accessible by a path through the directed graph. The environment node is responsible for reporting termination. The following diagram shows such a directed graph with four nodes, where node1 is the environment node with no incoming edges:



The DS algorithm to detect termination is to be run concurrently with the computation being carried out at each node. The computation sends messages over the edges from one node to another; the DS algorithm specifies additional statements that must be executed as part of the processing of messages by the sender and the receiver. The algorithm assumes that for each edge in the graph from node i to node j, there is a *back edge* that carries a special type of message called a *signal* from j to i. The back edges are shown by thick arrows in the following diagram:



A further assumption is that all the nodes except for the environment node are initially inactive, meaning that they are not performing any computation but are merely waiting to receive messages. The computation of the distributed system is initiated when the environment node sends messages on its outgoing edges. When a node that is not an environmental node receives its first message on any incoming

edge, it can begin its computation. Eventually the computation in each node terminates and it will no longer send messages, although if it receives more messages it may be restarted. At all times, a node is able to receive, process and send signals, as required by the termination algorithm.

Under this set of assumptions, we want to develop an algorithm in which the environment node announces termination of the system if and only if the computation has terminated in all the nodes.

Preliminary algorithm

For each message received by a destination node, it is required to eventually send a signal on the corresponding back edge to the source node. The difference between the number of messages received on an incoming edge E of node i and the number of signals sent on the corresponding back edge is denoted $inDeficit_i[E]$. The difference between the number of messages sent on outgoing edges of node i and the number of signals received on back edges is denoted $outDeficit_i$. $inDeficit_i[E]$ must be known for each incoming edge separately, while for outgoing edges it is sufficient to maintain the sum of the deficits over all the edges. When a node terminates it will no longer send messages; the sending of signals will continue as long as $inDeficit_i[E]$ is nonzero for any incoming edge. When $outDeficit_{env} = 0$ for the environment node, the algorithm announces termination.

Let us start with a preliminary algorithm (Algorithm 11.1). There are three variables for keeping track of deficits. outDeficit and the array inDeficit[E] were described above. In addition to the array of incoming deficits, the algorithm needs the sum of all these deficits:

$$\sum_{E \in incoming} inDeficit_i[E].$$

No confusion will result if we use inDeficit without an index to represent this sum. (Alternatively, the sum could be generated as needed in send signal.)

There are four parts to this algorithm that must be integrated into the underlying computation of each node:

send message When the underlying computation sends a message (whatever it is), an additional statement is executed to increment the outgoing deficit.

receive message When the underlying computation receives a message (whatever it is), additional statements are executed to increment the incoming deficit on that edge, as well as the total incoming deficit.

Algorithm 11.1: Dijkstra–Scholten algorithm (preliminary)	
	integer array[incoming] inDeficit ← [0,. . . ,0] integer inDeficit ← 0 integer outDeficit ← 0
	send message
p1:	send(message, destination, myID)
p2:	increment outDeficit
	receive message
p3:	receive(message, source)
p4:	increment inDeficit[source] and inDeficit
	send signal
p5:	when inDeficit > 1 or (inDeficit = 1 and isTerminated and outDeficit = 0)
p6:	E ← some edge E with inDeficit[E] ≠ 0
p7:	send(signal, E, myID)
p8:	decrement inDeficit[E] and inDeficit
	receive signal
p9:	receive(signal, _)
p10:	decrement outDeficit

send signal This is an additional process added to the program at each node. The statements of this process may be executed if the condition in the when clause is true; otherwise, the process is blocked. Whenever the incoming deficit is nonzero, the node may send a signal on the back edge, but it does not send the final signal until the underlying computation has terminated and the outgoing deficit is zero. It is assumed that a node can decide if its local computation has terminated; this is denoted by calling the boolean-valued function isTerminated.

receive signal This is also an additional process that is executed whenever a signal is received; it decrements the outgoing deficit.

For the environment node, the only field needed is the counter for the outgoing deficit (Algorithm 11.2).

Correctness of the preliminary algorithm

We now prove that if the computation terminates at all nodes, eventually the environment node announces termination. For the purpose of simplifying the invariants

Algorithm 11.2: Dijkstra–Scholten algorithm (env., preliminary)	
	integer outDeficit ← 0
	computation
p1:	for all outgoing edges E
p2:	send(message, E, myID)
p3:	increment outDeficit
p4:	await outDeficit = 0
p5:	announce system termination
	receive signal
p6:	receive(signal, source)
p7:	decrement outDeficit

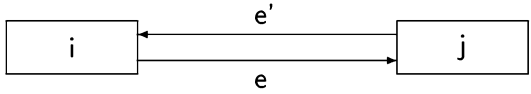
and their proofs, it will be convenient to assume that communications are synchronous, so that every message and signal sent is immediately received; by doing this, we do not have to account for messages that are in transit. Since we are proving a liveness formula with "eventually," this does not affect correctness, because we have assumed that each individual message and signal is eventually received.

Notation: $inDeficit_i$ is the value of inDeficit at node i and similarly for $outDeficit_i$.

Lemma 11.1 At each node i, $inDeficit_i \geq 0$ and $outDeficit_i \geq 0$ are invariant, as is the equality of the sums of the two deficits over *all* nodes:

$$\sum_{i \in nodes} inDeficit_i = \sum_{i \in nodes} outDeficit_i. \tag{11.1}$$

Proof: Let i and j be arbitrary nodes such that e is the edge from node i to j, and e' is the matching back edge for sending signals from j to i:



Let n be the number of messages received by j on e and n' the number of signals sent by j on e'; then $inDeficit_j[e] = n - n'$. But n is also the number of messages sent by node i on e and n' is also the number of signals received by i on e' so $outDeficit_i[e] = n - n'$, where the notation $outDeficit_i[e]$ means that we are counting the contribution of edge e to the total $outDeficit_i$. We have shown that for an arbitrary edge e, $inDeficit_j[e] = outDeficit_i[e]$. Since each edge is an outgoing edge for exactly one node and an incoming edge for exactly one node, summing over all the edges in the graph gives

$$\sum_{j \in nodes} \sum_{e \in incoming_j} inDeficit_j[e] = \sum_{i \in nodes} \sum_{e \in outgoing_i} outDeficit_i[e]. \tag{11.2}$$

By definition,

$$inDeficit_i = \sum_{e \in incoming_i} inDeficit_i[e] \tag{11.3}$$

$$outDeficit_i = \sum_{e \in outgoing_i} outDeficit_i[e], \tag{11.4}$$

Substituting (11.3) and (11.4) into (11.2) proves the invariance of (11.1).

For any edge e, $inDeficit_j[e] \geq 0$ is invariant, because $inDeficit_j[e]$ is decremented only in send signal and only after explicitly checking that its value is positive. Since $outDeficit_i[e] = inDeficit_j[e]$ for the node j at the other end of e, $outDeficit_i[e] \geq 0$ is also invariant. The invariance of $inDeficit_i \geq 0$ and $outDeficit_i \geq 0$ follows by summing the individual deficits of the edges. ∎

Theorem 11.2 If the system terminates, the source node eventually announces termination.

Proof: The system terminates when the underlying computation terminates at each node; therefore, no more messages will be sent so neither $inDeficit_i$ nor $outDeficit_i$ increase after termination. By the condition in send signal, each non-environment node will continue sending signals until

$$inDeficit_i > 1 \lor (inDeficit_i = 1 \land outDeficit_i = 0)$$

becomes false. Using the invariants $inDeficit_i \geq 0$ and $outDeficit_i \geq 0$ of Lemma 11.1, it can be shown (exercise) that the negation of this formula is equivalent to:

$$inDeficit_i = 0 \lor (inDeficit_i \leq 1 \land outDeficit_i > 0). \tag{11.5}$$

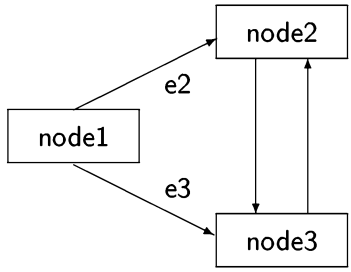
If $inDeficit_i = 0$, the formula is true regardless of the truth of $outDeficit_i > 0$), so this formula is in turn equivalent to:

$$inDeficit_i = 0 \lor (inDeficit_i = 1 \land outDeficit_i > 0).$$

From this we deduce $inDeficit_i \leq outDeficit_i$ as follows: if $inDeficit_i = 0$ the formula follows from $outDeficit_i \geq 0$, while if $inDeficit_i = 1$, from the second disjunct we have $outDeficit_i > 0$ which is the same as $outDeficit_i \geq 1$.

We have shown that at termination, $inDeficit_i \leq outDeficit_i$ holds for all non-environment nodes. For the environment node, $inDeficit_i = 0$ (since there are no incoming edges) and the invariant $outDeficit_i \geq 0$ also imply that $inDeficit_i \leq outDeficit_i$. Since this holds for all i, it follows from (11.2) that $inDeficit_i$ must be equal to $outDeficit_i$ at all nodes, in particular for the environment node where $inDeficit_i = 0$. Therefore, $outDeficit_i = 0$ and the node announces termination. ∎

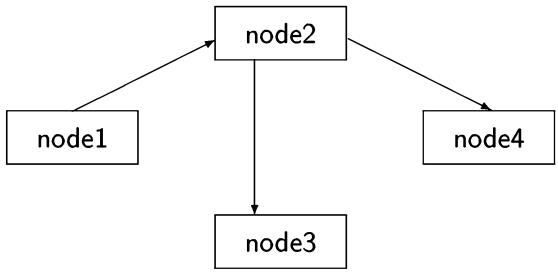
The algorithm is not safe. Consider the following set of nodes and edges:



Let node1 send messages to both node2 and node3, which in turn send messages to each other. At both nodes, $inDeficit_i = 2$ and furthermore, $inDeficit_2[e2] = 1$ at node2 and $inDeficit_3[e3] = 1$ at node3. By the statements at p5 and p6 in Algorithm 11.1, both can send signals to node1, which will now have $outDeficit_i = 0$ and can announce termination although the other two nodes have not terminated.

11.2 The Dijkstra–Scholten algorithm

Let us first consider the case where the directed graph is a tree. In a tree, each node except for the root has exactly one parent node, and furthermore, a tree necessarily has leaves, which are nodes with no outgoing edges. The following diagram shows a tree constructed from our directed graph:



The root is the environment node node1 and the edges are a subset of the edges of the graph. If the graph is a tree, it is trivial to detect termination. When a leaf terminates, it sends a signal to its parent. A non-leaf node waits for signals from each of its children and then sends a signal to its parent. The root has no parent, so when it receives signals from all its children, it can conclude that the distributed system has terminated.

The tree shown in the diagram is called a *spanning tree* because every node in the graph is included in the tree. The Dijkstra–Scholten algorithm implicitly constructs

Algorithm 11.3: Dijkstra–Scholten algorithm

```
        integer array[incoming] inDeficit ← [0,...,0]
        integer inDeficit ← 0
        integer outDeficit ← 0
        integer parent ← −1
    send message
p1: when parent ≠ −1          // Only active nodes send messages
p2:     send(message, destination, myID)
p3:     increment outDeficit
    receive message
p4: receive(message,source)
p5: if parent = −1
p6:     parent ← source
p7: increment inDeficit[source] and inDeficit
    send signal
p8: when inDeficit > 1
p9:     E ← some edge E for which
            (inDeficit[E] > 1) or (inDeficit[E] = 1 and E ≠ parent)
p10:    send(signal, E, myID)
p11:    decrement inDeficit[E] and inDeficit
p12: or when inDeficit = 1 and isTerminated and outDeficit = 0
p13:    send(signal, parent, myID)
p14:    inDeficit[parent] ← 0
p15:    inDeficit ← 0
p16:    parent ← −1
    receive signal
p17: receive(signal, _)
p18: decrement outDeficit
```

a spanning tree from the directed graph. By "implicitly," we mean that the tree is not held in any actual data structure, but it can be deduced from the internal states of the nodes. The trivial algorithm for termination in a tree can then be executed on the spanning tree.

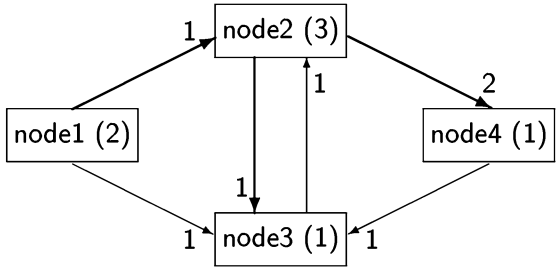
The source field of the *first* message to arrive at a node defines the parent of that node. We will call that incoming edge the *parent edge* of the node. The preliminary algorithm is modified so that the *last* signal from a node is sent on its parent edge; the parent then knows that no more signals will ever be received from the node. A node sends its last signal only when $outDeficit_i$ has become zero, so it becomes a leaf node in the spanning tree of non-terminated nodes.

The modifications are shown in Algorithm 11.3. The new variable parent stores the identity of the parent edge. The value -1 is used as a flag to indicate that the parent edge is not yet known. send message is modified to restrict the sending of messages to nodes that have already received a message and thus have an edge to their parent nodes. received message is modified to save the parent edge when the first message is received. send signal is modified to make sure that the last signal is sent on the parent edge; signals can be sent along any edge with an outstanding deficit (including the parent), as long as the final signal is saved for the parent node. When terminating, a node resets its parent variable because it may receive new messages causing it to be restarted.

Here a partial scenario:

Action	node1	node2	node3	node4
1 ⇒ 2	(-1,[],0)	(-1,[0,0],0)	(-1,[0,0,0],0)	(-1,[0],0)
2 ⇒ 4	(-1,[],1)	(1,[1,0],0)	(-1,[0,0,0],0)	(-1,[0],0)
2 ⇒ 3	(-1,[],1)	(1,[1,0],1)	(-1,[0,0,0],0)	(2,[1],0)
2 ⇒ 4	(-1,[],1)	(1,[1,0],2)	(2,[0,1,0],0)	(2,[1],0)
1 ⇒ 3	(-1,[],1)	(1,[1,0],3)	(2,[0,1,0],0)	(2,[2],0)
3 ⇒ 2	(-1,[],2)	(1,[1,0],3)	(2,[1,1,0],0)	(2,[2],0)
4 ⇒ 3	(-1,[],2)	(1,[1,1],3)	(2,[1,1,0],1)	(2,[2],0)
	(-1,[],2)	(1,[1,1],3)	(2,[1,1,1],1)	(2,[2],1)

The first column show the actions: n ⇒ m means that node n sends a message to node m. The other columns show the local data structures at each node: (parent, inDeficit[E], outDeficit). The values of the array inDeficit[E] are shown in increasing numerical order of the nodes. (The variable containing the sum of the values in the array can be easily computed as needed.) The data structures upon completion of this part of the scenario are graphically shown in the following diagram:



The outgoing deficits are shown in parentheses next to the node labels, while the incoming deficits are shown next to the edges.

We leave it as an exercise to expand the scenario to include decisions to terminate, and sending and receiving signals.

Correctness of the Dijkstra–Scholten algorithm

The proof of the liveness of the algorithm is almost the same as it is for the preliminary algorithm. We have delayed the sending of the last signal on the parent edge, but eventually it is sent, maintaining the liveness property.

Let us now prove the safety of the the algorithm. Define a non-environment node as *active* if and only if $parent \neq -1$.

Lemma 11.3 $inDeficit_i = 0 \rightarrow outDeficit_i = 0$ is invariant at non-environment nodes.

Proof: send message can make $outDeficit_i = 0$ false by sending a message, but it never does so unless the node is active. A node is not active initially and becomes active in receive message when inDeficit[source] is incremented; therefore, the antecedent must be false. $inDeficit_i = 0$ becomes true only after waiting for $outDeficit_i$ to become zero, so the truth of the formula is again maintained. ∎

Lemma 11.4 The edges defined by the parent variables in each node form a spanning tree of the active nodes with the environment node as its root. Furthermore, for each active node, $inDeficit_i \neq 0$.

Proof: A non-environment node can become active only by receiving a message, and this causes parent to be set to the parent edge. Therefore the the parent edges span the active nodes. Do these edges form a tree? Yes, because parent is set to a non-negative value when the first message is received and is never changed again as long as as the node is active.

A node can become inactive only by resetting inDeficit to zero. By Lemma 11.3, this implies that $outDeficit_i = 0$, so the children of this node must not be active. Therefore, this node must have been a leaf of the spanning tree of active nodes, and its removal maintains the property that all active nodes are in the tree. ∎

Theorem 11.5 If the environment node announces termination, then the system has terminated.

Proof: If the environment node announces termination, $outDeficit_{env} = 0$. By Lemma 11.4, if there were active nodes, they would form a spanning tree and at least one child of the environment node would have $inDeficit_i \neq 0$, which contradicts $outDeficit_{env} = 0$. ∎

Performance

A problem with the DS algorithm is that the number of signals equals the number of messages that are sent during the entire execution of the system. But consider a distributed system that is shutting down after it has been executing for several days sending billions of messages; a similarly large number of signals will have to be sent. In such a computation, it should be possible to reduce the number of signals.

A first idea is to reduce the deficit as much as possible in a signal. The algorithm for send signal becomes:

```
when inDeficit > 1
   (E, N) ← selectEdgeAndNum
   send(signal , E, myID, N)
   inDeficit [E] ← inDeficit [E] − N
   inDeficit ← inDeficit − N
```

where selectEdgeAndNum is:

```
if E ≠ parent and inDeficit [E] ≥ 1
   return (E, inDeficit [E])
else if inDeficit [parent] > 1
   return (parent, inDeficit [parent]−1)
```

The algorithm for receive signal is changed to:

```
receive (signal , source, N)
outDeficit ← outDeficit − N
```

A further improvement in the algorithm can be obtained by initializing all the parent edges to point to the environment node with ID zero, forming a "spanning bush" rather than creating a spanning tree on-the-fly. The initialization for non-environment nodes now becomes:

```
integer parent ← 0
integer array [incoming] inDeficit ← [1, 0, ..., 0]
integer inDeficit ← 1
integer outDeficit ← 0
```

In the exercises, you are asked to show the correctness of these modifications and to discuss the conditions under which performance is improved.

11.3 Credit-recovery algorithms

The DS algorithm can be difficult to implement if the deficit counts become too large to store in simple integer variables. *Credit-recovery algorithms* [53, 34] attempt to avoid this problem by using clever representations of the deficits.

In a credit-recovery algorithm, the environment node starts out with a unit "sum of money" or "weight," $W = 1.0$. When it sends a message to a node, it "lends" the destination node part of the weight. Non-environment nodes are active if they possess a nonzero amount of weight; in turn, they share this weight when sending messages to other nodes. When a non-environment node becomes terminated, it returns all its weight to the environment node. Once all the weights have been recovered, the environment node can declare global termination.

Algorithm 11.4 is for the environment node. Every outgoing message takes with it half the weight. Once the system has been initialized, the environment node simply waits for the weight that it has lent out to be recovered. This is done by signals sent to the environment node.

Algorithm 11.5 for the non-environment nodes is very similar. A non-environment node starts with zero weight and becomes active when it receives its first message together with an initial weight, which can then be lent to other nodes. When the node terminates, it returns its current weight directly to the environment node.

Algorithm 11.4: Credit-recovery algorithm (environment node)	
	float weight ← 1.0
	computation
p1:	for all outgoing edges E
p2:	weight ← weight / 2.0
p3:	send(message, E, weight)
p4:	await weight = 1.0
p5:	announce system termination
	receive signal
p6:	receive(signal, w)
p7:	weight ← weight + w

Algorithm 11.5: Credit-recovery algorithm (non-environment node)

```
        constant integer parent ← 0 // Environment node
        boolean active ← false
        float weight ← 0.0
    send message
p1: if active                    // Only active nodes send messages
p2:     weight ← weight / 2.0
p3:     send(message, destination, myID, weight)
    receive message
p4: receive(message, source, w)
p5: active ← true
p6: weight ← weight + w
    send signal
p7: when terminated
p8:     send(signal, parent, weight)
p9:     weight ← 0.0
p10:    active ← false
```

While we have shown non-environment nodes waiting until they terminate to return their weight, the algorithm could have enabled or required them to return excess weight earlier. For example, in Mattern's algorithm, weight received while a node is active is immediately returned to the environment node:

```
receive (message, source, w)
if active then
    send(signal , parent, w)
else
    active ← true
    weight ← w
```

Just as the deficit counters in the DS algorithm can grow to be very large, the values of weight can grow to be very small. The first node to receive a message from the environment node receives a value of 2^{-1}. If it now sends out one million messages, the values of weight will become $2^{-2}, 2^{-3}, \ldots, 2^{-1000001}$. By storing just the negative exponent, this is no more difficult than storing the value 1000000 in outDeficit. The problem arises when arbitrary weights are added in the environment node, leading to values such as: $2^{-1}+2^{-15}+2^{-272}+\cdots+2^{-204592}+\cdots+2^{-822850}$. In the exercises you are asked to explore various data structures for improving the space efficiency of the algorithm.

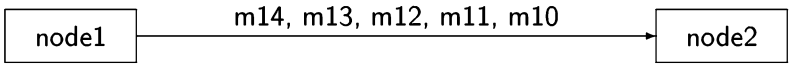
11.4 Snapshots

There is no real meaning to the global state of distributed systems, because global properties cannot be measured. The internal clocks of the processors at each node run independently and cannot be fully synchronized, and precise coordination is impossible because communications channels transfer data at uncertain rates. Nevertheless, there is meaning to the concept of a *global snapshot*: a consistent recording of the states of all the nodes and channels in a distributed system.

The state of a node is defined to be the values of its internal variables, together with the sequences of messages that have been sent and received along all edges incident with the node. (It is convenient to identify channels with the corresponding edge in the directed graph.) The state of an edge is defined as the sequence of messages sent on the edge but not yet delivered to the receiving node. For a snapshot to be consistent, each message must be in exactly one of these states: sent and in transit in an edge, or already received. It is not required that all the information of a snapshot be gathered at one particular instant, only that the assembly of the information over time be such that consistency is achieved. Algorithms for snapshots are useful because they generalize algorithms for several specific problems, such as detection of termination and deadlock.

The algorithm we present was developed by K. Mani Chandy and Leslie Lamport [22]. Unlike other distributed algorithms in this book, the CL algorithm works only if the channels are FIFO, that is, if the messages are delivered in the order they were sent.

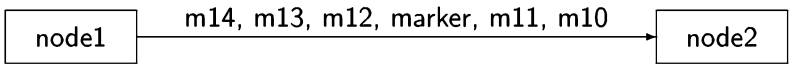
Consider two nodes and the stream of messages sent from node1 to node2:



Suppose that both of these nodes are instructed to take a snapshot. (We leave for now the question of how this occurs.) Presumably we want the snapshot to indicate that node1 has sent fourteen messages (m1, ..., m14), that node2 has received nine messages (m1, ..., m9), and that messages m10 through m14 are still on the edge. But node1 has no idea which messages have been received and which are still on the edge, and similarly, node2 can only know which messages it has received, not which messages are on the edge.

To ensure consistency when a snapshot is taken, an additional message called a *marker* is sent on each edge, in order to place a boundary between messages sent before the snapshot is taken and messages sent after it is taken. Suppose again that both nodes have been instructed to take a snapshot. Let us suppose further

that node1 received this command after sending message m11, but before sending message m12. It will send the marker to node2 immediately upon reception of the snapshot command, so the messages on the edge will be as follows:



node1 records its state immediately when the snapshot command is given; its state, therefore, includes messages m1 through m11. node2 also records its state, so the messages received by then—messages m1 through m9—are part of its state. This leaves messages m10 and m11, which have been sent but not received, to be assigned to the state of the edge. Receiving nodes are responsible for recording the state of an edge, which is defined as the set of messages received after it has recorded its state but before it has received the marker.

There is an additional possibility: a node may receive a marker before it is instructed to take a snapshot. In that case, the edge will be considered empty, as all messages received before the marker are considered to be part of the state of the receiving node.

As with the DS algorithm, it is convenient to assume that an environment node is responsible for initiating the algorithm, although the algorithm is very flexible and only requires that every node be reachable from some node that spontaneously decides to record the state. In fact, several nodes could concurrently decide to record the state and the algorithm would still succeed. The environment node will initiate the snapshot by sending a marker on each of its outgoing edges.

```
for all outgoing edges E
    send(marker, E, myID)
```

Algorithm 11.6 is the Chandy–Lamport (CL) algorithm for global snapshots. It consists of modifications to the sending and receiving of messages by the underlying algorithm, as well as a process that receives the markers. There is also a process that waits until all markers have been received and then records the state as described below.

To simplify the presentation, we do not store the contents of the messages, only their number within the sequence. Since FIFO channels are assumed these numbers suffice to specify which messages are sent and received. The internal state consists of simply the (number of the) last message sent on each outgoing edge—stored in the variable lastSent during send message, and the (number of the) last message received on each incoming edge—stored in the variable lastReceived during receive message. When the first marker is received, the state of the outgoing

Algorithm 11.6: Chandy–Lamport algorithm for global snapshots

```
        integer array[outgoing] lastSent ← [0, ..., 0]
        integer array[incoming] lastReceived ← [0, ..., 0]
        integer array[outgoing] stateAtRecord ← [−1, ..., −1]
        integer array[incoming] messageAtRecord ← [−1, ..., −1]
        integer array[incoming] messageAtMarker ← [−1, ..., −1]
    send message
p1: send(message, destination, myID)
p2: lastSent[destination] ← message
    receive message
p3: receive(message,source)
p4: lastReceived[source] ← message
    receive marker
p5: receive(marker, source)
p6: messageAtMarker[source] ← lastReceived[source]
p7: if stateAtRecord = [−1,...,−1]      // Not yet recorded
p8:     stateAtRecord ← lastSent
p9:     messageAtRecord ← lastReceived
p10:    for all outgoing edges E
p11:        send(marker, E, myID)
    record state
p12: await markers received on all incoming edges
p13: recordState
```

edges is recorded in stateAtRecord. All elements of the array are initialized to -1 and this is also used as a flag to indicate that the marker has not been received. (The assignments at p8 and p9 are array assignments; similarly, the test at p7 compares two arrays, although an additional boolean flag would suffice.)

For incoming edges, two array variables are needed: messageAtRecord stores the (number of the) last message received on each edge before the state was recorded, and messageAtMarker stores the (number of the) last message received on each edge before the first marker was received.

When the *first* marker is received, it initiates the recording of the state. For outgoing edges this is easy: the state is simply the last message sent on each edge. Similarly, for the incoming edge upon which the first marker was received, all messages received before the marker are part of the state of the node. For other incoming edges, it is possible that additional messages will be received after the state is recorded, but before the marker is received. The state of the edge is defined as the difference between the last message received before the node recorded its

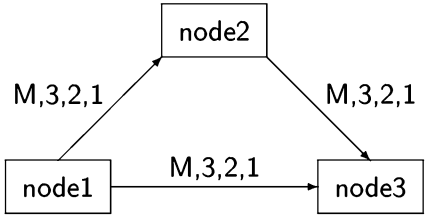
state (in messageAtRecord) and the last message received before the marker was received on this edge (in messageAtMarker).

When marker messages have been received from each incoming edge, the node can record its state, which consists of the following data:

- stateAtRecord[E]: the last message sent on each outgoing edge E.
- messageAtRecord[E]: the last message received on each incoming edge E.
- For each incoming edge E, if messageAtRecord[E] is not equal to messgeAtMarker[E], then the messages from messageAtRecord[E]+1 to messgeAtMarker[E] form the state of edge E.

The recorded global state is composed of the state recorded at the individual nodes. An algorithm similar to the DS algorithm can be used to send all the state information back to an environment node.

Let us construct a scenario for an example on the following graph:



Each of the nodes sends three messages 1, 2 and 3 in that order and then a marker M. In the display of the scenario, we have abbreviated the variables: ls for lastSent; lr for lastReceived; sr for stateAtRecord; mr for messageAtRecord; mm for messageAtMarker. Variables that have not changed from their initial values are indicated by a blank cell. The scenario starts by sending all three messages from node1 to node2, where they are received. Then three messages are sent from node1 to node3 and from node2 to node3, but they are not yet received. We display the scenario starting from the first state in this table:

Action	node1					node2				
	ls	lr	sr	mr	mm	ls	lr	sr	mr	mm
	[3,3]					[3]	[3]			
1M⇒2	[3,3]		[3,3]			[3]	[3]			
1M⇒3	[3,3]		[3,3]			[3]	[3]			
2⇐1M	[3,3]		[3,3]			[3]	[3]			
2M⇒3	[3,3]		[3,3]			[3]	[3]	[3]	[3]	[3]

(To save space, the data structures of node3, which are empty, are omitted.) node1, the source node, decides to initiate a snapshot. It sends markers to node2 and node3 (denoted 1M⇒2 and 1M⇒3), and records its own state as having sent three messages on each of its outgoing edges. node2 receives the marker (2⇐1M) and records its state: it has sent three messages and received three messages. We can see that there are no messages on the edge from node1 to node2. Finally, node2 sends a marker to node3 (2M⇒3).

The scenario continues as follows:

Action	node3				
	ls	lr	sr	mr	mm
3⇐2					
3⇐2		[0,1]			
3⇐2		[0,2]			
3⇐2M		[0,3]			
3⇐1		[0,3]		[0,3]	[0,3]
3⇐1		[1,3]		[0,3]	[0,3]
3⇐1		[2,3]		[0,3]	[0,3]
3⇐1M		[3,3]		[0,3]	[0,3]
		[3,3]		[0,3]	[3,3]

(The data structures for node1 and node2, which do not change, are omitted.)

node3 receives the three messages from node2 and updates its lastReceived variable. Then it reads the marker sent by node2 and records its state; again the state shows that the edge from node2 is empty. Finally, node3 receives the three messages from node1 (updating lastReceived as usual), and then receives the marker, recording its state. Since a marker has already been received by this node, messageAtRecord is *not* updated (p9), but messageAtMarker is updated to reflect the messages received on this edge (p6). The difference between the first components of these two variables indicates that the three messages sent from node1 to node3 are considered to have been on that edge when the snapshot was taken.

Correctness of the Chandy–Lamport algorithm

Theorem 11.6 If the environment node initiates a snapshot, eventually the algorithm displays a consistent snapshot.

Proof: The termination of the algorithm follows by the connectedness of the graph. The environment node sends a marker to each child node; when the marker

is received by any node, it immediately sends a marker to each of its children. By induction a marker is sent on each outgoing edge and hence received on each incoming edge, eventually causing display state to be executed at each node.

Since we assume that messages (and markers) are never lost, the only way that a snapshot could be inconsistent is if some message m appears in the state of i, but does not appear in exactly one of the states of the destination node or the edge. There are four cases, depending on whether m was sent before or after the marker from node i to node j, and whether it was received before or after node j recorded its state.

The first two cases are those where the message m was sent before i sent the marker.

Case 1: If m was received before j recorded its state, it will be stored in lastReceived by receive message and appear in the state of node j only.

Case 2: If m was received after j recorded its state, it will be stored in lastReceived by receive message but not in messageAtRecorded. Eventually the marker from node i will be received, so m will have a value greater than the value messageAtRecorded and less than or equal to the value of messageAtMarker; it will appear only in the state of the edge from i to j.

The other two cases are those where the message m was sent after i sent the marker to j. In these cases, the message is *not* part of the state of i, so we have to ensure that it does not appear in the state of j or in the state of the edge from i to j.

Case 3: m was received before j recorded its state. But this is impossible because j already recorded its state when it received the marker sent before m.

Case 4: m was received after j recorded its state. Clearly, m will not be recorded in the state of j or in the state of the edge. ∎

In the exercises you are asked to show that the state displayed by the snapshot need not be an actual state that occurred during the computation.

Transition

This chapter and the preceding one have presented algorithms for the solution of three classical problems in distributed programming: mutual exclusion, termination detection and snapshots. The algorithms have been given under certain assumptions about the topology of the network; furthermore, we have assumed that nodes do not fail and that messages are reliably received. In the next chapter, we show how reliable algorithms can be built even if some nodes or channels are not reliable.

Exercises

Dijkstra–Scholten

1. How many spanning trees are there for the example with four nodes? Construct scenarios that create each of the trees. (Assume that node1 remains the environment node.)

2. Given the details of the derivation of Equation 11.5 in Theorem 11.2.

3. Complete the scenario on page 245 assuming that no more messages are sent.

4. Prove the correctness of the modifications to the DS algorithm discussed on page 247.

5. Under what conditions does the modified DS algorithm lead to an improvement in performance?

6. Develop an algorithm that enables the environment node to collect a full description of the topology of the distributed system.

Credit-recovery

7. When an active node receives a message with a weight, it can be added to the current node's weight or it can be returned immediately to the environment node. Discuss the advantages and disadvantages of each possibility.

8. Develop efficient data structures for storing arbitrary sums of weights.

   (a) (Huang) Store fixed ranges of nonzero values, thus $\{2^{-4}, 2^{-5}, 2^{-7}, 2^{-8}\}$ would be stored as $(4, 1101)$.

   (b) (Mattern) Store the values as a set and perform implicit addition as new values are received; for example, if 2^{-8} is received and added to the above set, we get the smaller set $\{2^{-4}, 2^{-5}, 2^{-6}\}$.

Chandy–Lamport

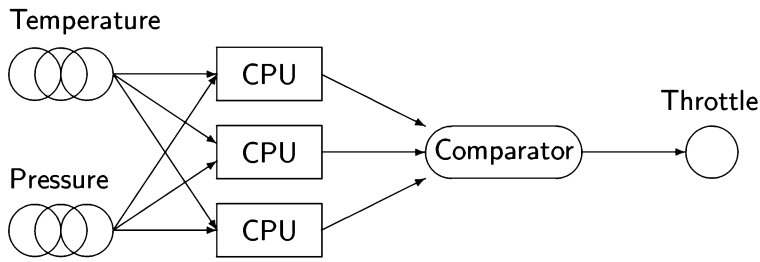
9. Construct a scenario such that the state displayed by the snapshot is not a state that occurred during the computation.

10. Where in the proof of correctness have we implicitly used the fact that the channels must be FIFO?

12 Consensus

12.1 Introduction

One of the primary motivations for building distributed systems is to improve reliability by replicating a computation in several independent processors. There are two properties that we can hope to achieve in a reliable system: a system is *fail-safe* if one or more failures do not cause damage to the system or to its users; a system is *fault-tolerant* if it continues to fulfil its requirements even if there are one or more failures. A distributed system is not automatically fail-safe or fault-tolerant. The RA algorithm for distributed mutual exclusion requires the cooperation of all the processes and will deadlock if one of them fails. The following diagram shows the general architecture of a reliable system:



Input sensors are replicated, as are the CPUs performing the computation. The outputs are compared and an algorithm such as majority voting is used to decide what command to send to control the system.

The simplicity of the diagram masks the complexity of the difficulties that must be overcome in designing a reliable system:

- When the input sensors are replicated, they may not all give exactly the same data. Voting on the outcome is no longer a trivial comparison of two digital values.
- A faulty input sensor or processor may not fail gracefully. It may produce spurious data or values that are totally out of the range considered by the algorithms.

- If all processors are using the same software, the system is not tolerant of software bugs. If several different programs are used, they may give slightly different values on the same data. Worse, different programmers are prone to make the same misinterpretations of the program specifications.

The design of such systems is beyond the scope of this book. Here, we will focus on the specific problem of achieving *consensus* in a distributed system; each node chooses an initial value and it is required that all the nodes in the system decide on one of those values. If there are no faults in the system, there is a trivial algorithm: each node sends its choice to every other node and then an algorithm such as majority voting is performed. Since all nodes have the same data and execute the same algorithm, they all decide upon the same value and consensus is achieved.

We will consider two types of faults: *crash failures* in which a node simply stops sending messages, and *byzantine failures* in which a faulty node may send arbitrary messages. The name is taken from the classic algorithm for consensus by Leslie Lamport, Robert Shostak and Marshall Pease called the *Byzantine Generals algorithm* [42]. After presenting this algorithm, we show a simple *flooding algorithm* that achieves consensus in the presence of crash failures. We conclude with another algorithm for consensus under byzantine failure developed by Piotr Berman and Juan Garay [11]. This algorithm, called the *King algorithm*, is much more efficient than the Byzantine Generals algorithm in terms of the number of messages exchanged.

We will prove the correctness of the algorithms for the minimum number of nodes for which the algorithms work; the full proofs are inductive generalizations of the proofs we give and can be found in the original articles and in textbooks [4, 48].

12.2 The problem statement

The problem of consensus was originally stated in a rather graphic setting, which has become so entrenched in the literature that we will continue to use it:

> A group of Byzantine armies is surrounding an enemy city. The balance of force is such that if all the armies attack together, they can capture the city; otherwise, they must all retreat to avoid defeat. The generals of the armies have reliable messengers who successfully deliver any message sent from one general to another. However, some of the generals may be *traitors* endeavoring to bring about the defeat of the Byzantine armies. Devise an algorithm so that all *loyal* generals come to a consensus on a plan. The final decision should be almost

the same as a majority vote of their initial choices; if the vote is tied, the final decision is to retreat.

Historical note: The Byzantine Empire existed from the year 323 until it was finally destroyed by the Ottoman Turks in 1453. Its capital city Constantinople (now Istanbul) was named after Constantine, the founder of the empire. Its history has given rise to the word *byzantine* meaning devious and treacherous political maneuvering, though they were probably no worse than any comparable political entity. To maintain the verisimilitude of the story, I will call the nodes of the system by real names of Byzantine emperors.

In terms of distributed systems, the generals model nodes and the messengers model communications channels. While the generals may fail, we assume that the messengers are perfectly reliable. We will model two types of node failures:

Crash failures A traitor (failure node) simply stops sending messages at any arbitrary point during the execution of the algorithm.

Byzantine failures A traitor can send arbitrary messages, not just the messages required by the algorithm.

In the case of crash failures, we assume that we *know* that the node has crashed; for example, there might be a *timeout* value such that the receiving node knows that any message will arrive within that amount of time. If we have no way to decide if a node has crashed or if a message is merely delayed, the consensus problem is not solvable [48, Chapter 17]. Byzantine failures are more difficult to handle; since it is sufficient that there exist one scenario for the algorithm to be incorrect, we must take into account a "malicious" traitor who designs exactly the set of messages that will break the algorithm.

The requirement that the consensus be "almost" the same as the majority vote ensures that the algorithm will be applicable and that a trivial solution is not given. A trivial solution, for example, is an algorithm stating that all loyal generals decide to attack, but this does not model a system that actually computes a value. Suppose that we have 10 generals of whom 2 are traitors. If the initial choices of the loyal generals are 6 to attack and 2 to retreat, the traitors could cause some generals to think that the choices were split 8–2 and others to think 6–4, but the majority vote is to attack in any case and the algorithm must also ensure that this decision is reached. If, on the other hand, the initial choices were 4–4 or 5–3, it is possible for the traitors to affect the final outcome, as long as there is consensus. This is meaningful, for if four *non-faulty* computers say to open the throttle and four say to close the throttle, then it probably doesn't make a difference, as long as they all choose the same command.

The algorithms are intended to be executed concurrently with the underlying computation whenever there is a need to reach consensus concerning the value of an item of data. The set of messages used is disjoint from the set used by the underlying computation. The algorithms are synchronous in the sense that each node sends out a set of messages and then receives replies to those messages. Since different types of messages are not received concurrently, we need not write a specific message type; the send statement includes only the destination node ID followed by the data sent, and the receive statement includes only variables to store the data that is received.

12.3 A one-round algorithm

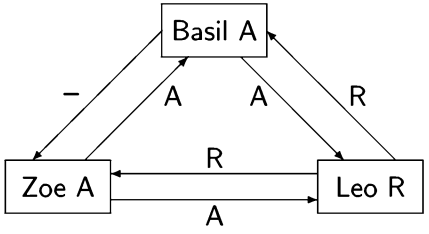
Let us start with the obvious algorithm:

Algorithm 12.1: Consensus—one-round algorithm

```
                    planType finalPlan
                    planType array[generals] plan
p1: plan[myID] ← chooseAttackOrRetreat
p2: for all other generals G
p3:     send(G, myID, plan[myID])
p4: for all other generals G
p5:     receive(G, plan[G])
p6: finalPlan ← majority(plan)
```

The values of planType are A for attack and R for retreat. Each general chooses a plan, sends its plan to the other generals and receives their plans. The final plan is the majority vote among all plans, both the general's own plan and the plans received from the others.

Suppose now that there are three generals, two of whom—Zoe and Leo—are loyal, and the third—Basil—is a traitor. Basil and Zoe choose to attack, while Leo chooses to retreat. The following diagram shows the exchange of messages according to the algorithm, where Basil has crashed after sending a message to Leo that he has chosen to attack, but before sending a similar message to Zoe:



The following tables show the content of the array plan at each of the loyal nodes, as well as the result of the majority vote:

Leo	
general	plan
Basil	A
Leo	R
Zoe	A
majority	A

Zoe	
general	plans
Basil	–
Leo	R
Zoe	A
majority	R

By a majority vote of 2–1, Leo chooses A. Zoe chooses R, because ties are (arbitrarily) broken in favor of R. We see that if a general crashes, it can cause the remaining loyal generals to fail to come to a consensus. It should be clear that this scenario can be extended to an arbitrary number of generals. Even if just a few generals crash, they can cause the remaining generals to fail to achieve consensus if the vote is very close.

12.4 The Byzantine Generals algorithm

The problem with the one-round algorithm is that we are not using the fact that certain generals are loyal. Leo should somehow be able to attribute more weight to the plan received from the loyal general Zoe than from the traitor Basil. In a distributed system an individual node cannot know the identities of the traitors directly; rather, it must ensure that the plan of the traitors cannot cause the loyal generals to fail to reach consensus. The Byzantine Generals algorithm achieves this by using extra rounds of sending messages: a first round in which each general sends its own plan, and subsequent rounds in which each general sends what it received from other generals about their plans. By definition, loyal generals always relay exactly what they received, so that if there are enough loyal generals, they can overcome the attempts of the traitors to prevent them from reaching a consensus.

Algorithm 12.2 is the two-round Byzantine Generals algorithm. The first round is the same as before; at its conclusion, the array plan holds the plans of each of the generals. In the second round, these plans are then sent to the other generals. Obviously, a general doesn't have to send her own plan to herself, nor does she have to send back to another general what he reported about himself. Therefore, each round reduces by one the number of messages that a general needs to send.

The line p8: send(G', myID, G, plan[G]) means: send to general G' that I (myID) received the plan stored in plan[G] from general G. When such a message is re-

Algorithm 12.2: Consensus—Byzantine Generals algorithm

```
                planType finalPlan
                planType array[generals] plan
                planType array[generals, generals] reportedPlan
                planType array[generals] majorityPlan

p1:  plan[myID] ← chooseAttackOrRetreat

p2:  for all other generals G                    // First round
p3:     send(G, myID, plan[myID])
p4:  for all other generals G
p5:     receive(G, plan[G])

p6:  for all other generals G                    // Second round
p7:     for all other generals G' except G
p8:        send(G', myID, G, plan[G])
p9:  for all other generals G
p10:    for all other generals G' except G
p11:       receive(G, G', reportedPlan[G, G'])

p12: for all other generals G                    // First vote
p13:    majorityPlan[G] ← majority(plan[G] ∪ reportedPlan[*, G])

p14: majorityPlan[myID] ← plan[myID]             // Second vote
p15: finalPlan ← majority(majorityPlan)
```

ceived, it is stored in reportedPlans, where the value of the array element reportedplan[G, G'] is the plan that G reported receiving from G'.

Voting is done in a two-stage procedure. For each other general G, a majority vote is taken of the plan received directly from G—the value of plan[G]—together with the reported plans for G received from the other generals—denoted in the algorithm by reportedPlan[*,G]. This is taken as the "real" intention of G and is stored in majorityplan[G]. The final decision is obtained from a second majority vote of the values stored in majorityPlan, which also contains plan[myID].

Don't worry if the algorithm seems confusing at this point! The concept will become clearer as we work through some examples.

12.5 Crash failures

Let us first examine the Byzantine Generals algorithm in the case of crash failures with three generals, one of whom is a traitor. Here are the data structures of the two loyal generals for the same scenario as in Section 12.3, where Basil crashes after sending the (first-round) message to Leo, but before sending it to Zoe:

Leo				
general	plan	reported by		majority
		Basil	Zoe	
Basil	A		–	A
Leo	R			R
Zoe	A	–		A
majority				A

Zoe				
general	plan	reported by		majority
		Basil	Leo	
Basil	–		**A**	A
Leo	R	–		R
Zoe	A			A
majority				A

The second column shows the general's own choice of plan and the plans received directly from the other generals, while the third and fourth columns show the plans reported in the second round. The last column shows the majority vote for each general. Obviously, a loyal general's own plan is correct; it is not sent and no report is received. For the other loyal general (Leo for Zoe and Zoe for Leo), the correct plan is received on the first round. Basil, having crashed during the first round, does not send second-round messages.

Basil sends a message to one of the loyal generals, Leo, and Leo relays this information to Zoe in the second round (denoted by **A**). If a general sends even one message before crashing, all the loyal generals receive the same report of this plan, and the majority vote will be the same for each of them.

Let us check one more scenario, in which the traitor sends out all of its first-round messages and one of its second-round messages before crashing. Basil has sent to Leo a message reporting that Zoe's plan is to attack, but crashes before sending the report on Leo to Zoe. There is only one missing message:

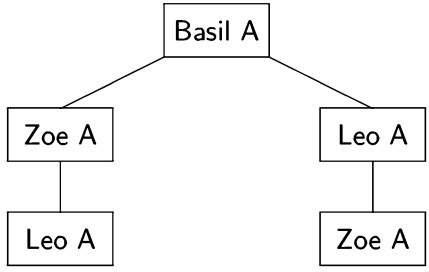
Leo				
general	plan	reported by		majority
		Basil	Zoe	
Basil	A		A	A
Leo	R			R
Zoe	A	A		A
majority				A

Zoe				
general	plan	reported by		majority
		Basil	Leo	
Basil	A		A	A
Leo	R	–		R
Zoe	A			A
majority				A

Again, both loyal generals have consistent data structures and come to the same decision about the final plan.

12.6 Knowledge trees

To prove the correctness of the Byzantine Generals algorithm, we will use *knowledge trees.*[1] A knowledge tree stores the data that is known ***about*** *the general listed at its root.* Therefore, the tree is not a data structure that exists locally at any node, but rather a virtual global data structure obtained by integrating the local data structures. The following diagram shows the knowledge tree *about* Basil, assuming that Basil has chosen A and that all messages, both from Basil and from the other nodes, have been sent and received:



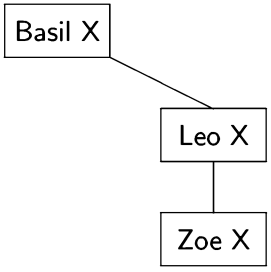
[1]These data structures were first introduced in [65], where they were called *virtual trees.* They are *not* the same as the *exponential information gathering trees* that are shown in most presentations of the Byzantine Generals algorithm, for example, in [48, Section 6.2]. Dynamic visualizations of knowledge trees are displayed when constructing scenarios of the algorithm with DAJ.

The root of the tree shows that Basil has chosen to attack. The first level below the root shows the results of the first round of messages: Basil sends his plan to both Leo and Zoe. The second level below the root shows the results of the second round of messages: Zoe sends to Leo that Basil's plan is to attack and Leo sends to Zoe that Basil's plan is to attack. Both Zoe and Leo receive two messages about Basil, and both messages are the same.

We can use knowledge trees to prove the correctness of Algorithm 12.2 under crash failures when there are three generals, one of whom is a traitor. We show that in any scenario, both Leo and Zoe come to the same conclusion about Basil's plan.

If Basil sends no messages, both Zoe and Leo know nothing about Basil's plan, but they correctly send their own plans to each other. Together with their own plans, the two generals have the same set of plans and reach the same decision.

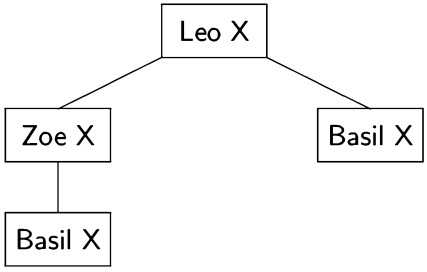
If Basil sends exactly one first-round message before crashing, then since both Zoe and Leo are loyal, the one who receives it (say, Leo) will report the message to the other. Exactly one branch of the tree will be constructed, as shown in the following diagram where we have represented an arbitrary plan chosen by Basil as X:



Both Zoe and Leo have exactly one message each about Basil's plan and will vote the same way.

If Basil sends two first-round messages, the result is the first tree in this section with A replaced by the arbitrary X. Both Zoe and Leo have received the same two messages about Basil's plan and come to the same decision.

We also have to consider the possibility that a crash of Basil before a second-round message can cause Leo to make an inconsistent decision about Zoe, or vice versa. Here is the knowledge tree *about* Leo that results if Basil crashes before sending the second-round message to Zoe:

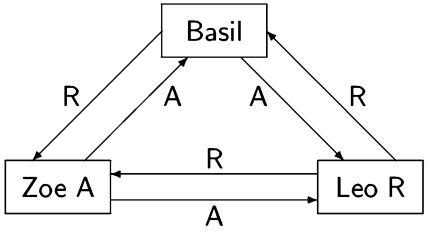


Leo of course knows his own plan X and Zoe knows the same plan having received it during the first round directly from the loyal Leo. Therefore, both Leo and Zoe come to the same decision about Leo's choice.

12.7 Byzantine failures with three generals

The two-round Byzantine Generals algorithm is much more complicated than it need be for a system that suffers only from crash failures. (A simpler algorithm is given in Section 12.9.) It is only when we consider Byzantine failures that the cleverness of the algorithm becomes apparent. Recall that a byzantine failure is one in which a node can send *any* message that it pleases, and that an algorithm is incorrect if there is even one set of malicious messages that causes it to fail. In the context of the story of the Byzantine Generals, a traitor is allowed to send an attack or retreat message, regardless of its internal state. In fact, we will not even show a choice of an initial plan for a traitor, because that plan can be totally ignored in deciding which messages are sent.

Here are the messages of the first round of the example from the previous section, where instead of crashing the traitor Basil sends an R message to Zoe:



The data structures are as follows and the two loyal generals will make inconsistent final decisions:

Leo	
general	plans
Basil	A
Leo	R
Zoe	A
majority	A

Zoe	
general	plans
Basil	R
Leo	R
Zoe	A
majority	R

We are not surprised, because the one-round algorithm was not correct even in the presence of crash failures.

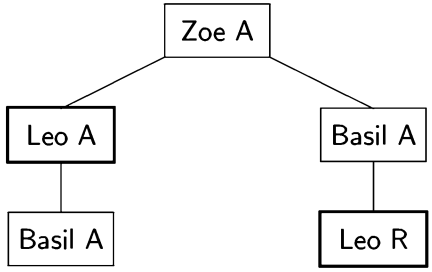
Consider now the two-round algorithm. In the first round, Basil sends an A message to both Zoe and Leo; in the second round, he correctly reports to Zoe that Leo's plan is R, but erroneously reports to Leo that Zoe's plan is **R**. The following data structure results:

Leo				
general	plans	reported by		majority
		Basil	Zoe	
Basil	A		A	A
Leo	R			R
Zoe	A	**R**		R
majority				R

Zoe				
general	plans	reported by		majority
		Basil	Leo	
Basil	A		A	A
Leo	R	R		R
Zoe	A			A
majority				A

Basil's byzantine failure has caused Leo to make an erroneous decision about Zoe's plan (the one-one tie is broken in favor of retreat). The two loyal generals reach inconsistent final decisions, so we conclude that the algorithm is incorrect for three generals of whom one is a traitor.

Let us look at the failure using the knowledge tree *about* Zoe:



Zoe chose A and sent A messages to Leo and Basil. While the loyal Leo reported Zoe's choice correctly to Basil, the traitor Basil falsely reported to Leo that Zoe sent him R. Leo has two conflicting reports about Zoe's plan (thick frames), leading to the wrong decision.

12.8 Byzantine failures with four generals

The Byzantine Generals algorithm for consensus is correct if there are three loyal generals and one traitor. When the first votes are taken to decide what each general has chosen, there will be two loyal reports against one report from the traitor, so the loyal generals will agree. We will show a scenario for the algorithm and then use knowledge trees to prove its correctness.

Our fourth general will be named John and we will change the traitor to Zoe. Basil and John choose A while Leo chooses R. Here is a partial data structure of the loyal general Basil; it shows only messages received from the loyal generals, not those from the traitor Zoe:

Basil					
general	plan	reported by			majority
		John	Leo	Zoe	
Basil	A				A
John	**A**		**A**	?	A
Leo	R	R		?	R
Zoe	?	?	?		?
majority					?

Basil receives the correct plan of loyal general John, both directly from John himself as well as indirectly from Leo (bold); the report from the traitor Zoe can at worst make the vote 2–1 instead of 3–0, but the result is always correct. Similarly, Basil has two correct reports of the plan of Leo.

Basil now has three correct votes—his own and the ones from John and Leo—as do the other two loyal generals. But it is premature to conclude that they come to a consensus in the final vote, because Zoe could send messages convincing some of them that her plan is A and others that it is R. It remains to show that the three loyal generals come to a consensus about the plan of Zoe.

Let us continue our example and suppose that Zoe sends first-round messages of R to Basil and Leo and A to John; these are relayed *correctly* in the second round by the loyal generals. Basil's data structure now becomes:

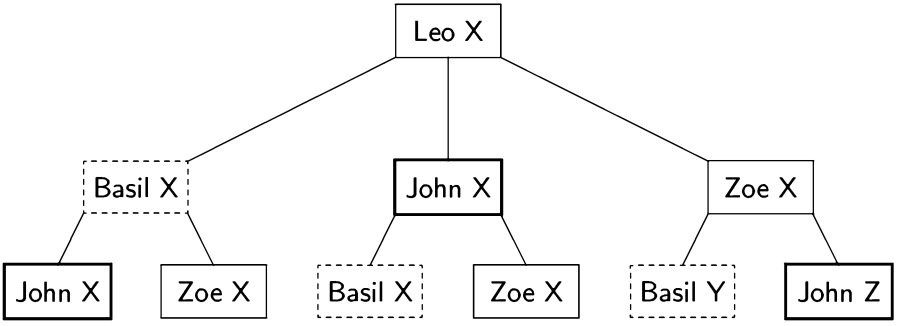
Basil					
general	plans	reported by			majority
		John	Leo	Zoe	
Basil	A				A
John	A		A	?	A
Leo	R	R		?	R
Zoe	**R**	**A**	**R**		R
					R

Clearly, Zoe can send arbitrary messages to the loyal generals, but during the second round these messages are accurately reported by all the loyal generals (bold), leading them to the same majority vote about Zoe's plan. In the example, the final decision taken by all the loyal generals will be a 2–1 vote in favor of R for Zoe's plan.

Thus the traitor cannot cause the loyal generals to fail to come to a consensus; at worst, their decision may be slightly influenced. For example, if Zoe had sent attack instead of retreat to Basil, it can be shown that the final decision would have been to attack (exercise). If the loyal generals initially choose the same plan, the final decision would be this plan, regardless of the actions of the traitor.

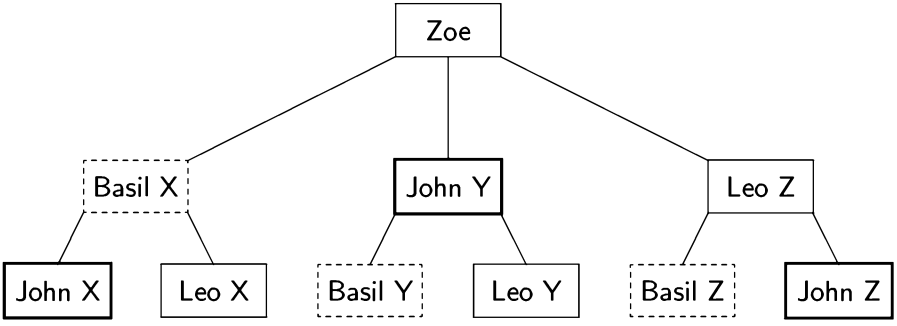
Correctness

We can use knowledge trees to show the correctness of the Byzantine Generals algorithm. Consider an arbitrary loyal general, say, Leo. Whatever plan X that Leo chooses, he correctly relays it to the other generals. The other loyal generals correctly report it to each other as X, though the traitor Zoe can report it as arbitrary plans Y and Z. Here is the knowledge tree *about* Leo that results:



From the tree it is easy to see that both John (thick frames) and Basil (dashed frames) each received two reports that Leo chose X. Therefore, the messages Y and Z cannot influence their votes.

Let us now examine the knowledge tree *about* the traitor Zoe, who can choose to send first-round messages with arbitrary plans X, Y and Z to the other three generals:



The contents of these messages are accurately relayed by the loyal generals during the second round. As you can see in the diagram, all three generals received exactly one each of the plans X, Y and Z; therefore, they all come to the same conclusion about the plan of Zoe.

Complexity

The Byzantine Generals algorithm can be generalized to any number of generals. For every additional traitor, an additional round of messages must be sent. The total number of generals must be at least $3t + 1$, where t is the number of traitors.

The total number of messages sent by each general is $(n - 1) + (n - 1) \cdot (n - 2) + (n - 2) \cdot (n - 3) + \cdots$, because it sends its plan to every other general $(n - 1)$, a

second-round report of each of the $(n-1)$ generals to $(n-2)$ generals, and so on. The total number of messages is obtained by multiplying this by n, the number of generals, giving:

$$n \cdot [(n-1) + \sum_{k=1}^{t} (n-k) \cdot (n-k-1)].$$

As can be seen from the following table, the algorithm quickly becomes impractical as the number of traitors increases:

traitors	generals	messages
1	4	36
2	7	392
3	10	1790
4	13	5408

12.9 The flooding algorithm

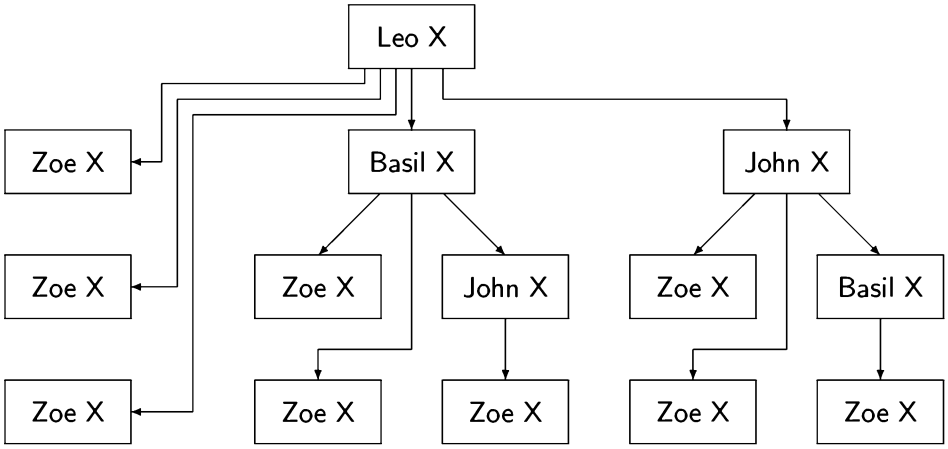
There is a very simple algorithm for consensus in the presence of crash failures. The idea is for each general to send over and over again the *set* of plans that it has received:

Algorithm 12.3: Consensus—flooding algorithm

```
                planType finalPlan
                set of planType plan ← { chooseAttackOrRetreat }
                set of planType receivedPlan
p1:  do t + 1 times
p2:     for all other generals G
p3:        send(G, plan)
p4:     for all other generals G
p5:        receive(receivedPlan)
p6:        plan ← plan ∪ receivedPlan
p7:  finalPlan ← majority(plan)
```

It is sufficient that a single such message from a loyal general reach every other loyal general, because once a loyal general has determined the plan of another loyal general, it retains that information. If there are $t+1$ rounds of sending and receiving messages in the presence of at most t traitors, then one such message must have been sent and received without crashing.

Let us examine how the algorithm works with four nodes—two of whom traitors—and three rounds. The following diagram shows a portion of the knowledge tree *about* Leo, assuming that no processes crash. There is not enough room to display the entire tree, so we display only those nodes that affect the decision of an arbitrary loyal general Zoe *about* Leo:

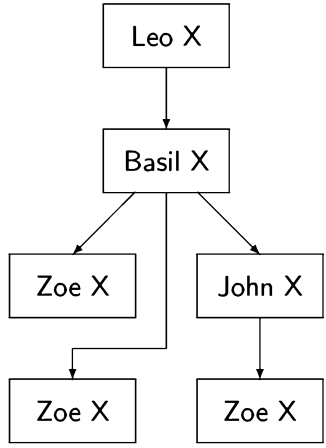


Leo chooses X. Each row below the box for Leo shows the messages sent and received during a round. In the first round, Leo sends his plan to the other three generals. Basil and John receive Leo's plan and send it to Zoe. In the third round, Leo's plan is still stored in the sets plan for Basil and John, so they both send it again to Zoe. On the left, we see the messages that Zoe receives directly from Leo on the three rounds.

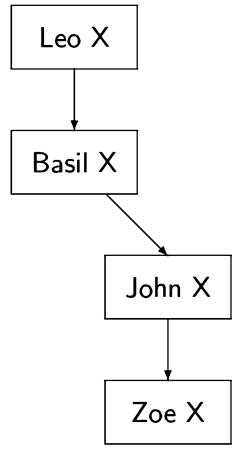
Is there a scenario in which at most two out of the three generals Leo, Basil and John crash, and Zoe cannot determine the plan of Leo? If not, then since Zoe and Leo were chosen arbitrarily, this implies that all loyal generals receive the same set of plans and can come to a consensus by majority vote.

We will make the assumption that no general crashes immediately after activation or after receiving a message without sending at least one message. The algorithm still works and we leave it as an exercise to extend the proof of correctness for these cases.

Leo sends at least one message; if he sends a message to Zoe before crashing, then Zoe knows his plan and correctness follows. If not, suppose that one of these messages went to Basil. The knowledge tree *about* Leo is a subtree of the tree shown in the following diagram:



In the second round, if Basil sends a message to Zoe, all is well, and we don't care if he crashes immediately afterwards, or if he never crashes and it is John who is the second traitor. However, Basil may send a message to John and then crash before sending a message to Zoe. The following diagram results:



But we have already "used up" the two traitors allowed in the statement of the problem, so John must be loyal and he sends Leo's plan to Zoe.

It can be proved [48, Section 6.2] that for any number of nodes of which t are subject to crash failure, the algorithm reaches consensus in $t + 1$ rounds. The idea of the proof is similar to the argument for four generals and two traitors: since there are more rounds than traitors, a loyal general must eventually send the set of plans it knows to all other generals, and from then on, the loyal generals exchange this information in order to reach consensus.

12.10 The King algorithm

The Byzantine Generals algorithm requires a large number of messages, especially as the number of traitors (and thus generals) increases. In this section, we describe another algorithm for consensus, the *King algorithm*, which uses many fewer messages. However, the algorithm requires an extra general per traitor, that is, the number of generals n must be at least $4t + 1$, where t is the number of traitors, instead of the $3t + 1$ required by the Byzantine Generals algorithm. We will present the algorithm for the simplest case of one traitor and $4 \cdot 1 + 1 = 5$ generals.

The algorithm is based upon the fact that a small number of traitors cannot influence the final vote if there is an overwhelming majority for one plan. If there are four loyal generals and the vote is 4–0 or 3–1, then the single traitor cannot influence the outcome. Only if the vote is tied at 2–2 can the traitor cause consensus to fail by sending A to some generals and R to others.

The algorithm is called the King algorithm because in each round one of the generals is given a special status as a king whose vote is more important than the vote of the other generals. To preserve the distributed nature of the algorithm, we will not assume that the identity of the king is known to all other nodes. All we need is that each node knows whether it is or is not a king on any particular round. Furthermore, to preserve the property that the system be fault-tolerant under arbitrary failure, we will not assume that the king node does not fail. That is, whatever method is used to designate the king, it is possible that the king will be a traitor.

However, if two generals take the role of king one after the other, then we are assured that at least one of them will be loyal. When a loyal general is king, he will cause the other generals to come to a consensus. If he is the second king, the final vote will be according to that consensus. If he is the first king, the loyal generals will have an overwhelming majority in favor of the consensus, so that even if the second king is the traitor, he cannot cause the loyal generals to change their plans.

Algorithm 12.4 is the King algorithm. As before, each general chooses a plan and sends it to each other general. After receiving all these messages, each general has five plans and stores the result of a majority vote in myMajority. In addition, the number of votes for the majority (3, 4 or 5) is stored in the variable votesMajority.

The second round starts with the king (only) sending his plan to the other generals. The choice of a king is decided according to some arbitrary, but fixed, algorithm executed by all the generals; for example, the generals can be ordered in alphabetical order and the king chosen according to this order. When a node receives the king's plan, it checks votesMajority, the number of votes in favor of myMajority; if the majority was overwhelming (greater than $\lfloor n/2 \rfloor + t$, here greater than 3), it ignores the king's plan, otherwise, it changes its own plan to the king's plan.

Algorithm 12.4: Consensus—King algorithm

```
                planType finalPlan, myMajority, kingPlan
                planType array[generals] plan
                integer votesMajority, kingID
p1:  plan[myID] ← chooseAttackOrRetreat

p2:  do two times
p3:     for all other generals G            // First and third rounds
p4:        send(G, myID, plan[myID])
p5:     for all other generals G
p6:        receive(G, plan[G])
p7:     myMajority ← majority(plan)
p8:     votesMajority ← number of votes for myMajority

p9:     if my turn to be king               // Second and fourth rounds
p10:       for all other generals G
p11:          send(G, myID, myMajority)
p12:       plan[myID] ← myMajority
        else
p13:       receive(kingID, kingPlan)
p14:       if votesMajority > 3
p15:          plan[myID] ← myMajority
           else
p16:          plan[myID] ← kingPlan

p17: finalPlan ← plan[myID]                // Final decision
```

Finally, the entire algorithm is executed a second time during which the king will be a different general.

Let us work through an example. We need five generals, so Mike will join as the fifth general, and we will assume that he is the traitor, so that we are only interested in the data structures of the other four generals. Let us suppose that the initial plans are evenly divided, with Basil and John choosing attack and Leo and Zoe choosing retreat. Let us further suppose that the traitor Mike sends attack to two of the generals and retreat to the other two in an attempt to foil the consensus. Upon completion of the first round, the data structures will be as follows:

Basil							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
A	A	R	R	R	R	3	

John							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
A	A	R	A	R	A	3	

Leo							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
A	A	R	A	R	A	3	

Zoe							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
A	A	R	R	R	R	3	

Consider the case where the first king is a loyal general, say Zoe. Zoe sends R, her myMajority; since no general had computed its myMajority with an overwhelming majority (votesMajority>3), they all change their own plans to be the same as the king's:

Basil							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
R							R

John							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
	R						R

Leo							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
		R					R

Zoe							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
				R			

In the third round, the plans are sent again and the variables myMajority and votesMajority recomputed:

Basil							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
R	R	R	?	R	R	4–5	

John							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
R	R	R	?	R	R	4–5	

Leo							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
R	R	R	?	R	R	4–5	

Zoe							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
R	R	R	?	R	R	4–5	

We have not shown the messages sent by the traitor Mike; regardless of what he sends, all the loyal generals agree that all of them have chosen R, so they all set myMajority to R and votesMajority to four (or five if the traitor happened to send R). Both four and five votes are considered an overwhelming majority, so it doesn't matter if the king in the fourth round is a traitor or not, because his plan will be ignored, and all the loyal generals will come to a consensus on R.

Consider now the case where the first king is the traitor Mike. In his role as king, the traitor Mike can, of course, send any messages he wants:

Basil							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
R							R

John							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
	A						A

Leo							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
		A					A

Zoe							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
				R			R

During the third round, the plans are again exchanged and the votes recomputed, resulting in the following data structures:

Basil							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
R	A	A	?	R	?	3	

John							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
R	A	A	?	R	?	3	

Leo							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
R	A	A	?	R	?	3	

Zoe							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
R	A	A	?	R	?	3	

All loyal generals have the same set of plans tied at two so, clearly, whatever messages the traitor sends will affect the value of myMajority at each node. But regardless of whether myMajority is for A or R, for each loyal general votesMajority will be three, that is, the choice is *not* overwhelming. Now the general chosen to be king in the fourth round will be a loyal general, say Zoe, so whatever Zoe sends as the king's plan, say A, will be adopted by all the other loyal generals in statement p16: plan[myID]←kingPlan:

Basil							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
A							A

John							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
	A						A

Leo							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
		A					A

Zoe							
Basil	John	Leo	Mike	Zoe	myMajority	votesMajority	kingPlan
				A			

These plans become the final plans, so they come to a consensus on A.

Correctness

Lemma 12.1 If a king is loyal, then the values of *plan*[*myID*] are equal for all loyal generals after an even (second or fourth) round.

Proof: The proof is by cases on the set of values of *plan*[*myID*] held by the loyal generals at the beginning of the associated odd (first or third) round.

Case 1: If the values of *plan*[*myID*] were equal for all loyal generals, then so were those of *myMajority* and the majorities were overwhelming; therefore, the plan sent by the king will not change them.

Case 2: If the values of *plan*[*myID*] were split 3–1, the values of *myMajority* were the same for all generals, although some majorities may have been overwhelming and some not. The plan sent by the loyal king is the common value of *myMajority*, so it doesn't matter if the value is accepted or rejected by each loyal general.

Case 3: If the values of *plan*[*myID*] were split 2–2, the values of *myMajority* may have been different for different generals, but *no* majority was overwhelming, so the plan sent by the loyal king is accepted by all the loyal generals. ∎

Theorem 12.2 The King algorithm achieves consensus with four loyal generals and one traitor.

Proof: If the second king is loyal, the result follows from Lemma 12.1. If the first king is loyal, Lemma 12.1 shows that the values of *plan*[*myID*] will be equal for the loyal generals at end of the second round and thus the beginning of third round. Therefore, all loyal generals will compute the same *myMajority*; since it is an overwhelming majority, the plan sent by the traitor king will not change the common value in *plan*[*myID*]. ∎

Complexity

While the King algorithm requires an extra general per traitor, the message traffic is much reduced. In the first round, each general sends a message to each other general, a total of $n \cdot (n - 1)$ messages. But in the second round, only the king sends messages, so the additional number of messages is $n - 1$. The total number of pairs of rounds is $t + 1$, giving a total message count of $(t + 1) \cdot (n + 1) \cdot (n - 1)$. The following tables compare the number of generals and messages, on the left for the Byzantine Generals algorithm and on the right for the King algorithm:

traitors	generals	messages
1	4	36
2	7	392
3	10	1790
4	13	5408

traitors	generals	messages
1	5	48
2	9	240
3	13	672
4	17	1440

We can see that in terms of the message traffic, the King algorithm remains reasonably practicable for longer than the Byzantine Generals algorithm as the number of failure nodes increases, although the total number of nodes needed will make it more expensive to implement.

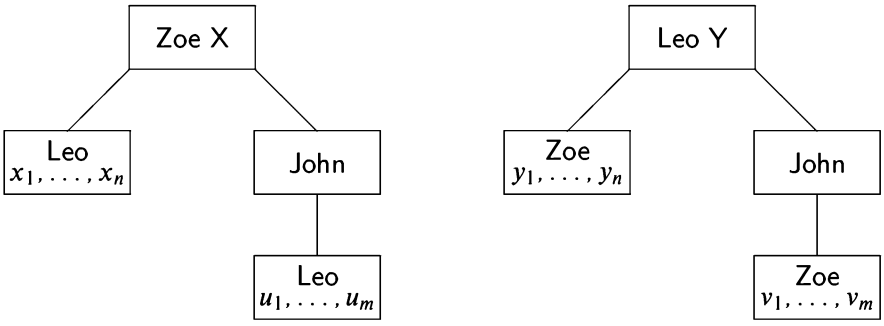
12.11 Impossibility with three generals^A

It can be proved [48, Section 6.4] that there is no algorithm that achieves consensus in the presence of byzantine failures when the number of generals is less than $3t+1$. Let us use knowledge trees to prove this for $t = 1$.

Theorem 12.3 With three generals, one of whom is a traitor, it is impossible to achieve consensus in the presence of Byzantine failures.

Proof: We first define what we mean by an algorithm for this problem. We assume that each general chooses a plan and sends one or more messages. For a loyal general, the messages it sends are either the plan that it chose or the plan received from another general. A traitor is allowed to send any messages whatsoever. We do not specify the number of rounds or the number of messages per round, but it doesn't really matter, because any message coming directly from a loyal general is correct and any message coming from a traitor can be arbitrary by the assumption of byzantine failures.

Let us assume that John is the traitor, that Zoe and Leo are loyal and they choose X and Y respectively. The knowledge trees for Zoe and Leo are:



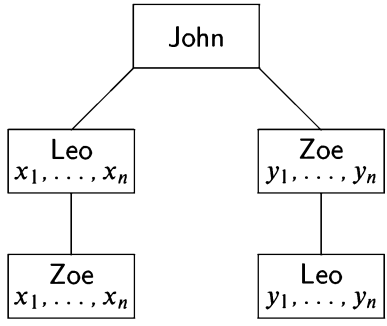
The decision by a loyal general is made applying a function f to the set of plans stored in local data or received as messages. For any algorithm to work, a loyal general must correctly infer the plans of the other loyal generals, so it must be true that:

$$f(\{x_1, \ldots, x_n\} \cup \{u_1, \ldots, u_m\}) = X, \tag{12.1}$$

$$f(\{y_1, \ldots, y_n\} \cup \{v_1, \ldots, v_m\}) = Y, \tag{12.2}$$

whatever the values of the sets $\{u_1, \ldots, u_m\}$ *and* $\{v_1, \ldots, v_m\}$.

Zoe and Leo must also come to a consensus on a plan for John. John sends a set of messages to Zoe and another set to Leo; both loyal generals relay the messages to the other. By "maliciously" choosing messages to send, the traitor can cause its knowledge tree to be:



Substituting $\{y_1, \ldots, y_n\}$ for $\{u_1, \ldots, u_m\}$ in (12.1) shows that Leo decides

$$f(\{x_1, \ldots, x_n\} \cup \{y_1, \ldots, y_n\}) = X,$$

and substituting $\{x_1, \ldots, x_n\}$ for $\{v_1, \ldots, v_m\}$ in (12.2) shows that Zoe decides

$$f(\{y_1, \ldots, y_n\} \cup \{x_1, \ldots, x_n\}) = Y.$$

Consensus is not achieved. ∎

Transition

There are many algorithms for consensus differing in their assumptions about the possible faults and in the importance assigned to various parameters of efficiency. We have described three algorithms: the original Byzantine Generals algorithm which is quite efficient in the presence of crash failures but much less so in the

presence of Byzantine failures; the King algorithm which showed that efficiency tradeoffs exist (better message efficiency in return for more processors per faulty processor); and the flooding algorithm which showed that crash failures are much easier to overcome than Byzantine failures.

The Byzantine Generals algorithm was originally developed during a project to build a reliable computing system for spacecraft. The reliability demands of real-time embedded systems have driven the development of constructs and algorithms in concurrent and distributed programming. Our final chapter will investigate the principles of real-time systems, especially as they pertain to issues of concurrent programming.

Exercises

1. On page 269, we claimed that if Zoe had sent attack instead of retreat to Basil, the decision would have been to attack. Work out the scenario for this case and draw the data structures.

2. We have been using the statement for all *other* generals. What would happen if a node sent its plan to itself?

3. (Ben-David Kolikant) In the Byzantine Generals algorithm, suppose that there is exactly one traitor and that Zoe's data structures are:

Zoe					
general	plan	reported by			majority
		Basil	John	Leo	
Basil	R		A	R	?
John	A	R		A	?
Leo	R	R	R		?
Zoe	A				A
					?

   (a) What can you know about the identity of the traitor?

   (b) Fill in the values marked ?.

   (c) Construct a minimal scenario leading to this data structure.

   (d) For this scenario, describe the data structures of the other generals.

4. Draw diagrams of data structures that are consistent with the knowledge trees on 270.

5. The Byzantine Generals algorithm can be generalized to achieve consensus in the presence of t traitors, if the total number of generals is at least $3t + 1$. For each additional traitor, an additional round of messages is exchanged. Implement the algorithm for seven generals, of whom two are traitors. Construct a faulty scenario for the case where there are only six generals.

6. Derive a formula for the maximum number of messages sent in the flooding algorithm for n nodes.

7. Prove that consensus is reached in the flooding algorithm even if a process crashes upon activation or after receiving a message, but before sending any messages of its own.

8. Give a scenario showing that the flooding algorithm is not correct if it is run for only t rounds instead of $t + 1$ rounds.

9. Construct a scenario for the King algorithm in which the initial decisions of the loyal generals are three for attack and one for retreat.

10. Suppose that two of five generals are traitors. Construct a scenario for the King algorithm in which consensus is not achieved. Draw diagrams of the data structures.

11. Construct scenarios that show the three cases discussed in Lemma 12.1 of the proof of correctness of the King algorithm.

13 Real-Time Systems

13.1 Introduction

A *reactive system* is one that runs continuously, receiving inputs from and sending outputs to hardware components. If a reactive system must operate according to strict constraints on its response time, it is called a *real-time system*. Modern computer systems are more likely than not to be reactive systems: a word processor can be considered to be a reactive system because it waits for input from hardware components (the keyboard and the mouse) and produces output on other hardware components (the screen and the hard disk). Nevertheless, the study of reactive systems in general, and real-time systems in particular, primarily concerns *embedded computer systems*, which have a computer as only one of a set of components that together comprise a system. Examples are: aircraft and spacecraft flight control systems, industrial plant controllers, medical imaging systems and monitors, and communications networks. Most of these systems are real-time systems, because given an input they must produce an output within a precisely-defined period of time, ranging from a few milliseconds for aerospace systems, to perhaps a few seconds for the controller of an industrial plant.

The difficulty in designing real-time systems lies in satisfying the strict requirement to meet processing deadlines. Here are two examples of challenging, complex computer systems which are *not* real-time:

- Simulation of the weather to develop a forecast. This requires the use of powerful supercomputers, but it is not a critical failure if the answers are received ten seconds or even ten minutes late.
- An airline reservation system. A large network of computers, databases and communications channels is needed to implement the system. The response time requirement for such a system is usually expressed statistically, rather than in terms of strict deadlines; for example, it may be required that 95% of the transactions are completed within 5 seconds and 99% within 15 seconds.

We don't want to imply that these systems shouldn't be designed to meet their deadlines all the time; however, there can be some flexibility in satisfying response-time

requirements if it would be prohibitively expensive to guarantee that all responses are produced within the target deadlines. A real-time system, on the other hand, need not be complex or high-performance. Checking the radioactivity levels in a nuclear power plant or monitoring the vital signs of a patient may require only the simplest processing, but a delay in decreasing the power level of the plant or in raising an alarm if the patient's vital signs deteriorate can literally be fatal.

This chapter surveys a range of topics concerning real-time systems, with emphasis on those related to synchronization issues. Detailed treatments of real-time systems can be found in the textbooks by Liu [47] and Burns and Wellings [19]. We start with a section that defines the temporal characteristics of real-time systems, followed by a description of two famous examples of problems that arose in real-time systems. Sections 13.4–13.6 present three different approaches to writing software for real-time systems: synchronous, asynchronous and interrupt driven. Almost invariably, some tasks performed by a real-time system are more important or more urgent than others. Priority assignments enable the designer to specify these constraints, but the interaction between priority and synchronization leads to a difficult problem called priority inversion. Section 13.7 discusses priority inversion, together with techniques to overcome it, priority inheritance and priority ceiling locking; Section 13.8 shows how priority inversion and inheritance can be modeled in Spin.

Real-time systems have requirements for moving data from processes reading sensors to processes computing algorithms and from there to processes controlling actuators. These are classical producer–consumer problems, but with the added requirement that blocking synchronization constructs like semaphores cannot be used. Section 13.9 presents Simpson's four-slot algorithm for ensuring wait-free atomic communication of data from a producer to a consumer.

The topic of languages and tools for specifying and implementing real-time systems is a current topic of research. Section 13.10 presents an overview of the Ravenscar profile, which is the specification of a subset of the Ada language designed for the performance requirements of real-time systems. Section 13.11 briefly introduces UPPAAL, a tool for verifying real-time algorithms specified as timed automata. The final section is a short introduction to scheduling algorithms.

Software problems occurring in spacecraft have a particularly high profile and are widely reported, and the danger and expense of exploring space ensure that such problems are carefully analyzed. We bring several examples of "bugs in space" that we hope will motivate the study of concurrency.

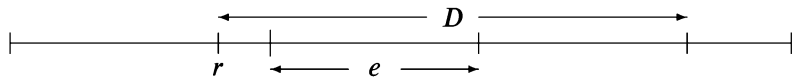
The Ada Real-Time Annex[L]

The Ada language specification contains several annexes that describe features of the language that go beyond a core common to all implementations. Throughout this chapter, we shall frequently mention the Real-Time Annex, which specifies constructs relevant to real-time programming. This makes Ada an excellent platform for building real-time systems, because real-time issues can be handled within the framework of a portable, standard language, rather than a specific operating system. For details, see [7, Chapter 16] and [19].

13.2 Definitions

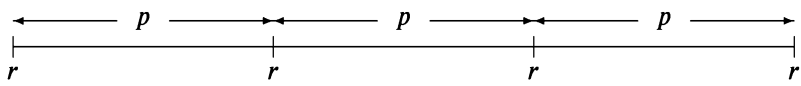
Processes in real-time systems are called *tasks*.

With each task in a real-time system is associated a set of parameters specifying its timing requirements. The *release time* r is the time when the task joins the ready queue of tasks ready to execute. The *execution time* e is the maximum duration of the execution of the task. The *response time* of a task is the duration of time from its release time r until it has completed its execution; the maximum response time allowed for a task is called its *(relative) deadline* D:

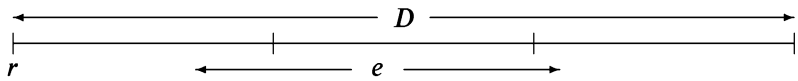


The response time for this execution of the task is measured from the release time r until the right end of the arrow for e indicating that the task is completed. Clearly, the response time is less than the deadline in this case.

Many tasks in a real-time system are *periodic*, meaning that there is a fixed interval between release times called the *period* p:



Real-time systems can be designed so that all tasks are released at the beginning of a period. A further simplification occurs when the deadline of a task is a multiple of its period, and if the periods of all tasks are multiples of the smallest period:



In addition to periodic tasks, a real-time system may have *sporadic* and *aperiodic* tasks which are released at arbitrary times. The difference between the two is that sporadic tasks have hard deadlines while aperiodic tasks have soft deadlines.

A task in a real-time system is categorized as having a *hard* or *soft* deadline, depending on the importance attached by the designer to meeting the deadline. (The terminology is extended so that a real-time system is hard if it contains tasks with hard deadlines.) The classification of a task is not simply a binary decision between hard and soft. Some tasks like those for flight control of an aircraft must meet their deadlines every time or almost every time. Of course it makes no sense to have a task that never need meet its deadline, but there are tasks like the self-test of a computer that can be significantly delayed without affecting the usefulness of a system. In the middle are tasks that can fail to meet their deadlines occasionally, or fail to meet them with a specified probability, for example, the task may fail to meet its deadline at most 5% of the time. An example would be the telemetry task of a spacecraft: an occasional delay in the transmission of telemetry data would not endanger the mission.

The designer of a real-time system has to decide what happens if a deadline is not met. Should the task continue executing, thereby delaying other tasks? Or should the task be terminated if it has not completed by its deadline (thereby freeing up CPU time for other tasks) because a late result is of no use?

13.3 Reliability and repeatability

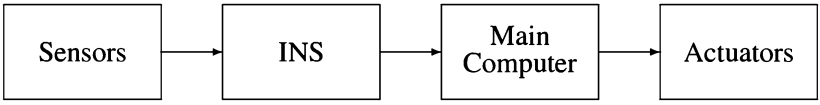
We have become tolerant of unreliable desktop software that crashes frequently. This attitude is simply unacceptable for embedded systems that control critical tasks, especially for systems that cannot be debugged. I call a computer system for a rocket launcher a "disposable computer" because you run the software once and then "throw away" the computer! There is no analog to pressing ctrl-alt-del. Two famous examples highlight the problems involved. The first launch of the Ariane 5 rocket failed because of a chain of mistakes related to issues of testing, verification and reliability. The first launch of the space shuttle was delayed because of a minor problem that had never occurred in countless hours of testing.

The Ariane 5 rocket

The first launch of the Ariane 5 rocket was carried out by the European Space Agency in 1996 from Kourou in Guyana. Thirty-seven seconds after launch, the rocket veered on its side and began to break up; it was then destroyed as a safety

measure. Investigation revealed that the failure was due to problems with the software that controlled the rocket.

The Ariane rocket uses an *inertial navigation system (INS)* to sense the position of the rocket. Data from motion sensors are processed by the INS and relayed to the main computer which issues commands to actuators that control the nozzles of the engines:



The sequence of events that led to the destruction of the Ariane 5 was as follows. A runtime error occurred during a data conversion operation in the INS, causing it to terminate execution. The error also occurred on the backup computer since both were executing the same program. The error was reported to the main computer which erroneously interpreted it as legal data; the main computer then gave an extreme command to the actuators, causing the rocket to go off course and break up.

The root of the software problem was traced to a decision to reuse the INS of the earlier Ariane 4 rocket. The new, larger rocket had a different trajectory than the Ariane 4, and one sensor value could no longer fit into 16-bits. To save CPU time, the value to be converted from 64- to 16-bits was not checked prior to the conversion, nor was there any provision for exception handling more sophisticated than reporting the error, nor had the omission of the check been justified on physical grounds. Finally, the INS software was not revalidated for the Ariane 5 under the assumption that it was unchanged from the software that had been validated for the Ariane 4 and had worked for many years.

The decision not to revalidate the software was not a simple oversight. Embedded systems cannot be tested as such: you can't launch thousands of rockets costing hundreds of millions of dollars each just to debug the software. Something has to be simulated somehow in a lab, and the better the fidelity of the simulation, the more it costs and the harder it is to perform. The difficult of testing software for embedded systems emphasizes the importance of both formal approaches to verification and the practices of software engineering. The full report of the investigating committee of the Ariane 5 failure has been published [45]; it is recommended reading for aspiring software engineers.

The space shuttle

Just before the first launch of the space shuttle, a fault occurred in the backup computer indicating that it could not receive data from the main computers. While the problem did not seem to be serious, the decision was made not to launch the new spacecraft as long as a fault was known to exist. The flight was delayed for two days until the problem was diagnosed [62].

The fault was eventually traced back to the scheduling algorithms used in the main computer. Two algorithms were used, one for periodic tasks related to flight control, and one for other, less critical, tasks. The algorithms should have been consistent, but investigation showed that in one case they were not, and this eventually caused the timing errors that were observed. The ultimate cause of the error was what can be called *degradation of assumptions*, where an initially valid assumption used by the programmers became incorrect as the software was modified.

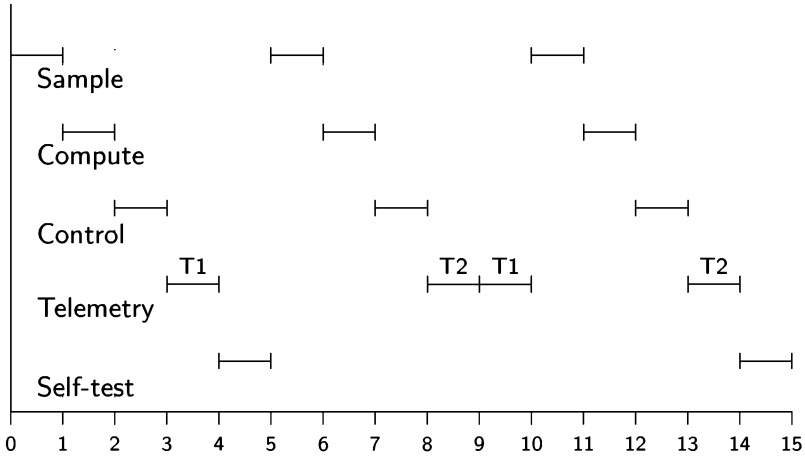
Strangely enough, the fault could only occur if one of the computers was turned on during a single 15 millisecond window of each second. The following diagram will give you an indication of how short that interval is:

The workaround to the fault was simply to turn the computer off and then back on again; there was only a 15 in 1000 (1 in 67) chance of falling into the window again. This also explains why the fault was not discovered in countless hours of testing; normally, you turn on a system and run hours of tests, rather than turning it on and off for each new test.

13.4 Synchronous systems

In a synchronous system, a hardware clock is used to divide the available processor time into intervals called *frames*. The program must then be divided into tasks so that every task can be completed *in the worst case* during a single frame. A scheduling table is constructed which assigns the tasks to frames, so that all the tasks in a frame are completed by the end of the frame. When the clock signals the beginning of the frame, the scheduler invokes the tasks as described in the table. If a task is too long to fit in one frame, it must be artificially split into smaller subtasks that can be individually scheduled.

The following diagram shows a typical, but extremely simple, assignment of tasks from an aircraft system to frames:



There are three critical flight-control tasks: Sample, Compute and Control; these tasks can each be executed within a single frame and they must be executed every five frames. We show two other tasks that are less critical: a Telemetry task that takes two frames and must be executed twice every fifteen frames, and a Self-test task that takes one frame and must be executed once every ten frames. We have managed to assign tasks to frames, breaking up the Telemetry task into two subtasks Telemetry 1 and Telemetry 2 that are scheduled separately. This of course requires the programmer to save the state at the end of the first subtask and restore it at the beginning of the second subtask.

Here is a scheduling table assigning tasks to frames:

0	1	2	3	4
Sample	Compute	Control	Telemetry 1	Self-test

5	6	7	8	9
Sample	Compute	Control	Telemetry 2	Telemetry 1

10	11	12	13	14
Sample	Compute	Control	Telemetry 2	Self-test

Algorithm 13.1 for scheduling the tasks is very simple. The assumption underlying this algorithm is that every task can be executed within a single frame. In fact, it has to take a bit less time than that to account for the overhead of the scheduler algorithm. Since the programming of a task may involve if and loop statements, it may not be possible to precisely calculate the length of time it takes to execute; therefore, it is prudent to ensure that there is time left over in a frame after a task

Algorithm 13.1: Synchronous scheduler	
	taskAddressType array[0..numberFrames-1] tasks ← [task address,. . . ,task address] integer currentFrame ← 0
p1:	loop
p2:	await beginning of frame
p3:	invoke tasks[currentFrame]
p4:	increment currentFrame modulo numberFrames

executes. The statement await beginning of frame is intended to ensure that each task starts when a frame begins, so that time-dependent algorithms can work as designed. It would normally be implemented as waiting for an interrupt from a hardware clock.

The simplicity of the synchronous scheduler is deceiving. All the hard work is hidden in dividing up the computation into tasks and constructing the scheduling table. These systems are very fragile in the sense that it is quite likely that as the system is modified, a task may become too long, and dividing it up may cause difficulties in re-assigning tasks to frames. A further disadvantage is that quite a lot of processor time may be wasted. The worst-case estimate of task duration must be used to ensure that it can be run within a single frame; if the average duration is much smaller, the processor will spend a lot of time waiting for the clock to tick. The frame duration is a basic parameter of the system. If it is made long to accommodate tasks of long duration, the system will not use processor time efficiently. If it is made short, it will become difficult to fit tasks into frames, and they must be divided into subtasks with the accompanying overhead and loss of reliability and maintainability.

On the surface, it may seem that synchronization is not a problem in synchronous systems, because at any time at most one task is executing and the order of execution of the tasks is known. For example, here is a system with a pair of producer–consumer algorithms:

Algorithm 13.2: Producer–consumer (synchronous system)					
queue of dataType buffer1, buffer2					
sample		**compute**		**control**	
	dataType d		dataType d1, d2		dataType d
p1:	d ← sample	q1:	d1 ← take(buffer1)	r1:	d ← take(buffer2)
p2:	append(d,buffer1)	q2:	d2 ← compute(d1)	r2:	control(d)
p3:		q3:	append(d2,buffer2)	r3:	

We assume that each task executes to completion during a frame, so there is no danger that one task might increment the count of elements in the buffer while another is decrementing it. Digging deeper, we find that this is far from true. First, tasks—like the Telemetry task in the example—may have to be split between frames. The programmer of the task cannot assume that a global variable retains its value from one subtask to another, because arbitrary tasks may be scheduled between the two. Second, the system is certain to undergo modification as time passes. We might initially assume that there is no need to synchronize the buffer used for passing data between the Sample task and the Compute task, and between the Compute task and the Control task, since they always occur one after another. This assumption may become invalid, or, more probably, it may be retained (because you don't want to modify code that has worked reliably for a long time), thus constraining future modifications to the program.

Synchronous systems are primarily used in control systems, where the algorithms require strict adherence to a timing specification. Feedback loops typically involve computations of the form $x_t = f(x_{t-1})$, where the value of a variable at time t depends on its value at time $t - 1$; the computation of the function f may not be very complex, but it is essential that it be carried out at precise intervals.

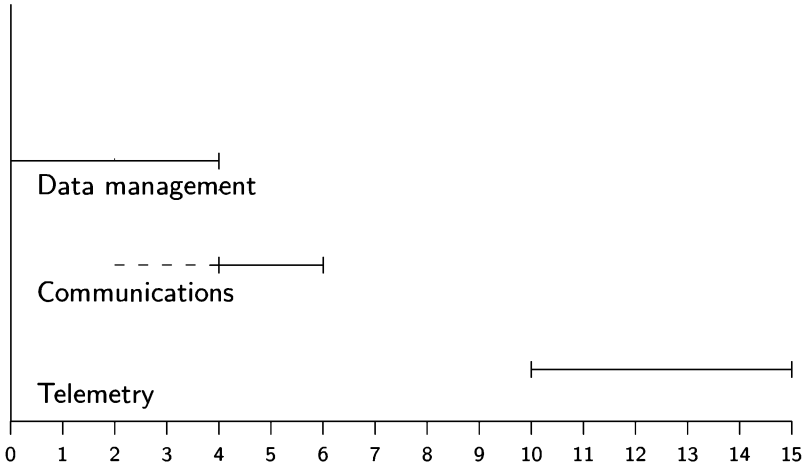
13.5 Asynchronous systems

In an asynchronous system tasks are not required to complete execution within fixed time frames. Instead, each task executes to completion and then the scheduler is called to find the next task, if any, ready to execute:

Algorithm 13.3: Asynchronous scheduler

```
                queue of taskAddressType readyQueue ← ...
                taskAddressType currentTask
    loop forever
p1:     await readyQueue not empty
p2:     currentTask ← take head of readyQueue
p3:     invoke currentTask
```

In the following example, the dashed line starting at time 2 indicates that the Communications task is ready to execute, but it must wait until the completion of the Data management task in order to begin execution. The Telemetry task is released at time 10 and can be executed immediately:



What we haven't shown is how tasks join the queue. One possibility is for one task to request that another be executed by adding it to the queue. This may occur, for example, as the result of receiving input data. Alternatively, a task may request that it be periodically executed, by adding itself to the queue together with an indication of the earliest next time at which it can be run. The scheduler, rather than simply removing and invoking the head of the queue, searches for a task whose time of next execution has passed.

Asynchronous systems are highly efficient, because no processor time is wasted: if a task is ready to run and the processor is free, it will be started immediately. On the other hand, it can be difficult to ensure that deadlines are met in an asynchronous system. For example, if the Communications task rescheduled itself to execute at time 11, it would have to wait until time 15 when the execution of the Telemetry task is completed.

Priorities and preemptive scheduling

Asynchronous systems of the type described are not practical for real-time systems, because a predefined limit would have to be set of the length of a task to ensure that important tasks can be executed when needed. There must be a way to ensure that more important tasks can be executed at the expense of less important tasks. This can done by defining priorities and using preemptive scheduling.

A priority is a natural number assigned to a task; higher-priority tasks are assumed to be more important or more urgent than lower-priority tasks. In *preemptive scheduling*, a task will not be executed if there is a higher-priority task on the ready queue. This is implemented by defining *scheduling events* that cause the

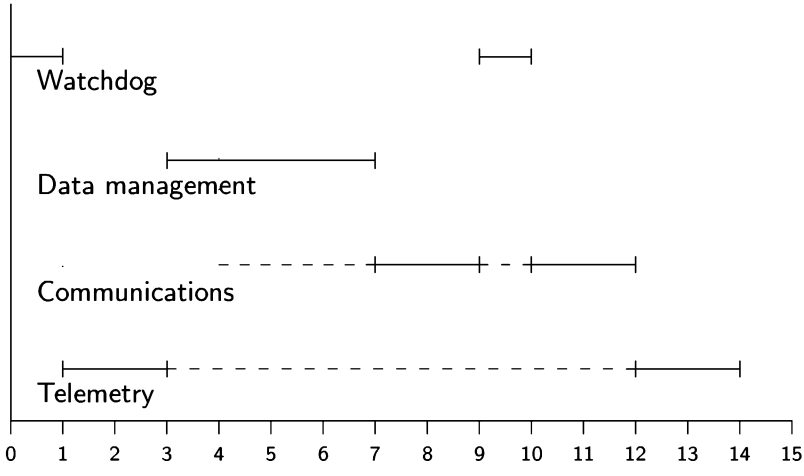
scheduler to be invoked. Whenever a task becomes ready by being appended to the queue, a scheduling event occurs. A scheduling event can also be forced to occur at other times—for example, as the result of a clock interrupt—to ensure a more fair assignment of processor time.

The preemptive scheduler checks the queue of ready tasks; if there is a task on the queue whose priority is higher than the currently running task, it is allowed to run:

Algorithm 13.4: Preemptive scheduler

```
            queue of taskAddressType readyQueue ← ...
            taskAddressType currentTask
    loop forever
p1:   await a scheduling event
p2:   if currentTask.priority < highest priority of a task on readyQueue
p3:      save partial computation of currentTask and place on readyQueue
p4:      currentTask ← take task of highest priority from readyQueue
p5:      invoke currentTask
p6:   else if currentTask's timeslice is past and
            currentTask.priority = priority of some task on readyQueue
p7:      save partial computation of currentTask and place on readyQueue
p8:      currentTask ← take a task of the same priority from readyQueue
p9:      invoke currentTask
p10:  else resume currentTask
```

Here is a timing diagram of the execution of tasks with preemptive scheduling:



In the diagram, higher priority is denoted by a higher position on the y-axis. The diagram shows the Telemetry task starting its execution at time 1, and then being preempted at time 3 by the Data management task. The dashed line starting at time 4 indicates that the Communications task is ready to execute, but it will not preempt the higher-priority Data management task. Only when it completes at time 7 will the Communications task begin executing, and only when it completes at time 12 will the low-priority Telemetry task finally be resumed.

The Watchdog is a task that is executed periodically with the highest priority in order to check that certain other tasks run sufficiently often. Suppose we wish to ensure that the important Data management task runs at least once every nine units of time. We define a global boolean variable ran which is set to true as the last statement of the Data management task:

Algorithm 13.5: Watchdog supervision of response time			
boolean ran ← false			
data management		**watchdog**	
	loop forever		loop forever
p1:	do data management	q1:	await ninth frame
p2:	ran ← true	q2:	if ran is false
p3:	rejoin readyQueue	q3:	response-time overflow
p4:		q4:	ran ← false
p5:		q5:	rejoin readyQueue

If the watchdog task executes twice without an intervening execution of the Data management task, the variable ran will remain false, indicating that the task has either failed to execute or failed to be completed within the required time span.

Algorithm 13.4 also shows how processor time can be shared among tasks of equal priority by time-slicing. Each task is granted a period of time called a *timeslice*; when a task has been computing at least for the duration of its timeslice, another ready task of equal priority (if any) is allowed to execute. As a rule, tasks of equal priority are executed in FIFO order of their joining the ready queue; this is called *round-robin scheduling* and it ensures that no task is ever starved for CPU time (provided, of course, that higher-priority tasks do not monopolize the CPU).

With proper assignment of priorities, preemptive schedulers do a good job of ensuring adequate response time as well as highly efficient use of processor time.

Asynchronous systems have to contend with difficult problems of synchronization. By definition, a preemptive scheduler can interrupt the execution of a task at any point, so the task cannot even assume that two adjacent statements will be executed one after another. In fact, this is an important justification for the definition of

the concurrency model in terms of arbitrary interleaving. Since we do not know precisely when an interrupt occurs, it is best to simply assume that it can occur at any time.

13.6 Interrupt-driven systems

An *interrupt* is the invocation of a software task by the hardware. The software task itself is called an *interrupt handler*. Since many real-time systems are embedded systems, they may be wholly or partially built around interrupts. In fact, it is convenient to consider an interrupt handler to be a normal task whose priority happens to be higher than that of any normal software task. This ensures that interrupt handlers can execute as critical sections with no danger of being interrupted by other tasks. If there are several interrupts, the hardware can be programmed to *mask* interrupts during the execution of an interrupt handler, thus ensuring mutual exclusion. Alternatively, interrupts themselves may have their own priorities that can be utilized in the design of the system: a higher-priority interrupt can be assured that it will not be preempted by a lower-priority one.

However, the problem remains of how the interrupt handlers synchronize and communicate with the software tasks. For example, suppose that an interrupt signals that data may be read from a temperature sensor; the interrupt handler will read the data and store it in a global buffer for use by another task. This is a producer–consumer problem and we must ensure synchronization between the two tasks. Of course, we could use a synchronization construct like a semaphore to ensure that the consumer does not read from an empty buffer and that the producer does not write to a full buffer. However, there is a lot of overhead in the use of semaphores, and, furthermore, semaphores are a relatively high-level construct that may not be appropriate to use in low-level interrupt handlers.

For these reasons, non-blocking producer–consumer algorithms are preferred for interrupt handling in real-time systems. One possibility is for the producer to throw away the new data if the buffer is full:

Algorithm 13.6: Real-time buffering—throw away new data

queue of dataType buffer ← empty queue

	sample		**compute**
	dataType d		dataType d
	loop forever		loop forever
p1:	d ← sample	q1:	await buffer not empty
p2:	if buffer is full do nothing	q2:	d ← take(buffer)
p3:	else append(d, buffer)	q3:	compute(d)

The compute task waits for data, either in a busy-wait loop or by being blocked. The sample task is invoked by an interrupt whenever data arrives, so there is the possibility that the buffer will be full. If so, it simply throws away the new data.

You should be quite familiar with throwing away new data: if you type on your keyboard so fast that the keyboard buffer fills up, the interrupt handler for the keyboard simply throws the data away (with an annoying beep). This alternative is used in communications systems that depend on acknowledgement of received messages, and retrying in the presence of errors. The system would be able to report that it has successfully received messages 1..n, but that, sorry, it threw away the later messages that now have to be resent.

Rather than throw away new data, real-time control systems overwrite old data:

Algorithm 13.7: Real-time buffering—overwrite old data

queue of dataType buffer ← empty queue

sample	compute
dataType d	dataType d
loop forever	loop forever
p1: d ← sample	q1: await buffer not empty
p2: append(d, buffer)	q2: d ← take(buffer)
p3:	q3: compute(d)

Interrupt overflow in the Apollo 11 lunar module

The absolutely higher priority given to interrupts over software tasks can cause problems, especially when they are invoked by hardware components over which the designer has little control. This was vividly demonstrated during the first landing on the moon. The radar of the lunar module was programmed to raise interrupts during the descent, but as the module neared the moon's surface, the number of interrupts increased so fast that that they tied up 15% of the resources of the computer system. Figure 13.1 shows what happened. The counter interrupts are unpredictable. The design presumed that they would not occur too often, but in this case, too many interrupts caused a Main task to be delayed and not to complete its execution before the check by the Watchdog.

In the case of the Apollo 11 landing module, the computer reinitialized itself three times during a 40-second period, causing a warning light to appear. Fortunately, NASA engineers recognized the source of the problem, and knew that it would not affect the success of the landing.

For more information about the Apollo computer system, see [66].

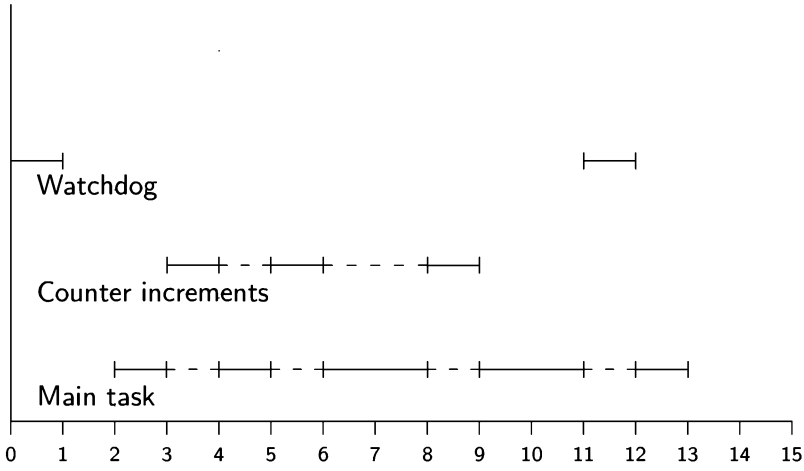
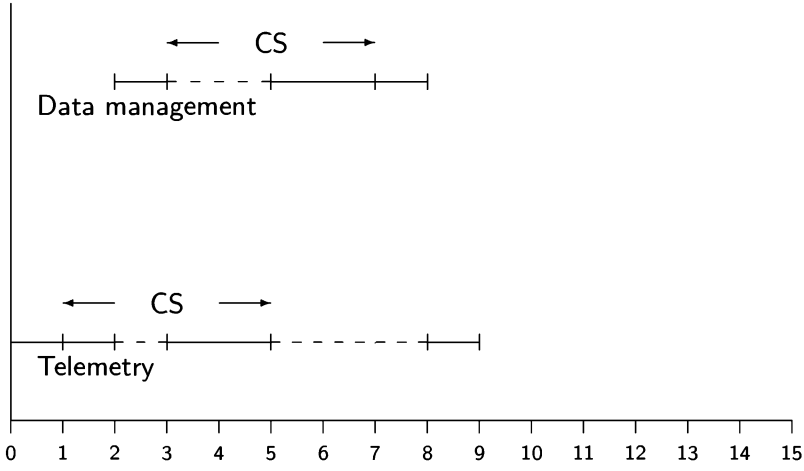


Figure 13.1: Interrupt overflow

13.7 Priority inversion and priority inheritance

Preemptive scheduling based upon priorities can interact with synchronization constructs in unforeseen ways; in particular, *priority inversion* [59] can arise. Consider the following timing diagram:

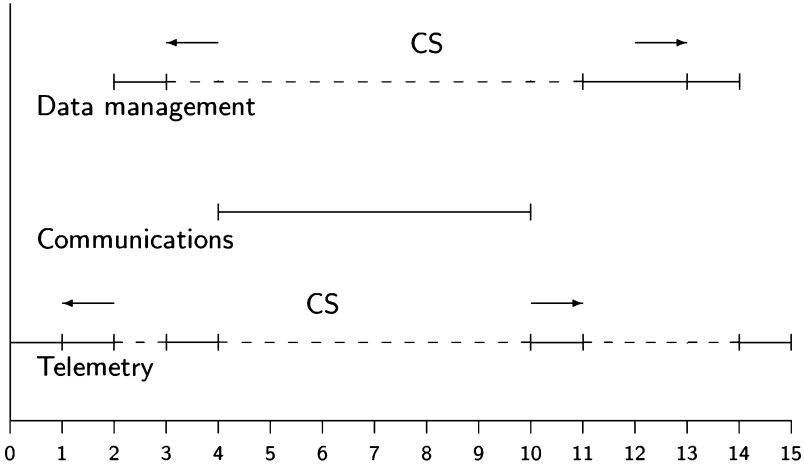


The low-priority Telemetry task begins executing and at time 1 enters its critical section. At time 2, it is preempted by the high-priority Data management task.

This poses no problem until time 3, when the Data management task attempts to enter a critical section that is to be executed under mutual exclusion with the critical section of the Telemetry task. The Data management task will block at the entry to its critical section, releasing the processor. The Telemetry task now resumes its critical section which completes at time 5. The Data management task can now preempt it and execute to completion, at which time the Telemetry task can be resumed.

At first glance, this seems to be a design defect, because a low-priority task is being executed in preference to a high-priority task, but there is really no other option. By definition, if a task starts a critical section, it must complete it before another task can start its critical section. Real-time systems designers must take to heart the suggestion that critical sections should be as short as possible. If critical sections are short, perhaps only taking a value from a buffer, then their maximum durations are easy to compute and take into account when analyzing if the response time requirements of the system are satisfied.

Priority inversion results if there is a third task whose priority is between those of the other two tasks, and which does not attempt to enter the critical section:



When the Communications task becomes ready at time 4, it preempts the Telemetry task and executes to completion at time 10; then the Telemetry task can resume its critical section. Only at time 11 when the critical section is exited can the high-priority Data management task continue. Its completion at time 14 will be way past its deadline and this will be detected by a Watchdog task (not shown in the diagram).

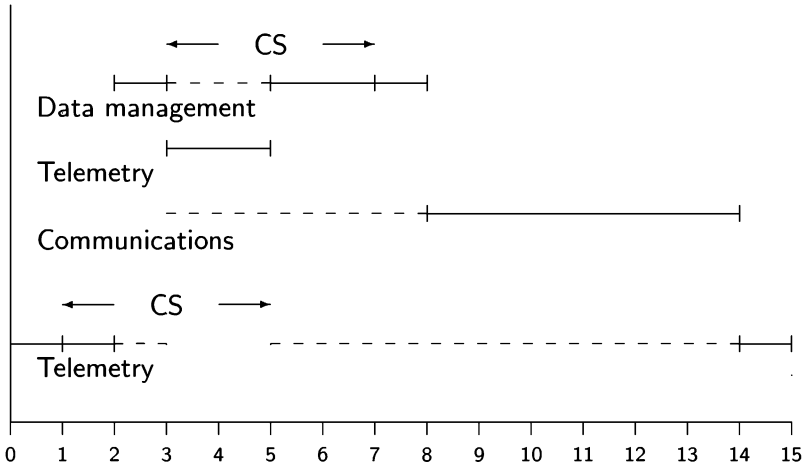
Since the medium-priority task may execute for an arbitrarily long amount of time, the high-priority task is delayed for this amount of time, defeating the concept of

preemptive scheduling. Priority inversion must be prevented so that the analysis of the performance of a real-time system can be done independently for each priority level, considering only tasks with higher priorities and the time spent executing critical sections.

Priority inheritance

Priority inversion can be solved by a mechanism called *priority inheritance*. Whenever a task p is about to be blocked waiting to enter a critical section, a check is made if a lower-priority task q is in its critical section; if so, q's priority is temporarily raised to be greater than or equal to the priority of p. Task q is said to *inherit* the priority of task p, enabling q to complete its critical section at a high priority, so that p will not be unnecessarily delayed.

In the following diagram, when the Data management task tries to enter its critical section at time 3, the Telemetry task will have its priority raised to be equal to the priority of the Data management task:



(For clarity in the diagram, this part of the time line of the Telemetry task is shown slightly below that of the Data management task.)

During the period from time 3 to time 5, the Telemetry task executes its critical section at the higher priority, and therefore is not preempted by the Communications task. Upon completion of the critical section at time 5, it relinquishes the higher priority and settles down to wait until time 14 when the other two tasks have been completed.

Priority inversion from queues

Priority inversion can also be caused by tasks waiting on queues of monitors or protected objects (Section 7.10). Normally these queues are implemented as FIFO queues. This means that a low-priority task could be blocked on a queue ahead of a high-priority task. The Real-Time Annex of Ada enables you to specify **pragma** Priority_Queuing; queues associated with entries of protected objects are then maintained in order of priority. However, if the queues are long, the overhead associated with maintaining these ordered queues may be significant.

Priority ceiling locking

For real-time systems with a single CPU, priority inversion can be avoided using an implementation technique called *priority ceiling locking*. This technique is appropriate when synchronization is performed by monitors or protected objects, where the statements that must be executed under mutual exclusion are clearly identified as belonging to a specific software component.

In priority ceiling locking, such component is assigned a *ceiling priority* which is greater than or equal to the highest priority of any task that can call its operations. When a task does call one of these operations, it inherits the ceiling priority. By definition, no other task that can invoke an operation of the component has the same or a higher priority, so by the normal rules of preemptive scheduling, no other task will ever preempt the task during its execution of the operation. This is sufficient to prevent priority inversion.

Not only will there be no priority inversion, but ceiling priority locking implements mutual exclusion without any need for additional statements! This follows from the same consideration as before: no task can ever preempt a task executing at the ceiling priority.

The Real-Time Annex of the Ada programming language specifies that ceiling priorities must be implemented for protected objects.

The Mars Pathfinder

The description of priority inversion in this section was adapted from a problem that occurred on the Mars Pathfinder mission. This small spacecraft landed on Mars in 1997 and—although designed to last only one month—continued to work for three months, sending back almost ten thousand pictures and large numbers of measurements of scientific data. The computer on the spacecraft used a real-time

operating system with preemptive scheduling, and global data areas were protected by semaphores.

The Pathfinder and its computer system were extremely successful, but there was an incident of priority inversion. The symptom of the problem was excessive resets of the software, causing a loss of valuable data. Using a duplicate system on the ground, the problem was finally diagnosed as priority inversion. Fortunately, the operating system implemented priority inheritance and the software was designed to allow remote modification. Once priority inheritance was specified, there were no more resets causing loss of data. For further details, see [35].

13.8 The Mars Pathfinder in Spin^L

While Spin cannot model the timing aspects of real-time systems, it can nevertheless be used to model qualitative aspects of such systems. Conceptually, the problem that occurred in the Mars Pathfinder is that a "long" computation in a task of medium priority was able to block the completion of a "short" computation in the critical section of a lower-priority task; it is not necessary to define precisely what is meant by "long" and "short."

Listing 13.1 shows a Promela model of the Pathfinder software. There are three processes: Data and Telem which alternate between executing critical and non-critical sections, and Comm which executes a long computation. The state of each process is stored in the corresponding variables data, telem and comm. The values of these variables are taken from the **mtype** definition and include idle and blocked which model the scheduler, and three states to model the various computational states: nonCS, CS and long.

The implementation of entering and exiting the critical section is shown in Listing 13.2. The two operations are defined as **atomic** so that no interleaving can occur between the operations on the binary semaphore and the change in the state of the process.

In order to model priority scheduling, we use the **provided** clause of Promela. When a process declaration is suffixed with a **provided** clause, the expression in the clause is implicitly added as a constraint on the execution of any statement in the process. We have defined the symbol ready(p) for the expression that checks that the state of process p is neither idle nor blocked. The **provided** clause for the medium-priority process Comm ensures that a statement of the process can only execute if the Data process is *not* ready, and similarly, the **provided** clause for the low-priority process Telem ensures that a statement of the process can only execute if both other processes are *not* ready.

```
1  mtype = { idle, blocked,  nonCS, CS, long };
2  mtype data = idle, comm = idle, telem = idle;
3  #define ready(p)  (p != idle  && p != blocked)
4
5  active proctype Data() {
6      do
7      ::  data = nonCS;
8          enterCS(data);
9          exitCS(data);
10         data = idle;
11     od
12 }
13
14 active proctype Comm() provided (!ready(data)) {
15     do
16     ::  comm = long;
17         comm = idle;
18     od
19 }
20
21 active proctype Telem()
22         provided (!ready(data) && !ready(comm)) {
23     do
24     ::  telem = nonCS;
25         enterCS(telem);
26         exitCS(telem);
27         telem = idle;
28     od
29 }
```

Listing 13.1: A Promela program for the Mars Pathfinder

What can this model show about the correctness of the program? Suppose we add the assertion:

```
assert( ! (telem == CS) );
```

between lines 7 and 8 of the program. This claims that when the high-priority process wants to enter its critical section, the low-priority process cannot be in its critical section. Clearly, this assertion is false, but that is not considered to be a problem, because critical sections are designed to be short.

```
1  bit sem = 1;
2  inline enterCS(state) {
3      atomic {
4          if
5          :: sem == 0 ->
6              state = blocked;
7              sem != 0;
8          :: else ->
9          fi ;
10         sem = 0;
11         state = CS;
12     }
13 }
14
15 inline exitCS(state) {
16     atomic {
17         sem = 1;
18         state = idle
19     }
20 }
```

Listing 13.2: Entering and exiting the critical section

Unfortunately, however, the following assertion is also false:

```
assert( ! (telem == CS && comm == long) );
```

Not only is the low-priority process Telem in its critical section, but it will not be allowed to execute because the medium-priority process Comm is executing a "long" computation. You can check this by running Spin in its interactive simulation mode. Execute a few steps of Telem until it completes execution of enterCS and then execute one step of Comm. A few steps of Data lead now to the following system state:

```
data = blocked, comm = long, telem = CS, sem = 0
```

The interactive simulation offers the following two choices (using the format of the jSpin display):

```
choice 1: 1 (Comm) [comm = idle]
choice 2: 0 (Data) [((! sem=0))]
```

Choice 1 represents the medium-priority process Comm completing its long computation and returning to the idle state, while Choice 2 represents the high-priority process Data evaluating the semaphore and continuing execution if it is non-zero. We have priority inversion: process Telem, which is the only process that can set the semaphore to a nonzero value, is not among the processes that can be selected to run because the medium-priority Comm is executing a long computation.

The problem can be solved by priority inheritance. This can be modeled by defining the symbol:

```
#define inherit(p) (p == CS)
```

and changing the **provided** clauses to:

```
active proctype Comm()
    provided (!ready(data) && !inherit(telem))

active proctype Telem()
    provided (!ready(data) &&
              (! ready(comm) || inherit (telem)))
```

This will ensure that the low-priority process Telem has the highest priority when it is in its critical section. Executing a safety verification run in Spin proves that the assertion

```
assert( ! (telem == CS && comm == long) );
```

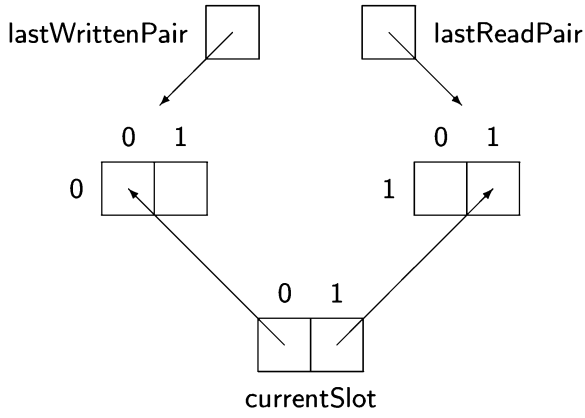
is never violated.

13.9 Simpson's four-slot algorithm^A

In Section 2.9 we discussed *atomic* variables, which are variables that can be read from and written to in a single CPU instruction. On most computer systems, the hardware provides for atomicity at the level of bits or words, so that if we want atomic access to multiword variables (like records) a software mechanism must be provided. For communicating values from interrupts, or more generally in real-time systems, the use of synchronization mechanisms like semaphores and monitors is not appropriate, because they can block the tasks. Here we present Simpson's algorithm [61],[1] which enables consistent access to memory by a reading task and a writing task without blocking either one.

[1]More precisely, this is called Simpson's *asynchronous communication mechanism*.

The data structures of the algorithm are illustrated in the following diagram:



At the center are shown four *slots*, variables of the type that must be passed from the writer to the reader. The slots are arranged in two pairs of two slots each. The array currentSlot contains the indices of the current slot within each pair; in the diagram, the current slot of pair 0 is 0 and the current slot of pair 1 is 1.

In Algorithm 13.8, the two-dimensional array data stores the four slots; the first index is the pair number and second is the slot number within the pair. Note that the indices are bit values and the complement of b is computed as $1 - b$.

p1, p4 and p11, p12 are the normal statements of a solution to the producer–consumer problem. The rest of the algorithm is devoted to computing indices of the pairs and slots.

If a writer task is not active, clearly, the reader tasks will continue to read the freshest data available. If a reader task is not active, the writer task uses the slots of one of the pairs alternately, as shown by the assignment to writeSlot in p3. The pair index is computed in p2 as the complement of lastReadPair and after the write is stored in lastWrittenPair.

Consider now what happens if a write operation commences during the execution of a read operation. lastReadPair contains the index of the pair being read from and the writer writes to its complement; therefore, it will not interfere with the read operation and the reader will obtain a consistent value. This holds true even if there is a sequence of write operations during the read operation, because the values will be written to the two slots of 1−lastReadPair. Although the reader may not read the latest value, it will obtain a consistent value.

What happens if a read operation commences during the execution of a write operation? "During the execution of a write operation" means after executing p2

Algorithm 13.8: Simpson's four-slot algorithm

```
        dataType array[0..1,0..1] data ← default initial values
        bit array[0..1] currentSlot ← { 0, 0 }
        bit lastWrittenPair ← 1, lastReadPair ← 1
    writer
    bit writePair, writeSlot
    dataType item
    loop forever
p1:     item ← produce
p2:     writePair ← 1− lastReadPair
p3:     writeSlot ← 1− currentSlot[writePair]
p4:     data[writePair, writeSlot] ← item
p5:     currentSlot[writePair] ← writeSlot
p6:     lastWrittenPair ← writePair
    reader
    bit readPair, readSlot
    dataType item
    loop forever
p7:     readPair ← lastWrittenPair
p8:     lastReadPair ← readPair
p9:     readSlot ← currentSlot[readPair]
p10:    item ← data[readPair, readSlot]
p11:    consume(item)
```

but before executing p6. It is possible that lastReadPair and lastWrittenPair have different values, so that after the writer executes p2 and the reader executes p7, both writePair and readPair contain the same value (exercise). But p3 and p9 ensure that the reader and writer access separate slots within the pair so there is no interference.

All the values in the algorithm, except for the data type being read and written, are single bits, which can be read and written atomically. Furthermore, only one task writes to each global variable: lastReadPair by the reader and the others by the writer. The overhead of the algorithm is fixed and small: the memory overhead is just four bits and the three extra slots, and the time overhead is four assignments of bit values in the writer and three in the reader. At no time is either the reader or the writer blocked.

A Promela program for verifying Simpson's algorithm is included in the software archive.

13.10 The Ravenscar profile^L

There is an obvious tradeoff between powerful and flexible constructs for concurrency, and the efficiency and predictability required for building real-time systems. Ada—a language designed for developing high-reliability real-time systems—has a very rich set of constructs for concurrency, some of which are not deterministic and are difficult to implement efficiently. The designer of a hard real-time system must use only constructs known to be efficiently implemented and to have a predictable execution time.

The *Ravenscar profile* [17] is a specification of a set of concurrency constructs in Ada to be used in developing hard real-time systems. The intention is that the profile be accepted as a standard so that programs will be portable across implementations. Note that the portability refers not to the language constructs themselves, which are already standardized, but to the assurance that this set of constructs will be efficiently implemented and that their execution times will be fixed and predictable for any particular selection of hardware and runtime system. This predictability ensures that programs developed according to the profile can be scheduled using real-time scheduling algorithms such as those described in the next section.

Another aim of the Ravenscar profile is to enable verification of programs by static analysis. Static analysis refers to techniques like model checking and SPARK (Appendix B.4) that are designed to verify the correctness of a program without execution and testing. There are also tools that can statically analyze a scheduling algorithm and the timing constraints for a set of tasks (periods, durations and priorities), in order to determine if hard deadlines can be met. The examples of space system given in this chapter show that testing by itself is not sufficient to verify real-time systems.

To make a system amenable to real-time scheduling algorithms, the Ravenscar profile ensures that the number of tasks is fixed and known when the software is written, and further that tasks are non-terminating. Tasks must be statically declared and after initiation must consist of a non-terminating loop. The scheduler of the runtime system is specified to use **pragma** FIFO_Within_Priorities, meaning that the running task is always one with the highest priority and that it runs until it is blocked, at which point other tasks with the same priority are run in FIFO order.

The rendezvous construct (Section 8.6) is incompatible with the requirements for predictability, so all synchronization must be carried out using low-level constructs: variables defined with **pragma** Atomic (Section 2.9), suspension objects (see below) or a very restricted form of protected objects (Section 7.10).

The requirement for predictability imposes the following restrictions on protected objects: only one entry per protected object, only one task can be blocked upon an entry and a barrier must contain only boolean variables or constants. Interrupt handlers can be written as procedures in protected objects. The runtime system must implement **pragma** Ceiling_Locking to ensure that mutual exclusion in access to a protected object is implemented efficiently by priority ceiling locking rather than by blocking. Static analysis can then ensure that deadlines are met because the delay in entering a protected object depends only on the fixed characteristics of tasks with higher priority.

Clearly, these restrictions severely limit the possible ways of writing concurrent programs in Ada (see [17] for examples of programming style according to the Ravenscar profile). But it is precisely these restrictions that make it possible to verify the correct performance of hard real-time systems.

A recent development is the integration of the Ravenscar profile into the SPARK language and toolset [5].

Suspension objects

Suppose that we just want a task to signal the occurrence of one event in order to unblock another task waiting for this event. Of course we could use a binary semaphore or a protected object:

Algorithm 13.9: Event signaling	
binary semaphore s ← 0	
p	**q**
p1: if decision is to wait for event p2: wait(s)	q1: do something to cause event q2: signal(s)

The problem is that a semaphore is associated with a set or queue of blocked tasks, and a protected entry can also have parameters and statements. The overhead of executing a semaphore wait statement or the barrier and body of an entry call is inappropriate when the requirement is for a simple signal. Furthermore, the barrier of a protected entry is limited in the kinds of conditions it can check.

The Real-Time Annex of Ada defines a new type called a *suspension object* by the following package declaration:

```
package Ada.Synchronous_Task_Control is
  type Suspension_Object is limited private;
  procedure Set_True(S : in out Suspension_Object);
  procedure Set_False(S : in out Suspension_Object);
  function Current_State(S : Suspension_Object)
    return Boolean;
  procedure Suspend_Until_True(
    S : in out Suspension_Object);
private
  -- not specified by the language
end Ada.Synchronous_Task_Control;
```

The suspension object itself can be implemented with a single bit that is set to false by a task wishing to wait and then set to true by a task that signals.

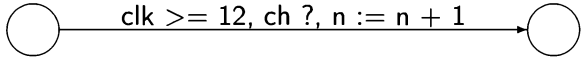
Algorithm 13.10: Suspension object—event signaling	
Suspension_Object SO ← false (by default)	
p	**q**
p1: if decision is to wait for event p2: Suspend_Until_True(SO)	q1: do something to cause event q2: Set_True(SO)

Suspend_Until_True(SO) causes the task to block until SO is true; it then resets SO to false, in preparation for the next time it needs to wait for the event.

13.11 UPPAAL^L

Software tools have been developed to aid in the specification and verification of real-time systems. UPPAAL is similar to Spin, performing simulations and model checking on system specifications. However, it uses *timed automata*, which are finite-state automata that include timing constraints. In UPPAAL, specifications are written directly as automata, rather than as programs in a language like Promela. The automata may be written as text or described using a graphical tool.

A system is specified as a set of tasks, one automaton for each task. There are declarations of global variables for clocks, channels and integer numbers. The automata are built up from states and transitions. Here is an example of a transition:



The first expression is a guard on the transition, which will only be taken when the guard evaluates to true. The guard also includes an input operation on a channel; communication between tasks is synchronous as with rendezvous in Promela. When the transition is taken, the assignment statement (called a reset) is executed.

Correctness specifications are expressed in *branching temporal logic*.

For more details see [43] and the UPPAAL website.

13.12 Scheduling algorithms for real-time systems

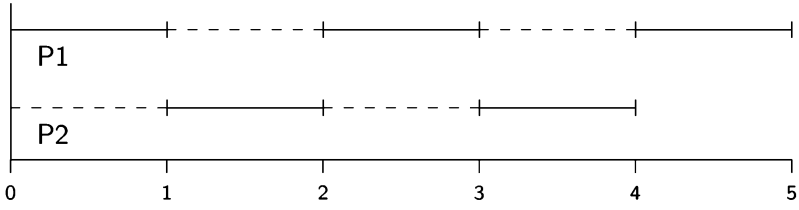
Asynchronous systems have the advantage of flexibility and high utilization of the CPU. In such systems, priorities must be assigned to tasks, and a preemptive scheduler ensures that the processor is assigned to a task with the highest priority. The system designer must be able to assign priorities and to prove that all tasks will meet their deadlines; if so, the assignment is called *feasible*. There are various scheduling algorithms and a large body of theory describing the necessary and sufficient conditions for feasible assignments to exist. In this section, we present two of these algorithms, one for a fixed assignment of priorities and the other for dynamic assignment of priorities.

Let us start with an example. Suppose that there are two tasks P1 and P2, such that

$$(p_1 = D_1 = 2, e_1 = 1), \ (p_2 = D_2 = 5, e_2 = 2).$$

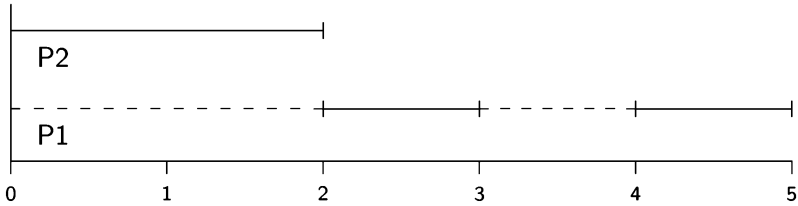
P1 needs to be executed every two units of time and it requires one unit during that period, while P2 requires two units in every five-unit period.

There are two possibilities for assigning priorities: P1 can have a higher priority than P2 or vice versa. The first priority assignment is feasible. P1 completes its execution during the first unit of the first two-unit interval, leaving an an additional unit for P2. At the end of two units, the higher-priority task P1 preempts P2 for its next interval, after which P2 continues with its second unit.



However, the second priority assignment is not feasible. If the priority of P2 is higher than that of P1, it will execute for a two-unit period during which P1 cannot

preempt it. Therefore, P1 has not received its one-unit period during this two-unit interval as required.



The rate monotonic algorithm

The *rate monotonic (RM)* algorithm assigns fixed priorities to tasks in inverse order of the periods p_i, that is, the smaller the period (the faster a task needs to be executed), the higher its priority, regardless of the duration of the task. In our example, $p_1 < p_2$, so task P1 receives the higher priority. Under certain assumptions, the RM algorithm is an optimal fixed-priority algorithm, meaning that if there exists a feasible fixed-priority assignment for a set of tasks, then the assignment given by the RM algorithm is also feasible [47, Section 6.4].

The earliest deadline first algorithm

The *earliest deadline first (EDF)* algorithm is an example of an algorithm based upon dynamic modification of task priorities. When a scheduling decision must be made, the EDF assigns the highest priority to the task with the closest deadline. The EDF algorithm is optimal: if it is feasible to schedule a set of tasks, then EDF also gives a feasible schedule. It can be proved that the algorithm is feasible if and only if the processor utilization of the set of tasks, defined as $\sum_i (e_i/p_i)$, is less than or equal to one [47, Section 6.3].

EDF is not always applicable because hardware tasks like interrupt routines may have fixed priorities. In addition, the algorithm may impose a serious overhead on the real-time scheduler, because the deadlines have to be recomputed at each scheduling event.

Transition

Concurrency has been a basic property of computer systems since the first multitasking operating systems. As the examples in this chapter have shown, even well-engineered and well-tested systems can fall victim to synchronization problems

that appear in unexpected scenarios. As computer systems become increasingly integrated into critical aspects of daily life, it becomes more and more important to ensure the safety and reliability of the systems.

Virtually every language or system you are likely to encounter will contain some variant of the constructs you have studied in this book, such as monitors or message passing. I hope that the principles you have studied will enable you to analyze any such construct and use it effectively.

The most important lesson that has been learned from the theory and practice of concurrency is that formal methods are essential for specifying, designing and verifying programs. Until relatively recently, the techniques and tools of formal methods were difficult to use, but the advent of model checking has changed that. I believe that a major challenge for the future of computer science is to facilitate the employment of formal methods in ever larger and more complex concurrent and distributed programs.

Exercises

1. Does the following algorithm ensure that the task compute executes once every period units of time?

Algorithm 13.11: Periodic task	
	constant integer period ← ...
	integer next ← currentTime
	loop forever
p1:	delay for (next − currentTime) seconds
p2:	compute
p3:	next ← next + period

   The function currentTime returns the current time and you can ignore overflow of the integer variables.

2. In Algorithm 13.4 for preemptive scheduling, a single ready queue is used; the algorithm searches for tasks of certain priorities on the queue. Modify the algorithm to use separate ready queues for each level of priority.

3. When a task is preempted, it is returned to the ready queue. Should it be placed at the head or tail of the queue (for its priority)? What about a task that is released from being blocked on a semaphore or other synchronization construct?

4. In Ada, you can specify that tasks are stored on the queues associated with entries of tasks and protected objects in order of ascending priority, rather than in FIFO order:

```
pragma Queuing_Policy(Priority_Queuing)
```

   Explain the connection between this and priority inversion.

5. Prove that protected objects can be implemented using priorities and ceiling priority locking with no additional locking.

6. Upon completion of a critical section, a low-priority task will lose its inherited priority and will likely be preempted by the high-priority task whose priority it inherited. Should the low-priority task be placed at the head or tail of the ready queue (for its priority)?

7. Construct a full scenario of Simpson's algorithm showing how readPair and writePair can get the same value without there being interference between the reader and the writer.

8. Solve the critical section problem using a suspension object.

   The following three exercises are taken from Liu [47]. In every case, $D_i = p_i$ and $r_i = 0$ unless otherwise indicated.

9. Let P1, P2 and P3 be three *nonpreemptable* tasks whose timing constraints are:

$$(p_1 = 10, e_1 = 3, r_1 = 0),$$
$$(p_2 = 14, e_2 = 6, r_2 = 2),$$
$$(p_3 = 12, e_3 = 4, r_3 = 4).$$

   Show that the schedule produced by the EDF algorithm is not feasible, but that there is a feasible schedule. Can this schedule be generated by a priority-driven scheduling algorithm?

10. Let P1, P2 and P3 be three tasks whose timing constraints are:

$$(p_1 = 4, e_1 = 1),\ (p_2 = 5, e_2 = 2),\ (p_3 = 20, e_3 = 5).$$

   Find the RM schedule for these tasks. At what times is the processor idle?

11. Let P1, P2 and P3 be three tasks. For which of the three following timing constraints is the RM algorithm feasible? For which is the EDF algorithm feasible?

$$\begin{array}{llll} \text{(a)} & (p_1 = 8, e_1 = 3), & (p_2 = 9, e_2 = 3), & (p_3 = 15, e_3 = 3), \\ \text{(b)} & (p_1 = 8, e_1 = 4), & (p_2 = 12, e_2 = 4), & (p_3 = 20, e_3 = 4), \\ \text{(c)} & (p_1 = 8, e_1 = 4), & (p_2 = 10, e_2 = 2), & (p_3 = 12, e_3 = 3). \end{array}$$

A The Pseudocode Notation

The algorithms are written in a pseudocode notation that is intended to be easy to translate into a programming language. It not only allows a language-independent presentation of the models and algorithms, but also enables us to abbreviate computations that are trivial for a programmer to supply. Examples are:

```
number[i] ← 1 + max(number)
```

in the bakery algorithm (Algorithm 5.2), and

```
for all other nodes
```

in the Byzantine Generals algorithm (Algorithm 12.2).

Structure

The title area gives the name and reference number of the algorithm. The following area is used for the declaration of global variables. Then come the statements of the processes. For two processes, which are named p and q, the statements are written in two columns. Algorithms for N processes are written in a single column. The code for each process is identical except for the ID of each process, written i in the concurrent algorithms and myID in the distributed algorithms.

In the distributed algorithms, the algorithm is given for each *node*. The node itself may contain several processes which are denoted by giving them titles; see Algorithm 11.3 for an example.

Monitors and other global subprograms that can be executed by any process appear between the global data declarations and the statements of the processes.

Syntax

Text from the symbol // until the end of a line is a comment.

Data declarations are of the form:

```
type-name variable-name ← initial value
```

The most common type is integer, which represents any integer type; implementations can use other types such as **byte** or **short**. In some algorithms, like producer–consumer algorithms, a data type for arbitrary elements is needed, but its precise specification is not important. In these cases, a name ending in Type is used, usually dataType.

Arrays are declared by giving the element type, the index range and the initial values:

```
integer array [1.. n] number ← [0,...,0]
```

Elementary abstract data types like queues and sets are used as needed.

The arrow ← is used in assignments, so that we don't have to choose between languages that use = and those that use :=. In expressions, mathematical symbols like =, ≠ and ≤ are used.

Indentation indicates the substatements of compound statements.

if statements are conventional. We are quite flexible in the syntax of loop statements, from conventional while statements to abbreviations like do two times. loop forever is used in most concurrent algorithms to denote the indefinite looping of a process.

Semantics

Numbered statements represent *atomic operations* as in Promela; if a continuation line is needed, it is not numbered.

Assignment statements and expression evaluations are atomic statements. The atomicity of evaluating the expression in a condition includes the decision to take one branch or another; for example, in the statement if a<b then s1 else s2, if the value of a is in fact less than the value of b, evaluating a<b will cause the control pointer of the process to point to s1.

Note that loop forever, else and goto are not numbered; they serve only to specify the control flow and do not denote executable statements.

Synchronization constructs

The statement `await boolean-valued-expression` is an implementation-independent notation for a statement that waits until the expression becomes true. This can be implemented (albeit inefficiently) by a busy-wait loop that does nothing until the condition is true. Note that in Promela, an `await` statement can be implemented by simply writing an expression:

```
turn == 1
```

The notation `ch⇐x` indicates that the value of `x` is sent on the channel `ch`, and similarly, the notation `ch⇒y` means that the value of the message received from the channel `ch` is assigned to the variable `y`. In CSP and Promela these are denoted `ch!x` and `ch?y`, but the use of arrows clarifies the data flow.

B Review of Mathematical Logic

Mathematical logic is used to state correctness properties and to prove that a program or algorithm has these properties. Fortunately, the logic needed in this book is not very complex, and consists of the propositional calculus (which is reviewed in this appendix) and its extension to temporal logic (Chapter 4). Variables in concurrent algorithms typically take on a small number of values, so that the number of *different* states in a computation is finite and relatively small. If a complicated calculation is required, it is abstracted away. For example, a concurrent algorithm may calculate heat flow over a surface by dividing up the surface into many cells, computing the heat flow in each and then composing the results. The concurrent algorithm will *not* concern itself with the calculations using real numbers, but only with the division of the calculation into tasks, and the synchronization and communications among the tasks.

For more on mathematical logic in the context of computer science, see my textbook [9].

B.1 The propositional calculus

Syntax

We assume the existence of an unbounded set of atomic propositions $\{p, q, r, \ldots\}$, one unary operator $\neg$ (*not*), and four binary operators, $\vee$ (*disjunction*, *or*), $\wedge$ (*conjunction*, *and*), $\rightarrow$ (*implication*) and $\leftrightarrow$ (*equivalence*). A *formula* of the propositional calculus is constructed inductively as the smallest set such that an atomic proposition is a formula, and if f and g are formulas, then so are $(\neg f)$, $(f \vee g)$, $(f \wedge g)$, $(f \rightarrow g)$ and $(f \leftrightarrow g)$.

In an implication $(p \rightarrow q)$, p is called the *antecedent* and q is called the *consequent*.

As with expressions in programming languages, parentheses can be omitted by assigning a precedence to each operator: $\neg$ has the highest precedence, followed

in descending order by $\wedge$, $\vee$, $\rightarrow$ and $\leftrightarrow$. We can further omit parentheses since it follows from the definition of the semantics of formulas that all the binary operators are associative except for $\rightarrow$. Therefore, the meaning of the formula $(p \vee q) \vee r$ is the same as that of $p \vee (q \vee r)$, and it can be written $p \vee q \vee r$. However, $p \rightarrow q \rightarrow r$ must be written as $(p \rightarrow q) \rightarrow r$ or $p \rightarrow (q \rightarrow r)$, because the meanings of the two formulas are different.

Here are some examples of formulas of the propositional calculus:

$$p, \quad p \rightarrow (q \rightarrow p), \quad (\neg p \rightarrow \neg q) \leftrightarrow (q \rightarrow p), \quad (p \vee q) \rightarrow (p \wedge q), \quad p \rightarrow \neg p.$$

Semantics

The semantics of a formula is obtained by defining a function v, called an *assignment*, that maps the set of atomic propositions into $\{T, F\}$ (or $\{true, false\}$). Given an assignment, it can be inductively extended to an *interpretation* of a formula (which we will also denote by v) using the following rules:

A	$v(A_1)$	$v(A_2)$	$v(A)$
$\neg A_1$	T		F
$\neg A_1$	F		T
$A_1 \vee A_2$	F	F	F
$A_1 \vee A_2$	otherwise		T
$A_1 \wedge A_2$	T	T	T
$A_1 \wedge A_2$	otherwise		F
$A_1 \rightarrow A_2$	T	F	F
$A_1 \rightarrow A_2$	otherwise		T
$A_1 \leftrightarrow A_2$	$v(A_1) = v(A_2)$		T
$A_1 \leftrightarrow A_2$	$v(A_1) \neq v(A_2)$		F

A formula f is said to be *true* in an interpretation v if $v(f) = T$ and *false* if $v(f) = F$. Under the assignment $v(p) = T, v(q) = F$, the formula $p \rightarrow (q \rightarrow p)$ is true, as can be seen by inductively checking the interpretation using the above table: $v(q \rightarrow p) = T$ since $v(q) = F$ and $v(p) = T$, from which it follows that $v(p \rightarrow (q \rightarrow p)) = T$. On the other hand, the formula $(p \vee q) \rightarrow (p \wedge q)$ is false in the same interpretation, since $v(p \vee q) = T$ and $v(p \wedge q) = F$, which is precisely the one case in which the implication is false.

A formula f is *satisfiable* if it is true in *some* interpretation, and it is *valid* if it is true in *all* interpretations. Similar definitions can be given in terms of falsehood: A formula f is *unsatisfiable* if it is false in *all* interpretations, and it is *falsifiable* if it

is false in *some* interpretation. The two sets of definitions are *dual*: f is satisfiable if and only if $\neg f$ is falsifiable, and f is valid if and only if $\neg f$ is unsatisfiable. It is easy to show that a formula is satisfiable or falsifiable since you only have to come up with one interpretation in which the formula is true or false, respectively; it is difficult to show that a formula is valid or unsatisfiable since you have to check *all* interpretations.

We have already shown that $(p \vee q) \rightarrow (p \wedge q)$ is not valid (falsifiable) by finding an assignment in which it is false. We leave it to the reader to check that $p \rightarrow (q \rightarrow p)$ is valid, while $(p \vee q) \rightarrow (p \wedge q)$, though not valid, is satisfiable.

Logical equivalence

Two formulas are *logically equivalent* to each other if their values are the same under all interpretations. Logical equivalence is important, because logically equivalent formulas can be substituted for one another. This is often done implicitly, just as we replace the arithmetic expression $1 + 3$ by the expression $2 + 2$ without explicit justification.

The two formulas $p \rightarrow q$ and $\neg p \vee q$ are logically equivalent because they are both false in any interpretation in which $v(p) = T, v(q) = F$, and are both true under all other interpretations. Other important equivalences are *deMorgan's laws*: $\neg(p \wedge q)$ is logically equivalent to $\neg p \vee \neg q$ and $\neg(p \vee q)$ is logically equivalent to $\neg p \wedge \neg q$. For a more complete list, see [9, Section 2.4].

B.2 Induction

The concept of proof by numerical induction should be familiar from mathematics. Suppose that we want to prove that a formula $F(n)$ containing a variable n is true for all possible values of n in the non-negative integers. There are two steps to the proof:

(a) Prove $F(0)$. This is called the *base case*.

(b) Assume that $F(m)$ is true for all $m \leq n$ and prove $F(n + 1)$. This is called the *inductive step* and the assumption $F(m)$ for $m \leq n$ is called the *inductive hypothesis*.

If these are proved, we can conclude that $F(n)$ is true for all values of n. This form of induction is called *generalized induction*, because we assume $F(m)$ for *all* numbers m less than $n + 1$ and not just for n.

Let us use generalized induction to prove the formula $F(n)$: if n is a positive even number, then it is the sum of two odd numbers:

(a) We need to prove two base cases: $F(1)$ is trivially true because 1 is not even, and $F(2)$ is true because $2 = 1 + 1$.

(b) Let us assume the inductive hypothesis $F(m)$ for $m \leq n$ and prove $F(n+1)$. The proof is divided into two cases:

  (1) $n + 1$ is odd. In this case, the proof is trivial because the antecedent is false.

  (2) $n+1$ is even and $n+1 > 2$. But $n-1$ is also even and $n-1 \geq 2$, so by the inductive hypothesis, there are two odd numbers $2i + 1$ and $2j + 1$ such that $n - 1 = (2i + 1) + (2j + 1)$. A bit of arithmetic gives us:

  $$n+1 = (n-1)+2 = (2i+1)+(2j+1)+2 = (2i+1)+(2(j+1)+1),$$

  showing that $n + 1$ is also the sum of two odd numbers.

Computational induction is used to prove programs; the induction is on the states of a computation rather than on natural numbers, but the concept is similar because the states are also arranged in an increasing sequence.

Frequently, we will want to prove invariants, which are formulas that are valid in every state of a computation. These will be proved using induction on the computation: the base case is to show that the formula is true in the initial step; the inductive step is to show that if the formula is assumed to be true in a state, it remains true in any successor state.

B.3 Proof methods

Model checking

f is valid if and only if $\neg f$ is unsatisfiable, so in order to show that f is valid, search for an interpretation that satisfies $\neg f$. If you find one, f is falsifiable, that is, it is not valid; if not, $\neg f$ is unsatisfiable, so f is valid. In mathematical logic, this dual approach is used in important techniques for proving validity: semantic tableaux, analytic tableaux and resolution.

The advantage of this method is that the search can be systematic and efficient, so that it can be mechanized by a computer program. Software systems called *model*

checkers combine a (concurrent) program together with the negation of a logical formula expressing a correctness property, and then search for a satisfying scenario. If one is found, the correctness property does not hold, and the scenario produced by the model checker is a falsifying interpretation. If an exhaustive search turns up no such scenario, the program is correct.

Deductive proof

The more classical approach to showing that a formula is valid is to prove it. We assume that certain formulas, called *axioms*, are valid, and that certain *rules* can be used to infer new true formulas from existing ones. A *proof* is a sequence of formulas, each one of which is either an axiom, or a formula that can be inferred from previous formulas using a rule. A formula has been proved if it is the last formula in a proof sequence. For a proof system to be useful it must be *sound* and preferably *complete*: a proof system is sound if every formula it proves is valid and it is complete if every valid formula can be proved. There is an axiom system for propositional LTL that is sound and complete (see [9, Section 12.2]).

Material implication

Many correctness properties can be expressed as implications $p \rightarrow q$. An examination of the table on page 322 shows that the only way that $p \rightarrow q$ can be false is if $v(p) = T$ and $v(q) = F$. This is somewhat counterintuitive, because it means, for example, that the formula

$$(1 + 1 = 3) \rightarrow (1 + 1 = 2)$$

is true. This definition, called *material implication*, is the one that is used in mathematics.

Suppose that $p \rightarrow q$ is true in a state and that we want to prove that it remains true in any successor state. We need only concern ourselves with: (a) states in which $v(p) = v(q) = T$, but "suddenly" in the next state $v(q)$ becomes false while $v(p)$ remains true, and (b) states in which $v(p) = v(q) = F$, but "suddenly" in the next state $v(p)$ becomes true while $v(q)$ remains false. If we can show that neither of these situations can occur, the inductive step has been proved.[1]

If the correctness claim is an implication of the form "the computation is at statement p3 implies that *flag* = 1," the entire verification is reduced to two claims:

[1] It is also possible that $v(p) = F$ and $v(q) = T$, but " suddenly" $v(p)$ becomes true *and* $v(q)$ becomes false.

(a) any statement of process p leading to the control pointer pointing to p3 cannot be taken if $flag = 1$ is false or if the statement leads to a state in which $flag = 1$ becomes or remains true, and (b) if the control pointer of process p is at statement p3, then no other process can falsify $flag = 1$.

Psychologists have investigated the difficulties of reasoning with material implication using *Wason selection tasks*. Suppose that the following four cards are placed before you:

$p3$	$p5$	$flag = 1$	$flag = 0$

On one side of each card is written the current location of the computation (denoted $p3$ if the control pointer is at statement p3 and similarly for $p5$), and on the other side the value of $flag$. What is the smallest set of cards that you need to turn over to check that they all describe true instances of the claim "if the computation is at statement p3, then $flag = 1$"? The answer is that you need to turn over the first card (to ensure that the true antecedent is associated with a true consequent on the other side of the card) and the fourth card (to ensure that the false consequent is associated with a false antecedent on the other side of the card). If the antecedent is false (the second card), it doesn't matter what the consequent is, and if the consequent is true (the third card), it doesn't matter what the antecedent is.

B.4 Correctness of sequential programs

Let us assume that a program P contains only one read statement as its first statement, and one write statement as its last statement:

$$\mathsf{read(x1,x2,\ldots,xM)}, \quad \ldots, \quad \mathsf{write(y1,y2,\ldots,yN)}.$$

There are two definitions of correctness of P. Informally, they are:

Partial correctness *If* P halts, the answer is "correct."

Total correctness P *does* halt and with the answer is "correct."

These concepts can be formalized mathematically. We use the notation $\bar{x}$ and $\bar{y}$ to denote sequences of (mathematical) variables that contain the values of the (program) variables (x1,x2,...,xM) and (y1,y2,...,yN), and similarly for sequences of values $\bar{a}$ and $\bar{b}$. Let $A(\bar{x})$ be a logical formula with free variables $\bar{x}$ called the *precondition*, and $B(\bar{x}, \bar{y})$ be a logical formula with free variables $\bar{x}, \bar{y}$ called the *postcondition*.

Partial correctness

Program P is *partially correct with respect to* $A(\bar{x})$ *and* $B(\bar{x}, \bar{y})$ if and only if for all values of $\bar{a}$:

if $A(\bar{a})$ is true,
and $\bar{a}$ are the input values read into (x1,x2,. . . ,xM),
and P terminates writing out the values $\bar{b}$ of (y1,y2,. . . ,yN),
then $B(\bar{a}, \bar{b})$ is true.

Total correctness

Program P is *totally correct with respect to* $A(\bar{x})$ *and* $B(\bar{x}, \bar{y})$ if and only if for all values of $\bar{a}$:

if $A(\bar{x})$ is true,
and $\bar{a}$ are the input values read into (x1,x2,. . . ,xM),
then P terminates writing out the values $\bar{b}$ of (y1,y2,. . . ,yN),
and $B(\bar{a}, \bar{b})$ is true.

Consider the following program, and take a few minutes to try to understand what the program does before reading on:

Algorithm B.1: Verification example

```
                    integer x1, integer x2
                    integer y1 ← 0, integer y2 ← 0, integer y3
p1:  read(x1,x2)
p2:  y3 ← x1
p3:  while y3 ≠ 0
p4:      if y2+1 = x2
p5:          y1 ← y1 + 1
p6:          y2 ← 0
p7:      else
p8:          y2 ← y2 + 1
p9:      y3 ← y3 − 1
p10: write(y1,y2)
```

The program is partially correct with respect to $A(x1, x2)$ defined as *true* and $B(x1, x2, y1, y2)$ defined as

$$(x1 = x2 \cdot y1 + y2) \wedge (0 \leq y2 < x2).$$

The program computes the result and the remainder of dividing the integer $x1$ by the integer $x2$. It does this by counting down from x1 in the variable y3, adding one to the remainder y2 on each execution of the loop body. When the remainder would become equal to the divisor x2, it is zeroed out and the result y1 is incremented.

It is true that if zero or a negative number is read into $x2$, then the program will not terminate, but termination is not required for partial correctness; the only requirement is that *if* the program terminates, *then* the postcondition holds. On the other hand, the program *is* totally correct with respect to the precondition $A(x1, x2)$ defined as $(x1 \geq 0) \wedge (x2 > 0)$ and the same postcondition.

This emphasizes that it is always wrong to speak of a *program* as being correct. A program can only be correct with respect to its specification, here its pre- and postconditions.

The problem with the concepts of partial and total correctness is that they are appropriate only for programs that terminate. While there are concurrent programs that terminate (such as scientific simulations that use parallelism to compute the behavior of complex systems), most concurrent programs (such as operating systems and real-time controllers) are designed to be non-terminating. The term *reactive* is often used to emphasize that the program is designed to repeatedly react to external stimuli (such clicking a button on a mouse) or to repeatedly sample an external sensor. If one of these systems ever does terminate, we say that the computer has "crashed" and consider it a bug! A new definition of correctness is needed for concurrent programs.

Verification in SPARK^L

Most of the proof of the algorithm for integer division can be mechanically carried out by SPARK [5]. Listing B.1 shows the algorithm written in the SPARK language, which is a subset of Ada augmented with annotations. (Some of the variable names have been changed for clarity.) The annotation **derives**, together with the mode declarations **in** and **out** of the parameters, is used for checking information flow. **derives** states that the values of Q and R that are returned from the procedure are obtained from the values of X1 and X2 that enter the procedure.

The proof annotations are the precondition **pre** that states that the dividend X1 is non-negative and that the divisor X2 is positive, and the postcondition **post**

```
--# main_program;
procedure Divide(X1, X2: in Integer ; Q, R : out Integer )
--# derives Q, R from X1, X2;
--# pre (X1>=0) and (X2>0);
--# post (X1=Q*X2+R) and (X2>R) and (R>=0);
is
    N: Integer ;
begin
   Q := 0; R := 0; N := X1;
   while N /= 0
   --# assert (X1=Q*X2+R+N) and (X2>R) and (R>=0);
   loop
      if R+1 = X2 then
         Q := Q + 1; R := 0;
      else
         R := R + 1;
      end if;
      N := N - 1;
   end loop;
end Divide;
```

Listing B.1: A SPARK program for integer division

that states the relation among the variables required for the correctness of integer division. The **assert** annotation specifies what must be true upon entering the **while** loop, namely that the division relation hold as long as the value of N is added to Q*X2+R. Clearly, when leaving the loop, the value of N will be zero and the relation will hold among the variables that are the parameters of the procedure.

When SPARK is run, it verifies the information flow and then generates four *verification conditions* that must be proved in order to prove the partial correctness of the procedure. A verification condition is a formula that describes the execution of a program along a path in the program. The four conditions are:

- From the precondition at the beginning of the procedure until the assertion:

$$(X1 \geq 0) \wedge (X2 > 0) \rightarrow$$
$$(X1 = Q \cdot X2 + R + N) \wedge (X2 > R) \wedge (R \geq 0).$$

- From the assertion until the postcondition at the end of the procedure (using the knowledge that the **while** loop terminates, so $N = 0$):

$$(X1 = Q \cdot X2 + R + N) \wedge (X2 > R) \wedge (R \geq 0) \wedge (N = 0) \rightarrow$$
$$(X1 = Q \cdot X2 + R) \wedge (X2 > R) \wedge (R \geq 0).$$

- From the assertion, around the loop via the **then** branch, and back to the assertion (using the knowledge that the condition of the **if** statement is true):

$$(X1 = Q \cdot X2 + R + N) \wedge (X2 > R) \wedge (R \geq 0) \wedge (R + 1 = X2) \rightarrow$$
$$(X1 = Q' \cdot X2 + R' + N') \wedge (X2 > R') \wedge (R' \geq 0).$$

  The primed variables indicate denote the new values of the variables after executing the assignments in the loop body.

- From the assertion, around the loop via the **else** branch, and back to the assertion (using the knowledge that the condition of the **if** statement is false):

$$(X1 = Q \cdot X2 + R + N) \wedge (X2 > R) \wedge (R \geq 0) \wedge (R + 1 \neq X2) \rightarrow$$
$$(X1 = Q' \cdot X2 + R' + N') \wedge (X2 > R') \wedge (R' \geq 0).$$

The *simplifier* tool of SPARK can reduce the first three formulas to true. It does this by substituting the expressions for the variables in the assignment statements, and then simplifying the formulas using elementary knowledge of arithmetic. For example, substituting the initial values into the first formula gives:

$$(X1 \geq 0) \wedge (X2 > 0) \rightarrow$$
$$(X1 = 0 \cdot X2 + X1) \wedge (X2 > 0) \wedge (0 \geq 0).$$

It does not take much knowledge of arithmetic to conclude that the formula is true.

The condition that goes through the **else** branch cannot be proved by the simplifier. Substituting Q for Q', $R+1$ for R' and $N-1$ for N, the formula that must be proved is:

$$(X1 = Q \cdot X2 + R + N) \wedge (X2 > R) \wedge (R \geq 0) \wedge (R + 1 \neq X2) \rightarrow$$
$$(X1 = Q \cdot X2 + R + 1 + N - 1) \wedge (X2 > R + 1) \wedge (R + 1 \geq 0).$$

The first subformulas on either side of the implication are equivalent, so this reduces to:

$$(X2 > R) \wedge (R \geq 0) \wedge (R + 1 \neq X2) \rightarrow (X2 > R + 1) \wedge (R + 1 \geq 0).$$

But this is easy for us to prove. By the properties of arithmetic, if $X2 > R$ then $X2 = R + k$ for $k \geq 1$. From the antecedent we know that $R + 1 \neq X2$, so $k > 1$, proving that $X2 > R + 1$. Finally, from $R \geq 0$, it is trivial that $R + 1 \geq 0$.

Another component of SPARK, the *proof checker*, can be used to partially automate the proof of verification conditions such as this one that the simplifier cannot perform.

C Concurrent Programming Problems

This appendix contains a list of problems to be solved by writing concurrent programs. For completeness, the list includes problems presented in the text, together with a reference to the sections where they were first posed.

1. The critical section problem (Section 3.2).
2. The producer–consumer problem (Section 6.7).
3. The problem of the readers and writers (Section 7.6). When solving the problem, you must specify if you want to give precedence to readers or to writers, or to alternate precedence to ensure freedom from starvation.
4. Multiple readers and writers. In the problem of the readers and writers, the two classes of processes were distinguished by allowing multiple readers but not multiple writers. We now allow multiple writers. Develop an algorithm to coordinate reading and writing, paying special attention to freedom from starvation. (This problem is also known as the unisex bathroom problem and the baboon bridge-crossing problem.)
5. Conway's problem (Section 8.2).
6. The sleeping barber. Consider the following scenario for Algorithm 6.7 for the producer–consumer problem:

n	producer	consumer	Buffer	notEmpty
1	**append(d, Buffer)**	wait(notEmpty)	[ ]	0
2	**signal(notEmpty)**	wait(notEmpty)	[1]	0
3	append(d, Buffer)	**wait(notEmpty)**	[1]	1
4	append(d, Buffer)	**d ← take(Buffer)**	[1]	0
5	**append(d, Buffer)**	wait(notEmpty)	[ ]	0

Line 5 is the same as line 1 and the scenario can be extended indefinitely.

Every time that the consumer process executes wait, the value of the semaphore is nonzero so it never blocks. Nevertheless, both processes must execute semaphore operations, which are less efficient than ordinary statements. Develop an algorithm to solve the producer–consumer problem so that the consumer executes wait only if it actually has to wait for the buffer to be nonempty.

7. (Patil, Parnas) The cigarette smokers problem. The problem concerns three resources, three servers and three clients. Server S12 can supply resources r_1 and r_2, server S13 can supply resources r_1 and r_3, and server S23 can supply resources r_2 and r_3. Similarly, client C12 needs resources r_1 and r_2, client C13 needs resources r_1 and r_3, and client C23 needs resources r_2 and r_3.[1] The servers supply resources one at a time and notify accordingly, so clients cannot simply wait for pairs of resources to become available. For example, server S23 may execute signal(r2) to indicate that resource r_2 is available, but this may unblock client C12 waiting for its second semaphore or client C23 waiting for its first semaphore. The original statement of the problem asks for a solution using only semaphore operations without conditional statements.

8. (Manna and Pnueli) Computation of the binomial coefficient:

$$\binom{n}{k} = \frac{n!}{(n-k)! \cdot k!} = \frac{n \cdot (n-1) \cdot \cdots \cdot (n-k+1)}{1 \cdot 2 \cdot \cdots \cdot k}.$$

Let one process compute the numerator and the other the denominator. Prove that $i!$ divides $j \cdot (j+1) \cdot \cdots \cdot (j+i-1)$. Therefore, the numerator process can receive partial results from the denominator process and do the division immediately, keeping the intermediate results from becoming too big. For example, $1 \cdot 2$ divides $10 \cdot 9$, $1 \cdot 2 \cdot 3$ divides $10 \cdot 9 \cdot 8$ and so on.

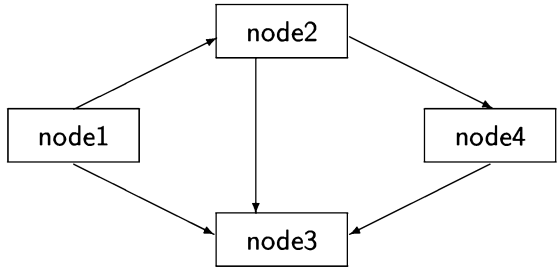
9. The dining philosophers (Section 6.9).

10. The roller-coaster problem. There are n people at an amusement park who can decide they want to ride the roller-coaster, or they can be engaged in other activities and may never try to ride the roller-coaster. A single-car roller-coaster ride begins its run only when exactly r passengers are ready. Develop an algorithm to synchronize the actions of the roller-coaster and the passengers. Generalize the program to multiple roller-coaster cars; however, the cars cannot pass each other.

[1] The problem was originally described in terms of smokers needing tobacco, paper and matches; in the interests of public health, it is presented in terms of abstract resources.

11. (Trono) The Santa Claus problem. Develop an algorithm to simulate the following system: Santa Claus sleeps at the North Pole until awakened by either all of the nine reindeer, or by a group of three out of ten elves. He then performs one of two indivisible actions: If awakened by the group of reindeer, Santa harnesses them to a sleigh, delivers toys, and finally unharnesses the reindeer who then go on vacation. If awakened by a group of elves, Santa shows them into his office, consults with them on toy R&D, and finally shows them out so they can return to work constructing toys. A waiting group of reindeer must be served by Santa before a waiting group of elves. Since Santa's time is extremely valuable, marshaling the reindeer or elves into a group must not be done by Santa.

12. Consider an arbitrary directed graph with no cycles and at least one source node, for example:



The graph is intended to represent precedence requirements: the computation at a node cannot begin until the computations at previous nodes have been completed. Develop an algorithm that takes a representation of a precedence graph as an input and generates a synchronization skeleton that ensures that the precedence requirements are respected.

13. Barriers.[2] In Section 6.6 we showed how to use semaphores to solve order-of-execution problems and the previous problem asked you to generalize this to arbitrary precedence relations. The *barrier* problem is another generalization, in which a group of processes have to repeatedly synchronize phases of their computation (Algorithm C.1). Barrier synchronization is important in parallel scientific computation on multiprocessors. Solve the problem efficiently using the constructs in Section 3.10.

14. Sorting algorithms. Section 6.6 briefly mentioned mergesort, but many other sorting algorithms can be written to run concurrently, in particular those like quicksort that work by divide and conquer.

[2]There is no connection between this problem and the concept of barriers of protected objects (Section 7.10).

Algorithm C.1: Barrier synchronization	
global variables for synchronization	
	loop forever
p1:	wait to be released
p2:	computation
p3:	wait for all process to finish their computation

15. (Hoare) Develop an algorithm for a server that minimizes the amount of seek time done by the arm of a disk drive. A simple server could satisfy requests for data in decreasing order of distance from the current position of the arm. Unfortunately, this could starve a request for data if requests for close tracks arrive too fast. Instead, maintain two queues of requests: one for requests for data from track numbers less than the current position and one for requests with higher track numbers. Satisfy all requests from one queue before considering requests from the other queue. Make sure that a stream of requests for data from the current track cannot cause starvation.

16. Matrix multiplication (Sections 8.3 and 9.4).

17. (Roussel) Given two binary trees with labeled leaves, check if the sequence of labels is the same in each tree. For example, the two trees defined by the expressions $(a, (b, c))$ and $((a, b), c)$ have the same sequence of leaves. Two processes will traverse the trees concurrently, sending the leaf labels in the order encountered to a third process for comparison.

18. (Dijkstra) Let S and T be two disjoint sets of numbers. Develop an algorithm that modifies the two sets so that S contains the $|S|$ smallest members of $S \cup T$ and T contains the $|T|$ largest members of $S \cup T$. One process will find the largest element in S and send it to a second process that returns the smallest element in T.

19. (Conway) The Game of Life. A set of cells is arranged in a (potentially infinite) rectangular array so that each cell has eight neighbors (horizontally, vertically and diagonally). Each cell is *alive* or *dead*. Given an initial finite set of alive cells, compute the configuration obtained after a sequence of generations, using a separate process for each cell or group of cells. The rules for passing from one generation to the next are:

    - If a cell is alive and has fewer than two live neighbors, it dies.
    - If it has two or three live neighbors, it continues to live.
    - If it has four or more live neighbors, it dies.

- A dead cell with exactly three live neighbors becomes alive.

20. Given a binary tree with nodes labeled by numbers, compute the sum of the labels in the tree.

21. Given a binary tree representing the parse tree of an arithmetical or logical expression, evaluate the expression.

22. Computation of all prime numbers from 2 to n. First prove that if k is not a prime, it is divisible by a prime $p(k) \leq \lfloor \sqrt{k} \rfloor$. The algorithm called the Sieve of Eratosthenes can be written as a concurrent program using dynamic creation of processes. Allocate a process to delete all multiples of 2. Whenever a number is discovered to be prime by all existing processes, allocate a new process to delete all multiples of this prime.

23. Image processing is a natural application area for concurrent programming. Typically, discrete integral and differential operators are applied independently at each pixel in the image. For example, to smooth an image, replace the pixel at $x_{i,j}$ with the weighted average of the pixel and its four neighbors:

$$(4 \cdot x_{i,j} + x_{i+1,j} + x_{i,j+1} + x_{i-1,j} + x_{i,j-1})/8.$$

To sharpen an image, replace the pixel with:

$$(4 \cdot x_{i,j} - x_{i+1,j} - x_{i,j+1} - x_{i-1,j} - x_{i,j-1})/8.$$

24. Solutions to the critical section problem enable processes to obtain exclusion access to a single resource. In the *resource allocation problem*, there is a set of n (identical) resources that m processes can request, use and then return. (Of course the problem is meaningful only if $m > n$; otherwise, a resource could be allocated permanently to each process.) A server allocates available resources and blocks clients when no more resources are available. Solve the resource allocation problem, first for the case where each process requests only a single resource at a time, and then for the case where a process can request $k \leq n$ resources at a time. The latter problem is difficult to solve because of the need to ensure atomicity of execution from the request to the allocation of the resources, and furthermore, if the process is blocked due to lack of resources, the request must be retried or resubmitted when resources become available.

25. The *stable marriage problem* concerns matching two sets of items, in a manner that is consistent with the preferences of each item [30]. A real-world application is matching between hospitals offering residencies and medical students applying to the hospitals. The problem itself is conventionally expressed in terms of matching men and women:

> Given a set of n men and n women, each man lists the women in order of preference and each woman lists the men in order of preference. A *matching* is a list of n pairs (m, w), where the first element is from the set of men and second from the set of women. A matching is *unstable* if there exists a pair of matches (m_1, w_1) and (m_2, w_2) such that m_1 prefers w_2 to w_1 and also w_2 prefers m_1 to m_2. A matching is *stable* if it is not unstable. Find an algorithm to construct a stable matching.

Here is an example of preference lists, where decreasing preference is from left to right in each line:

Man	List of women			
1	2	4	1	3
2	3	1	4	2
3	2	3	1	4
4	4	1	3	2

Woman	List of men			
1	2	1	4	3
2	4	3	1	2
3	1	4	3	2
4	2	1	4	3

Any matching containing the pairs $(1, 1)$ and $(4, 4)$ is unstable, because man 1 prefers woman 4 to woman 1 and woman 4 prefers man 1 to man 4. There are two stable matchings for this instance of the problem:

$$\{(1,4),(2,3),(3,2),(4,1)\}, \quad \{(1,4),(2,1),(3,2),(4,3)\}.$$

Algorithm C.2: Gale–Shapley algorithm for a stable marriage

```
                integer list freeMen ← {1,...,n}
                integer list freeWomen ← {1,...,n}
                integer pair-list matched ← ∅
                integer array[1..n, 1..n] menPrefs ← ...
                integer array[1..n, 1..n] womenPrefs ← ...
                integer array[1..n] next ← 1
p1: while freeMen ≠ ∅, choose some m from freeMen
p2:     w ← menPrefs[m, next[m]]
p3:     next[m] ← next[m] + 1
p4:     if w in freeWomen
p5:         add (m,w) to matched, and remove w from freeWomen
p6:     else if w prefers m to m' // where (m',w) in matched
p7:         replace (m',w) in matched by (m,w), and remove m' from freeMen
p8:     else // w rejects m, and nothing is changed
```

Algorithm C.2 is the *Gale–Shapley* algorithm for constructing a stable matching. (The implementation of `w prefers m to m'` is not given; it is easy to program using the preference matrices or by constructing auxiliary arrays that directly return the answer to this question.) The algorithm has the remarkable property that for any instance of the problem, it will find a stable matching, and the same matching will be found regardless of the order in which the men are chosen.

Develop a concurrent algorithm to solve the stable marriage problem. Is your algorithm significantly more efficient than a sequential one?

26. The *n-Queens problem* is to place n queens on an $n \times n$ board so that no queen can take another according to the rules of chess. Here is an example of a solution of the 8-queens problem:

	1	2	3	4	5	6	7	8
1	Q							
2							Q	
3					Q			
4								Q
5		Q						
6				Q				
7						Q		
8			Q					

Develop a concurrent algorithm to find one solution (or all solutions) to the n-queens problem. There are several different ways of utilizing the inherent concurrency in the problem. A separate process can search for a solution starting from each of the n possible assignments of a queen to the first column. Or, each time a queen is successfully placed on the board, a new process can be allocated to continue expanding this potential solution.

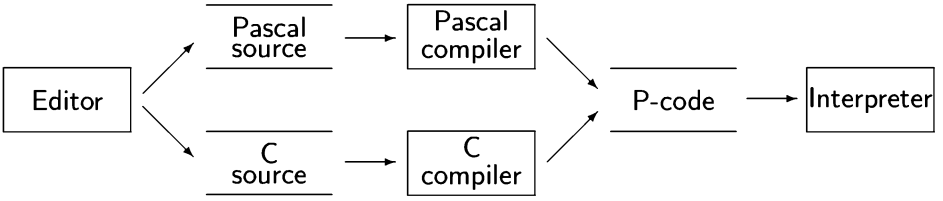
27. Develop a distributed algorithm for leader election in a ring of n nodes. In a ring, each process can communicate only with its successor and predecessor nodes (or, in another version of the problem, only with its successor node). Eventually, exactly one node outputs a notification that it is the leader.

D Software Tools

In this appendix we give a brief overview of software tools that can facilitate studying concurrent and distributed programming. For further information, see the documentation associated with each tool.

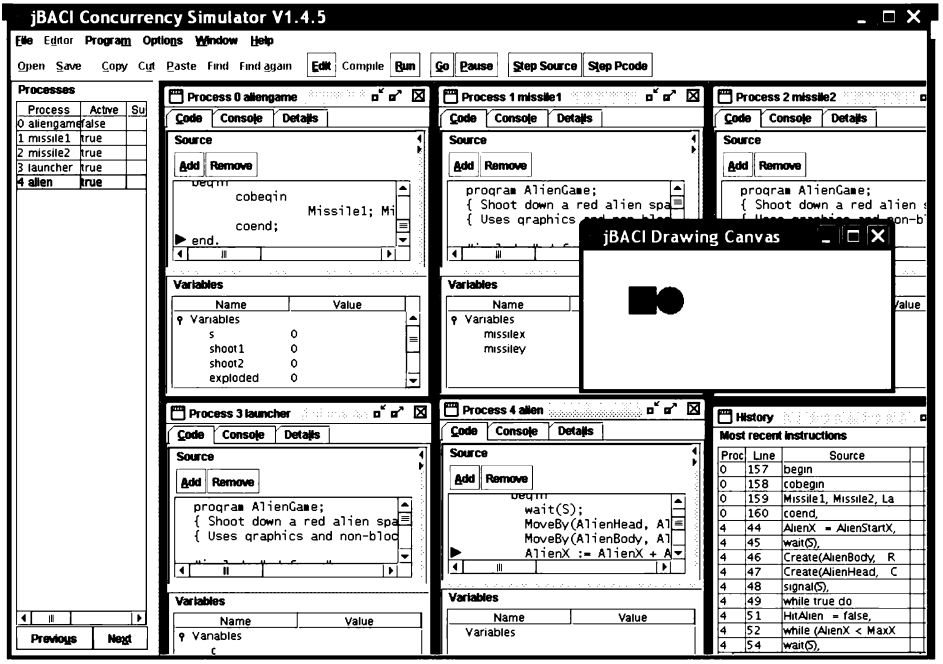
D.1 BACI and jBACI

BACI, the *Ben-Ari Concurrency Interpreter*, was developed by Tracy Camp and Bill Bynum [20]. The following diagram shows the architecture of BACI.



The compilers compile the source program into P-Code, an intermediate code for a virtual stack machine. The P-Code is then interpreted by the interpreter. The compilers for Pascal and C and the interpreter are written in C; the compilers are built using the generators lex and yacc (flex and bison can also be used). The software has been carefully designed so as to minimize system-dependent aspects, and prebuilt versions for many systems are available on the BACI website. The compilers and the interpreter are run as command-line tools; you can interactively run a concurrent program step-by-step or set breakpoints to examine the values of the variables at any step. There is also a graphical user interface, but this is available for UNIX systems only.

David Strite has developed a new interpreter for P-Code and a GUI that is written in Java and thus portable over all platforms. I used this software as the basis of jBACI, which is an integrated development environment (IDE) for concurrent programming. The graphical user interface of jBACI includes menus, mnemonics and accelerator keys. Many aspects of the software, especially of the user interface, are configurable at either compile time or upon initialization.



After you open a source file, it can be edited in a large panel that appears below the toolbar. (This mode is not shown in the above screenshot.) Then you can select Compile and the appropriate BACI compiler is invoked as a subprocess. When the program has been compiled successfully, select Run to invoke the interpreter. The program can be interpreted by selecting Go. Pause will halt the execution, which can be restarted by Go or restarted from the beginning by selecting Run.

The atomic statements of BACI are the individual P-Code instructions, but in Strite's interpreter and in jBACI you can interleave source code statements as if they were atomic statements. Step Source considers a source code *line* as an atomic statement, while Step PCode considers a P-Code instruction to be an atomic statement.

> To run the programs in this textbook in the intended manner, make sure that each assignment statement and each condition in a control statement are written on a single line.

The process table on the left side of the screen enables you to select the process from which to execute the next statement.

There is a process window for each process, displaying the source code and the P-Code for the process, as well as console output from the process and the values

of its variables. Breakpoints can be set by clicking on a source code or P-Code line in a process window and then selecting the Add button above the display of the code. A red dot will appear in front of the line. To remove a breakpoint, click on the line and select Remove.

Other optional windows can be displayed which show the global variables, the Linda tuple space, a history of the last statements executed, and a console containing the output from all processes.

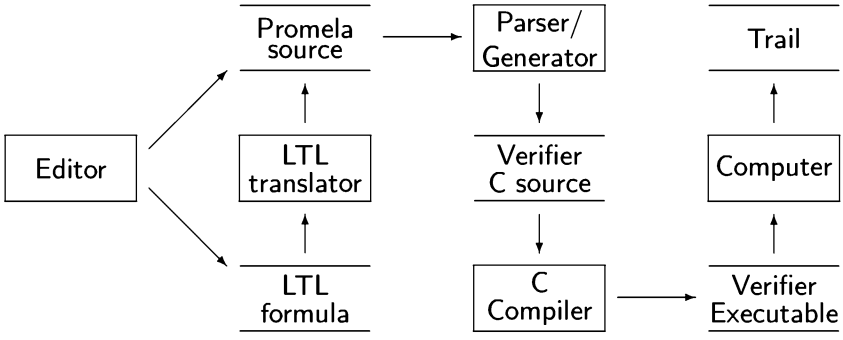
As part of the jBACI package, I have modified the compilers and interpreters to support a simplified version of the Linda model of synchronization. In addition, some graphics commands from the Java Swing library are made available at the source code level, so that you can study synchronization issues within the framework of simple game-like displays.

D.2 Spin and jSpin

The Spin Model Checker was developed by Gerard J. Holzmann [33]. It is written in C and the source code is available together with prebuilt executable files for many computers and systems. Spin has a command-line interface using options to configure the software. Two GUIs exist: XSpin written by Holzmann in Tcl/Tk, and jSpin written by this author in Java.

Spin can be used as a concurrency simulator. In *random simulation* mode the interleaving as well as the resolution of nondeterminism in if and do statements is performed randomly, while in *interactive simulation* mode the user is prompted to guide the selection of the next statement to execute.

Verification is performed as shown in the following diagram:



First, Spin is executed to analyze the Promela source code and to generate source code in C for a verifier that is optimized for the specific program and the options

that you have chosen. The Promela source code may be augmented with a **never** claim produced by translating a formula in linear temporal logic; Spin includes such a translator, although other translators can be used.

When the verifier program has been generated, it is compiled with a C compiler and linked as usual to produce an executable verifier. Running the verifier will result in a notice that no errors were found or an indication of the error. In the latter case, the verifier writes a coded file called the *trail*, which can be used to guide Spin to reproduce the computation that led to the error.

There are three modes of verification in Spin (safety, acceptance and non-progress), and one of these must be selected. Safety mode is used to verify safety properties like freedom from deadlock, assertions and invariants, such as those expressed by $\Box$-formulas. Acceptance mode is used to verify liveness properties such as freedom from starvation which can be expressed with $\Diamond$-formulas. Non-progress mode is an alternate method of verifying liveness properties without explicitly writing LTL formulas.

You can select if (weak) fairness is to be assumed during verification.

The jSpin user interface

The user interface of jSpin consists of a single window with menus, a toolbar and three adjustable text areas. The left area is an editor for Promela source code; the right area is used for the output of the Spin simulation (random, guided or obtained from a trail file); the bottom area displays messages from Spin. Spin option strings are automatically supplied.

When running a random simulation, the output appears in the right text area; the degree of detail can be set through menu selections. For guided simulation, the alternatives for the next statement to execute are displayed in the lower text area and a popup window enables selection of one of the alternatives. The Spin output is filtered so as not to overwhelm the user; the filtering algorithm works on output lines of Java type `String` and can be easily changed by modifying the class `Filter` of jSpin. A special feature enables the user to interactively select which variables will be displayed.

The menu bar contains a text field for entering LTL formulas. Invoking `Translate` negates the formula and translates it to a **never** claim. When `Verify` is selected, the claim together with the Promela program is passed to Spin.

jSpin Version 2.2

File Edit Spin Convert Options Settings Help | LTL formula <>nostarve

Open | Check | Random | Interactive | Trail | Weak fairness | Safety | Verify | Stop | Translate | Clear | Load | nostarve.prp | Max

-dekker.pml

```
1  /* Dekker's algorithm */
2
3  #define NOSTARVE
4  #include "critical.h"
5
6  bool    wantp = false, wantq = false;
7  byte    turn = 1;
8
9  active proctype p() {
10     do
11     :: wantp = true;
12        do
13        :: !wantq -> break;
14        :: else ->
15           if
16           :: (turn == 1)
17           :: (turn == 2) ->
18              wantp = false;
19              (turn == 1);
20              wantp = true
21           fi
22        od;
23        critical_section('p');
24        wantp = false;
25        turn = 2
26     od
27 }
```

```
1 (q)  wantq = 0          1   0   2   0   1
1 (q)  turn = 1           1   0   2   0   0
0 (p)  ((turn==1))        1   0   1   0   0
1 (q)  wantq = 1          1   0   1   0   0
1 (q)  (!(wantp))         1   0   1   0   1
0 (p)  wantp = 1          1   0   1   0   1
q in CS
1 (q)  printf('MSC: %c   1   0   1   1   1
1 (q)  critical = (cri    1   0   1   1   1
0 (p)  ((turn==1))        1   1   1   1   1
Proces Statement          P1inCS  critica turn  wantp  wantq
1 (q)  assert((critica    1   1   1   1   1
0 (p)  ((turn==1))        1   1   1   1   1
1 (q)  ((_pid==1))        1   1   1   1   1
1 (q)  P1inCS = 1         1   1   1   1   1
0 (p)  ((turn==1))        1   1   1   1   1
1 (q)  critical = (cri    1   1   1   1   1
1 (q)  wantq = 0          1   0   1   1   1
0 (p)  (!(wantq))         1   0   1   1   0
1 (q)  turn = 1           1   0   1   1   0
p in CS
0 (p)  printf('MSC: %c   1   0   1   1   0
0 (p)  critical = (cri    1   0   1   1   0
0 (p)  assert((critica    1   1   1   1   0
1 (q)  wantq = 1          1   1   1   1   0
0 (p)  critical = (cri    1   1   1   1   1
1 (q)                     1   0   1   1   1
0 (p)                     1   0   1   1   1
```

```
c:\spin\spin -g -l -p -r -s -X -u250 dekker.pml ... done!
```

jSpin is written in Java for portability and is distributed under the GNU General Public License. All aspects of jSpin are configurable: some at compile time, some at initialization through a configuration file and some at runtime.

How Spin verifies properties

The description of verification by model checking in Section 4.6 is over-simplified, because Spin does not check a property $\Box F$ by constructing all states and checking F on each state. Instead, states are constructed incrementally and the property is checked as each state is constructed. If $\neg F$ is true in a state, then the formula $\Box F$ is falsified and there is no reason to construct the rest of the states.

A correctness property expressed as a formula in temporal logic is negated and added to the Promela program in a form that enables it to be executed in parallel with the program. So, to prove $\Box F$, we add $\neg \Box F$ to the program. This formula can be written as $\Diamond \neg F$, which can be considered as a "program":

---

loop forever
  if $\neg F$ is true in this state then break

One step of this "program" is run after every step of the Promela program; if the condition is ever true, the "program" terminates and Spin has found a counterexample to the correctness claim $\Box F$.

It may help to understand this process if we repeat the explanation on a specific example, Dekker's algorithm described in Section 4.7. Consider the correctness claim given by the formula $F = criticalp + criticalq \leq 1$. $\Box F$ means that something good (at most one process in its critical section) is always true; that is, true in every state. What does $\neg \Box F$ mean? It means that it is not true that in every state F is true, so, eventually, there must be a state in which F is false, that is, a state in which $criticalp + criticalq > 1$ is true, meaning that more than one process is in its critical section, violating mutual exclusion. The corresponding "program" is:

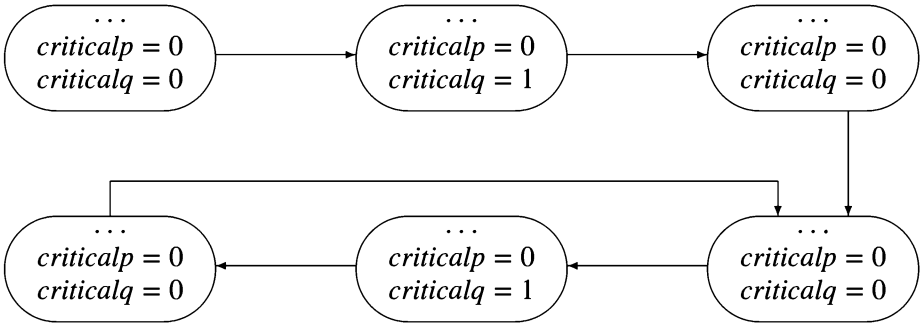
```
loop forever
    if criticalp + criticalq > 1 in this state then break
```

If Spin successfully finds a state in which $criticalp + criticalq > 1$ is true, then we have failed to prove $\Box(criticalp + criticalq \leq 1)$; conversely, if Spin fails to find a state in which $criticalp + criticalq > 1$ is true, then we have successfully proved $\Box(criticalp + criticalq \leq 1)$.

For liveness properties, the situation is somewhat more complex. Consider the correctness claim of freedom from starvation $\Diamond(criticalp > 0)$. Negating this gives $\Box(criticalp \leq 0)$. Now it is not necessary for $criticalp \leq 0$ to be true in *all* states for a counterexample to exist. All that is necessary is that there exist *some* (potentially infinite) computation in which $criticalp \leq 0$ is true in all states of that computation. A potentially infinite computation in a finite program is one that contains a cycle of states that can be repeated indefinitely. So Spin looks for *acceptance cycles*:

```
loop forever
    if criticalp > 0 then abandon this computation
    else if this state has been encountered then break
```

A computation in which starvation does not occur is uninteresting as we are trying to see if there exists *some* computation that can starve process p. However, we find a counterexample if a state is reached that has been previously encountered and $criticalp \leq 0$ has remained true throughout. This is demonstrated in the following diagram; the values of the variables criticalp and criticalq are explicitly given, while the ellipses represent all other elements of the state.



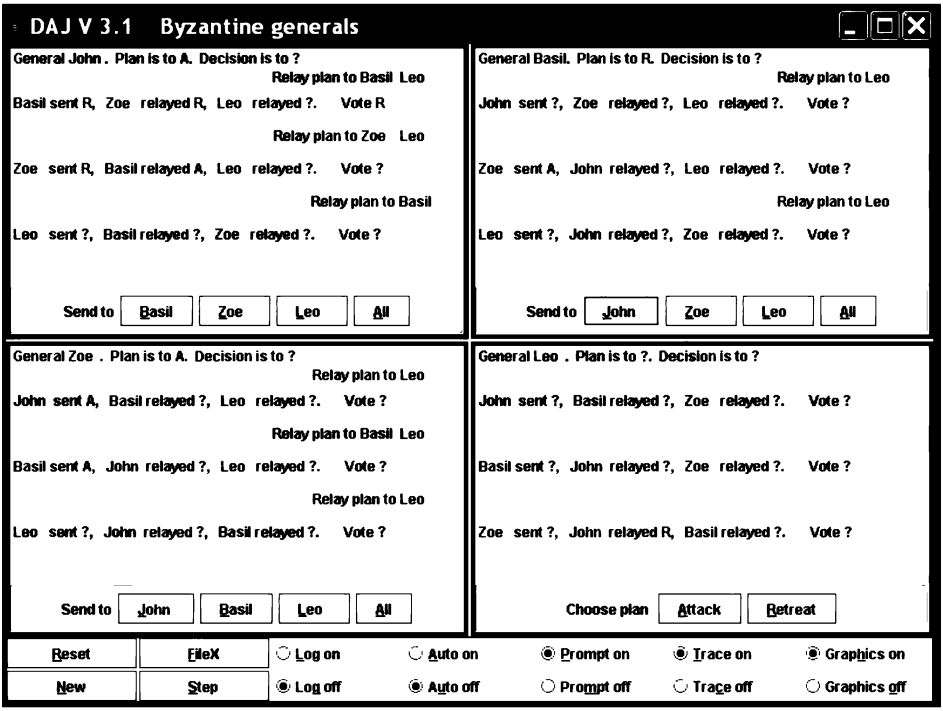
This computation goes through three states followed by a cycle representing a potentially infinite repetition of three other states. While the critical section of process q is repeatedly entered, the critical section of process p is never entered. Such a cycle is called an acceptance cycle and proves that there exists a computation that starves process p.

The "programs" constructed from correctness claims are actually nondeterministic finite automata which can be executed in parallel with the Promela programs, a process called *model checking*. For a brief introduction to model checking see [9, Section 12.5]; more advanced treatments are given in [51, Chapter 5] and [24]. The details of the construction of the automata in Spin and an explanation of the concept of acceptance cycles (and the related non-progress cycles) can be found in the Spin reference manual [33].

D.3 DAJ

DAJ is an *interactive*, *visual study aid* for learning distributed algorithms [8]. *Interactive*, because you must explicitly specify every step of the interleaved execution sequence. *Visual*, because the state of the nodes is continuously displayed. *Study aid*, because DAJ solves one of the most difficult problems encountered by students of these algorithms by automatically doing the necessary "book-keeping."

The program is an interactive state machine that displays the state of execution of an algorithm as you construct a scenario. The structure of the display for each algorithm is identical. Most of the screen is devoted to a grid of two to six panels, each one displaying the local state of a node in the system. The data for each node is color-coded, with a separate color for each node. Each panel contains a pair of lines for data relating to the node itself, followed by a pair of lines for data relating to each of the other nodes. The first line of a pair contains the data, followed by a second line which lists the outstanding messages that must be sent.



At the bottom of the screen is a line of buttons that globally affect all the nodes, while at the bottom of each node panel is a line of buttons for choosing a step of the algorithm. The contents of these buttons change according to the state of the node. To perform a step of an algorithm, select a button in one of the nodes. A sequence of such steps results in sending a message to another node, and the data structures of both the sending and receiving nodes are updated. Some buttons may not be active, although this is not denoted by any visual change in the buttons. It is a test of your understanding of the algorithm that you *not* click on a non-active node, though if you do so, the data structure is not changed.

For example, in the Byzantine Generals algorithm, a possible panel for General John is structured as follows:

General John. Plan is to A. Decision is to ?. Relay plan to Basil Leo
Basil send R. Zoe relayed R. Leo relayed ?. Vote R. Relay plan to Zoe Leo
Zoe send R. Basil relayed A. Leo relayed ?. Vote ?. Relay plan to Basil
Leo send ?. Basil relayed ?. Zoe relayed ?. Vote ?.
Plan of Basil Zoe Leo Me

The first line displays John's plan A and eventually his decision; the second line shows that he has yet to relay his initial choice to Basil and Leo. Since he has already relayed it to Zoe, the button for sending to Zoe will be inactive. The second pair of lines shows that Basil has sent his plan R to John, and that Zoe has relayed that she thinks that Basil's plan is R. This information is sufficient to enable John to carry out the vote concerning Basil's plan, as shown by the notation `Vote R`. The second line of prompts shows that John still has to relay the plan he received from Basil to Zoe and Leo. The line of buttons would be the second in a sequence for relaying message: `Send to`, `Plan of`, `Which is to`.

The Reset button returns all the nodes to their initial state. The New button will terminate the execution of the algorithm and return to the algorithm selection menu. The buttons FileX, Step, Log on/off and Auto on/off are used for reading and writing the log file, so that you can save a scenario and automatically rerun it. Prompt on/off selects whether to display the prompt lines, and Trace on/off selects whether to display the action trace window. Normally, prompts and traces will be turned on, but they can be turned off for assessment. Graphics on/off selects whether to display the graphics window. Currently, the visualizations that have been implemented are: the global queue of the Ricart–Agrawala algorithm (Section 10.3), the knowledge trees of the Byzantine Generals algorithm (Section 12.6) and the spanning tree of the Dijkstra–Scholten algorithm (Section 11.1).

A trace of all actions is displayed so that you and your instructor can discuss the sequence of actions leading to a particular state. The log file can also be used to analyze scenarios offline.

Ten distributed algorithms are currently implemented, including the following algorithms described in this book: the Ricart–Agrawala algorithm, the Dijkstra–Scholten algorithm, the Chandy–Lamport algorithm, the Byzantine Generals algorithm, the Berman–Garay King algorithm and the Huang–Mattern algorithm. The source code is structured so that you can implement other algorithms with only an elementary knowledge of Java: you need to implement a state machine for the

algorithm and to construct the strings to display in the panels. Knowledge of the Swing libraries for constructing a user interface is not needed, because the uniform interface is constructed in an abstract superclass common to all the algorithms.

E Further Reading

Undergraduate textbooks on concurrency are [2, 31, 49]. [2] is strong on systems and languages, but has much less material on verification. [31] shows how to implement concurrent algorithms in Java, while [49] combines CSP-based theory with Java programming. Advanced textbooks on concurrency are [3, 23, 50, 51, 57]; they emphasize the formal verification of programs.

The discussion of distributed algorithms in Chapters 10–12 just begins to scratch the surface of this fascinating topic. Extensive presentations can be found in [4, 48].

A summary of Leslie Lamport's work on mutual exclusion and an extensive list of references is given in [1].

My textbook on mathematical logic [9] provides the background needed to study advanced books on concurrency.

The monitor construct is widely used, but there are so many options for defining its semantics that it can be difficult to understand. These issues are fully analyzed in [15]. Much material on monitors can be found in the historical collection of Per Brinch Hansen [13].

A good analysis of the confusing terminology of concurrency is given in [16].

References to specific languages and systems have been given throughout the book. Here we summarize the main ones: Ada [7, 18]; Java [31, 44]; Linda [28]; Promela and Spin [33]. Advanced programming techniques in Promela are described in [56].

Operating systems are a major application of concurrency. Two popular textbooks are [60, 63]. You will also find the historical collection [14] interesting. Textbooks on real-time systems are [19, 47].

Reports of concurrency problems in spacecraft [35, 62, 66] are extremely interesting and make for motivational reading!

Websites

This is a list of websites for the languages, tools and systems mentioned in the book. This list also includes the archive of Edsger W. Dijkstra's technical notes and letters on verification and concurrency, Leslie Lamport's list of his works (many of which can be downloaded), and the Oxford University Computing Laboratory's website of links on concurrency and formal methods.

The URLs here and in the bibliography were checked on 15 September 2005.

Ada	`http://www.sigada.org/`
BACI	`http://www.mines.edu/fs_home/tcamp/baci/`
CSP	`http://archive.comlab.ox.ac.uk/csp.html`
DAJ	`http://stwww.weizmann.ac.il/g-cs/benari/daj/`
Dijkstra	`http://www.cs.utexas.edu/users/EWD/`
GNAT	`http://libre.adacore.com/`
Java	`http://java.sun.com`
JPF	`http://javapathfinder.sourceforge.net/`
JavaSpaces	`http://java.sun.com/developer/products/jini/`
jBACI	`http://stwww.weizmann.ac.il/g-cs/benari/jbaci/`
jSpin	`http://stwww.weizmann.ac.il/g-cs/benari/jspin/`
Lamport	`http://research.microsoft.com/users/lamport/`
MPI	`http://www-unix.mcs.anl.gov/mpi/`
occam	`http://www.wotug.org/`
Oxford	`http://archive.comlab.ox.ac.uk/`
pthreads	`http://www.opengroup.org/`
PVM	`http://www.csm.ornl.gov/pvm/`
SAnToS	`http://www.cis.ksu.edu/santos/`
SMV	`http://www-2.cs.cmu.edu/~modelcheck/smv.html`
NuSMV	`http://nusmv.irst.itc.it/`
SPARK	`http://www.sparkada.com`
Spin	`http://www.spinroot.com`
STeP	`http://www-step.stanford.edu/`
TLA	`http://research.microsoft.com/users/lamport/tla/tla.html`
UPPAAL	`http://www.uppaal.com/`

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