MA 232 Linear Algebra **HW 5** **Due: Oct 31, 2022 by 11:59 pm**

Name (Printed): ______________________________

Pledge and Sign: ______________________________

Upload solutions to Gradescope by the due date. Assign solution pages to corresponding problems. You need to pledge and sign on the cover page of your solutions. You may use this page as the cover page.

Legibility, organization of the solution, and clearly stated reasoning where appropriate are all important. Points will be deducted for sloppy work or insufficient explanations.

1. Consider the vectors $\mathbf{b} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$, $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

(a) [6 pts.] Find the projection $\mathbf{p}$ of $\mathbf{b}$ onto the subspace spanned by $\mathbf{a}_1$ and $\mathbf{a}_2$.

(b) [4 pts.] Find the error vector $\mathbf{e} = \mathbf{b} - \mathbf{p}$, and verify that it is orthogonal to both $\mathbf{a}_1$ and $\mathbf{a}_2$ by computing the corresponding dot products.

2. [10 pts.] Find the line $y = C + Dx$ that best fits the data
$(x, y) = \{(0, 1), (1, 8), (2, 8), (3, 20)\}$.

3. [10 pts.] Use Gram–Schmidt Process to find an orthonormal basis for the subspace spanned by $\mathbf{a} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{c} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{d} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$.

(This subspace is actually the whole $\mathbb{R}^4$ but that does not change anything for the Gram–Schmidt process.)

Partially **check your result** by computing the four dot products

$\mathbf{q_2} \cdot \mathbf{q_1}$, $\quad \mathbf{q_2} \cdot \mathbf{q_2}$, $\quad \mathbf{q_2} \cdot \mathbf{q_3}$, $\quad \mathbf{q_2} \cdot \mathbf{q_4}$.