MA 232 Linear Algebra **HW 6** **Due: Nov 11, 2022 by 11:59pm**

Name (Printed): ______________________________

Pledge and Sign: ______________________________

Upload solutions to Gradescope by the due date. Assign solution pages to corresponding problems. You need to pledge and sign on the cover page of your solutions. You may use this page as the cover page.

Legibility, organization of the solution, and clearly stated reasoning where appropriate are all important. Points will be deducted for sloppy work or insufficient explanations.

1. Let $A = \begin{bmatrix} 0 & 3 & 1 \\ 1 & 0 & 2 \\ 5 & 6 & 3 \end{bmatrix}$.

(a) [5 pts.] Use the cofactor formula in row 1 to find $|A|$. Then use the cofactor formula in row 2. Make sure you get the same answer.

(b) [5 pts.] Reduce A to its upper triangular form U without using row scaling (that is, only using row swaps and subtracting a multiple of one row from another row). Use the fact that $|A| = \pm$ (product of diagonal entries of U), where the sign depends on number of row swaps, to confirm your answer in part (a).

2. Let $A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix}$.

(a) [4 pts.] Use Cramer's Rule to find all solutions of $A\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

(b) [6 pts.] Use the cofactor formula to find A^{-1}.

3. Assume L is lower triangular and S is symmetric. Assume both L and S are invertible. Also assume the 3×3 matrix Q is orthogonal, that is $Q^T Q = I$.

$$L = \begin{bmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{bmatrix} \qquad S = \begin{bmatrix} a & b & d \\ b & c & e \\ d & e & f \end{bmatrix}$$

(a) [3 pts.] Which three cofactors of L are 0? Use it to argue that L^{-1} must be lower triangular.

(b) [3 pts.] Which cofactors of S are equal? Use it to argue that S^{-1} is symmetric.

(c) [2 pts.] Use the the facts that $Q^T Q = I$, and $(Q^{-1})^T = (Q^T)^{-1}$ to show that Q^{-1} is an orthogonal matrix.

(d) [2 pts.] There are only two possibilities for $\det(Q)$. What are those numbers? [Recall that det (Q)=det (Q^T)]