Eigenvalues and eigenvectors.

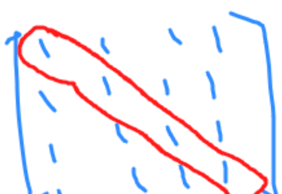
$Ax = \lambda x$ $\quad$ λ eigenvalue of A
$\quad$ x eigenvector of A.

$(A-\lambda I)x = 0$ $\leftarrow$ means Nullspace of $A-\lambda I$ is nonzero
so $A-\lambda I$ is <u>singular</u>.

what if A is already singular?
Then $\lambda = 0$ is an eigenvalue

Moreover: multiplicity of $\lambda=0$ is <u>at least</u> the $\dim N(A)$.

(More generally, multiplicity of eigenvalue λ is $\geq \dim N(A-\lambda I)$.)

<u>Ex.</u> $A =$ 

rank $A = 1$ so $\dim N(A) = 4-1 = 3$, so there are at least three eigenvalues 0.
sum of eigenvalues $=$ sum of diag. entries, so
$\lambda_4 = 1+1+1+1-0 = 4$.

A $n \times n$ matrix

n real and complex eigenvalues (possibly repeating)

real

complex non-real

not studied systematically in this course

$\underbrace{\lambda_1, \lambda_1, \ldots}_{k}$ $\quad \lambda_2, \lambda_2, \ldots$ $\quad \ldots\ldots$

at least one eigenvector

at most k eigenvectors

If there are n lin. indep. eigenvectors then matrix is diagonalizable.

- For that, A must have n eigenvalues (possibly repeating)
- For that, it is enough to have n different eigenvalues.

the case when not is not covered in this course

(Look up "Jordan normal form")

Eigenvectors that correspond to diff. eigenvalues are lin. independent!

$$c_1 v_1 + \ldots + c_m v_m = 0$$

Apply A to both sides