MA503: Homework 9

Exercise 9.1. [10pts] Let F be a field and $f(x) = c_n x^n + c_{n-1}x^{n-1} + \cdots + c_1 x + c_0 \in F[x]$ an irreducible polynomial of degree n. Divide $f(x)$ by its leading coefficient c_n to get a monic polynomial

$$g(x) = \tfrac{1}{c_n} f(x) = x^n + \cdots + \tfrac{c_1}{c_n} x + \tfrac{c_0}{c_n} \in F[x]$$

Prove that for every $h_1(x), h_2(x) \in F[x]$

$$h_1(x) \equiv_{f(x)} h_2(x) \quad \Leftrightarrow \quad h_1(x) \equiv_{g(x)} h_2(x).$$

(That's why we can always assume that the modulus is monic.)

Exercise 9.2. [10pts] Let F_1, F_2 be subfields of a field E. Prove that $F = F_1 \cap F_2$ is a subfield of E.

Exercise 9.3. [20pts] Let $f(x) = x^2 + x + 2 \in \mathbb{Z}_3[x]$.

(a) Show that $f(x)$ is irreducible. Hence, $E = F[x]/f(x)$ is a field.
(b) Is $x^3 - x^2 - 1$ trivial in E, or not? Why?
(c) $x^3 + 2x = 2x^2$ in E, or not? Why?
(d) Find the multiplicative inverse of $x + 1$ in E.
(e) $\chi(E) =$
(f) $|E| =$
(g) Find the order of $x + 2$ in E.
(h) Is x a primitive root in E?