MA 232 Linear Algebra **HW 7** **Due: Nov 18, 2022 by 11:59 pm**

Name (Printed): ____________________

Pledge and Sign: ____________________

Upload solutions to Gradescope by the due date. Assign solution pages to corresponding problems. You need to pledge and sign on the cover page of your solutions. You may use this page as the cover page.

Legibility, organization of the solution, and clearly stated reasoning where appropriate are all important. Points will be deducted for sloppy work or insufficient explanations.

1. Let $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and $I + A = \begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix}$.

(a) [6 pts.] Compute the eigenvalues and eigenvectors of A and $I + A$.

(b) [2 pts.] Find a relationship between eigenvectors and eigenvlaues of A and those of $I + A$.

(c) [2 pts.] Prove the relationship you found in Part (b) for an arbitrary $n \times n$ matrix A.

2. (a) [4 pts.] Diagonalize the matrix $A = \begin{bmatrix} 0.6 & 0.2 \\ 0.4 & 0.8 \end{bmatrix}$.

(b) [4 pts.] Use the diagonalization you obtained in (a) to find a formula for A^k.

(c) [2 pts.] Compute $A^\infty = \lim_{k \to \infty} A^k$.

3. Consider the triangular matrix $A = \begin{bmatrix} a & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$.

(a) [2 pts.] Find eigenvalues of A.

(b) [2 pts.] If $a \neq 1, 2$, explain why A is diagonalizable.

(c) [3 pts.] If $a = 1$, is A diagonalizable?

(d) [3 pts.] If $a = 2$, is A diagonalizable?