Peter Rauscher

HW 3

I pledge my honor that I have abided by the Stevens Honor System.

1)

a) $S=\begin{bmatrix}1&2&5\\2&6&2\\5&2&10\end{bmatrix} \rightarrow \left[\begin{array}{ccc|ccc}1&0&0&1&2&5\\0&1&0&2&6&2\\0&0&1&5&2&10\end{array}\right] \xrightarrow{R_2-2R_1} \left[\begin{array}{ccc|ccc}1&0&0&1&2&5\\2&1&0&0&2&-8\\0&0&1&5&2&10\end{array}\right] \xrightarrow{R_3-5R_1}$

$\left[\begin{array}{ccc|ccc}1&0&0&1&2&5\\2&1&0&0&2&-8\\5&0&1&0&-8&-15\end{array}\right] \xrightarrow{R_3+4R_2} \left[\begin{array}{ccc|ccc}1&0&0&1&2&5\\2&1&0&0&2&-8\\5&-4&1&0&0&-47\end{array}\right]$

$L=\begin{bmatrix}1&0&0\\2&1&0\\5&-4&1\end{bmatrix} \quad D=\begin{bmatrix}1&0&0\\0&2&0\\0&0&-47\end{bmatrix} \quad L^T=\begin{bmatrix}1&2&5\\0&1&-4\\0&0&1\end{bmatrix}$

$S=LDL^T=\begin{bmatrix}1&0&0\\2&1&0\\5&-4&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&2&0\\0&0&-47\end{bmatrix}\begin{bmatrix}1&2&5\\0&1&-4\\0&0&1\end{bmatrix}$

b) By the definition of symmetry, a symmetric matrix S is only symmetric if $S=S^T$. Consider the matrix A^TA where A is any matrix. For A^TA to be symmetric, we must show

$$(A^TA)^T=A^TA$$

$$(A^T)^TA^T=A^TA$$

$$AA^T=A^TA$$

$$A^TA=A^TA$$

So for any matrix A, the matrix A^TA is symmetric

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2)

$$A=\begin{bmatrix}1 & 2 & x_1\\ 2 & 3 & x_2\\ 4 & 0 & x_3\end{bmatrix}\begin{bmatrix}-7\\4\\1\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$=\begin{bmatrix}-7+8+x_1\\-14+12+x_2\\-28+0+x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}-1\\2\\28\end{bmatrix}$$

$$A=\begin{bmatrix}1 & 2 & -1\\ 2 & 3 & 2\\ 4 & 0 & 28\end{bmatrix}$$

3)

$$\begin{aligned}1x+2y-1z&=1\\3x+5y+2z&=3\\2x+1y+13z&=2\end{aligned}\longrightarrow\left[\begin{array}{ccc|c}1&2&-1&1\\3&5&2&3\\2&1&13&2\end{array}\right]\xrightarrow{R_2-3R_1}$$

$$\rightarrow\left[\begin{array}{ccc|c}1&2&-1&1\\0&-1&5&0\\2&1&13&2\end{array}\right]\xrightarrow{R_3-2R_1}\left[\begin{array}{ccc|c}1&2&-1&1\\0&-1&5&0\\0&-3&15&0\end{array}\right]\xrightarrow{R_3-3R_2}\left[\begin{array}{ccc|c}1&2&-1&1\\0&-1&5&0\\0&0&0&0\end{array}\right]$$

Here, the free variable is $z=0$ (does not belong to a pivot column) so the system becomes

$$x+2y=1$$
$$-y=0$$

$y=0$ $\qquad$ $x+2(0)=1$

$x=1$

So a particular solution x_p is $\begin{bmatrix}1\\0\\0\end{bmatrix}$

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3 cont) $x+2y-z=0$
$-y+5z=0$

Let z be any number c, since it is a free variable

$x+2y-c=0$
$5c-y=0$
$y=5c$

$x+10c-c=0$
$x+9c=0$
$x=-9c$

So $x_p+x_n=\begin{bmatrix}1\\0\\0\end{bmatrix}+z\begin{bmatrix}-9\\5\\1\end{bmatrix}$