Peter Rauscher

HW 6

I pledge my honor that I have abided by the Stevens Honor System. Peter Rauscher

1a) $|A| = \begin{vmatrix} 0 & 3 & 1 \\ 1 & 0 & 2 \\ 5 & 6 & 3 \end{vmatrix} = 0 \cdot C_{11} + 3 \cdot C_{12} + 1 \cdot C_{13}$

$= 3 \cdot C_{12} + C_{13}$

$= 3 \cdot (-1)^{1+2} \cdot \begin{vmatrix} 1 & 2 \\ 5 & 3 \end{vmatrix} + (-1)^{1+3} \cdot \begin{vmatrix} 1 & 0 \\ 5 & 6 \end{vmatrix}$

$= -3 \cdot (1 \cdot 3 - 2 \cdot 5) + (1 \cdot 6 - 0 \cdot 5)$

$= (-3)(-7) + 6$

$= 27$

Now, using row 2

$|A| = \begin{vmatrix} 0 & 3 & 1 \\ 1 & 0 & 2 \\ 5 & 6 & 3 \end{vmatrix} = 1 \cdot C_{21} + 0 \cdot C_{22} + 2 \cdot C_{23}$

$= C_{21} + 2 \cdot C_{23}$

$= (-1)^{2+1} \cdot \begin{vmatrix} 3 & 1 \\ 6 & 3 \end{vmatrix} + 2(-1)^{2+3} \cdot \begin{vmatrix} 0 & 3 \\ 5 & 6 \end{vmatrix}$

$= (-1)(3 \cdot 3 - 1 \cdot 6) + (2)(-1)(0 \cdot 6 - 3 \cdot 5)$

$= -3 + 30$

$= 27$

1b) $\begin{bmatrix} 0 & 3 & 1 \\ 1 & 0 & 2 \\ 5 & 6 & 3 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & 1 \\ 5 & 6 & 3 \end{bmatrix} \xrightarrow{R_3 - 5R_1} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & 1 \\ 0 & 6 & -7 \end{bmatrix} \xrightarrow{R_3 - 2R_2} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & -9 \end{bmatrix}$

$|A| = 1 \cdot 3 \cdot (-9) \cdot (-1)^{1 \text{ row swap}} = 27$

2a) $Ax = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$$\begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$x_1 = \frac{\det B_1}{\det A} = \frac{-1}{2}$ $\quad x_2 = \frac{\det B_2}{\det A} = \frac{3}{2}$ $\quad x_3 = \frac{\det B_3}{\det A} = \frac{0}{-4} = 0$

$$|A| = 2 \cdot \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} + 0(-1)\begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix}$$

$$= 2(1-4) + (2-0)$$

$$= -6 + 2 = -4$$

$$|B_1| = \begin{vmatrix} 1 & 0 & 1 \\ 2 & 1 & 2 \\ 3 & 2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = (1-4) + (4-3) = -2$$

$$|B_2| = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 2 \\ 0 & 3 & 1 \end{vmatrix} = 2\begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix} - \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 0 & 3 \end{vmatrix} = -8 - 1 + 3 = -6$$

$$|B_3| = \begin{vmatrix} 2 & 0 & 1 \\ 1 & 1 & 2 \\ 0 & 2 & 3 \end{vmatrix} = 2\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} - 0\begin{vmatrix} 1 & 2 \\ 0 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} = -2 + 2 = 0$$

$$x = \begin{bmatrix} -1/2 \\ 3/2 \\ 0 \end{bmatrix}$$

2b) $A^{-1} = \frac{1}{\det(A)} C^T = \frac{1}{4} \begin{bmatrix} -3 & -2 & 1 \\ 1 & -2 & 3 \\ -2 & 4 & -2 \end{bmatrix}$

$$A^{-1} = \begin{bmatrix} -3/4 & -1/2 & 1/4 \\ 1/4 & -1/2 & 3/4 \\ -1/2 & 1 & -1/2 \end{bmatrix}$$

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3a) $C_L = \begin{bmatrix} cf & -bf & be-cd \\ 0 & af & -ea \\ 0 & 0 & ac \end{bmatrix}$

So C_{21} or b, C_{31} or d, and C_{32} or e are the zero cofactors of L. L^{-1} is the transpose of the cofactor matrix C_L above divided by the determinant of L, but anything divided by zero is zero, so the zeros of C_L^T are in C_{12}, C_{13}, and C_{23}, and the C_L^T matrix would look like

$$\begin{bmatrix} & 0 & 0 \\ \cdots & & 0 \\ & & \end{bmatrix}$$ which is clearly of lower triangular form.

3b) $C_S = \begin{bmatrix} cf-e^2 & ed-bf & be-cd \\ ed-bf & af-d^2 & bd-ae \\ be-cd & bd-ae & ac-b^2 \end{bmatrix}$ $C_{23} = C_{32} = A$, $C_{12} = C_{21} = B$, $C_{13} = C_{31} = D$

X =

See above for definitions of A, B and D. We can generalize the matrix S^{-1} as

$$S^{-1} = \frac{1}{\det S} C_S^T = \frac{1}{\det S} \begin{bmatrix} C_{11} & B & D \\ B & C_{22} & A \\ D & A & C_{33} \end{bmatrix}$$

If S^{-1} is symmetric, its transpose is the same as above. $(S^{-1})^T$ can be written as

$$(S^{-1})^T = \frac{1}{\det S} \begin{bmatrix} C_{11} & B & D \\ B & C_{22} & A \\ D & A & C_{33} \end{bmatrix}$$ So S^{-1} is clearly symmetric.

3c) A matrix is orthoganal if the product of its transpose and itself is the identity matrix I.

It is given that $Q^TQ = I$

So, $(Q^TQ)^{-1} = I^{-1}$ recall $I^{-1} = I$

$(Q^T)^{-1}Q^{-1} = I$

We are also given that $(Q^T)^{-1} = (Q^{-1})^T$

So,

$$(Q^{-1})^T Q^{-1} = I$$

Clearly, the product of Q^{-1} and its transpose is the identity matrix I, therefore Q^{-1} is orthoganal.

3d) We are given that we may assume Q is orthoganal or $Q^TQ = I$.

$\det(Q^TQ) = \det(I) = 1$

$\det(Q^T)\det(Q) = 1$

We know $\det(Q^T) = \det(Q)$, so

$\det(Q)\det(Q) = 1$

$\det(Q)^2 = 1$

Therefore, the determinant of Q can be either 1 or -1.