MA 232 Linear Algebra **HW 8** **Due: Friday Dec 2, 2022 by 11:59 pm**

Name (Printed):

Pledge and Sign:

Upload solutions to Gradescope by the due date. Assign solution pages to corresponding problems. You need to pledge and sign on the cover page of your solutions. You may use this page as the cover page.

Legibility, organization of the solution, and clearly stated reasoning where appropriate are all important. Points will be deducted for sloppy work or insufficient explanations.

1. Consider the matrix $S = \begin{bmatrix} 4 & 8 & 0 \\ 8 & 25 & 9 \\ 0 & 9 & 25 \end{bmatrix}$.

(a) [5 points] Show that S is a positive definite matrix.

(b) [5 points] Find the Cholesky factorization and the Cholesky factor for S.

2. Consider the quadratic form $f(x, y) = x^2 + xy + y^2$.

(a) [3 points] Show that the symmetric matrix S such that $f(x, y) = \begin{bmatrix} x & y \end{bmatrix} S \begin{bmatrix} x \\ y \end{bmatrix}$ is positive definite.

(b) [7 points] Find the semiaxes and their directions from the eigenvalues and eigenvectors of the matrix S.

3. [10 points] Suppose a differentiable function $f(x, y, z)$ has a critical point at $(0, 0, 0)$. Suppose further the derivatives of $f(x, y, z)$ at $(0, 0, 0)$ are as follows:

$$\frac{\partial^2 f}{\partial x^2} = -2, \quad \frac{\partial^2 f}{\partial y^2} = -2, \quad \frac{\partial^2 f}{\partial z^2} = -2,$$

$$\frac{\partial^2 f}{\partial x \partial y} = 1, \quad \frac{\partial^2 f}{\partial x \partial z} = a, \quad \frac{\partial^2 f}{\partial y \partial z} = 1.$$

Find the range of values of a that guarantee that $(0, 0, 0)$ is a point of local maximum of f.