Econometric modelling and forecasting of wholesale electricity prices

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1. Introduction

Most economic and policy decisions involve formulating expectations and forming beliefs about the future, but the future is uncertain. The energy sector is no exception to this: most readers are probably aware of predictions, periodically updated, on the peak in oil production or may have heard about future scenarios on energy sources in relation to global warming. Optimisation of investments in fossil-fueled power plants, in photovoltaic panels or in energy storage facilities requires forecasts about costs and revenues over a time horizon corresponding to the life cycle of the equipment.

In the electricity sector, forecasting efforts are relevant on a shorter horizon too, given the volatile and spiky character of wholesale electricity prices. Forecasting on a daily horizon is useful for production and supply decisions by power-generating companies and utilities in wholesale electricity auctions, as well as for portfolio optimisation by electricity traders, who need to allocate their investment shares among different energy commodity contracts. Pricing of power derivatives needs a correct representation of the stochastic process driving the underlying asset. These examples suggest that forecasts constitute fundamental inputs to energy company decision-making. Academics, policy-makers and practitioners have shown interest towards forecasting electricity market variables, starting from the 1990s, when the liberalisation process began in countries such as the UK and the Scandinavian ones. The electricity market forecasting literature flourished around the turn of the new millennium.[2]

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[1] Valuable research assistance by Emmanuel Senyo Fianu and Khadija Merzougui is gratefully acknowledged.

[2] Examples include Bunn (2000), Ramanathan et al. (1997), Johnson and Barz (1999), Szkuta et al. (1999).

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This chapter is an introduction to modelling and forecasting market-clearing electricity prices determined in liberalised wholesale power exchanges, on a short-term (days to weeks) horizon. Wholesale electricity prices are benchmarks for retail price setting by regulatory authorities and for balancing prices, and hence enter indirectly in the computation of production costs by energy-using business firms. Electricity prices pose a number of modelling and forecasting challenges due to the peculiar physical and economic features of electricity that lie behind the simultaneous presence of mean reversion, spikes, long memory, volatility clustering, regime switches, and negative prices.

The two main econometric approaches to modelling and forecasting electricity prices are encompassed in the chapter. One is based on studying the price-determination mechanisms, namely the relationships between the price and its fundamentals (structural econometric models). Another approach extrapolates future values of the electricity prices by relying on their past values (reduced-form econometric models).

The chapter builds on previous reviews on electricity price forecasting, published in books, such as Bunn (2004), Weron (2007), Zareipour (2008), as well as in scholarly journals, including Bunn (2000), Aggarwal et al. (2009a,b), Chan et al. (2012), Weron (2014). Alternatives to econometric models are only shortly covered in this chapter. The use of big data and the related computational intelligence approach is the subject of another chapter in this handbook. Economic models, including strategic interaction models and agent-based simulation models, are briefly discussed as they are built for policy-assessment and interpretive goals and can supply useful predictive information only on a longer time horizon. From the viewpoint of an economist, the econometric approach strikes the best compromise between forecasting performance and the need to understand the underlying economic mechanisms. The economic approach proves inadequate in terms of short-term forecasting performance. At the opposite stands the computational intelligence approach, which may be superior in terms of forecasting ability but offers less insight for the economist and the policy-maker.

The structure of the chapter is the following. Section 2 reviews the main stylised facts that models of the electricity market are challenged to reproduce, as well as practical information on data processing. Section 3 is dedicated to econometric models of wholesale electricity prices, illustrating the most commonly used univariate structural and reduced-form models. Section 4 briefly presents and discusses the economic approach to modelling electricity prices. Conclusions are outlined in Section 5, along with suggestions for further reading.

The learning goals pursued by this chapter are essentially three. The first is to provide readers with an overview of the main stylised facts on wholesale electricity prices, robustly verified in samples from the main power exchanges.

Second, the chapter offers insights on how to translate generic features of wholesale electricity prices, rooted in the economics of power generation and demand, into empirically successful econometric models. A third learning goal concerns comparisons and selection among alternative econometric models of wholesale electricity prices. The emphasis is on concepts and ideas about modelling, while estimation and testing issues are left in the background. In outlining the chapter, a choice has been made in favour of a wide-ranging detour of modelling strategies, rather than on specific energy policy—evaluation issues. The reason is that the handbook is targeted at readers with engineering backgrounds, who may have been exposed to stochastic analysis and may capitalise on such knowledge stock. Modelling tools learned in this chapter constitute the starting point for pricing power derivatives and can be useful, to some extent, in the analysis of storable energy commodity prices.

Readers can fully profit from this chapter if they possess knowledge about power system economics and basic notions of probability theory, statistics, and regression analysis. Nonetheless, the chapter will provide the essential background references to books and scholarly articles.

2. Stylised facts

To select an empirically sound econometric model for electricity prices, it is instructive to review the main statistical properties of the data, robustly assessed in the empirical literature on electricity markets. Such a review allows to understand which statistical properties a model is expected to reproduce.

In wholesale electricity markets, market-clearing prices exhibit rich structures, involving time-varying mean and variance, mean reversion, multiple periodic patterns, volatility clustering, spikes, and negative prices. All these properties, examined in the following subsections, are due to some distinctive features of electricity that make it different from other commodities.

Maintaining a constant balance between demand and supply in the power transmission grid is a technical requirement for security reasons. Low elasticity of demand with respect to price and high consumer switching costs imply that the adjustment towards market equilibrium is carried out along the supply function, with its *hockey stick* shape. At times, market clearance occurs at the intersection between an inelastic demand curve and a very steep supply curve, so that any small shock to either side of the market gives rise to a wide adjustment in the equilibrium price. Shocks cannot be smoothed out also because technologies for the storage of electrical energy are not yet economically viable. What can currently be stored are the resources used for power generation, such as water behind dams and fossil fuels. The related generation technologies require capital outlays that are more accessible to large players, exacerbating the influence of individual moves on the market equilibrium.

In illustrating each stylised fact, this section will also provide information on preliminary data treatment and testing. Before moving on to the description of stylised facts, some remarks are needed on the choice of the variable to be modelled, on measurement issues and on data sources.

2.1 Measurement issues

Analysing wholesale electricity prices is enticing from the viewpoint of an economist interested in the functioning of markets for non-storable commodities. Indeed, the frequency of the data is rather high — up to half hours in the UK market — and regulatory constraints are exogenous to the market mechanism in the reference time frame, although they matter in shaping the short-term dynamics of wholesale prices. Hence, forecasting wholesale electricity prices benefits from a wealth of data on measurable items, such as a relatively long stream of electricity prices, fuel prices, power demand, supply from renewables, the status of the grid and so forth.

Conversely, the challenges in forecasting retail tariffs are rather different: drivers of political decision-making may be hardly observable or quantifiable, and the low frequency of tariff revisions implies that the time series of retail tariffs in the post-liberalisation age are rather short.

Predicting wholesale electricity prices involves some preliminary considerations regarding the choice of the variable to be modelled. Issues to be tackled include the correct frequency of observations for the so-called day-ahead market, the spatial resolution of the data and the range of feasible price values.

2.1.1 The frequency of observations

Most power exchanges house 24 or 48 auctions per day in their day-ahead segments. Each auction sets the price for a given time interval within a day. Typically, in the day-ahead segment, the deadline for submitting sealed bids is common to all auctions and expires the day before delivery. Hence, in formulating bids for all the 24 (or 48) auctions, buyers and sellers use information available 1 day ahead. Any correlation between market-clearing prices in different auctions for the same day cannot be interpreted as serial correlation. In other terms, the price at 4 p.m. is correlated with the price at 3 p.m. because bids submitted for delivery at 3 p.m. and 4 p.m. are based on the same information set. This line of reasoning has motivated most researchers to avoid considering the day-ahead market data as a hourly-frequency time series. Rather, it is more correct to focus on each hourly auction as a separate time series or to consider the set of 24 (or 48) hourly prices, observed over time, as a longitudinal panel. An alternative to these solutions is to focus on daily statistics, for instance, by modelling prices of baseload and peakload contracts or modelling the average daily price. In some applications, a weighted average has been considered, with weights given by load values to emphasise hours when energy consumption is larger. Some authors have also used the median daily price, with a caveat: the hourly auction that yields the median price may not be the same every day. For the aforementioned reasons, the electricity markets literature mostly focuses on forecasting daily-frequency time series.[3]

2.1.2 The spatial resolution of the data

The spatial resolution of the analysis is yet another issue influencing the definition of the electricity price variable to be modelled. Most power exchanges adopt a nodal or zonal price system, triggering a number of questions: is there a location that is more representative than the others? Should one model and forecast the prices of all locations simultaneously? Whether a location is more or less representative of a zonal price system depends on the goals of the forecaster. If the grid is characterised by sufficient transmission capacity and low congestion frequencies, prices at all locations have very similar statistical properties.

Averaging zonal or nodal prices is a way to account for spatial information without selecting any specific zone or node. However, averaging provides limited information on market conditions in regions that are often congested out of the grid, whose prices are quite far from the national average. Nonetheless, the electricity price in one location incorporates information from other locations if there is network connection. The equilibrium in one location is affected by the possible lack thereof in a connected location. For instance, excess supply in one market zone flows through the cable connecting it to another zone, whose price is therefore depressed. Power-generating companies can take these patterns into account when formulating their bidding strategies. Advanced models forecast the whole set of locational prices through models that account, implicitly or explicitly, for their simultaneous and dynamic correlations.

2.1.3 The range of feasible price values

Additional considerations need to be made about the feasible range of values that an electricity price series can take. An appropriate model, indeed, yields predictions within the feasible range. In general, prices belong to the set of real-valued positive numbers. Econometric predictions based on linear models may yield negative values as the error terms in econometric models are drawn from probability distributions defined over an infinite support. This problem has traditionally been circumvented by building models of the logarithm of price, that can assume negative values even if prices are not negative. This is no longer a cogent issue in electricity price forecasting since the appearance of negative prices (see Section 2.6), but scholars have been discussing whether

[3] A similar problem is faced in modelling balancing prices because they are determined through continuous trading and there is no proper 'daily' balancing price.

it is more appropriate to model price levels or log prices, with Weron (2007) providing elements in favour of modelling price levels.

Some remarks are also useful on measures of electricity price dynamics. Scholars engaged in electricity price modelling and forecasting in the early 1990s had backgrounds in asset and commodity price modelling, hence a natural focus of their analysis was on returns, which measure the ability of an asset or commodity to store and increase its value across time. Returns can be measured by log-price differences $\log P_t - \log P_{t-1}$, that approximate relative price differences $\frac{P_t - P_{t-1}}{P_{t-1}}$. Returns have a different meaning in the case of electricity because the electricity generated and exchanged at different points in time is not the same commodity. Hence, the relative price change cannot be considered as a return in the same sense as in financial and commodity markets.

In illustrating the stylised facts, reference will be made to Figs. 15.1−15.3 that depict day-ahead electricity prices determined on the German-Austrian segment of EPEX, respectively, over a year, a month, and a day.[4] That market is among the most liquid in Europe and belongs to a power exchange which is considered the core of a possibly fully integrated EU market for electricity. The figures showcase a number of properties, such as periodic patterns, volatility clustering, spikes and negative prices, to be described in the upcoming subsections. Further illustrations will be based on data from the Scandinavian power exchange NordPool, the Dutch APX and the French Powernext, based on some of the author's publications.

2.2 Seasonality and periodic patterns

Wholesale electricity prices exhibit three main periodic patterns: seasonal, weekly and intra-day. Seasonal patterns refer to differences across seasons of the year. Electricity demand, indeed, can be higher in months when air conditioning is more requested, so that one can identify higher average prices during the summer (for cooling purposes) and/or during the winter (for heating). In Fig. 15.1, higher average prices are observed in late fall and during the winter. Differences in average prices across the week are due to the changing level of economic activity. Typically, prices on Saturday and Sunday are lower, with peaks reached on Tuesday, Wednesday and Thursday. Fig. 15.2 makes this clear: EPEX baseload and peakload prices determined in Germany and Austria in November 2017 peaked on 8, 15, 29 November (Wednesdays) and on 21 November (Tuesday), whereas troughs can be spotted during weekends.

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[4] EPEX is a power exchange including markets from France, Switzerland, Austria, Germany and Luxembourg.

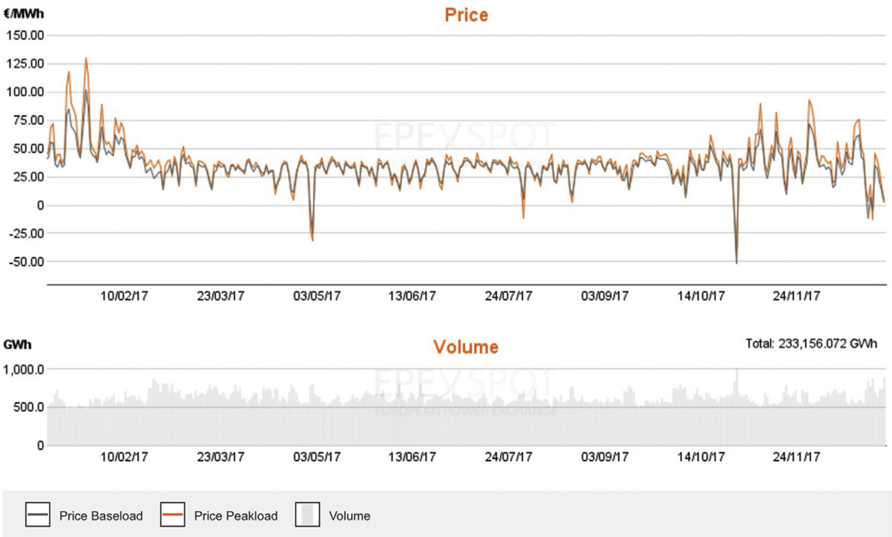

FIGURE 15.1
EPEX baseload and peakload day-ahead prices (in Eur/MWh) and volumes (in GWh) between 1 January and 31 January 2017, for Germany and Austria. *Source: EPEX website.*

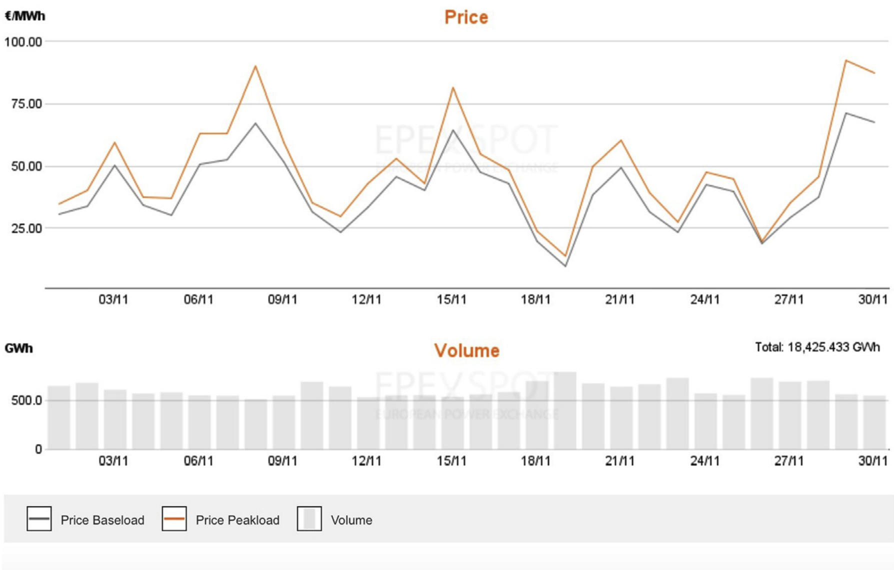

FIGURE 15.2
EPEX baseload and peakload day-ahead prices (in Eur/MWh) and volumes (in GWh) between 1 and 30 November 2017, for Germany and Austria. *Source: EPEX website.*

FIGURE 15.3
EPEX hourly, baseload, and peakload day-ahead prices (in Eur/MWh) and volumes (in GWh) on 17 January 2017, for Germany and Austria. *Source: EPEX website.*

Similarly, one can spot price peaks around midday and mid-afternoon, as well as lower prices during the night. Fig. 15.3 refers to hourly, baseload, and peakload day-ahead prices on 17 January 2017 (a Friday) for the German-Austrian market. Prices go up and down across hours in an asymmetric sinusoidal fashion, reaching the highest values at 10 a.m. and 7 p.m. and the lowest at 3 a.m. Notice that the price triples between 6 a.m. and 9 a.m. Holiday effects are also detected in the literature.[5]

It is worth recalling here that electricity demand is a derived demand. The supply curve, instead, is more stable across seasons and week days, as it mainly depends on fuel prices that are fixed by long-term contracts — although periodicities in renewables (including hydropower and photovoltaics) and market power exercise can induce some recurrent shifts in the supply curve as well. Also, periodic and seasonal patterns in electricity markets are neither deterministic nor symmetric.

Periodic patterns can be detected by visual inspection of time series plots and autocorrelograms or, more formally, by performing tests on autocorrelation coefficients. The sample autocorrelation function is defined as follows:

$$\mathrm{ACF}(k) = \frac{\gamma(k)}{\sigma^2} \tag{15.1}$$

[5] In many power exchanges, the traditional day-night pattern has been upset in recent years when a large inflow of photovoltaic power, characterised by low marginal costs, has shaved the price peaks.

where ACF(k) stands for autocorrelation function at lag k (typically measured in hours or days in the case of electricity markets); $\gamma(k)$ is the covariance between prices at times t and $t-k$; σ^2 is the variance of the time series. The autocorrelogram is the graphical representation of the autocorrelation function.

In a time series observed at a daily frequency, because of weekly patterns, we expect to find autocorrelation coefficients that are significantly different from zero at lag seven and multiples. If we consider hourly time series, instead, significant autocorrelations are found at lag 24 and multiples. By contrast, in a white noise process, sample autocorrelation coefficients are not significantly different from zero at all lags. Peaks in the autocorrelogram at lag seven and multiples (daily data) or at lag 24 and multiples (hourly data) are expected. Some instances of autocorrelograms of electricity log-returns, peaking at lag seven and multiples, are shown in Fig. 15.4.

The main difficulty in dealing with the seasonal component lies in the asymmetric and stochastic nature of seasonal patterns. In econometric modelling, one way of accounting for the presence of seasonals and periodic drivers

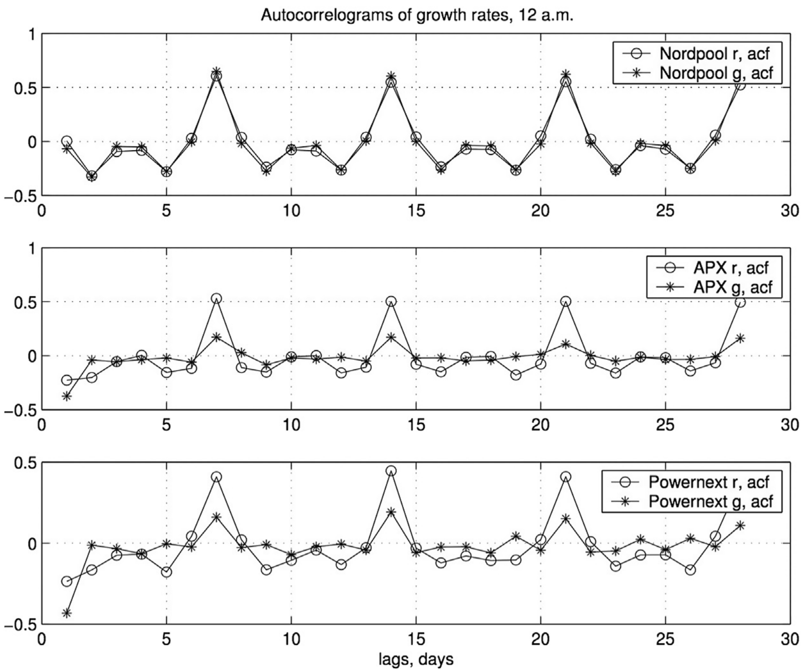

FIGURE 15.4

Autocorrelograms of log-returns and volume rates of change, for NordPool (1 Jan 1997—31 Dec 2002), APX (6 Jan 2001—31 Dec 2004), Powernext (1 Feb 2002—31 Jan 2007), 12 a.m. auctions. Autocorrelation coefficients are significant if their absolute value is greater than 0.0592 (NordPool), 0.0513 (APX), 0.0601 (Powernext). *Source: author's own elaborations* on data from NordPool, APX, and Powernext *markets*.

consists of including autoregressive terms, e.g. prices at lag 7 (daily data) or at lag 24 (hourly data), as well as deterministic regressors, for instance dummies or trigonometric functions. Alternatively, the seasonal/periodic component can be estimated and expunged from the data prior to fitting parametric or non-parametric statistical models. This latter approach postulates the existence of a trend-seasonal-stochastic decomposition that can assume an additive formulation:

$$P_t = T_t + S_t + T_t \tag{15.2}$$

where P_t is the electricity price at time t, T_t is the trend component at time t, S_t is the seasonal component at time t, and Y_t is the residual stochastic noise; or a log-additive formulation[6]

$$lnP_t = lnT_t + lnS_t + lnT_t \tag{15.3}$$

Wavelets are increasingly used for the trend-seasonal-stochastic decomposition. A wavelet is a mathematical tool that allows the decomposition of signals and their reconstruction. See the study by Janczura et al. (2013) for a review of deseasonalisation methods, Chui (2016) for a textbook on wavelets and Fianu (2017) for an application.

2.3 Mean reversion

A time series displays mean reversion if it tends to gravitate around its mean. To put it otherwise, the higher (lower) the price is, the more likely it is to decrease (increase). Hence, a mean-reverting series of prices does not persist away from its mean for long spells. The mean towards which prices revert can be constant or time-varying. Evidence of mean reversion in electricity prices can be found in the large majority of empirical works on the subject (see Bunn, 2000 and Weron, 2007 for surveys), with very few exceptions (e.g. DeVany and Walls, 1999; Leon and Rubia, 2004).[7] Fig. 15.1 shows that along a whole year of observation (2017 in this example), prices do not display any specific trend.

Electricity prices are mean-reverting for at least two reasons. One is that demand is influenced by economic activity and weather, both of which are mean-reverting in the very short term. Another reason is that, following an increase in demand, higher prices also stimulate a surge in supply.

In applied works, mean reversion is usually tested in relation to covariance stationarity. A stochastic process is covariance stationary if its expected value does not depend on time and if the autocovariance function is finite and depends only

[6] The log-additive decomposition is equivalent to the multiplicative decomposition $P_t = T_t \cdot S_t \cdot Y_t$.
[7] Weron (2007) uses the terms mean reversion and anti-persistence as synonyms.

on the time lags. Mean reversion is a necessary — but not sufficient — condition for covariance stationarity. Indeed, a mean-reverting process with a constant mean is covariance stationary, yet a mean-reverting process can also revert to a time-varying mean, e.g. when the time series is driven by a deterministic linear trend.

Testing covariance stationarity is useful also to avoid estimating biased econometric relationships. Indeed, estimates of the relationship between two variables, e.g. the wholesale electricity price and the supply from wind power, are biased upwards if both variables are driven by increasing trends, as the estimated partial correlation is partly due to the common or correlated trends and only partly reflecting the economic cause-effect mechanisms linking the two variables. Unbiased estimates can be obtained by appropriately transforming the variables, to make them stationary. Which transformation is appropriate depends on the nature of the underlying trend: deterministic or stochastic; linear or non-linear. To make the time series stationary in the case of a stochastic trend, it is necessary to take the first difference of the variable.[8] In case of a deterministic trend, the variable has to be regressed on the linear trend and the fitted value subtracted from the observed time series. Rejecting the hypothesis of stochastic and deterministic trends would motivate the researcher to opt for mean-reverting econometric models.

The most common stationarity tests, or *unit root tests*, are tests of the hypothesis that the partial autocorrelation coefficient is equal to 1. This is the logic followed by the Augmented Dickey Fuller (ADF) test and by the Phillips-Perron test. The ADF algorithm performs a regression of the first-differenced variable of interest on its first-order lag. The null hypothesis of non-stationarity is that the coefficient associated to the first-order lag is not statistically different from zero. The null is rejected if the test statistic is greater than the critical value at the given significance level. The test statistic is sensitive to deterministic sources of non-stationarity and to serial correlation in the error term, motivating the inclusion of additional regressors. Including a linear trend in the regression allows to test whether the underlying trend, if any, is stochastic or deterministic in nature. A deterministic trend is suggested if the inclusion of a trend causes the null hypothesis to be rejected.

Alternatively, the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test (Kwiatkowski et al., 1992) tests the null hypothesis that an observable time series is stationary around a deterministic trend (i.e. trend-stationary) against the alternative of a stochastic trend. The test statistic is obtained by modelling the series as the sum of deterministic trend, random walk and a stationary error term, and

[8] It may not be sufficient if the first differences are driven by a stochastic trend. This is a rare occurrence, yet it has been observed in some macroeconomic time series, such as unemployment.

a Lagrange multiplier statistic is used to test the null hypothesis that the random walk has zero variance.

By performing both the KPSS and ADF-related tests, one can obtain indications concerning the causes of non-stationarity (stochastic vs. deterministic trends), although results can be influenced by structural breaks that cause the parameters of the data-generating process to change over time. Also, one can obtain hints on the possible presence of long memory (or long-range dependence), namely a property of a time series such that autocorrelations decay slowly, in power law fashion (see also fractional integration models in Section 3.1.1).

2.4 Spikes and time-varying volatility

Even if electricity prices are stationary in mean, their variance is not constant over time. This occurs because of heteroskedasticity, volatility clustering and spikes. In Fig. 15.1, German-Austrian EPEX day-ahead prices for baseload and peakload contracts spike up to around 125 EUR/MWh in January and above 75 EUR/MWh in December 2017, while staying between 25 and 50 EUR/MWh in the other months. Also, prices appear to be more volatile in January, February, November and December than in the other seasons. In other words, volatility is clustered.

One of the most distinctive features of electricity prices is the existence of wide, sudden and short-lived excursions, commonly named spikes. Spikes are most frequently positive and occur when the gap between the system capacity and the electricity demand is relatively small, for instance, in peak hours. Larger spike magnitudes are expected when the excess capacity is more limited because of the convexity of the supply stack. Spikes are short-lived because transmission system operators quickly react to fix the shortages in generation or transmission capacity that caused them, that may be critical for the security and reliability of the system.

Spikes can be treated as sheer outliers, as commonly done in macroeconometrics where extreme values tend to be attributed to measurement errors. Alternatively, they can be treated as genuine economic fluctuations caused by extreme but recurring market conditions. Such fluctuations can be random, as when a power plant fails or there is an unexpected weather shock, or strategic, as with capacity withholding. Nevertheless, it is common to perform econometric estimates only after filtering out spikes or to use stochastic processes that generate spikes endogenously (see Sections 3.1.3 and 3.1.4). In fact, the mean reversion observed in electricity price series is partly due to the sudden ups and downs generated by spikes. Spike modelling and removal allow to identify the 'genuine' degree of mean reversion.

There is no univocal definition of a spike in the literature. The features that price spikes are assumed to possess are (1) a value higher than a threshold and (2) a reversal to lower values after a very short period of time. The empirical identification of the spikes changes depending on the assumed threshold and duration, and indeed the literature offers several methods for identification. Thresholds can be fixed or variable. The threshold can concern the price level, its absolute change or its rate of change. When a fixed threshold is assumed, all prices (or price changes) exceeding a given threshold are classified as spikes. A variable threshold is defined in terms of some statistics, e.g. prices above a certain quantile or exceeding a given number of standard deviations away from the mean. Such threshold can be computed iteratively, namely recomputed after each spike removal and replacement. Another family of methods involves estimating an econometric model and identifying as spikes all observations that lie outside of a given percent prediction interval. Other approaches mandate filtering the prices according to several thresholds and then selecting the one leading to the best match with respect to some statistics, e.g. the kurtosis. More sophisticated methods are based on wavelet decompositions, wherein a smoother time series is obtained by leaving out the highest one or two detail levels.

Once the spikes are identified, they have to be replaced by values that can be considered 'normal' because gaps in time series would prevent the correct estimation of their dynamic properties. In replacing the identified price spikes, one should have in mind a counterfactual in which the unusual event that triggered the spike has not occurred – be it a unit failure, an extraordinary surge in renewables or an extreme weather event. Various techniques for spike replacement have been proposed and applied. Such techniques are often purely heuristic, but in principle, they should be consistent with a credible theoretical explanation for the spikes. Spikes can be replaced by the chosen threshold, one of the neighbouring prices, the mean of the two neighbouring prices, values observed in similar market conditions, e.g. the median of all prices having the same weekday and month, or the mean of the deseasonalised prices. Not all these methods are innocuous with respect to the statistical properties of the data and to the ensuing estimation of econometric models. Janczura et al. (2013) provide a comprehensive review of methods for spike identification and replacement.

More generally, electricity prices are characterised by heteroskedasticity and volatility clustering. A variable is heteroskedastic if it displays different variances in different subsamples. A particular form of heteroskedasticity is volatility clustering so that 'large changes tend to be followed by large changes of either sign and small changes tend to be followed by small changes' (Mandelbrot, 1963). A specific evidence of time-dependence in volatility is provided by the *inverse leverage effect*, according to which the effects of positive and negative shocks on the conditional variance are asymmetric. Estimates by Knittel and Roberts (2005) and others have shown that positive shocks amplify conditional volatility more than negative shocks of the same magnitude, possibly because of convexity in the supply stack.

Heteroskedasticity is commonly tested through the White test. Behind the test lies the insight that if the variance of a regressand is constant, conditional on its regressors (e.g. the variance of electricity prices conditional on price fundamentals), its correlation with the regressors is not statistically significant from zero. Accordingly, the test relies on an auxiliary regression. First, the variance of the error term is proxied by means of the squared residuals; second, the squared residuals are regressed onto the set of regressors augmented with their squares and cross-products; third, a Lagrange multiplier test statistic is built by multiplying the coefficient of determination (R^2) from the auxiliary regression times the sample size. A relatively large R^2 runs against the null hypothesis of homoskedasticity. The test statistics follows a Chi-squared distribution. Another approach to detecting volatility clustering requires inspecting the autocorrelogram of some measure of volatility, such as absolute or squared returns, as well as by testing the statistical significance of their autocorrelation coefficients, using the Ljung-Box Q statistic. Under the null hypothesis, autocorrelations of the squared or absolute returns are equal to zero. Rejecting the null points to volatility clustering.

Models in the generalized autoregressive conditionally heteroskedastic (GARCH) family and models with stochastic volatility (Section 3.1.2) are appropriate when heteroskedasticity is detected.

2.5 Skewness and excess kurtosis

As for many commodities, electricity prices and returns are characterised by non-Gaussian empirical probability distributions. Departures from normality are testified by skewness and positive excess kurtosis.

Positive skewness means that the mass of the distribution is left of the mean, whereas the opposite holds with negative skewness. Excess kurtosis for a normal variate is equal to 0, whereas positive values indicate that the tails of the probability density function taper off more slowly than for a normal variate. The presence of spikes is one obvious reason for excess kurtosis. Fig. 15.1 suggests that extreme fluctuations in prices are rare, but not negligible (excess kurtosis), and that prices move upwards more frequently than downwards (positive skewness).

In order to test if Gaussian distributions provide an adequate description of the probability distribution of electricity prices and returns, one can use goodness of fit (GoF) statistics, such as the Kolmogorov–Smirnov, Cramer-von Mises and Anderson–Darling statistics.

Such statistics measure the distance between the empirical distribution function $F_n(x)$, for sample size n, and the fitted distribution function $F(x)$. The Kolmogorov–Smirnov D statistic is defined as the maximum absolute deviation between the theoretical and empirical cumulative distribution functions (CDFs):

$$D = sup_x|F_n(x) - F(x)| \tag{15.4}$$

The Cramer-von Mises Q statistic is based on the quadratic deviations between theoretical and empirical CDFs:

$$Q = n\int_{-\infty}^{\infty}\{F_n(x) - F(x)\}^2\psi(x)dF(x) \tag{15.5}$$

where $\psi(x)$ denotes a function that gives weights to the squared differences contained in the aforementioned formula. The Anderson-Darling A^2 statistic results when $\psi(x) = [F(x)\{1 - F(x)\}]^{-1}$. The empirical values of the GoF statistics need to be compared to the limiting values for a given confidence level (Feller, 2015; Stephens, 1974).

Many probability laws are compatible with the evidence of heavy tails and asymmetry. Table 15.1 reports the formulas for the most commonly tested probability density functions in the electricity price literature. Such laws can be obtained under the assumption that the change in price is equal to the sum of independent and identically distributed (i.i.d.) random variables or shocks.

Given n i.i.d. shocks, not restricted to having finite moments, the generalised central limit theorem states that the distribution of the sum converges to an α-stable distribution as $n \to \infty$ (Samorodnitsky and Taqqu, 1994). The α-stable distribution features four parameters (scale, location, skewness, and tail). The tail parameter, $\alpha \in (0,2]$, tunes the steepness of the tails. If $\alpha = 2$, the decay is exponential and the Gaussian distribution results, but $\alpha < 2$ implies that its tails are described by a power (or Pareto) law and that the variance is infinite.

The class of hyperbolic distributions includes further candidates to describe heavy-tailed electricity returns. The generalised hyperbolic law is obtained as a variance-mean mixture of Gaussian variates, if the mixing density is a generalised inverse Gaussian (GIG) with λ in the set of real numbers.[9] By assigning specific values to λ, special cases are obtained. If the mixing distribution is a GIG with $\lambda = 1$, the hyperbolic distributions results. The normal inverse Gaussian (NIG) law obtains in the case $\lambda = -1/2$.

[9] In the cited cases, a mixture of distributions is obtained by assuming that a parameter of the probability distribution of the shocks is itself a random variable.

Table 15.1 Probability density functions of distributional laws estimated in the literature on wholesale electricity prices.

Law	Pdf or characteristic function	location	Scale	Tail	Skewness
Hyperbolic	$f_H(x) = \frac{\sqrt{\alpha^2-\beta^2}}{2\alpha\sigma K_1\left(\sigma\sqrt{\alpha^2-\beta^2}\right)} e^{-\alpha}\sqrt{\alpha^2+(x-\mu)^2}+\beta(x-\mu)$ where K_1: modified Bessel function of the second kind	μ	σ	α	β
Normal Inverse Gaussian	$f_{NIG}(x) = \frac{\alpha\sigma}{\pi} \frac{K_1\left(\alpha\sqrt{\sigma^2+(x-\mu)^2}\right)}{\sqrt{\sigma^2+(x-\mu)^2}} e^{\sigma}\sqrt{\alpha^2-\beta^2}+\beta(x-\mu)$	μ	σ	α	β
Asymmetric Exponential Power	$f_{AEP}(x) = \frac{1}{C} e^{-\left(\frac{1}{b_\ell}\left\lvert\frac{x-\mu}{a_\ell}\right\rvert^{b_\ell}\theta(\mu-x)+\frac{1}{b_r}\left\lvert\frac{x-\mu}{a_r}\right\rvert^{b_r}\theta(\mu-x)\right)}$ where $\theta(.)$: Heaviside function $C = a_\ell A_0(b_\ell) + a_r A_0(b_r); A_k(y) = y^{\frac{k+1}{y}-1}\Gamma\left(\frac{k+1}{y}\right)$	μ	a_ℓ, a_r	b_ℓ, b_r	$\lvert a_\ell - a_r\rvert$, $\lvert b_\ell - b_r\rvert$
α-stable	(log characteristic function) $\log\phi(t) = -\sigma^\alpha\lvert t\rvert^\alpha\left\{1 + i\beta sign\,(t)\tan\frac{\pi t}{2}\left[(\sigma\,\lvert t\rvert)^{1-\alpha} - 1\right]\right\}$ $+ i\mu t\,,\ \alpha \neq 1$ $\log\phi(t) = -\sigma\lvert t\rvert\left\{1 + i\beta sign\,(t)\frac{2}{\pi}\log(\sigma\,\lvert t\rvert)\right\}$ $+ i\mu t,\ \alpha = 1$	μ	σ	α	β

Characteristic functions are reported when the probability density functions do not have closed-form expressions. Parameters tuning the location, the scale, the tail decay and the skewness are explicitly indicated.

A further class of distributions is defined if one assumes that shocks $x \sim$ i.i.d.$N(0,h\psi)$, where $h \sim$ i.i.d. exponential. As shown by Fu et al. (2005), if $\psi \geq 0$, this model yields an exponential power (or Subbotin) distribution with shape parameter that is positive and inversely related to ψ (Subbotin, 1923). The exponential power (EP) distribution reduces to a Laplace if the shape parameter is equal to one and to a normal if it equals 2. As the shape parameter becomes smaller, the density becomes heavier-tailed and more sharply peaked. West (1987) represented the exponential power distribution as a scale mixture of normals with an α-stable mixing distribution. The EP family has been generalised to allow for asymmetry by Bottazzi and Secchi (2011), who introduced the asymmetric exponential power (AEP) family, whose density is characterised by two positive shape parameters (one for each tail), two positive scale parameters and one position parameter. The EP distribution is a special case when the left and right shape and scale parameters are equal.

To date, there is no systematic comparison among distribution laws for a comprehensive set of power exchanges. The α-stable model was shown to outperform the hyperbolic and NIG distributions by Rachev et al. (2004) for EEX daily price differences and by Weron (2008) for EEX and NordPool data. Work by Weron (2007) reports estimates of the α-stable, hyperbolic and NIG models on data from several markets (EEX, Omel, PJM, NEPOOL)

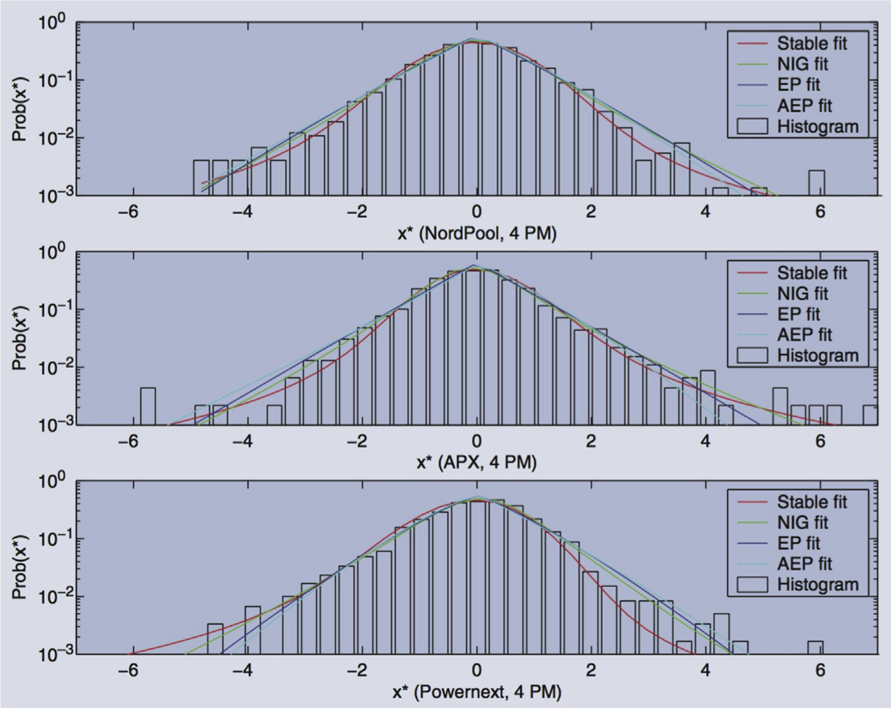

FIGURE 15.5

Histograms of log-returns from power exchanges: NordPool (1 January 1997—31 December 2002), APX (6 January 2001—31 December 2004), Powernext (1 February 2002—31 January 2007), 4 p.m. auctions and superimposed density fit (α-stable, NIG, exponential power, asymmetric exponential power). AEP, asymmetric exponential power; NIG, normal inverse Gaussian. *Source: Sapio, A., 2012. Modeling the distribution of day-ahead electricity returns: a comparison. Quant. Financ. 12 (12), 1935—1949.*

and using various measures of price returns. The results vary across countries and are affected by how the returns are defined. Sapio (2012) covers three markets (NordPool, APX, Powernext) and considers three definitions of price returns (log-returns, percentage returns, price differences). The α-stable and NIG outperform the EP and AEP distributions. Histograms of log-returns from some power exchanges and superimposed fitted densities are reported in Fig. 15.5. Further references can be found in the cited articles.

The departures from Gaussianity described previously, along with the other statistical properties to be described, motivate the adoption of models incorporating heteroskedasticity, jumps and regime switches (Sections 3.1.2, 3.1.3, 3.1.4).

2.6 Negative prices

A final, most peculiar feature of electricity price series is the phenomenon of *negative prices*. The study by Sewalt and De Jong (2003) is perhaps the earliest dedicated to this issue, illustrating negative prices in the Dutch power exchange.

Negative prices are presently permitted in several European power exchanges, such as EPEX, the Nordic Power Exchange and markets in Belgium, The Netherlands and Great Britain, as well as in the US (New York Independent System Operator, PJM, the California ISO and the Electric Reliability Council of Texas [ERCOT]) and Australia.

Brijs et al. (2015) report that in the period 1 December 2012–30 November 2013, negative prices occurred in Belgium, Germany, the Netherlands and France, respectively, in 6.78%, 21.08%, 6.73% and 0.42% of the hourly sessions, mostly in the night and early morning hours, with frequency peaks during weekends and public holidays (see also Genoese et al. 2010; Keles et al., 2012; Fanone et al., 2013). On the other side of the Atlantic, negative prices were observed in ERCOT around 10% of the time between 2008 and 2011 (Huntowski et al., 2012). Negative prices can therefore be substantial in magnitude but are typically short-lived. Fig. 15.1 is consistent with the evidence cited previously: in 2017 the peakload contracts were negatively priced in 4 days, whereas this occurred three times in the case of baseload contracts. Prices went as low as nearly −50 EUR/MWh, still far from the record negative value of −500 EUR/MWh on 4 October 2009 in the hour between 2 and 3 a.m.

Non-experts are puzzled upon hearing that electricity prices can be negative, but the frequency with which negative prices occur and their correlation with market fundamentals suggest that negative prices are not merely accidental outliers. Negative bids can be rationalised in specific circumstances.

From the viewpoint of a supplier, a negative price for a commodity reveals a willingness to pay to keep power plants running. Indeed, by placing a negative bid, suppliers undercut rivals and secure acceptance of their offered power. Such a move can be optimal when there is excess local electricity supply, for instance, because transmission capacity is insufficient given the geographical location of non-flexible supply (e.g. renewables). Shut-down costs for a conventional power plant unit (e.g. coal, nuclear) make it expensive for generating companies to interrupt production when residual demand is very low and inflexible, motivating the need to maintain a steady generation level. Similarly, to save on ramp-up costs, it may be rational to purchase the excess power when electricity demand is low. Another rationale concerns units subject to must-run obligations, for instance, because they are contracted to provide reserve capacity or because of rules that prioritise their supply, as with wind-based generation or other sources with limited storage possibilities. Policy tools can also lead generators to bid a negative price to undercut rivals and generate a positive revenue (e.g. the production tax credit received by wind power in the US: Fink et al., 2009; Huntowski et al., 2012). See also the analysis in Gerster (2016).

Consistently, the existing evidence shows that negative prices are more likely to occur at times when interconnection fails, when low system load is combined with moderate generation from intermittent renewables or when load is moderate and renewables generation is abundant. Errors in forecasting demand and renewable generation can moreover affect the probability of negative prices in the balancing market. These considerations challenge researchers to formulate models that incorporate the endogenous occurrence of negative prices.

3. Econometric modelling and forecasting

Econometrics is the application of statistical methods to economic data in order to learn about their properties and relationships. Most econometrics revolves around regression analysis, which is a set of statistical processes for estimating relationships among variables. Econometric estimates allow to assess the correlation between couples of variables, e.g. between the wholesale electricity price and the wind power in-feed, conditional on the values of all other explanatory variables (partial correlation). However, correlation does not imply causation, and appropriate methods need to be applied to identify causal effects.

The branch of econometrics that is most relevant for the study of electricity markets is time series econometrics, namely the analysis of series of data points indexed in time order. Regression analysis applied to time series starts from the observation that the dependent variable — in our case, the wholesale electricity price — is serially correlated, i.e. it correlates with its past values, as well as with current and past values of other variables. Time series econometric models, in essence, comprise stochastic difference equations. The dependent variable can be a scalar (univariate time series) or a vector of variables (multivariate time series). Less frequently, electricity prices have been studied by means of econometric models for longitudinal panel data, namely data sets including observations about multiple units (individuals, market zones, regions, countries) tracked over time. Useful textbooks on time series analysis are written by Box and Jenkins (1970), Brockwell and Davis (2002) and Enders (2008).

Econometric models for electricity prices can be classified along several dimensions. The most relevant classification here is between *structural models* and *reduced-form models*. Reduced-form models, to be reviewed in Section 3.1, include univariate time series models such as autoregressive integrated moving average models and their special cases (ARMA, AR), GARCH, regime-switching models and jump-diffusion models. In structural models (Section 3.2), the electricity price is represented as a function of market fundamentals, such as demand and fuel costs.

3.1 Reduced-form models

3.1.1 ARMA models

An autoregressive model assumes that the random variable of interest depends on a linear combination of its past values. Let P_t denote the electricity price at time t. An autoregressive model of order p (AR(p)) for the electricity price reads

$$\phi(B)P_t = \epsilon_t \tag{15.6}$$

where B is the backward shift operator, or lag operator, such that $B^k P_t \equiv P_{t-k}$; $\phi(B)$ is a polynomial of order p in B; ϵ_t is a white noise (or i.i.d.) error term with constant mean μ and variance σ^2. The $\phi(B)$ polynomial is often specified to account for periodic patterns, e.g. $\phi(B) = 1 - \phi_1 B - \phi_7 B^7$ allows to estimate the first-order and seventh-order autocorrelation in daily-frequency electricity prices. In the latter case, the AR model can be written as follows:

$$P_t = \phi_1 P_{t-1} + \phi_7 P_{t-7} + \epsilon_t \tag{15.7}$$

If we further assume that the error term is serially correlated up to q lags, we obtain an autoregressive-moving average, or ARMA(p,q), as follows:

$$\phi(B)P_t = \theta(B)\epsilon_t \tag{15.8}$$

where $\theta(B)$ is a polynomial of order q in B. Coefficients in the $\theta(B)$ polynomial capture the serial correlation in the error term.

The ARMA model is covariance stationary when the roots of the $\varphi(B)$polynomial lie outside the unit circle. The simplest case of a non-stationarity ARMA model, or *integrated* ARMA model, is as follows:

$$(1 - B)P_t = \theta(B)\epsilon_t \tag{15.9}$$

or $P_t = P_{t-1} + \epsilon_t + \theta_1\epsilon_{t-1} + \cdots + \theta_q\epsilon_{t-q}$. In this case, the current price is perfectly correlated with the lagged price. The $\varphi(B) = 1 - B$ polynomial has a root equal to one (unit root).

AR models in research on electricity markets are sometimes presented as the discretised versions of continuous-time stochastic processes originally used in finance. Typically, a mean-reverting AR model is derived from the Ornstein–Uhlenbeck (OU) process, first introduced by Vasicek (1977). An integrated AR(1) process instead corresponds to a Brownian motion.

Generalisations of the univariate ARMA-type model have been used in the literature. In a first one, the model is specified in terms of the logarithm of prices:

$$\phi(B)\ln P_t = \theta(B)\epsilon_t \tag{15.10}$$

The logarithmic transform results in a smaller variance, which is useful in the analysis of a very volatile variable and allows to meet the non-negativity constraint on prices (but see Section 2.6 on negative prices).[10]

A second generalisation augments the ARMA model by means of exogenous variables (contemporaneous and/or lagged), giving rise to the ARMAX model.[11] For instance, if X_{it} denotes the i-th generic exogenous variable observed at time t and $\gamma_i(B)$ is the associated coefficient, we have

$$\phi(B)P_t = \sum_{i=1}^{k} \gamma_i(B)X_{it} + \theta(B)\epsilon_t \tag{15.11}$$

The matrix of exogenous variables can include dummies, or sinusoidal functions, that account for various seasonal and periodic patterns (if not pre-filtered), as well as variables that are relevant for policy reasons (e.g., congestion measures, renewables in-feed).

Another generalisation consists in providing separate models for the short- and long-term components of the electricity price. The short-term component is typically the error term. For instance, in the 2-factor model by Lucia and Schwartz (2002), the long-term component is a mean-reverting AR, the error term is an arithmetic Brownian motion and the Wiener processes driving them are correlated.

A last generalisation is based on the concept of fractional integration. In a stationary ARMA model, autocorrelations decay exponentially. However, it has been observed that in samples of electricity prices, the rate of decay in autocorrelations is slower than that in an exponential law. This is an instance of *long memory* or *long-range dependence*.

The AutoRegressive Fractional Integration Moving Average(p, d, q) model (ARFIMA) seeks to model such slower-decaying autocorrelations appropriately:

$$\phi(B)(1-B)^d P_t = \theta(B)\epsilon_t \tag{15.12}$$

where $d \in [0,1]$ is a coefficient measuring the degree of fractional integration. If its point estimate is in the $(0,1)$ interval and is significantly different from 0 and 1, then the data display *fractional integration* and the autocorrelogram decays in power law fashion. Polynomials $\phi(B)$ (of order p) and $\theta(B)$ (of order q) are defined as previously, and ϵ_t is an i.i.d. error term with mean μ and variance σ^2. The autocorrelogram generated by an ARFIMA process has a power law

[10] Ironically, in the early days of the liberalised industry, the model specified on price levels was discarded precisely because it could predict negative prices, that were not yet allowed.
[11] The X in ARMAX originates from the notation commonly used to denote a matrix of exogenous control variables, namely X.

profile, with a fast decay at the beginning and then a plateau. Useful references for this model are the study by Hosking (1981) and the book edited by Doukhan et al. (2003).

With respect to the list of stylised facts provided in Section 2, the limitations of ARMA-type models are well understood. A non-Gaussian distribution of the error term is a necessary condition for the model to reproduce the spikes detected in the literature. Also, the model is characterised by conditional homoskedasticity as the variance of the error term is assumed constant and does not allow to reproduce the empirical evidence of volatility clustering.

3.1.2 Heteroskedastic and stochastic volatility models

The inability of ARMA-type models to account for time-dependence and clustering in volatility has been tackled through models that incorporate conditional heteroskedasticity. In conditionally heteroskedastic models, the conditional variance (e.g., σ^2 in an AR(p) process) is not constant. In one strand of models, the conditional variance depends on some of the variables of interest; other models assume it is serially correlated.

A model corresponding to the first case is the stochastic variance AR(p) model described in Ghysels et al. (1996). In the model, the variance of the error term is a function of lagged prices, and hence, it is time varying:

$$\phi(B)(P_t - \mu) = \sigma_t \epsilon_t \tag{15.13}$$

with

$$\sigma_t \equiv (\psi(B)P_t)^\gamma$$

where ϵ_t is now an i.i.d. error term with 0 mean and unit variance. The exponent $\gamma > 0$ and polynomial $\psi(B)$ tune the dependence of the conditional variance on the previous prices.

The Auto-Regressive Conditionally Heteroskedastic (ARCH) model has been introduced by Engle (1982), originally for applications in finance, to model data characterised by serially correlated conditional variance. Specifically, the conditional variance of the error term reads:

$$\sigma_t^2 = \omega + \alpha(B)\epsilon_t^2 \tag{15.14}$$

with constant coefficients ω and the terms in the $\alpha(B)$ polynomial. Because the conditional variance is positive by definition, the ARCH model imposes non-negativity constraints upon coefficients. The ARCH process is weakly stationary if and only if $\sum_{i=1}^{q} \alpha_i < 1$. The process is characterised by positive excess kurtosis even if ϵ_t is Gaussian.

More generally, the conditional variance can depend on a moving average component, as proposed by Bollerslev (1986), leading to a Generalised Auto-Regressive Conditionally Heteroskedastic (GARCH) model:

$$\sigma_t^2 = \omega + \alpha(B)\epsilon_t^2 + \beta(B)\sigma_t^2 \qquad (15.15)$$

where $\alpha(B)$ and $\beta(B)$ are polynomials of order p and q, respectively. Non-negativity constraints on coefficients hold for the GARCH model as well.

Applications in electricity econometrics couple GARCH processes for the conditional variance with an ARMA process for the price mean, giving rise to ARMA–GARCH models. These models, thus, include two equations: one for the mean and one for the conditional variance that are estimated jointly.[12] The equation for the mean can also be an ARFIMA process, as in the study by Gianfreda and Grossi (2012) among others, as well as regime-switching and jump-diffusion models (Sections 3.1.3 and 3.1.4).

Several generalisations of the GARCH model have been considered in the literature, to encompass some empirical facts or to allow testing theoretical hypotheses. One approach modifies the equation for the mean, leading to the GARCH-in-mean model, wherein the equation for the mean includes a measure of volatility, such as the square root of the conditional variance (Engle et al., 1987). Convexity in the electricity supply stack implies that in volatile market sessions, the average price is higher. Hence, the coefficient associated to the volatility term in the mean equation is expected positive. Other approaches involve the equation for the conditional variance. In the exponential GARCH (Nelson, 1991), the conditional variance depends on both the magnitude and the sign of past residuals. Technically speaking, this model allows to overcome the non-negativity restriction on coefficients. More importantly, it allows to model the inverse leverage effect (see Section 2.4). The threshold GARCH (TGARCH) model (Zakoian, 1994) seeks to match the empirical evidence, suggesting the existence of discontinuities in the time series behaviour of electricity prices. As market conditions change, it often appears that market-clearing prices transition to different regimes, characterised by different means, variances, as well as autoregressive and heteroskedasticity properties.

Stochastic variance and GARCH-type models are well suited to reproduce volatility clustering and long tails, as well as mean reversion when coupled with ARMA models for the mean. Yet, despite efforts to model discontinuities (e.g. the TGARCH model), these models do not provide a good-enough match to the evidence of spikes and structural changes in the process driving the electricity price. These considerations have paved the way for two further model families: regime-switching (RS) and jump-diffusion models.

[12] In finance, where the interest is on returns, the empirical finding on random walk prices, i.e. unit roots in the time series of prices, has motivated a white noise equation for the mean return.

3.1.3 Regime switching models

RS models, introduced in economics by Hamilton (1989), are motivated by the observation that the dynamics of electricity prices, rather than evolving smoothly across time, seems to undergo abrupt and reversible qualitative changes. In other words, the price undergoes different and recurring regimes during its evolution. Interpreting the price dynamics in terms of switching between regimes allows to rationalise pieces of evidence such as spikes and volatility clustering.

An RS model is defined by (1) the number of regimes, (2) the properties of the stochastic process guiding prices within each regime and (3) a stochastic process driving transitions between regimes. Regimes are at least two, but it is not infrequent to assume more. Regimes can correspond to specific time periods or network congestion times, can be determined by a latent variable or endogenously identified by means of thresholds concerning some variables that are relevant for the analysis. In Markov RS models, transitions are modelled as a Markov chain with given transition probabilities.[13] In threshold autoregressive models, transitions between regimes are dictated by movements of a variable across pre-defined thresholds. If the threshold is defined in terms of the price itself, one speaks of a self-exciting threshold autoregressive (SETAR) model. In both types of models, the threshold values can be supplied by the modeller, based on theoretical considerations, or estimated from the data.

In a Markov RS model featuring two regimes ('base' and 'spike'), the electricity price is driven by a mean-reverting process in the 'stable' regime and by a white noise in the 'spike' regime. Transitions are Markovian. Formally,

$$\begin{cases} \phi(B)P_t = \epsilon_{st,t} & \text{Stable regime} \\ P_t = \epsilon_{sp,t} & \text{Spike regime} \end{cases} \tag{15.16}$$

where $\epsilon_{st,t}$ and $\epsilon_{sp,t}$ are i.i.d. Gaussian variates with mean μ_{st} and μ_{sp}, respectively, in the stable and in the spike regime, with $\mu_{st} < \mu_{sp}$. The autoregressive polynomial $\phi(B)$ is defined as in Section 3.1.1. The transition matrix Π is a 2-by-2 matrix including probabilities $\pi(i,j)$ that the price is in state j at time t, conditional on it being in state i at time $t-1$. By construction, the probability of staying in state i is $\pi(i,i) = 1-\pi(i,j)$, whereas the probability of remaining in state j is $\pi(j,j) = 1-\pi(j,i)$. The switching probabilities are assumed constant. A model of this kind has been presented by Huisman and De Jong (2003) among others.

[13] According to the Markov property of a stochastic process, the conditional probability distribution of future states of the process does not depend on the sequence of events that preceded it.

The aforementioned RS model allows to reproduce the upward spikes and the subsequent return to the stable regime. Richer dynamics can be obtained by postulating that prices can experience negative spikes even if no positive spike occurred before. The Markov RS model with three regimes, proposed by Huisman and Mahieu (2003), posits a regime $R(0)$ with mean-reverting prices, a regime $R(+1)$ with white noise prices and a regime $R(-1)$ again with mean-reverting prices, but with a lower long-term mean. Formally,

$$\begin{cases} P_t = \epsilon_{1,t} & R(+1) \\ \phi_0(B)P_t = \epsilon_{0,t} & R(0) \\ \phi_{-1}(B)P_t = \epsilon_{-1,t} & R(-1) \end{cases} \tag{15.17}$$

where the autoregressive polynomials $\phi_0(B)$ and $\phi_{-1}(B)$ differ across regimes, as do the Gaussian error terms $\epsilon_{-1,t}$, $\epsilon_{0,t}$, $\epsilon_{1,t}$ and their means and variances. Transition probabilities are assumed constant and are summarised by a 3-by-3 transition matrix.

An instance of a SETAR model has been illustrated by Lucheroni (2012):

$$\begin{cases} \phi_0(B)P_t = \epsilon_{0,t} & u(BP_t) < T \\ \phi_1(B)P_t = \epsilon_{1,t} & u(BP_t) \geq T \end{cases} \tag{15.18}$$

where $u(.)$ is a function defined over lagged values of the electricity price (notice the lag operator B), and T is a price threshold, above which the parameters of the stochastic process driving the electricity price change (the autoregressive polynomial from $\phi_0(B)$ to $\phi_1(B)$; the error term from $\epsilon_{0,t}$ to $\epsilon_{1,t}$).

Limitations of RS models have been discussed in the literature. One is that with constant transition probabilities, prices may remain in the spike regime longer than empirically plausible. In other words, switches cannot be memoryless, and the likelihood of a downward switch should be more likely after a transition from the stable to the spiky regime. Research on RS models has explored various solutions to achieve realistic transition patterns.

It is worth noting that the choice of stochastic processes in the different regimes is not constrained by the aforementioned examples. Following previous instances (e.g. Klaassen, 2002; De Jong, 2006), Cifter (2013) has built a Markov Switching-GARCH model wherein the market undergoes two regimes (low volatility, high volatility), and the price is driven by GARCH processes in both regimes, featuring different parameters and different assumptions on the distribution of the error term (e.g. Student t). RS models with long memory have been estimated by Haldrup and Nielsen (2006). Many other variations have been envisaged and applied to data samples. A useful reference in this respect is the study by Janczura and Weron (2010), who have reviewed and compared various RS models for wholesale electricity prices.

3.1.4 Jump-diffusion models

In a jump-diffusion model, the price dynamics is driven by a mean-reverting component and by a jump component represented by a Poisson process. The jumps are realisations of the Poisson process that can be drawn from Gaussian or from heavy-tailed probability distributions. In the simplest version of the model, the Poisson process is homogeneous: jump probabilities do not vary over time.

A basic formulation for the jump-diffusion model is the following:

$$\phi(B)P_t = \epsilon_t + Q_t \tag{15.19}$$

where $Q_t \equiv \sum_{i=1}^{N_t} q_{it}$ is a compound Poisson process, N_t is a Poisson process with rate λ, q_{it} is an i.i.d. random variable, independent of N_t. The error term ϵ_t and the jump component Q_t fulfil different roles: the former accounts for the high-frequency, low-magnitude random fluctuations, whereas the jump component allows to reproduce the stylised fact of infrequent yet large-price excursions.

More refined versions of the jump-diffusion model applied to electricity prices have relaxed some assumptions on jump frequencies, size and duration. Non-homogeneous Poisson processes allow to more closely track the evidence of higher spike frequencies in times of peaking demand, e.g. by using a periodic intensity function. A signed Poisson process is appropriate if anti-spikes are to be reproduced, for instance, to achieve a better fit for a time series displaying negative prices. Signed jumps and deterministic sequences of positive and negative jumps also allow for a speed of reversion to the 'base' price level that is more in line with the empirical evidence. The mean-reverting and jump components are usually posited to be mutually independent, yet it can make sense to assume that jump intensity and duration depend on market conditions: in times of tight balance between demand and supply, jumps may be higher, but market players and regulatory authorities are expected to react faster, thereby causing a faster reversion to the previous price level. Jump size can be modelled as proportional to the current price level. Readers can refer to to the studies Weron et al. (2004), Cartea and Figueroa (2005), Geman and Roncoroni (2006) and Meyer-Brandis and Tankov (2008) for early and influential applications of jump-diffusion modelling to wholesale electricity prices.

3.2 Structural models

In structural models of electricity prices, also called fundamental models, the relationship between the electricity price and its determinants is derived from the market-clearing condition that supply and demand of electricity must be equal at all times. Let D_t and S_t denote, respectively, the demand and supply of electricity at time t in a given market. Assume that demand is price-inelastic and driven by a stochastic process, whereas supply is an increasing function of the electricity price P_t and of other fundamentals (in matrix X_t), such as $S_t = S(P_t, X_t)$ with $\frac{\partial S(\cdot)}{\partial P} > 0$ and $S(.)$ invertible with respect to P_t (and stochastic). Given the market-clearing condition

$$D_t = S(P_t; X_t), \tag{15.20}$$

the electricity price can be derived as

$$P_t = S^{-1}(D_t; X_t) \tag{15.21}$$

where S^{-1} is the *inverse supply function*, also known as *bid stack*, as it tracks the merit order of the supply bids.

A whole family of econometric models can be derived from the aforementioned equation, each member corresponding to a different functional form for $S(.)$, a different set of fundamentals X_t and different properties of the stochastic process guiding D_t. Carmona and Coulon (2014) review this modelling approach. Here we shall illustrate some examples.

In the study by Barlow (2002), the supply function is a shifted power function, as follows:

$$y \equiv S(P_t) = a_0 - b_0 P_t^{\alpha} \tag{15.22}$$

where $\alpha < 0$. Parameter a_0 is interpreted as the maximum supply available in the market, given the generation capacity. The inverse supply function reads

$$S^{-1}(y) = \left(\frac{a_0 - y}{b_0}\right)^{\frac{1}{\alpha}} \tag{15.23}$$

for $y < a_0$. Demand D_t is driven by a mean-reverting stochastic process, such as an OU process, around a long-term mean or trend.

If demand and supply are in equilibrium ($D_t = S_t$), the following holds:

$$P_t = \begin{cases} \left(\dfrac{a_0 - D_t}{b_0}\right)^{\frac{1}{\alpha}} & D_t < a_0 - \epsilon_0 b_0 \\ \epsilon_0^{\frac{1}{\alpha}} & D_t \geq a_0 - \epsilon_0 b_0 \end{cases} \tag{15.24}$$

where ϵ_0 is the price level corresponding to the discontinuity. The aforementioned functional form lends itself to generating spiky time series as the price increases continuously as demand grows, only to jump once demand hits the $a_0 - \epsilon_0 b_0$ threshold. Parameter α tunes the steepness of the price-demand relationship. A smaller α implies that spikes are more pronounced.

Kanamura and Ohashi (2007, 2008) retained the assumption of a mean-reverting demand process and assumed a hockey stick supply curve, namely a piecewise function punctuated by two thresholds: a linear function below the lower and above the higher and a quadratic function between the thresholds (see Fig. 2 in Kanamura and Ohashi, 2008). Formally:

$$P_t = S^{-1}(y)\begin{cases} \alpha_1 + \beta_1 y & y \leq z - s \quad \text{Lower linear regime} \\ a + by + cy^2 & z - s < y < z + s \quad \text{Quadratic regime} \\ \alpha_2 + \beta_2 y & y \geq z + s \quad \text{Upper linear regime} \end{cases} \tag{15.25}$$

where the definitions of coefficients are omitted from the text for the sake of space.[14]

The lower linear regime is identified as a non-spiky regime, whereas spikes are generated in the regions of the supply curve corresponding to the quadratic regime and to the upper linear regime. The price process is obtained by replacing D_t instead of y in the inverse supply curve.

Interestingly, Kanamura and Ohashi (2008) show that demand fluctuations along such piece-wise quadratic bid stack give rise to an RS model. In other words, they are able to identify a correspondence between the transition probabilities of an RS model and the parameters of the bid stack.

A similar model is proposed by Mari and Tondini (2010), who represent the supply stack by means of a power function, wherein the exponent β_t is driven by a Markov process assuming two values, and the scale parameters a_t and h_t capture, respectively, random shocks and the seasonal variation in the data:

$$P_t = h_t \left(\frac{q_t}{a_t}\right)^{\beta_t} \tag{15.26}$$

where q_t indicates the power volume, equal to inelastic load D_t in the market equilibrium. The exponent β_t assumes values below one if the supply stack is concave. When $\beta_t = \beta_0$, the supply curve is nearly flat, whereas $\beta_t = \beta_1 > \beta_0$ occurs in a steep region of the supply stack.

Further developments of structural models of electricity prices have specified how additional explanatory variables, such as system capacity and fuel prices, can be included in the model. Skantze et al. (2000) and Cartea and Villaplana (2008) have modelled the electricity price as a function of both demand and system capacity:

$$P_t = exp(aD_t + bC_t) \tag{15.27}$$

[14] Given $x_1 = z - s$ and $x_2 = z + s$, parameters a, b, x and α_2 are defined as follows: $a \equiv \alpha + \beta_1 x_1 - bx_1 - cx_2^1$, $b = \frac{x_2\beta_1 - x_2 beta_2}{x_2 - x_1}$, $c = \frac{\beta_2 - b}{2x_2}$, $\alpha_2 = -\beta_2 x_2 + \alpha + bx_2 + cx_2^2$. See Kanamura and Ohashi (2008).

where C_t is system capacity and $a > 0$ and $b < 0$ are constant coefficients (not corresponding to the ones used previously). The negative sign of the capacity coefficients conveys the insight that a smaller system capacity makes it more likely that the system will run into scarcity and that prices will spike.

There are a number of limitations in such approach, as discussed by Carmona and Coulon (2014). One is that capacity data may not be available, and even if they are, capacity may be only one of the possible determinants of spikes. Additional variables, e.g. transmission limits or ramp-up costs, should be included. Second, a change in capacity may imply both a shift in the supply stack and a change in its slope, depending on the technology whose capacity is changing (baseload or peaking) and on the market power exercised by power-generating companies. Finally, the model in the aforementioned equation does not exclude that demand may exceed available capacity.

Solutions to these issues have required taking different routes. One involves devising more sophisticated functional forms for the bid stack that satisfy the constraint $D_t \leq C_t$. Other authors have built models specified with respect to the demand-capacity ratio D_t/C_t, the reserve margin $C_t - D_t$ or the percentage reserve margin one D_t/C_t. Other works have treated capacity as part of an unobservable noise that includes other similar effects, such as outages, maintenance and so forth.

The inclusion of fuel prices requires a preliminary step, wherein the examination of the power market in question should reveal whether a single fuel is predominantly the marginal, price-setting source (single-fuel models) or if the dynamics of two or more fuels need to be modelled explicitly (multiple-fuel models). Examples of a single-fuel model are reported in Pirrong and Jermakyan (2008) and Eydeland and Geman (1999), who assumed the following functional form for the inverse supply curve:

$$S^{-1}(y) = F_t^{\gamma}\varphi(y) \tag{15.28}$$

where F_t is the fuel price, $\gamma \geq 1$ its associated coefficient and $\varphi(.)$ an increasing function. The interested reader will find details about the two approaches in the study by Carmona and Coulon (2014).

3.3 Assessing forecasting accuracy

The most successful forecasting model is the one that minimizes some measure of distance between the actual and the predicted prices, over a given time window. Because discrepancies between actual and predicted values can be both negative and positive, merely summing them would not provide a meaningful measure of forecasting quality. The most frequently used measures of point forecasting accuracy rely on absolute deviations or on quadratic deviations. The mean absolute error and the root mean square error are respectively defined as follows:

$$\text{MAE} = \frac{1}{H}\sum_{h=1}^{H}\left|P_h - \widehat{P}_h\right| \tag{15.29}$$

$$\text{RMSE} = \sqrt{\frac{1}{H}\sum_{h=1}^{H}\left(P_h - \widehat{P}_h\right)^2} \tag{15.30}$$

where H denotes the forecasting horizon.

The forecasting quality measure can be computed as a proportion of the actual price levels, as with the mean absolute percentage error:

$$\text{MAPE} = \frac{1}{24}\sum_{h=1}^{24}\frac{\left|P_h - \widehat{P}_h\right|}{P_h} \tag{15.31}$$

or as a proportion of the daily average price (mean daily error):

$$MDE = \sqrt{\frac{1}{\overline{P}_{24}}MAE} \tag{15.32}$$

A forecasting accuracy measure which is more robust to outliers is the median.

A forecasting accuracy measure which is more robust to outliers is the Median Absolute Error, defined as

$$\text{MedAE} = \frac{\text{MAE}}{\widetilde{P}_H} \tag{15.33}$$

where $\widetilde{P}_H$ is the median price observed over time horizon H.

Before selecting the best performing model, some issues need to be solved. First, which forecasting accuracy measure should be privileged, when measures provide different rankings? Second, how large should gaps in forecasting accuracy before a model can be declared the best?

With respect to the first question, the availability of various forecasting accuracy measures gives rise to a trade-off: the ranking they provide may not be univocal, but at the same time, the risk of picking up the 'wrong' model is diversified. Answering the second question requires running statistical tests, such as the one proposed by Diebold and Mariano (1995), which is a test of the hypothesis that the mean of the loss differential series (the difference in loss functions computed for each model being compared) is zero. Refer to the study by Diebold (2015) for more details. An alternative test was designed by Hansen et al. (2011), with the null hypothesis that predictions from a model do not contain additional information with respect to those from the competing model. Mariano (2002) overviews the main tools for forecasting accuracy testing.

3.4 Discussion

Despite the wealth of econometric works on wholesale electricity prices, a comprehensive comparison among models in terms of predictive ability is missing. Several case studies are included in the book by Weron (2007), and many scholarly articles have performed comparisons (from Crespo-Cuaresma et al., 2004; Knittel and Roberts, 2005; Kosater and Mosler, 2006; Bowden and Payne, 2008, to more recent efforts). The general picture, as suggested by Weron (2014), is that sophisticated models, such as RS and jump-diffusion models, do not yield accurate predictions beyond the qualitative matching of stylised facts, unless they are combined with methods that exploit information from multiple time series and spatial dependencies (see also Section 5). Interestingly, the incorporation of heavy-tailed error terms in linear time series models does not seem to improve the forecasting performance with respect to their Gaussian counterparts, as shown by Weron and Adam (2008). It is hard to draw general conclusions because the models have been estimated on samples from different countries in different time periods, with varying data treatment procedures and econometric specifications (in terms of the explanatory variables being included and their definitions), as well as uncertainty in the comparative data quality. Ranking among models can change over different forecasting horizons.

Structural econometric models and their predictions are very useful for policy-evaluation purposes, as they allow for an improved understanding of the relationship between prices and market fundamentals. The emphasis of reduced-form models on the use of past information allows to save on data requirements as an excellent fit can be obtained by using information contained in a single time series but does not provide clear and immediate insights on the underlying market mechanism. It is precisely the amount of data required that penalizes the use of structural models in forecasting activities. In particular, there is a trade-off between realism in the assumptions on the supply function (that requires a preliminary estimation of the supply function shape using data on individual bids, as in the study by Coulon and Howison, 2009) and data requirements. Relying on simultaneous relationships between price and market fundamentals (demand, fuel prices, reserve margins and so forth) implies that to forecast prices, one needs to forecast fundamentals as well. Yet, forecasting performances can, in some cases, be superior to those obtained by reduced-form models, as with the model by Kanamura and Ohashi (2008) outperforming jump diffusions.

A better balance between forecasting performance and policy-making support can be achieved through a systematic study on how to convert a structural model into a reduced-form model and back. Theoretical work on time series solves this in the case of linear models, whereas equivalence relationships between structural and reduced-form models under non-linearity are less obvious. An instance of such equivalence is provided by Kanamura and Ohashi (2008), who derived RS transition probabilities from a structural econometric model. Such explorations allow to better grasp the informational content of reduced-form coefficients.

4. Economic models of the electricity market: some notes

After illustrating the econometric approach to electricity price modelling and forecasting, it is worth having a brief look at the *economic approach*. Although it provides limited answers to the problem of forecasting, the economic approach is closely related to structural econometric modelling and is part of the educational background of many energy policy-makers, regulators, academics and practitioners.

In the economic approach, the modeller builds a set of equations describing the decision processes of individuals on the demand side (residential, commercial and industrial energy users) and on the supply side (power-generating companies, retailers). In such models, individuals make decisions (on prices, quantities) to achieve their own objectives (in terms of profits, costs, utility) while satisfying given constraints (budget constraints, plant capacity, transmission limits). Model solutions, computed through algebra or via computer simulations, relate market − clearing prices with their underlying determinants or fundamentals. Traditionally, and in introductory textbooks, model equations are assumed deterministic, whereas academic and applied research often relies on stochastic formulations. Forecasts are performed through simulations of the model after inputing parameter values.

The economic modelling and forecasting approach includes strategic interaction models and agent-based simulation models. Both types of models assume that power-generation companies compete in an oligopoly: few firms produce and sell electricity to a large number of consumers. Strategic interaction and agent-based models (ABMs), however, differ in how they solve the trade-off between mathematical simplicity and the realism of the assumptions, with specific reference to the assumptions on rationality of decision-makers and on learning.

4.1 Strategic interaction models

In strategic interaction models, oligopolistic firms are assumed to make decisions under conditions of full rationality, by solving constrained optimality problems, e.g. cost minimisation and profit maximisation, conditional on conjectured or observed decisions by their competitors. Firms act strategically, motivating the use of game theory as an analytical tool.

Strategic interaction models differ in terms of assumptions about the decision variable (price, quantity, capacity), the timing of decisions (simultaneous, sequential), the degree of detail in representing the system under study and the specific functional forms and parametrisations that are adopted. Models representing the market and the grid in greater detail are typically aimed at supporting short-term decisions, whereas the goal of simpler models is to examine the longer term implications of market conditions for investment decisions and public policy. A comprehensive review of strategic interaction models for electricity generation is included in studies by Ventosa et al. (2005) and Smeers (1999).[15] von der Fehr and Harbord (1993) have modelled the electricity market as a Bertrand duopoly. Duopolistic power generators set their own supply prices, given conjectures on the opponents' simultaneous moves, to maximize profits, while facing a capacity constraint and a regulatory price cap. Demand is price-inelastic. The model results show that with sufficient available capacity, the power-generating company with the lowest marginal cost bids slightly below the marginal costs of the competitor, thereby winning the whole market. Predicting the electricity price then boils down to predicting the marginal cost of generation (or the fuel price) for the less-efficient producer. However, when demand is higher and the capacity constraint at the system level is binding, it pays off for generators to ask the price cap. The authors performed the aforementioned comparative statics analysis without modelling the stochastic process driving electricity demand, which is responsible for switches between periods of low and high demand. In essence, given information on capacity constraints, electricity price prediction through the von der Fehr-Harbord model is a combination of fuel price and electricity demand forecasts. A related modelling approach relies on auction theory, as illustrated by Fabra et al. (2002), lending itself to theoretical comparisons of existing and proposed pricing rules (for instance, uniform price vs. pay-as-bid auctions in the paper by Ren and Galiana, 2004a; 2004b). Empirical predictions rooted in this literature, on the explanatory power of marginal costs, have been outlined and tested by Bunn et al. (2015).

In Cournot models of the electricity market, power-generating companies set their power supplies simultaneously to maximise profits, given their cost functions, their plant capacities, a downward-sloping demand function and conditional on conjectures about their competitors' decisions. Each company sets a reaction function, summarising the optimal supply for each possible level of the competitors' supplied quantities. Solving the system of reaction functions yields the optimal supplies for all generators. The electricity price is obtained upon solving the market equilibrium equation (demand = supply). The equilibrium electricity price is a function of power-generation costs (fuel prices) and of the parameters related to the price-elasticity of demand. Borenstein et al. (1999), Willems (2002) and Acemoglu et al. (2017) among others have

[15] For a more general treatment of strategic interaction models, it is worth reading textbooks by Tirole (1988) and Cabral (2000).

adopted this framework. Cournot models are sometimes extended to include a large number of perfectly competitive ‘fringe’ producers, in addition to the few strategic generating companies. This is equivalent to a shift in the demand function and may improve forecasting performances in markets characterised by a core-fringe configuration.

Stackelberg models give up the simultaneity assumption and posit that one generating company, the leader, sets the supply quantity and that its competitors (followers) make their decisions only after observing the leader’s. The leader is usually the former regulated monopolist. The leader-follower framework is not a formally accurate representation of how electricity auctions work, as real-world bids are submitted simultaneously in uniform price auctions. Yet, the model captures the notion that through informal exchanges of information, the former monopolist can influence the decisions by smaller competitors that have accessed the market in the wake of its liberalisation. An instance of a Stackelberg model for the electricity market is included in Floro (2009).[16]

A more realistic depiction of supply in electricity markets is given by supply function equilibrium (SFE) models. SFE models assume that generators simultaneously submit individual supply functions, specifying the quantities of electricity they have available to supply at each price level, given a downward-sloping demand function. Parameters of the individual supply functions are set optimally (profit maximisation) and strategically (i.e. considering possible reactions by competitors). The market operator collects all individual supply functions and sums them up to obtain a market-wide supply function. The electricity price is computed as the outcome of the market equilibrium condition. In SFEs, demand is typically assumed to be a random variable, conditional on the electricity price. In particular, if the demand function is linear, the intercept is assumed to be a random variable. Unlike most other strategic interaction models, SFEs are analytically solvable and guarantee a stable and unique equilibrium only under certain conditions, related to the slopes of demand and supply functions. A comprehensive mathematical treatment of SFE models for uniform price auctions is provided by Baldick et al. (2004), whereas Holmberg (2009) deals with pay-as-bid auctions. Applications are found in the study by Green and Newbery (1992), Sapio et al. (2009) and Ciarreta and Espinosa (2010) among others.

The comparative explanatory power of Bertrand, Cournot, Stackelberg and SFE models has been assessed only occasionally. Floro (2009) compared Cournot and Stackelberg models using data on the Italian power market, concluding

[16] Nekouei et al. (2015) and Neto et al. (2016), among others, have modified the model by assuming that the leader is a demand aggregator or a market monitor.

that a Stackelberg framework is the most relevant description of the power system in the southern regions. Willems et al. (2002) performed a comparison of Cournot and SFE models on German data, finding the two models to be similar in terms of explanatory performance, yet suggesting that Cournot models are better suited than SFEs in a short-run representation of the electricity market.

4.2 Agent-based simulation models

ABMs assume that market dynamics emerge from interactions among individuals who are boundedly rational and heterogeneous (in their endowments but possibly in their decision rules). ABMs give up the assumption of perfect rationality made by strategic interaction models: in dealing with complex decision settings, individuals are constrained by limited ability in processing information. Hence, as theorized by Herbert Simon (Simon, 1972), individuals may seek to optimise decisions on a simplified representation of the real-world system or adopt decisions that satisfy a minimal performance threshold, even if sub-optimal or driven by rules of thumb (so-called satisficing behaviour). Complexity is indeed relevant in a system involving a large number of spatially dispersed units and consumers, injecting and withdrawing electricity in a physical network subject to load-balancing constraints and spatial externalities. Moreover, ABMs typically emphasise the dynamic aspects of individual decisions. Power-generating companies in ABMs of the electricity market engage in repeated interaction, attempt to learn the most profitable bids and face inter-temporal goals and constraints. Modelling individual decision-making as the outcome of full optimization is therefore implausible.

Formally, the assumptions of heterogeneity and bounded rationality, and the emphasis on repeated interaction and dynamics, make it natural to describe the market as a system of non-linear dynamic equations. Such systems have closed-form solutions only under certain conditions that may not correspond to realistic market settings. Therefore, ABMs rely on computer simulations to study the properties of the market system being modelled.

Overviews on ABMs for electricity markets can be found in the study by Weidlich and Veit (2008) and Guerci et al. (2010), as well as in a recent book chapter by Tesfatsion (2018). Electricity markets are good candidates for the application of ABMs as long as research is driven by an interpretive motivation and policy-making goals. Indeed, the purpose of ABMs is mainly that of exploring the behaviour of a complex system, while forecasting is a secondary motivation for their use. Scholars in the ABM field have explored issues such as measures to mitigate market power exercise (Bower and Bunn, 2001; Bunn and Martoccia, 2005), comparisons among market architectures (e.g. pool vs. bilateral in Bower and Bunn, 2000) and the comparative effects of climate policies on the diffusion of renewables and on energy efficiency, which in turn affect electricity prices (see the review in Balint et al. 2017).

Artificial power exchanges have been developed to offer decision-support tools in the policy-making process, as with the AMES wholesale power market test bed (Agent-Based Modeling of Electricity Systems) described in the study by Li and Tesfatsion (2009) and Sun and Tesfatsion (2007). The Genoa Power Exchange documented in Cincotti and Gallo (2012) is another case in point.

The main drawbacks of ABMs with respect to short-term forecasting goals are their high computational cost and the requirement of highly detailed system operation data. The lack of closed-form solutions can also hamper their acceptance and diffusion in scholarly and policy circles, as cause-effect links may not emerge with enough clarity.

4.3 Discussion

The models in the economic approach can hardly provide reliable forecasts over a daily or hourly horizon. Indeed, they are mostly conceived as decision-support tools for economists and policy-makers. Because of their longer term orientation, economic models can provide reasonable approximations of the trends and seasonal patterns. Spiky behaviours in time series can also be reproduced, at least qualitatively, by models that feature threshold mechanisms (such as those of von der Fehr and Harbord 1993). Yet, economic models tends to smooth out extreme fluctuations in prices. This is clear if the assumed equations are linear, but even when random variables are included in the model, the models provide little or no intrinsic reasons to deviate from Gaussian assumptions. Linearity, often required to obtain closed-form solutions, confirms the fast-decaying Gaussian tails of the random shocks. Therefore, forecasting performance is particularly limited with respect to stylised facts such as heteroskedasticity and spikes. ABMs are better suited than strategic interaction models to yield heteroskedastic and heavy-tailed price series because of their non-linear dynamic equations.

Few works have compared the explanatory power of optimising models and ABMs with respect to electricity market dynamics, concluding in favour of ABMs (Saguan et al., 2006, Guerci and Sapio, 2011), although more research is needed before drawing robust conclusions. Structural econometric models, illustrated in Section 3.2, share some features with models in the economic approach but are better suited to short-term forecasting as they assume explicitly stochastic processes for demand and supply, while taking an agnostic standpoint on assumptions concerning behaviours and technological constraints.

5. Conclusion and further reading

This chapter has offered an introduction to econometric modelling and prediction of electricity prices determined in liberalised wholesale power exchanges, focusing on a high frequency of observation and on short-term forecasting horizons. The stylised facts on wholesale electricity prices (seasonality, mean reversion, volatility clustering, spikes, negative prices) have been reproduced, with varying success, by structural and reduced-form econometric models. The chapter has discussed the insights behind the modelling efforts, rooted in the economics of power systems, and the comparative merits of the reviewed models, along with some notes on strategic interaction and agent-based economic models of electricity markets. By way of a conclusion, let us provide some reading suggestions and an overview of more advanced econometric modelling approaches.

5.1 Applications of the reviewed models

The models reviewed in the chapter have been used in the literature for two main goals: assessing policy issues and pricing power derivatives. Policy issues tackled through econometric models include the measurement of market power exercise and the effects of environmental policies on wholesale electricity prices (penetration of renewables, nuclear decommissioning). Market power assessment has dominated the early literature, whereas studies on the so-called merit order effect of renewables have blossomed as soon as large-scale programmes to support renewables have been adopted in all major economies. Merit order effects are often estimated by means of simple ARMAX or structural models, whereas estimates through RS models have been published by Sapio (2015) and Lagarde and Lantz (2018). Strategic interaction and ABMs have been used as well for these purposes (see references in Sections 4.1 and 4.2). On the issue of power derivatives, useful readings are those of Eydeland and Geman (1999), Eydeland and Wolyniec (2003) and Pirrong and Jermakyan (2008).

5.2 Limitations

Reviewing the modelling and forecasting literature has revealed a number of limitations, which may highlight new opportunities for scholarly research. For one, a systematic comparative testing of the various probability density functions, reduced-form models and structural models, based on data sets from a wide range of countries, is still beyond reach. Comparisons exist, yet they are mostly focused on small sets of countries and concern few models. Data availability issues motivate this, with regards to both the data available to the individual research team and the relatively short observation period that characterised the samples analysed in the earlier literature. Forecasting competitions, such as the ones cited in the study by Weron (2014), help in refining the set of likely data-generating processes, yet they often involve models that bear little economic interpretation. With the availability of longer time series, opportunities for model comparisons increase, as well as the need to tackle structural change issues and the related difficulties in measuring institutional change. Replication studies are also essential in this respect, as they allow the very same analysis to be repeated on updated data sets (a special issue on replication has been published by the journal *Energy Economics*). Another limitation concerns the lack of a common international protocol for definition and measurement of variables. Often, the empirical tests of a theoretical hypothesis are not directly comparable across publications because the same theoretical determinant of electricity prices is measured differently.

5.3 Advanced models

Because of its introductory purpose, this chapter has focused on modelling and point forecasting the mean and the variance of the wholesale electricity prices through univariate models. Spurred by policy issues, such as regional market integration and the penetration of renewable energy sources, econometric modelling applications in recent years have moved in a number of directions, involving more sophisticated univariate models (quantile regression models); models for multiple time series (vector autoregressive [VAR] models; spatial econometric models) for joint modelling of different markets, market zones or market segments; and more advanced forecasting techniques (density forecasts). Such latest developments are briefly reviewed here, providing suggestions for further reading.

In the field of univariate reduced-form econometric models, quantile regression is increasingly considered in forecasting exercises (Hagfors et al., 2016) and in interpretive studies (Sapio, 2019) as it models the conditional probability distribution of electricity prices as a whole, quantile by quantile, relaxing the restrictive assumption that the error term is identically distributed at all points in the conditional distribution (Koenker and Bassett, 1978). Coefficients are allowed to vary across quantiles of the conditional price distribution. The coefficient estimated at a given quantile is interpreted as the marginal change of the dependent variable due to a marginal change in the given regressor, conditional on the dependent variable being at the specified quantile. The model is therefore naturally suited to deal with stylised facts, such as long tails and asymmetry, while also allowing to assess the autocorrelation properties of the data away from the mean. The existing applications of quantile regression to electricity prices have found stronger autocorrelation and stronger effects of reserve margins in the right tail, presumably due to market power.

Joint modelling of different markets, zones and segments has become useful for policy assessments. On the one hand, market coupling and transmission capacity investment programmes, adopted in the European Union, highlight the advantages — in forecasting and interpretive terms — of including time series of prices quoted in different markets in the same model. On the other hand, the increasing share of renewables in the day-ahead market segments calls for broadening the scope of the analysis to the balancing market, where despatchable units guarantee the reliability of the power system and can mitigate the missing money problem affecting power-generating companies, along with capacity auctions (Newbery, 2016).

VAR models generalise the univariate AR models by describing the dynamics of a vector of variables as a linear function of its past values (Lüktepohl, 2005 and Juselius, 2006 for textbooks). In regards to electricity markets, the variables in the vector can include electricity prices only, i.e. zonal day-ahead prices from the same hourly auction; day-ahead prices from different hourly auctions; prices from different segments of the same market and prices from interconnected markets. Deeper understanding of electricity price determinants, however, has pushed scholars to jointly model prices of different energy commodities (e.g. electricity prices, gas prices and oil prices), as well as prices from energy and non-energy markets (e.g. electricity prices, carbon prices, exchange rates, stock prices). VAR models are deployed to estimate the direction of causality and the propagation of shocks across interconnected markets, as well as the intra-day dynamics of day-ahead prices, the stability of the relationship between the wholesale electricity price and its fundamentals (e.g. fuel prices) and the stability of arbitrage relationships across segments (forward and day-ahead markets; day-ahead and balancing markets). When applied to data sets of individual bids, VAR models also allow to estimate leader-follower patterns followed by power-generating companies when setting their bidding strategies. The effects of market coupling programmes can be appropriately assessed through VAR models (Parisio and Pelagatti, 2019). Conditional heteroskedasticity can be taken into account by estimating multivariate GARCH models that allow to capture volatility transmission and contagion effects across markets. VAR models provide also the basis for factor models that are appropriate when the dimension of the vector of variables is high and techniques for dimensionality reduction (such as principal component analysis) are needed to avoid overfitting (Weron, 2014).

A spatial econometric model describes a vector of variables whose values are defined in space, measured in different locations and spatially dependent, conditional upon covariates (spatial autocorrelation). Suggested textbooks on spatial econometrics include Anselin (1988) and Arbia (2014). One key reason motivating the use of spatial econometrics in modelling electricity prices can be traced back to seminal work by Bohn et al. (1984) on nodal pricing. Nodal prices are computed as the solution of a system cost-minimisation programme, subject to various constraints. Optimal nodal prices vary stochastically across time and locations, and price differences between two nodes may emerge depending on events throughout the transmission grid. The reason is that whenever an additional power generator injects electricity into the system, the thermal constraints may become binding due to congestion, thereby shifting nodal prices away from marginal costs. Price differentials reflect the relative scarcity of electricity at neighbouring locations, so that, following Bushnell and Stoft (1996), the optimal nodal price at one location reflects the average nodal prices at neighbouring locations. Additional rationales for the use of spatial econometrics include the spatial dependencies in renewable energy sources. Wind turbines are usually clustered in wind farms or parks because the wind resource tends to be spatially concentrated. Wake effects and smoothing effects impact on the availability and volatility of wind power outputs. De Siano and Sapio (2019) review the existing applications of spatial econometrics to modelling electricity prices and loads.

Finally, in his review of the literature on electricity price forecasting, Weron (2014) highlights the importance of going beyond point forecasts in assessing the predictive ability of econometric models. In particular, density forecasts and forecast combinations have not been adequately exploited in the literature on electricity price forecasting. Density forecasts are motivated by the observation that the entire forecast density provides precious information to the modeller. Combining or averaging forecasts from different models has been shown to be superior to the use of individual models, in terms of accuracy, while mitigating the modelling risk that the modeller faces when selecting a specific forecasting method among the available ones.

6. Questions

1. Describe the reasons why scholars, practitioners and policy-makers are interested in econometric modelling and forecasting of wholesale electricity prices.
2. Which different modelling challenges are posed by data on wholesale electricity prices? How do they differ from those implied by retail prices?
3. Which economic mechanisms about electricity production and trading make the Gaussian distribution an empirically incorrect representation for wholesale electricity prices?
4. Why are electricity prices sometimes negative?
5. Illustrate the limitations of autoregressive models in reproducing the stylised facts about wholesale electricity prices.
6. Compare advantages and disadvantages of the reduced-form and structural approaches to electricity price modelling.
7. How do non-linear econometric models seek to reproduce the evidence of spiky and heteroskedastic time series?
8. Describe the similarities and differences between structural econometric models and strategic interaction models.

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