UNIVERSITY OF COPENHAGEN
DEPARTMENT OF ECONOMICS

Modelling Nordic Electricity Spot Prices using Markov Switching Stochastic Volatility Models

Seminar: Topics in Financial Econometrics

Caroline Marie Sachmann (PWB301) and Cecilie Walsted Gaarskær (MFL861)

Supervisor: Philipp Kless
Submitted on: December 1st, 2021
ECTS-point: 7.5
Characters (with spaces): 63,025 (26.3 pages)

Contributions:
PWB301: Section 2.1, 3.1, 3.3, 4.2, 5.1, 6.1, 6.3, 7.2.
MFL861: Section 2.2, 3.2, 4.1, 4.3, 5.2, 6.2, 7.1.
In collaboration: Abstract, Section 1 and 8.

Abstract

Electricity is used in all aspects of modern life, and the price of electricity is therefore non-trivial. To capture the electricity price dynamics, this paper applies the 2-state Markov Switching Stochastic Volatility (MS-SV) Autoregressive (AR) model suggested by Guidolin and Timmermann in 2005. Thereby, this paper contributes to the existing literature by closing the gap between simpler MS-SV models used on traditional financial commodities, such as the 2-state MS-SV AR model, and more advanced MS-SV models typically used to model the electricity price dynamics. Both an AR(1) model, a mean reverting MS-SV AR(4) model, an AR(1)-GARCH(1,1) and a 2-state MS-SV AR(1) model are compared to investigate, how the price dynamics are captured most accurately. Using the Nordic system price for the period January, 2019 to December, 2020, it was found that the latter model is capturing the price dynamics in accordance with observed market behaviour, which indicates the presence of different regimes in the pricing process. The 2-state MS-SV AR(1) model displays a dual regime pattern, as the unconditional probability of being in one of the regimes is almost the same. This is opposed to the usual finding in the literature, where the 2-state MS-SV models have a frequent base regime and an infrequent spike regime.

Table of content

1 Introduction

To imagine a world without the use of electricity is impossible as it is used in all aspects of modern life. Therefore, the price of electricity is non-trivial and as the spot electricity prices are soaring at the moment, it has attracted the attention of the media.
This paper pursues to estimate the dynamics of spot electricity prices[1]. Unlike other financial commodities, electricity cannot be stored and must be used right away. The demand for electricity shows a strong seasonal dependence both on a daily, weekly and annual basis. In addition, the process of the spot electricity prices displays sudden extreme, yet temporary changes in price, which are known as jumps and spikes (Janczura and Weron, 2010). To capture these features of the spot electricity pricing process, the Markov Switching Stochastic Volatility (MS-SV) model is commonly used in the literature. This class of MS-SV models is - unlike Autoregressive Conditional Heteroscedasticity (ARCH) type models - able to capture jumps in the pricing process. This is the case because it is considered as a shift to another regime, when a jump occurs in the pricing process. These MS-SV models have been widely applied in both economical and financial research since the introduction by Hamilton (1989) (Bazzi et al., 2017). Hamilton's MS-SV model has become popular among econometricians, after Hamilton used it to model the probability of a recession in the U.S. economy.

The two economists Massimo Guidolin and Allan Timmermann presented in their mutual papers from 2005 (Guidolin and Timmermann, 2005), 2007 (Guidolin and Timmermann, 2007) and 2008 (Guidolin and Timmermann, 2008) a simple 2-state MS-SV autoregressive (AR) model suited to capture the pricing process of financial commodities such as stocks and bonds. We want to examine whether their proposed MS-SV model is applicable to model other types of financial commodities such as the Nordic system price, which is the Nordic spot electricity price. The Nordic spot electricity price arise from the Nordic energy market, which is a unification of the Norwegian, Swedish, Finnish and Danish electricity market (Nordpool, 2020a).

This paper presents itself in the middle of the existing literature with simple 2-state MS-SV AR models applied to the pricing process of financial commodities like stocks and bonds on one side and highly advanced models used for modelling electricity prices on the other side. Contributing to closing this gap, we present a simple 2-state MS-SV AR(1) model in line with Guidolin and Timmermann to model the Nordic spot electricity prices covering the period from January 1st, 2019 to December 31th, 2020. The considered model is benchmarked against a simple AR(1) model to explore the importance of using a MS-SV model type for modelling the spot electricity price dynamics. The parameters of the MS-SV model is estimated via the Expectation Maximisation (EM) algorithm.

We find that the 2-state MS-SV AR(1) model proposed by Guidolin and Timmermann seems to capture the dynamics of the spot electricity prices to a larger extend than both a simple AR(1) model, a more complex mean reverting MS-SV AR(4) model introduced by Hamilton (1990) and an AR(1)-GARCH(1,1) model. Additionally, the volatility persistence of the 2-state MS-SV AR(1) model is in accordance with the observed market behaviour. This suggests that rather simple MS-SV models with dependent regimes can be used to correctly model the dynamics of Nordic spot electricity prices for the considered period of time.

[1] We have many names for the things we love. Therefore, we will throughout the paper use the following terms equivalently: log spot electricity price and log Nordic system electricity price. Likewise, the term spot electricity price and Nordic spot electricity price will be used synonymously with the term Nordic system spot price.

The remainder of this paper is structured in the following way: Section 2 places the paper into the literature of financial (regime switching) models and other related literature. Section 3 presents the theoretical context of the electricity market as a financial market. Next, Section 4 presents the relevant econometric theory as well as the chosen empirical strategy. The empirical analysis is outlined in Section 5, which is followed by a discussion and future research in Section 6 and 7 respectively. Finally, Section 8 concludes.

2 Literature Review

We will use the 2-state MS-SV AR(1) model to capture the dynamics of the Nordic spot electricity prices. Therefore, we provide a brief overview of how others have used similar model frameworks in the literature.

2.1 Financial Time Series Models

Among the variety of models considered in the field of financial econometrics, the popular Generalized ARCH (GARCH) model introduced by Bollerslev (1986) captures volatility clustering, which is a typical feature of financial data. This clustering behaviour is a sign of uncertainty at the given financial market and implies that the considered returns or prices could be time-dependent. This suggests that the underlying data generation process have a conditional heteroskedastic variance, which is captured by the GARCH model.
Another approach is to use the Stochastic Volatility (SV) model, where the stochastic volatility process is the modelling objective. Thus, the SV model replicates the correlation structures and marginal distributions found in financial data. A downside of the SV model is that the corresponding Maximum Likelihood Estimation (MLE) is more cumbersome than Quasi MLE (QMLE) for the GARCH model.

2.2 Markov Switching Stochastic Volatility Models and the Electricity Market

In 1989, James D. Hamilton introduced a model class, which can have one or more parameters switching between two or more states (Hamilton, 1989). In the simplest case, a 2-state model is considered and as the unobserved exogenous switching process between the two states is governed by a Markov chain, the model is known as a 2-state Markov Switching (MS) model. The MS model is one of the most popular non-linear time series models in the literature, because the framework can be applied to a wide range of econometric models such as the GARCH model, the Vector Autoregressive (VAR) model and the SV model. These models are known as the MS-GARCH model, MS-VAR model and the MS-SV model respectively.

The MS-SV model with one or more regimes following an univariate AR process has in various versions been used to analyse financial time series. This is exemplified by Guidolin and Timmermann (2005) who modelled returns of portfolios consisting of stocks and bonds using a MS-SV AR model with regime switching intercept, volatility and autoregressive parameters. Hamilton (1990) used a mean reverting MS-SV AR(4) model with regime switching means to capture discontinuous jumps in the target band of Federal Reserve for the Federal funds rate. Furthermore, Hamilton (1989) modelled the probability of a recession in the U.S. economy using this model type. In all of these cases, the respective authors found that their regime switching models were suited to capture the dynamics of the regarded financial time series.

In terms of modelling log spot electricity prices, the MS-SV model with either two or three regimes have been used by both Ethier and Mount (1998), Huisman and Mahieu (2003), Weron et al. (2004), Bierbrauer et al. (2004) and Bierbrauer et al. (2007). Ethier and Mount (1998) estimated a 2-state mean reverting MS-SV AR(1) model with regime switching mean values and variances. Ethier and Mount found it empirical evident that their model, with regime varying variances and means, was suitable to model Australian and American electricity spot price data. Huisman and Mahieu (2003) proposed a 3-state model with a mean reverting base regime, a spike regime containing a Poisson distributed component followed by a reversion regime, where the pricing process returns to the base regime.

Another approach was taken by Weron et al. (2004) who modelled an independent spike regime, where spikes were log-normally distributed. The associated base regime was governed by a mean reverting AR(1) process. This modelling approach was further investigated by Bierbrauer et al. (2004), who used Nord Pool log spot electricity prices for the three year period from 1997 to 2000. Bierbrauer and colleagues modelled the independent spike regime using a Pareto distribution, which underestimated the spike frequency and in turn overestimated the spike size compared to the log-normally distributed spike regime. The log-normal distribution was found to be more suitable for the spike regime than the Pareto distribution.

In 2007, Bierbrauer and co-workers proposed the above-mentioned 2-state regime switching model with an independent exponentially distributed spike regime (Bierbrauer et al., 2007). However, Bierbrauer and co-workers found that the 2-state MS-SV model with independent Gaussian spikes and base regime given by an AR(1) process outperforms similar regime switching models with another spike probability distribution (Bierbrauer et al., 2007). Additionally, Bierbrauer et al. (2007) compared the 2-state MS-SV model with independent regimes to both a similar 3-state MS-SV model as well as a standard mean reverting AR(1) model and a jump-diffusion model. Based on German log spot electricity price data for the period from October 2000 to September 2003, Bierbrauer et al. (2007) concluded that the 2-state MS-SV model with a base regime modelled by an AR(1) process and a Gaussian spike regime provides the best modelling performance.

From these papers it is evident that one of the first things to pinpoint when estimating a MS-SV model is the number of regimes or states used. The majority of the literature on MS-SV models applies a 2-state MS-SV model, because of a lower computational burden compared to MS-SV models with numerous states (Janczura and Weron, 2010).

Secondly, it should be determined whether one or more states of the MS-SV model should exhibit mean reversion, as it is a common feature of financial time series (Genon-Catalot et al., 2002). All of the mentioned papers on spot electricity price modelling apply model framework with this feature. In contrast, the MS-SV model proposed by Guidolin and Timmermann (2005) does not exhibit this feature.

Finally, it should be decided whether the considered regimes are dependent or independent. Dependent regimes share the same random noise process, but have one or more regime switching parameters (Janczura and Weron, 2010). Naturally, this leads to a simpler model framework which can be estimated directly using the EM algorithm. Independent regime models, on the other hand, provide a more flexible model framework, which is more in accordance with the observed spot electricity price evolution, as it occasionally exhibits radical changes (Bean et al., 2019). The downside of the independent regime models is that the EM algorithm is more burdensome to use.

With this in mind, we aim to apply the modelling methodologies of Guidolin and Timmermann (2005) to the Nordic spot electricity market. Thereby, we contribute to existing literature by closing the gap between the simpler MS-SV models used on traditional financial data and more advanced MS-SV models typically used to model spot electricity prices. Therefore, our paper aims to apply the 2-state MS-SV AR(1) model with dependent regimes as well as regime switching intercepts, autoregressive parameters and standard deviations as in Guidolin and Timmermann (2005) on the most recent data for the Nordic spot electricity prices.

To the best of our knowledge, this has not been done similarly since Mount and co-workers in 2006 proposed a mean reverting 2-state MS-SV AR(1) model with depended regimes and a regime switching intercept, variance and autoregressive parameter for log spot electricity prices (Mount et al., 2006). Further, the electricity reverse margin[2] and the electricity was included as regime switching, explanatory variables. Mount and co-workers used American PJM data on spot electricity prices covering the one year period from May 1999 to May 2000 and modelled the transition probabilities to be dependent on the electricity reserve margin. Thereby, Mount et al. (2006) demonstrated that when the reserve margin is included as a explanatory variable, their model correctly captures the price spikes in the infrequent high price regime, while the normal price evolution is captured by the low price base regime.

3 The Nordic Electricity Market

Many actors such as producers, transmission system operators (TSOs)[3] and traders are present at the Nordic electricity market. The Nordic electricity market is branched into different bidding areas. Denmark consists of two bidding areas, whereas Norway and Sweden subsist of five and four bidding areas respectively. On the other hand, do countries such as Finland, Estonia, Lithuania and Latvia consist of just one bidding area each (Nordpool, 2020a). These different bidding areas guarantee that regional market conditions affect the associated regional price, and that constraints in the transmission system are identified rapidly. The electricity from the Nordic electricity power market is traded at the Nordic Power Exchange, Nord Pool[4], which is considered as one of the leading electricity hubs in Europe in terms of traded volume (Nordpool, 2021b). Nord Pool offers different types of standardised contracts: physically settled spot contracts and financially settled futures contracts, forward contracts and other specialised contracts.

The Nordic electricity spot market is a day-ahead market. In 2019, the Nordic day-ahead market covered more than 90 percent of the electricity consumption in the Nordic countries according to Energistyrelsen (2020). The residual of the electricity consumption arise from electricity traded via the intraday market. A traditional financial commodity spot market is not possible due to technical constraints in the nature of the network and the characteristics of the spot electricity price, cf. Section 3.3.
In practice, the Nord Pool day-ahead market is a closed auction where energy for the subsequent 24 hours is

[2]The electricity reserve margin is the difference between the maximum demand for electricity (peak demand) and the available capacity.

[3]The TSOs ensure the supply of electricity and an operative transmission grid.

[4]20 different European countries trade at Nord Pool (Nordpool, 2020b). These countries are Norway, Sweden, Denmark, Finland, Estonia, Latvia, Lithuania, Austria, Belgium, Germany, Luxembourg, France, Poland, UK, the Netherlands, Croatia, Bulgaria, Hungary, Romania and Slovenia.

traded. This is unlike other financial products which are traded continuously. At the Nord Pool day-ahead market, each day is divided into 24 hourly spot contracts. Before noon at a given day, all market participants submit their bids for each delivery hour the subsequent day. Then bids are matched to maximise social welfare, while taking network constraints provided by the TSOs into account (Energistyrelsen, 2020).

Similar to Weron et al. (2004), Bierbrauer et al. (2004) and Bierbrauer et al. (2007), we use the daily average electricity spot price to model the electricity price dynamics. Where Bierbrauer et al. (2007) use data from the German EEX power market, we follow Weron et al. (2004) and Bierbrauer et al. (2004) by using Nord Pool data on the Nordic system price.

3.1 The Nordic System Price

Following regulation by the European Union, the Nordic system price was introduced as a reference price to ensure accuracy and integrity (Nordpool, 2021a). The hourly Nordic system price is for each of the bidding areas calculated as the equilibrium point between the aggregate supply and demand curves. Electricity flows between the Nordic countries and the Netherlands, Germany, Poland and the Baltics are incorporated into the Nordic system price calculation. The Nordic system price is therefore a clearing reference price for the Nordic region. It is calculated without peak load, adjustments within operation day and real-time adjustments, and it is calculated assuming no congestion in the Nordic transmission grid. This implies that the Nordic system price is the true theoretical equilibrium price (Weron et al., 2004).

3.2 Liberalisation in The Nordic Electricity Market

Up until the beginning of the 1990s, the Nordic electricity market was mainly regulated by local governments. Since then, implementation of European Union directives on market liberalisation have ensured a gradual liberalisation of the Nordic electricity market (Green, 2006). This has led to increased market inter-connectivity and thereby a more competitive electricity market, which has boosted electricity wholesale trading in most European countries. The liberalisation has instigated substantial elements of risk such as uncertain demand and risk in terms of price and volume volatility (Amundsen and Bergman, 2006). Of these risk factors, the price volatility is most pronounced.

Subsequently, regulations of the electricity market has continuously been imposed. This underlines the necessity of understanding and thus modelling the dynamics of the electricity spot price.

3.3 Stylized Facts of Spot Electricity Prices

The following stylized facts concerning spot electricity prices clarify why electricity as commodity distinguish itself from the other financial commodities. When modelling the Nordic electricity prices, these stylized facts must be taken into account to capture the correct dynamics of the pricing process.

Volatility

As electricity is non-storable, the corresponding demand and supply are constantly balanced on a razor's edge. In combination with problems regarding transmission and capacity, this constant balancing causes high price volatility, which is more conspicuous than for other commodities (Cartea and Figueroa, 2005).
In addition to the general price volatility, both extreme jumps and spikes are observed in the pricing process. While sudden cessations or failures in the power grid cause jumps in the pricing process, the price spikes arise due to unexpected changes in the demand. Hence, an unexpected excess (shortage of) demand for electricity entails a positive (negative) price spike. When modelling the spot electricity pricing process, the price jumps can be interpreted as unforeseen discontinuities, whereas price spikes can be described as non-Markovian behaviour (Bierbrauer et al., 2007).

Seasonality

Part of the price volatility can be explained by seasonality and it is therefore predictable. It is empirically evident that electricity spot prices exhibit substantial seasonal dependence both on an annual, weekly and daily basis. Beyond that large geographical variations are present. Thus, this seasonal dependence distinguishes the spot electricity prices from other financial commodities (Cartea and Figueroa, 2005).

Mean reversion

According to Cartea and Figueroa (2005), the electricity spot prices exhibit mean reversion to a greater extend than other financial commodities. While the price of other financial commodities typically reverts towards a single mean, the electricity spot prices display a possible multiple mean reversion.
Intuitively, this arise because electricity prices depend on the electricity generating process and therefore on the associated input factor[5]. According to Deane et al. (2015), electricity spot prices revert to different mean levels depending on the input factor that determines the marginal price of the commodity. This multiple mean reversion emerges because electricity origins from either renewables or fossil fuels such as oil and gas. If the electricity originates from renewables, the marginal production cost converges towards zero. Hence, the spot electricity price would do the same. Consequently, if the electricity originates from fossil fuels, the marginal production costs and therefore the spot electricity price would converge to the marginal price of the given input fuel.

4 Empirical Strategy

The following section introduces the methodology for deseasonalisaton of the Nordic spot electricity prices. The MS-SV model framework is presented and the requisite theory on the EM algorithm is provided for the chosen 2-state MS-SV model.

[5]This follows from theory on electricity's Merit Order Curve and the Duration Price Curve, see Energistyrelsen (2020).

4.1 Deseasonalising of Spot Electricity Prices

The first step in defining a model for the spot electricity price dynamics consists of finding an appropriate description of the seasonal pattern. It is consensus in the literature on modelling spot electricity prices that they exhibit seasonality (Janczura and Weron, 2010), (Weron et al., 2004), (Bierbrauer et al., 2004). By using the Nordic system price, variations in the data series occur only due to seasonality and not physical disruptions. Seasonality can arise from changes in seasons, temperature and climate conditions such as hydro units, snow and wind. These changes affect not only the supply, but also the behaviour of spot electricity prices. Therefore, deseasonalising of the spot electricity prices cleanse the price evolution for seasonal related noise and leaves only the stochastic part of the log price process.

Following the deseasonalising approach suggested by Weron et al. (2004), the daily, deseasonalised log price, d_t, can be obtained by computing

$$d_t = \log(P_t - \Omega_t - \Xi_t), \tag{1}$$

where P_t is the daily, average system price, Ω_t is an average weekday and Ξ_t is the annual cycle. Ω_t and Ξ_t are subtracted to account for the intra-weekly and annual variations. This implies that the potential issue of intra-daily variations is not addressed in this paper.

First, the intra-weekly pattern, Ω_t, is calculated using the approach outlined in the Appendix, Section A.1. Secondly, the annual cycle, Ξ_t can be approximated by a sinusoid with a linear trend to capture the seasonal variation cf. Weron et al. (2004) as

$$\Xi_t = A \sin\left(\frac{2\pi}{365}(t+B)\right) + Ct,$$

where the parameters A, B and C can be estimated via a Least Square estimation.
Another way of modelling the annual cycle is proposed by Lucia and Schwartz (2002). The annual variation is modelled by Lucia and Schwartz (2002) as a function with level shifts depending on the calendar month, and whether it is either a holiday or a weekend. This function takes the following form

$$\Xi_t = \psi + \kappa D_t + \sum_{i=2}^{12} \kappa_i M_{it}, \qquad \text{where}$$

$$D_t = \begin{cases} 1, & \text{if date } t \text{ is weekend or holiday} \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad M_{it} = \begin{cases} 1, & \text{if date } t \text{ belongs to calendar month } i \\ 0, & \text{otherwise.} \end{cases}$$

Here, the parameters ψ, κ and κ_i for $i = 2, 3, ..., 12$ are estimated using the usual Ordinary Least Squares (OLS) estimation method. Thereby, level variations from the different months as well as holidays and weekends are captured by the parameters κ_i and κ respectively. If the estimated parameters are insignificant, the model is re-estimated omitting these insignificant parameter estimates. This latter model is used as an approximation of the annual cycle. Finally, the daily, deseasonalised log prices, d_t, can be computed from Equation (1) using the annual cycle estimates of either Lucia and Schwartz (2002) or Weron et al. (2004). These daily, deseasonalised log prices are then compared to the actual log prices, $y_t = \log(P_t)$ to examine the effect of deseasonalising the log spot electricity prices.

4.2 The 2-state Markov Switching Stochastic Volatility Model

The non-linear and jumpy nature of spot electricity prices, presented by the stylized facts in Section 3.3, justify the use of MS-SV models, as it is considered as a shift to another regime, when a jump occurs in the electricity price process. An advantage of the MS-SV model is that it requires no specification of a threshold variable for when to change from one regime to another. Following the vast literature on modelling macro variables as well as spot electricity prices, a 2-state MS-SV model is considered. The concept is to consider the time series of spot electricity prices as two different regimes or states of the world with different underlying processes, where the duration and timing of the regime shifts are non-deterministic. The model is governed by a random, unobserved switching variable, $s_t = \{1, 2\}$, which takes the value $s_t = 1$ in the low volatility base regime and $s_t = 2$ in the high volatility spike regime, where jumps appear. The switching process $(s_t)_{t=1,2,...}$ is a Markovian process implying

$$(s_t \mid s_{t-1}, s_{t-2}, ..., s_1) \overset{d}{=} (s_t \mid s_{t-1}).$$

It follows that the probability of being in regime j at time t depends exclusively on the past regime i at time $t-1$, which can be expressed as $p_{ij} = P(s_t = j \mid s_{t-1} = i)$, where $i, j = 1, 2$. The corresponding transition matrix P collects the transition probabilities p_{ij} and is given by

$$P = \begin{pmatrix} P(s_t = 1 \mid s_{t-1} = 1) & P(s_t = 1 \mid s_{t-1} = 2) \\ P(s_t = 2 \mid s_{t-1} = 1) & P(s_t = 2 \mid s_{t-1} = 2) \end{pmatrix} = \begin{pmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{pmatrix} = \begin{pmatrix} p_{11} & 1-p_{22} \\ 1-p_{11} & p_{22} \end{pmatrix}.$$

Here, it is used that $p_{21} + p_{22} = 1 \iff p_{21} = 1 - p_{22}$ and analogously that $p_{12} = 1 - p_{11}$. The unconditional probability of being in regime i is according to Christiansen (2008) computed as

$$P(s_t = i) = \frac{1 - p_{jj}}{2 - p_{ii} - p_{jj}} \qquad \text{for} \quad i, j = 1, 2, \qquad \text{where} \quad i \neq j.$$

The finite state Markov switching variable, s_t, must be weakly mixing such that the Law of Large Numbers (LLN) applies in order to estimate the MS-SV model. s_t is weakly mixing when it is both aperiodic and irreducible. Irreducibility of s_t means that there must be a positive probability of returning to the other regime, such that s_t is never stuck in one of the regimes. Hence, s_t is irreducible when $p_{11}, p_{22} < 1$. Aperiodicity of s_t denote that there is a positive probability of remaining in one of the regimes, i.e. $p_{11}, p_{22} > 0$. Fulfilment of these two criteria further imply that all moments are finite.

Following the modelling approach by Guidolin and Timmermann (2005), the 2-state MS-SV AR(1) model is considered

$$y_t = \alpha_{s_t} + \phi_{s_t} y_{t-1} + \sigma_{s_t}\epsilon_t, \qquad \text{where} \quad \epsilon_t \sim iidN(0,1) \quad \text{and} \quad s_t \in \{1,2\}. \tag{M.2}$$

Here, y_t denotes the natural logarithm of the daily spot electricity prices, the initial y_0 is given and $t = 1, 2, ..., T$. The initial state s_1 is unknown and the processes (ϵ_t) and (s_t) are independent. It holds that the variances $\sigma_1^2, \sigma_2^2 > 0$. The considered model has both regime switching autoregressive parameter, ϕ_{s_t}, intercept, α_{s_t} and standard deviation, σ_{s_t}. The parameter estimates are obtained through log-likelihood estimation which is carried out using the EM algorithm.

To investigate the importance of using regime switching models for pricing electricity spot prices, the following non-regime switching AR(1) model is considered as benchmark

$$y_t = \alpha + \phi y_{t-1} + \rho_t, \tag{M.1}$$

where $t = 1, 2, ..., T$, $\rho_t \sim iidN(0, \sigma^2)$ is a white noise process and the initial value y_0 is given. Weakly mixing of this AR(1) model is ensured when $|\phi| < 1$ and estimation of the model parameters relies on usual OLS estimation.

4.3 The Expectation Maximisation Algorithm

As mentioned in Section 4.2, the switching process (s_t) is unobserved. This implies that the log-likelihood function is infeasible to evaluate the switching volatility process, σ_{s_t}, autoregressive process, ϕ_{s_t} and intercept, α_{s_t}, as they are unobserved for model (M.2). To handle the unobserved processes, the numerical optimisation procedure known as the EM algorithm is introduced. The EM algorithm is based on the idea of iteratively maximising an expected, conditional log-likelihood function based on an initial guess of the parameter estimates, $\tilde{\theta}$. In this case, the parameters to estimate in model (M.2) are given by $\theta = \{\alpha_1, \alpha_2, \phi_1, \phi_2, \sigma_1, \sigma_2\}$. The EM algorithm is a two-step numerical optimisation procedure with an expectation step (E-step) and a maximisation step (M-step).

The E-Step

In the E-step, the expected, conditional log-likelihood function, $L_{EM}(Y, \theta)$, is calculated from an initial guess of θ, $\tilde{\theta}$. $L_{EM}(Y, \theta)$ is computed solely based on the observed log spot price process $Y = (y_1, y_2, ..., y_T)$, because the process of the switching variable $S = (s_1, s_2, ..., s_T)$ is unobserved

$$
\begin{aligned}
L_{EM}(Y, \theta) &= E_{\tilde{\theta}}(\log f_\theta(Y, S, \theta \mid Y)) \\
&= c + \sum_{i,j=1}^{2} \log p_{ij} \sum_{t=2}^{T} p_t^*(i, j) + \sum_{j=1}^{2} \sum_{t=2}^{T} p_t^*(j) \log f_{\tilde{\theta}}(y_t \mid s_t = j),
\end{aligned}
\tag{2}
$$

where c is a positive constant. Cf. model (M.2), $\epsilon_t \sim iidN(0, 1)$ implying that the probability density function is Gaussian and therefore given by

$$
f_{\tilde{\theta}}(y_t \mid s_t = j) = \frac{1}{\sqrt{2\pi\sigma_j^2}} \cdot \exp\Big(\frac{(y_t - \phi_j y_{t-1} - \alpha_j)^2}{2\sigma_j^2}\Big), \qquad \text{where} \quad j = 1, 2.
$$

From Equation (2), the smoothed probability for regime j, $p_t^*(j) = P_{\tilde{\theta}}(s_t = j \mid Y)$, expresses the probability of being in regime j given the information of the entire data set, whereas $p_t^*(i, j) = P_{\tilde{\theta}}(s_{t-1} = i, s_t = j \mid Y)$ denotes the smoothed transition probability for $i, j = 1, 2$. These smoothed (transition) probabilities are obtained via the forward-backward algorithm, which combines backward and forward recursions.

The forward-backward algorithm computes the smoothed (transition) probabilities according to Hamilton (1994) and Pedersen and Rahbek (2017). Start by expressing the smoothed probabilities as

$$
p_t^*(j) = \frac{b_t(j) a_t(j)}{\sum_{i=1}^{2} b_t(i) a_t(i)}
$$

and the smoothed transition probabilities as

$$
p_t^*(i, j) = \frac{b_t(j) f_{\tilde{\theta}}(y_t \mid s_t = j) \tilde{p}_{ij} a_{t-1}(i)}{\sum_{k=1}^{2} b_t(k) a_t(k)}
$$

for $k, i = 1, 2$ denoting the two considered regimes.

The forward algorithm recursively calculates the sequence $a_t(\cdot)$ as

$$a_t(j) = \sum_{i=1}^{2} f_{\tilde{\theta}}(y_t \mid s_t = j)\tilde{p}_{ij} a_{t-1}(i),$$

where the initial value is given as $a_1(j) = f_{\tilde{\theta}}(y_1 \mid s_1 = j)v_j$ for $j = 1, 2$ and some fixed distribution $v = (v_1, v_2)$ on $\{1, 2\}$.

The backward algorithm computes the sequence $b_t(\cdot)$ as

$$b_t(i) = \sum_{j=1}^{2} f_{\tilde{\theta}}(y_{t+1} \mid s_{t+1} = j) b_{t+1}(j)\tilde{p}_{ij},$$

given that the initial value is set to $b_T(j) = 1$ for the $j = 1, 2$ considered regimes[6].
When $p_t^*(j)$ and $p_t^*(i, j)$ are known from the forward-backward algorithm, $L_{EM}(Y, \theta)$ can be computed.

The M-Step

In the M-step $L_{EM}(Y, \theta)$ is maximised over θ while keeping $\tilde{\theta}$ fixed. The smoothed (transition) probabilities, $p_t^*(j)$ and $p_t^*(i, j)$, are therefore also kept fixed when maximising $L_{EM}(Y, \theta)$

$$\hat{\theta} = \arg\max_{\theta} L_{EM}(Y, \theta). \tag{3}$$

Here, it is used that the deduced maximum $\hat{\theta}$ increases the likelihood value compared to the initial or previous guess, $\tilde{\theta}$. Hence, $L_{EM}(Y, \hat{\theta}) \geq L_{EM}(Y, \tilde{\theta})$. The two-step EM algorithm is repeated until $\left|\hat{\theta} - \tilde{\theta}\right| < \delta$, where δ is the chosen tolerance[7]. Then, the algorithm is interrupted and the maximum likelihood estimator is found to be $\hat{\theta}_{MLE} = \hat{\theta}$. If $\left|\hat{\theta} - \tilde{\theta}\right| \geq \delta$, the two EM algorithm steps are iterated with a new guess of $\tilde{\theta}$ given by $\hat{\theta}$, i.e. $\tilde{\theta} = \hat{\theta}$.

Test for Model Selection

To determine empirically which model is preferred, two different information criteria are calculated. An information criterion provides the trade-off between the number of regressors in the model and the resulting goodness-of-fit. The Akaike's Information Criterion (AIC) is given by $AIC = L + \frac{2 \cdot \iota}{T}$, where T refers to the number of observations, L is the log-likelihood value and ι is the number of estimated parameters.

As a cross-check procedure, the Schwarz Bayesian Information Criterion (BIC), $BIC = L + \frac{\iota \cdot \log(T)}{T}$, is computed. It holds for both AIC and BIC that the favoured model is the one with the lowest information criterion.

[6] Note, that dependence on θ is suppressed in the theory of the forward-backward algorithm to simplify notation.
[7] The chosen MS-SV model is estimated using the EM algorithm in R by applying the R-package *MSwM* developed by Sanchez-Espigares and Lopez-Moreno (2021), where the tolerence is $\delta = 1e^{-8}$.

5 Empirical Analysis

5.1 Data and Deseasonalisation

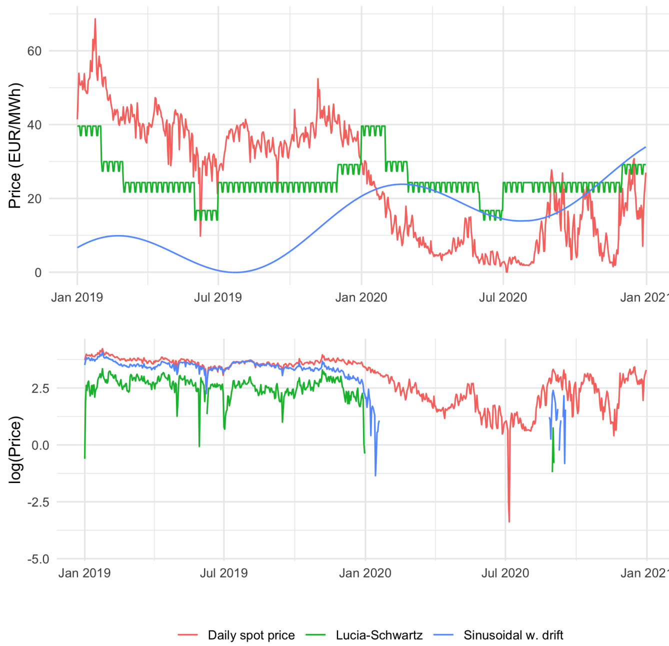
Daily Nordic spot electricity prices in EUR pr. megawatt hour (MWh) from Nord Pool, also known as the Nordic system price, is used to estimate and thereby investigate the modelling performance of the chosen MS-SV AR(1) model (M.2). This model is then benchmarked against the AR(1) model (M.1). We use the most recent data where a whole year is accessible. Therefore, we examine the two year period from January 1st, 2019 to December 31th, 2020. The length of the considered period of time is chosen in accordance with Weron et al. (2001), who base their analysis of the spot electricity price dynamics on a period consisting of two years.

We aim to deseasonalise the spot electricity price following the deseasonalising approach presented in Section 4.1. In Figure 5.1, the deseasonalising of the spot electricity prices are shown. The estimated annual cycle calculated using the technique of Weron et al. (2004), labelled *Sinusoidal w. drift*, seems to have a trend that overshoots the evolution of spot electricity prices. This can be seen from the top chart of Figure 5.1, as the annual sinusoidal cycle with a drift exceeds the actual daily spot prices in most of 2020. This results in undefined log spot electricity prices for the majority of 2020, which is apparent from the lower chart of Figure 5.1, as the natural logarithm is undefined for negative values.

The estimated model for the annual cycle according to Lucia and Schwartz (2002) is given by[8] $\Xi_t = \psi + \kappa D_t + \kappa_1 M_{i1} + \kappa_2 M_{i2} + \kappa_6 M_{i6} + \kappa_{12} M_{i12}$. Here, only significant parameter estimates are included. The annual cycle suggested by Lucia and Schwartz (2002) tends to overestimate the observed spot electricity pricing process in most of 2020, similar to the annual cycle approach by Weron et al. (2004).
Hence, also the deseasonalised log spot prices calculated using the method suggested by Lucia and Schwartz (2002) yields undefined log prices in the main part of 2020, as seen from the lower chart of Figure 5.1.

[8] In the estimation of the annual cycle cf. Lucia and Schwartz (2002), only the parameter estimates for the intercept, the non-working days as well as the months January, February, June and December are included. The rest of the parameter estimates are omitted, as they are insignificant at a 5 percent level of significance.

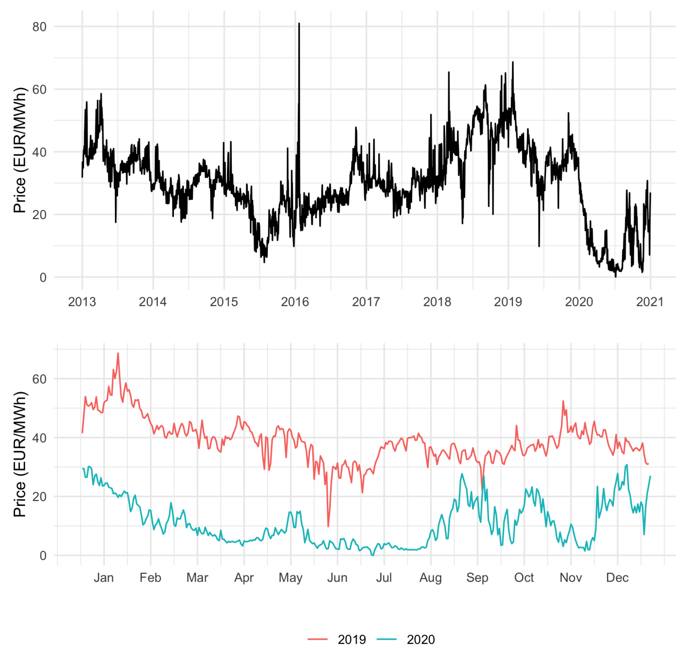
Figure 5.1: Deseasonalisation of the Spot Electricity Prices



Note: The upper chart displays how the estimated annual cycles using either the approach by Lucia and Schwartz (2002) or Weron et al. (2004) (labelled Sinusoidal w. drift) coincide with the actual evolution of the Nordic spot electricity prices. The lower chart shows the resulting (deseasonalised) log prices.
Source: Nord Pool and internal calculations.

Therefore, it is evident that neither of the deseasonalisation methods are adequate for our considered time series of spot electricity prices, as the negative deseasonalised spot prices yield data breach in the time series of the log spot electricity prices. This might be the case as our data does not exhibit a clear seasonal pattern, which can be seen from Figure 5.2. An obvious seasonal trend on an annual or a monthly level is not found neither in our considered period of time covering 2019 and 2020 nor in the years from 2013 to 2020, see Figure 5.2. Hence, the applied deseasonalisation methods are inapplicable in our case. The same conclusion was reached by Weron et al. (2004), who analysed the German spot electricity prices covering the period from January 1997 to April 2000. For this reason, the suggested models from Section 4.2 are estimated based on the raw log spot electricity prices, $y_t = \log(P_t)$. Furthermore, the Nordic system price is in itself a theoretical price, which means that it is already processed and cleaned to some extend, cf. Section 3.1. This substantiates why we continue our empirical analysis using the raw log Nordic spot electricity price.
Finally, we note that spot electricity prices do not display an obvious mean reverting behaviour for our chosen two year period as seen from the lower panel of Figure 5.2.

Figure 5.2: The Evolution of the Raw Spot Electricity Prices, 2013-2020



Source: Nord Pool and internal calculations.

5.2 Estimation of Model Parameters

The parameter estimates resulting from the OLS estimation of model (M.1) and the EM algorithm of model (M.2) are given in Table 5.1. The normality test of estimated residuals[9] of model (M.1), $\hat{\rho}_t$, is strictly rejected implying non-normally distributed residuals. This implies slower convergence of the parameter estimates towards the true parameter values. Additionally, the (M.1) model exhibits autocorrelated estimated residuals as the test of no autocorrelated residuals is strongly rejected[10]. Although the null of a unit root process is rejected on a 5 percent level of significance[11], the AR(1) model (M.1) appears misspecified.

The 2-state MS-SV AR(1) model (M.2) is weakly mixing as $0 < \hat{p}_{11}, \hat{p}_{22} < 1$. The regimes in the (M.2) model seem quite persistent as the probability of remaining in the same regime at time t as at time $t-1$ is $\hat{p}_{11} = 0.9724$ for regime $s_t = 1$ and $\hat{p}_{22} = 0.9646$ for regime $s_t = 2$. Regime $s_t = 1$ is the low volatility regime with a volatility of $\hat{\sigma}_1 = 0.0627$ and a mean of $E[y_t]_{s_t=1} = 3.4364$[12], which is relatively higher than the unconditional mean of the AR(1) model (M.1), $E[y_t] = 2.8736$. The unconditional mean of regime $s_t = 2$ in the (M.2) model is considerably lower with $E[y_t]_{s_t=2} = 2.1705$, whereas the volatility of $\hat{\sigma}_2 = 0.4131$ is

[9]The test for normality of the residual is described in detail in Section A.2.3 of the Appendix.

[10]The test for no residual autocorrelation is defined in Section A.2.1 of the Appendix.

[11]The LR test for a unit root is carried out in accordance with A.2.2 in the Appendix. In this case, $L_A = -126.25$, whereas the log-likelihood under the null hypothesis is given by $L_0 = 133.51$. As the LR test statistics is given by $LR = -2 \cdot (-133.51 - (-126.25)) = 14.53$, the null hypothesis of a unit root is rejected as the corresponding Dicky-Fuller distribution has a critical value of 9.13 on a 5 percent level of significance.

[12]The unconditional mean of the AR(1) model is the expectation of a convergent geometric series, i.e. $E[y_t] = \frac{\alpha}{1-\phi}$.

substantially larger in regime $s_t = 2$ relative to regime $s_t = 1$. The unconditional probability of being in one of the regimes is approximately the same as $P(s_t = 1) = 0.5616$ whereas $P(s_t = 2) = 0.4384$. This intuitively violates the notion of having a frequent *base* regime and a more infrequent *spike* regime. Rather, the considered 2-state MS-SV AR(1) model displays a low volatility regime with a relatively high $E[y_t]$ when $s_t = 1$ and a high volatility regime with a relatively low $E[y_t]$ when $s_t = 2$.

Based on both the AIC and BIC information criteria, it is evident that the 2-state MS-SV AR(1) model (M.2) is preferred to the non-regime switching AR(1) model (M.1). This underlines the necessity of using a multiple regime MS-SV model, when modelling electricity spot prices. Having this in mind, the properties of the 2-state MS-SV AR(1) model (M.2) is investigated.

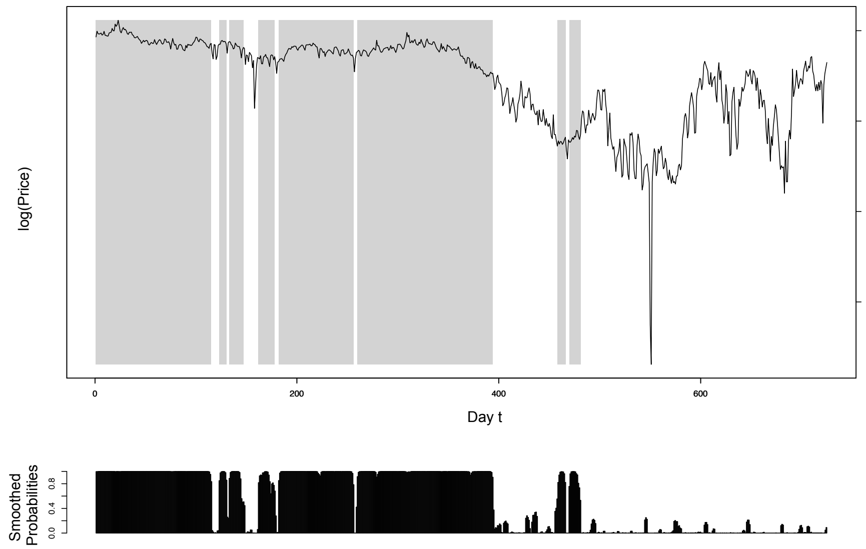
Table 5.1: Estimation Results of Model (M.1)-(M.2), 726 observations

	(M.1)	(M.2)	
Regime $s_t = i$		$i = 1$	$i = 2$
α	0.1270* (0.03321)	0.0378 (0.0264)	0.2368* (0.0596)
ϕ	0.9558* (0.01089)	0.9890* (0.0073)	0.8909* (0.0254)
σ	-	0.0627	0.4131
p_{ii}	-	0.9724	0.9646
p_{ij}	-	0.0276	0.0354
$P(s_t = i)$	-	0.5616	0.4384
$E[y_t]$	2.8736	3.4364	2.1705
Log-lik.	-126.25	-286.12	
AIC	258.49	-564.24	
BIC	272.25	-519.55	
No autocorrelation	[0.000]	-	
Normality	[0.000]	-	
T	726	726	
Sample start	2019-01-01	2019-01-01	2019-01-01
Sample end	2020-12-31	2020-12-31	2020-12-31

*Note: P-values in [·] for misspecification tests and standard errors in (·) for α and ϕ. * indicates that the parameter estimate is significant at a 5 percent level of significance.*
Source: Nord Pool and internal calculations.

The smoothed probabilities of the 2-state MS-SV AR(1) model (M.2) are plotted against the log spot electricity price, y_t, for the two regimes in Figure 5.3 and 5.4 respectively. According to these figures, the (M.2) model predicts most of 2019 to be modelled by the low volatility regime $s_t = 1$ with the relatively high unconditional mean of $E[y_t]_{s_t=1} = 3.4364$ besides a few exceptions. On the other hand, the high volatility regime $s_t = 2$ provides the best fit for the log price evolution in the majority of 2020. This indicates that when a spike appears in the pricing process, it is probable to observe another extreme value. This feature is known as volatility clustering and evident in financial commodities. Further, it should be noted that the high volatility regime $s_t = 2$ seems to capture both positive and negative spikes and not only the positive ones. This fact was addressed by Janczura and Weron (2010) and will be discussed further in Section 6. Nonetheless, it seems that the (M.2) model resemble the characteristic behaviour of the electricity prices as the high volatility regime $s_t = 2$ captures the spikes regardless of them being positive or negative.

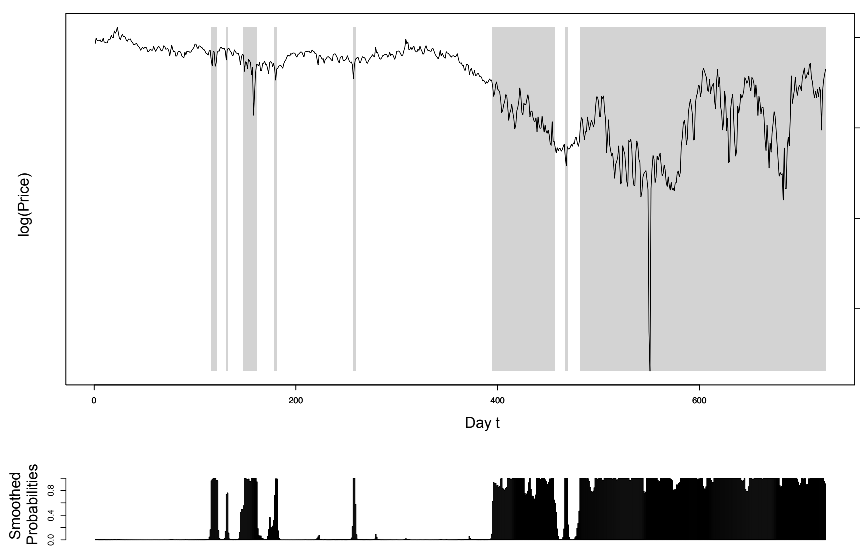
Figure 5.3: Smoothed Probabilities for Regime $s_t = 1$ of the (M.2) Model



Note: The x-axis denote the day t in the period from January 1st, 2019 ($t = 1$) to December 31th, 2020 ($t = 726$). The grey hatched area indicates when y_t is in regime $s_t = 1$.

Source: Nord Pool and internal calculations.

Figure 5.4: Smoothed Probabilities for Regime $s_t = 2$ of the (M.2) Model



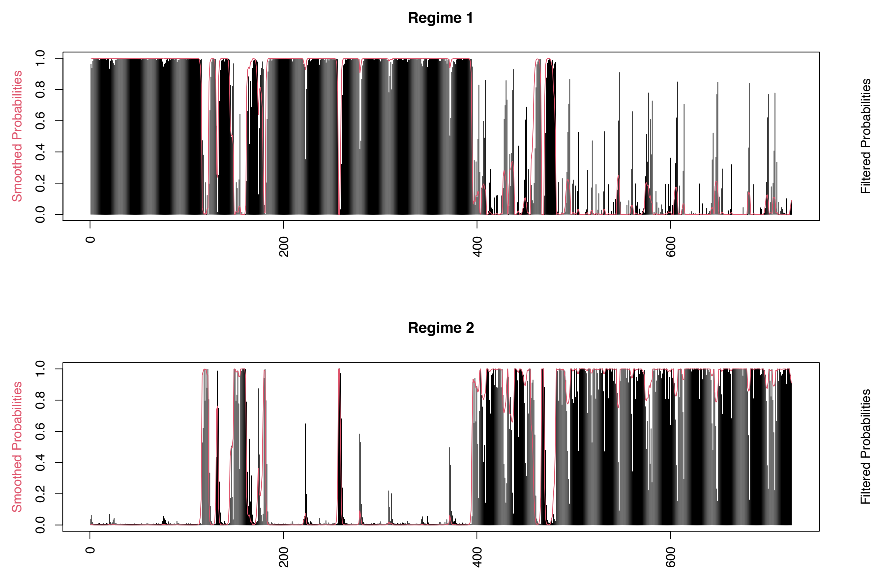
Note: The x-axis denote the day t in the period from January 1st, 2019 ($t = 1$) to December 31th, 2020 ($t = 726$). The grey hatched area indicates when y_t is in regime $s_t = 2$.

Source: Nord Pool and internal calculations.

Figure 5.5 depicts both the smoothed and the filtered probabilities for the regimes of the (M.2) model. The filtered probability is also known as the *real-time regime classification*, because it states the probability of being in the specific regime given the information up until time t, i.e. $P(s_t = i \mid y_0, y_1, ..., y_t; \theta)$. It is evident that the probability of being in a particular regime is more even, when considering the smoothed estimated probabilities relative to the filtered probabilities.

Finally, the regime switches implied by the (M.2) model seems dual from Figure 5.5 as empathised above. However, it can be argued that if the time series of the electricity spot prices was more equally volatile, then the 2-state MS-SV model would capture the high volatility regime as a proper spike regime rather than a rather protracted high volatility regime.

Figure 5.5: Smoothed and Filtered Probabilities for the Regimes of the (M.2) Model



Note: The x-axis denote the day t in the period from January 1st, 2019 ($t = 1$) to December 31th, 2020 ($t = 726$).
Source: Nord Pool and internal calculations.

Conclusively, we find that the models produce estimates for transition probabilities that can be interpreted according to market behaviour. This result resembles other papers in the field of MS-SV models e.g. Guidolin and Timmermann (2008) and Weron et al. (2004).

6 Discussion

6.1 Critique of the 2-state MS-SV Model

An immediate critique of the 2-state MS-SV model (M.2) is whether it is indeed able to capture the electricity spot price dynamics within the considered period of time. And additionally, to which extend the spot electricity price evolution in the chosen period of time is representative for the general price trend. The latter criticism arise as the COVID-19 pandemic has affected almost every aspect of the global economy in the majority of 2020. Therefore, it is reasonable to think that it has affected the evolution of the Nordic spot electricity prices as well.

A report on electricity prices from the International Energy Agency in December 2020 confirms that the demand for electricity in the Nordic regions has been decreasing throughout 2020 (IEA, 2020). The declining demand could ceteris paribus explain the relatively low spot electricity prices in 2020 compared to the period from 2013 to 2019 as seen from Figure 5.2. This indicates that the findings in our empirical analysis might not be representative of the Nordic electricity spot price dynamics within the last three to eight years. It would therefore be intriguing to investigate, whether replicating the analysis for an earlier two-year period would change the results.

Despite this critique, the empirical analysis shows this somewhat dual regime pattern, where the high volatility regime kicks in the beginning of 2020 as seen from Figure 5.4. Thereby, the (M.2) model captures the evident change in the volatility pattern, which is what the MS-SV model is constructed to do. This points to the strength of the 2-state MS-SV model (M.2), as it captures non-deterministic structural changes within the market economy. Hence, the (M.2) model still seems to explain the spot electricity price characteristics validly, which have important implications for policy objectives.

When it comes to the model selection between the AR(1) model (M.1) and the 2-state MS-SV model (M.2), the latter was chosen based on the information criteria AIC and BIC. This is underlined by the fact that model (M.1) appears misspecified. The 2-state MS-SV model (M.2) captures characteristic features of the spot electricity prices such as the sudden extreme, yet transitory spikes in the pricing process, which makes the commodity unique. This characteristic feature is not captured by model (M.1). It is widely agreed in the literature on modelling spot electricity prices that the preferred model is the one capturing these characteristics the best. This underlines the choice of the 2-state MS-SV model (M.2) compared to the non-regime switching AR(1) model.

However, we raise the concern of a potential unit root in regime $s_t = 1$ of model (M.2), as the autoregressive parameter estimate $\hat{\phi}_1 = 0.989$ is very close to 1, indicating the existence of a unit root process. If a test for a unit root could easily be carried out in a MS-SV model, we could with certainty state whether regime $s_t = 1$ in model (M.2) is a unit root process and therefore a random walk or not. As the implementation of the unit root test for the MS-SV model type is non-trivial and cumbersome, we have not examined it. Thus, such a test can be implemented using the testing framework suggested by Hall et al. (1999), but it is beyond the scope of this paper.

Nonetheless, when examining the time series of the log electricity spot price in Figure 5.3, the degree of auto-correlation seems adequate as the price process is quite stable, which could explain why $\hat{\phi}_1$ is almost one. This suggests that the (log) price at time t affects the (log) price at time $t + 1$ almost one-to-one in regime $s_t = 1$.

As mentioned in Section 5.2, the volatility regime $s_t = 2$ seems to capture both positive and negative spikes in the log electricity spot prices. This can be an argument against the (M.2) model as Janczura and Weron (2010) stated that models such as the considered 2-state MS-SV AR(1) model have a tendency not to classify the regimes correctly. This implies that sudden price drops i.e. negative spikes are modelled by the high volatility regime, $s_t = 2$, rather than the lower volatility regime, $s_t = 1$. This speaks in favour of a 3-state MS-SV model, where negative and positive spikes are modelled by separate regimes. This 3-state MS-SV

model will be discussed in Section 7.
However, it could be argued that the high volatility regime naturally should capture both the positive and negative spikes in the log pricing process when having a 2-state MS-SV model instead of just the positive spikes as stated by Janczura and Weron (2010). In this way, the low volatility regime ends up capturing solely the periods of low volatility instead of both the low volatility periods and the negative spikes. Following this argument, the (M.2) performs well, when it comes to identifying the high - and low volatility regimes and thereby capturing the volatility dynamics of the log spot electricity prices. Although the (M.2) model might be simple compared to other MS-SV models in the literature on modelling spot electricity prices, it does capture the characteristics of the spot electricity prices despite the complex pricing dynamics.

6.2 Should the 2-state MS-SV Model be Mean Reverting?

As examined in Section 5.1, there is no clear evidence of either a seasonal pattern or a mean reverting behaviour in the log spot electricity prices. For this reason, we have not incorporated a mean reverting term in the 2-state MS-SV AR(1) model (M.2). Nonetheless, it is of interest to investigate, whether a mean reverting 2-state MS-SV AR model provides a better fit for our considered log spot electricity time series covering the years 2019 and 2020. Therefore, we estimate the mean reverting 2-state MS-SV AR(4) model with state dependent means, which was suggested by Hamilton (1990). The Hamilton model is given by

$$y_t = \nu_{s_t} + \zeta_1(y_{t-1} - \nu_{s_{t-1}}) + \zeta_2(y_{t-2} - \nu_{s_{t-2}}) + \zeta_3(y_{t-3} - \nu_{s_{t-3}}) + \zeta_4(y_{t-4} - \nu_{s_{t-4}}) + \eta\upsilon_t \tag{M.3}$$

where ν_{s_t} is the unconditional, regime switching mean, $\upsilon_t \sim N(0, \sigma^2)$ and $s_t \in \{1, 2\}$. The initial state s_1 is unknown and the processes (υ_t) and (s_t) are independent. The Hamilton model (M.3) is estimated using the EM algorithm[13] which for regime $s_t = 1$[14] yields the following parameter estimates

$$y_t = \underset{(0.344)}{2.845} + \underset{(0.037)}{0.959}(y_{t-1} - \underset{(0.344)}{2.845}) - \underset{(0.051)}{0.275}(y_{t-2} - \underset{(0.344)}{2.845}) + \underset{(0.051)}{0.215}(y_{t-3} - \underset{(0.344)}{2.845}) + \underset{(0.037)}{0.070}(y_{t-4} - \underset{(0.344)}{2.845}) + \underset{(0.004)}{0.0761}\upsilon_t,$$

whereas the resulting parameter estimates for regime $s_t = 2$ amount to[15]

$$y_t = \underset{(0.332)}{2.836} + \underset{(0.037)}{0.959}(y_{t-1} - \underset{(0.332)}{2.836}) - \underset{(0.051)}{0.275}(y_{t-2} - \underset{(0.332)}{2.836}) + \underset{(0.051)}{0.215}(y_{t-3} - \underset{(0.332)}{2.836}) + \underset{(0.037)}{0.070}(y_{t-4} - \underset{(0.332)}{2.836}) + \underset{(0.004)}{0.0761}\upsilon_t.$$

Our estimates are peculiar. The parameter estimates suggest that the mean of the two regimes is almost identical. For regime $s_t = 1$ the estimated mean is $\hat{\nu}_1 = 2.845$, whereas it for regime $s_t = 2$ is $\hat{\nu}_2 = 2.836$. Compared to the unconditional means of $E[y_t]_{s_t=1} = 2.1705$ and $E[y_t]_{s_t=2} = 3.4364$ for the (M.2) model, the estimated regime switching means of the Hamilton model (M.3), $\hat{\nu}_1$ and $\hat{\nu}_2$, are situated just between those. The transition probability of remaining in one of the regimes is almost the same as $\hat{p}_{11} = 0.5042$ and $\hat{p}_{22} = 0.5061$, which indicates non-persistent regimes. Hence, the mean reverting model specification seem to dampen the probability of being in a certain regime compared to (M.2) model. The unconditional

[13] This model suggested by Hamilton (1990) is estimated in Python using the EM algorithm from the Python-package *statsmodels* developed by Seabold and Perktold (2010).

[14] Here, the standard errors are given in $(\cdot)$. All parameter estimates are significant at a 5 percent level of significance except for ζ_4, which is significant at a 10 percent level of significance.

[15] Again, the standard errors are given in $(\cdot)$. All parameter estimates are significant at a 5 percent level of significance except for ζ_4, which is significant at a 10 percent level of significance.

probabilities are also corresponding as $P(s_t = 1) = 0.4999$ and $P(s_t = 2) = 0.5001$. Thus, the parameter estimates of the (M.3) model show no evidence for mean reversion in the log electricity spot prices, as the regime switching means are identical and the unconditional probabilities of being in one of the regimes are the same. However, it seems like the Hamilton model (M.3) truly models a simple mean reverting AR(4) model.

Based on the information criterion, the non mean reverting 2-state MS-SV model (M.2) is favoured with an AIC-score of -564.24 compared to the AIC-score of 207.30 for the Hamilton model (M.3). This again supports that no mean reversion is present in the considered time series of log electricity spot prices, and that a MS-SV model should exhibit more than just regime switching means to capture the nature of these prices.

Both Bierbrauer et al. (2004) and Bierbrauer et al. (2007) found evidence of mean reversion in German and Nord Pool spot electricity prices respectively. Their conclusions were reached by analysing data from the end of the 1990s and beginning of the 2000s. Since then, the electricity market has emerged tremendously and become more integrated as stated in Section 3. This development implies that electricity generation originates from various input factors, which causes a dual mean reversion pattern. Thus, the mean reverting behaviour of the spot electricity prices, which was found suitable two decades ago, is not relevant anymore according to our analysis. This finding is in accordance with the Lucas critique, which states that it is not good practice to transfer econometric models suitable for one period of time to another period of time without further considerations (Telatar et al., 2007).
This is in line with Nomikos and Andriosopoulos (2012) and Geman (2005) who conclude that the mean reverting behaviour of electricity spot prices is not found in newer analyses modelling these prices. Additionally, Nomikos and Andriosopoulos (2012) emphasise that the actual evidence of mean reversion in energy commodities such as electricity spot prices comes from the associated energy futures.

The lacking evidence for mean reversion of the log electricity spot prices could be due to the fact that the Hamilton model (M.3) was not intended to model spot electricity price dynamics. Rather, the Hamilton model was suggested for modelling discontinuous shifts in the Treasury bill rates caused by changes in the Federal Reserve's target band for the Federal funds rate.
It was stated as a stylized fact in Section 3.3 that the spot electricity pricing process exhibits *dual* mean reversion. Hence, depending on which regime the process is in, the spot electricity pricing process will converge towards different mean levels. Therefore, it seems reasonable that the (M.2) model predicts two almost equally likely states with rather different unconditional means instead of a frequent, low volatility base regime and an infrequent spike regime. This underlines that the (M.2) model captures the characteristics of the electricity spot price to a larger extend than the Hamilton model (M.3) does.

For this reason, it should hold that electricity arise from one main input factor in the majority of 2019, where the electricity spot prices are modelled by regime $s_t = 1$ of the (M.2) model cf. Figure 5.3. Likewise, the electricity should originate from another input factor in the majority of 2020, where the electricity spot prices are modelled by regime $s_t = 2$ of the (M.2) model cf. Figure 5.4. Consequently, this indicates that a larger fraction of the electricity in 2020 arose from renewables compared to 2019, as renewables have substantially lower marginal costs than fossil fuels such as oil or gas. Whatever the circumstances, the (M.2) model is preferred based on our analysis.

6.3 Is an AR(1)-GARCH(1,1) Model better at Capturing the Spot Electricity Price Dynamics?

Nomikos and Andriosopoulos (2012) states that innovations for all energy log-price time series exhibit volatility clustering, which makes GARCH type models an obvious choice for modelling the spot electricity prices. To investigate their statement, an AR(1)-GARCH(1,1) model is considered as an alternative benchmark model. This model is chosen as it uses the AR(1) process to specify the spot electricity price process similar to the (M.2) model. The AR(1)-GARCH(1,1) model is given by

$$\begin{aligned} y_t &= \gamma + \tilde{\phi} y_{t-1} + \tilde{\epsilon}_t \\ \tilde{\epsilon}_t &= \tilde{\sigma}_t z_t \\ \tilde{\sigma}_t^2 &= \omega + \tilde{\alpha}\tilde{\epsilon}_{t-1}^2 + \tilde{\beta}\tilde{\sigma}_{t-1}^2 \end{aligned} \tag{M.4}$$

Here, $\tilde{\epsilon}_t \mid \mathcal{F}_{t-1} \sim N(0, \tilde{\sigma}_t^2)$, $z_t \sim N(0,1)$ and $\mathcal{F}_{t-1}$ denotes the filtration at time $t-1$, i.e. all the available information at time $t-1$. As the conditional variance of the innovations $\tilde{\epsilon}_t$ must be positive, it implies the following parameter restrictions $\omega > 0$, $\tilde{\alpha} \geq 0$ and $\tilde{\beta} \geq 0$. The AR(1)-GARCH(1,1) model (M.4) is estimated[16] using ML estimation due to the non-linear parameters. However, the functional form of the log-likelihood function does not correspond to the underlying true distribution, if z_t is not standard normally distributed. Therefore, QMLE is performed to obtain consistent parameter estimates and robust standard errors despite the potentially wrong distributional assumption of z_t. The resulting parameter estimates[17] of the AR(1)-GARCH(1,1) model are given in Table A.1, which can be found in Section A.3 of the Appendix. It is evident that the estimated $\hat{z}_t$ is non-normally distributed, which underlines the necessity of using QMLE. The estimated residuals are highly autocorrelated because the test of no autocorrelation of the residuals is strongly rejected. It is therefore impossible to conclude upon the test of no-ARCH effects[18]. This implies autocorrelation in the squared residuals, which as well is a sign of ARCH effects. The (M.4) is therefore either too simple or it has the incorrect functional form. In any case the AR(1)-GARCH(1,1) model (M.4) appears misspecified.

Hence, the suggested AR(1)-GARCH(1,1) model (M.4) does not seem to capture the spot electricity price dynamics to a larger extend than the preferred 2-state MS-SV AR(1) model. This is again empathised by the information criterion, which are substantially lower for the 2-state MS-SV AR(1) model (M.2) than for the AR(1)-GARCH(1,1) model (M.4).

7 Future Research

In the following we highlight the most relevant subjects for future research given our empirical analysis and discussion. These include forecasting electricity derivatives to examine the pricing performance of the regarded models and various model extensions.

[16] Estimation of the AR(1)-GARCH(1,1) model (M.4) requires that the drift criterion applies to the process $(y_t)_{t=0,1,2,\ldots}$. When the drift criterion holds cf. Assumption I.3.2 and Theorem I.3.2 in Pedersen and Rahbek (2020a), y_t is stationary and weakly mixing such that the LLN applies. It is further required that the Central Limit Theorem (CLT) holds, which it does when Theorem II.4.1 in Pedersen and Rahbek (2020b) is satisfied.

[17] The QMLE is carried out in R using the package *rugarch*, which is developed by Ghalanos (2020).

[18] The test for no-ARCH effects is described in Section A.2.4 of the Appendix.

7.1 Electricity Price Forecasting

Forecasting electricity spot prices is of particular interest due to the characteristic nature of these prices as discussed in Section 3.3. And as argued by Nomikos and Andriosopoulos (2012), the need for risk management in the field of electricity prices is of upsurge interest due to the volatile market environment. Thus, accurate electricity price forecasting is of interest both at a corporate - and a government level to manage the risk of unforeseen large price fluctuations and therefore unforeseen large expenses. Buying electricity derivatives such as electricity futures is way of hedging this price risk.

By using the considered models in this paper to forecast the electricity price and thereby the price of electricity derivatives such as electricity futures, the pricing performance of the models can be evaluated. We would expect that the 2-state MS-SV model (M.2) delivers a more accurate out-of-sample pricing compared to the baseline AR(1) model (M.1), the mean reverting Hamilton model (M.3) and the AR(1)-GARCH(1,1) model (M.4) in accordance with their examined modelling performance.

7.2 Model Extensions: The 3-state MS-SV Model and Independent Regimes

For future research it would be intriguing to investigate whether extending the preferred 2-state MS-SV model (M.2) with an additional state or with independent states would improve the modelling precision. Huisman and Mahieu (2003) proposes a 3-state MS-SV model for the electricity price dynamics, where a third reversing jump regime is considered in addition to a base regime and a regime modelling an initial jump, which can be either positive and negative. Bierbrauer et al. (2007) did not find any evidence for choosing this model instead of a 2-state MS-SV model, when modelling the German EEX Power Market electricity spot price dynamics. However, it should be noted that the MS-SV models examined by Bierbrauer et al. (2007) were defined with mean reverting states as oppose to our (M.2) model and therefore, their conclusions cannot be generalized to our case.

The dependent regime MS-SV models examined in this paper are computationally simpler to estimate than the MS-SV models with independent regimes. On the other hand, the independent regime MS-SV model does provide a more flexible and therefore realistic model framework, because the spot electricity pricing process fundamentally changes its dynamics occasionally (Janczura and Weron, 2010), which can be captured by this model type. For instance, Bierbrauer et al. (2004) model the electricity price dynamics with an independent 2-state MS-SV model with a mean reverting base regime and a spike regime following either a Pareto, log-normal or Gaussian distribution. Bierbrauer and co-workers conclude that these models provide more modelling accuracy than the 3-state MS-SV model proposed by Huisman and Mahieu (2003). Thus, a similar comparison would be interesting to carry out in our modelling setting with a MS-SV AR(1) model.

8 Concluding Remarks

We conclude that the 2-state MS-SV AR(1) model (M.2) of the form suggested by Guidolin and Timmermann (2005) seems to accurately capture the dynamics of the Nordic spot electricity prices for the period from January, 2019 until December, 2020. It was evident that the Nordic spot electricity prices did not show adequate signs of seasonality as neither of the employed deseasonalisation methods were applicable. Instead of deseasonalised log prices, the logarithm of the raw Nordic spot electricity price was used.

Despite the simple model specification and regime structure, the 2-state MS-SV AR(1) model (M.2) was found to capture the nature of the Nordic spot electricity prices to a larger extend than both the simplistic AR(1) model (M.1), the mean reverting 2-state MS-SV AR(4) model (M.3) and the AR(1)-GARCH(1,1) model (M.4). This supports the notion of having different regimes, but no ordinary mean reversion in the pricing process. The volatility process of the (M.2) model seemed rather persistent, which is in accordance with the observed market behaviour.

However, the 2-state MS-SV AR(1) model (M.2) showed a dual regime pattern, as the unconditional probability of being in one of the regimes was almost the same. This is not in accordance with the usual finding in the literature, where the 2-state MS-SV models have a frequent base regime and an infrequent spike regime. Thus, this suggests a dual mean reversion of the Nordic spot electricity price process, because the electricity arise from different input factors with different marginal prices.

References

Amundsen, E. S. and L. Bergman (2006). Why has the nordic electricity market worked so well? *Utilities policy 14*(3), 148–157.

Bazzi, M., F. Blasques, S. J. Koopman, and A. Lucas (2017). Time-varying transition probabilities for markov regime switching models. *Journal of Time Series Analysis 38*(3), 458–478.

Bean, N., A. Lewis, and G. Nguyen (2019). Estimation of markovian-regime-switching models with independent regimes. *arXiv preprint arXiv:1906.07957*.

Bierbrauer, M., C. Menn, S. T. Rachev, and S. Trück (2007). Spot and derivative pricing in the eex power market. *Journal of banking & finance 31*(11), 3462–3485.

Bierbrauer, M., S. Trück, and R. Weron (2004). Modeling electricity prices with regime switching models. In *International Conference on Computational Science*, pp. 859–867. Springer.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of econometrics 31*(3), 307–327.

Cartea, A. and M. G. Figueroa (2005). Pricing in electricity markets: a mean reverting jump diffusion model with seasonality. *Applied Mathematical Finance 12*(4), 313–335.

Christiansen, C. (2008). Level-arch short rate models with regime switching: Bivariate modeling of us and european short rates. *International Review of Financial Analysis 17*(5), 925–948.

Deane, P., S. Collins, B. Ó. Gallachóir, C. Eid, R. Hartel, D. Keles, and W. Fichtner (2015). Quantifying the" merit-order" effect in european electricity markets.

Energistyrelsen (2020). Electricity market design. https://ens.dk/sites/ens.dk/files/Globalcooperation/layouted_electricity_market_design_v3.pdf. Accessed: November 2021.

Ethier, R. and T. Mount (1998). Estimating the volatility of spot prices in restructured electricity markets and the implications for option values. Technical report, PSerc Working Paper 98-31.

Geman, H. (2005). Energy commodity prices: Is mean-reversion dead? *The Journal of Alternative Investments 8*(2), 31–45.

Genon-Catalot, V., C. Laredo, and M. Nussbaum (2002). Asymptotic equivalence of estimating a poisson intensity and a positive diffusion drift. *The Annals of Statistics 30*(3), 731–753.

Ghalanos, A. (2020). *rugarch: Univariate GARCH models.* R package version 1.4-4.

Green, R. (2006). Electricity liberalisation in europe—how competitive will it be? *Energy Policy 34*(16), 2532–2541.

Guidolin, M. and A. Timmermann (2005). Economic implications of bull and bear regimes in uk stock and bond returns. *The Economic Journal 115*(500), 111–143.

Guidolin, M. and A. Timmermann (2007). Asset allocation under multivariate regime switching. *Journal of Economic Dynamics and Control 31*(11), 3503–3544.

Guidolin, M. and A. Timmermann (2008). International asset allocation under regime switching, skew, and kurtosis preferences. *The Review of Financial Studies 21*(2), 889–935.

Hall, S. G., Z. Psaradakis, and M. Sola (1999). Detecting periodically collapsing bubbles: a markov-switching unit root test. *Journal of Applied Econometrics 14*(2), 143–154.

Hamilton, J. D. (1989). A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica: Journal of the econometric society*, 357–384.

Hamilton, J. D. (1990). Analysis of time series subject to changes in regime. *Journal of econometrics 45*(1-2), 39–70.

Hamilton, J. D. (1994). *Time series analysis.* Princeton University Press.

Huisman, R. and R. Mahieu (2003). Regime jumps in electricity prices. *Energy economics 25*(5), 425–434.

IEA (2020). Electricity market report - december 2020. https://www.iea.org/reports/electricity-market-report-december-2020/2020-global-overview-prices. Accessed: November 2021.

Janczura, J. and R. Weron (2010). An empirical comparison of alternate regime-switching models for electricity spot prices. *Energy economics 32*(5), 1059–1073.

Lucia, J. J. and E. S. Schwartz (2002). Electricity prices and power derivatives: Evidence from the nordic power exchange. *Review of derivatives research 5*(1), 5–50.

Mount, T. D., Y. Ning, and X. Cai (2006). Predicting price spikes in electricity markets using a regime-switching model with time-varying parameters. *Energy Economics 28*(1), 62–80.

Nomikos, N. and K. Andriosopoulos (2012). Modelling energy spot prices: Empirical evidence from nymex. *Energy Economics 34*(4), 1153–1169.

Nordpool (2020a). Annual review 2020. https://www.nordpoolgroup.com/49eea7/globalassets/download-center/annual-report/annual-review-2020.pdf. Accessed: November 2021.

Nordpool (2020b). History. https://www.nordpoolgroup.com/About-us/History/. Accessed: November 2021.

Nordpool (2021a). Nordpool system price. https://www.nordpoolgroup.com/trading/Day-ahead-trading/Price-calculation/. Accessed: November 2021.

Nordpool (2021b). Nordpool trading figures. https://www.nordpoolgroup.com/message-center-container/newsroom/exchange-message-list/2021/q1/nord-pool-announces-2020-trading-figures/. Accessed: November 2021.

Pedersen, R. S. and A. Rahbek (2017). Part v: Finite sv: Em-algorithm-based estimation. *Unpublished lecture note from the course Financial Econometrics A, University of Copenhagen.*

Pedersen, R. S. and A. Rahbek (2020a). Part i: An introduction to the theory of non-linear time series with focus on arch of order one. *Unpublished lecture note from the course Financial Econometrics A, University of Copenhagen.*

Pedersen, R. S. and A. Rahbek (2020b). Part ii: General arch and garch. *Unpublished lecture note from the course Financial Econometrics A, University of Copenhagen*.

Sanchez-Espigares, J. A. and A. Lopez-Moreno (2021). *MSwM: Fitting Markov Switching Models*. R package version 1.5.

Seabold, S. and J. Perktold (2010). statsmodels: Econometric and statistical modeling with python. In *9th Python in Science Conference*.

Telatar, E., F. Telatar, and N. Bolatoglu (2007). A regime switching approach to the feldstein–horioka puzzle: Evidence from some european countries. *Journal of Policy Modeling 29*(3), 523–533.

Weron, R., M. Bierbrauer, and S. Trück (2004). Modeling electricity prices: jump diffusion and regime switching. *Physica A: Statistical Mechanics and its Applications 336*(1-2), 39–48.

Weron, R., B. Kozłowska, and J. Nowicka-Zagrajek (2001). Modeling electricity loads in california: a continuous-time approach. *Physica A: Statistical Mechanics and its Applications 299*(1-2), 344–350.

A Appendix

A.1 Calculating the Intra-Weekly Pattern, Ω_t, for the Deseasonalisation of Spot Electricity Prices

The intra-weekly pattern is calculated by using a centered 7 day moving average technique as done by Weron et al. (2001) to remove the weekly component and reduce the resulting noise for each of the T Nordic spot electricity prices $\{x_1, x_2, ..., x_T\}$

$$m_t = \frac{1}{7} \cdot (x_{t-3} + x_{t-2} + ... + x_{t+2} + x_{t+3}), \qquad \text{where} \quad t = 4, 5, ..., T-3.$$

Then, the average deviation for the *k'th* weekday, $k = 1, 2, ..., 7$, is computed as

$$w_k = (x_{k+7l} - m_{k+7l}), \qquad \text{where} \quad 3 < k + 7l \leq T - 3.$$

Here, l denotes the week number and $k + 7l$ therefore referes to the *k'th* weekday in week number l. Thereby, w_1 and w_5 refer to the average deviation for mondays and fridays of the given period respectively. For each of the k weekdays, the weekly component Ω_t is then found as

$$\Omega_k = w_k - \frac{1}{7} \sum_{i=1}^{7} w_i, \qquad \text{where} \quad k = 1, 2, ..., 7.$$

Hence, at day t, which is the *k'th* weekday, the belonging average weekly component is given by $\Omega_t = \Omega_k$.

A.2 Misspecification Tests

A.2.1 Test for No Residual Autocorrelation

Using a Breusch-Godfrey Lagrange Multiplier (LM) test, $LM = T \cdot R^2$, the test for no residual autocorrelation of order 1-5 is carried out. Here, T is the number of observations used for estimating model (M.1), whereas R^2 results from an auxiliary regression. In this auxiliary regression, the estimated residual from model (M.1) is regressed on the lagged, estimated residuals of order 1-5. The test statistics is asymptotically $\chi^2(5)$-distributed under the null hypothesis of no autocorrelation of the estimated residuals with a critical value of 11.07 on a 5 percent level of significance. If the null hypothesis is rejected, model (M.1) is dynamically incomplete. Otherwise, the test indicates that the model (M.1) is well-specified. Equivalently, this test can be applied to the (M.4) model.

A.2.2 Likelihood Ratio Test for a Unit Root

Testing for a unit root in the model (M.1) is of importance as a unit root process implies that shocks to the innovation ρ_t create non-transitory dynamic effects. To perform this LR test, there must be no autocorrelation of the residual. The two-sided LR test statistics is $LR(\pi = \alpha = 0) = -2(logL_0 - logL_A)$, where $\pi = \rho - 1$, $logL_A$ is the log-likelihood of the (M.1) model and $logL_0$ is the log-likelihood of the (M.1) model under the null hypothesis where $\pi = \alpha = 0$. Under the null hypothesis, the test statistics follows a Dickey-Fuller (DF) distribution, DF_c^2, with a critical value of 9.13 on a 5 percent level of significance. If the null hypothesis cannot be rejected, it indicates the presence of a unit root.

A.2.3 Test for Normality of the Error Term

The speed of convergence of the parameter estimates towards the true parameter values is faster, if the error term is normally distributed. Therefore, a test for normality of the error carried out by comparing the skewness (S) and kurtosis (K) of the standard normal distribution, where $S = 0$ and $K = 0$, to S and K of the estimated residuals. The test statistics follows a χ^2-distribution under the null with a critical value of 3.84 on a 5 percent level of significance and is given by $\xi_S = \frac{T}{6} \cdot S^2 \to \chi^2(1)$ and $\xi_K = \frac{T}{24} \cdot (K-3)^2 \to \chi^2(1)$.

A.2.4 Test for No-ARCH Effects

A Breusch-Godfrey LM test, $\xi_{ARCH} = T \cdot R^2$, is used to test for no-ARCH effects of order 2. Again, T is the number of observations used for estimating model (M.4) and R^2 arise from the following auxiliary regression

$$\hat{\tilde{\epsilon}}_t^2 = \tau_0 + \tau_1 \hat{\tilde{\epsilon}}_{t-1}^2 + \tau_2 \hat{\tilde{\epsilon}}_{t-2}^2 + \mu_t.$$

Here, $\hat{\tilde{\epsilon}}_t^2$ is the estimated residual from model (M.4) and μ_t is a suprise term, $E[\mu_t \mid \mathcal{F}_{t-1}] = 0$. The null hypothesis of no ARCH effects is $H_0 = \tau_1 = \tau_2 = 0$, which is tested against the alternative hypothesis $H_A = \tau_i \neq 0$, where $i = 1, 2$. Under the null ξ_{ARCH} asymptotically follows a $\chi^2(2)$ distribution. Before performing this test, the test for no residual autocorrelation must be carried out. If the residuals are autocorrelated, the squared residuals are autocorrelated as well, which makes it impossible to conclude upon the test of no-ARCH effects.

A.3 Estimation Results of the AR(1)-GARCH(1,1) Model (M.4)

Table A.1: Estimation Results of the AR(1)-GARCH(1,1) Model (M.4), 726 observations

	(M.4)
γ	3.6373* (0.0537)
$\tilde{\phi}$	0.9407* (0.3349)
ω	0.0005 (0.0007)
$\tilde{\alpha}$	0.2656** (0.1530)
$\tilde{\beta}$	0.7334* (0.1981)
Log-lik.	-286.72
AIC	-0.78
BIC	-0.74
$\tilde{\alpha} + \tilde{\beta}$	0.999
No autocorrelation	[0.000]
No ARCH 1-2	[0.592]
Normality	[0.000]
T	726
Sample start	2019-01-01
Sample end	2020-12-31

*Note: P-values in [·] for misspecification tests and robust standard errors in (·). * indicates that the parameter estimate is significant at a 5 percent level of significance, whereas ** indicates that parameter estimate is significant at a 10 percent level of significance.*

Source: Nord Pool and internal calculations.