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Applying ARMA–GARCH approaches to forecasting short-term electricity prices

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ABSTRACT

Accurately modeling and predicting the mean and volatility of electricity prices can be of great importance to value electricity, bid or hedge against the volatility of electricity prices and manage risk. The paper applies various autoregressive moving average (ARMA) models with generalized autoregressive conditional heteroskedasticity (GARCH) processes, namely ARMA–GARCH models, along with their modified forms, ARMA–GARCH-in-mean (ARMA–GARCH-M), to model and forecast hourly ahead electricity prices. In total, 10 different model structures are adopted, and this paper thus conducts a comprehensive investigation on the ARMA–GARCH based time series forecasting of electricity prices. Multiple statistical measures are employed to evaluate the modeling sufficiency and predication accuracy of the ARMA–GARCH(-M) methods. The results show that the ARMA–GARCH-M models are in general an effective tool for modeling and forecasting the mean and volatility of electricity prices, while ARMA–SGARCH-M models are simple and robust and the ARMA–GJRGARCH-M model is very competitive. In addition, we observe that hourly electricity prices exhibit apparent daily, weekly and monthly periodicities, and have the nonlinear and asymmetric time-varying volatility together with an inverse leverage effect.
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1. Introduction

In recent years, electricity markets in many countries have been deregulated to introduce competition in supply and demand activities. In a deregulated electricity market, generators compete to sell electricity and at the same time suppliers to consumers compete to purchase electricity. The supply and demand activities of electricity force the electricity market to reach an equilibrium price so that the trades can happen. However, the demand of electricity is influenced by some social and economic activities and by weather conditions. Particularly electricity cannot be physically stored in a direct way, and thus production and consumption as well as equilibrium prices have to be continuously balanced to smooth out supply and demand shocks. It is evident that the introduction of deregulation makes electricity prices become uncertain and have a high volatility. The volatility of electricity prices is as comparable as other financial markets like stocks or other commodities (Escribano et al., 2002). To accurately value electricity, bid or hedge against the volatility of electricity prices and manage risk, generators and suppliers in deregulated electricity markets extensively use modeling and prediction techniques to characterize and understand the behavior of electricity prices and estimate electricity prices (Carolina et al., 2011; Fanone et al., 2013).

Autoregressive integrated moving average (ARIMA) models with autoregressive conditional heteroskedastic (ARCH) (Engle, 1982) or generalized autoregressive conditional heteroskedastic (GARCH) (Bollerslev, 1986) processes are the widely used approaches to modeling the mean and volatility of electricity prices. For GARCH models, one of their advantages over ARCH models is parsimony which implies that fewer model parameters are needed to conduct estimation. In the literature, the GARCH models frequently used are the general GARCH model (Garcia et al., 2005) and the exponential GARCH (EGARCH) model (Bowden and Payne, 2008). Note that GARCH models have a number of variants and several of them can model asymmetric time-varying volatility. As such, it is necessary to comprehensively evaluate and compare these models in an attempt to select an appropriate one among them. The goal of this article is to show our effort in this aspect. Specifically, we use 10 different ARMA–GARCH(-M) approaches to model the mean and volatility of electricity prices from the New England electricity market, employ various criteria to evaluate the modeling sufficiency of in-sample electricity prices, and then use the built models to perform the prediction for out-of-sample electricity prices.

The rest of the article is organized as follows. Section 2 reviews the current literature related to modeling the mean and volatility of electricity prices. Section 3 describes the general GARCH methodology and their typical variants. Section 4 presents the criteria adopted to analyze model specification and measure modeling sufficiency. Section 5 applies 10 different ARMA–GARCH(-M) approaches to model the mean and volatility of hourly electricity spot prices from the New England electricity market, and analyzes the obtained results. Section 6 uses the established ARMA–GARCH(-M) models to conduct the out-of-sample electricity price prediction, and the prediction

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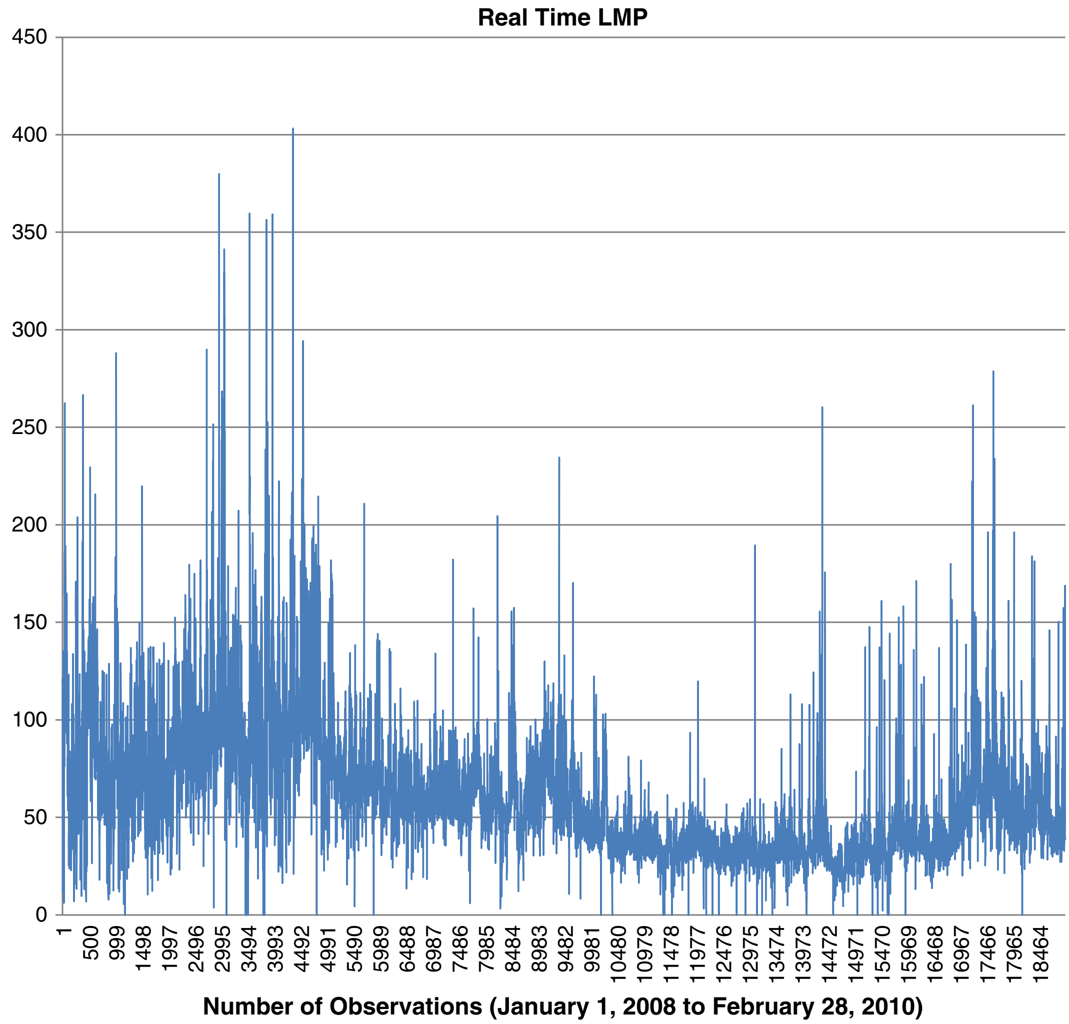

Fig. 1. Diagrammatic trend of chosen LMP data ($/MWh).

accuracy of different models is compared. Finally, Section 7 concludes this study by summarizing the major discoveries and contributions.

2. Literature review

The methods applied to predict electricity prices are pretty diverse. Aggarwal et al. (2009) classify these methods into three groups with each divided into several subsets. The first group is based on game theory, and it includes several equilibrium models such as Nash equilibrium, Cournot model (Siriruk and Valenzuela, 2011), Bertrand model, and supply function equilibrium (Bajpai and Singh, 2004). The models in this group are able to model the strategies of market participants and identify optimal solutions (Ladjicia and Boudour, 2011; Pozo et al., 2011). The second group involves simulation. The simulation models (Bastian et al., 1999; Deb et al., 2000; Lise et al., 2010) can mimic the actual dispatch, and consider the physical status of electricity system together with system operating requirements and constraints. The prediction of electricity prices based on a simulation model intends to solve a security constrained optimal power flow within the entire system range. As Aggarwal et al. (2009) comment, although the simulation methods have the potentials to provide detailed insights into system prices, these methods suffer from two drawbacks, namely the implementation complication of simulation model and the high computational cost.

The third group is time series forecasting methods. These methods use the past behavior of electricity prices and some exogenous variables to forecast future electricity prices. In this group, two types of models are essential which are artificial intelligence techniques and conventional statistical models. Artificial intelligence techniques such as artificial neural networks are able to extract a nonlinear relationship governing inputs and outputs, and then provide prediction. Although artificial intelligence techniques can give the accurate prediction of electricity prices (Amjady and Keynia, 2011; Catalão et al., 2007; Lin et al., 2010; Szkuta et al., 1999; Yamin et al., 2004), one of its critical deficiencies is that the function forms built by them are implicit and the further analysis on the function forms such as sensitivity analysis is difficult. Traditional statistical models well known and widely applied in practice include regression, autoregressive (AR), moving average (MA), autoregressive moving average (ARMA), autoregressive integrated moving average (ARIMA) (Box et al., 1994), and ARMA with exogenous variables (ARMAX) models (Weron and Misiorek, 2005). These models

Table 1
Basic statistics of chosen LMP data ($/MWh).

Minimum	0
Maximum	403.23
Average	61.09
Standard deviation	33.4735
Kurtosis	6.691
Skewness	1.8264

Table 2
Fitted mean equation parameters in ARMA–GARCH models.

	SGARCH	QGARCH	GJRGARCH	EGARCH	NGARCH
Constant	109.5761	40.6728	31.4803	34.8652	40.6556
	5.0754	1.3235	2.0351	2.0969	1.3938
AR(1)	0.3596	0.2968	0.3003	0.2994	0.2999
	0.0119	0.0134	0.0132	0.0127	0.0136
AR(11)	0.1168	0.0628	0.0858	0.0911	0.0691
	0.0063	0.0062	0.0068	0.0063	0.0064
AR(24)	0.1107	0.1102	0.1094	0.1078	0.1072
	0.0055	0.0052	0.0056	0.0054	0.0053
AR(48)	0.0220	0.0351	0.0324	0.0261	0.0284
	0.0045	0.0043	0.0045	0.0042	0.0043
AR(72)	0.0273	0.0331	0.0348	0.0381	0.0341
	0.0051	0.0047	0.0050	0.0048	0.0048
AR(96)	0.0439	0.0311	0.0388	0.0342	0.0299
	0.0054	0.0055	0.0054	0.0048	0.0051
AR(120)	0.0341	0.0358	0.0381	0.0368	0.0338
	0.0049	0.0046	0.0047	0.0048	0.0047
AR(144)	0.0223	0.0266	0.0226	0.0286	0.0265
	0.0049	0.0045	0.0048	0.0048	0.0047
AR(168)	0.0372	0.0403	0.0471	0.0423	0.0421
	0.0050	0.0047	0.0050	0.0048	0.0047
AR(216)	0.0415	0.0201	0.0125	0.0146	0.0191
	0.0049	0.0041	0.0046	0.0045	0.0044
AR(264)	0.0347	0.0239	0.0279	0.0350	0.0238
	0.0050	0.0049	0.0050	0.0049	0.0050
AR(336)	0.0263	0.0243	0.0274	0.0254	0.0248
	0.0050	0.0046	0.0049	0.0046	0.0048
AR(504)	0.0328	0.0365	0.0424	0.0376	0.0362
	0.0049	0.0048	0.0049	0.0048	0.0047
AR(672)	0.0800	0.0429	0.0493	0.0583	0.0465
	0.0054	0.0048	0.0053	0.0045	0.0049
AR(840)	0.0395	0.0376	0.0378	0.0380	0.0421
	0.0051	0.0045	0.0046	0.0046	0.0045
MA(1)	−0.3748	−0.4480	−0.4510	−0.4600	−0.4632
	0.0153	0.0162	0.0162	0.0153	0.0162
MA(2)	−0.2282	−0.2919	−0.2832	−0.2842	−0.3131
	0.0136	0.0141	0.0144	0.0139	0.0142
MA(3)	−0.2376	−0.2895	−0.2746	−0.2628	−0.3072
	0.0119	0.0121	0.0124	0.0121	0.0123
MA(4)	−0.2112	−0.2634	−0.2423	−0.2391	−0.2850
	0.0113	0.0117	0.0120	0.0114	0.0119
MA(5)	−0.1911	−0.2157	−0.2040	−0.1996	−0.2328
	0.0107	0.0108	0.0112	0.0108	0.0112
MA(6)	−0.1622	−0.1771	−0.1699	−0.1660	−0.1899
	0.0104	0.0099	0.0105	0.0102	0.0103
MA(7)	−0.1325	−0.1288	−0.1324	−0.1408	−0.1476
	0.0098	0.0093	0.0100	0.0095	0.0097
MA(8)	−0.1285	−0.1042	−0.1157	−0.1225	−0.1164
	0.0097	0.0087	0.0096	0.0089	0.0092
MA(9)	−0.1161	−0.0853	−0.0994	−0.1051	−0.0898
	0.0091	0.0081	0.0092	0.0086	0.0086
MA(10)	−0.0804	−0.0624	−0.0747	−0.0799	−0.0699
	0.0083	0.0075	0.0080	0.0077	0.0078

(Note: In each cell, the first and second rows are the estimated parameter value and the corresponding standard error, respectively.)

Table 3
Fitted variance equation parameters in ARMA-GARCH models.

	SGARCH	QGARCH	GJRGARCH	EGARCH	NGARCH
Constant	6.5732	10.2311	6.9088	0.4131	7.0076
	0.4634	0.4873	0.3831	0.0195	0.3324
ARCH(1)	0.4594	0.4395	0.2049	0.4180	0.3514
	0.0231	0.0180	0.0131	0.0143	0.0155
GARCH(1)	0.6488	0.6189	0.6509	0.9220	0.6045
	0.0142	0.0109	0.0111	0.0039	0.0109
Phi	N/A	2.9160	0.4392	N/A	N/A
		0.1322	0.0240		
Theta	N/A	N/A	N/A	0.3523	0.5134
				0.0171	0.0204

(Note: In each cell, the first and second rows are the estimated parameter value and the corresponding standard error, respectively.)

have certain advantages for electricity price forecasting such as the simplicity and explicitness of model structures, the accuracy of prediction results, and the accessibility of application software like SAS® and MINITAB®. Although these models can predict electricity prices, they assume that electricity prices have the homoskedastic property, and thus ignore the heteroskedasticity of data – an essential feature of data non-stationarity in time series forecasting.

The models reviewed above focus on the prediction of electricity price mean. Recently, some researchers have suggested modeling the volatility of electricity prices. On the one hand, it can generalize the time series modeling approach of electricity prices by relaxing the homoskedastic assumption. On the other hand, it can facilitate the valuing and trading of futures and options related to electricity. While modeling the volatility of electricity prices, ARCH and GARCH models are adopted.

The literature on using GARCH methods to model and predict the conditional volatility of electricity prices is growing quickly. Escribano et al. (2002) use GARCH approaches with a jump-diffusion intensity parameter to model daily spot prices of different electricity markets. They investigate the different behavior of electricity prices and quantify the roles of different characteristics (seasonality, mean-reversion, volatility, and jumps) in each individual market. Knittel and Roberts (2005) also include GARCH and jump processes in their model specification for the empirical examination of hourly electricity prices. Their findings reveal several characteristics unique to electricity prices including an inverse leverage effect. Garcia et al. (2005) focus on day-ahead forecasts of electricity prices with high volatility periods using the GARCH methodology. According to historical data from the mainland Spain and California electricity markets, they illustrate that the generalized heteroskedastic error specification is strongly supported by hourly electricity price data and significantly improves both the goodness of fit and the out-of-sample predictive accuracy. Following the standard practice of modeling volatility, Koopman et al. (2007) conduct the research on the conditional mean and volatility of electricity price innovations. The importance of regression effects, periodicity, long memory, and volatility in electricity spot prices is highlighted, and a simultaneous model for these features is proposed with the parameters in this model being jointly estimated by the method of approximate maximum likelihood. Bowden and Payne (2008) examine the in- and out-of-sample forecasting performance of three time series models (ARIMA, ARIMA–EGARCH, and ARIMA–EGARCH-M) for short-term electricity prices. The results from the ARIMA models disclose the presence of autoregressive conditional heteroskedasticity. The EGARCH specification recognizes the asymmetric time-varying volatility, demonstrates the presence of an inverse leverage effect in electricity prices, and outperforms the other models according to the out-of-sample forecasting performance. Diongue et al. (2009) investigate conditional mean and volatility forecasts of electricity spot market prices using a dynamic model following a k-factor GIGARCH process. They apply this method to the German electricity market to forecast spot prices, and claim that the k-factor GIGARCH process is a suitable tool to forecast spot prices according to the root mean squared error (RMSE) criteria. By using wavelet transform combined with ARIMA and GARCH

Table 4
Modeling sufficiency evaluation of ARMA-GARCH models.

	SGARCH	QGARCH	GJRGARCH	EGARCH	NGARCH
Adjusted R^2	0.8299	0.8262	0.8270	0.8273	0.8260
Overall F	2953*	2781*	2797*	2803*	2777*
	(<0.0001)**	(<0.0001)**	(<0.0001)**	(<0.0001)**	(<0.0001)**
AIC	131,172	130,370	130,600	130,474	130,150
BIC (SBC)	131,397	130,603	130,833	130,707	130,383

(Note: * is the value of *F* statistics of each cell and ** is the *P*-value.)

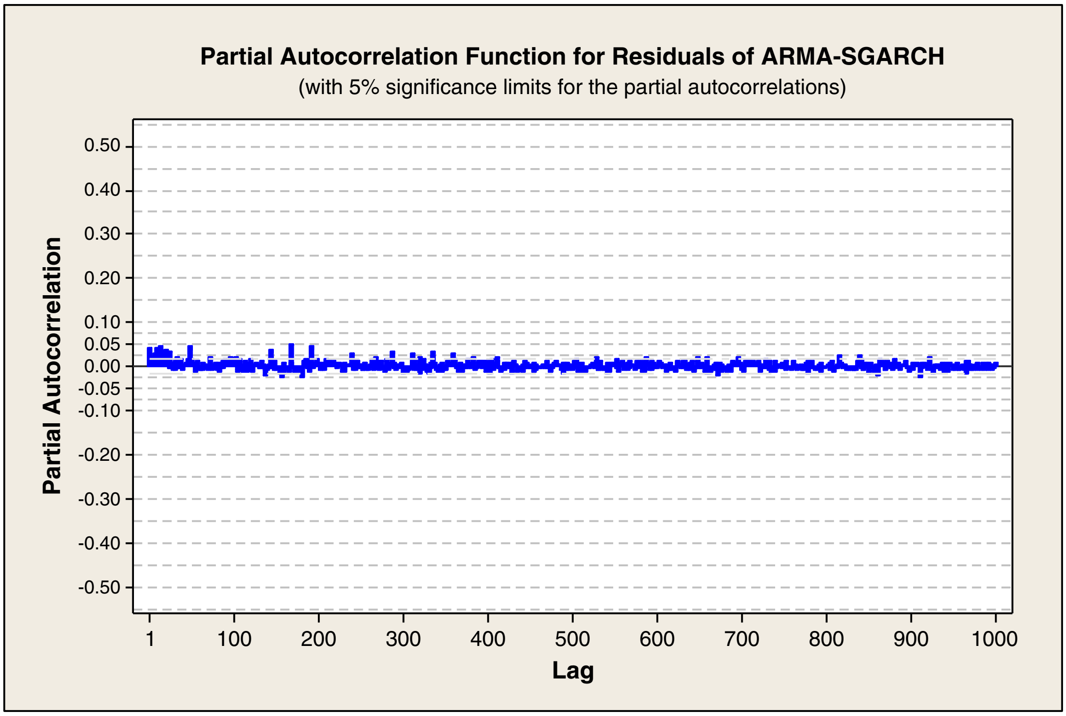

Fig. 2. PACF chart of residuals from ARMA–SGARCH model.

models, Tan et al. (2010) capture the complex characteristics of non-stationarity, nonlinearity and high volatility in day-ahead electricity prices.

Although the literature presented above has shown that electricity prices have the time-varying volatility along with an inverse leverage effect, GARCH models frequently used are the general GARCH model and the EGARCH model. Compared with other forecasting methods such as simulation and intelligence techniques, ARMA–GARCH models have the advantages of accuracy and explicitness as well as being able to accommodate the heteroskedasticity. Note that GARCH models have a number of variants and several of them can model asymmetric time-varying volatility. The goal of this article is to comprehensively evaluate

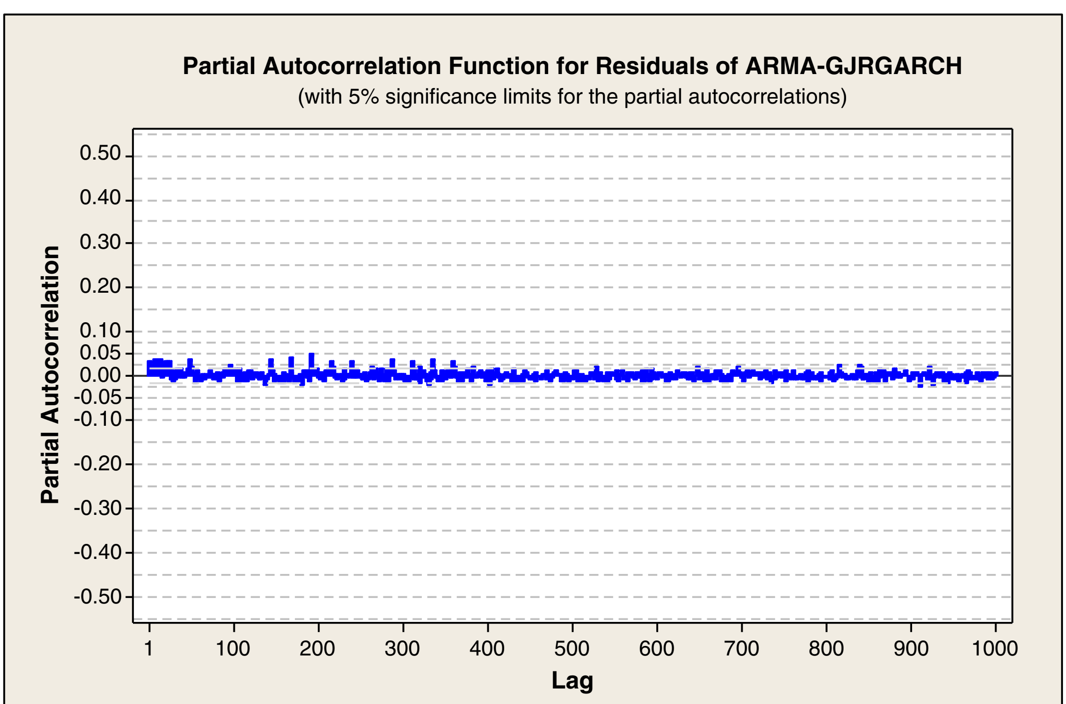

Fig. 3. PACF chart of residuals from ARMA–GJRGARCH model.

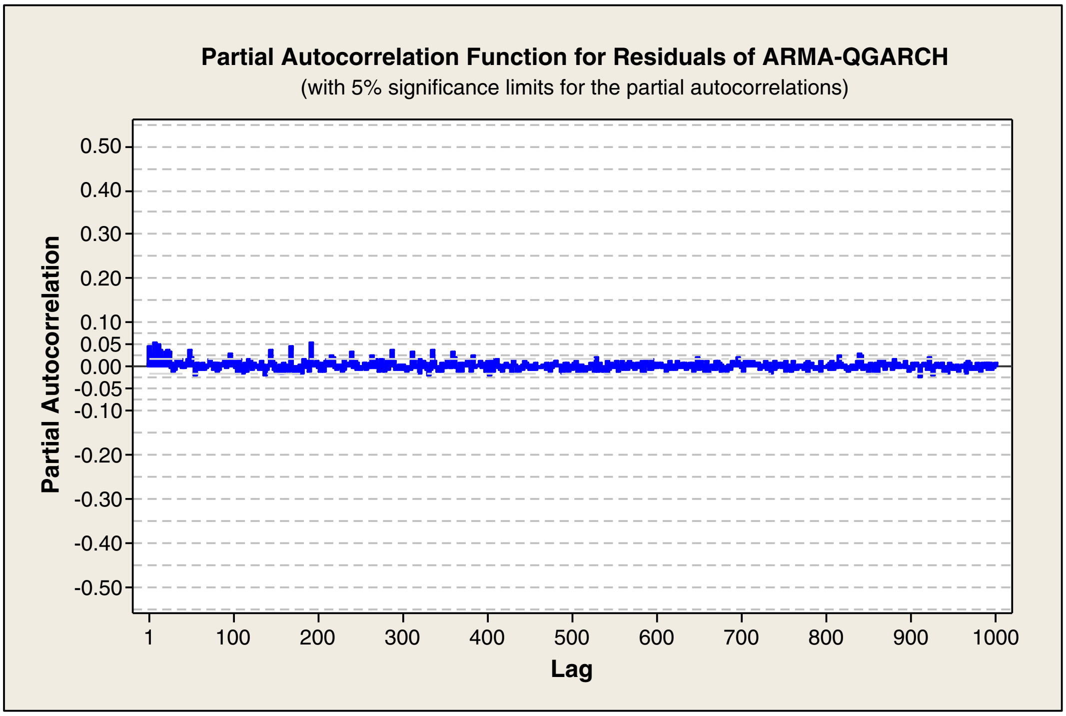

Fig. 4. PACF chart of residuals from ARMA–QGARCH model.

and compare these models in an attempt to select an appropriate one among them.

3. ARIMA–GARCH(-M) methodology

3.1. Conventional ARIMA–GARCH models

ARIMA approach developed by Box and Jenkins (1976) is a class of stochastic models used to analyze time series data. Consider the following autoregressive moving average model denoted as ARMA(p, q)

$$y_t = \delta + \sum_{i=1}^{p} \phi_i y_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t, \qquad (1)$$

where δ is a constant term, ϕ_i the ith autoregressive coefficient, θ_j the jth moving average coefficient, and ε_t the error term at time t. p and q are called the orders of autoregressive and moving average terms,

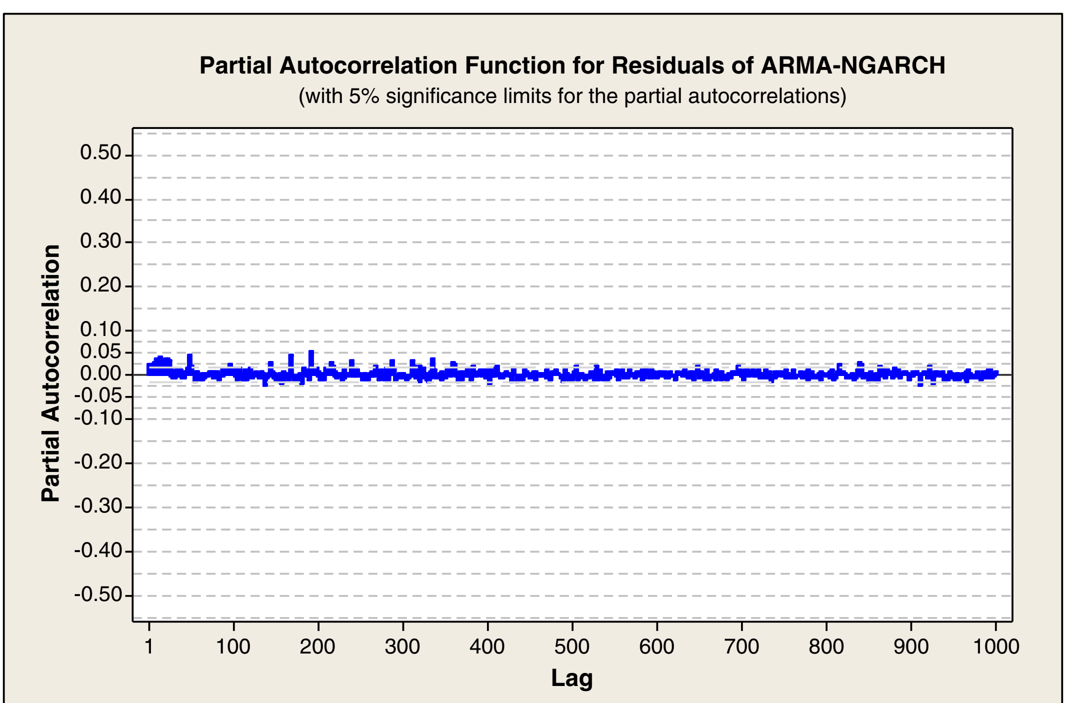

Fig. 5. PACF chart of residuals from ARMA–EGARCH model.

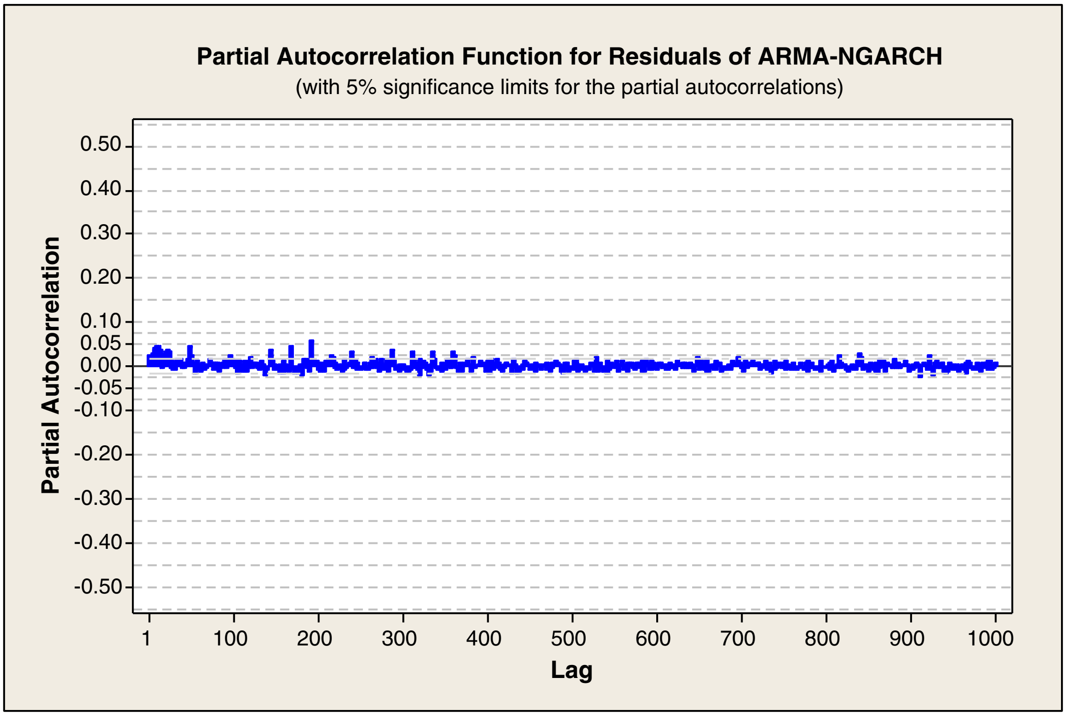

Fig. 6. PACF chart of residuals from ARMA–NGARCH model.

respectively. When the backshift operator B is applied, Eq. (1) can be written as

$$\left(1-\sum_{i=1}^{p}\phi_i B^i\right)(y_t-\mu)=\left(1+\sum_{j=1}^{q}\theta_j B^j\right)\varepsilon_t, \tag{2}$$

where $B(y_t-\mu)=y_{t-1}-\mu$ and $B\varepsilon_t=\varepsilon_{t-1}$.

The general scheme of ARIMA application includes four steps which are the identification of model structures, the estimation of model parameters, the validation of fitted models, and the forecast of future values. The identification of models is carried out by the careful inspection of main characteristics of time series data which is hourly electricity prices in this paper. The autocorrelation and partial autocorrelation functions, namely ACF and PACF, are basic instruments to identify stationary series. In the SAS® software, the procedure Proc ARIMA can be used to build the underlying ARMA models. The orders of the autoregressive and moving average terms can be determined using the extended sample autocorrelation function (ESACF). After the model structures have been specified, the parameters of these models can be estimated by a maximum likelihood function. In the third step, the methods of diagnosis checks can be used to analyze whether residuals satisfy the requirements of a Gaussian white noise process: zero mean, constant variance, uncorrelated process and normal distribution. In the fourth step, the established model can be used to conduct prediction. In this step, an obvious feature is that predictions are less accurate as the forecast lead time becomes larger.

For traditional ARIMA models, error term ε_t has zero mean and is homoskedastic plus the serial uncorrelated property. When time series data exhibits the conditional heteroskedastic feature, ARCH models proposed by Engle (1982) are more appropriate and should be adopted. ARCH models can accommodate the serial correlation in volatilities which changes over time. In an ARCH model, ε_t is transformed as

$$\varepsilon_t=\sqrt{v_t}z_t, \tag{3}$$

where z_t is a Gaussian white noise sequence with mean = 0, variance = 1, and $V(\varepsilon_t|\psi_{t-1})=v_t$. ψ_{t-1} represents the information known before time t. Assume that v_t is dependent on l previous errors and can be estimated by the following equation,

$$v_t=\varsigma_0+\eta_1\varepsilon_{t-1}^2+\eta_2\varepsilon_{t-2}^2+\cdots+\eta_l\varepsilon_{t-l}^2, \tag{4}$$

where ς_0 and η_i are constant coefficients. In this case, ε_t is said to follow an autoregressive conditional heteroskedastic process of order l, expressed as ARCH(l).

GARCH models are a generalized version of ARCH models and developed by Bollerslev (1986). In a GARCH model, the current conditional variance depends not only on l previous errors but also on k previous conditional variances. That is, Eq. (4) becomes the following form,

$$v_t=\varsigma_0+\sum_{i=1}^{k}\varsigma_i v_{t-i}+\sum_{i=1}^{l}\eta_i\varepsilon_{t-i}^2, \tag{5}$$

where ς_i are constant coefficients. In Eq. (5), error term ε_t is said to follow a GARCH process of orders k and l, denoted by GARCH(k, l). A fact that should be borne in mind when modeling conditional variances is that ς_0, ς_i and η_i are non-negative. In the conventional GARCH model or Eq. (5), the influence of ε_t is symmetric. That is, positive and negative shocks to electricity prices with the same magnitude produce the same amount of volatility.

3.2. Typical variants of GARCH models

GARCH models have some typical variants such as EGARCH (Nelson, 1991), nonlinear GARCH (NGARCH) (Engle and Ng, 1993), quadratic GARCH (QGARCH) (Sentana, 1995), and Glosten–Jagannathan–Runkle GARCH (GJRGARCH) models (Glosten et al., 1993). These models use a nonlinear function to model time-varying volatilities as well as leverage effects in which conventional GARCH models cannot capture. We use EGARCH models as an example to explain the leverage effects.

Table 5
Fitted mean equation parameters in ARMA-GARCH-M models.

	SGARCH	QGARCH	GJRGARCH	EGARCH	NGARCH
Constant	109.9454	98.1807	104.3812	106.0747	100.6331
	1.6124	3.4881	3.7158	3.6381	3.6652
AR(1)	0.3516	0.2378	0.2359	0.2337	0.2346
	0.0119	0.0125	0.0124	0.0122	0.0126
AR(11)	0.1014	0.1030	0.1030	0.1011	0.1038
	0.0062	0.0053	0.0053	0.0052	0.0054
AR(24)	0.1073	0.1329	0.1319	0.1314	0.1330
	0.0055	0.0045	0.0045	0.0045	0.0045
AR(48)	0.0232	0.0258	0.0253	0.0249	0.0258
	0.0045	0.0050	0.0050	0.0050	0.0050
AR(72)	0.0297	0.0398	0.0397	0.0395	0.0399
	0.0051	0.0057	0.0057	0.0057	0.0057
AR(96)	0.0443	0.0347	0.0340	0.0338	0.0347
	0.0055	0.0051	0.0052	0.0051	0.0052
AR(120)	0.0343	0.0454	0.0457	0.0459	0.0456
	0.0050	0.0058	0.0059	0.0058	0.0059
AR(144)	0.0237	0.0412	0.0415	0.0419	0.0415
	0.0049	0.0058	0.0058	0.0058	0.0058
AR(168)	0.0388	0.0562	0.0557	0.0554	0.0562
	0.0050	0.0056	0.0056	0.0056	0.0056
AR(216)	0.0397	0.0535	0.0527	0.0527	0.0533
	0.0051	0.0051	0.0051	0.0051	0.0051
AR(264)	0.0334	0.0373	0.0369	0.0368	0.0372
	0.0050	0.0052	0.0052	0.0052	0.0052
AR(336)	0.0268	0.0403	0.0400	0.0408	0.0402
	0.0050	0.0051	0.0051	0.0051	0.0051
AR(504)	0.0322	0.0553	0.0544	0.0537	0.0548
	0.0049	0.0046	0.0046	0.0046	0.0046
AR(672)	0.0786	0.0429	0.0426	0.0421	0.0428
	0.0055	0.0058	0.0058	0.0058	0.0058
AR(840)	0.0406	0.0525	0.0513	0.0519	0.0521
	0.0051	0.0055	0.0055	0.0055	0.0055
MA(1)	−0.3869	−0.4334	−0.4537	−0.4614	−0.4470
	0.0153	0.0141	0.0144	0.0136	0.0149
MA(2)	−0.2384	−0.3021	−0.3185	−0.3221	−0.3104
	0.0137	0.0103	0.0104	0.0096	0.0109
MA(3)	−0.2439	−0.2762	−0.2904	−0.2924	−0.2828
	0.0120	0.0079	0.0080	0.0075	0.0085
MA(4)	−0.2171	−0.2630	−0.2746	−0.2749	−0.2683
	0.0115	0.0073	0.0074	0.0068	0.0078
MA(5)	−0.1967	−0.2139	−0.2239	−0.2226	−0.2185
	0.0109	0.0072	0.0074	0.0068	0.0076
MA(6)	−0.1640	−0.1933	−0.2017	−0.2004	−0.1974
	0.0105	0.0067	0.0068	0.0064	0.0071
MA(7)	−0.1302	−0.1698	−0.1767	−0.1750	−0.1738
	0.0097	0.0066	0.0067	0.0064	0.0069
MA(8)	−0.1210	−0.1469	−0.1518	−0.1498	−0.1503
	0.0096	0.0065	0.0065	0.0062	0.0068
MA(9)	−0.1014	−0.1209	−0.1244	−0.1223	−0.1241
	0.0090	0.0061	0.0061	0.0060	0.0063
MA(10)	−0.0675	−0.0959	−0.0983	−0.0969	−0.0982
	0.0079	0.0062	0.0061	0.0061	0.0063
Gamma0	−0.2629	−0.1077	−0.1530	−0.1837	−0.1256
	0.0263	0.0105	0.0104	0.0094	0.0134

(Note: In each cell, the first and second rows are the estimated parameter value and the corresponding standard error, respectively.)

Table 6
Fitted variance equation parameters in ARMA-GARCH-M models.

	SGARCH	QGARCH	GJRGARCH	EGARCH	NGARCH
Constant	6.5617	8.2447	9.5593	−1.8706	11.3120
	0.4273	0.6722	0.9834	0.0051	0.6200
ARCH(1)	0.4596	0.4449	0.6460	0.4034	0.3585
	0.0213	0.0001	0.0001	0.0018	0.0001
GARCH(1)	0.6497	0.6430	0.6574	4.3583	0.6150
	0.0128	0.0001	0.0001	0.0029	0.0002
Phi	N/A	−3.3134	−0.4279	N/A	N/A
		0.0085	0.0006		
Theta	N/A	N/A	N/A	−0.3982	−0.4693
				0.0044	0.0003

(Note: In each cell, the first and second rows are the estimated parameter value and the corresponding standard error, respectively.)

Table 7
Modeling sufficiency evaluation of ARMA-GARCH models.

	SGARCH	QGARCH	GJRGARCH	EGARCH	NGARCH
Adjusted R^2	0.8309	0.8331	0.8335	0.8335	0.8331
Overall F	2875*	3245*	3252*	3252*	3245*
	(<0.0001)**	(<0.0001)**	(<0.0001)**	(<0.0001)**	(<0.0001)**
AIC	131,084	142,202	142,170	142,170	142,204
BIC (SBC)	131,317	142,412	142,380	142,380	142,414

(Note: * is the value of *F* statistics of each cell and ** is the *P*-value.)

EGARCH models are developed by Nelson (1991). Formally, an EGARCH model with orders of k and l, expressed as EGARCH(k, l), is defined as follows,

$$\log(v_t) = \varsigma_0 + \sum_{i=1}^{k} \varsigma_i \log(v_{t-i}) + \sum_{i=1}^{l} \eta_i g(z_{t-i}), \tag{6}$$

where $g(z_t) = \theta z_t + \gamma[|z_t| - E(|z_t|)]$. The value of γ is usually set to 1. The GARCH model in Eq. (5) has the non-negative constraints on parameters ς_i and η_i. However, in the EGARCH model, the restrictions on these parameters are not needed. Nelson and Cao (1992) argue that the non-negativity constraints in the conventional GARCH model, namely Eq. (5), are too restrictive. In EGARCH, the standardized variable z_t follows a standard normal distribution with $E(|z_t|) = \sqrt{\frac{2}{\pi}}$ (Johnson and Kotz, 1970). The EGARCH(1,1) model thus becomes,

$$\log(v_t) = \varsigma_0 + \varsigma_1 \log(v_{t-1}) + \eta_1 \theta z_{t-1} + \eta_1 \gamma[|z_{t-1}| - \sqrt{\frac{2}{\pi}}]. \tag{7}$$

The parameter $\eta_1\theta$ captures the leverage effect. For "good news" ($z_{t-1} > 0$), the impact of the innovation ε_t is $(\eta_1\theta + \eta_1\gamma)z_{t-1}$; for "bad news" ($z_{t-1} < 0$), it is $(\eta_1\theta - \eta_1\gamma)z_{t-1}$. If $\eta_1\theta = 0$, $\log(v_t)$ symmetrically responds to z_{t-1}.

It is often observed that downward movements of prices in the equity market are followed by higher volatilities than upward movements of the same magnitude (Nelson, 1991). This phenomenon is called the leverage effect. In the electricity market, electricity prices have the opposite effect known as the "inverse leverage effect". The "inverse leverage effect" asserts that electricity price volatility tends to arise more with positive shocks than negative shocks. To produce an inverse leverage effect, $\eta_1\theta$ needs to be positive (Bowden and Payne, 2008). These positive shocks represent an unexpected positive electricity demand shock. In the short run, as demand increases, additional generators with higher marginal costs are used to meet the increased demand which induces the high price volatility. When $\eta_1\theta$ is negative, a leverage effect will appear (Schmitt, 1996).

NGARCH is developed by Engle and Ng (1993), and it has the following mathematical form,

$$v_t = \varsigma_0 + \varsigma_1 v_{t-1} + \eta_1 (\varepsilon_{t-1} - \theta\sqrt{v_{t-1}})^2, \tag{8}$$

where ς_0, $\varsigma_1 \geq 0$ and $\eta_1 > 0$. θ reflects the leverage effect for future volatility. NGARCH is known as a nonlinear asymmetric GARCH model.

QGARCH model proposed by Sentana (1995) can be applied to model asymmetric effects of positive and negative shocks. The most common form of QGARCH model is in the form of,

$$v_t = \varsigma_0 + \varsigma_1 v_{t-1} + \eta_1 \varepsilon_{t-1}^2 + \phi \varepsilon_{t-1}, \tag{9}$$

where φ is a constant.

GJRGARCH model, which is developed by Glosten et al. (1993), is similar to QGARCH. It is also used to model the asymmetry in the GARCH process. The mathematical form of GJRGARCH model is as follows,

$$v_t = \varsigma_0 + \varsigma_1 v_{t-1} + \eta_1 \varepsilon_{t-1}^2 + \phi \varepsilon_{t-1}^2 I_{t-1}, \tag{10}$$

where $I_{t-1} = 0$ if $\varepsilon_{t-1} \geq 0$, and $I_{t-1} = 1$ if $\varepsilon_{t-1} < 0$.

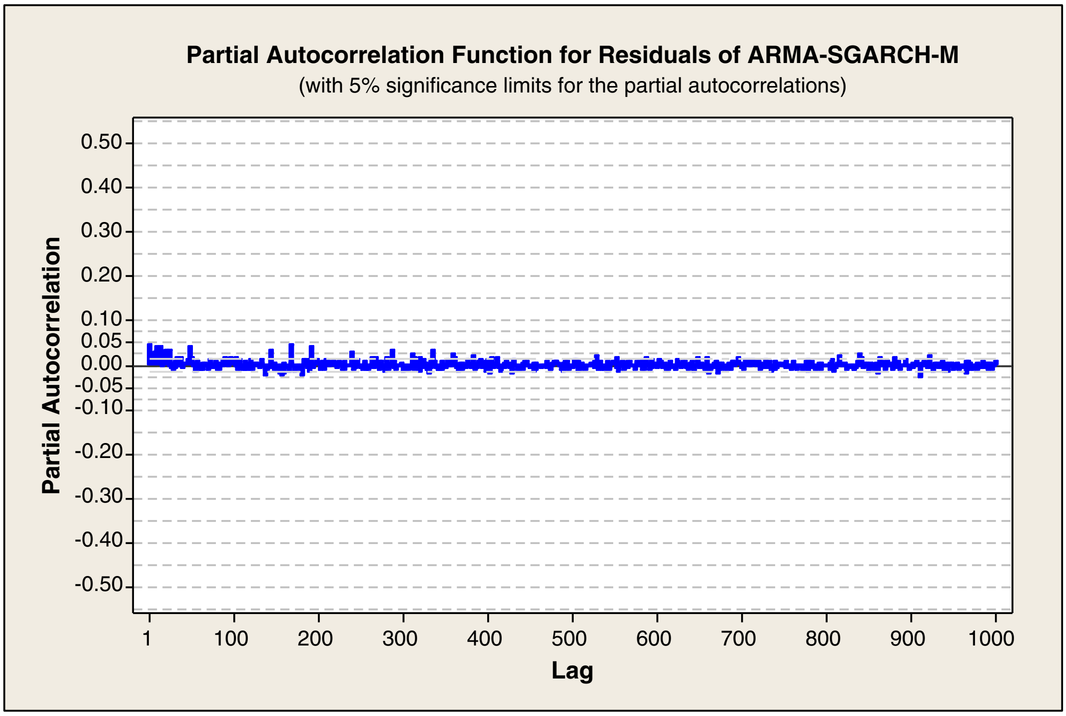

Fig. 7. PACF chart of residuals from ARMA–SGARCH-M model.

Using the GARCH-M model framework, we further explore the impact of the conditional volatility of electricity prices on the mean of electricity prices in this paper. A GARCH-M model adds a heteroskedasticity term into the mean equation to show the influence of volatility on mean prediction. The GARCH model could be in any form such as NGARCH or EGARCH. For example, for an ARMA–QGARCH(1,1)-M model, the specified mathematical form is,

$$y_t = \delta + \sum_{i=1}^{p} \phi_i y_{t-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} + \gamma_0 \sqrt{v_t} + \varepsilon_t, \tag{11}$$

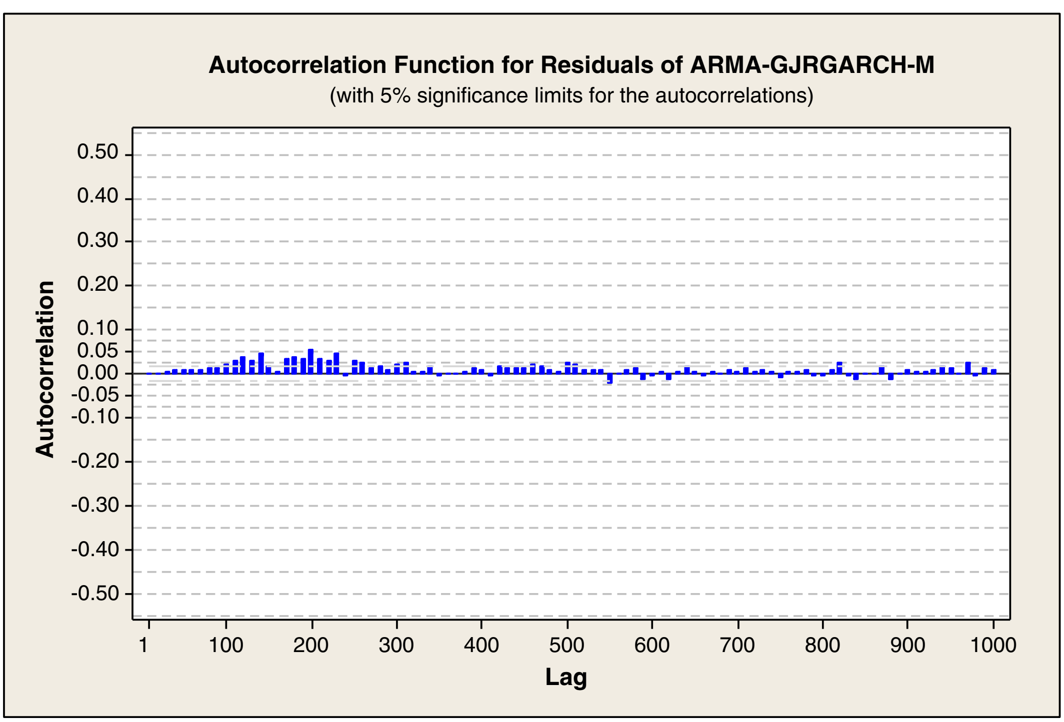

Fig. 8. PACF chart of residuals from ARMA–GJRGARCH-M model.

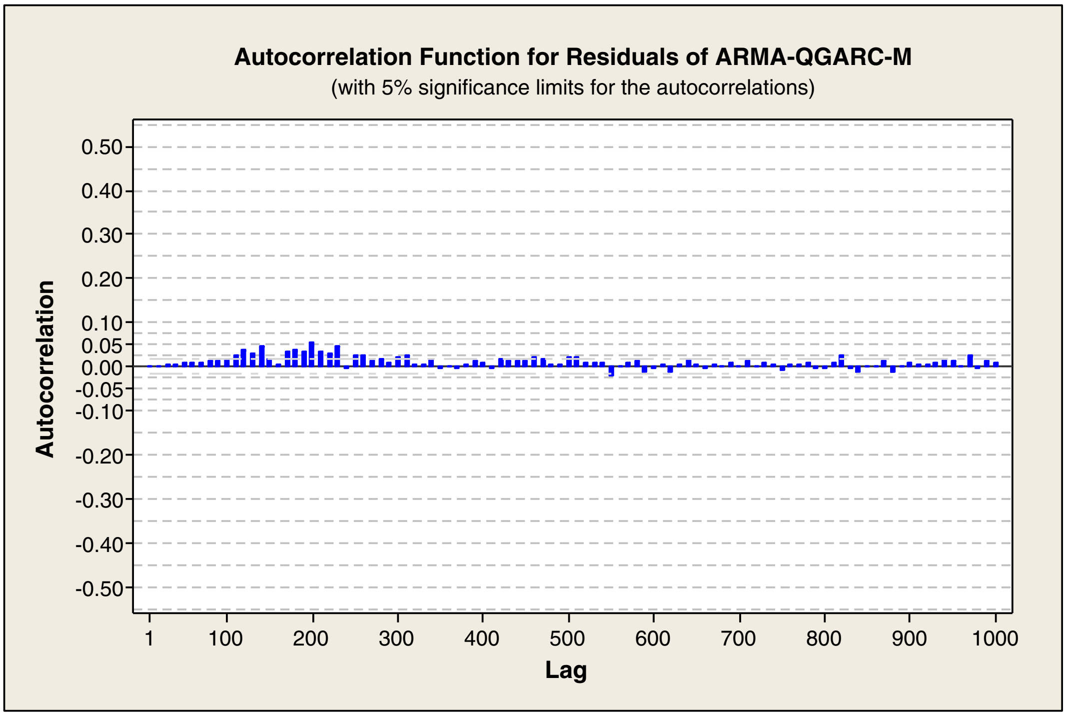

Fig. 9. PACF chart of residuals from ARMA–QGARCH-M model.

$$v_t = \varsigma_0 + \varsigma_1 v_{t-1} + \eta_1 \varepsilon_{t-1}^2 + \phi \varepsilon_{t-1}, \tag{12}$$

where γ_0 is a coefficient.

4. Evaluation methods of model sufficiency and prediction accuracy

4.1. Evaluation methods of model sufficiency

The evaluation methods of model sufficiency can give critical guidance to the appropriate choice of models. These methods can be divided into two categories. One is used to evaluate the goodness of fit of fitted models and it includes (adjusted) R^2, *F*-test, Akaike's Information Criterion (AIC), and Schwarz's Bayesian Information Criterion (SC, BIC or SBC). Another category is the diagnostic checking of fitted models. Some methods can be used to check fitted models. In this work, ACF and PACF plots are used to diagnose whether there exist autocorrelation and partial autocorrelation among residuals.

For nonlinear ARIMA–GARCH modeling, AIC suggested by Akaike (1973) and BIC also called Schwarz criterion (SBC) (Schwarz, 1978) are extensively adopted to guide the choice of alternative models (Verbeek, 2004). AIC and BIC use a likelihood function to select the best fitted model. The two criteria represent a trade-off between 'fit' measured by the log likelihood value and 'parsimony' as measured by the number of free parameters. The models with smaller AIC or BIC values are usually preferred. The main difference between AIC and BIC is that BIC places more emphasis on the number of free parameters. While the two criteria differ in their trade-off between fit and parsimony, the BIC criterion can be preferred (Verbeek, 2004) and the AIC criterion tends to asymptotically result in overparameterized models (Hannan, 1980).

4.2. Measurement of prediction performance

Four measurement statistics are used to examine the prediction accuracy of different models, and they are root mean squared error (RMSE), mean absolute error (MAE), mean absolute percentage error (MAPE), and Theil's inequality coefficient (TIC). In general form, RMSE, MAE, MAPE, and TIC are computed as follows,

$$MAE = \frac{1}{n}\sum_{i=1}^{n}\left|y_{pi} - y_{ai}\right|, \tag{13}$$

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{y_{pi} - y_{ai}}{y_{ai}}\right|, \tag{14}$$

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(y_{pi} - y_{ai}\right)^2}, \tag{15}$$

$$TIC = \frac{\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(y_{pi} - y_{ai}\right)^2}}{\sqrt{\frac{1}{n}\sum_{i=1}^{n}{y_{pi}}^2} + \sqrt{\frac{1}{n}\sum_{i=1}^{n}{y_{ai}}^2}}, \tag{16}$$

where n is the number of observations in a testing dataset, and y_{ai} and y_{pi} represent the actual and predicted values, respectively. Both RMSE and MAE statistics depend on the scale of variable, and they should only be used to compare the forecasts of the same variables across different models. In such cases, the smaller is the error, the better is the forecasting performance. MAPE and TIC are insensitive to the scale of variable. The smaller is the MAPE and TIC, the better is the forecast. Normally, TIC yields a range of values between zero and one with zero indicating a perfect fit of the forecasted values to the actual.

5. Empirical experiences

5.1. Data description

The data used in this study is hourly real time location-based marginal prices (LMP) in ISO New England market from January 1,

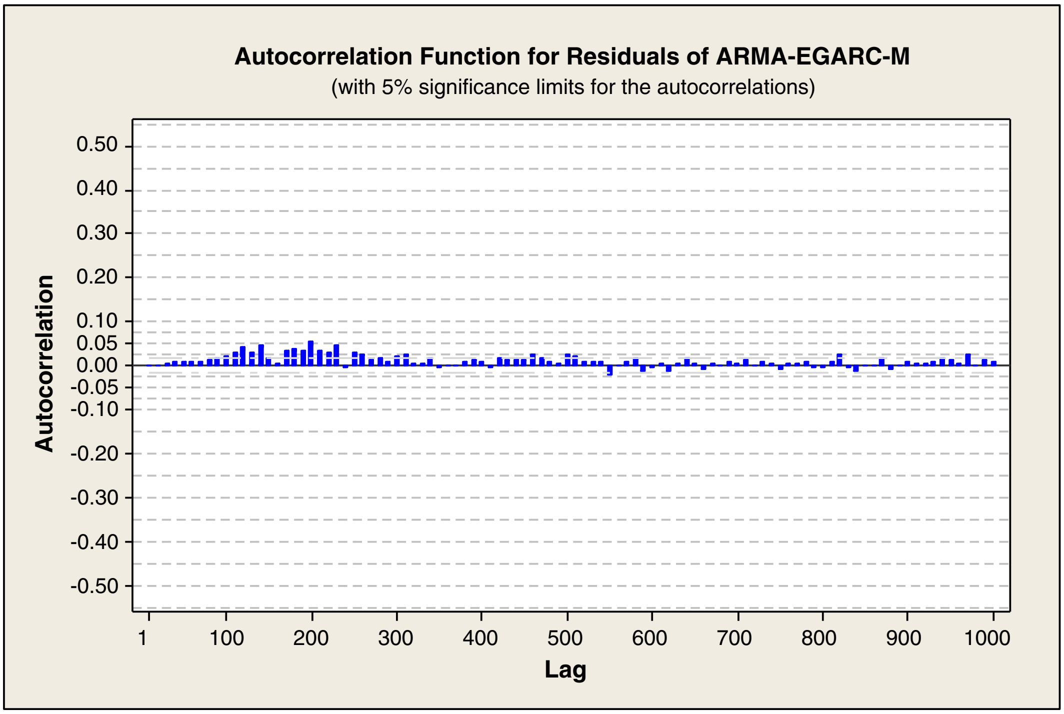

Fig. 10. PACF chart of residuals from ARMA–EGARCH-M model.

2008 to February 28, 2010 for a total of 18,960 observations. ISO New England manages a few wholesale electric power markets in the United States that allow generators to sell their electricity to marketers who then sell the electricity to end users such as businesses and households. LMP implies the cost of supplying the next increment of electricity to a specified grid at a given pricing node (Bowden and Payne, 2008). For the investigated data, the first two years of data (17,544 observations) are used to fit different ARMA–GARCH(-M) models, and the remaining (1416 observations) are used to test the predication accuracy of the fitted models. Fig. 1 shows the change trend of the chosen hourly real time electricity prices. Table 1 reports their summary statistics. According to Table 1, it can be seen that the electricity prices range from 0 to \$403.23/MWh with the mean and standard deviation being 61.09 and 33.4735, respectively. It should be

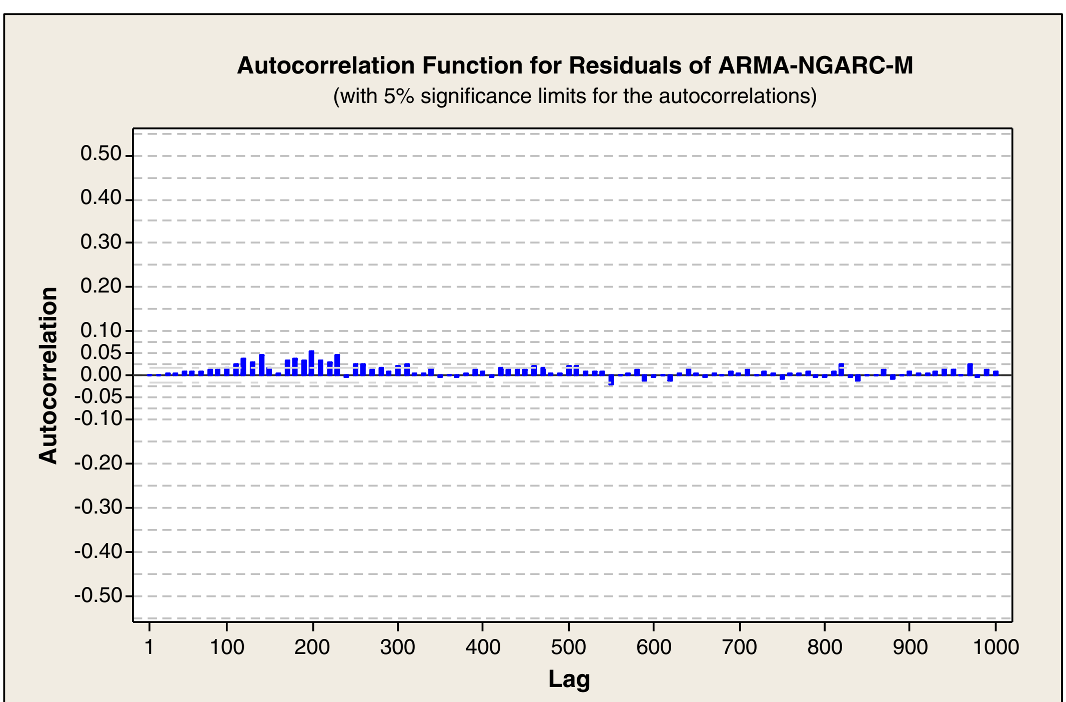

Fig. 11. PACF chart of residuals from ARMA–NGARCH-M model.

Table 8
Modeling sufficiency evaluation of ARMA-GARCH models.

	RMSE	MAE	MAPE	TIC
ARMA-SGARCH	7.5462	0.1251	14.6838	0.1172
ARMA-QGARCH	7.4579	0.1230	14.7201	0.1180
ARMA-GJRGARCH	7.4238	0.1225	14.6868	0.1177
ARMA-EGARCH	7.4677	0.1237	14.7260	0.1178
ARMA-NGARCH	7.4710	0.1232	14.7612	0.1182
ARMA-SGARCH-M	7.4388	0.1220	14.4774	0.1165
ARMA-QGARCH-M	7.5655	0.1274	14.3942	0.1150
ARMA-GJRGARCH-M	7.5258	0.1265	14.3506	0.1148
ARMA-EGARCH-M	7.6427	0.1298	14.5258	0.1154
ARMA-NGARCH-M	7.5546	0.1271	14.4015	0.1151

noted that the electricity prices in our data are all positive, but in general it is possible that some electricity prices could be negative due to the high cost and the low profit during a certain time period. Moreover, Table 1 shows that the electricity prices are positively skewed and leptokurtic, and thus non-normally distributed. In Fig. 1, it is also observed that the electricity prices manifest the high frequency and extreme values. In the literature review, we find that these features of electricity prices in the electricity market of New England are similar to those of other electricity markets (Bowden and Payne, 2008). Although in this paper we discuss only the electricity prices from one market, the obtained results, to some extent, still have the general guidance and meanings for other electricity markets.

5.2. Model establishment

5.2.1. Specification of time series mean model structure

The SAS® version 9.1 is used to fit ARMA–GARCH and ARMA–GARCH-M models. We use the procedure Proc ARIMA and Proc Model in the SAS software to build an ARIMA–GARCH model. For the residuals from the built model, ACF and PACF figures are drawn. After a thorough analysis, the ARMA part of the model is refined to the following form.

$$\begin{aligned}&(1-\phi_1 B-\phi_{11}B^{11}-\phi_{24}B^{24}-\phi_{48}B^{48}-\phi_{72}B^{72}-\phi_{96}B^{96}-\phi_{120}B^{120}-\phi_{144}B^{144}\\&-\phi_{168}B^{168}-\phi_{216}B^{216}-\phi_{336}B^{336}-\phi_{504}B^{504}-\phi_{672}B^{672}-\phi_{840}B^{840})(y_t-\mu)\\&=\delta+\left(1+\theta_1B^1+\theta_2B^2+\theta_3B^3+\theta_4B^4+\theta_5B^5+\theta_6B^6+\theta_7B^7+\theta_8B^8+\theta_9B^9+\theta_{10}B^{10}\right)\varepsilon_t\end{aligned} \tag{17}$$

In Eq. (17), the lagged terms $\phi_{24}B^{24}$, $\phi_{48}B^{48}$, $\phi_{72}B^{72}$, $\phi_{96}B^{96}$, $\phi_{120}B^{120}$, $\phi_{144}B^{144}$, and $\phi_{216}B^{216}$ represent the time lags of 1, 2, 3, 4, 5, 6 and 9 days of electricity prices, and in this paper we categorize them to the daily periodicities of electricity prices; the lagged terms $\phi_{168}B^{168}$, $\phi_{336}B^{336}$, and $\phi_{504}B^{504}$ imply the time lags of 1, 2 and 3 weeks, and we regard them as the weekly periodicities of electricity prices; the lagged terms $\phi_{672}B^{672}$ and $\phi_{840}B^{840}$ imply the time lags of 4 and 5 weeks, and we treat them as the monthly periodicities of electricity prices.

5.2.2. Parameter estimation of ARMA–GARCH models

Tables 2 and 3 summarize the estimated parameters of the 5 ARMA–GARCH models. In Tables 2 and 3, for each parameter, the first row shows the estimated value and the second row presents the standard error of

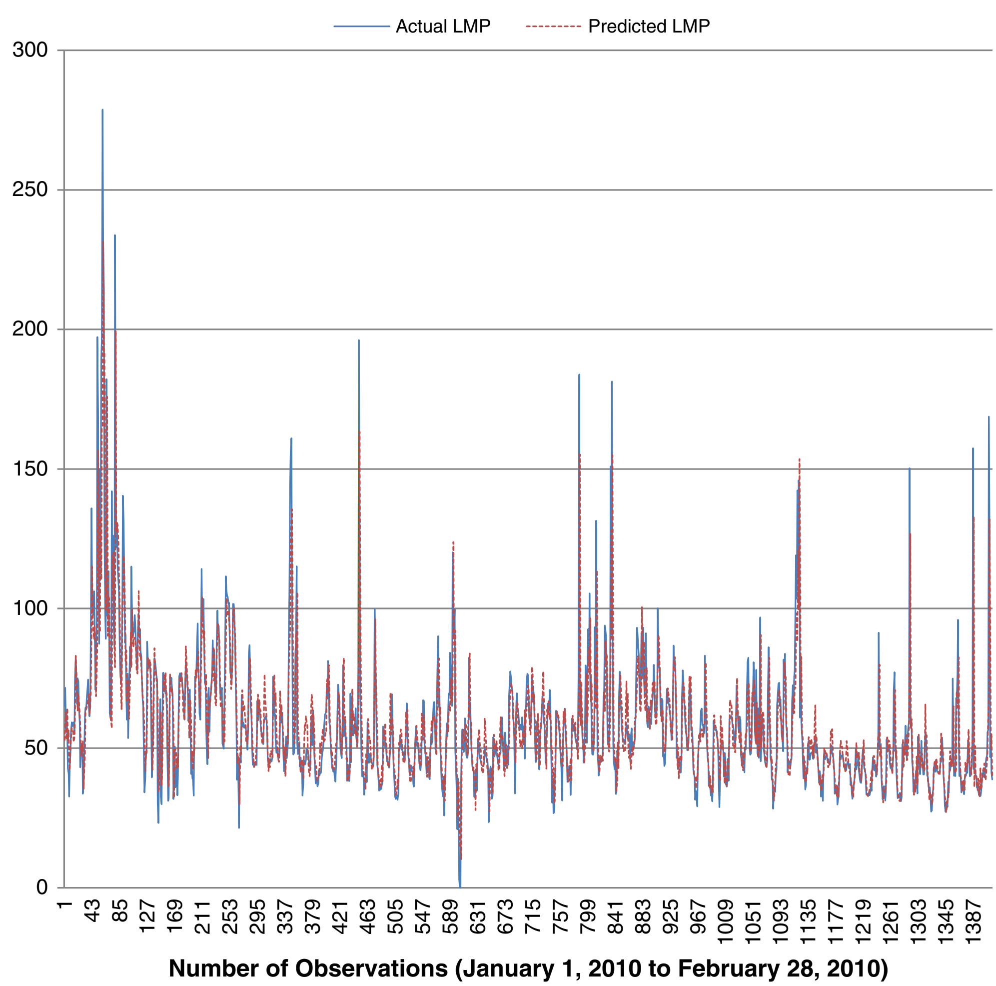

Fig. 12. Diagrammatic trend of actual and predicted LMP for ARMA–SGARCH model.

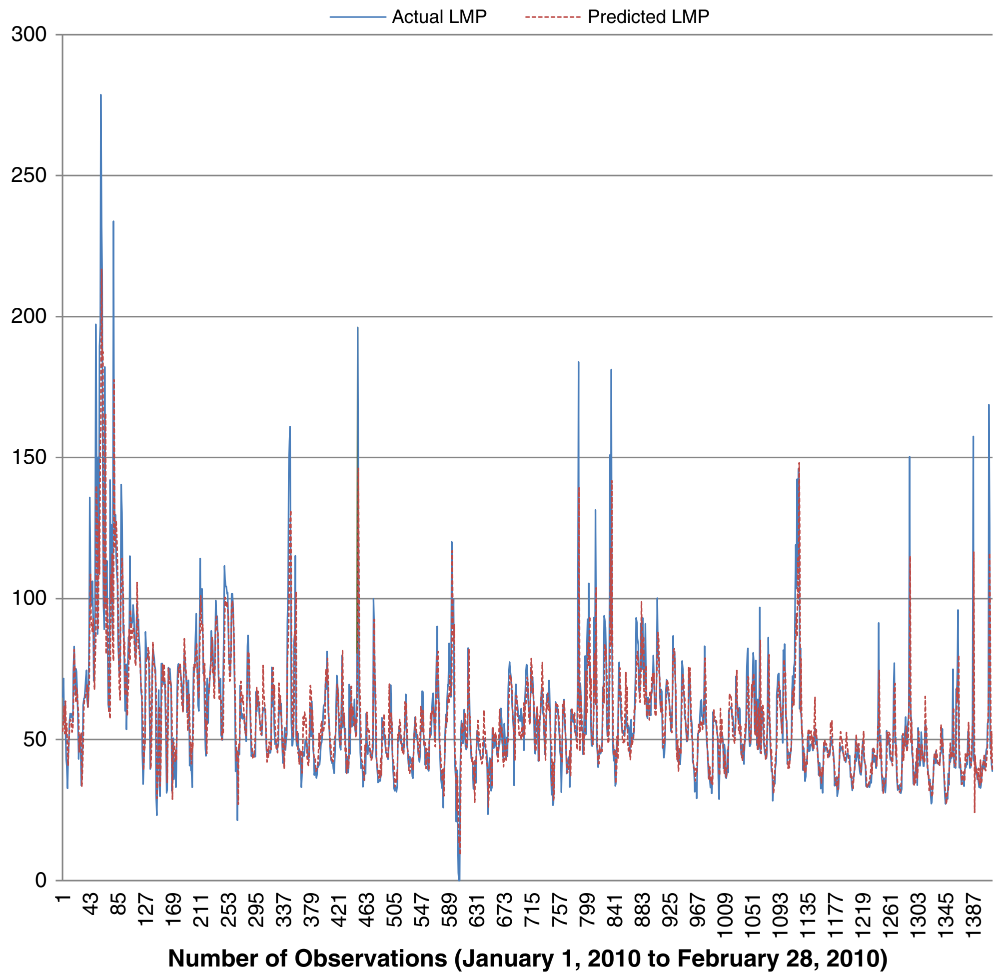

Fig. 13. Diagrammatic trend of actual and predicted LMP for ARMA–SGARCH-M model.

this estimate. In Table 2, Constant, AR(i) and MA(j) represent μ, the coefficient of the ith autoregressive term, and the coefficient of the jth moving average term, respectively. For instance, in the fitted ARMA–QGARCH model, 0.2968 is the estimated value of parameter AR(1), namely ϕ_1 in Eq. (17), and 0.0134 is the standard error of this estimated parameter. According to the t-test, the estimated parameters in these two tables are all highly significant at the level of 0.1%. The high significance of estimated parameters in the ARMA component adequately discloses that electricity prices have the apparent daily, weekly and monthly periodicities. In Table 3, Constant, ARCH(1), and GARCH(1) denote ς_0, η_1, and ς_1; phi and theta mean ϕ and θ, respectively. The high significance of estimated parameters in the GARCH component shows that the volatilities of hourly electricity prices are time-varying. The results from NGARCH, QGARCH, EGARCH and GJRGARCH models indicate that the volatilities of electricity prices have the nonlinear and asymmetric property and there exists an inverse leverage effect. For example, for the ARMA–EGARCH model, the estimated values of parameters θ and η_1 are positive. The findings about the nonlinear and asymmetric property and the inverse leverage effect are consistent with those of other references (Bowden and Payne, 2008; Knittel and Roberts, 2005).

We use the methods of rigorous statistical hypothesis testing introduced in Section 4 to evaluate the fitted models. The obtained results are in Table 4 and Figs. 2–6. Table 4 includes the values of adjusted R^2, F-test, AIC, and BIC. Figs. 2–6 show the PACF plots of residuals. In this work, both ACF and PACF are adopted to analyze the autocorrelation of residuals, and the results from ACF and PACF are found to be consistent. For the purpose of brevity, we only display the PACF figures in the paper since they can more accurately tell which lags are correlated with the current observation. In Table 4, the values of adjusted R^2 reveal that the electricity prices are well fitted by ARMA–GARCH models. Although the ARMA–SGARCH model has the highest adjusted R^2 value, there are no apparent differences among the adjusted R^2 values of the 5 fitted ARMA–GARCH models and they are all located in the interval from 0.82 to 0.83. The *p-values* of F-statistic are in the parenthesis and these *p-values* are less than 0.0001, confirming that the fitted 5 GARCH models are highly significant.

As for AIC and BIC values, it can be noted that the values from ARMA–QGARCH are the highest, and the values from ARMA–QGARCH, ARMA–GJRGARCH, ARMA–EGARCH, and ARMA–NGARCH models are close to the ones from ARMA–NGARCH model, the smallest ones. Normally, a smaller AIC or BIC value implies a better fitted model. The results from adjusted R^2, AIC and BIC are slightly mixed. The values of adjusted R^2 demonstrate that ARMA–SGARCH model is the best fitted one, but AIC and BIC values show that ARMA–SGARCH model is the worst one. Note that AIC and BIC are more effective measurement methods in time series modeling and we therefore believe that according to AIC and BIC values, ARMA models with nonlinear and asymmetric GARCH processes have more potentials to model electricity prices.

5.2.3. Parameter estimation of ARMA–GARCH-M models

We next explore the impact of the conditional volatility of electricity prices on the mean of respective hourly electricity prices using the 5 different ARIMA–GARCH-M models. However, while simultaneously

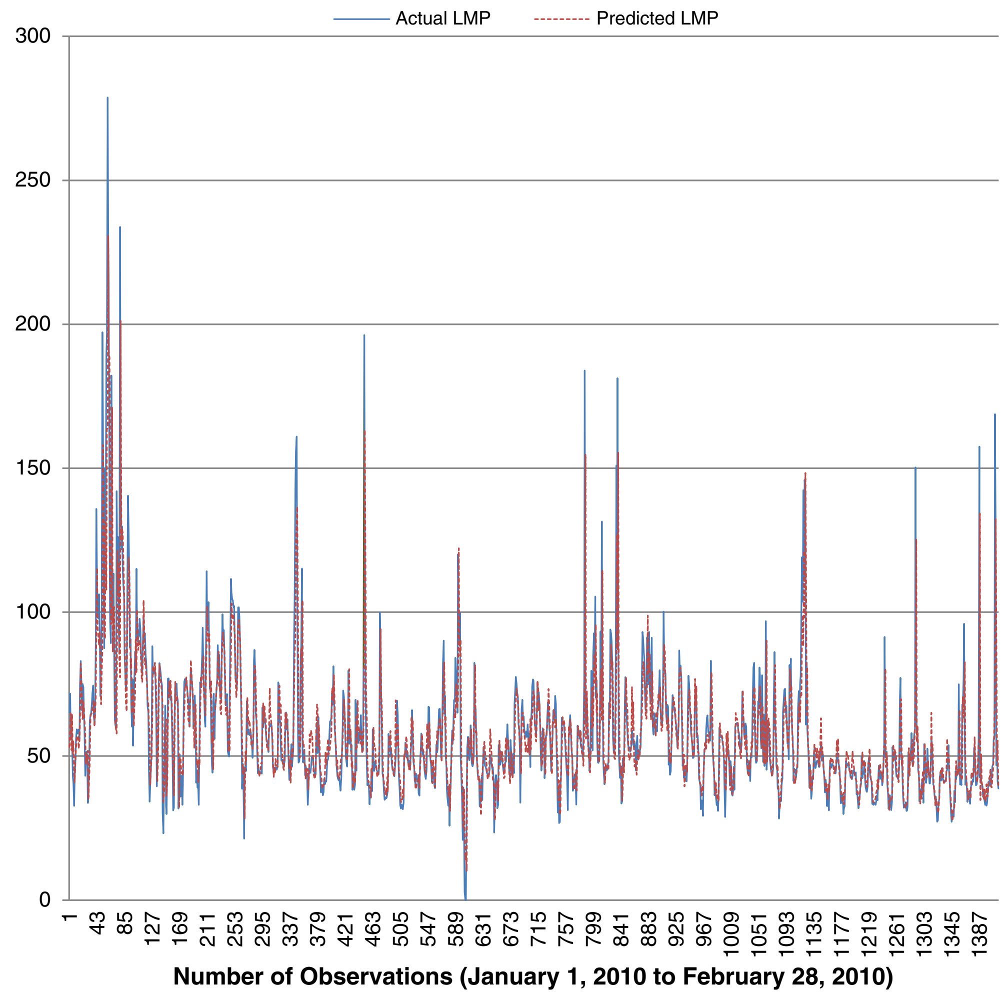

Fig. 14. Diagrammatic trend of actual and predicted LMP for ARMA–GJRGARCH model.

modeling the ARMA–QGARCH-M, ARMA–GJRGARCH-M, ARMA–EGARCH-M and ARMA–NGARCH-M models, Proc Model does not converge appropriately and leads to a bias problem. To avoid such a problem, we adopt an approximate method. We first use Proc Model to build ARMA–GARCH models and then manually add the term of variance square root to the ARMA equation. After that, we use Proc Model to separately estimate the parameters again for the ARMA equation with the term of variance square root and the GARCH equation. Although this is an approximate method, it can help us to fulfill the comprehensive comparison and generate some insights into these models.

The results of parameter estimation are reported in Tables 5 and 6. In Table 5, Gamma0 represents γ_0 in Eq. (11). In both tables, for each parameter, similarly, the first row shows the estimated values and the second row provides the standard error values of these estimates. Although the results of *t*-test for the estimated parameters are not shown in these tables, their respective *p-values* are all less than 0.1% and thus approve the high significance of these parameter estimates. Similar to the 5 ARMA–GARCH models, the high significance of estimated parameters from the 5 ARMA–GARCH-M models shows that electricity prices have the apparent daily, weekly and monthly periodicities and the nonlinear and asymmetric time-varying volatilities. Nevertheless, the ARMA–GARCH-M models have one more term which indicates the influence of volatilities of electricity prices on mean electricity prices. The estimated coefficient values of this term range from -0.2629 to -0.1077, and they are highly significant. This phenomenon demonstrates that the volatilities of electricity prices can negatively influence mean electricity prices.

Table 7 reports the evaluation results of modeling sufficiency. In this table, we can find that the values of adjusted R^2 are all greater than 0.83. Among the 5 fitted models, the ARMA–GJRGARCH-M and ARMA–EGARCH-M models have the highest adjusted R^2 values which imply that these two models may be the best fitted models. However, we also notice that the adjusted R^2 values of all the 5 models are very close and there are no significant differences among them. In addition, the values of adjusted R^2 from the 5 ARMA–GARCH-M models are all greater than the values of adjusted R^2 from the 5 ARMA–GARCH models. To some extent, it discloses that the electricity prices are better fitted by ARMA–GARCH-M models. The *p-values* of *F*-statistic in Table 7 are placed in the parenthesis, and they are all less than 0.0001. This confirms the high significance of the 5 fitted ARMA–GARCH-M models.

The AIC and BIC values from the ARMA–GARCH-M models are very close except those from the ARMA–SGARCH-M model which are the smallest ones. What we need to emphasize is that the ARMA–SGARCH-M model is simultaneously modeled and the rest of the 4 models are approximately modeled, which may negatively impact their modeling effectiveness and forecasting accuracy. Figs. 7–11 depict the PACF plots of residuals from the 5 ARMA–GARCH-M models which disclose that there do not exist significant autocorrelations among the residuals of these models.

In Tables 3 and 6, the obtained coefficients of GARCH models show that the shocks to conditional variance are persistent, in the sense that they remain important for the forecasts of all horizons. This phenomenon indicates the presence of an approximate unit root in the autoregressive polynomial for the conditional second order

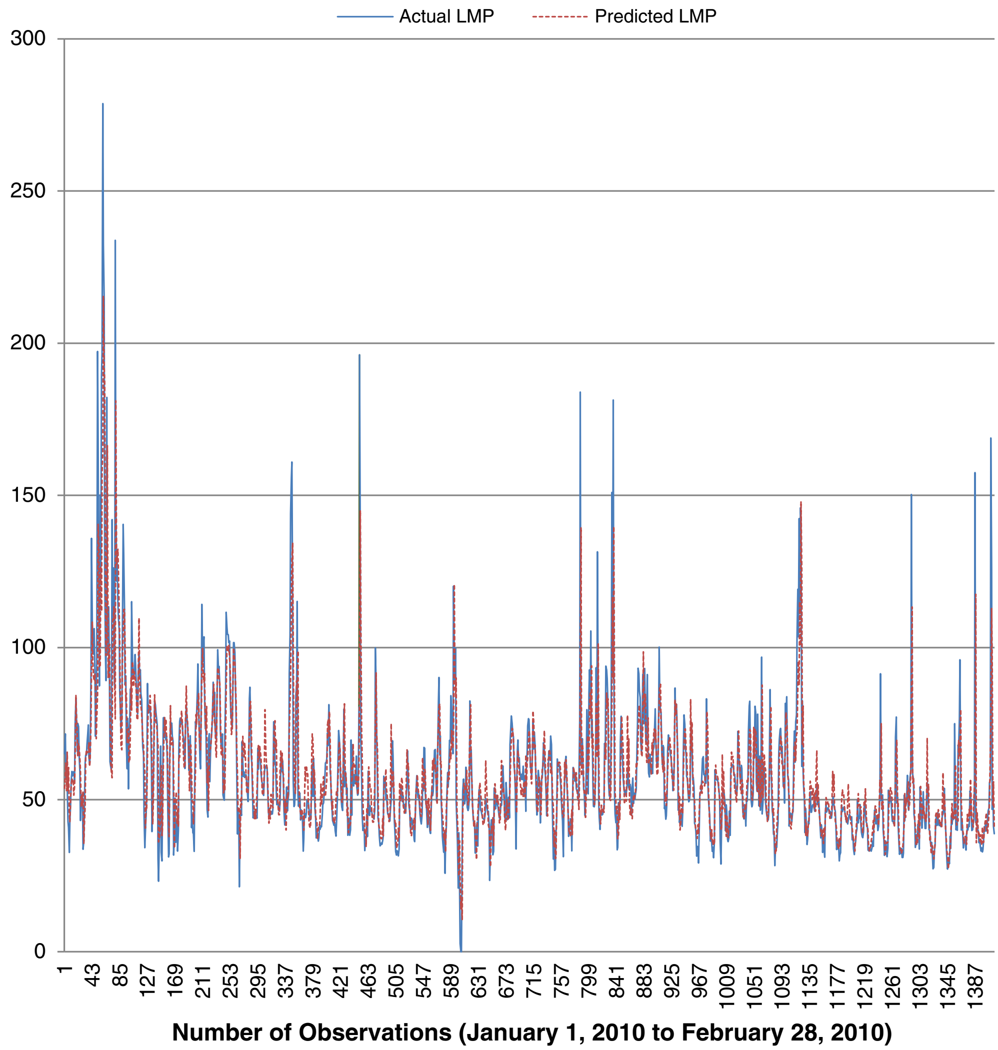

Fig. 15. Diagrammatic trends of actual and predicted LMP for ARMA–GJRGARCH-M model.

moments of electricity prices, as in the so-called integrated GARCH (IGARCH) class of models proposed in Engle and Bollerslev (1986).

5.3. Comparison and analysis of prediction results

The 10 ARMA–GARCH(-M) models reported above are estimated using the hourly real time electricity prices over the period of January 1, 2008 to December 31, 2009. To investigate the out-of-sample prediction accuracy, we conduct one-step-ahead forecasts for the electricity prices from January 1, 2010 to February 28, 2010. The four evaluation statistics of forecasting accuracy from Section 4.2 are used to examine the prediction accuracy of different models. Table 8 displays the results.

RMSE and MAE statistics are both based on absolute values, and MAPE and TIC statistics however are based on relative values. Therefore, it is not surprising to see that the results of RMSE and MAE are consistent, and those of MAPE and TIC are normally similar. For all the 4 measuring indices adopted, the smaller are the values, the more accurate are the forecasts. In Table 8, for the 5 ARMA–GARCH models, the ARMA–SGARCH model has the smallest MAPE and TIC values which are 14.6838 and 0.1172, respectively, and the ARMA–GJRGARCH model has the smallest RMSE and MAE values which are 7.4388 and 0.1220, respectively. Thus, the ARMA–SGARCH and ARMA–GJRGARCH models have the best forecasting performance among the 5 ARMA–GARCH models. As for the 5 ARMA–GARCH-M models, the ARMA–SGARCH-M model has the smallest RMSE and MAE values which are 7.4238 and 0.1220, respectively, and the ARMA–GJRGARCH-M model has the smallest MAPE and TIC values which are 14.3506 and 0.1148, respectively. Thus the ARMA–SGARCH-M and ARMA–GJRGARCH-M models are the two best fitted models among the 5 ARMA–GARCH-M models. As far as all the 10 fitted ARMA–GARCH(-M) models are concerned, the ARMA–GJRGARCH has the smallest RMSE value (i.e., 7.4238), the ARMA–SGARCH-M model has the smallest MAE value (i.e., 0.1220), and ARMA–GJRGARCH-M model has the smallest MAPE and TIC values equal to 14.3506 and 0.1148, respectively.

For the 10 models, the values for each performance index are however close and there are no significant differences. The maximum differences (between the highest and lowest index values) are around 3% in terms of RMSE, MAPE, and TIC, and around 7% in terms of MAE. As such, if we balance the predication accuracy with the complexity of model construction, the ARMA–SGARCH-M model appears to be the one that is preferred. This is because the model structure has the advantage of simplicity and robustness, and it generates overall good prediction accuracy – it has the lowest MAE value and relatively good performances in terms of RMSE, MAPE, and TIC as well. Certainly, the ARMA–GJRGARCH-M model is also a good contender because it produces the lowest MAPE and TIC values among this group of 10 models.

For the 4 models mentioned above, we provide 4 charts, namely Figs. 12–15, to illustrate the forecasting effects of these models. By inspecting these 4 figures, we see that although the actual electricity prices are volatile, the change trend of forecasting values accurately follows that of the actual price data.

6. Conclusions

In a deregulated electricity market, hourly ahead price forecasting is critical for market participants. An accurate hourly ahead price forecasting can help power suppliers to adjust their bidding strategies to achieve the maximal revenue, and meanwhile consumers can derive a plan to minimize their cost and to protect themselves against high prices. In the future, electricity markets will be further deregulated and thus become fully privatized, and the private trading of electricity along with its future contracts and options will be intensified. Therefore, the empirical time series modeling and forecasting of electricity prices will be more important. Due to some complex factors, accurately forecasting electricity prices however is difficult. This paper makes its unique contribution in this aspect by examining the modeling capabilities and forecasting performances of the 10 ARMA–GARCH(-M) approaches for hourly electricity prices in the electricity market of ISO New England. Among the 10 GARCH model structures, the NGARCH(-M), QGARCH(-M) and GJRGARCH(-M) approaches are for the first time adopted to model hourly electricity prices, to the best of our knowledge.

By empirically modeling electricity prices from the New England electricity market, we note that in general electricity prices exhibit time-varying volatility and apparent daily, weekly and monthly periodicities, and the ARMA–GARCH(-M) models are effective for modeling the mean of electricity prices and the volatility which has the nonlinear and asymmetric property together with an inverse leverage effect. The ARMA–GARCH-M models are more suitable than the ARMA–GARCH models. In particular, the ARMA–SGARCH-M model almost outperforms all the 5 ARMA–GARCH models, and the ARMA–GJRGARCH-M has the smallest values of MAPE and TIC plus the highest value of adjusted R^2 among the 10 fitted models. Certainly, our results are based on only an electricity market and it is necessary to conduct more empirical analysis on other electricity markets in the future. Additionally, electricity demand depends on some factors such as weather variables and holidays, and we may investigate the possibility to include them as exogenous variables in the future work.

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