

## Algebra Prelim Syllabus Topics:

### Notes:

I tried to organize the old prelim questions in their specific topics but it was challenging to do that without trying to solve them outright.

The starred questions are in more than one subtopic of the big topics.

The questions that didn't have a specific subtopic went into the basic definitions and examples sections.

### Saved Prelims for Practice: August 2008, January 2016, August 2018

#### Group Theory:

Basic definitions and examples: J20Q1, A19Q2, J19Q1, J18Q2, J17Q1, A16Q2, J15Q1, A13Q2, J12Q2, A11Q1&, J10Q1, A09Q1\*, J14Q1

Lattice of subgroups/normal subgroups: A17Q1\*, A14Q1, A13Q1&, J11Q1, A10Q1

Quotient groups: A17Q1\*, A10Q2

Isomorphism theorems

Characterization of products (*I treated this kind of as the characterization of groups*): A16Q1, J16Q2, J14Q2, J07Q2

Lagrange's Theorem

Cauchy's Theorem

Cayley's Theorem

Structure of finitely generated abelian groups

Group actions: A20Q1, A17Q2, J16Q1, J10Q1, A09Q2&, J09Q1\*, A08Q2\*

Class equation: A08Q2\*

Sylow's Theorems: A20Q2, J20Q2\*, A18Q1, J18Q1, J17Q2\*, J15Q2\*, J13Q2(a), J12Q1, A11Q2\*, A09Q1(a)(c)&, J08Q1, J07Q1

Jordan-Hölder Theorem

Simple groups: A19Q1, J19Q2, A12Q1, J09Q1\*, J09Q2, A08Q1

Solvable groups: J17Q2\*&, J08Q2&, A12Q2

Semidirect products: J20Q2\*, A18Q5, J15Q2\*, J13Q1, A11Q2\*&, A14Q2, J11Q2

Free groups

Presentations of groups

Nilpotent (I added): J13Q2(b), A09Q1(b)

**ALL GROUP THEORY QUESTIONS (UNSORTED):** A20Q1, A20Q2, J20Q1, J20Q2, A19Q1, A19Q2, J19Q1, J19Q2, A18Q1, A18Q5, J18Q1, J18Q2, A17Q1, A17Q2, J17Q1, J17Q2, A16Q1, A16Q2, J16Q1, J16Q2, J15Q1, J15Q2, A14Q1, A14Q2, J14Q1, J14Q2, A13Q1, J13Q1, J13Q2, A12Q1, A12Q2, J12Q1, J12Q2, A11Q1, A11Q2, J11Q1, J11Q2, A10Q1, A10Q2, J10Q1, J10Q2, A09Q1, A09Q2, J09Q1, J09Q2, A08Q1, A08Q2, J08Q1, J08Q2, J07Q1, J07Q2

#### Ring Theory:

Basic definitions and examples: A20Q3, J19Q3, **A18Q2**, **A18Q4**, A16Q3, A13Q3, A10Q3

Lattice of subrings/ideals: A14Q3

Prime ideals: A12Q3, J11Q3, J10Q3, A09Q3, J09Q3

Quotient rings

Chain conditions

Rings of fractions: J15Q3\*

Chinese Remainder Theorem

Euclidean domains: J18Q3, J15Q3\*, **A08Q3\***, J20Q3

PID's: J17Q3, J14Q3, **A08Q3\***

UFD's: A19Q3, **J16Q3**, J13Q3, A11Q3, J07Q3, J12Q3

Polynomial rings

Irreducibility criteria for polynomials

**ALL RING THEORY QUESTIONS (UNSORTED):** A20Q3, J20Q3, A19Q3, J19Q3, **A18Q2**, **A18Q4 (?)**, J18Q3, J17Q3, A16Q3, **J16Q3**, J15Q3, A14Q3, J14Q3, A13Q3, J13Q3, A12Q3, J12Q3, A11Q3, J11Q3, A10Q3, J10Q3, A09Q3, J09Q3, **A08Q3**, J08Q3, J07Q3

### Modules and Linear Algebra:

Basic definitions and examples: J19Q4, **A18Q6**, J18Q4, A16Q4, A13Q4, A12Q4, A11Q4, **A10Q4**, **A09Q4**, **J09Q4**, **A08Q4**, [J07Q4 (not in our syllabus anymore)]

Lattice of submodules: J14Q4

Quotient modules

The matrix of a linear transformation

Minimal polynomial of a transformation: A19Q4\*

Cayley-Hamilton Theorem over a commutative ring

Trace and determinant: J17Q4, J08Q4

Dual spaces

Modules over a PID: J20Q4, **J16Q4**

Rational canonical form: J11Q4\*, A14Q4

Jordan canonical form: A20Q4, A19Q4\*, J15Q4, J12Q4, J11Q4\*

**ALL MODULES AND LIN ALG QUESTIONS (UNSORTED):** A20Q4, J20Q4, A19Q4, J19Q4, **A18Q6**, J18Q4 (?), J17Q4, A16Q5, **J16Q4**, J15Q4, A14Q4, J14Q4, A13Q4, A12Q4, J12Q4, A11Q4, J11Q4, A10Q4, A09Q4, J09Q4, **A08Q4**, J08Q4, J07Q4

### Field Theory:

Basic definitions and examples: J20Q5, A19Q5, A17Q5, **J16Q5**, A14Q5, A12Q5, J07Q5

Field extensions: A13Q5, A11Q5, A11Q6

Simple extensions

Algebraic extensions: J17Q5\*

Transcendental extensions

Separable extensions: **J18Q5\***, **J09Q5**

Cyclotomic extensions: **A20Q5**, **J19Q5**, **A17Q6\***, **J13Q6**, **A12Q6**, **J10Q5**

Solution of the Greek construction problems: **A13Q6**

Splitting fields and normality: **J17Q5\***, **J17Q6**, **A16Q6\***, **A10Q6**, **A08Q5**, **A08Q6\***, **J08Q5** (same as **A10Q6** but without hints or guidance), **J07Q6\***

Algebraic closure: **J11Q5**, **J08Q6\***

The Galois correspondence (FTGT): **J15Q6\***, **A10Q5(?)**, **A09Q6**, **J07Q6\***

Galois groups of extensions/polynomials: **A20Q6**, **J20Q6**, **A19Q6**, **J19Q6**, **A18Q3**, **J18Q5\***, **J18Q6**, **A17Q6\***, **A16Q6\***, **J16Q6**, **J15Q6\***, **J15Q5(?)**, **A14Q6**, **J14Q6**, **J12Q6\***, **J11Q6**, **J09Q6**, **A08Q6\***, **J08Q6\***, **J07Q6\***

Solvable and radical extensions: **J12Q6\***, **J10Q6**

The insolvability of the quintic

Fundamental Theorem of Algebra

Finite fields: **J13Q5**, **J12Q5**, **A09Q5**

Frobenius endomorphism: **A16Q5**

**ALL FIELD THEORY QUESTIONS (UNSORTED):** **A20Q5**, **A20Q6**, **J20Q5**, **J20Q6**, **A19Q5**, **A19Q6**, **J19Q5**, **J19Q6**, **A18Q3**, **J18Q5**, **J18Q6**, **A17Q5**, **A17Q6**, **J17Q5**, **J17Q6**, **A16Q5**, **A16Q6**, **J16Q5**, **J16Q6**, **J15Q5**, **J15Q6**, **A14Q5**, **A14Q6**, **J14Q6**, **A13Q5**, **A13Q6**, **J13Q5**, **J13Q6**, **A12Q5**, **A12Q6**, **J12Q5**, **J12Q6**, **A11Q5**, **A11Q6**, **J11Q5**, **J11Q6**, **A10Q5**, **A10Q6**, **J10Q5**, **J10Q6**, **A09Q5**, **A09Q6**, **J09Q5**, **J09Q6**, **A08Q5**, **A08Q6**, **J08Q5**, **J08Q6**, **J07Q5**, **J07Q6**

**Miscellaneous:**

Applications of Zorn's Lemma: **J14Q5**

**Ones I couldn't sort:** **A17Q3**, **A17Q4**