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Geometry/Topology

Ph.D. Preliminary Exam Department of Mathematics University of Colorado Boulder

January, 2017

INSTRUCTIONS:

- 1. Answer each of the six questions on a separate page. Turn in a page for each problem even if you cannot do the problem.
- 2. Label each answer sheet with the problem number.
- 3. Put your number, not your name, in the upper right hand corner of each page. If you have not received a number, please choose one (1234 for instance) and notify the graduate secretary as to which number you have chosen.

Problem 1. Compute the fundamental group of $\mathbb{R}^3 - C$ where

$$C = \{(x, y, z) \mid x = 0, y^2 + z^2 = 2\}.$$

(Hint: Consider a tube around C whose inner hole has been filled by a disk.)

Problem 2. Let $p: X \to Y$ be a continuous closed surjection.

- (a) Let $U \subset X$ be an open set which contains $p^{-1}(\{y\})$. Prove that there is an open neighborhood W_y of y such that $p^{-1}(W_y) \subset U$. (Hint: Consider $X \setminus U$.)
- (b) Recall that X is normal if the one point sets in X are closed and, for every pair disjoint of closed sets A and B, there exists disjoint open sets U and V such that $A \subset U$ and $B \subset V$. Show that if X is normal, then so is Y.

Problem 3. Let $p: \widetilde{X} \to X$ be the universal cover of a connected and locally-path connected space X and let $A \subset X$ be a connected and locally path-connected subspace. Let \widetilde{A} be a path component of $p^{-1}(A)$.

- (a) Show that $\widetilde{A} \to A$ is a covering space.
- (b) Prove that the image of

$$\pi_1(\widetilde{A}, \widetilde{a}_0) \to \pi_1(A, a_0)$$

coincides with the kernel of $\iota_*: \pi_1(A, a_0) \to \pi_1(X, a_0)$, where $\widetilde{a}_0 \in \widetilde{A}$ is any basepoint, $a_0 = p(\widetilde{a}_0)$, and $\iota: A \hookrightarrow X$ is the canonical embedding.

Problem 4. (a) Show that there is no immersion $S^1 \to \mathbb{R}$.

(b) Consider the function $f: \mathbb{R}^3 \to \mathbb{R}^2$, $(x, y, z) \mapsto (x^3 z, xy + z)$. At which point is f a submersion? Determine the regular values of f.

Problem 5. Define $\omega = dx_1 \wedge dx_2 + dx_3 \wedge dx_4 + dx_5 \wedge dx_6$ as a 2-form on \mathbb{R} . Show that no diffeomorphism $\varphi : \mathbb{R}^6 \to \mathbb{R}^6$ satisfying $\varphi^* \omega = \omega$ can map the unit sphere S^5 to a sphere of radius $r \neq 1$.

Hint: consider $\omega \wedge \omega \wedge \omega$.

Problem 6. Recall that an n-dimensional manifold M is called parallelizable if its tangent bundle is trivial. Which of the following manifolds are parallelizable? Provide a short justification of your answer in a sentence.

- (i) The *n*-torus $(S^1)^n = \mathbb{R}^n/\mathbb{Z}^n$ (where \mathbb{Z}^n is the subgroup of the additive group of \mathbb{R}^n consisting of points whose coordinates are all integers);
- (ii) the sphere S^2 ;
- (iii) the real projective plane \mathbb{RP}^2 .

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7)	(a) Since U is open, then X/U is closed, and since p: x = y
	is a surjective, closed map we have that p(X(U) is closed.
	Take Wy = I \p (x(u). We obviously have that yewy
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	p(x) & Y \ p(x \ u), so x & x \ U. Thus x & u and p' (Wy) & U
	(b) Since a is all all is like
	(b) Since p is closed and surjective, we get that the one
	point sets are closed in y for free. Now suppose A, B = y are disjoint closed sets. Since p is confirmous, we then
	have that p'(A) and p'(B) are closed in X. So there
	exist upen sets U, V = x, UNV=d p-(A) & U and p-(B) & V
	Now, for each act, p'({a}) & u so by part (a) there
	is some wa = y such that p'(wa) =U. Define the sets
	WA := U Wa, WB = U Wb. Then we have that A & WA,
	BEWS and that Up and WB are open. with p'(W) EU
	and p'(WB) EV. Well then p'(WANWB) & p'(WB) & UNV20
	So Wy 1 WB = & and we have that y is normal.

Top / Poff Gree Prelim Evan January 2017 3) (a) Let \$\bar{A}\$ be a path component of \$p^*(A)\$ and consider the costainan \$p_{\beta} : \bar{A}\$ and of the contents map \$p\$. The any \$x \times A there is some everally contents out can take some smaller path connected acquisitionhood of \$x\$. Then V is everly conceed and the path components of \$p^*(V)\$ are mapped homeomorphically onto \$V\$. Since \$A\$ is a path component of \$p^*(A)\$, the path components of \$p^*_{\beta}(V)\$ are mapped homeomorphically onto \$V\$. Thus \$p^*_{\beta} : \bar{A}\$ is a covering map and \$A^* \text{A}\$ is a covering map and \$A^* \text{A}\$ is a covering space. (b) Consider the following diagram: 1 \text{A} \$(\bar{A}, \bar{a}_0)\$ then there exists \$p(\beta(\bar{A})\$ such that \$q_{\beta}p(\bar{A})\$! (\bar{A})\$ but then we have theat this is a short exact sequence (inty) : Note, \$\frac{1}{2} = (\beta(\bar{a}_1)^2 = (\beta(\bar{a}_1)^2 + (\beta(\bar{a}_1)^2)^2) = \beta(\beta(\bar{a}_1)^2) = \beta(\bar{a}_1)^2		
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Conversely, let [d] & ker (ι_{x}). Then $\iota_{x} = \lfloor \iota_{od} \rfloor = \lfloor \iota_{a} \rfloor$. But $\lceil \iota_{a} \rceil = \rho_{x} (\gamma_{x} (\tilde{\chi}_{a}))$, so there is a loop $\tilde{\alpha}$ in $\tilde{\chi}$ at \tilde{a}_{o} such that $\rho_{x}[\tilde{\alpha}] = \lceil \iota_{ao} \rceil$. Since \tilde{A} is a path component and $\tilde{a}_{o} \in \tilde{A}$, we must have $\lceil \tilde{\alpha} \rceil \in \gamma_{x} (\tilde{A}_{ao})$. Therefore, $q_{x}[\tilde{\alpha}] = \lceil \tilde{\alpha} \rceil$	Harrier and the state of the st	
such that $p_{*}[\tilde{x}] = [c_{a}]$. Since \tilde{A} is a path component and $\tilde{a} \in \tilde{A}$, we must have $[\tilde{x}] \in \tilde{X}(\tilde{A}, \tilde{a})$. Therefore, $q_{*}[\tilde{x}] = [x]$	So $im(q_*) \in ker(i_*)$.	
such that $p_{*}[\tilde{\alpha}] = [c_{*}]$. Since \tilde{A} is a path component and $\tilde{\alpha}_{*} \in \tilde{A}$, we must have $[\tilde{\alpha}] \in \tilde{A}(\tilde{A}, \tilde{\alpha}_{*})$. Therefore, $q_{*}[\tilde{\alpha}] = [\alpha]$		
a & A, we must have [a] & M (A a). Therefore, que [a] = [a]	Conversely, let [d] 6 ker (Lx). Then Lx = [cod) = [Cas]. But	
such that prod = LCas . Since A is a path component and \[\tilde{a} \in \tilde{A}, \tilde{we must have } \left[\tilde{a} \right] \in \tilde{A}, \tilde{A} \tilde{a} \right]. Therefore, \[\tilde{a} \right] = [\tilde{a} \right] \] and \[\land \left[\tilde{a} \right] \[\tilde{a} \right] \tilde{a} \right] \tilde{a} \right] \[\tilde{a} \right] \tilde{a} \right] \[\tilde{a} \right] \tilde{a} \right] \tilde{a} \right] \tilde{a} \right] \tilde{a} \right] \[\tilde{a} \right] \tilde{a} \right] \tilde{a} \right] \[\tilde{a} \right] \tilde{a} \right] \tilde{a} \right] \[\tilde{a} \right] \tilde{a} \right] \tilde{a} \right] \[\tilde{a} \right] \tilde{a} \right] \[\tilde{a} \right] \tilde{a} \right] \[\tilde{a} \right] \\ \tilde{a} \right] \tilde{a} \right] \[\tilde{a} \right] \tilde{a} \right] \[\tilde{a} \right] \\ \tilde{a} \right] \tilde{a} \right] \[\tilde{a} \right] \\	ICa G (W (x a)), so there is a loop & in X at a	
and [a] 6 in(q _A). and ker(L _X) & in(q _X).	such that pola - LCao . Since A is a path component and	
and $Lal = im(q_x)$. and $Ker(L_x) = im(q_x)$.	a & A, we must have [a & M, (A, a). Therefore, que [a] = [a]	
	and [a] 6 in (qx). and Ker (Lx) & in (qx).	
	II	

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5) Su	ppose 4: R' -> R' is a dillement to be 5
_ h	some other sphere S. Let was dx'sdx' + dx 3 dx 4 dx 5 A
Th	en we have that
	w= dx Ndx Ndx Ndx Ndx Ndx
a	differentian is the standard volume form on go. Since 4 is
	diffeomorphism, we then get that when 4" = w
	Vol(3) = 1 5 w/w/w
	= 1 5 4 (61 (61 (6)
	= = = = = = = = = = = = = = = = = = =
	= 6 Js W1W1W
	2 Je William 2
	= Vo((5 ⁵),
-	~ 1 1 1 1 1 1
00	I must be a sphere with the same radius as Hence, the radius of 3 is 1.
	Prence, Ane radius of J. 1.
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(a) (a) Se	We know that of smooth r T(Î) m;)	for the produce manifolds	ut space of a	ny finite
	is a paralleliza we have $T(s')$ $T((s')^n)$			
	T((s')")	$= \underbrace{\mathcal{T}(\hat{\mathfrak{G}} S')}_{= \hat{\mathfrak{G}} \mathcal{T}(S')}$ $= \hat{\mathfrak{G}}(S' \hat{\mathfrak{G}} \mathcal{R})$	J	
36	(S')" is parallel	= (S')" DR"		
(b) How	Every parallelize sever, by the Home can exist on	sable manifold lain Bull Theor	admits a smu em, we know eme, 5° is not	that no such parallelizable
	Every parallelizable ntable, so if a			
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