# Definitions

## Syntax of Terms

### Expressions

expression $e$ ::= $x$
| $C$
| $(e_i \; e_j)$
| $(\lambda x.e)$
| let $x = e_i$ in $e_j$

### Patterns[1]

pattern $p$ ::= $x$
| $C$
| $(p_i \; p_j)$

### Monotypes[2]

monotype $t$ ::= $v$
| $C$
| $(t_i \; t_j)$
| $(t; \; p)$

### Dependent Types

dependent-type $\delta$ ::= $t$
| mapval $(x_0 : t_0)$ $(x_1 : t_1)$ .. $(x_i : t_i)$ . $t_j$

### Polytypes

polytype $\sigma$ ::= $\delta$
| forall $v_0$ $v_1$ .. $v_i$ . $\delta$

[1] Some special cases exist, but these are just sugar; e.g., $x + 2$ may be used instead of $(Succ\ (Succ\ x))$

[2] Again, some special cases exist, but these are just sugar; e.g., $t_0 \rightarrow t_1$ may be used instead of $((\rightarrow\ t_0)\ t_1)$

## Syntax of Rules

predicate ::= $x : t \in \Gamma$
  $|$  $Inst_d(\delta) = t$
  $|$  $Inst_p(\sigma) = t$
  $|$  $v = newvar$
  $|$  $t_0 = t_1$

judgement ::= $e : \sigma$
  $|$  $x : Gen(t)$

premise ::= predicate
  $|$  judgement

conclusion ::= judgement

rule ::= $\dfrac{\text{premise}}{\text{conclusion}}$ [Rule]

## Rules

$$\frac{x : \sigma \in \Gamma \qquad t = \mathrm{Inst}(\sigma)}{\Gamma \;\vdash\; x : t}\ [\mathrm{Var}]$$

$$\frac{\Gamma \;\vdash\; e_0 : t_0 \qquad \Gamma \;\vdash\; e_1 : t_1 \qquad t_2 = \mathit{newvar} \qquad \mathrm{Unify}(t_0,\ t_1 \to t_2)}{\Gamma \;\vdash\; (e_0\ e_1) : t_2}\ [\mathrm{App}]$$

$$\frac{t_0 = \mathit{newvar} \qquad \Gamma, x : t_0 \;\vdash\; e_0 : t_1}{\Gamma \;\vdash\; (\lambda x.e_0) : t_0 \to t_1}\ [\mathrm{Abs}]$$

$$\frac{\Gamma \;\vdash\; e_0 : t_0 \qquad \Gamma, x : \mathit{Gen}(t_0) \;\vdash\; e_1 : t_1}{\Gamma \;\vdash\; \text{let } x = e_0 \text{ in } e_1 : t_1}\ [\mathrm{Let}]$$

$$\frac{\Gamma, \Gamma' \;\vdash\; e_1 : t_1 \ ...\ \Gamma, \Gamma' \;\vdash\; e_n : t_n \qquad \Gamma, \Gamma'' \;\vdash\; e_0 : t_0}{\Gamma \;\vdash\; \text{rec } v_1 = e_1 \text{ and ... and } v_n = e_n \text{ in } e_0 : t_0}\ [\mathrm{Rec}]$$

## Example

Let $\text{Uint} = \{0 : \text{Uint}, (\text{Succ } n) : \text{Uint}\}$
Let $\text{Array} = \{[] : [a; 0], (::) : a \to [a; n] \to [a; (\text{Succ } n)]\}$
Let $\Gamma = \{\mathit{tail} : [a; (\text{Succ } n)] \to [a; n]\} \cup \text{Uint} \cup \text{Array}$
Show $\Gamma \ \vdash \ \mathit{tail}\ (0 :: (0 :: [])) : [\text{Uint}; (\text{Succ } 0)]$

To do this, we will first prove the following lemma:

$$\Gamma \ \vdash \ 0 :: (0 :: []) : [\text{Uint}; (\text{Succ } (\text{Succ } 0))]$$

$$\cfrac{\cfrac{(::) : a \to [a; n] \to [a; (\text{Succ } n)] \qquad 0 : \text{Uint} \qquad v_0 = \mathit{newvar} \qquad a \to [a; n] \to [a; (\text{Succ } n)] = \text{Uint} \to v_0}{((::)\ 0) : v_0}\ [\text{Ap}}{v_0 = [a; n] \to [a; (\text{Succ } n)]}\ [\text{Id}]$$

$$\cfrac{[\mathit{tail} : \forall a. \prod_{n:\text{Uint}} .[a; (\text{Succ } n)] \to [a; n]]^1 \qquad [v; (\text{Succ } x)] \to [v; x] = \text{Inst}(\forall a. \prod_{n:\text{Uint}} .[a; (\text{Succ } n)] \to [a; n])}{\mathit{tail} : [v; (\text{Succ } x)] \to [v; x]}$$